Chapter One:

Basic Concepts

1.1 If 60 C of charge pass through an electric conductor in 30 seconds, determine the current in the conductor. CS

SOLUTION:

1.1 $Q = 60C$ $\Delta t = 30\,s$ $I = \frac{Q}{\Delta t} = \frac{60}{30}$ $\boxed{I = 2A}$

1.2 In an electric conductor, a charge of 300 C passes any point in a 5-s interval. Determine the current in the conductor.

SOLUTION:

1.2 $Q = 300C$ $\Delta t = 5s$ $I = \dfrac{Q}{\Delta t}$ $\boxed{I = 60A}$

1.3 The current in a conductor is 1.5 A. How many coulombs of charge pass any point in a time interval of 1.5 min?

SOLUTION:

1.3 $I = 1.5\,A$ $\Delta t = 1.5\,min = 90\,s$ $Q = I(\Delta t)$ $\boxed{Q = 135\,C}$

1.4 Determine the number of coulombs of charge produced by a 12-A battery charger in an hour.

SOLUTION:

1.4 $I = 12\,A \quad \Delta t = 1\text{ hour} = 60\text{ min} = 3600\,s$

$$Q = I(\Delta t) = 12(3600) \qquad \boxed{Q = 43.2\,kC}$$

1.5 A lightning bolt carrying 20,000 A lasts for 70 μs. If the lightning strikes a tractor, determine the charge deposited on the tractor if the tires are assumed to be perfect insulators.

SOLUTION:

1.5 $I = 20{,}000\ A$ $\Delta t = 70\ \mu s$ $Q = I(\Delta t) = (20k)(70\mu)$

$\boxed{Q = 1.4\ C}$

1.6 If a 12-V battery supplies 10 A, find the amount of energy delivered in 1 hour.

SOLUTION:

1.6 $V = 12V$ $I = 10A$ $\Delta t = 1$ hour $= 3600s$

$P = VI = 12(10) = 120W$ $W = P(\Delta t) = 120(3600)$

$\boxed{W = 432 \text{ kJ}}$

1.7 Determine the energy required to move 240 C through 6 V. **CS**

SOLUTION:

1.7 $Q = 240\ C$ $V = 6V$ $W = QV = 240(6)$ $\boxed{W = 1440J}$

1.8 Five coulombs of charge pass through the element in Fig. P1.8 from point A to point B. If the energy absorbed by the element is 120 J, determine the voltage across the element.

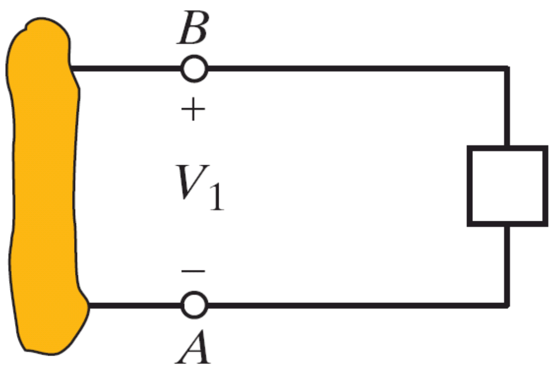
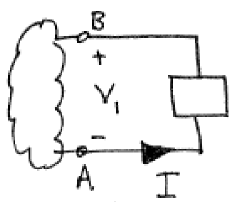


Figure P1.8

SOLUTION:

1.8



$Q = 5C$

$W = 120J$

For passive sign convention:

$W = -V_1 Q$ $\quad V_1 = -W/Q$

$\boxed{V_1 = -24V}$

1.9 The charge entering an element is shown in Fig. P1.9. Find the current in the element in the time interval $0 \leqq t \leqq 0.5$ s. [*Hint:* The equation for $q(t)$ is $q(t) = 1 + (1/0.5)t, t \geqq 0$.]

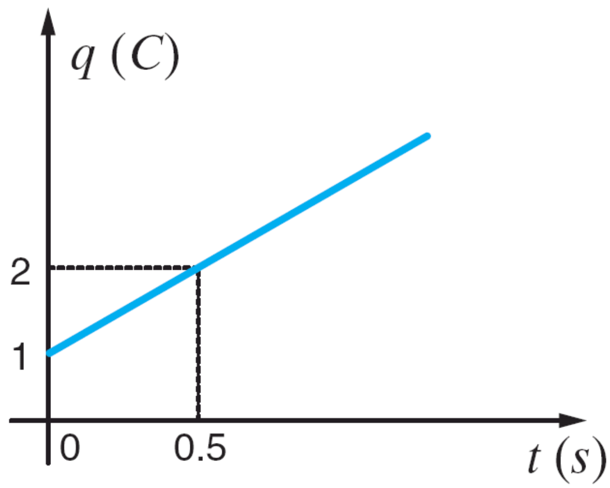


Figure P1.9

SOLUTION:

1.9 $0s \leq t \leq 0.5s$ $\quad q(t) = 1 + 2t \quad i(t) = \frac{dq}{dt} = 2\ A$

$$\boxed{i(t) = 2\ A}$$

1.10 Determine the amount of power absorbed or supplied by the element in Fig. P1.10 if

(a) $V_1 = 9$ V and $I = 2$ A.

(b) $V_1 = 9$ V and $I = -3$ A.

(c) $V_1 = -12$ V and $I = 2$ A.

(d) $V_1 = -12$ V and $I = -3$ A.

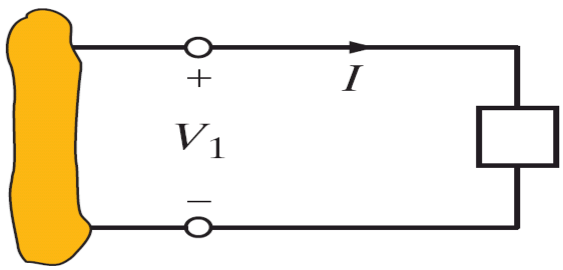
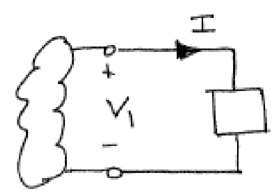


Figure P1.10

SOLUTION:

1.10 a) $V_1 = 9V$, $I = 2A$



For passive sign convention, $P = V_1 I$ is power absorbed.

$P = V_1 I = 9(2) = 18W$ absorbed $\boxed{P = 18W \text{ absorbed}}$

b) $V_1 = 9V$, $I = -3A$

$P = 9(-3) = -27W$ $\boxed{P = 27W \text{ supplied}}$

c) $V_1 = -12V$, $I = 2A$

$P = -24W$ $\boxed{P = 24W \text{ supplied}}$

d) $V_1 = -12V$, $I = -3A$

$P = +36W$ $\boxed{P = 36W \text{ absorbed}}$

1.11 Determine the magnitude and direction of the voltage across the elements in Fig. P1.11.

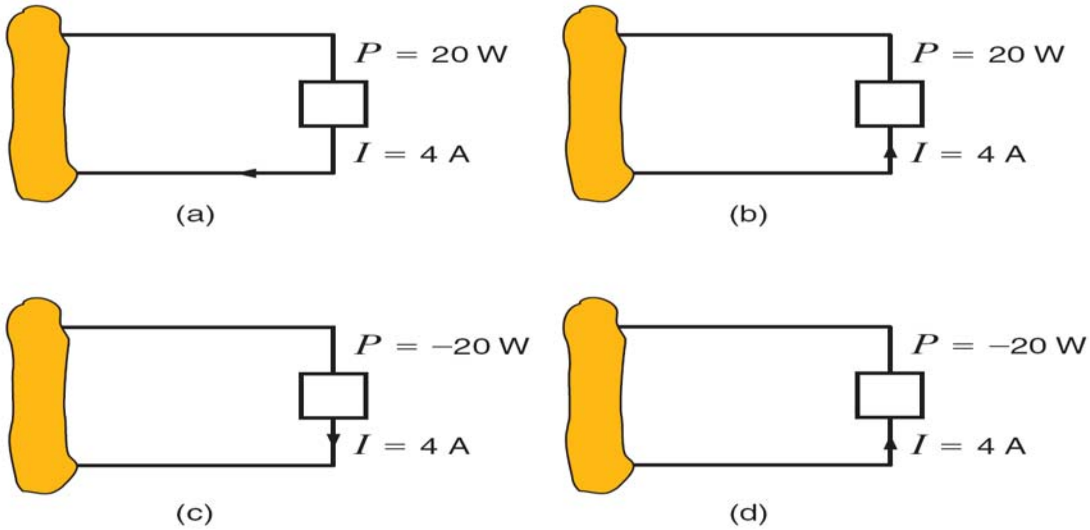
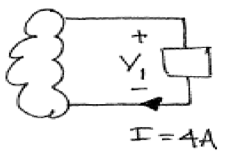


Figure P1.11

SOLUTION:

1.11 a)

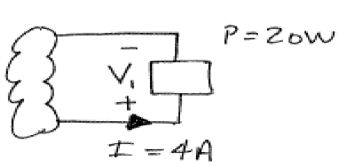


Passive sign convention

$P = VI \qquad V = P/I = \frac{20}{4} = 5V$

$\boxed{V_1 = 5V}$

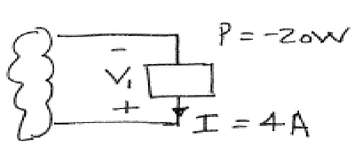
b)



$P = VI \qquad V = P/I = \frac{20}{4} = 5V$

$\boxed{V_1 = 5V}$

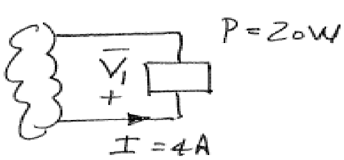
c)



$V = P/I = -5V$ {Element supplies power!!

$\boxed{V_1 = 5V}$

d)



same as part b).

$\boxed{V_1 = 5V}$

1.12 Determine the missing quantity in the circuits in Fig. P1.12.

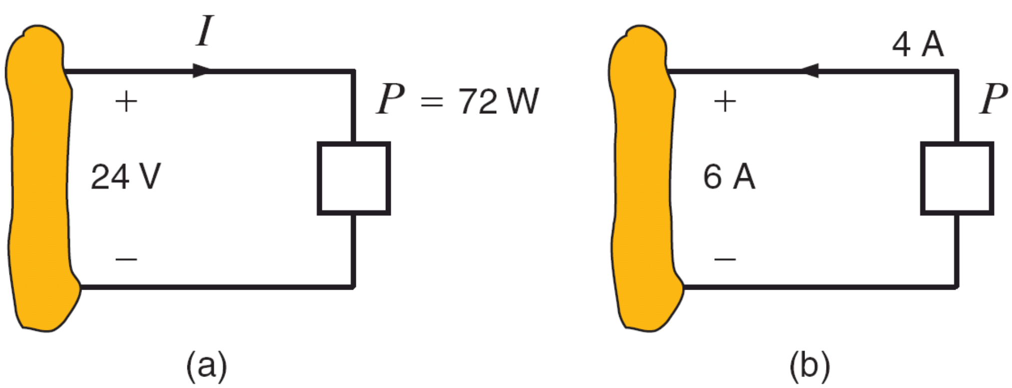
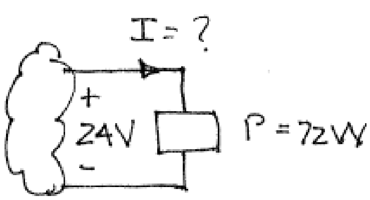


Figure P1.12

SOLUTION:

1.12 a)

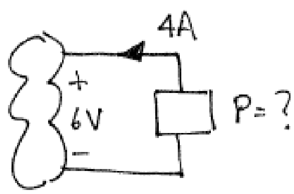


Passive sign convention

$P = VI \qquad I = P/V = 72/24 = 3A$

$\boxed{I = 3A}$

b)



V & I are in the active sign convention

$P = -VI = -(6)(4) = -24W$

$\boxed{P = -24W \text{ or } 24W \text{ supplied}}$

1.13 Determine the missing quantity in the circuits in Fig. P1.13.

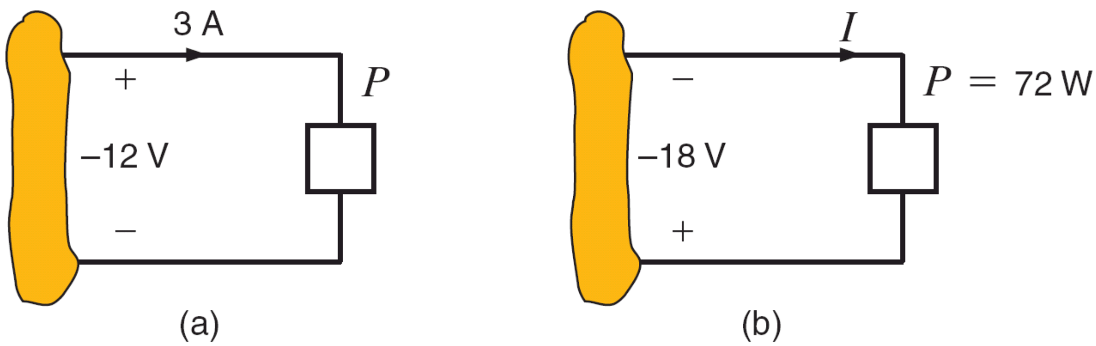
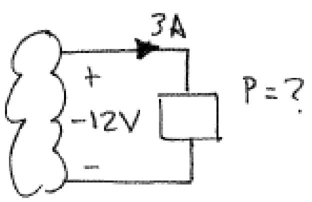


Figure P1.13

SOLUTION:

1.13 a)



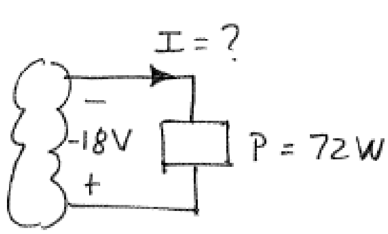
V & I are defined in passive sign convention

$P = VI = -36W$

Since P is negative, power is actually supplied

$\boxed{P = -36W \text{ or } 36W \text{ Supplied}}$

b)



V & I defined in active sign convention

$P = -VI \qquad I = \frac{-P}{V} = \frac{-72}{-18}$

$\boxed{I = 4A}$

1.14 Determine the missing quantity in the circuits in Fig. P1.14. **CS**

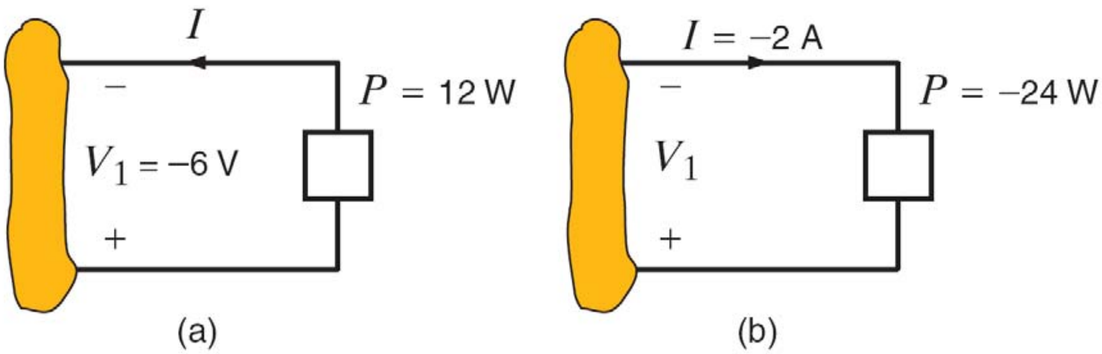
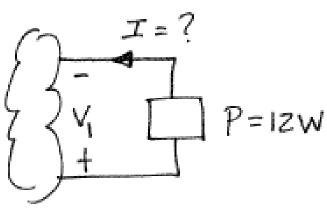


Figure P1.14

SOLUTION:

1.14 a)



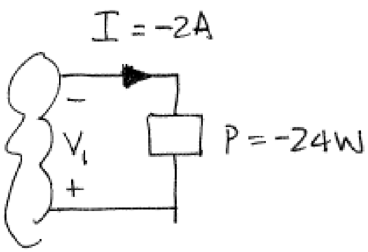
$V_1 = -6V$

V_1 & I defined as passive sign convention.

$P = V_1 I \qquad I = P/V_1 = \frac{12}{-6} = -2A$

$\boxed{I = -2A}$

b)



V_1 & I defined as active sign convention

$P = -V_1 I \qquad V_1 = -P/I = -\left[\frac{-24}{-2}\right] = -12V$

$\boxed{V_1 = -12V}$

1.15 Two elements are connected in series, as shown in Fig. P1.15. Element 1 supplies 24 W of power. Is element 2 absorbing or supplying power, and how much? **CS**

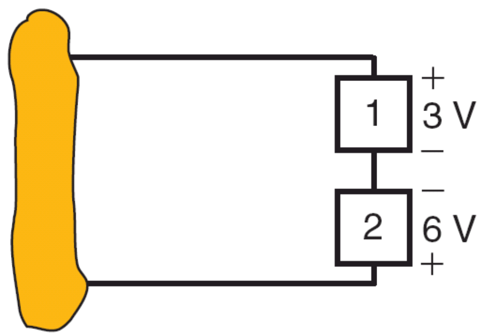
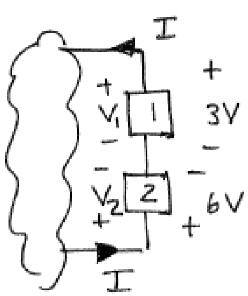


Figure P1.15

SOLUTION:

1.15



$P_1 = 24\,W$ supplied.

Using active sign convention for element 1

$P = V_1 I \Rightarrow I = P/V_1 = 8\,A$

(Note I is defined for active sign convention for element 1!)

In element 2, V & I are defined as passive sign convention.

$P_2 = V_2 I = (6)(8) = 48\,W$

$\boxed{P_2 = 48\,W \text{ absorbed}}$

1.16 Determine the power supplied to the elements in Fig. P1.16.

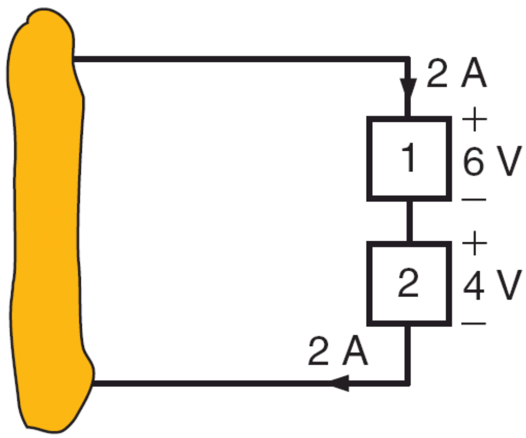
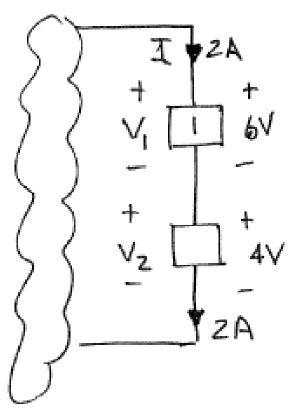


Figure P1.16

SOLUTION:

1.16



For element 1, V_1 and I are defined in passive sign convention. So, power supplied to the element is,

$$P_1 = +V_1 I = (2)(6) = 12W$$

$$\boxed{P_1 = 12W}$$

Element2 has V_2 & I defined in the passive sign convention also.

$$P_2 = V_2 I = (2)(4) = 8W$$

$$\boxed{P_2 = 8W}$$

1.17 Determine the power supplied to the elements in Fig. P1.17.

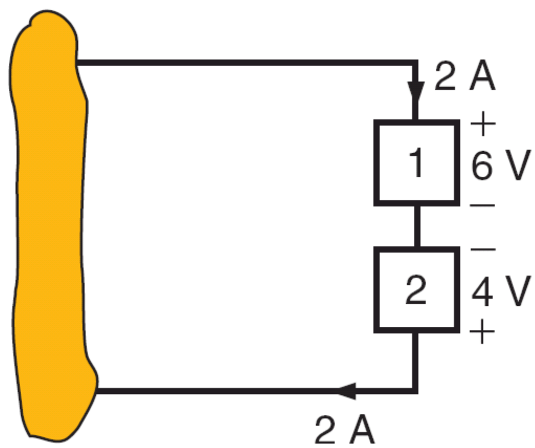
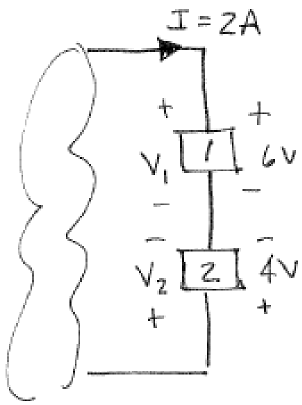


Figure P1.17

SOLUTION:

1.17



For element 1: V_1 & I are defined in the passive sign convention. So, power supplied to element 1 is,

$$P_1 = V_1 I = 6(2) = 12W$$

$$\boxed{P_1 = 12W}$$

For element 2: V_2 & I are defined in the active sign convention Power supplied to element 2 is

$$P_2 = -V_2 I = -4(2) = -8W$$

$$\boxed{P_2 = -8W}$$

1.18 Determine the power supplied to the elements in Fig. P1.18.

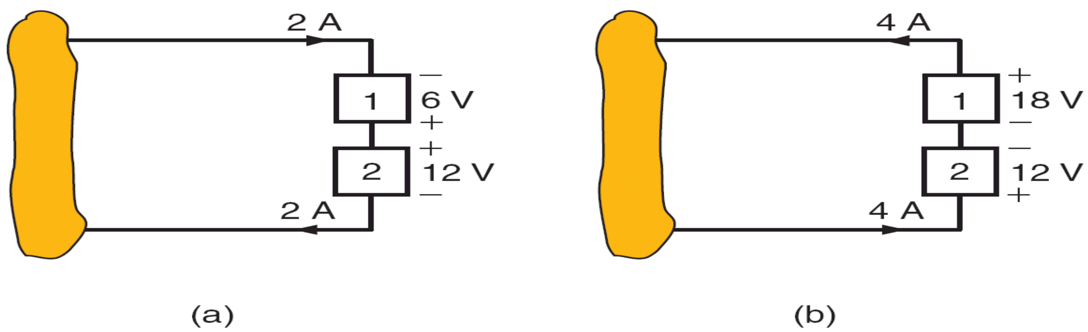
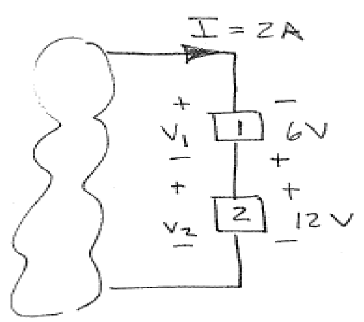


Figure P1.18

SOLUTION:

1.18 a)



In both elements, voltages and currents are defined in passive sign convention.

For element 1, power supplied is

$P_1 = V_1 I = (-6)(2) = 12W$

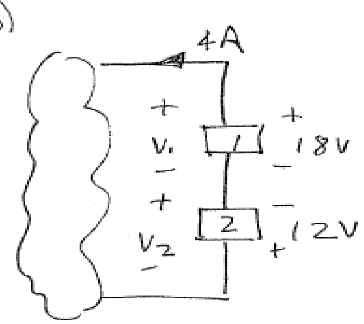
$\boxed{P_1 = 12W}$

For element 2,

$P_2 = V_2 I = 12(2) = 24W$

$\boxed{P_2 = 24W}$ absorbed

b)



In both elements, voltages and currents are defined in active sign convention

For element 1: $V_1 = +18V$

$P_1 = -V_1 I = -(18)(4) = -72W$

$\boxed{P_1 = -72W}$ absorbed

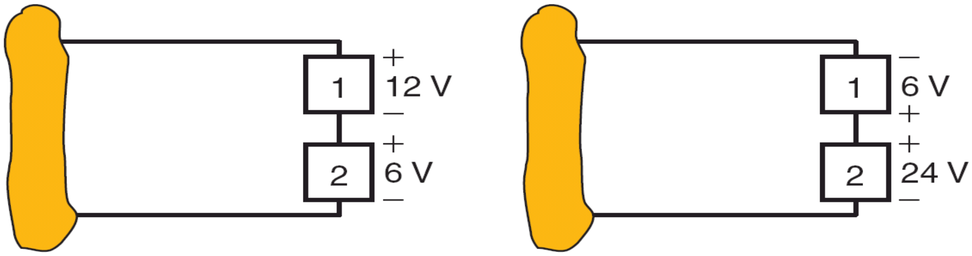
For element 2: $V_2 = -12V$

$P_2 = -V_2 I = -(-12)(4) = +48W$

$\boxed{P_2 = 48W}$ absorbed

1.19 (a) In Fig. P1.19(a), $P_1 = 36$ W. Is element 2 absorbing or supplying power, and how much?

(b) In Fig. P1.19(b), $P_2 = -48$ W. Is element 1 absorbing or supplying power, and how much?

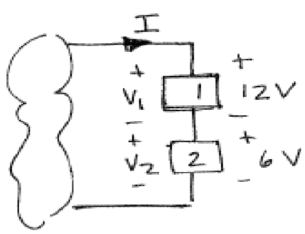


(a) (b)

Figure P1.19

SOLUTION:

1.19a) $P_1 = 36W$



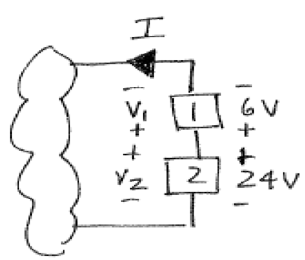
By default, using passive sign convention. Since P_1 is positive, I flows as shown on circuit diagram.

$P_1 = V_1 I \quad I = P_1/V_1 = 36/12 = 3\text{ A} \quad I = 3\text{A}$

For element 2, V_2 & I are defined in passive sign convention,

$P_2 = V_2 I = 6(3) = 18W$ $\boxed{P_2 = 18W \text{ absorbed}}$

b) $P_2 = -48W$



Again, passive sign convention is the default. Since $P_2 < 0$, element 2 supplies power and I flows as shown.

$P_2 = -V_2 I = -24 I \quad I = -48/-24 = 2A$

For element 1, V_1 & I are defined in passive sign convention. Power absorbed is

$P_1 = V_1 I = 6(2) = 12W$ $\boxed{P_2 = 12W \text{ absorbed}}$

1.20 Two elements are connected in series, as shown in Fig. P1.20. Element 1 supplies 24 W of power. Is element 2 absorbing or supplying power, and how much?

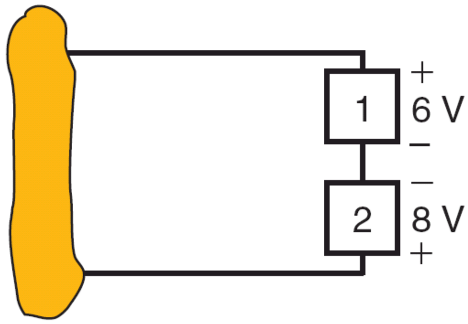
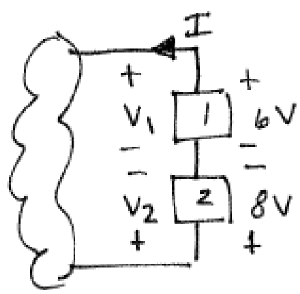


Figure P1.20

SOLUTION:

1.20 Element 1 supplies 24W.



For element 1 supplying power, I must flow as shown.

$$I = P_1 / V_1 = \frac{24}{6} = 4A$$

In Element 2, V_2 & I are defined as the passive sign convention power absorbed is

$$P_2 = V_2 I = 8(4) = 32W$$

$\boxed{P_2 = 32W \text{ absorbed}}$

1.21 Two elements are connected in series, as shown in Fig. P1.21. Element 1 supplies 24 W of power. Is element 2 absorbing or supplying power, and how much? **CS**

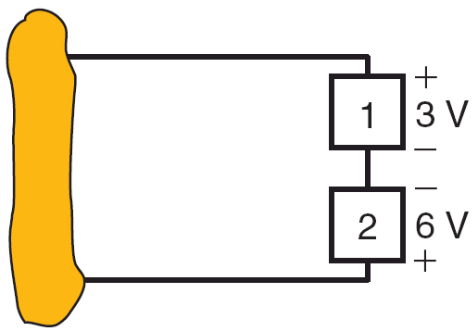
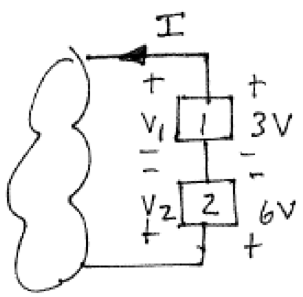


Figure P1.21

SOLUTION:

1.21 Element 1 supplies 24W. Since supplying power, I must flow as shown.



$I = P_1/V_1 = 24/3 = 8\text{ A}$ $\qquad I = 8\text{A}$

For element 2, V_2 & I obey passive sign convention. Power absorbed is

$P_2 = V_2 I = 6(8) = 48\text{W}$ $\qquad \boxed{P_2 = 48\text{W absorbed}}$

1.22 Two elements are connected in series, as shown in Fig. P1.22. Element 1 absorbs 36 W of power. Is element 2 absorbing or supplying power, and how much?

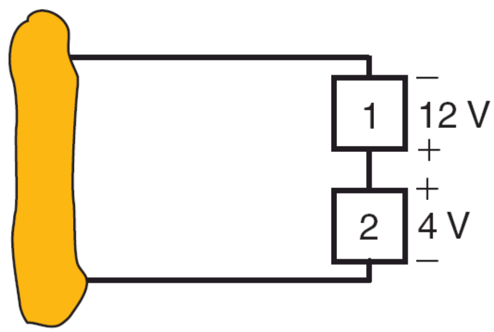
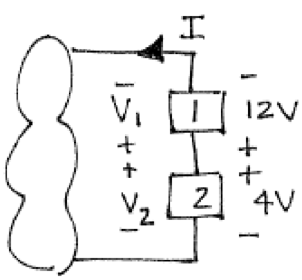


Figure P1.22

SOLUTION:

1.22 Element 1 absorbs 36 W. For absorbing power, I must flow as shown in the diagram.



$P_1 = V_1 I \qquad I = P_1/V_1 = 36/12 \qquad I = 3\text{ A}$

For element 2, V_2 & I are defined in active sign convention, Power supplied is

$P_2 = V_2 I = 4(3) = 12\text{ W}$

$\boxed{P_2 = 12\text{ W supplied}}$

1.23 Determine the power that is absorbed or supplied by the circuit elements in Fig. P1.23.

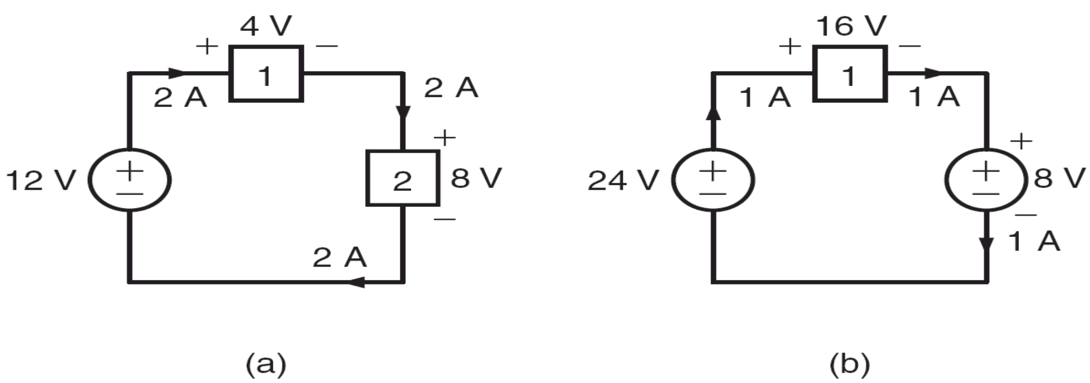
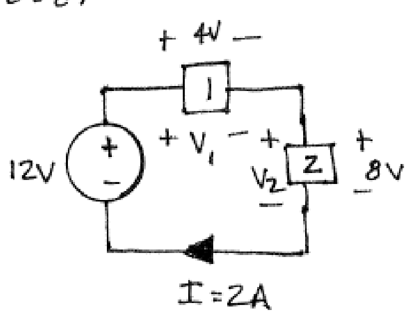


Figure P1.23

SOLUTION:

1.23 a)



Both element 1 and 2 voltages and currents are defined in passive sign convention.

$P_1 = V_1 I = 4(2) = 8W$ $\boxed{P_1 = 8W \text{ absorbed}}$

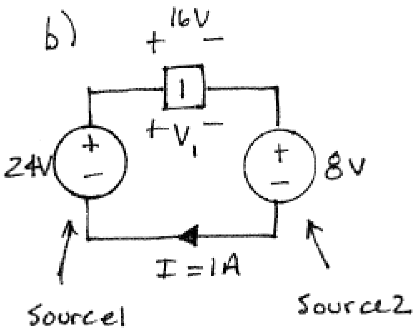
$P_2 = V_2 I = 8(2) = 16W$ $\boxed{P_2 = 16W \text{ absorbed}}$

Voltage source must supply sum $P_1 + P_2$ for power balance

$P_{12V} = P_1 + P_2 = 24W$ $\boxed{P_{12V} = 24W \text{ supplied}}$

Continued on next page.

b)



V_1 & I in passive sign convention

$P_1 = V_1 I = 16(1) = 16W$ $\boxed{P_1 = 16W \text{ absorbed}}$

For source 1, V & I in active sign convention.

$P_{24V} = 24(1) = 24W$ $\boxed{P_{24V} = 24W \text{ supplied}}$

For source 1, V & I are defined in passive sign convention.

$P_{8V} = VI = 8(1) = 8W$ $\boxed{P_{8V} = 8W \text{ absorbed}}$

CHECK: for power balance, $P_{supplied} = P_{absorbed}$

$24 = 8 + 16 = 24$ ✓

$P_{24V} = P_{8V} + P_1$

1.24 Find the power that is absorbed or supplied by the network elements in Fig. P1.24. **PSV**

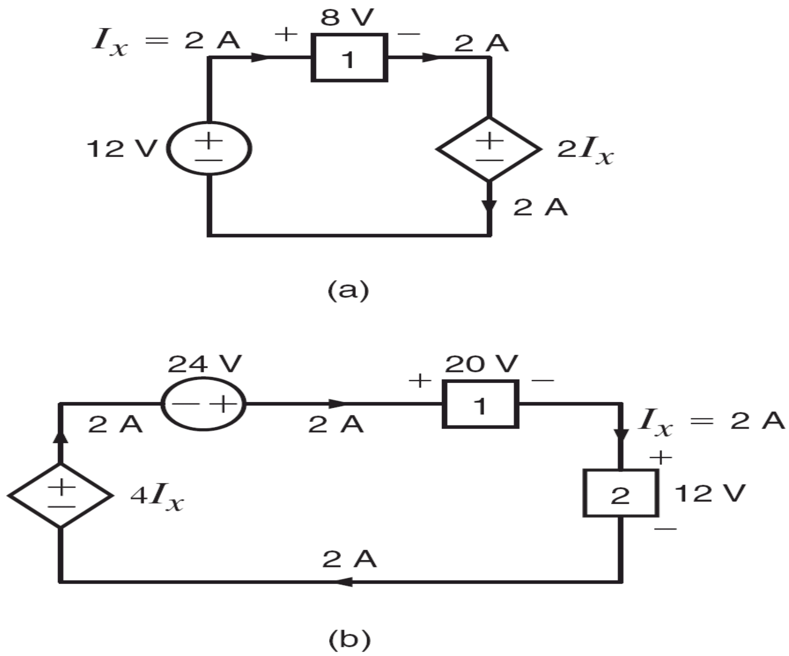
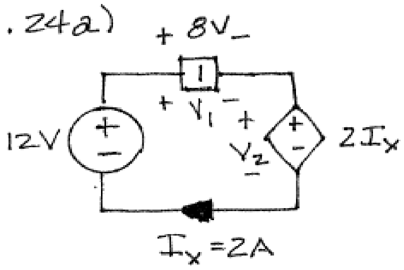


Figure P1.24

SOLUTION:

1.24a)



Voltages and currents for element 1 and for the dependent source are defined in the passive sign convention.

$P_1 = V_1 I_x = 8(2) = 16\text{W}$ $\boxed{P_1 = 16\text{W absorbed}}$

$P_2 = V_2 I_x = (2I_x) I_x = 4(2) = 8\text{W}$

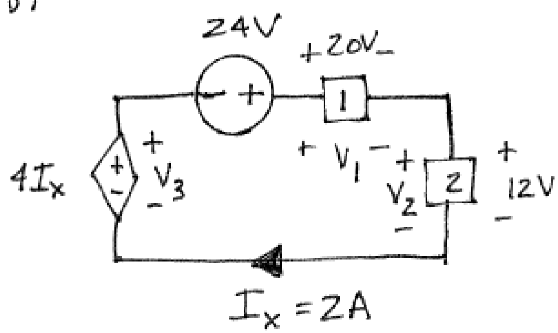
$\boxed{P_2 = 8\text{W absorbed}}$

For the dependent source, V & I are defined in the active sign convention.

$P_{12V} = 12 I_x = 12(2) = 24\text{W}$ $\boxed{P_{12V} = 24\text{W supplied}}$

Continued on next page.

b)



V & I are defined in the passive sign convention for elements 1 and 2; and in the active sign convention in both the dependent and independent source.

$P_1 = V_1 I_x = 20(2) = 40W$

$P_2 = V_2 I_x = 12(2) = 24W$

$P_3 = V_3 I_x = 4I_x^2 = 4(2)^2 = 16W$

$P_{24V} = 24(I_x) = 24(2) = 48W$

$$\boxed{\begin{array}{l} P_1 = 40W \text{ absorbed} \\ P_2 = 24W \text{ absorbed} \\ P_3 = 16W \text{ supplied} \\ P_{24V} = 48W \text{ supplied} \end{array}}$$

1.25 Is the source V_S in the network in Fig. P1.25 absorbing or supplying power, and how much?

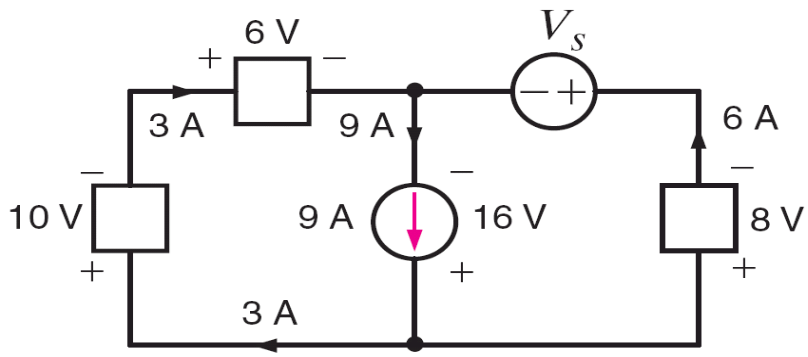
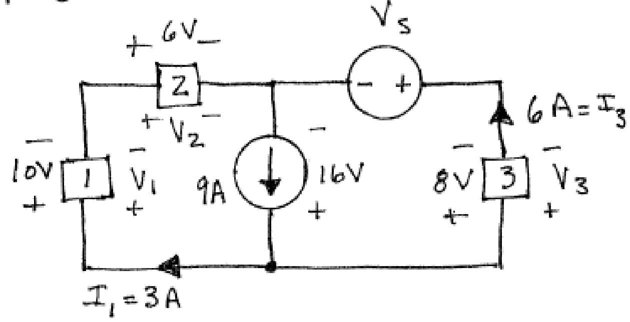


Figure P1.25

SOLUTION:

1.25



V & I for elements 1, 2, 3 defined in passive sign convention.

$P_1 = V_1 I_1 = 10(3) = 30W$ absorbed

$P_2 = V_2 I_1 = 6(3) = 18W$ absorbed

$P_3 = V_3 I_3 = 8(6) = 48W$ absorbed

Current source V & I defined in active sign convention

$P_{9A} = 9(16) = 144W$ supplied.

Power balance requires power supplied = power absorbed.
Assume V_s supplies power.

$$P_{9A} + P_{V_S} = P_1 + P_2 + P_3 \Rightarrow 144 + P_{V_S} = 30 + 18 + 48 = 96W$$

$$P_{V_S} = -48W$$

Since $P_{V_S} < 0$, V_s absorbs power $\boxed{P_{V_S} = 48W \text{ absorbed}}$

1.26 Find V_x in the network in Fig. P1.26.

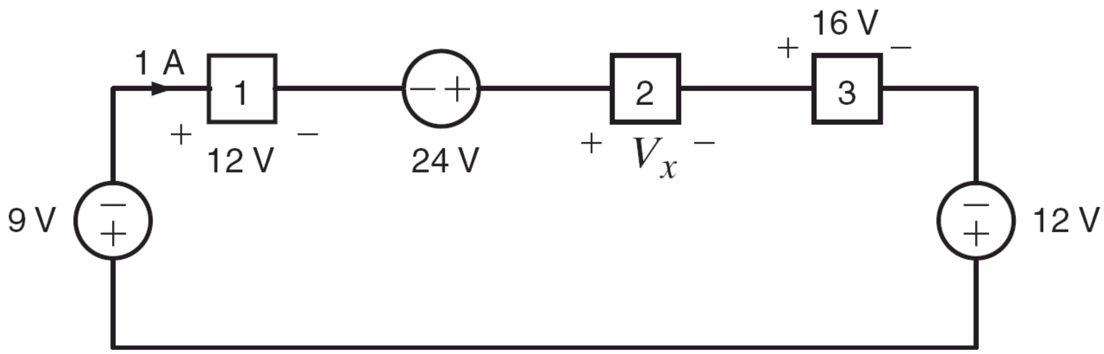
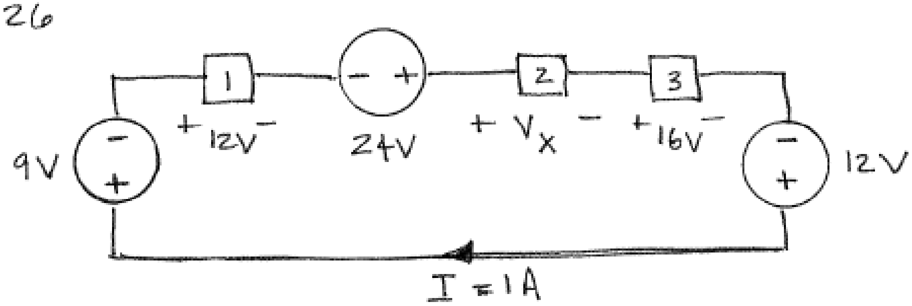


Figure P1.26

SOLUTION:

1.26



Passive sign convention: Elements 1, 2, 3 and 9-V source

Active sign convention: 24-V and 12-V source.

Power balance: $P_{24V} + P_{12V} = P_{9V} + P_1 + P_2 + P_3$

$$24I + 12I = 9I + 12I + V_x I + 16I$$

$$36 = 37 + V_x$$

$$\boxed{V_x = -1V}$$

1.27 Find V_x in the network in Fig. P1.27. PSV

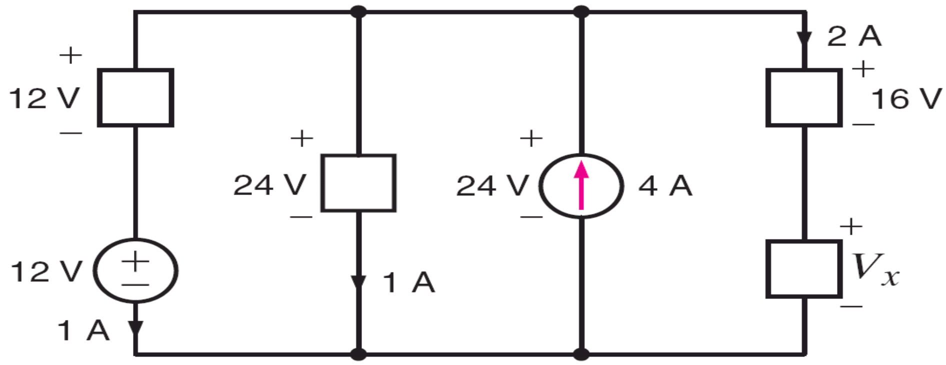
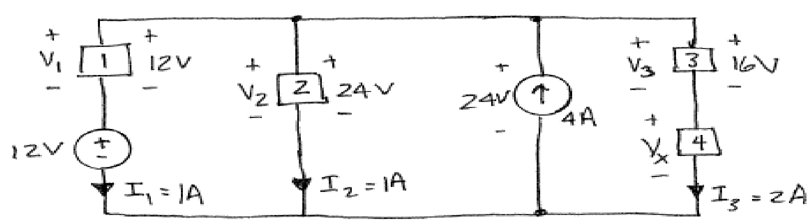


Figure P1.27

SOLUTION:

1.27



Passive sign convention: Elements 1, 2, 3, 4 and 12-V source.

$P_1 = V_1 I_1 = 12(1) = 12W$ $\qquad$ $P_1 = 12W$ absorbed

$P_2 = V_2 I_2 = 24(1) = 24W$ $\qquad$ $P_2 = 24W$ absorbed

$P_3 = V_3 I_3 = 16(2) = 32W$ $\qquad$ $P_3 = 32W$ absorbed

$P_4 = V_4 I_3 = V_x I_3 = 2V_x$ $\qquad$ $P_4 = 2V_x$ absorbed

$P_{12V} = 12(I_1) = 12(1) = 12W$ $\qquad$ $P_{12V} = 12W$ absorbed

$P_{4A} = 24(4) = 96W$ $\qquad$ $P_{4A} = 96W$ supplied

Power balance requires $P_{supplied} = P_{absorbed}$.

$$P_{4A} = P_{12V} + P_1 + P_2 + P_3 + P_4$$

$$96 = 12 + 12 + 24 + 32 + 2V_x$$

$$\boxed{V_x = 8V}$$

1.28 Compute the power that is absorbed or supplied by the elements in the network in Fig. P1.28. CS

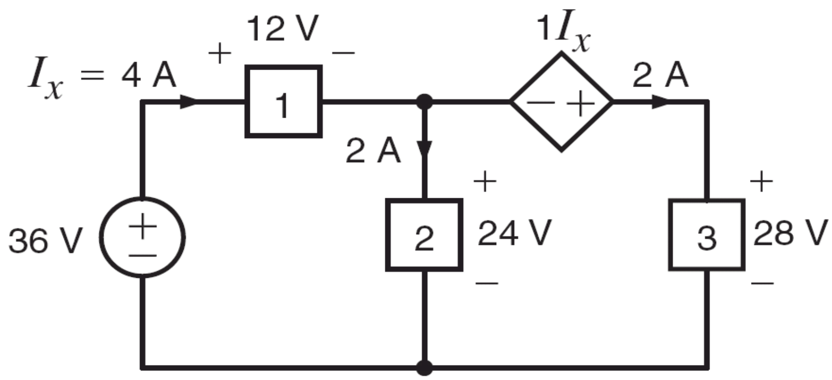
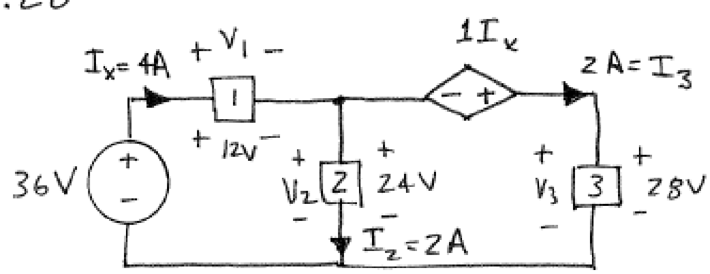


Figure P1.28

SOLUTION:

1.28



Passive sign convention?

Elements 1, 2, 3.

$P_1 = V_1 I_x = 12(4) = 48W$

$\boxed{P_1 = 48W \text{ absorbed}}$

$P_2 = V_2 I_2 = 24(2) = 48W$ $\boxed{P_2 = 48 W \text{ absorbed}}$

$P_3 = V_3 I_3 = 28(2) = 56W$ $\boxed{P_3 = 56W \text{ absorbed}}$

$P_{36V} = 36 I_x = 36(4) = 144W$ $\boxed{P_{36V} = 144 W \text{ supplied}}$

$P_{D.S.} = (1I_x) I_3 = 4(2) = 8W$ $\boxed{P_{D.S.} = 8W}$

1.29 Find I_o in the network in Fig. P1.29. CS

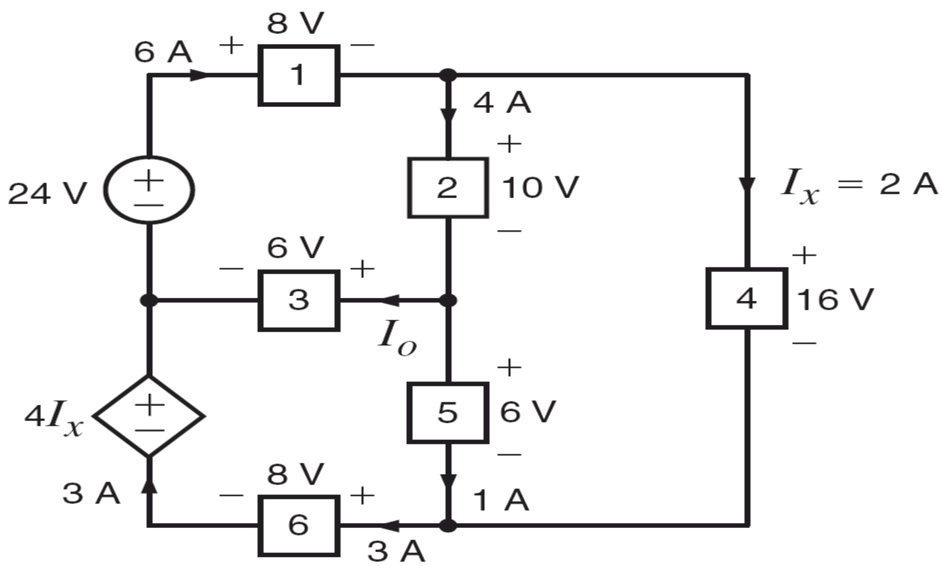
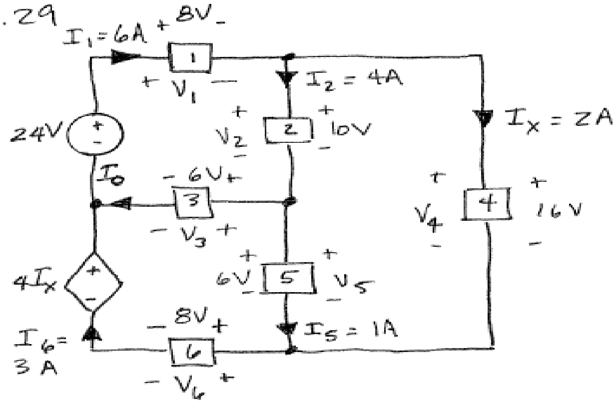


Figure P1.29

SOLUTION:

1.29



Passive sign convention:

Elements 1, 2, 3, 4, 5.

$P_1 = V_1 I_1 = 48\text{ W}$ absorbed

$P_2 = V_2 I_2 = 40\text{ W}$ absorbed

$P_3 = V_3 I_o = 6 I_o$ absorbed

$P_4 = V_4 I_x = 32\text{ W}$ absorbed

$P_5 = V_5 I_5 = 6\text{ W}$ absorbed

$P_6 = V_6 I_6 = 24\text{ W}$ absorbed

$P_{24V} = 24 I_1 = 144\text{ W}$ supplied $\quad P_{4I_x} = 4I_x I_6 = 24\text{ W}$ supplied

$$P_{supplied} = P_{absorbed}$$

$$P_{24V} + P_{4I_x} = P_1 + P_2 + P_3 + P_4 + P_5 + P_6$$

$$168 = 48 + 40 + 6I_o + 32 + 6 + 24$$

$$\boxed{I_o = 3\text{ A}}$$

1.30 Find I_x in the circuit in Fig. P1.30.

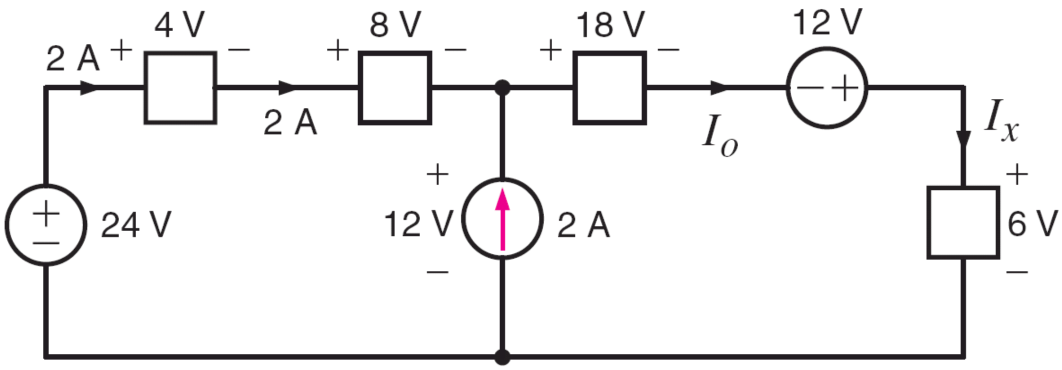
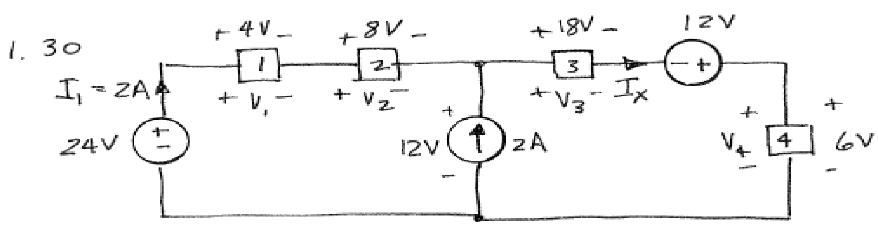


Figure P1.30

SOLUTION:



Passive Sign Convention: Elements 1, 2, 3, 4

$P_1 = V_1 I_1 = 8W$ absorbed $\quad P_2 = V_2 I_1 = 16W$ absorbed

$P_3 = V_3 I_x = 18 I_x$ absorbed $\quad P_4 = V_4 I_x = 6 I_x$ absorbed

$P_{24V} = 24 I_1 = 48W$ supplied $\quad P_{2A} = 12(2) = 24W$ supplied

$P_{12V} = 12 I_x$ supplied

$$P_{supplied} = P_{absorbed}$$

$$P_{12V} + P_{24V} + P_{2A} = P_1 + P_2 + P_3 + P_4$$

$$12 I_x + 48 + 24 = 8 + 16 + 18 I_x + 6 I_x$$

$$48 = 12 I_x$$

$$\boxed{I_x = 4A}$$

1.31 Find V_x in the network in Fig. P1.31.

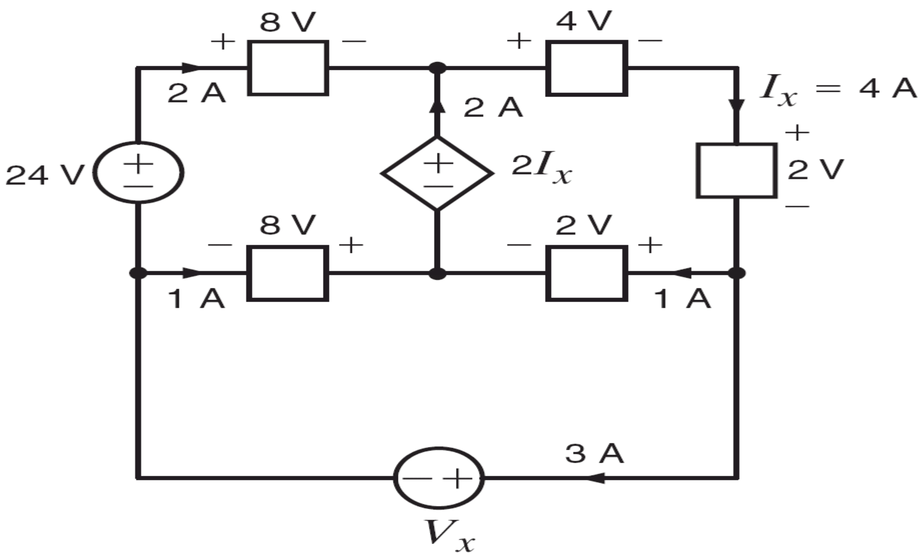
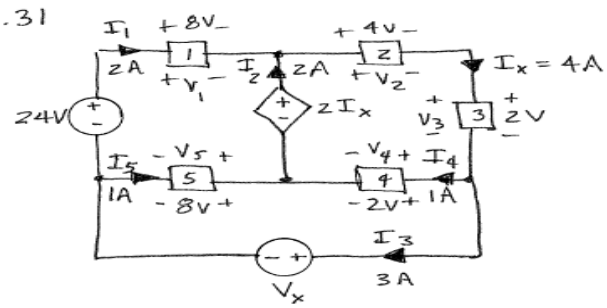


Figure P1.31

SOLUTION:

1.31



Passive sign convention:

Elements 1, 2, 3, 4, V_x

$P_1 = V_1 I_1 = 16\text{ W}$ absorbed

$P_2 = V_2 I_x = 16\text{ W}$ absorbed

$P_3 = V_3 I_x = 8\text{ W}$ absorbed

$P_4 = V_4 I_4 = 2\text{ W}$ absorbed

$P_{V_x} = V_x I_3 = 3V_x$ absorbed

$P_{24V} = 24 I_1 = 48\text{ W}$ supplied $\qquad P_5 = V_5 I_5 = 8\text{ W}$ supplied

$P_{2I_x} = 2 I_x I_2 = 16\text{ W}$ supplied

$P_{\text{supplied}} = P_{\text{absorbed}}$

$P_{24V} + P_5 + P_{2I_x} = P_1 + P_2 + P_3 + P_4 + P_{V_x}$

$48 + 8 + 16 = 16 + 16 + 8 + 2 + 3V_x$

$\boxed{V_x = 10\text{V}}$

1.32 Find I_s such that the power absorbed by the two elements in Fig. P1.32 is 24 W.

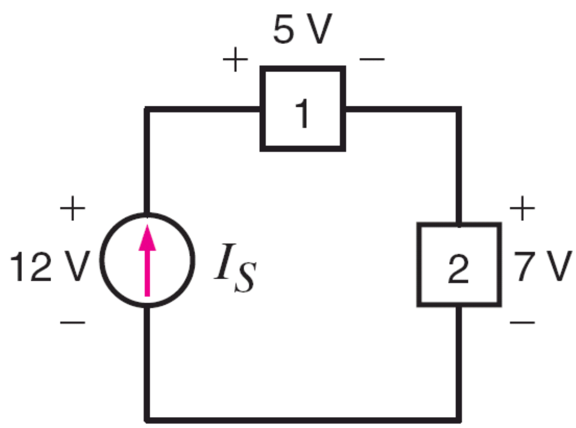
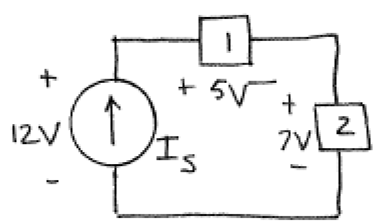


Figure P1.32

SOLUTION:

1.32



$$P_1 + P_2 = P_{I_S} = 24\text{W}$$

$$P_{I_S} = 12(I_S) = 24$$

$$\boxed{I_S = 2\text{ A}}$$

Chapter Two:

Resistive Circuits

2.1 Find the current I and the power supplied by the source in the network in Fig. P2.1. **CS**

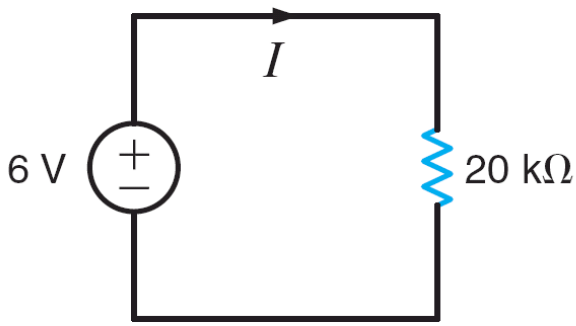
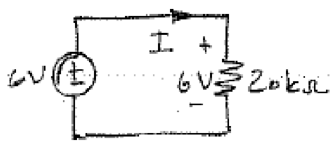


Figure P2.1

SOLUTION:

2.1 Find I & power supplied by source.



$P = VI \qquad I = V/R = \dfrac{6}{20 \times 10^3} = 300\,\mu A$

$P = 6(300 \times 10^{-6}) = 1.8\,mW$ (supplied)

Since voltage polarity & I do not obey passive sign convention, power calculated above is supplied!

2.2 In the network in Fig. P2.2, the power absorbed by R_x is 20 mW. Find R_x. **CS**

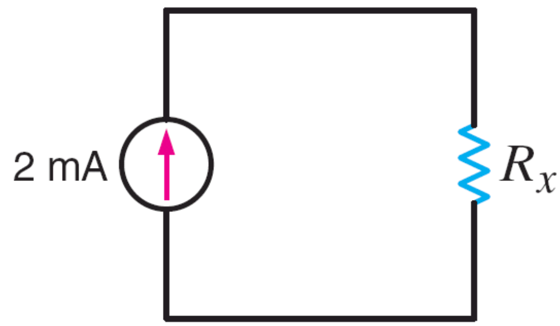
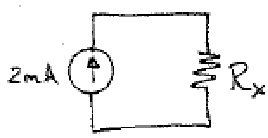


Figure P2.2

SOLUTION:

2.2 Find R_x if Power absorbed is 20mW



$P = I^2 R_x$ $I = 2mA$ $R_x = \dfrac{20\times10^{-3}}{(2\times10^{-3})^2} \Rightarrow R_x = 5k\Omega$

2.3 Find the current I and the power supplied by the source in the network in Fig. P2.3.

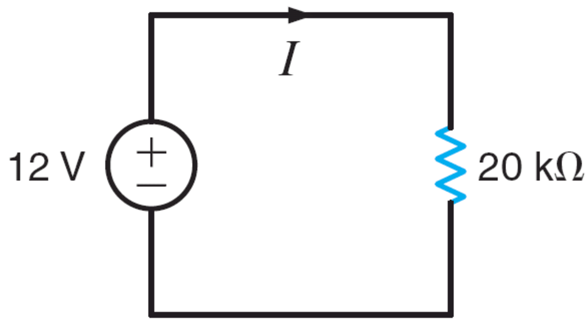
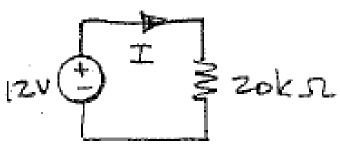


Figure P2.3

SOLUTION:

2.3 Find I and power supplied by source

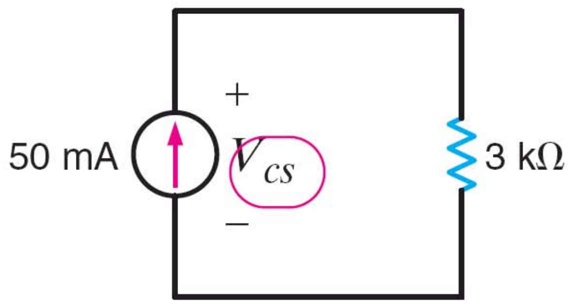


$I = V/R = \dfrac{12}{20 \times 10^3} \Rightarrow I = 600\mu A$

$P = VI \Rightarrow P = (12)(600 \times 10^{-6}) \Rightarrow P = 7.2\,mW$

Since voltage polarity & current direction for source do not obey passive sign convention, the power above is supplied.

2.4 In the circuit in Fig. P2.4, find the voltage across the current source and the power absorbed by the resistor.

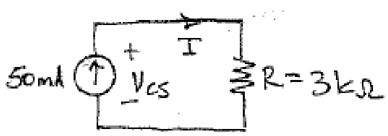


AU: Should "cs", subscript here be italic or roman?

Figure P2.4

SOLUTION:

2.4 Find V_{cs} and power absorbed by R



$V_{cs} = IR = (50\times10^{-3})(3\times10^{3})$

$V_{cs} = 150\text{V}$

$P_R = V_{cs} I = (150)(50\times10^{-3}) = 7.5\text{W}$

At R, V_{cs} and I obey passive sign convention, so, P_R is absorbed.

2.5 If the 5-kΩ resistor in the network in Fig. P2.5 absorbs 200 mW, find V_S.

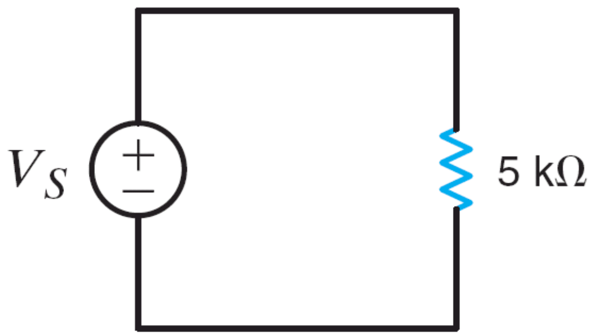
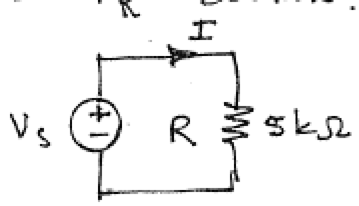


Figure P2.5

SOLUTION:

2.5 $P_R = 200\text{mW}$. Find V_s.



$$P_R = \frac{V_s^2}{R} = 0.2$$

$$V_s = \sqrt{(0.2)(5000)}$$

$$\boxed{V_s = 31.6\text{ V}}$$

2.6 In the network in Fig. P2.6, the power absorbed by G_x is 20 mW. Find G_x. **PSV**

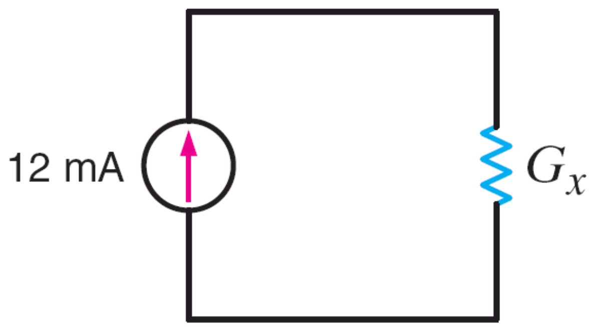
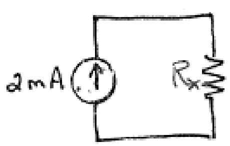


Figure P2.6

SOLUTION:

2.6 $P_G = 20\,mW$. Find R_x



$P_G = R_x I^2 = 0.02$ $R_x = \frac{0.02}{(2\times10^{-3})^2} \rightarrow \boxed{R_x = 5k\Omega}$

2.7 A model for a standard two D-cell flashlight is shown in Fig. P2.7. Find the power dissipated in the lamp.

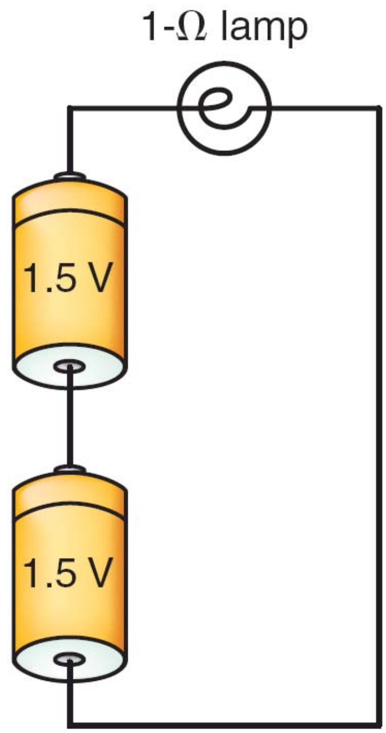
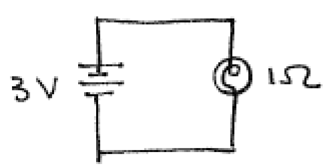


Figure P2.7

SOLUTION:

2.7 Find P_R.



$P_R = V^2/R = \dfrac{3^2}{1}$ $\boxed{P_R = 9W}$

2.8 An automobile uses two halogen headlights connected as shown in Fig. P2.8. Determine the power supplied by the battery if each headlight draws 3 A of current.

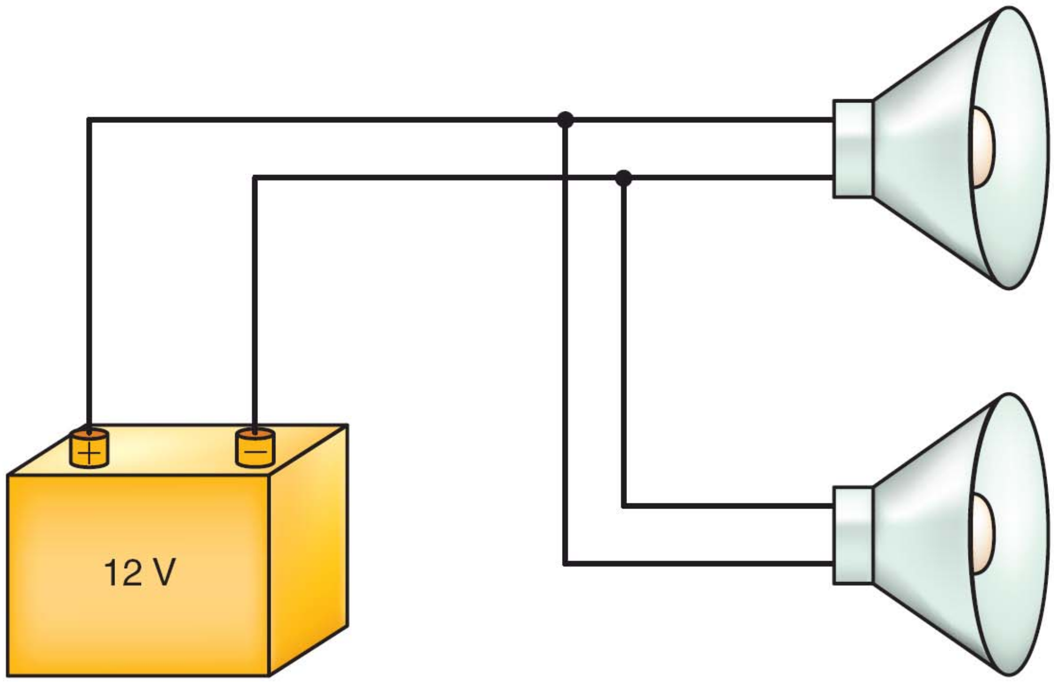
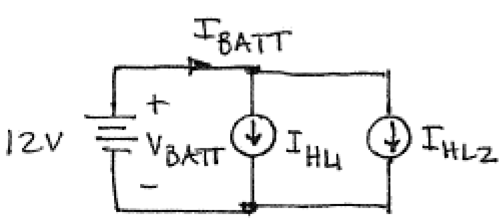


Figure P2.8

SOLUTION:

2.8 Find P_{BATT}.



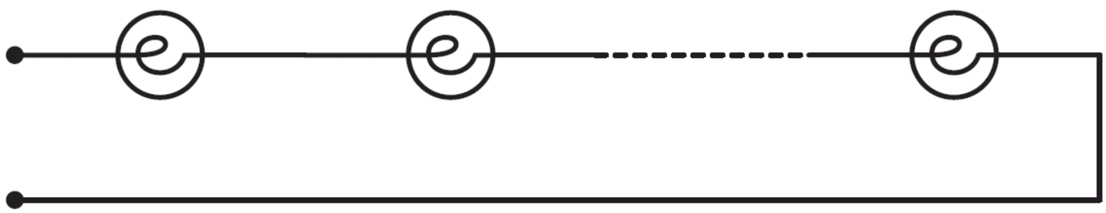
$I_{HL1} = I_{HL2} = 3\text{ A}$

$I_{BATT} = I_{HL1} + I_{HL2} = 6\text{A}$

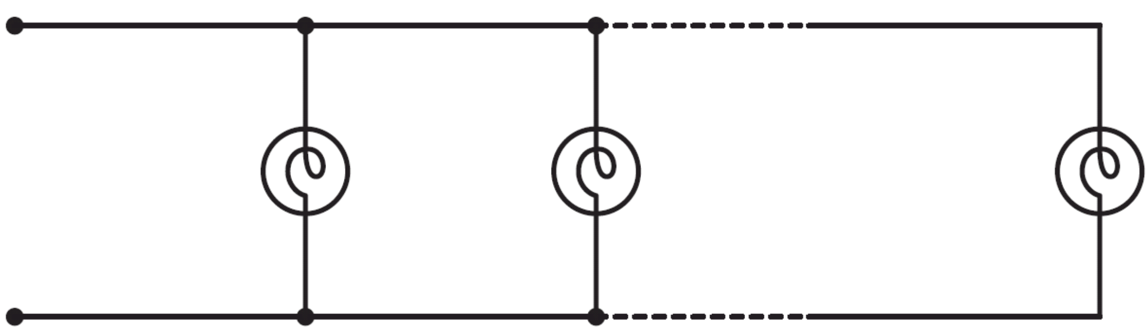
$P_{BATT} = V_{BATT} I_{BATT} = 12(6)$

$\boxed{P_{BATT} = 72\text{W}}$

2.9 Many years ago a string of Christmas tree lights was manufactured in the form shown in Fig. P2.9a. Today the lights are manufactured as shown in Fig. P2.9b. Is there a good reason for this change?



(a)



(b)

Figure P2.9

SOLUTION:

2.9 Why connect Christmas tree lights in parallel rather than series?

If a bulb fails as a open circuit (common failure), the series connect conducts no current and all bulbs are off. In the parallel connection, only the failed bulb is off, all others still function.

2.10 Find I_1 in the network in Fig. P2.10. CS

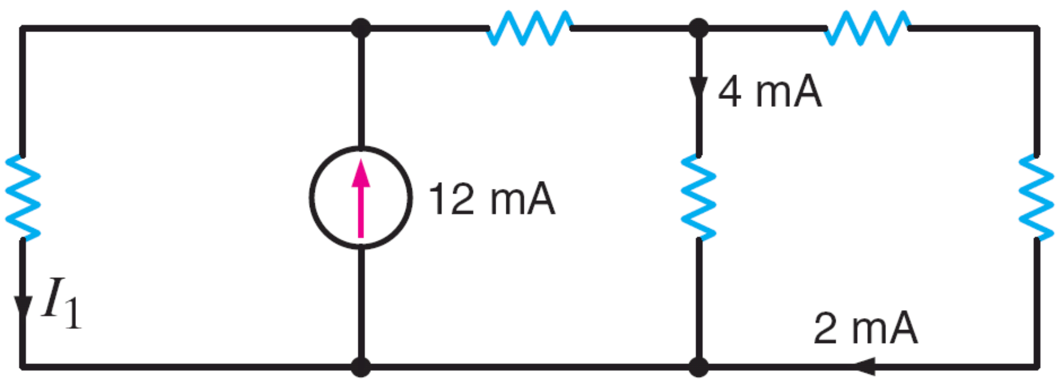
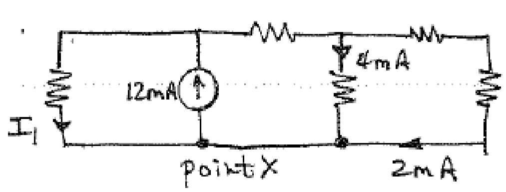


Figure P2.10

SOLUTION:

2.10 Find I_1



KCL at point X: all currents enter

$$I_1 - 12\times10^{-3} + 4\times10^{-3} + 2\times10^{-3} = 0$$

$$\boxed{I_1 = 6\,\text{mA}}$$

2.11 Find I_1 and I_2 in the circuit in Fig. P2.11. **CS**

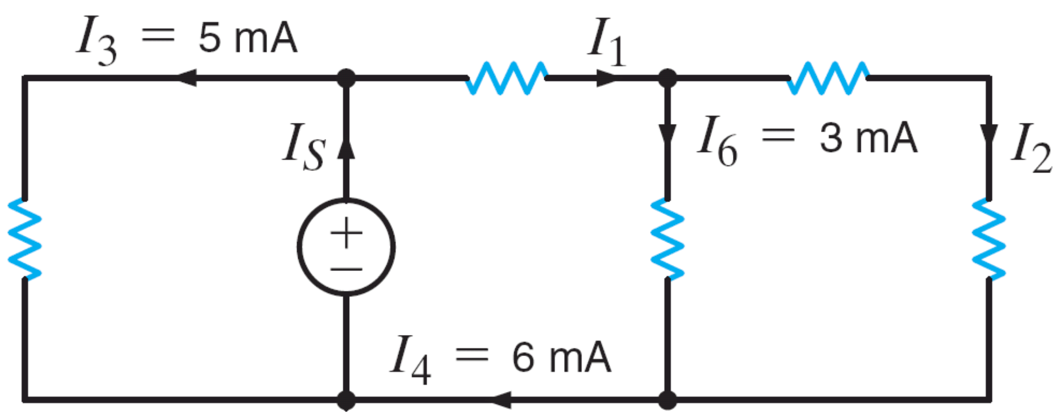
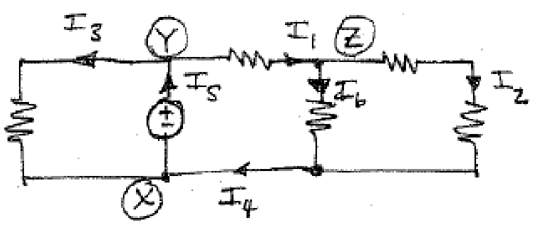


Figure P2.11

SOLUTION:

2.11 Find I_1 and I_2



$I_3 = 5mA \quad I_6 = 3mA \quad I_4 = 6mA$

KCL @ X : all currents enter

$I_3 - I_S + I_4 = 0$

$I_S = 5\times10^{-3} + 6\times10^{-3} \Rightarrow I_S = 11mA$

KCL @ Y : all currents enter

$-I_3 + I_S - I_1 = 0 \Rightarrow I_1 = I_S - I_3 = 11\times10^{-3} - 5\times10^{-3} \Rightarrow \boxed{I_1 = 6\,mA}$

KCL @ Z : all current enter

$I_1 - I_6 - I_2 = 0 \Rightarrow I_2 = I_1 - I_6 = 6\times10^{-3} - 3\times10^{-3} \Rightarrow \boxed{I_2 = 3\,mA}$

2.12 Find I_o and I_1 in the circuit in Fig. P2.12.

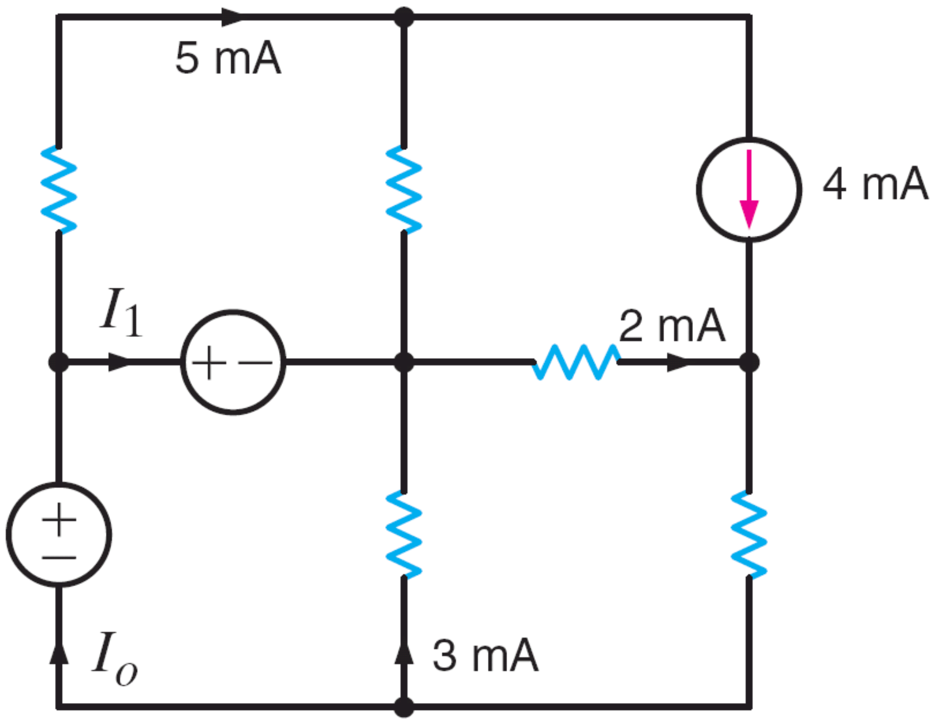
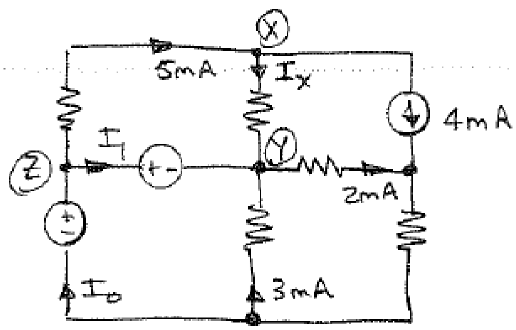


Figure P2.12

SOLUTION:

2.12 Find I_o and I_1



KCL @ X: currents enter

$5\times10^{-3} - 4\times10^{-3} - I_x = 0 \Rightarrow I_x = 1\,mA$

KCL @ Y: current enter

$I_x - 2\times10^{-3} + 3\times10^{-3} + I_1 = 0 \Rightarrow \boxed{I_1 = -2\,mA}$

KCL @ Z: currents enter

$-5\times10^{-3} - I_1 + I_o = 0 \Rightarrow \boxed{I_o = 3\,mA}$

2.13 Find I_x in the circuit in Fig. P2.13.

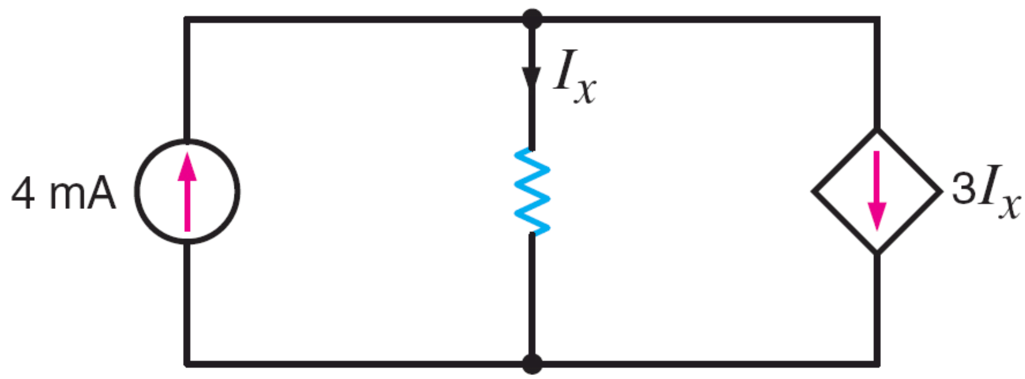
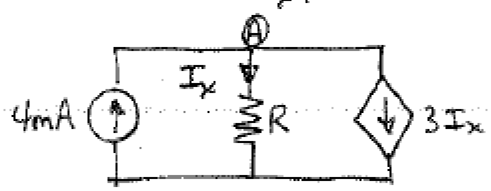


Figure P2.13

SOLUTION:

2.13 Find I_x.



KCL @ A: currents leaving

$$-4\times10^{-3} + I_x + 3I_x = 0$$

$$4I_x = 4\times10^{-3} \qquad \boxed{I_x = 1\,\text{mA}}$$

2.14 Find I_x in the circuit in Fig. P2.14. PSV

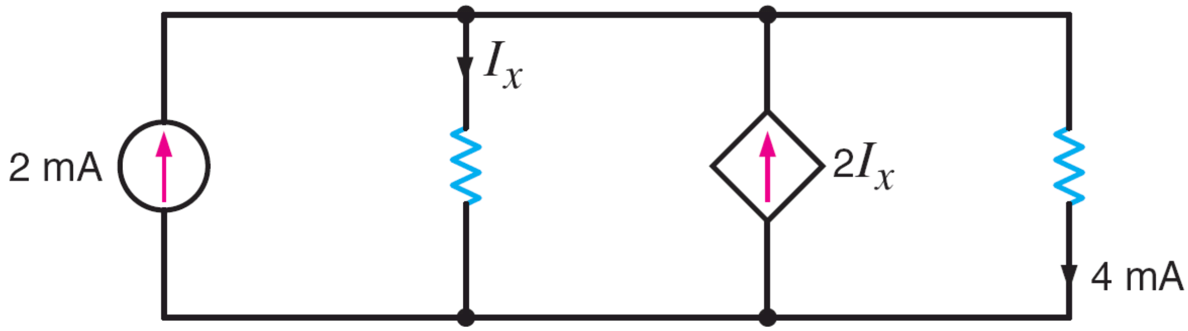
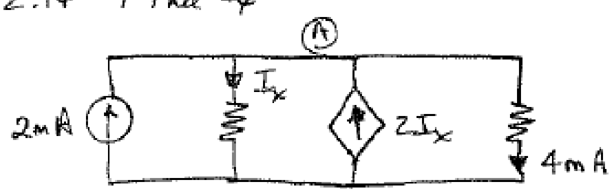


Figure P2.14

SOLUTION:

2.14 Find I_x



KCL @ A : currents leaving

$$-2\times10^{-3} + 4\times10^{-3} + I_x - 2I_x = 0$$

$$\boxed{I_x = 2\,\text{mA}}$$

2.15 Find I_x in the circuit in Fig. P2.15. cs

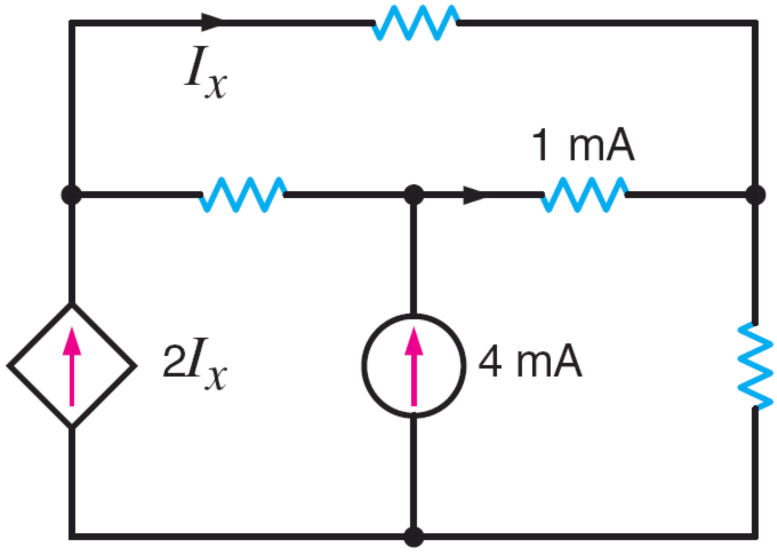
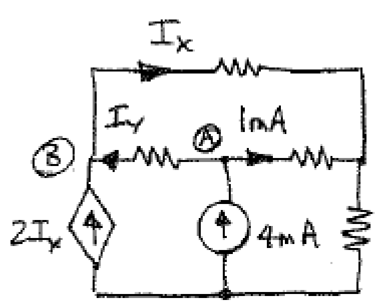


Figure P2.15

SOLUTION:

2.15 Find I_x.



KCL @ A: currents leaving

$-4\times10^{-3} + 10^{-3} + I_y = 0$ $\quad I_y = 3\text{ mA}$

KCL @ B: currents entering

$I_y + 2I_x - I_x = 0$ $\quad \boxed{I_x = -3\text{ mA}}$

2.16 Find V_x in the circuit in Fig. P2.16. **CS**

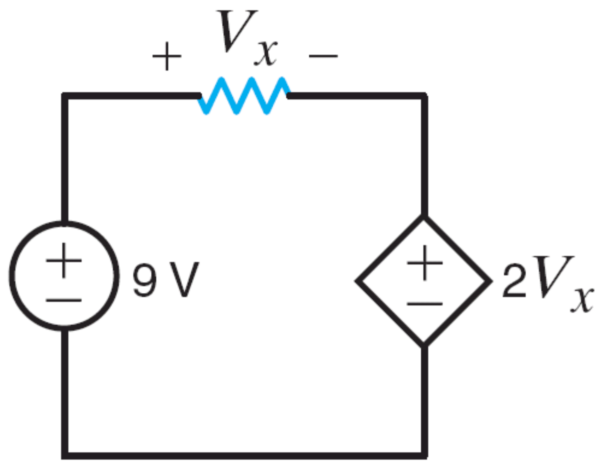
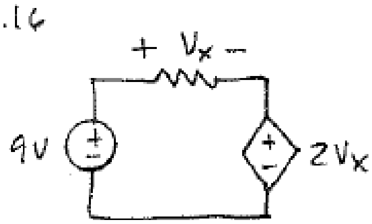


Figure P2.16

SOLUTION:

2.16



Find V_x.

KVL: $-9 + V_x + 2V_x = 0 \Rightarrow \boxed{V_x = 3V}$

2.17 Find V_{fb} and V_{ec} in the circuit in Fig. P2.17.

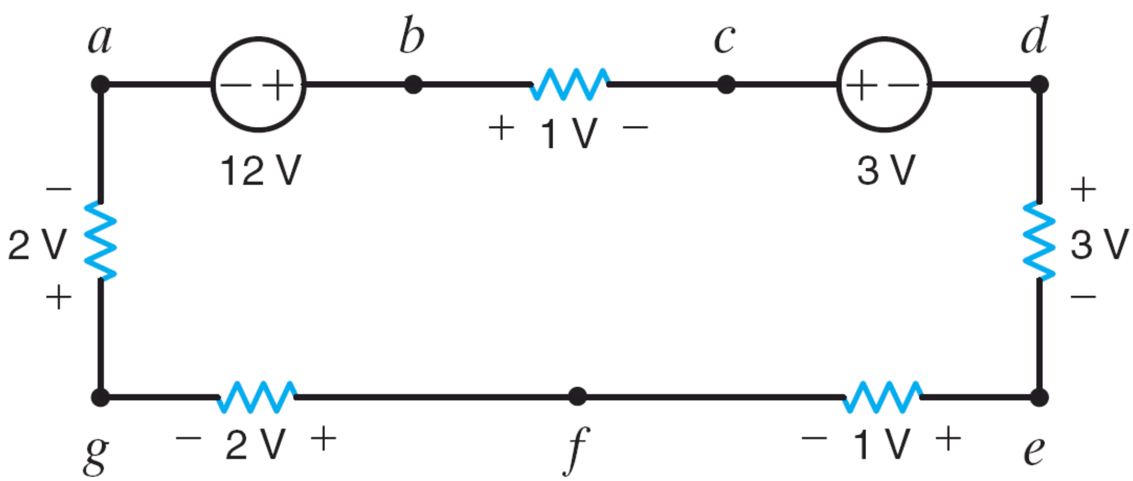
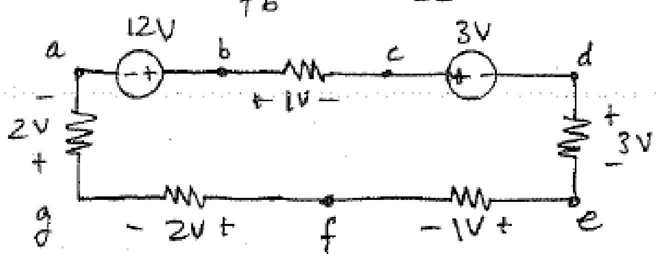


Figure P2.17

SOLUTION:

2.17 Find V_{fb} and V_{ec}



KVL along abfga:

$-12 + V_{bf} + 2 + 2 = 0$

$V_{bf} = 8V \Rightarrow \boxed{V_{fb} = -8V}$

KVL along cdec: $3 + 3 + V_{ac} = 0 \Rightarrow \boxed{V_{ec} = -6V}$

2.18 Find V_{ac} in the circuit in Fig. P2.18. **CS**

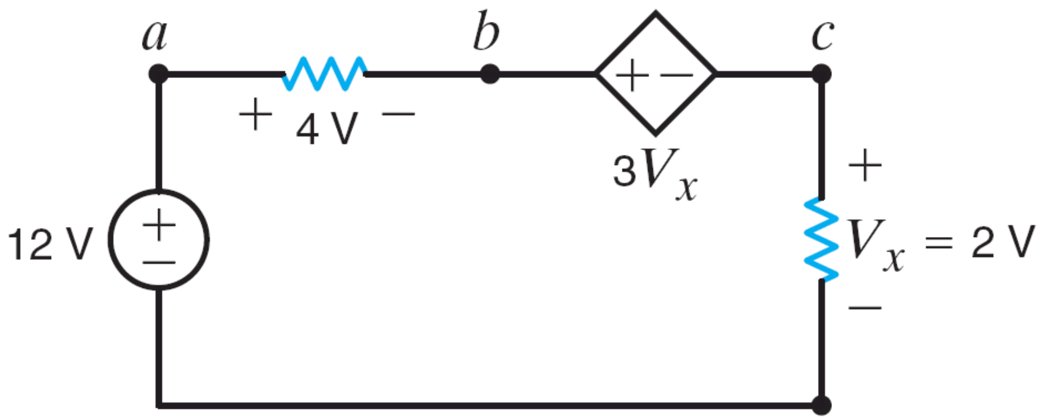
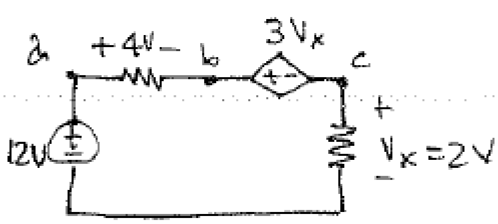


Figure P2.18

SOLUTION:

2.18 Find V_{ac}



$V_{ac} = V_{ab} + V_{bc} = 4 + 3V_x$

$V_x = 2\,V$

$\boxed{V_{ac} = 10V}$

2.19 Find V_{da} and V_{be} in the circuit in Fig. P2.19.

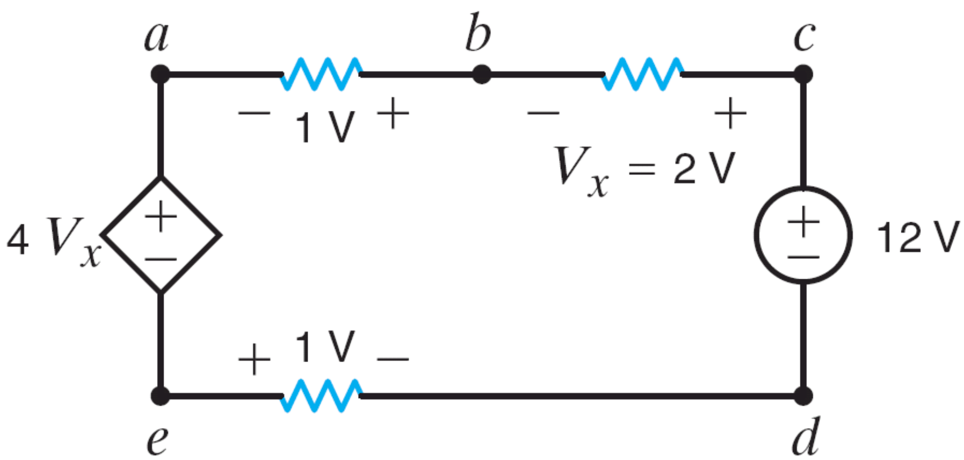
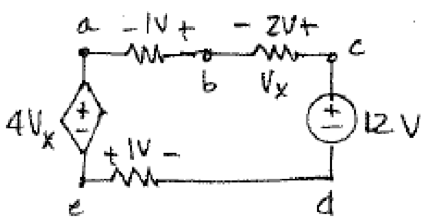


Figure P2.19

SOLUTION:

2.19 Find V_{da} and V_{be}



$V_{da} = V_{dc} + V_{cb} + V_{ba} = -12 + 2 + 1 = -9$

$\boxed{V_{da} = -9\,V}$

$V_{be} = V_{bc} + V_{cd} + V_{de} = -2 + 12 - 1 = 9$

$\boxed{V_{be} = 9\,V}$

2.20 The 10-V source absorbs 2.5 mW of power. Calculate V_{ba} and the power absorbed by the dependent voltage source in Fig. P2.20.

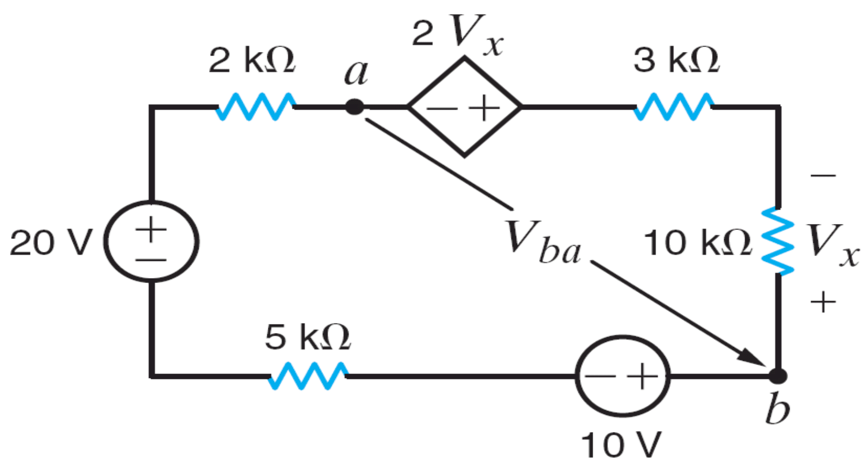
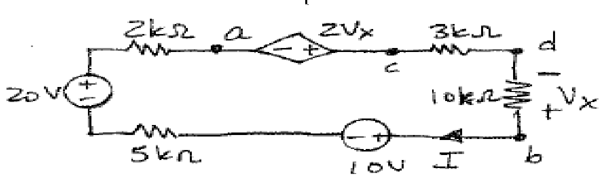


Figure P2.20

SOLUTION:

2.20 $P_{10V} = 2.5mW$ absorbed. Find V_{ba} & power absorbed by dependent source



$P_{10V} = 10I = 2.5mW$

$I = 250\mu A$

$V_{ba} = V_{bd} + V_{dc} + V_{ca}$

$V_{bd} = -I(10\times10^3) = -2.5V$

$V_{dc} = -I(3\times10^3) = -0.75V$

$V_{ca} = 2V_x \qquad V_x = V_{bd} = -2.5V$

$V_{ca} = -5V$

$\boxed{V_{ba} = -8.25V}$

$P_{DS} = -(2V_x)(I) \qquad V_x = V_{bd} = -2.5V \qquad I = 250\mu A$

$\boxed{P_{DS} = 1.25mW}$

2.21 Find V_o in the network in Fig. P2.21.

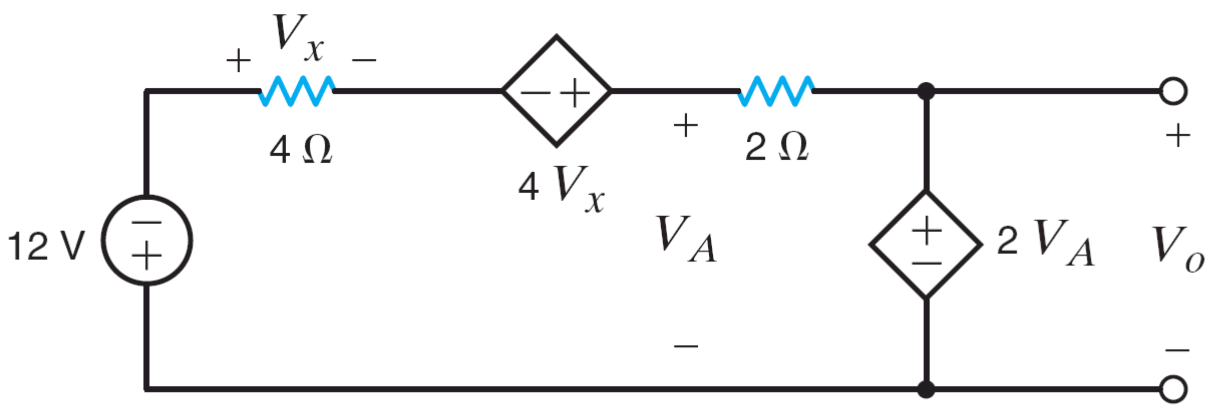
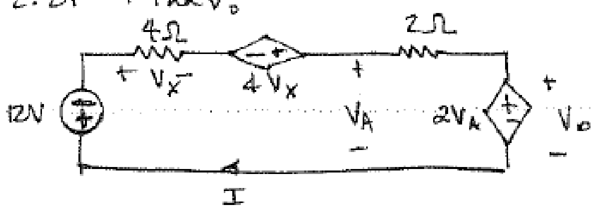


Figure P2.21

SOLUTION:

2.21 Find V_o



KVL:

$$12 + 4I - 4V_x + 2I + 2V_A = 0 \quad (1)$$

$$V_x = 4I \quad (2)$$

$$V_A = 2I + 2V_A \Rightarrow V_A = -2I \quad (3)$$

$$V_o = 2V_A \quad (4)$$

Substitute (2) & (3) into (1)

$$12 + 4I - 16I + 2I - 4I = 0$$

$$-14I = -12 \qquad I = 6/7 \text{ A} \Rightarrow V_A = -\frac{12}{7} \text{ V} \Rightarrow \boxed{V_o = -\frac{24}{7} \text{ V}}$$

2.22 Find V_o in the circuit in Fig. P2.22. **PSV**

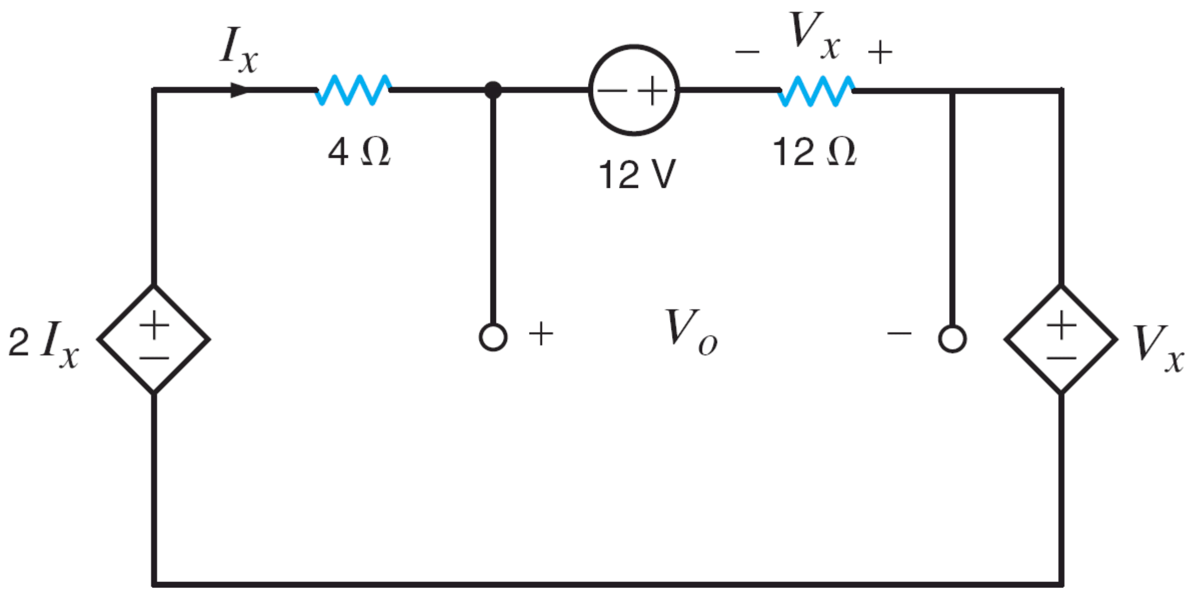
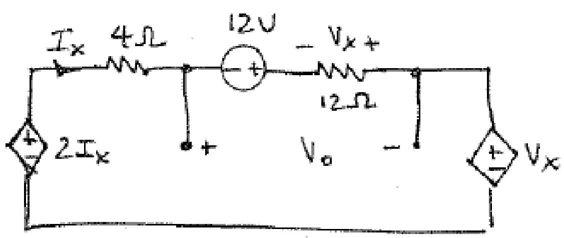


Figure P2.22

SOLUTION:

2.22 Find V_o



KVL :

$$4I_x - 12 + 12I_x + V_x - 2I_x = 0 \quad (1)$$

$$V_x = -12I_x \quad (2)$$

Substitute (2) into (1): $4I_x + 12I_x - 12I_x - 2I_x = 12$

$$2I_x = 12 \Rightarrow I_x = 6\text{ A}$$

$$V_o = -12 + 12I_x = -12 + 12(6) \Rightarrow \boxed{V_o = 60\text{V}}$$

2.23 Find V_{ac} in the network in Fig. P2.23.

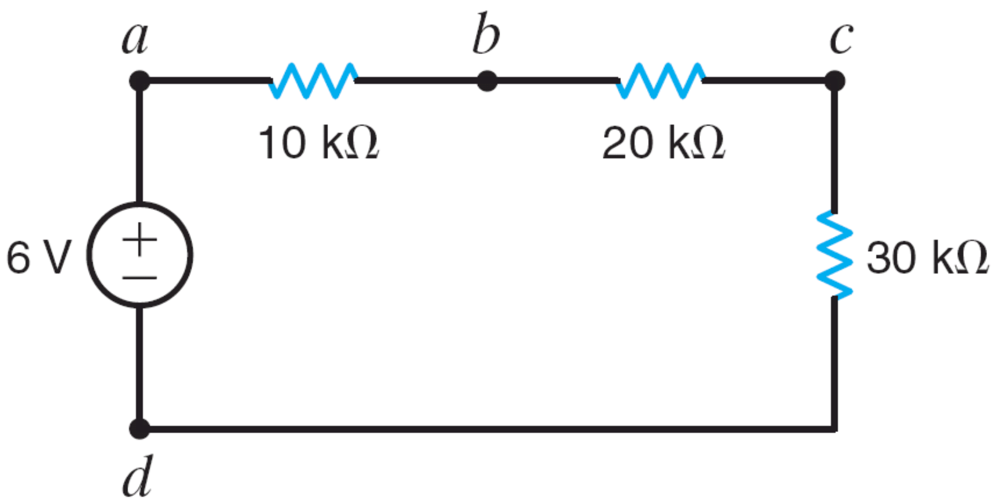
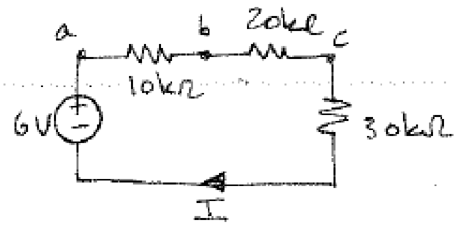


Figure P2.23

SOLUTION:

2.23 Find V_{ac}



KVL: $-6 + 10^4 I + 2\times10^4 I + 3\times10^4 I = 0$

$6\times10^4 I = 6 \Rightarrow I = 100\mu A$

$V_{ac} = V_{ab} + V_{bc} = 10^4 I + 2\times10^4 I$

$\boxed{V_{ac} = 3V}$

2.24 Find both I and V_{bd} in the circuit in Fig. P2.24.

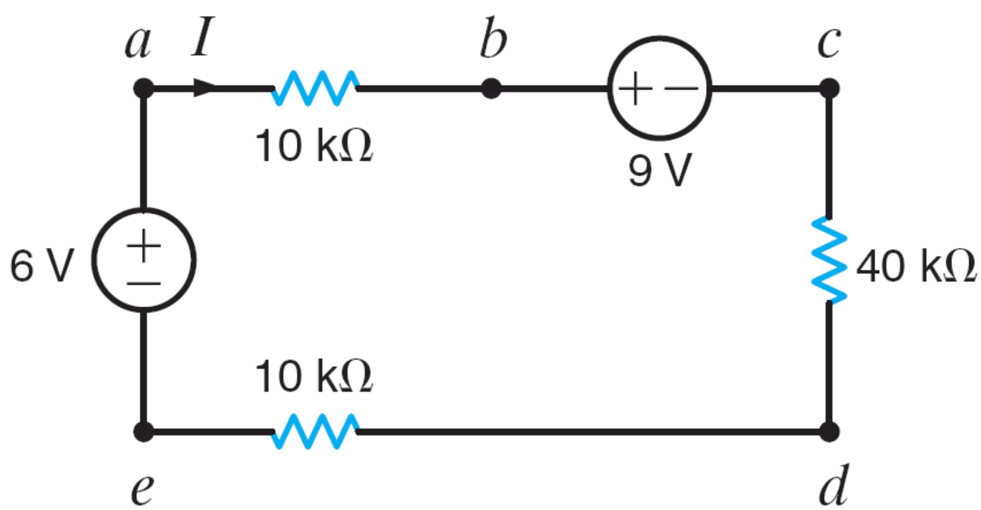
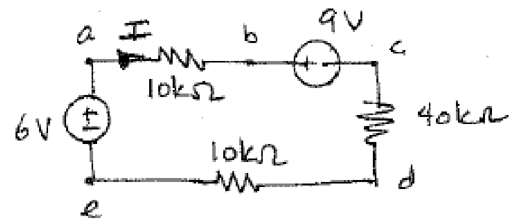


Figure P2.24

SOLUTION:

2.24 Find I and V_{bd}.



KVL: $-6 + 10^4 I + 9 + 4\times10^4 I + 10^4 I = 0$

$6\times10^4 I = -3$ $\quad \boxed{I = -50.0\,\mu A}$

$V_{bd} = V_{bc} + V_{cd} = 9 + 4\times10^4 I$

$\boxed{V_{bd} = 7\text{ V}}$

2.25 Find V_x in the circuit in Fig. P2.25. CS

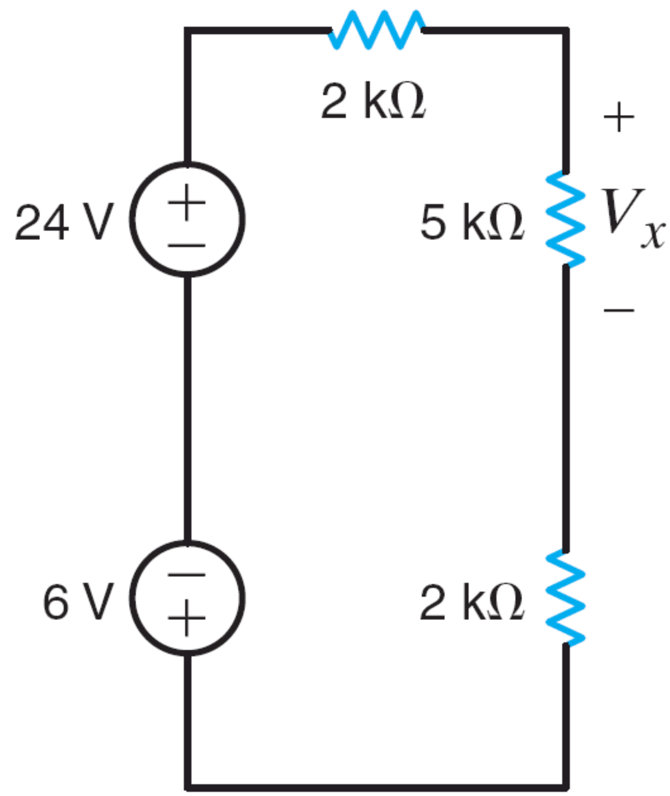
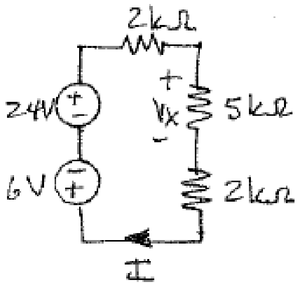


Figure P2.25

SOLUTION:

2.25 Find V_x



KVL: $6-24+I(2\times10^3)+I(5\times10^3)+I(2\times10^3)=0$

$9\times10^3 I = 18 \Rightarrow I = 2mA$

$V_x = 5\times10^3 I \Rightarrow \boxed{V_x = 10V}$

2.26 Find V_1 in the network in Fig. P2.26. PSV

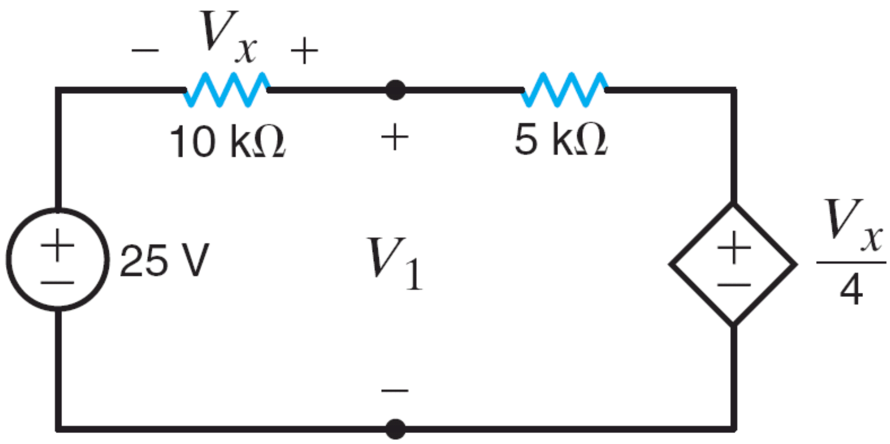
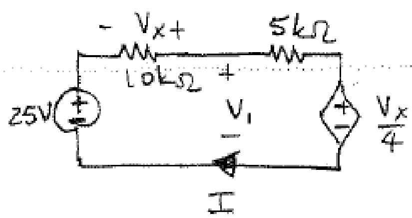


Figure P2.26

SOLUTION:

2.26 Find V_1.



KVL: $-25 + 10^4 I + 5\times10^3 I + V_x/4 = 0$ (1)

also: $V_x = -10^4 I$ (2)

Substitute (2) into (1):

$$-25 + I\left(10^4 + 5\times10^3 - 10^4/4\right) = 0$$

$$I = 2mA$$

$V_1 = 25 - 10^4 I \Rightarrow$ $\boxed{V_1 = 5V}$

2.27 Find the power absorbed by the 30-kΩ resistor in the circuit in Fig. P2.27. CS

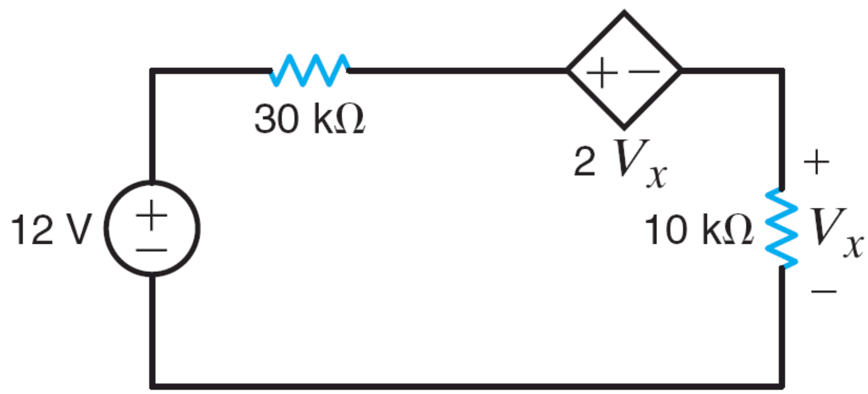
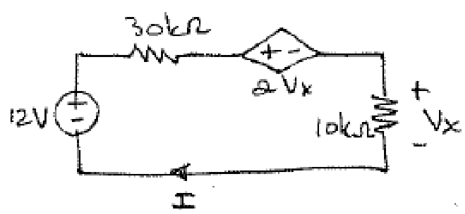


Figure P2.27

SOLUTION:

2.27 Find P_{30k} absorbed.



KVL: $-12 + 3\times10^4 I + 2V_x + 10^4 I$ (1)

also: $V_x = 10^4 I$ (2)

Substitute (2) into (1)

$I(3\times10^4 + 10^4 + 2\times10^4) = 12 \Rightarrow I = 200\mu A$

$P_{30k} = 3\times10^4 I^2 \Rightarrow \boxed{P_{30k} = 1.2\,mW}$

2.28 In the network in Fig. P2.28, if $V_x = 12$ V, find V_S.

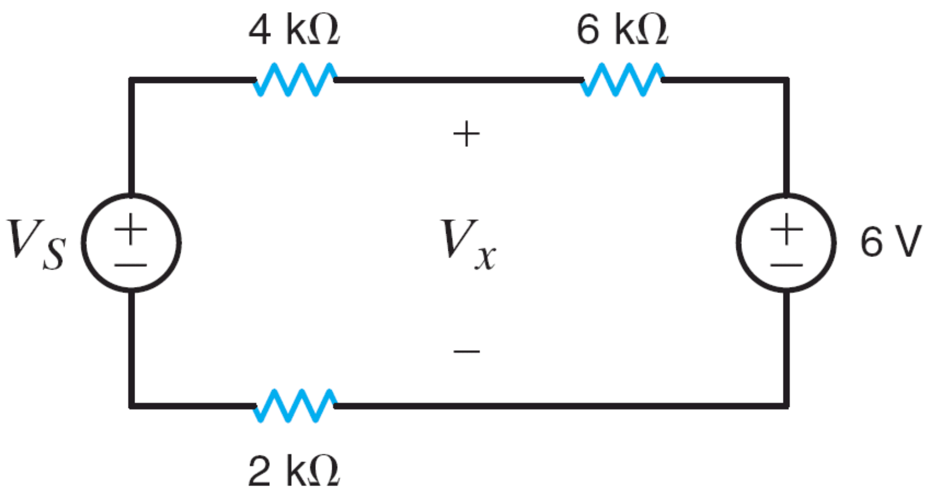
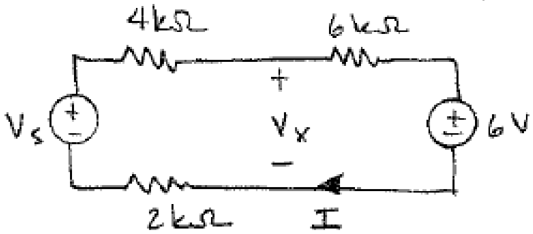


Figure P2.28

SOLUTION:

2.28 Find V_S if $V_x = 12$V



KVL: $-V_x + 6\times10^3 I + 6 = 0$

$$I = \frac{12-6}{6\times10^3} = 1\text{mA}$$

KVL: $-V_S + 4\times10^3 I + V_x + 2\times10^3 I = 0$

$$\boxed{V_S = 18\text{V}}$$

2.29 In the circuit in Fig. P2.29, $P_{3\,\mathrm{k}\Omega} = 12$ mW. Find V_S.

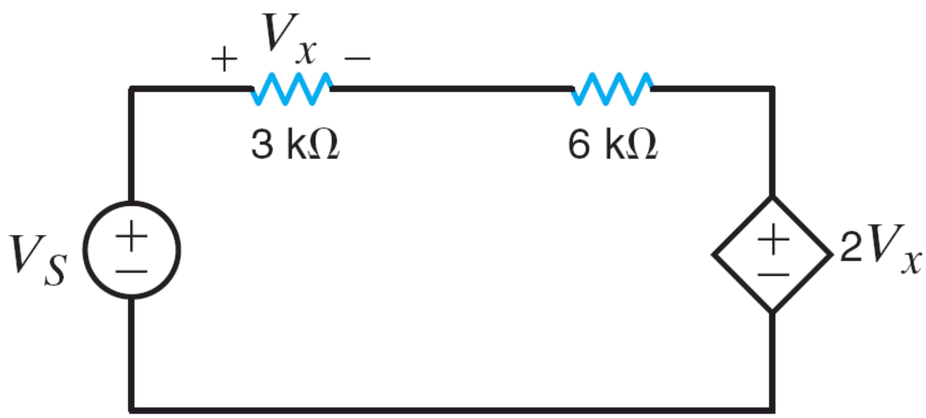
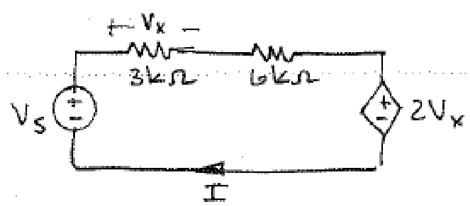


Figure P2.29

SOLUTION:

2.29 $P_{3k} = 12$ mW. Find V_s



$$P_{3k} = 12\,\text{mW} = I^2\,(3\times10^3)$$

$$I = \sqrt{\frac{12\times10^{-3}}{3\times10^{3}}} \Rightarrow I = 2\,\text{mA}$$

KVL: $-V_s + 3\times10^3 I + 6\times10^3 I + 2V_x = 0$ (1)

and: $V_x = I(3\times10^3)$ (2)

substitute (2) into (1): $I(3\times10^3 + 6\times10^3 + 6\times10^3) = V_s \Rightarrow \boxed{V_s = 30\text{V}}$

2.30 If $V_o = 4$ V in the network in Fig. P2.30, find V_S.

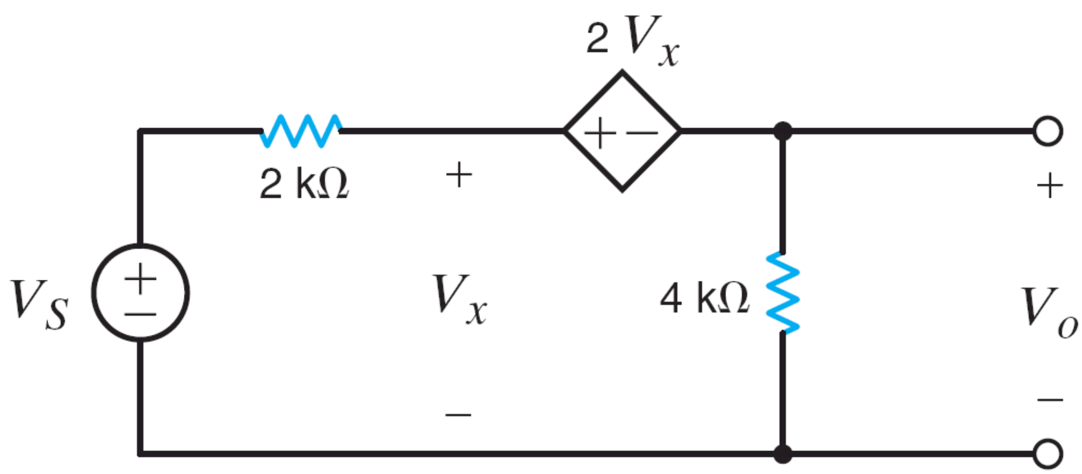
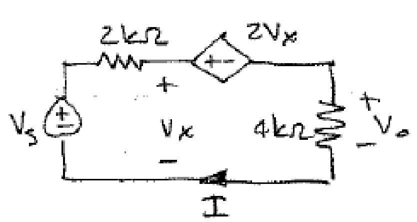


Figure P2.30

SOLUTION:

2.30 $V_o = 4V$, find V_s



KVL: $-V_s + 2\times10^3 I + 2V_x + 4\times10^3 I = 0$ (1)

also: $-V_x + 2V_x + V_o = 0 \Rightarrow V_x = -4V$ (2)

and $V_o = 4\times10^3 I \Rightarrow I = 1mA$ (3)

substitute (2) & (3) into (1): $I(6\times10^3) + 2V_x = V_s \Rightarrow \boxed{V_s = -2V}$

2.31 If $V_A = 12$ V in the circuit in Fig. P2.31, find V_S.

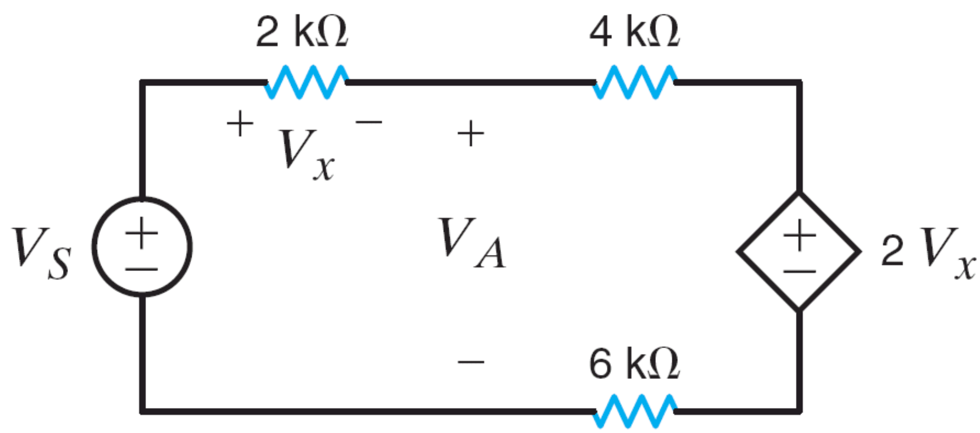
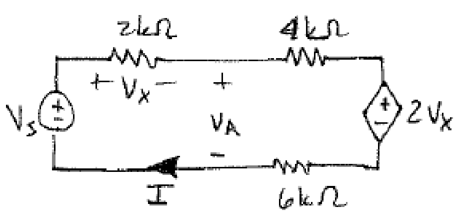


Figure P2.31

SOLUTION:

2.31 $V_A = 12V$. Find V_s.



KVL: $-V_S + 2\times10^3 I + 4\times10^3 I + 2V_X + 6\times10^3 I = 0$ (1)

KVL: $-V_A + 4\times10^3 I + 2V_X + 6\times10^3 I = 0$ (2)

also: $V_X = 2\times10^3 I$ (3)

Substitute (3) into (2): $-12 + I(4\times10^3 + 4\times10^3 + 6\times10^3) = 0$

$$I = \frac{6}{7} mA$$

Substitute (3) into (1): $I[2+4+4+6]\times10^3 = V_S$

$$\boxed{V_S = \frac{96}{7} V = 13.7 V}$$

2.32 A commercial power supply is modeled by the network shown in Fig. P2.32.

(a) Plot V_o versus R_{load} for $1\ \Omega \leq R_{\text{load}} \leq \infty$.

(b) What is the maximum value of V_o in (a)?

(c) What is the minimum value of V_o in (a)?

(d) If for some reason the output should become short circuited, that is, $R_{\text{load}} \rightarrow 0$, what current is drawn from the supply?

(e) What value of R_{load} corresponds to maximum power consumed?

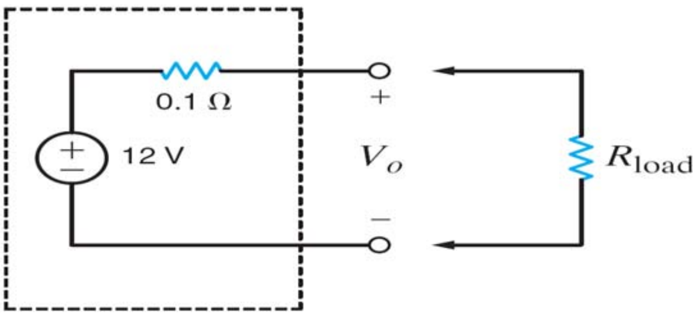
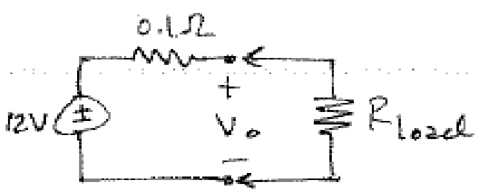


Figure P2.32

SOLUTION:

2.32 $1\Omega \leq R_{load} \leq \infty$



a) $V_o = 12\left[\dfrac{R_{load}}{R_{load}+0.1}\right]$

b) $V_{o\,max}$ occurs when $R_{load} = \infty$

$$V_o = 12\left[\frac{R_{load}}{0.1+R_{load}}\right]$$

$$\boxed{V_{o\,max} = 12V}$$

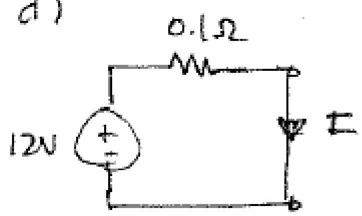
Continued on the next page.

c) $V_{o\,min}$ occurs when $R_{load} = 1\Omega$

$$V_{o\,min} = 12\left[\frac{1}{1+0.1}\right]$$

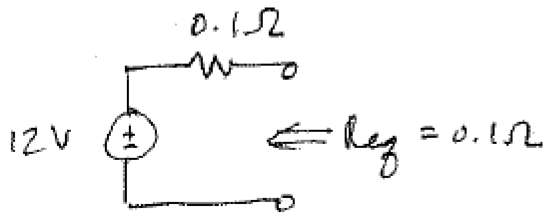
$$\boxed{V_{o\,min} = 10.9\,V}$$

d)



$$\boxed{I = \frac{12}{0.1} = 120A}$$

e) for max. power at R_{load}, $\boxed{R_{load} = R_{eq} = 0.1\Omega}$



2.33 A commercial power supply is guaranteed by the manufacturer to deliver 5 V ±1% across a load range of 0 to 10 A. Using the circuit in Fig. P2.33 to model the supply, determine the appropriate values of *R* and *V*.

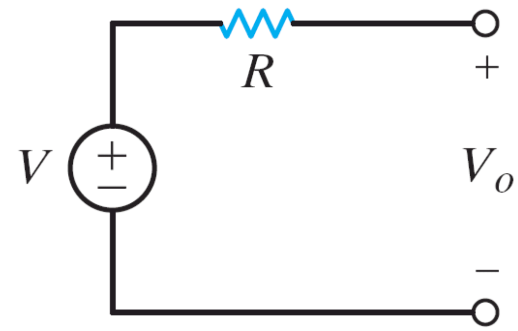
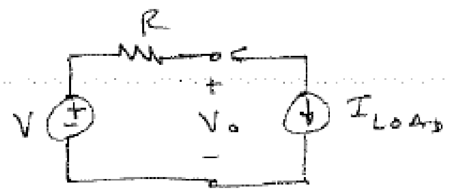


Figure P2.33

SOLUTION:

2.33. $V_o = 5 \pm 1\%$ for load of 0 to 10A. Find V & R.



at $I_{LOAD} = 0A$,

$$V_o = V = V_{omax} = 5(1.01)$$

$$\boxed{V = 5.05V}$$

at $I_{LOAD} = 10A$,

$$V_o = V - I_{LOAD}R = 5.05 - 10(R) = 5(0.99)$$

$$\boxed{R = 10 m\Omega}$$

2.34 A power supply is specified to provide 48 ± 2 V at 0–200 A and is modeled by the circuit in Fig. P2.34.

(a) What are the appropriate values for V and R?

(b) What is the maximum power the supply can deliver? What values of I_{load} and V_o correspond to that level?

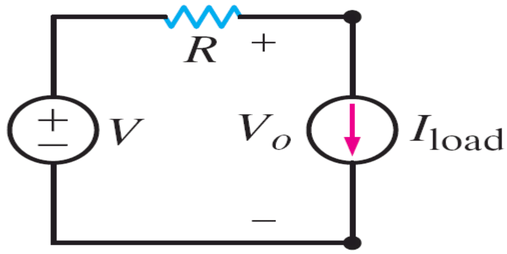


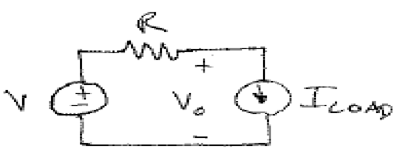
Figure P2.34

SOLUTION:

2.34 $V_o = 48 \pm 2V$ at I_{LOAD} from $0 \to 200A$.

a). Find V & R.

b). Find P_{LOAD} max & corresponding I_{LOAD} & V_o



a) At $I_{LOAD} = 0A$, $V_o = V_{omax} = 50V$

$$\boxed{V_o = V = 50V}$$

At $I_{LOAD} = 200A$,

$$V_o = V - I_{LOAD} R = 46V$$

$$\boxed{R = 20\,m\Omega}$$

b) Max P_{LOAD} occurs at

$$\frac{\partial P_{OUT}}{\partial I_{LOAD}} = 0 \qquad P_{LOAD} = I_{LOAD} V_o = I_{LOAD}(V - R I_{LOAD})$$

$$\frac{\partial P_{OUT}}{\partial I_{LOAD}} = V - 2R I_{LOAD} = 0 \Rightarrow I_{LOAD} = V/2R$$

at max power out: $I_{LOAD} = 1250A \leftarrow$ beyond specs

∴ Max power occurs at $I_{LOAD} = 200A$ & $V_o = 46V$

2.35 Although power supply loads are often modeled as either resistors or constant current sources, some loads are best modeled as constant power loads, as indicated in Fig. P2.35. Given the model shown in the figure,

(a) Write a V–I expression for a constant power load that always draws P_L watts.

(b) If $P_L = 40$ W, $V_{ps} = 9$ V and $I_o = 5$ A, determine the values of V_o and R_{ps}.

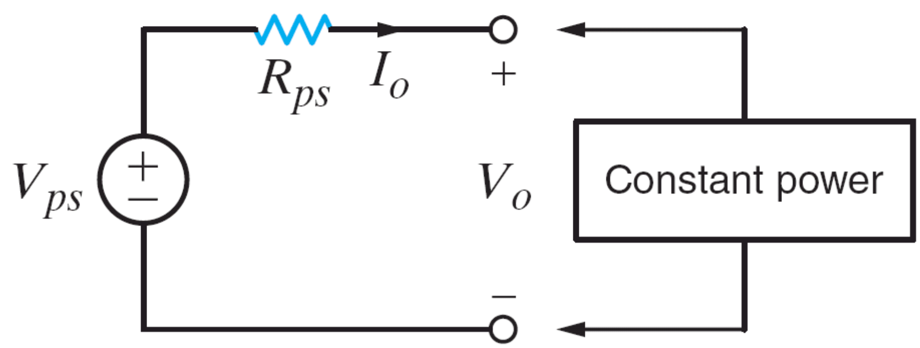
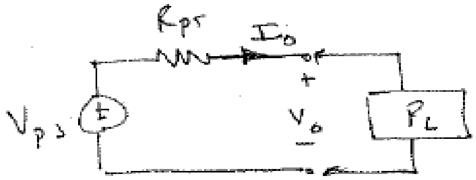


Figure P2.35

SOLUTION:

2.35 a) Write V-I expression for load @ $P = P_L$.

b) $P_L = 40\,W$ $V_{ps} = 9V$ $I_o = 5\,A$, find V_o & R_{ps}.



a) for the load $P_L = V_L I_L$

$$\boxed{V_L = P_L / I_L}$$

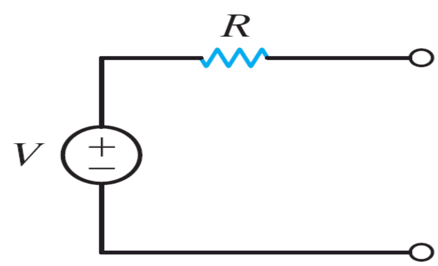
b) $V_o = P_L / I_o \Rightarrow \boxed{V_o = 8\,V}$

$$R_{ps} = \frac{V_{ps} - V_o}{I_o} = \frac{9-8}{5} \quad \boxed{R_{ps} = 0.2\,\Omega}$$

2.36 A student needs a 15-V voltage source for research. She has been able to locate two power supplies, a 10-V supply and a 5-V supply. The equivalent circuits for the two supplies are shown in Fig. P2.36.

(a) Draw an equivalent circuit for the effective 15-V supply.

(b) If she can tolerate a 0.5-V deviation from 15 V, what is the maximum current change the combined supply can satisfy?



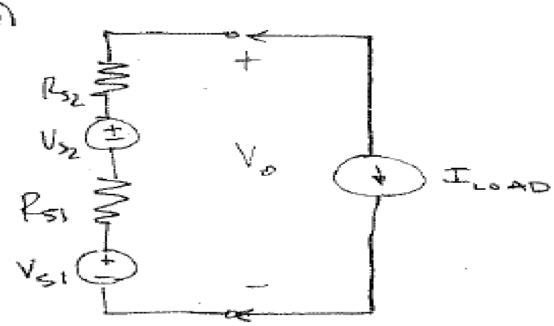
Voltage	5 V	10 V
Resistance	0.25 Ω	0.05 Ω

Figure P2.36

SOLUTION:

2.36 $V_{S1} = 5V$, $R_{S1} = 0.25\Omega$, $V_{S2} = 10V$ $R_{S2} = 0.05\Omega$

a)



b) $V_o = V_{S1} + V_{S2} - I(R_{S1} + R_{S2})$

$V_{o\,min} = 14.5 = 15 - I_o(0.3)$

$$\boxed{I_{o\,max} = \frac{0.5}{0.3}}$$

$$\boxed{I_{o\,max} = 1.67A}$$

2.37 Given the network in Fig. P2.37, we wish to obtain a voltage of $2\text{ V} \leq V_o \leq 9\text{ V}$ across the full range of the pot. Determine the values of R_1 and R_2.

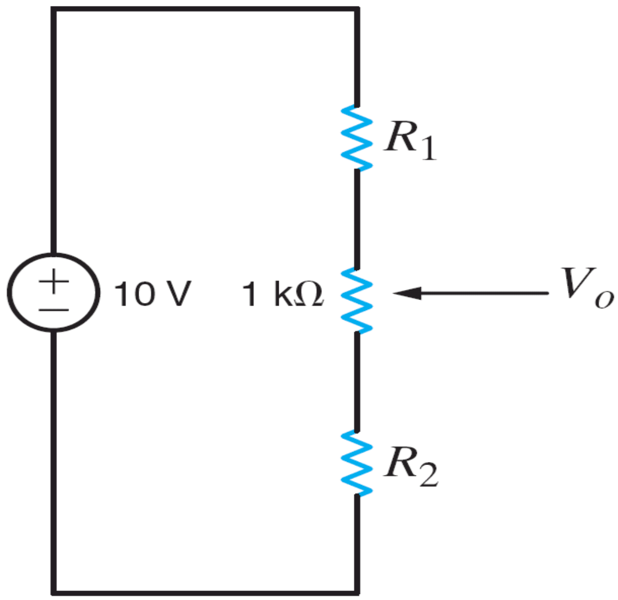
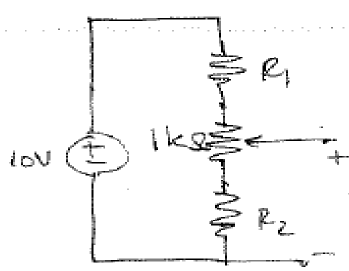


Figure P2.37

SOLUTION:

2.37 $\quad 2V \leq V_o \leq 9V \quad$ Find R_1 & R_2.



Case a: wiper at bottom of variable R.

$$V_o = 10\left[\frac{R_2}{R_1+R_2+1000}\right] = V_{o_{min}} = 2V$$

Case b: wiper at top of variable R,

$$V_o = 10\left[\frac{R_2+1000}{R_1+R_2+1000}\right] = V_{o\,max} = 9V$$

$$\frac{V_{o\,max}}{V_{o\,min}} = \frac{9}{2} = \frac{R_2+1000}{R_2} \Rightarrow \boxed{\begin{array}{l} R_2 = 286\,\Omega \\ R_1 = 144\,\Omega \end{array}}$$

2.38 Determine I_L in the circuit in Fig. P2.38.

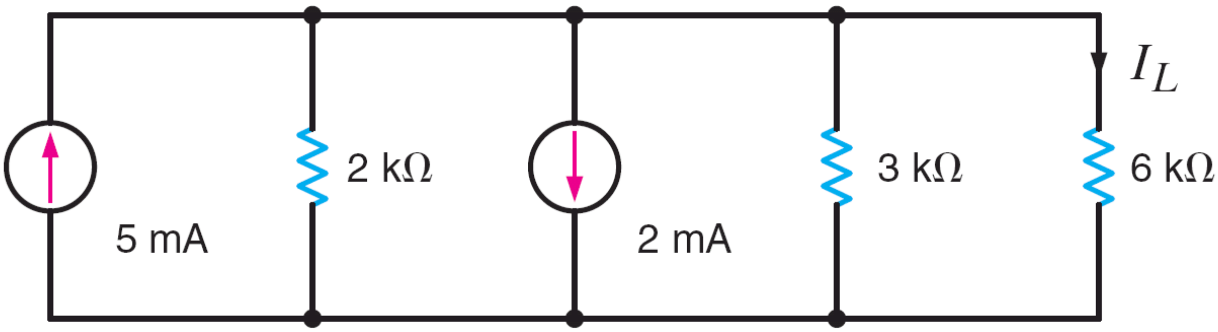
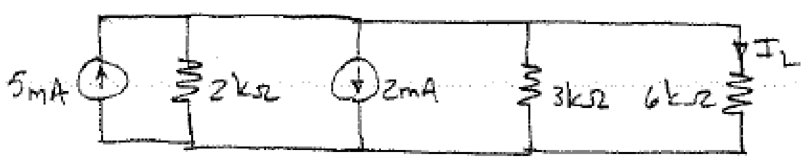


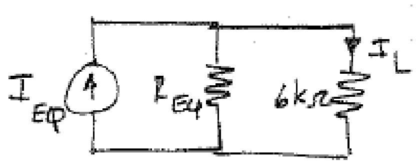
Figure P2.38

SOLUTION:

2.38 Find I_L.



⇓



$$I_{EQ} = 5\times10^{-3} - 2\times10^{-3} = 3\,mA$$

$$R_{EQ} = 2000 \,//\, 3000 = 1200\,\Omega$$

Current division: $I_L = I_{EQ}\left[\dfrac{R_{EQ}}{R_{EQ}+6000}\right]$ $\boxed{I_L = 0.5\,mA}$

2.39 Find V_o in the circuit in Fig. P2.39. **CS**

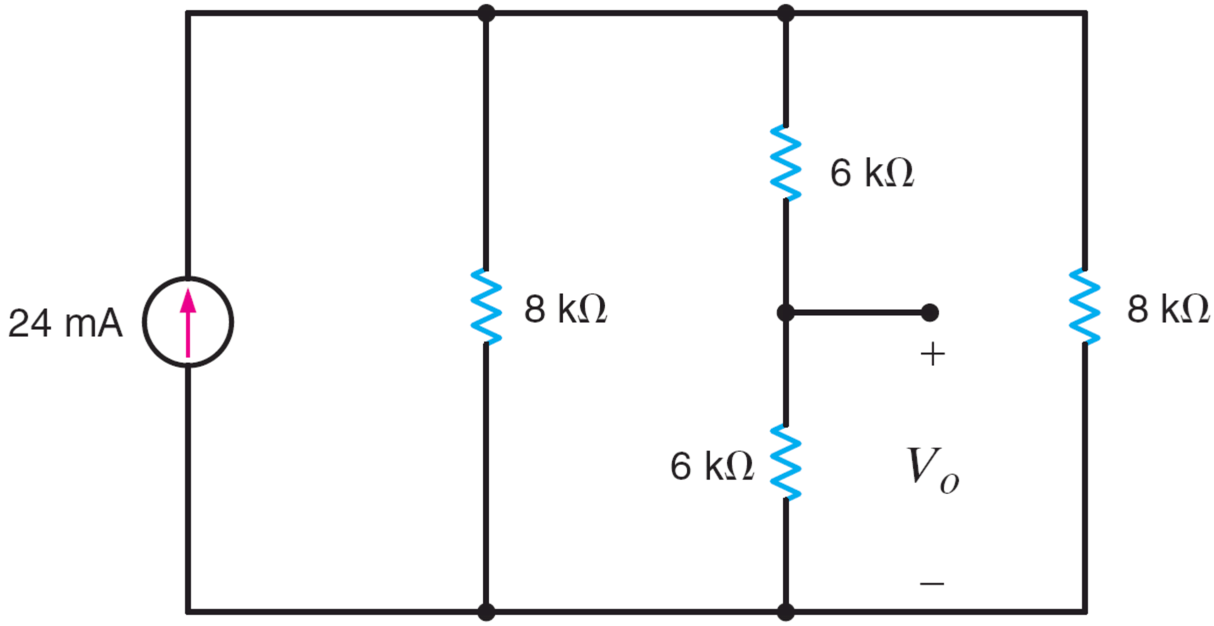
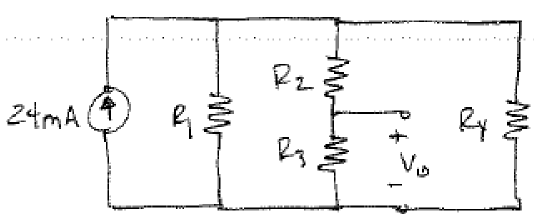


Figure P2.39

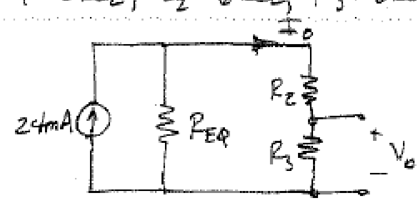
SOLUTION:

2.39 Find V_o

$R_1 = 8k\Omega$, $R_2 = 6k\Omega$, $R_3 = 6k\Omega$, $R_4 = 8k\Omega$



$\Rightarrow$



$R_{EQ} = R_1 /\!/ R_4 = 4k\Omega$

By current division: $I_o = 24\times10^{-3}\left[\dfrac{R_{EQ}}{R_{EQ}+(R_2+R_3)}\right] = 6mA$

$V_o = R_3 I_o$ $\quad\boxed{V_o = 36V}$

2.40 Find I_o in the network in Fig. P2.40. **PSV**

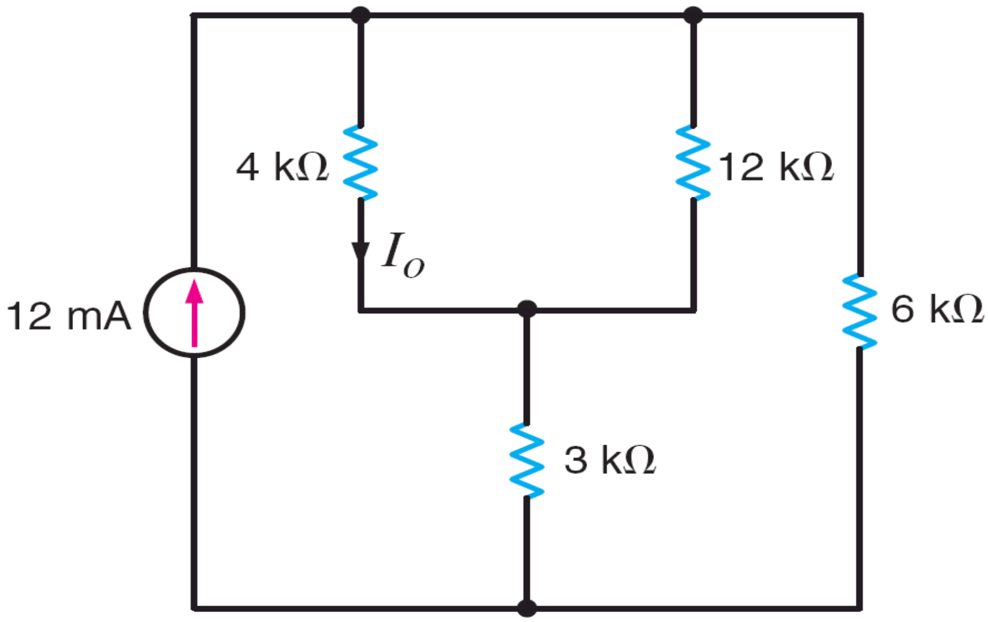
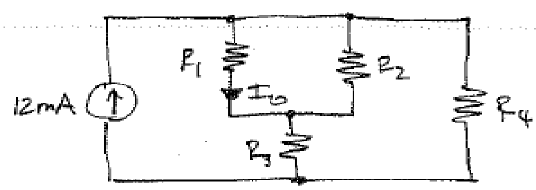


Figure P2.40

SOLUTION:

2.40 Find I_o

$R_1 = 4k\Omega$, $R_2 = 12k\Omega$, $R_3 = 3k\Omega$, $R_4 = 6k\Omega$

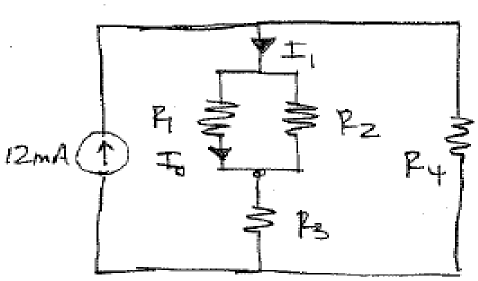


$\Rightarrow$

$$I_1 = 12\times10^{-3}\left[\frac{R_4}{R_4+R_{EQ1}}\right] = 6\,mA$$

Isolate R_1 & R_2

$$R_{EQ1} = (R_1 /\!/ R_2) + R_3 = 6k\Omega$$



Current division: $I_o = I_1\left[\dfrac{R_2}{R_1+R_2}\right]$

$$\boxed{I_o = 4.5mA}$$

2.41 Find V_o in the network in Fig. P2.41.

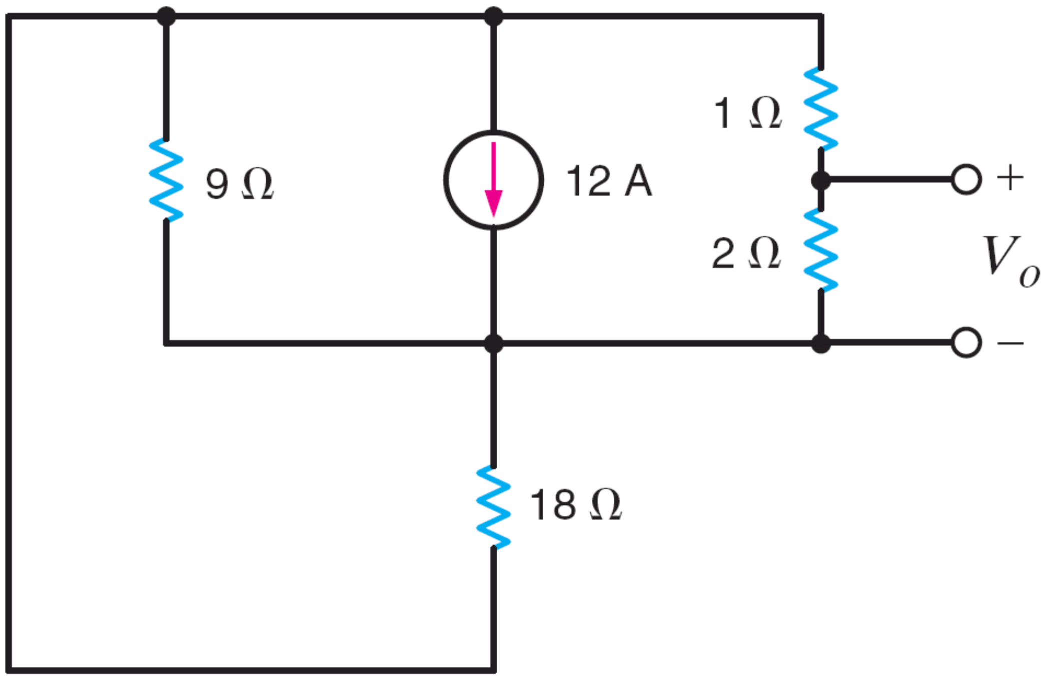
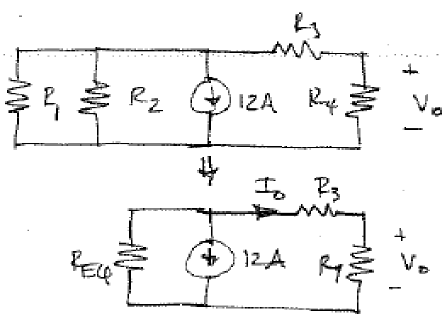


Figure P2.41

SOLUTION:

2.41 Find V_o (circuit is redrawn for readability)



$R_1 = 18\Omega,\ R_2 = 9\Omega,\ R_3 = 1\Omega,\ R_4 = 2\Omega$

$R_{EQ} = R_1 // R_2 = 6\Omega$

Current division: $I_o = -12\left[\dfrac{R_{EQ}}{R_{EQ} + (R_3 + R_4)}\right]$

$I_o = -8A$

$V_o = I_o R_4$ $\quad \boxed{V_o = -16V}$

2.42 In the network in Fig. P2.42, $P_{6\,k\Omega} = 96$ mW. Find I_S.

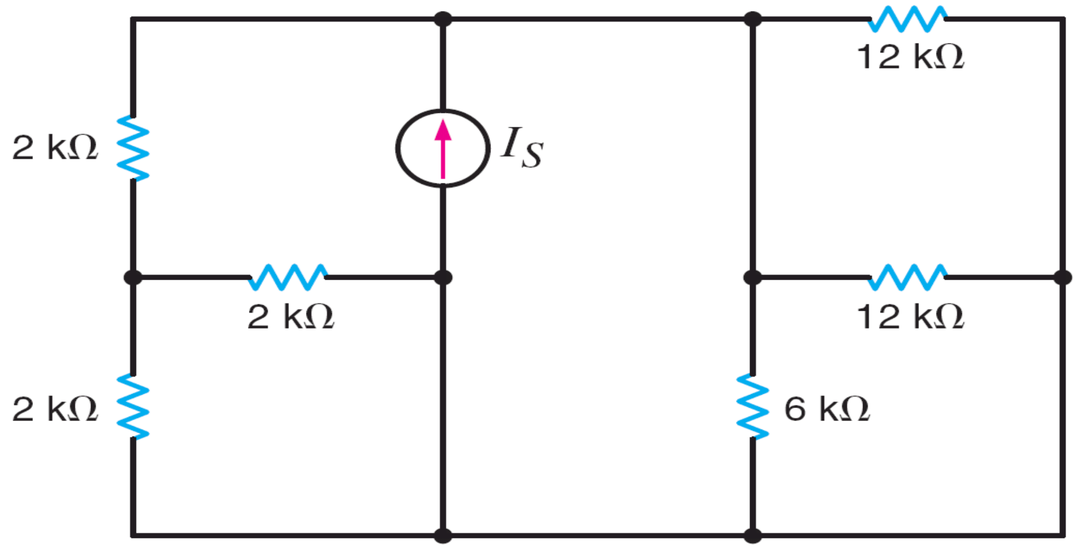
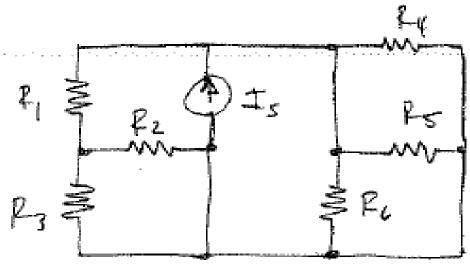


Figure P2.42

SOLUTION:

2.42 $P_{R6} = 96$mW. Find I_S.

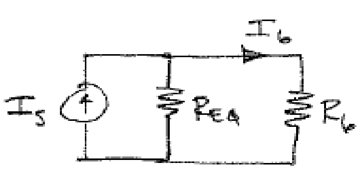
$R_1 = R_2 = R_3 = 2k\Omega \quad R_4 = R_5 = 12\,k\Omega \quad R_6 = 6\,k\Omega$



$\Rightarrow$

$R_{EQ1} = (R_3 // R_2) + R_1 = 3k\Omega$

$R_{EQ2} = R_4 // R_5 = 6k\Omega$



$R_{EQ} = R_{EQ1} // R_{EQ2} = 2k\Omega$

Current division: $I_6 = I_S \left[\frac{R_{EQ}}{R_{EQ} + R_6} \right] = \frac{I_S}{4}$

Also,

$P_{R_6} = I_6^2 R_6 = 96$mW $\qquad I_6 = 4$mA $\Rightarrow \boxed{I_S = 16mA}$

2.43 In the circuit in Fig. P2.43, $V_x = 12$ V. Find V_S.

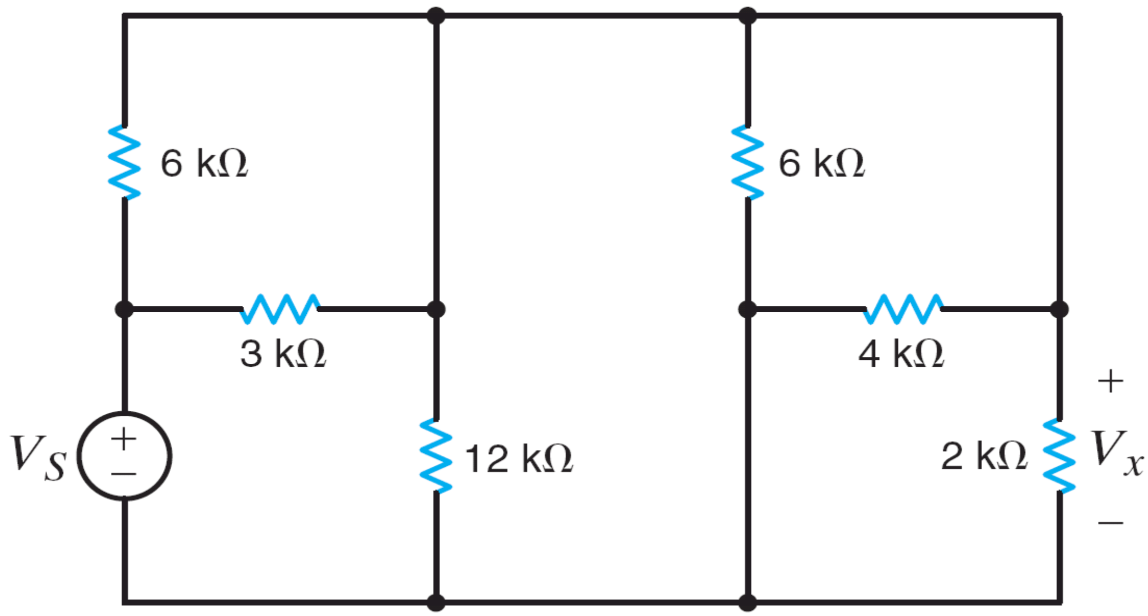
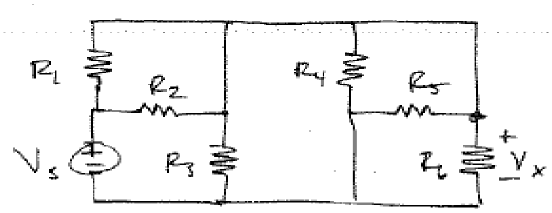


Figure P2.43

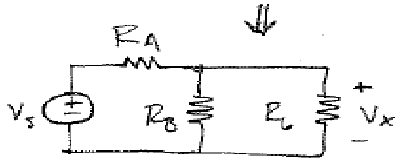
SOLUTION:

2.43 $V_x = 12V$, Find V_s.



$R_1 = R_4 = 6k\Omega$ $R_2 = 3k\Omega$ $R_3 = 12k\Omega$

$R_5 = 4k\Omega$ $R_6 = 2k\Omega$



$R_A = R_1 // R_2 = 2k\Omega$ $R_B = R_3 // R_4 // R_5 = 2k\Omega$

$R_C = R_B // R_6 = 1k\Omega$

Voltage ÷ : $V_x = V_s\left[\frac{R_C}{R_A + R_C}\right]$

$\boxed{V_s = 36V}$

2.44 In the circuit in Fig. P2.44, $V_x = 6$ V. Find I_S.

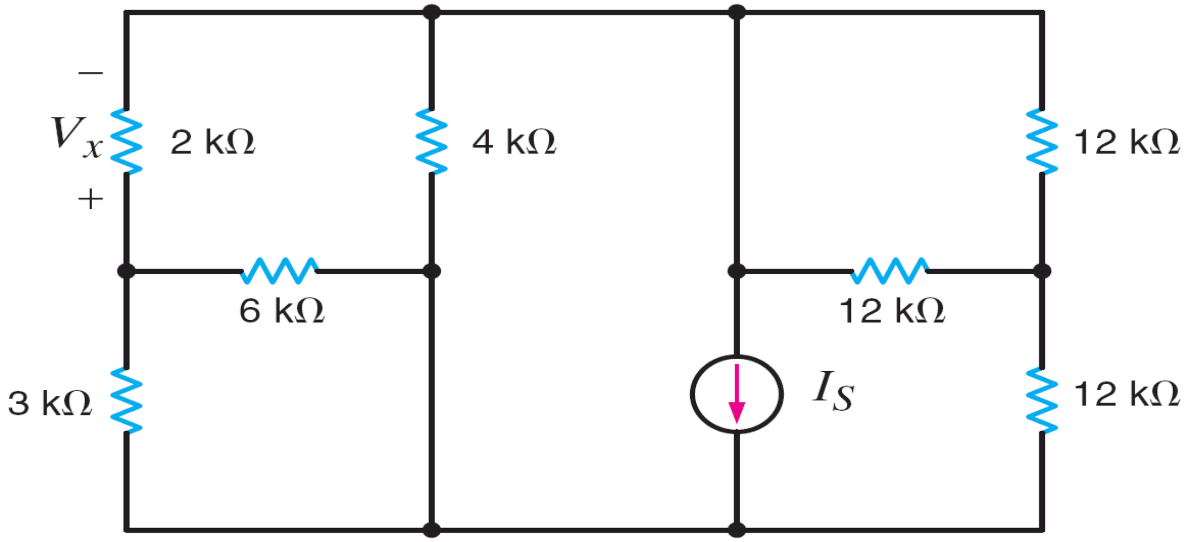
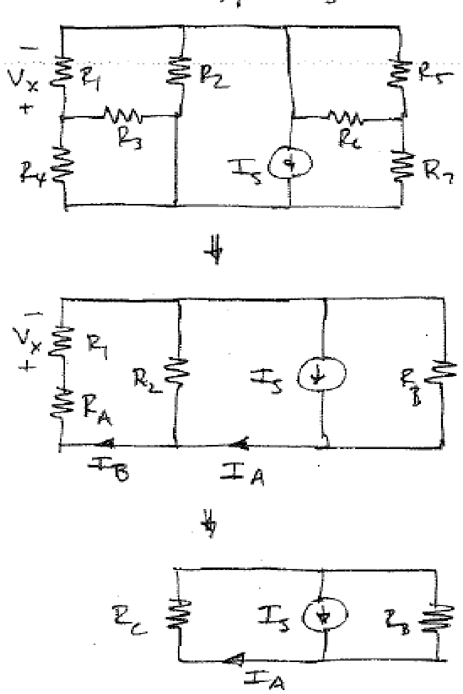


Figure P2.44

SOLUTION:

2.44. $V_x = 6\text{V}$, find I_s.



$R_1 = 2\text{k}\Omega$, $R_2 = 4\text{k}\Omega$, $R_3 = 6\text{k}\Omega$, $R_4 = 3\text{k}\Omega$

$R_5 = R_6 = R_7 = 12\text{k}\Omega$

$R_A = R_3 /\!/ R_4 = 2\text{k}\Omega$ $\quad R_B = R_7 + (R_5 /\!/ R_6) = 18\text{k}\Omega$

Current ÷ : $I_B = I_A\left[\dfrac{R_2}{R_2 + (R_A + R_1)}\right]$

$I_B = \dfrac{V_x}{R_1} = 3\text{mA}$ $\quad I_A = 6\text{mA}$

$R_C = R_2 /\!/ (R_A + R_1) = 2\text{k}\Omega$

$I_A = I_S\left[\dfrac{R_B}{R_B + R_C}\right] \Rightarrow \boxed{I_S = 6.67\text{mA}}$

2.45 Determine I_L in the circuit in Fig. P2.45.

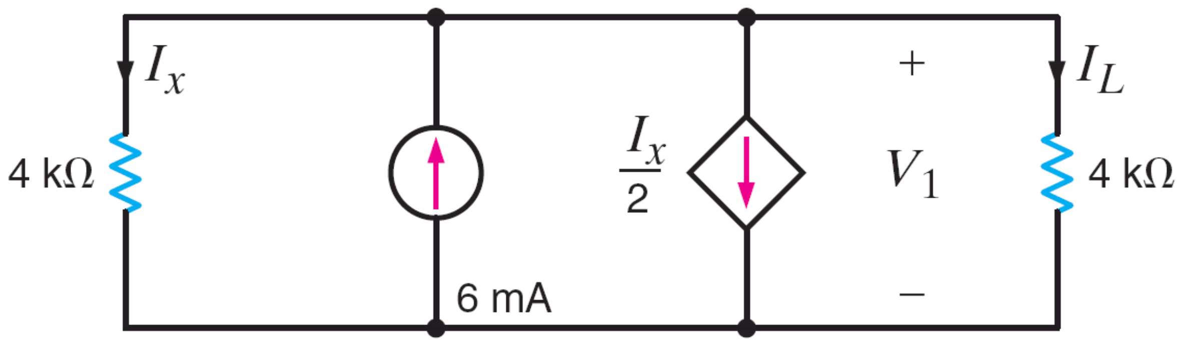
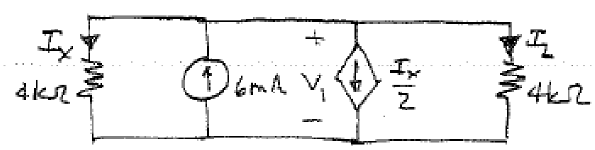


Figure P2.45

SOLUTION:

2.45 Find I_L



KCL: $\frac{V_1}{4\times10^3} + \frac{V_1}{4\times10^3} + \frac{I_x}{2} + (-6\times10^{-3}) = 0$

$I_x = \frac{V_1}{4\times10^3}$ $\quad I_L = \frac{V_1}{4\times10^3}$

$\boxed{I_L = 2.4mA}$

2.46 Determine I_L in the circuit in Fig. P2.46. **PSV**

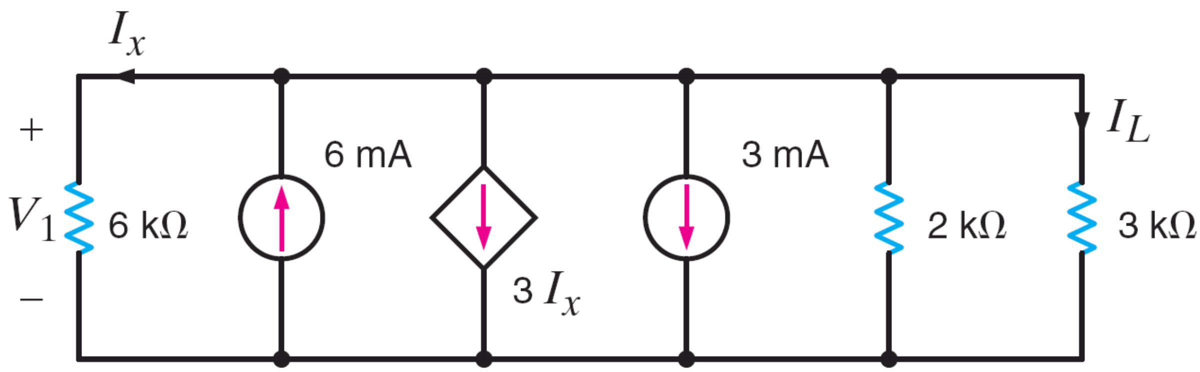
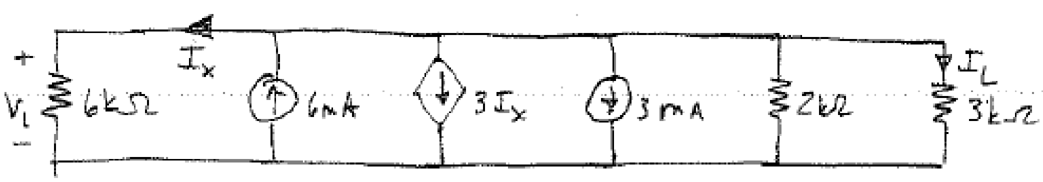


Figure P2.46

SOLUTION:

2.46 Find I_L



KCL: $$\frac{V_1}{6000} + 3I_x + 3\times10^{-3} + \frac{V_1}{2000} + \frac{V_1}{3000} - 6\times10^{-3} = 0 \qquad (1)$$

Also: $$I_x = \frac{V_1}{6000} \qquad (2)$$

Substitute (2) into (1) $\Rightarrow$ $V_1 = 2V$

$$I_L = \frac{V_1}{3000} \qquad \boxed{I_L = \frac{2}{3}\,mA}$$

2.47 Find R_{AB} in the circuit in Fig. P2.47. CS

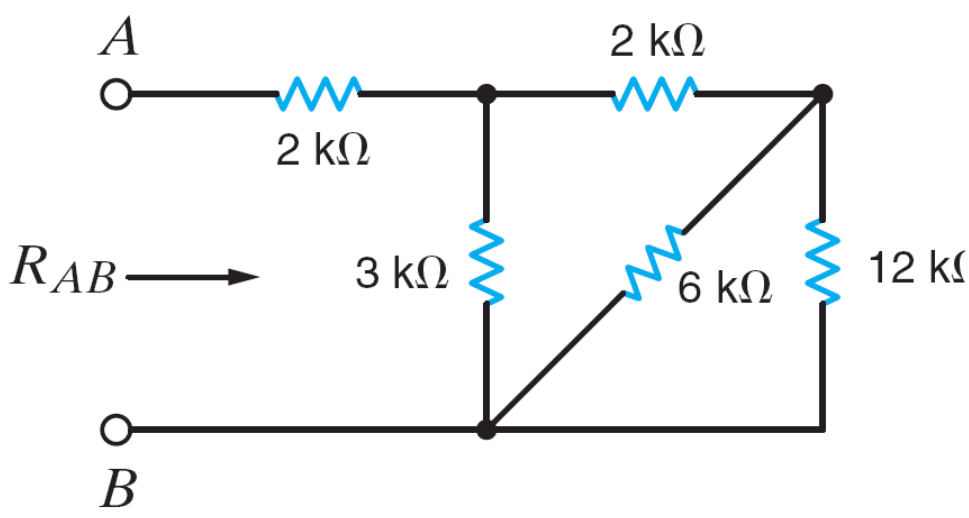
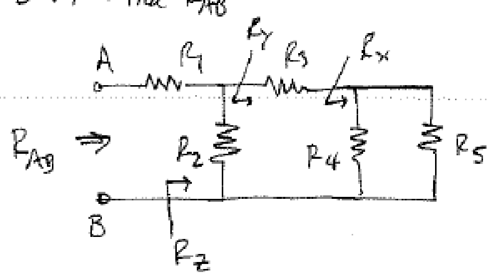


Figure P2.47

SOLUTION:

2.47 Find R_{AB}



$R_1 = R_3 = 2k\Omega \quad R_2 = 3k\Omega \quad R_4 = 6k\Omega \quad R_5 = 12k\Omega$

$R_X = R_Y /\!/ R_5 = 4k\Omega$

$R_Y = R_3 + R_X = 6k\Omega$

$R_Z = R_2 /\!/ R_Y = 2k\Omega$

$R_{AB} = R_1 + R_Z \qquad \boxed{R_{AB} = 4k\Omega}$

2.48 Find R_{AB} in the network in Fig. P2.48.

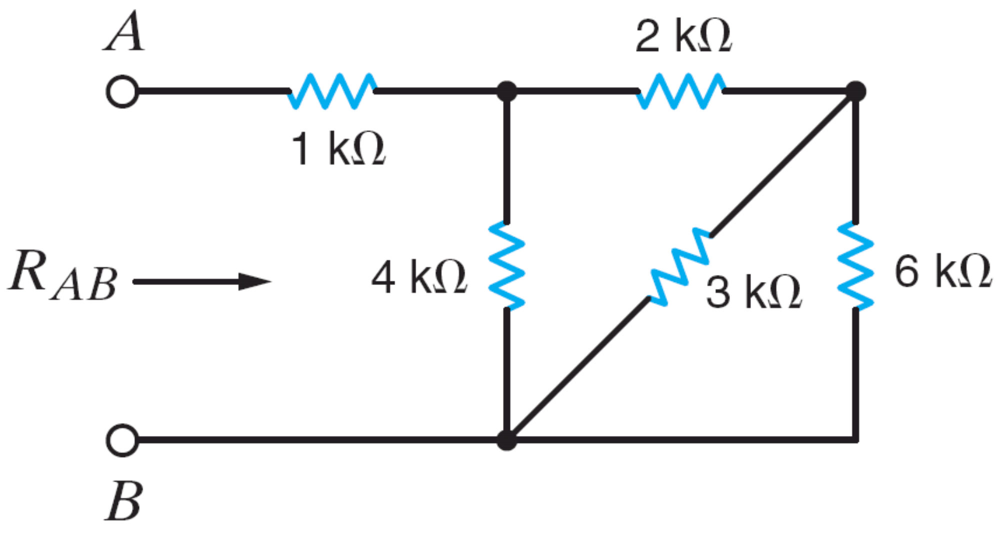
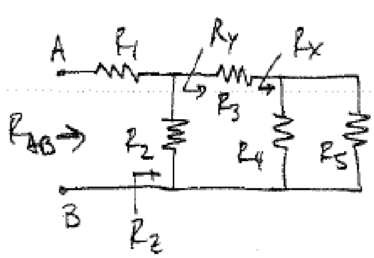


Figure P2.48

SOLUTION:

2.48 Find R_{AB}



$R_1 = 1k\Omega$, $R_2 = 4k\Omega$, $R_3 = 2k\Omega$ $R_4 = 3k\Omega$ $R_5 = 6k\Omega$

$R_X = R_4 // R_5 = 2k\Omega$

$R_Y = R_3 + R_X = 4k\Omega$

$R_Z = R_2 // R_Y = 2k\Omega$

$R_{AB} = R_1 + R_Z \Rightarrow \boxed{R_{AB} = 3k\Omega}$

2.49 Find R_{AB} in the circuit in Fig. P2.49.

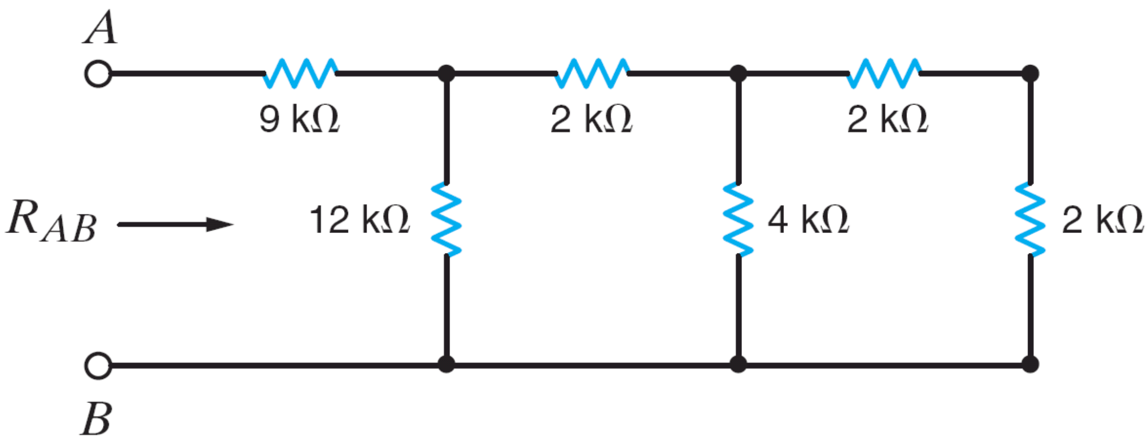
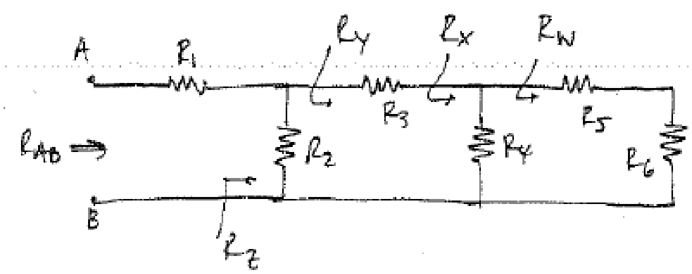


Figure P2.49

SOLUTION:

2.49 Find R_{AB}.



$R_1 = 9k\Omega$ $R_2 = 12k\Omega$ $R_3 = 2k\Omega$

$R_4 = 4k\Omega$ $R_5 = 2k\Omega$ $R_6 = 2k\Omega$

$R_W = R_5 + R_6 = 4k\Omega$

$R_X = R_4 // R_W = 2k\Omega$

$R_Y = R_3 + R_X = 4k\Omega$ $\quad R_Z = R_2 // R_Y = 3k\Omega$

$R_{AB} = R_1 + R_Z \Rightarrow \boxed{R_{AB} = 12k\Omega}$

2.50 Find R_{AB} in the circuit in Fig. P2.50. **PSV**

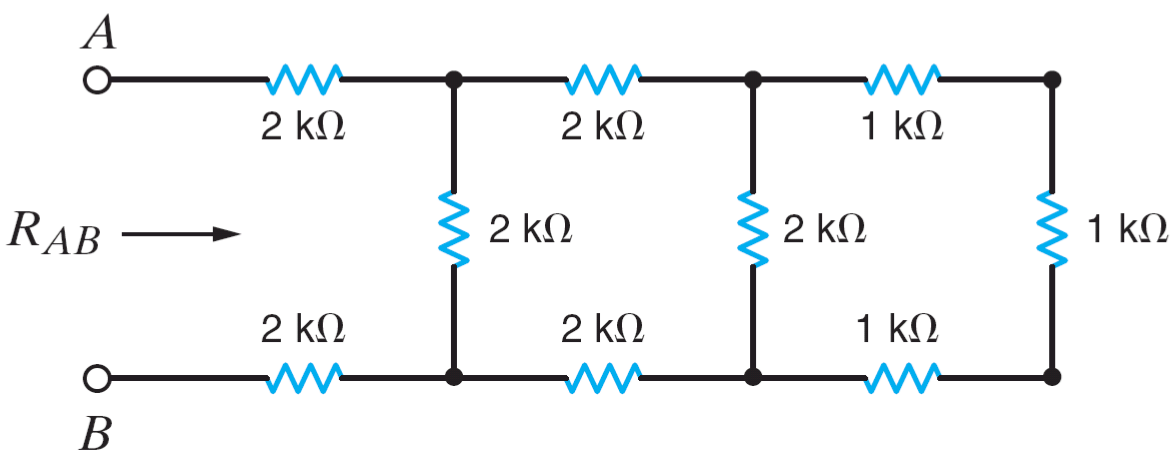
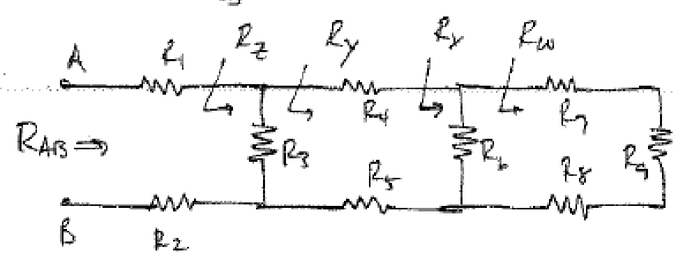


Figure P2.50

SOLUTION:

2.50 Find R_{AB}



$R_1 = R_2 = R_3 = R_4 = R_5 = R_6 = 2k\Omega$

$R_7 = R_8 = R_9 = 1k\Omega$

$R_w = R_7 + R_8 + R_9 = 3k\Omega$

$R_x = R_6 // R_w = \frac{6}{5} k\Omega = 1.2k\Omega$

$R_y = R_4 + R_x + R_5 = 5.2k\Omega$

$R_z = R_3 // R_y = 1.44k\Omega$

$R_{AB} = R_1 + R_z + R_2 = 5.44k\Omega$

$\boxed{R_{AB} = 5.44k\Omega}$

2.51 Determine R_{AB} in the circuit in Fig. P2.51.

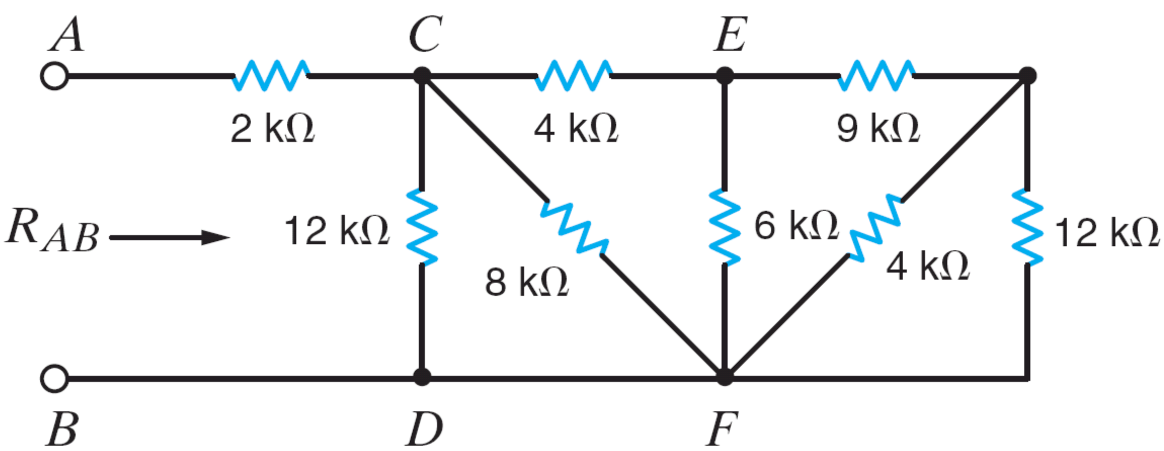
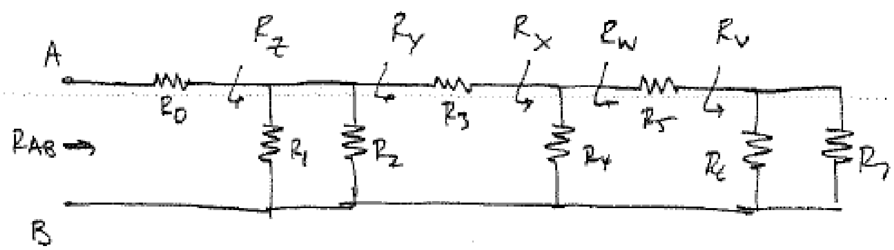


Figure P2.51

SOLUTION:

2.51 Find R_{AB}



$R_0 = 2k\Omega$, $R_1 = 12k\Omega$, $R_2 = 8k\Omega$, $R_3 = 4k\Omega$, $R_4 = 6k\Omega$, $R_5 = 9k\Omega$

$R_6 = 4k\Omega$ $\quad R_7 = 12k\Omega$

$R_V = R_6 /\!/ R_7 = 3k\Omega$ $\quad R_W = R_5 + R_V = 12k\Omega$ $\quad R_X = R_4 /\!/ R_W = 4k\Omega$

$R_Y = R_3 + R_X = 8k\Omega$ $\quad R_Z = R_1 /\!/ R_2 /\!/ R_Y = 3k\Omega$

$R_{AB} = R_0 + R_Z$ $\quad \boxed{R_{AB} = 5k\Omega}$

2.52 Find R_{AB} in the network in Fig. P2.52.

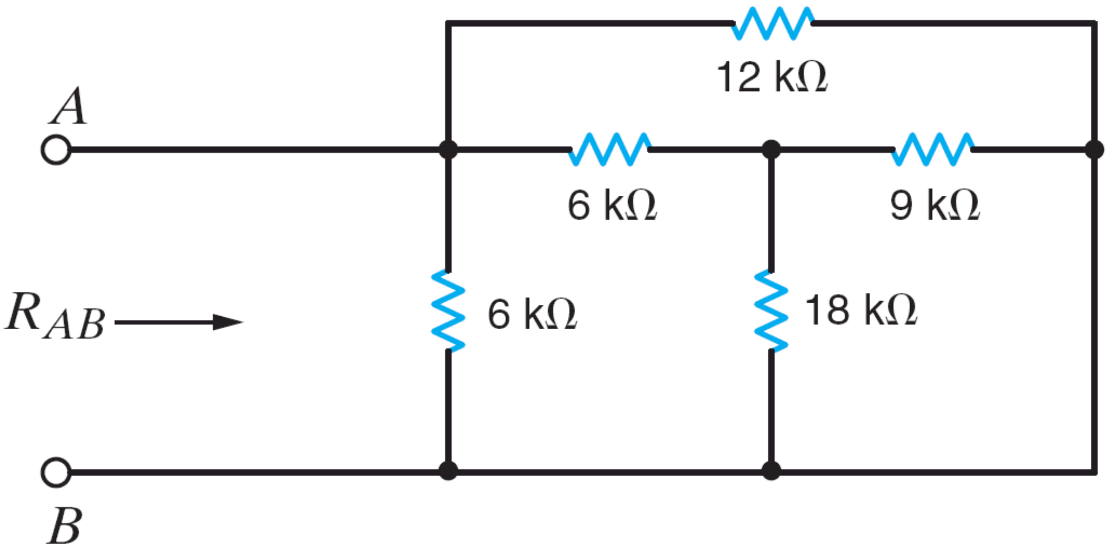
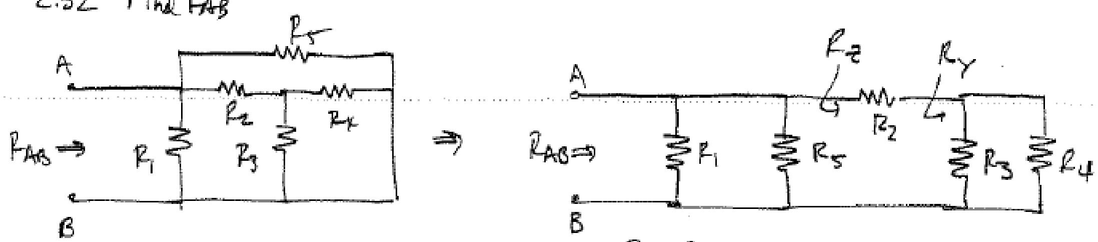


Figure P2.52

SOLUTION:

2.52 Find R_{AB}



$R_1 = R_2 = 6k\Omega$ $R_3 = 18k\Omega$ $R_4 = 9k\Omega$ $R_5 = 12k\Omega$

$R_Y = R_3 // R_4 = 6\,k\Omega$ $R_Z = R_2 + R_Y = 12k\Omega$

$R_{AB} = R_1 // R_5 // R_Z$ $\boxed{R_{AB} = 3\,k\Omega}$

2.53 Find R_{AB} in the network in Fig. P2.53. CS

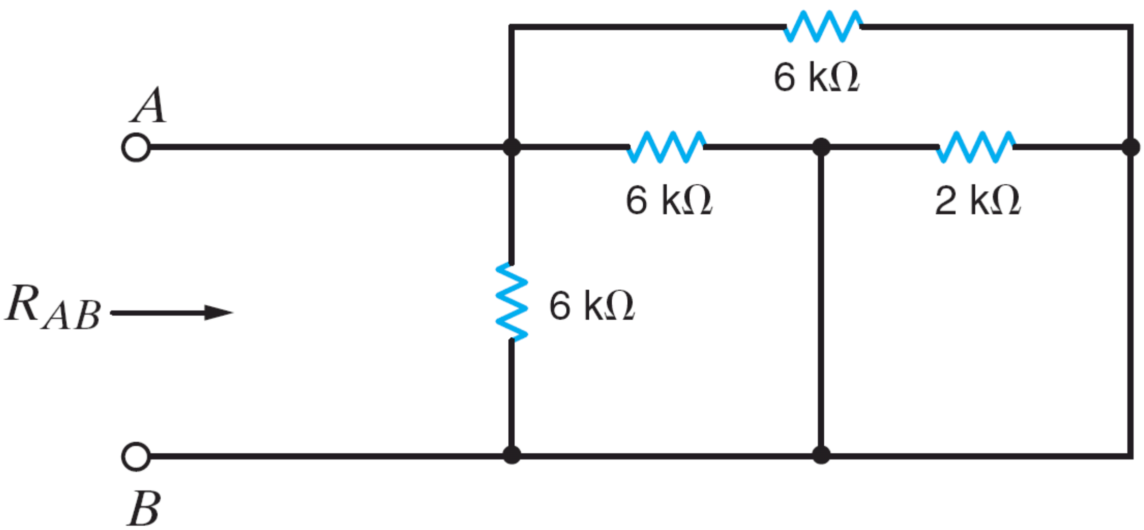
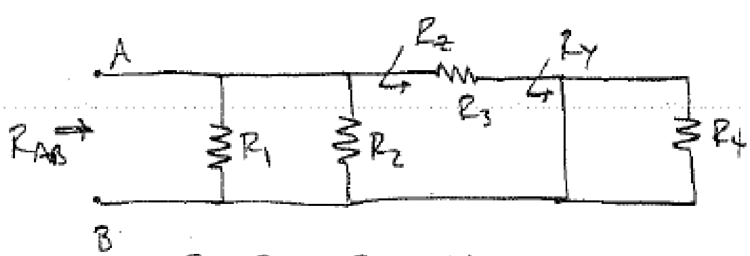


Figure P2.53

SOLUTION:

2.53 Find R_{AB} (circuit is redrawn)



$R_1 = R_2 = R_3 = 6k\Omega \quad R_4 = 2k\Omega$

$R_Y = 0 // R_4 = 0\Omega$

$R_Z = R_3 + R_Y = 6k\Omega$

$R_{AB} = R_1 // R_2 // R_Z$

$\boxed{R_{AB} = 2k\Omega}$

2.54 Find the equivalent resistance, R_{eq}, in the circuit in Fig. P2.54.

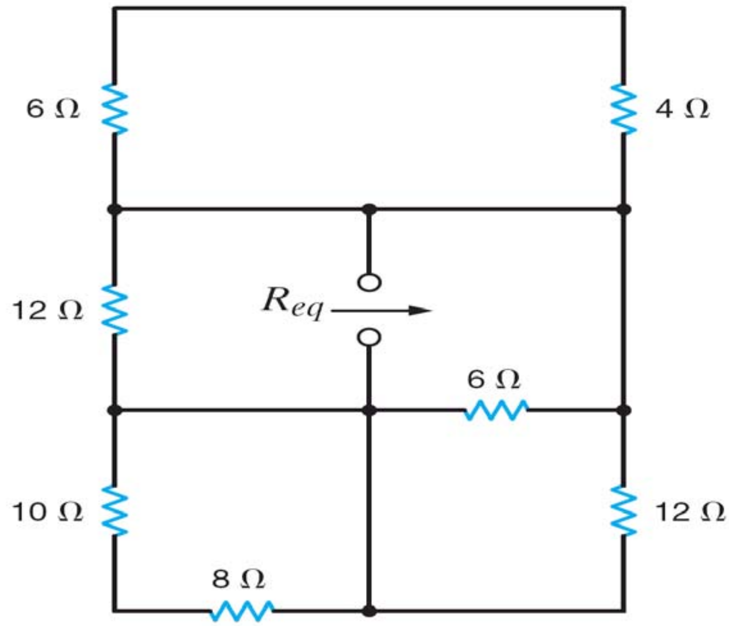
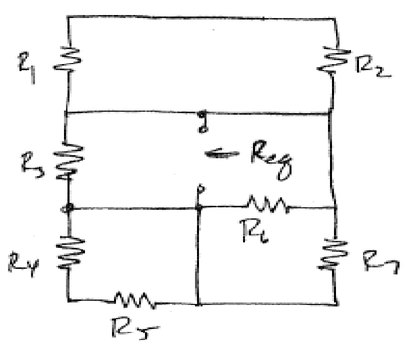


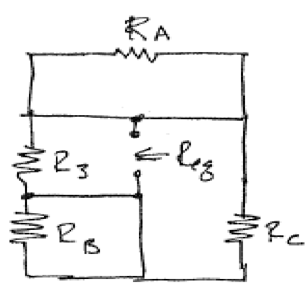
Figure P2.54

SOLUTION:

2.54 Find R_{eq}



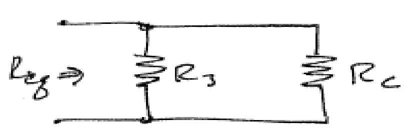
$\Rightarrow$



Note: R_A is shorted as is R_B !

$R_A = R_1 + R_2 \quad R_B = R_4 + R_5$

$R_C = R_6 \,//\, R_7$



$R_{eq} = R_3 \,//\, R_C = R_3 \,//\, R_6 \,//\, R_7$

Given: $R_3 = 12\Omega$, $R_6 = 6\Omega$ & $R_7 = 12\Omega$, $\boxed{R_{eq} = 3\Omega}$

2.55 Find the equivalent resistance, R_{eq}, in the network in Fig. P2.55.

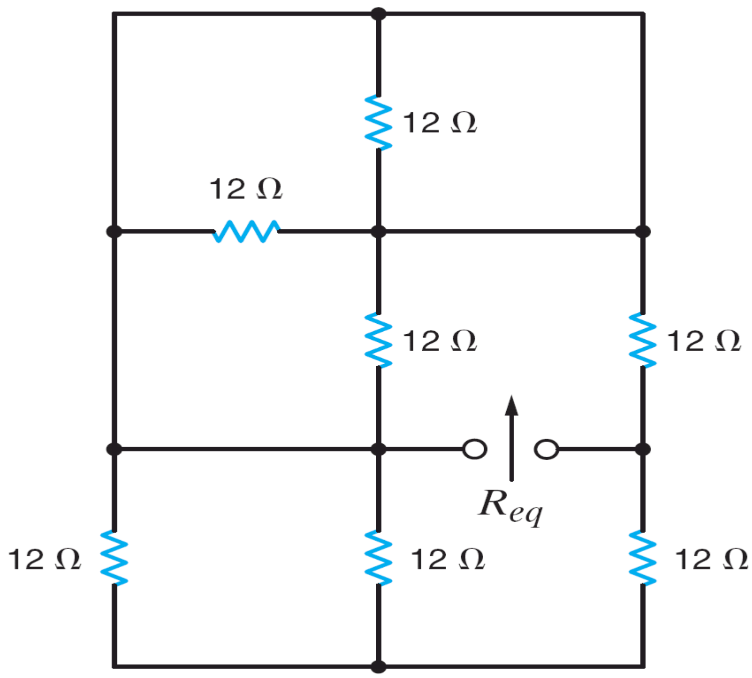
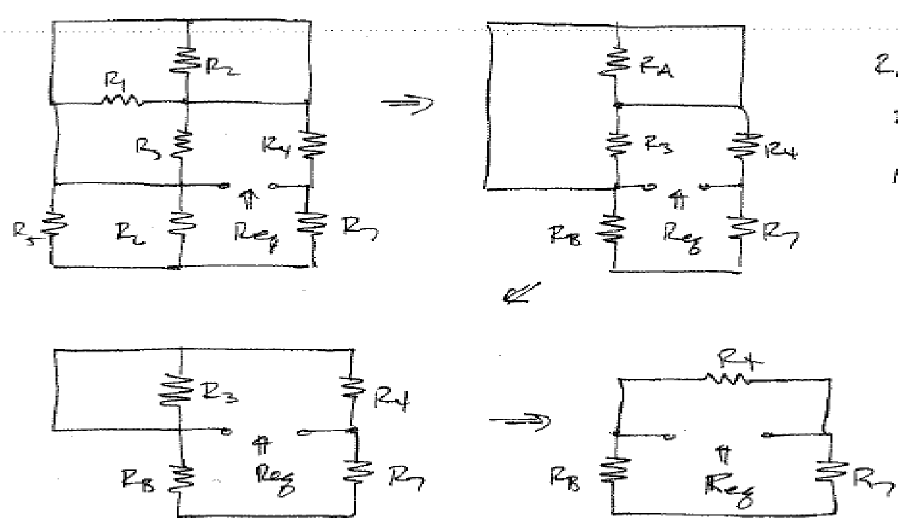


Figure P2.55

SOLUTION:



$R_A = R_1 // R_2$

$R_B = R_5 // R_6$

Note: R_A is shorted

R_3 is also shorted

$R_{eq} = R_4 // (R_B + R_7)$

$\boxed{R_{eq} = 7.2\ \Omega}$

2.56 Find the range of resistance for the following resistors.

(a) 1 kΩ with a tolerance of 5%

(b) 470 Ω with a tolerance of 2%

(c) 22 kΩ with a tolerance of 10%

SOLUTION:

2.56 a) $R = 1\,k\Omega$ @ $\pm 5\%$

b) $R = 470\,\Omega$ @ $\pm 2\%$

c) $R = 22\,k\Omega$ @ $+10\%$

Solution

a) $R_{min} = R\,(1 - tol.) = 1000\,(0.95) = 950\,\Omega$

$R_{max} = R\,(1 + tol.) = 1000\,(1.05) = 1050\,\Omega$

b) $R_{min} = 470\,(0.98) = 460.6\,\Omega$

$R_{max} = 470\,(1.02) = 479.4\,\Omega$

c) $R_{min} = 22\times10^3(0.9) = 19.8\,k\Omega$

$R_{max} = 22\times10^3(1.1) = 24.2\,k\Omega$

2.57 Given the network in Fig. P2.57, find the possible range of values for the current and power dissipated by the following resistors. **CS**

(a) 390 Ω with a tolerance of 1%

(b) 560 Ω with a tolerance of 2%

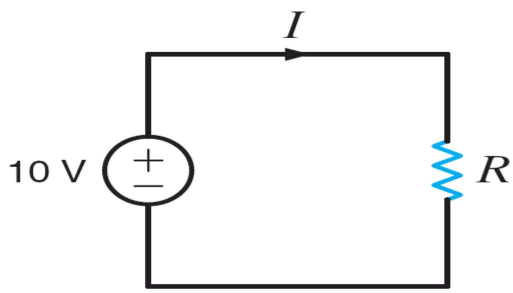
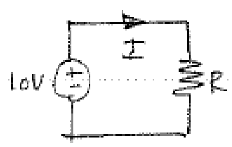


Figure P2.57

SOLUTION:

2.57 Find I and P ranges.



a) $R = 390\,\Omega$ @ $\pm 1\%$

$$I = \frac{10}{R} \qquad I_{max} = \frac{10}{R_{min}} \qquad I_{min} = \frac{10}{R_{max}}$$

$$R_{min} = 390(0.99) = 386.1\,\Omega \qquad I_{max} = 25.90\,mA$$

$$R_{max} = 390(1.01) = 393.9\,\Omega \qquad I_{min} = 25.39\,mA$$

$$P_{max} = I_{max}(10) \Rightarrow P_{max} = 259.0\,mW$$

$$P_{min} = I_{min}(10) \Rightarrow P_{min} = 253.9\,mW$$

b) $R = 560\,\Omega$ @ $\pm 2\%$

$$R_{min} = 560(0.98) = 548.8\,\Omega$$

$$R_{max} = 560(1.02) = 571.2\,\Omega$$

$I_{max} = 18.22\,mA$	$P_{max} = 182.2\,mW$
$I_{min} = 17.51\,mA$	$P_{min} = 175.1\,mW$

2.58 Given the circuit in Fig. P2.58,

(a) find the required value of R.

(b) use Table 2.1 to select a standard 10% tolerance resistor for R.

(c) calculate the actual value of I.

(d) determine the percent error between the actual value of I and that shown in the circuit.

(e) determine the power rating for the resistor R.

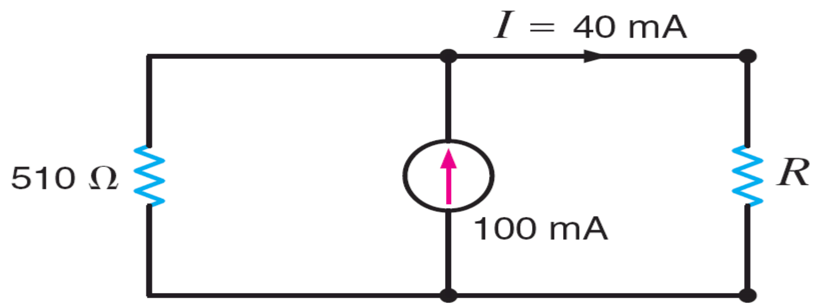
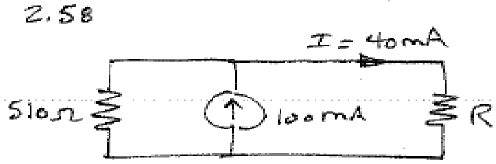


Figure P2.58

SOLUTION:

2.58



a) Find R

$$40\times10^{-3} = 100\times10^{-3}\left[\frac{510}{510+R}\right]$$

$$\boxed{R = 765\Omega}$$

b) From Table 2.1, best 10% tolerance choice is $\boxed{R = 820\Omega}$

c) $I = 10^{-1}\left[\frac{510}{510+820}\right]$ $\boxed{I = 38.3\text{mA}}$

d) percent error $= \left(\frac{\text{actual} - \text{target}}{\text{target}}\right)100 \Rightarrow$ % error $=$ $\boxed{-4.25\%}$

e) $P_R = I^2R = (38.3\times10^{-3})^2(820) = 1.2\text{W}$

$\boxed{\text{Recommend a 2-W resistor.}}$

2.59 The resistors R_1 and R_2 shown in the circuit in Fig. P2.59 are 1 Ω with a tolerance of 5% and 2 Ω with a tolerance of 10%, respectively.

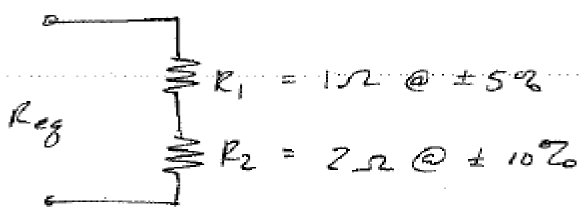
(a) What is the nominal value of the equivalent resistance?

(b) Determine the positive and negative tolerance for the equivalent resistance.

Figure P2.59

SOLUTION:

2.59



a) Nominal value for $R_{eq} = R_1 + R_2$ $\qquad R_{eq} = 3\Omega$

b) $R_{eq\,max} = R_1(1.05) + R_2(1.1) = 3.25\Omega$

$R_{eq\,min} = R_1(0.95) + R_2(0.9) = 2.75\Omega$

$+R_{eq}$ tolerance $= \dfrac{3.25-3}{3} = +8.33\%$

$-R_{eq}$ tolerance $= \dfrac{2.75-3}{3} = -8.33\%$

$\boxed{R_{eq} \text{ tolerance} = \pm 8.33\%}$

2.60 Find V_{ab} and V_{dc} in the circuit in Fig. P2.60.

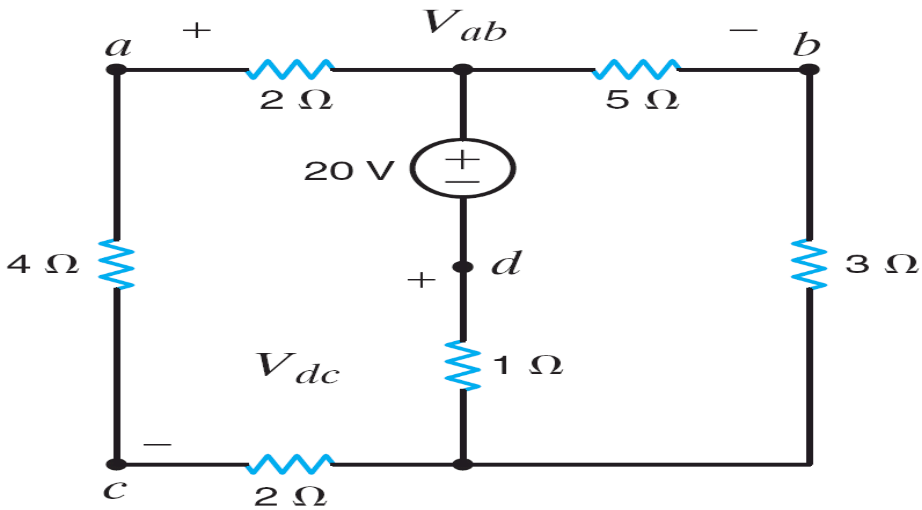
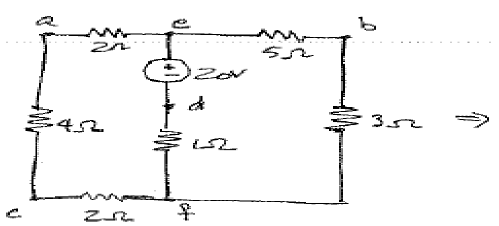
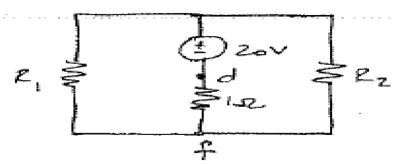


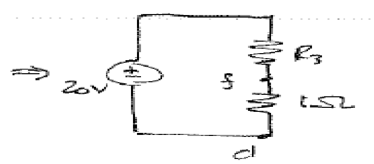
Figure P2.60

SOLUTION:

2.60 Find V_{ab} and V_{dc}







$R_1 = 2+4+2 = 8\Omega$

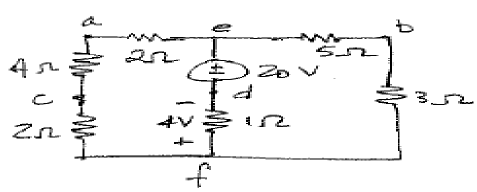
$R_2 = 5+3 = 8\Omega$

$R_3 = R_1 // R_2 = 4\Omega$

$V_{fd} = 20\left[\frac{1}{R_3+1}\right]$

$V_{fd} = 4V$

Now we have



$V_{ef} = 16V$

$V_{cf} = V_{ef}\left[\frac{2}{2+4+2}\right] = 4V$

$V_{dc} = V_{df} + V_{fc} = -4 + (-4) = -8V$

$\boxed{V_{dc} = -8V}$

$V_{ea} = V_{ef}\left[\frac{2}{2+2+4}\right] = 4V$ $\quad V_{eb} = V_{ef}\left[\frac{5}{5+3}\right] = 10V$

$V_{ab} = V_{ae} + V_{eb} = -4 + (10) = 6V$ $\quad \boxed{V_{ab} = 6V}$

2.61 Find I_1 and V_o in the circuit in Fig. P2.61.

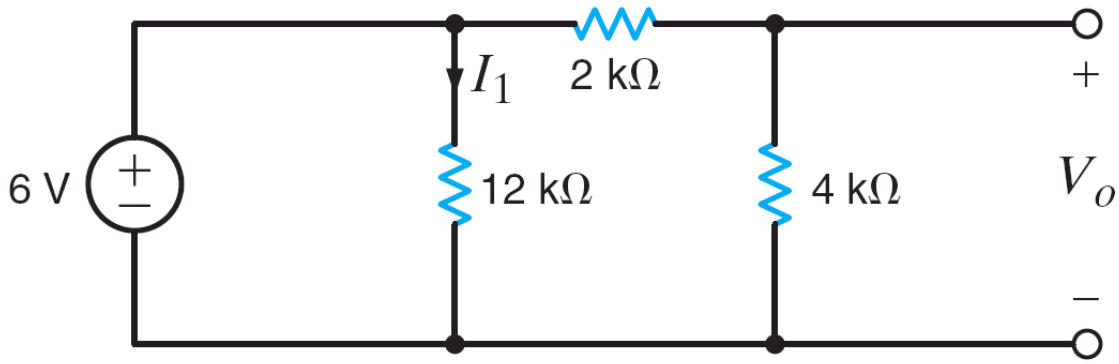
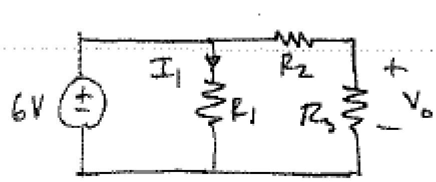


Figure P2.61

SOLUTION:

2.61 Find I_o & V_o



$I_1 = \frac{6}{R_1} \Rightarrow I_1 = 0.5\,mA$

$R_1 = 12\,k\Omega$
$R_2 = 2\,k\Omega$
$R_3 = 4\,k\Omega$

$V_o = 6\left[\frac{R_3}{R_2 + R_3}\right] \Rightarrow \boxed{V_o = 4V}$

2.62 Find I_1 and V_o in the circuit in Fig. P2.62.

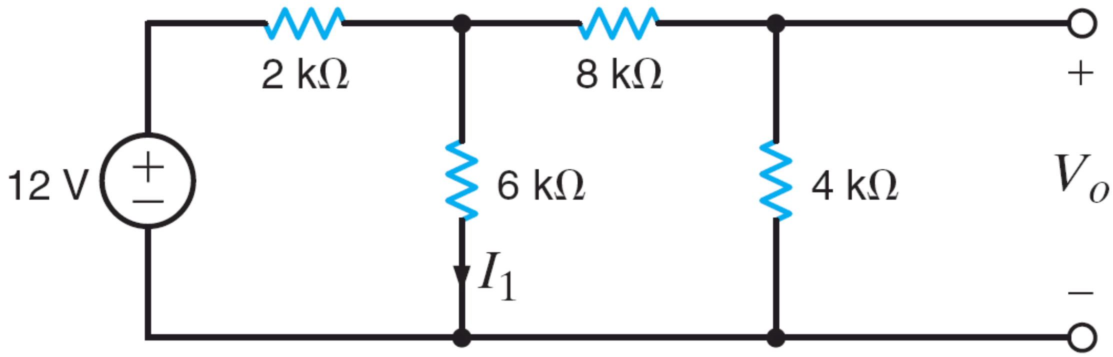
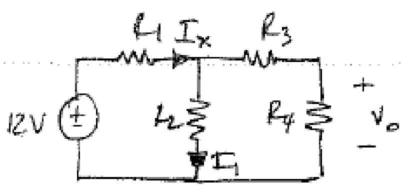


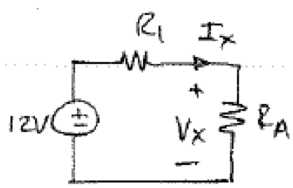
Figure P2.62

SOLUTION:

2.62 Find I_1 & V_o.



$\Rightarrow$



$I_x = \dfrac{12}{R_1 + R_x} = 2mA$

$V_x = 12\left[\dfrac{R_A}{R_A + R_1}\right] = 8V$

$R_1 = 2k\Omega$ $R_2 = 6k\Omega$

$R_3 = 8k\Omega$ $R_4 = 4k\Omega$

$R_A = R_2 // (R_3 + R_4)$

$R_A = 4k\Omega$

By current division: $I_1 = I_x\left[\dfrac{R_4 + R_3}{R_2 + R_4 + R_3}\right] \Rightarrow I_1 = 1.33\ mA$

By voltage division: $V_o = V_x\left[\dfrac{R_4}{R_4 + R_3}\right] \Rightarrow \boxed{V_o = 2.67V}$

2.63 Find I_o in the network in Fig. P2.63.

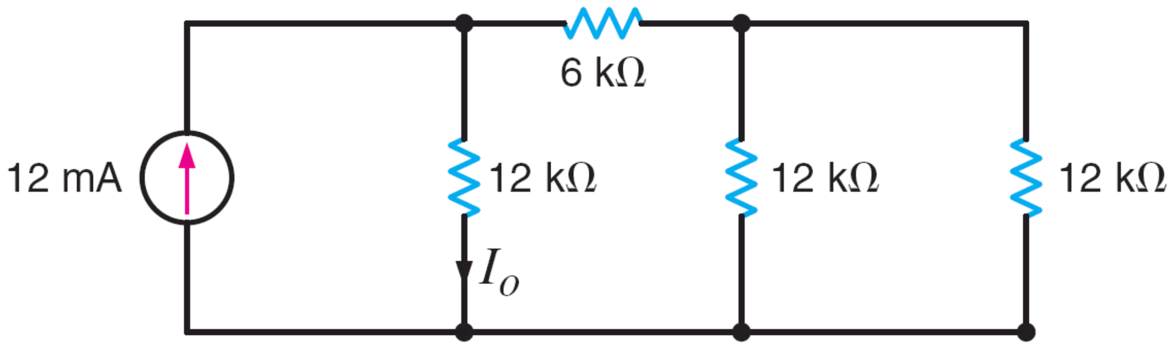
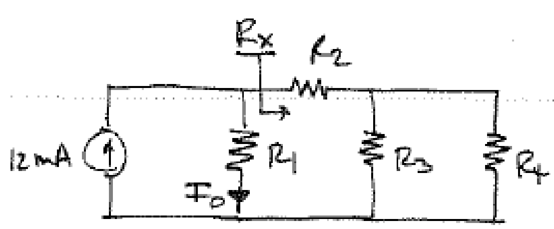


Figure P2.63

SOLUTION:

2.63 Find I_o



$R_1 = 12k\Omega \quad R_2 = 6k\Omega \quad R_3 = R_4 = 12k\Omega$

$$R_x = R_2 + (R_3 // R_4) = 12k\Omega$$

$$I_o = 12\times10^{-3}\left[\frac{R_x}{R_1 + R_x}\right]$$

$$\boxed{I_o = 6mA}$$

2.64 Find I_1 in the circuit in Fig. P2.64.

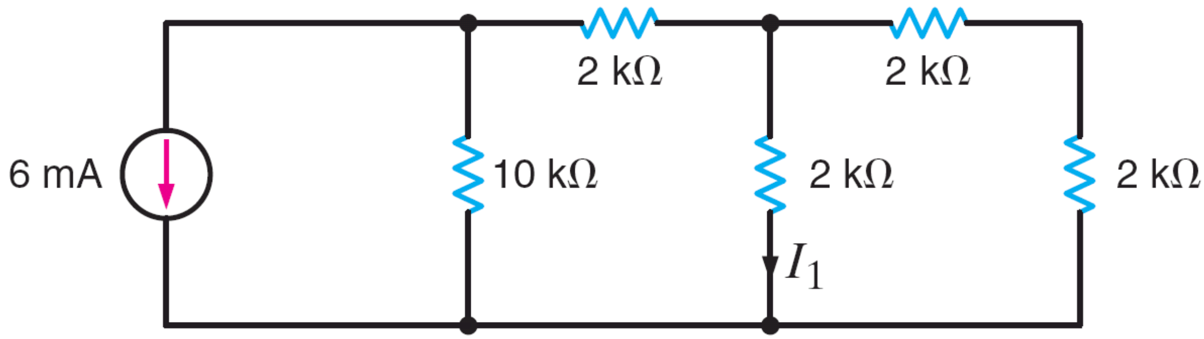
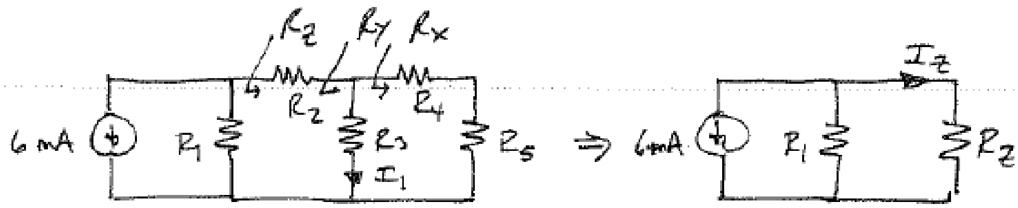


Figure P2.64

SOLUTION:

2.64 Find I_1



$R_1 = 10k\Omega$ $R_2 = R_3 = R_4 = R_5 = 2k\Omega$

$R_x = R_4 + R_5 = 4k\Omega$

$R_y = R_x // R_3 = 1.33k\Omega$

$R_Z = R_2 + R_y = 3.33k\Omega$

$I_1 = I_Z \left[\frac{R_x}{R_x + R_3} \right] \Leftarrow I_Z = -6 \times 10^{-3} \left[\frac{R_1}{R_1 + R_Z} \right] = -4.5mA$

$\boxed{I_1 = -3mA}$

2.65 Determine V_o in the network in Fig. P2.65. **PSV**

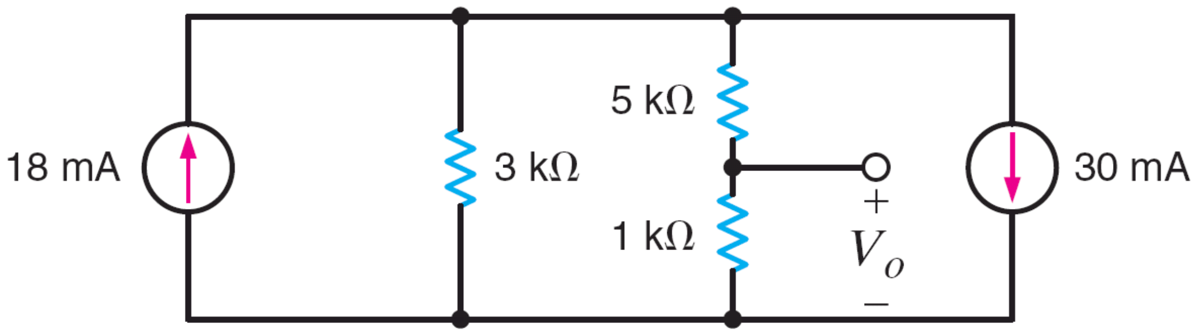
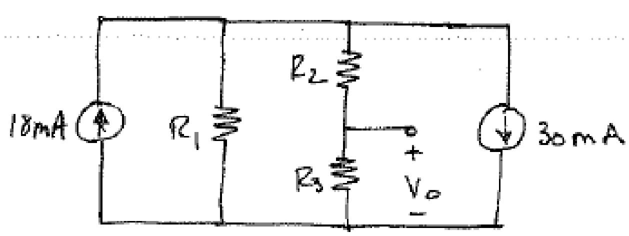


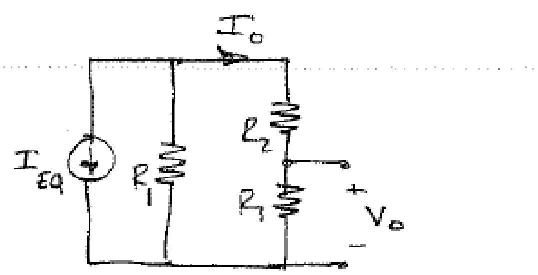
Figure P2.65

SOLUTION:

2.65 Find V_o:



$\Rightarrow$



$R_1 = 3k\Omega \quad R_2 = 5k\Omega \quad R_3 = 1k\Omega$

$I_{EQ} = 30\times10^{-3} - 18\times10^{-3} = 12mA$

$-I_o = I_{EQ}\left[\frac{R_1}{R_1+(R_2+R_3)}\right] \Rightarrow I_o = -4mA \quad V_o = I_oR_3 \quad \boxed{V_o = -4V}$

2.66 Determine I_o in the circuit in Fig. P2.66.

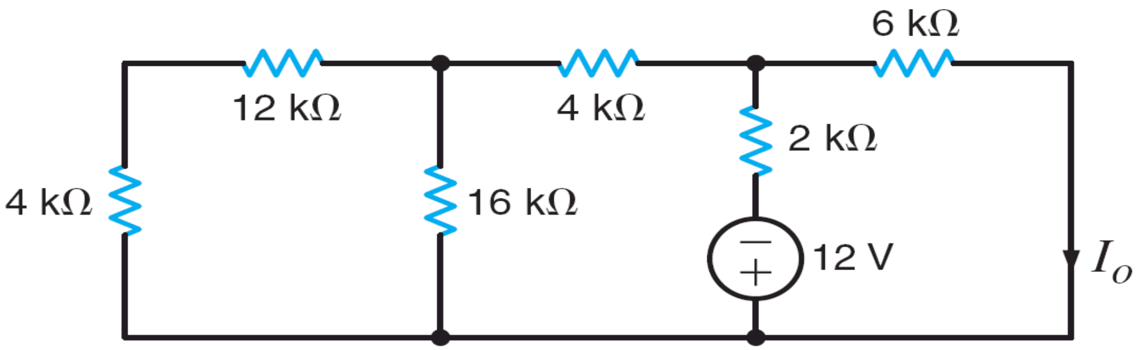
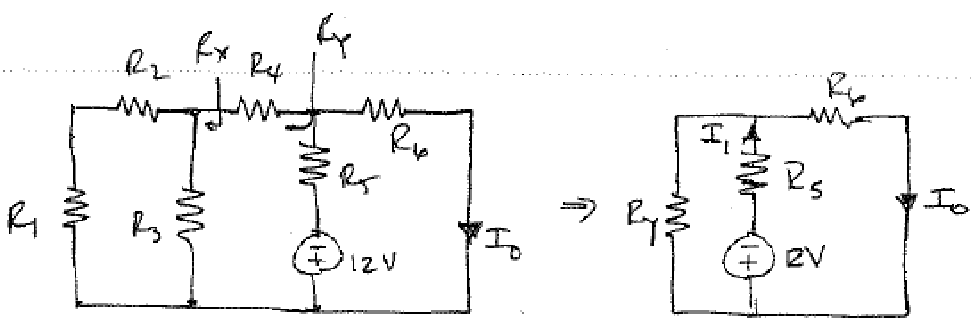


Figure P2.66

SOLUTION:

2.66 Find I_o.

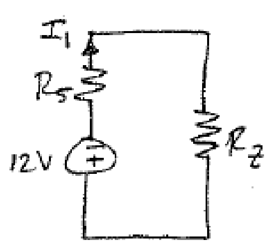


$R_1 = 4k\Omega$ $R_2 = 12k\Omega$ $R_3 = 16k\Omega$

$R_4 = 4k\Omega$ $R_5 = 2k\Omega$ $R_6 = 6k\Omega$

$R_Y = R_4 + R_Z$ $R_Z = (R_1 + R_2) // R_3$

$R_Y = 12k\Omega$ $R_Z = 8k\Omega$



$R_Z = R_Y // R_6 = 4k\Omega$

$I_1 = \dfrac{-12}{R_5 + R_Z} \Rightarrow I = -2mA$

By current division: $I_o = I_1 \left[\dfrac{R_Y}{R_Y + R_6} \right] \rightarrow \boxed{I_o = -1.33mA}$

2.67 Determine V_o in the network in Fig. P2.67.

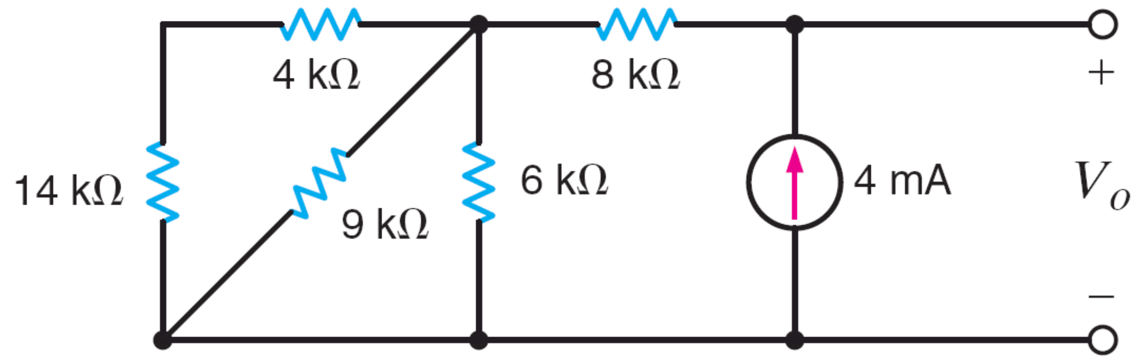
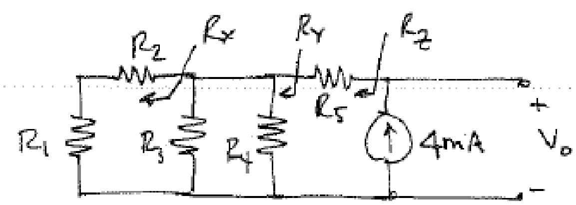


Figure P2.67

SOLUTION:

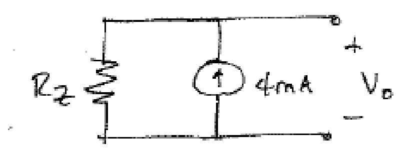
2.67 Find V_o



$R_1 = 14k\Omega$ $R_2 = 4k\Omega$ $R_3 = 9k\Omega$

$R_4 = 6k\Omega$ $R_5 = 8k\Omega$

$R_X = R_1 + R_2 = 18k\Omega$ $R_Y = R_3 // R_4 // R_X = 3k\Omega$ $R_Z = R_5 + R_Y = 11k\Omega$



$V_o = (4\times10^{-3})(R_Z)$ $\boxed{V_o = 44V}$

2.68 Find I_o in the circuit in Fig. P2.68. **CS**

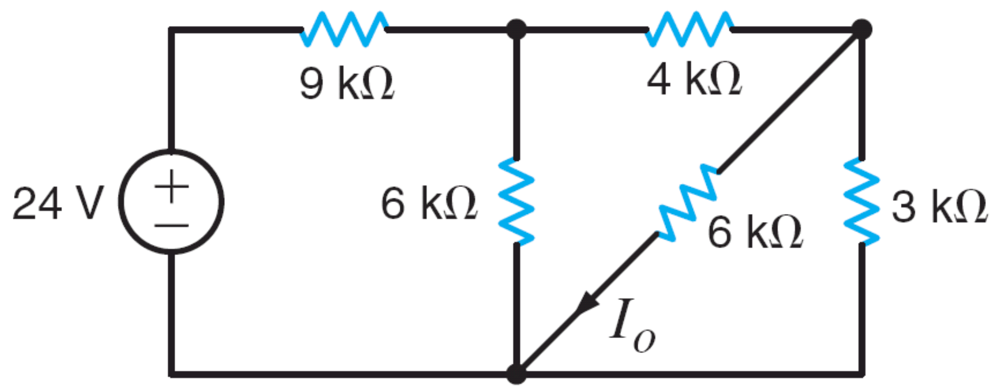
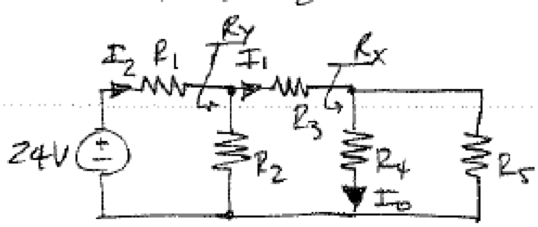


Figure P2.68

SOLUTION:

2.68 Find I_o

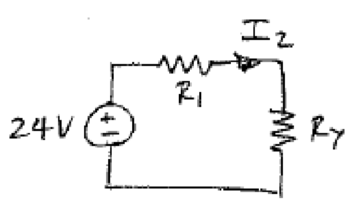


$R_1 = 9\,k\Omega \quad R_2 = 6\,k\Omega \quad R_3 = 4\,k\Omega$

$R_4 = 6\,k\Omega \quad R_5 = 3\,k\Omega$

$$I_1 = I_2\left[\frac{R_2}{R_2 + (R_3 + R_x)}\right] \quad \& \quad I_o = I_1\left[\frac{R_5}{R_4 + R_5}\right]$$

$$R_x = R_4 // R_5 = 2\,k\Omega \qquad R_y = R_2 // [R_3 + R_x] = 3\,k\Omega$$



$$I_2 = \frac{24}{R_1 + R_Y}$$

$I_2 = 2\,mA$

$I_1 = 1\,mA$

$$\boxed{I_o = 0.33\,mA}$$

2.69 Find the value of V_x in the network in Fig. P2.69 such that the 5-A current source supplies 50 W. PSV

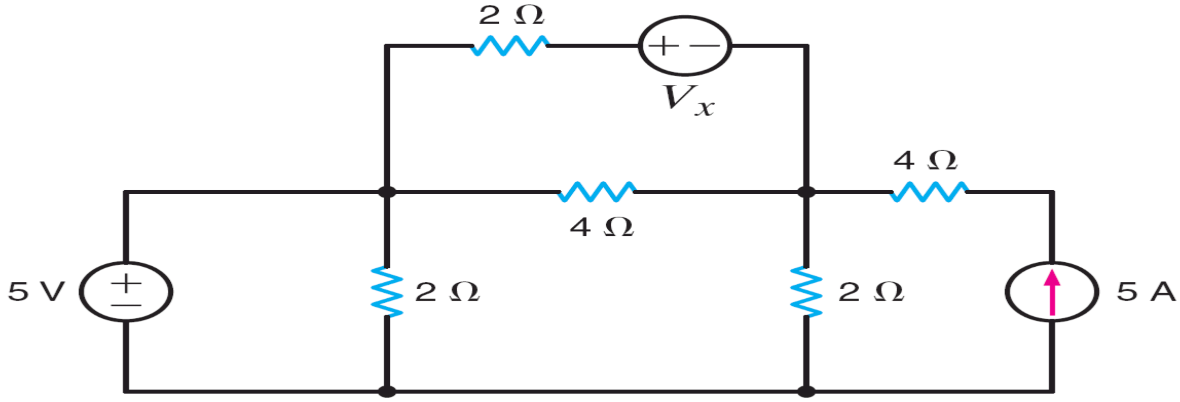
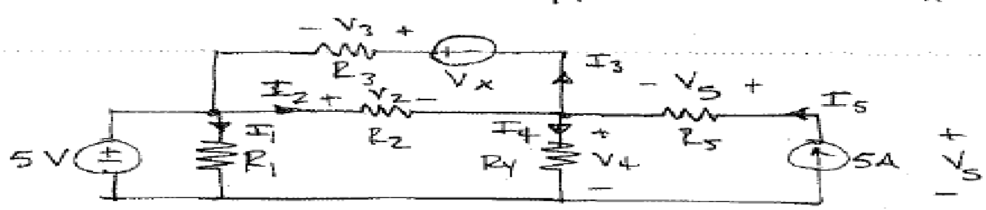


Figure P2.69

SOLUTION:

2.69 $P_{5A} = 50\text{W}$ supplied. Find V_x



$R_1 = 2\Omega$, $R_2 = 4\Omega$, $R_3 = 2\Omega$, $R_4 = 2\Omega$, $R_5 = 4\Omega$

$P_{5A} = (V_S)(5) = 50 \Rightarrow V_s = 10V$

$V_5 = I_s R_5 = 20V$

$V_4 = -V_5 + V_s = -10V$

$I_4 = \frac{V_4}{R_4} = -5A$

$V_2 = 5 - V_4 = 15V$

$I_2 = \frac{V_2}{R_2} = \frac{15}{4}A = 3.75A$

$I_3 = I_2 - I_4 + I_5 = 13.75A$

$V_3 = I_3 R_3 = 27.5V$

$V_x = V_3 + V_2 = 42.5V$

$\boxed{V_x = 42.5V}$

2.70 Find the value of V_1 in the network in Fig. P2.70 such that $V_a = 0$.

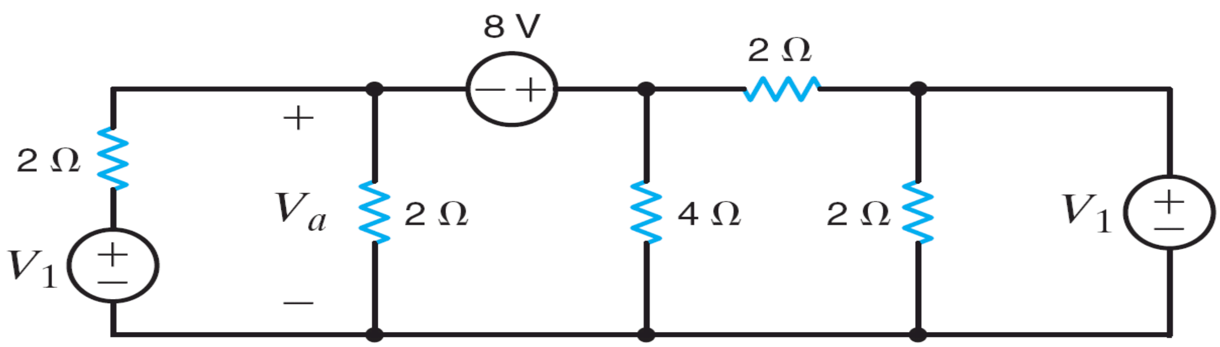
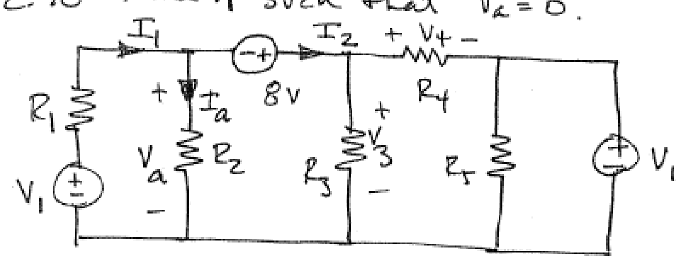


Figure P2.70

SOLUTION:

2.70 Find V_1 such that $V_a = 0$.



$R_1 = R_2 = R_4 = R_5 = 2\Omega$

$R_3 = 4\Omega$

$I_1 = I_a + I_2 \Rightarrow I_1 = I_2$ (By Ohm's Law, if $V_a = 0$, $I_a = 0$!)

$I_1 = \frac{V_1}{R_1} = \frac{V_1}{2}$

$I_2 = \frac{V_3}{R_3} + \frac{V_4}{R_4}$ $V_3 = 8\,V$ $V_4 = 8 - V_1$

So,

$I_2 = 6 - \frac{V_1}{2}$

$I_1 = I_2 \Rightarrow \frac{V_1}{2} = 6 - \frac{V_1}{2} \Rightarrow \boxed{V_1 = 6\,V}$

2.71 Find the value of V_x in the circuit in Fig. P2.71 such that the power supplied by the 5-A source is 60 W.

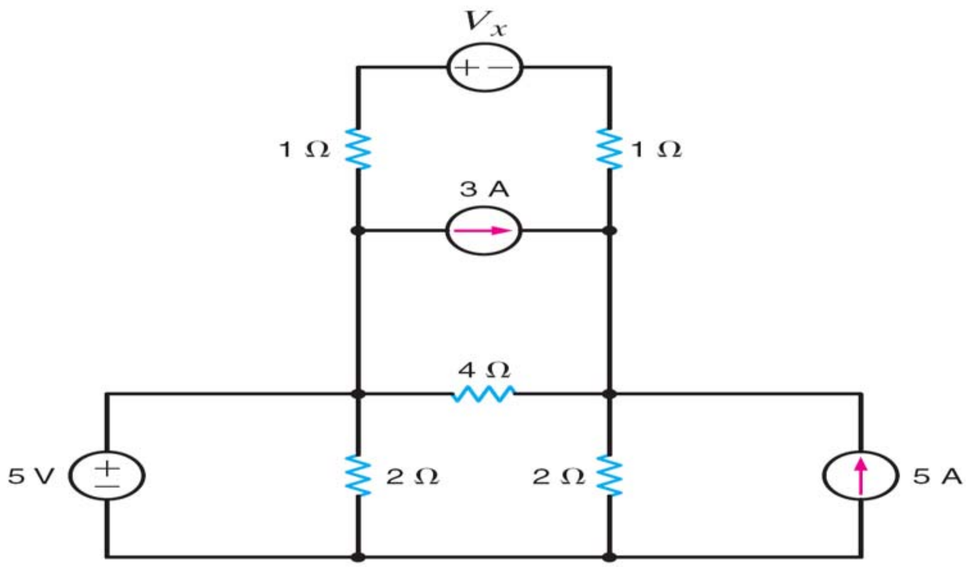
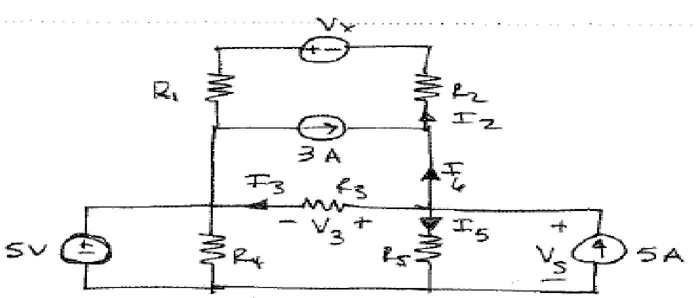


Figure P2.71

SOLUTION:

2.71 $P_{5A} = 60W$ supplied. Find V_x



$R_1 = R_2 = 1\Omega$

$R_3 = 4\Omega$

$R_4 = R_5 = 2\Omega$

$P_{5A} = 60 = 5 V_S \Rightarrow V_s = 12V$

$I_5 = \frac{V_S}{R_5} = 6A$

$V_3 = V_S - 5 = 7V$

$I_3 = V_3 / R_3 = \frac{7}{4} A$

$I_6 = 5 - I_3 - I_5 = -2.75A$

$I_2 = 3 + I_6 = 0.75A$

$V_x = I_2 R_1 - V_3 + I_2 R_2 = 0.75 - 7 + 0.75 = -5.5V$

$\boxed{V_x = -5.5V}$

2.72 Find the value of V_S in the network in Fig. P2.72 such that the power supplied by the current source is 0.

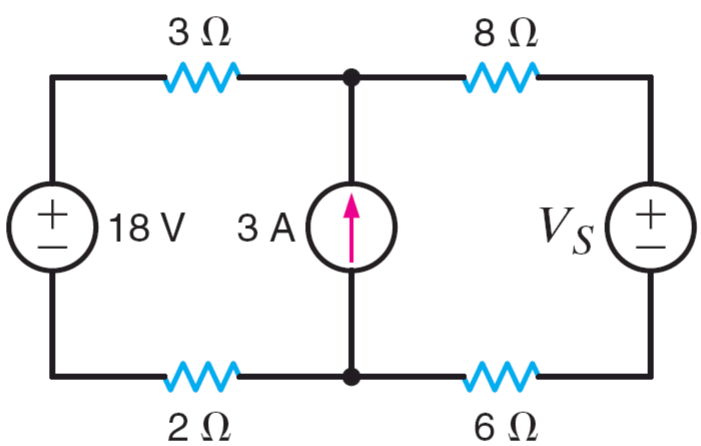
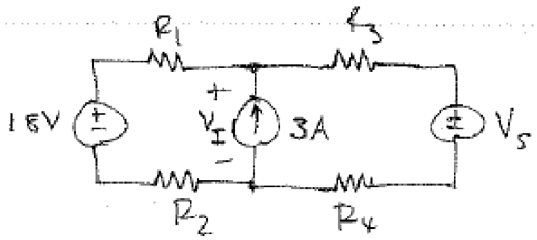


Figure P2.72

SOLUTION:

2.72 $P_{I_S} = 0\,W$. Find V_s

$P_{I_S} = 3V_I = 0 \Rightarrow V_I = 0$



KCL: $\frac{18}{R_1+R_2} + \frac{V_S}{R_3+R_4} + 3 = 0$

$\frac{18}{5} + \frac{V_S}{14} = -3$

$R_1 = 3\Omega$ $R_2 = 2\Omega$ $R_3 = 8\Omega$ $R_4 = 6\Omega$

$\boxed{V_S = -92.4V}$

2.73 Find V_o in the circuit in Fig. P2.73.

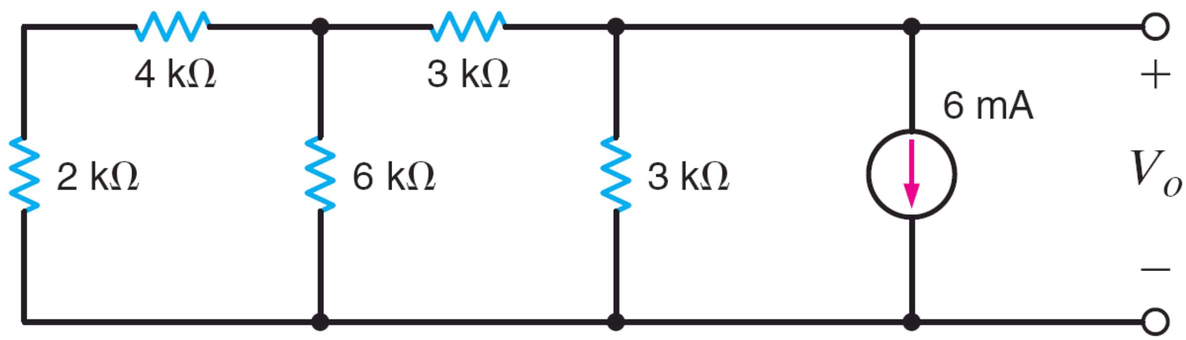
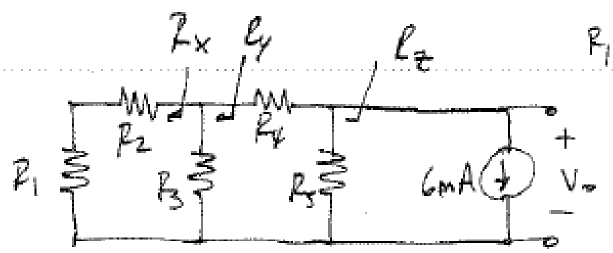


Figure P2.73

SOLUTION:

2.73 Find V_o

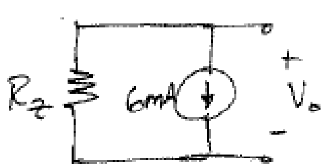


$R_1 = 2k\Omega$ $R_2 = 4k\Omega$ $R_3 = 6k\Omega$

$R_4 = 3k\Omega$ $R_5 = 3k\Omega$

$R_x = R_2 + R_1 = 6k\Omega$

$R_y = R_3 \,//\, R_x = 3k\Omega$ $R_z = (R_y + R_4) \,//\, R_5 = 2k\Omega$



$V_o = -6\times10^{-3} R_z$ $\boxed{V_o = -12V}$

2.74 Find I_o in the network in Fig. P2.74.

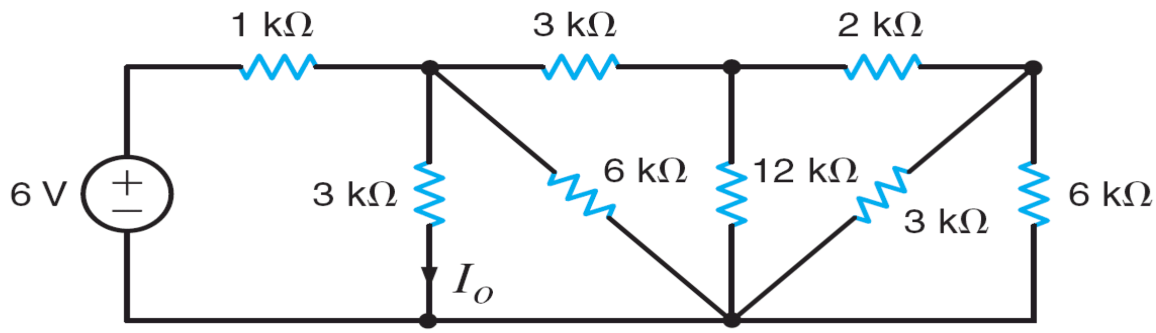


Figure P2.74

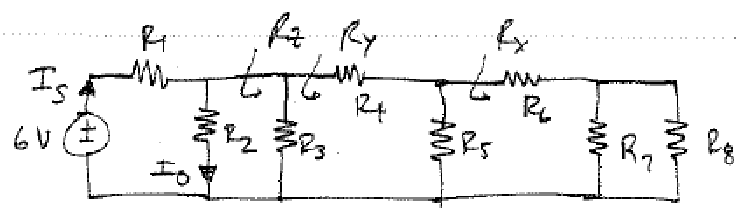
SOLUTION:

2.74 Find I_o

$R_1 = 1k\Omega \quad R_2 = R_4 = R_7 = 3k\Omega$

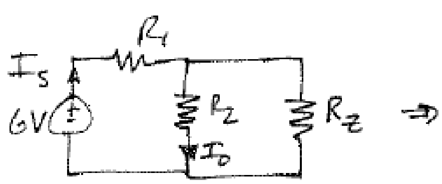
$R_3 = R_8 = 6k\Omega \quad R_5 = 12k\Omega$

$R_6 = 2k\Omega$

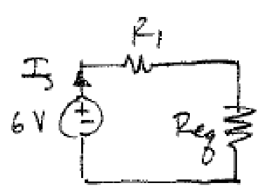


$R_X = R_6 + (R_7 // R_8) = 4k\Omega \quad R_Y = R_4 + (R_5 // R_X) = 6k\Omega$

$R_Z = R_3 // R_Y = 3k\Omega$



$\Rightarrow$



$R_{eq} = R_2 // R_Z = 1.5k\Omega$

$I_0 = I_S \left[\dfrac{R_Z}{R_Z + R_2} \right] \qquad I_S = \dfrac{6}{R_1 + R_{eq}} = 2.4mA$

$\boxed{I_0 = 1.2mA}$

2.75 Find I_o in the circuit in Fig. P2.75. **CS**

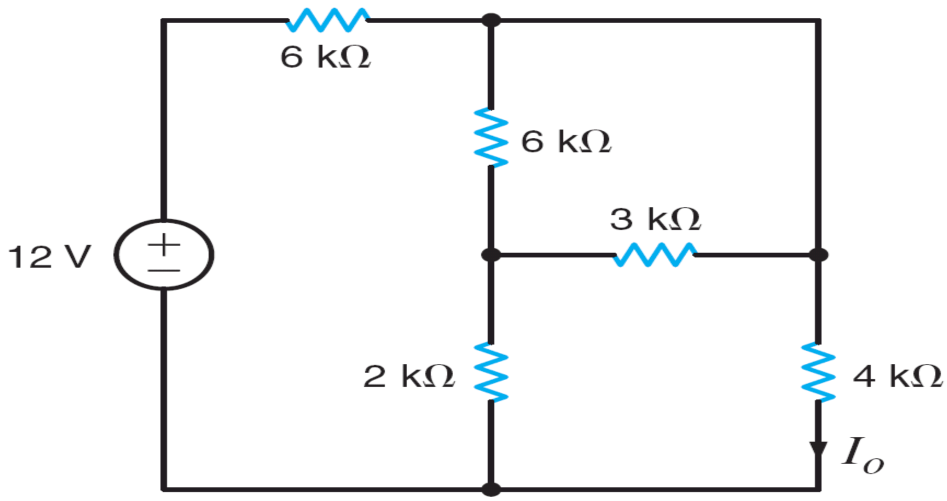
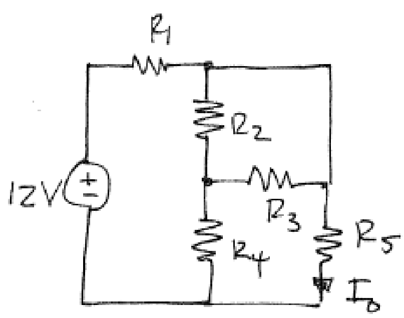


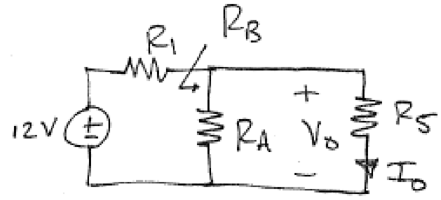
Figure P2.75

SOLUTION:

2.75 Find I_o.



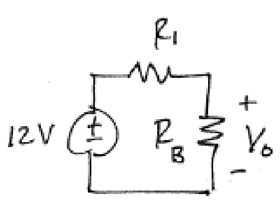
$\Rightarrow$



$R_1 = R_2 = 6\,k\Omega \quad R_3 = 3\,k\Omega \quad R_4 = 2\,k\Omega$

$R_5 = 4\,k\Omega \quad R_A = R_4 + (R_2 /\!/ R_3) = 4\,k\Omega$

$R_B = R_A /\!/ R_5 = 2\,k\Omega$



$V_o = 12\left(\frac{R_B}{R_B + R_1}\right) \quad V_o = 3V$

$I_o = \frac{V_o}{R_5} \Rightarrow \boxed{I_o = 0.75\,mA}$

2.76 Determine V_o in the circuit in Fig. P2.76. **PSV**

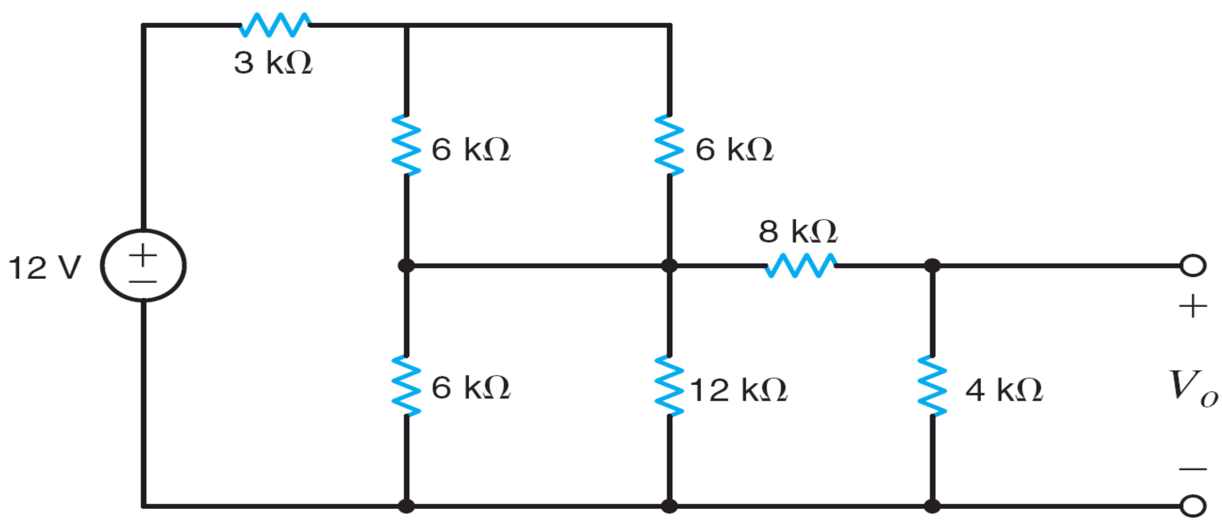
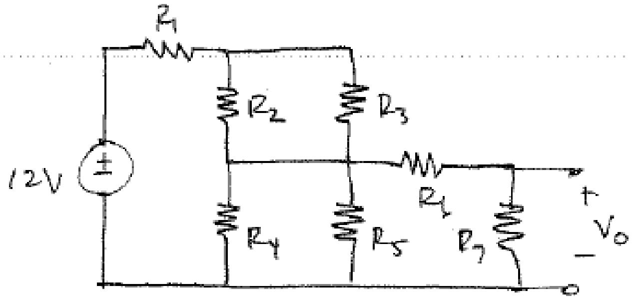


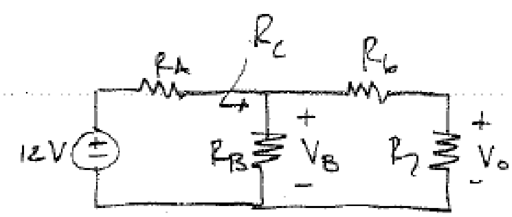
Figure P2.76

SOLUTION:

2.76 Find V_o



$\Rightarrow$



$R_1 = 3k\Omega$ $\quad R_2 = R_3 = R_4 = 6k\Omega$

$R_5 = 12k\Omega$ $\quad R_6 = 8k\Omega$ $\quad R_7 = 4k\Omega$

$R_A = R_1 + (R_2 // R_3) = 6k\Omega$

$R_B = R_4 // R_5 = 4k\Omega$

$R_C = R_B // (R_6 + R_7) = 3k\Omega$

$$V_B = 12\left(\frac{R_C}{R_C + R_A}\right) = 4V$$

$$V_o = V_B\left(\frac{R_7}{R_6 + R_7}\right) \Rightarrow \boxed{V_o = 1.33V}$$

2.77 Find V_o in the circuit in Fig. P2.77.

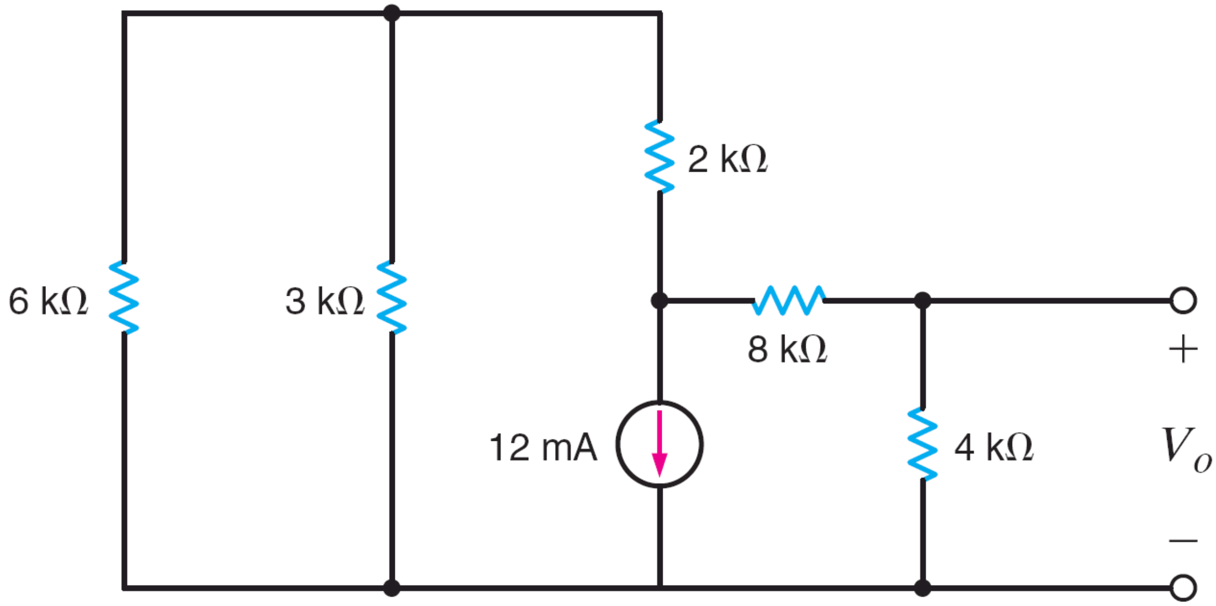
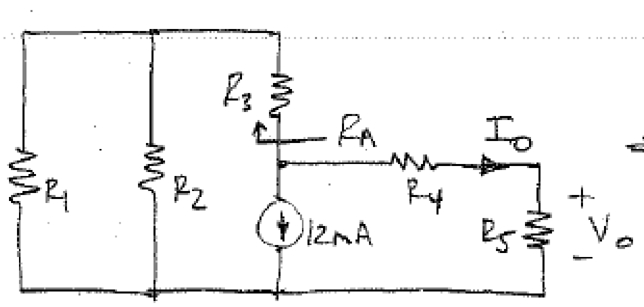
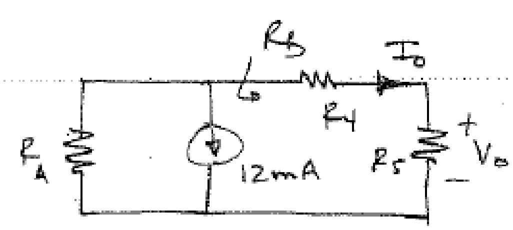


Figure P2.77

SOLUTION:

2.77 Find V_o





$R_1 = 6k\Omega \quad R_2 = 3k\Omega \quad R_3 = 2k\Omega$

$R_4 = 8k\Omega \quad R_5 = 4k\Omega$

$R_A = R_3 + (R_1 // R_2) = 4k\Omega$

$R_B = 12k\Omega$

$I_o = -12 \times 10^{-3} \left(\frac{R_A}{R_A + R_B} \right) = -3mA$

$V_o = I_o R_5$

$\boxed{V_o = -12V}$

2.78 Find V_o in the circuit in Fig. P2.78.

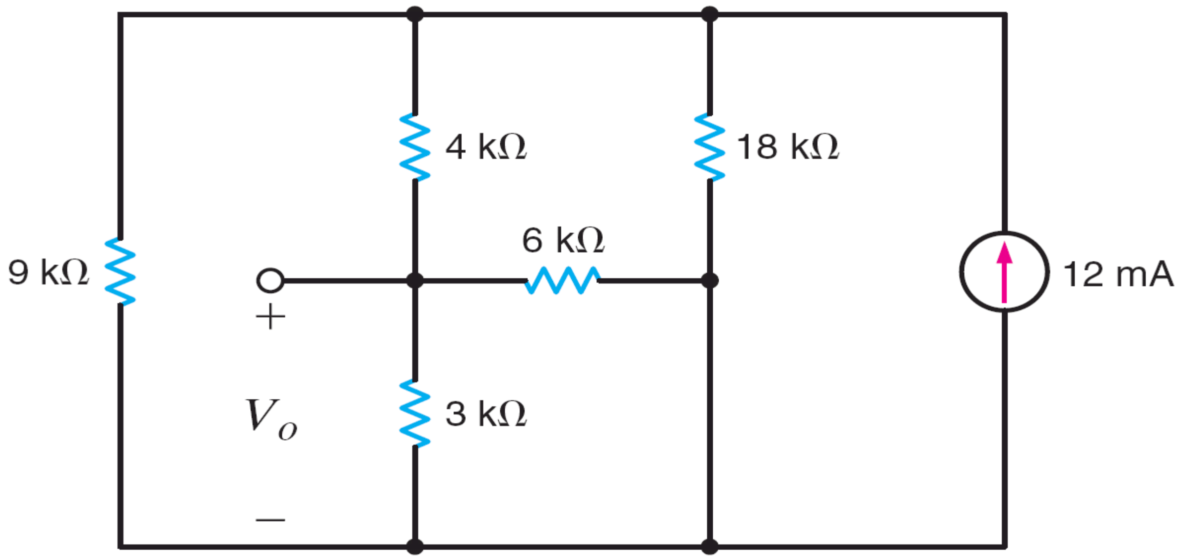
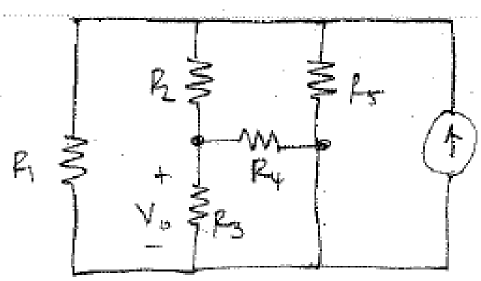


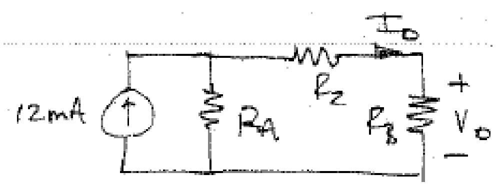
Figure P2.78

SOLUTION:

2.78 Find V_o.



$\Rightarrow$



$R_1 = 9k\Omega$ $R_2 = 4k\Omega$ $R_3 = 3k\Omega$

$R_4 = 6k\Omega$ $R_5 = 18k\Omega$

$R_A = R_1 // R_5 = 6k\Omega$

$R_B = R_3 // R_4 = 2k\Omega$

$I_o = 12 \times 10^{-3} \left(\dfrac{R_A}{R_A + (R_2 + R_B)} \right)$

$I_o = 6mA$

$V_o = R_B I_o \Rightarrow \boxed{V_o = 12V}$

2.79 Find I_o in the circuit in Fig. P2.79.

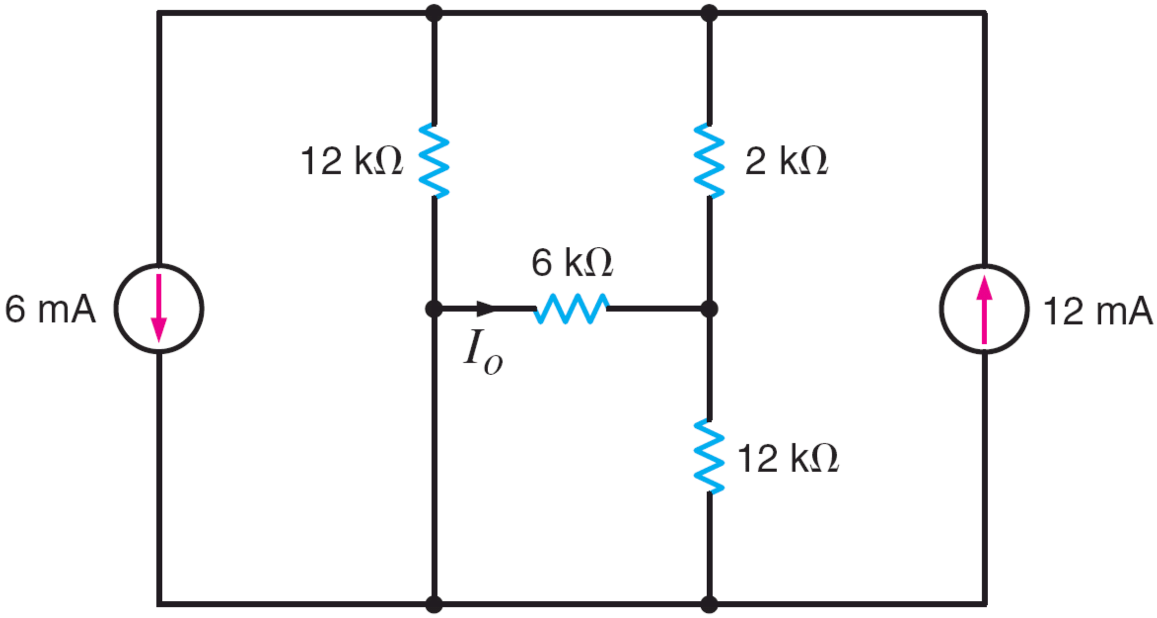
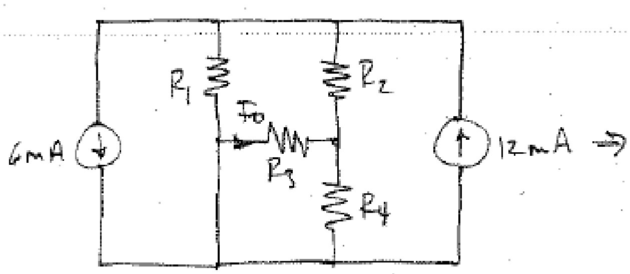
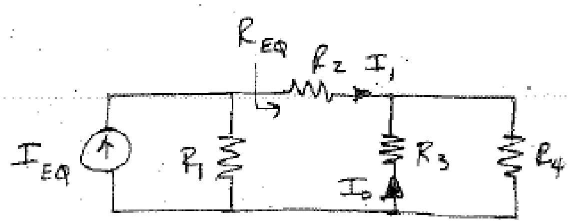


Figure P2.79

SOLUTION:

2.79 Find I_o





$R_1 = R_4 = 12k\Omega$, $R_2 = 2k\Omega$, $R_3 = 6k\Omega$

$$I_{EQ} = 12\times10^{-3} - 6\times10^{-3} = 6\,mA$$

$$R_{EQ} = R_2 + (R_3 // R_4) = 6k\Omega$$

$$I_1 = I_{EQ}\left[\frac{R_1}{R_1 + R_{EQ}}\right] = 4\,mA$$

$$I_o = -I_1\left[\frac{R_4}{R_3 + R_4}\right] \Rightarrow \boxed{I_o = -2.67\,mA}$$

2.80 Find I_o in the circuit in Fig. P2.80.

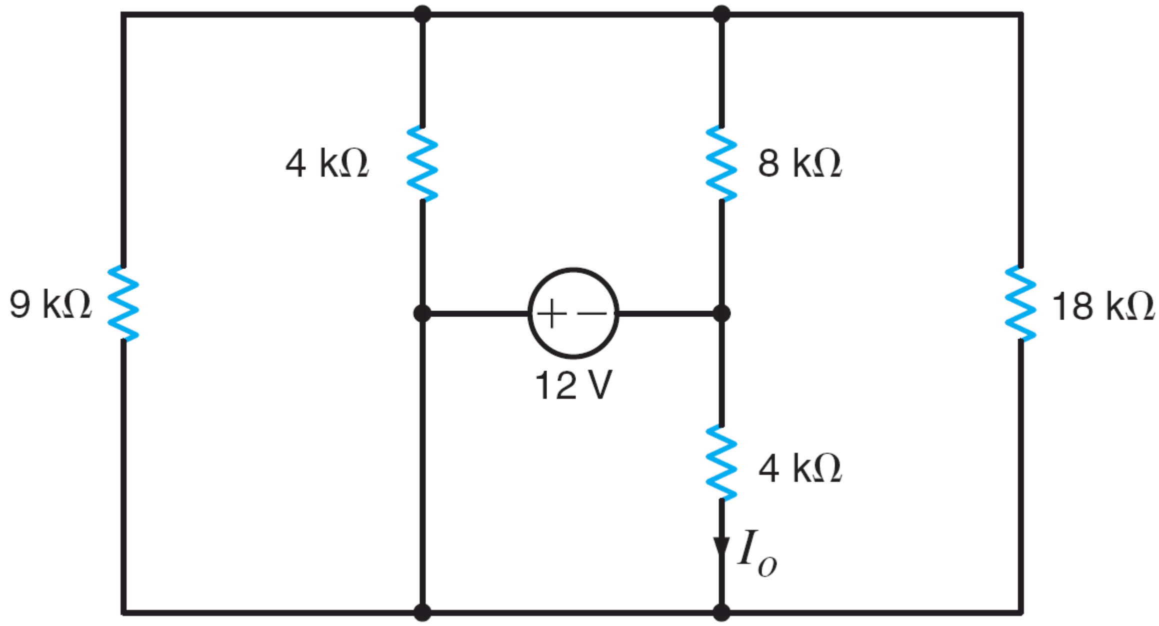
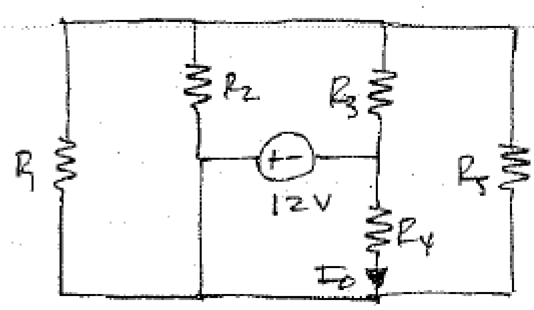


Figure P2.80

SOLUTION:

2.80 Find I_o.



$R_4 = 4k\Omega$

$$I_o = \frac{-12}{R_4} = -3mA$$

$$\boxed{I_o = -3mA}$$

2.81 Find I_o in the circuit in Fig. P2.81.

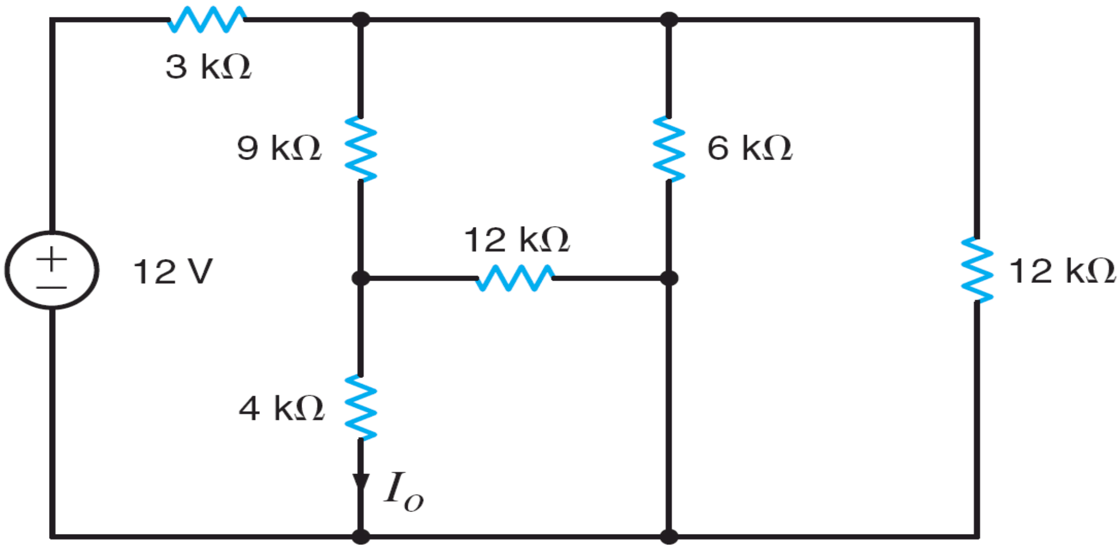
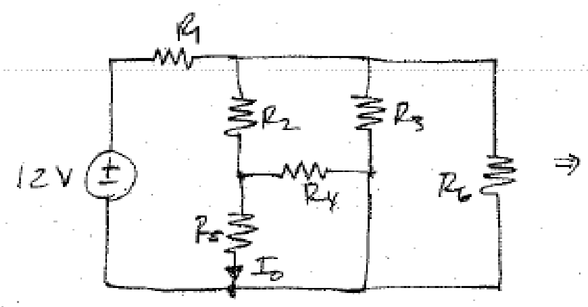


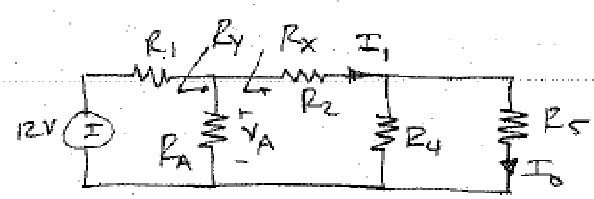
Figure P2.81

SOLUTION:

2.81 Find I_o



$\Rightarrow$



$R_1 = 3k\Omega$, $R_2 = 9k\Omega$, $R_3 = 6k\Omega$

$R_4 = 12k\Omega$, $R_5 = 4k\Omega$; $R_6 = 12k\Omega$

$R_A = R_3 // R_6 = 4k\Omega$

$R_X = R_2 + (R_4 // R_5) = 12k\Omega$

$R_Y = R_A // R_X = 3k\Omega$

$$V_A = 12\left[\frac{R_Y}{R_1 + R_Y}\right] = 6V \qquad I_1 = \frac{V_A}{R_X} = \frac{1}{2}\, mA$$

$$I_o = I_1\left[\frac{R_4}{R_4 + R_5}\right] \Rightarrow \boxed{I_o = 375\,\mu A}$$

2.82 Find V_o in the circuit in Fig. P2.82. **PSV**

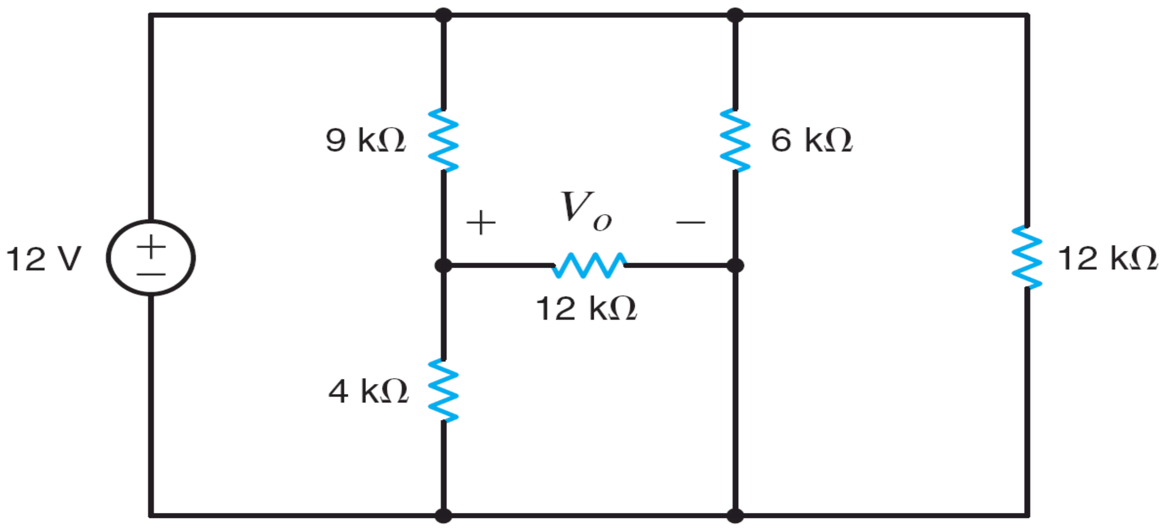
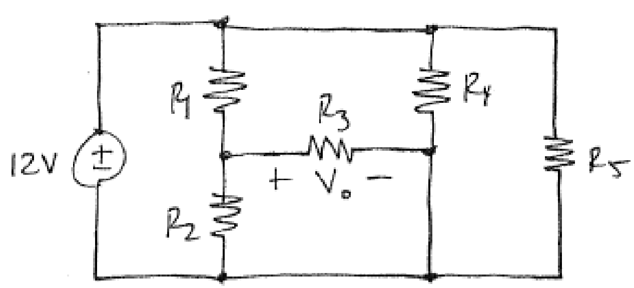


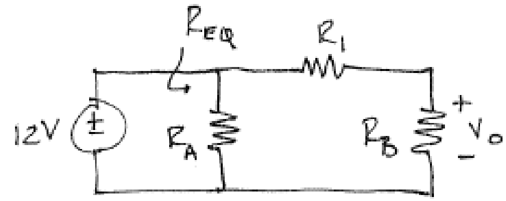
Figure P2.82

SOLUTION:

2.82 Find V_o.



$\Rightarrow$



$R_1 = 9k\Omega$, $R_2 = 4k\Omega$, $R_3 = 12k\Omega$

$R_4 = 6k\Omega$ $R_5 = 12k\Omega$

$R_{EQ} = R_A \,//\, (R_1 + R_B)$

$R_A = R_4 // R_5 = 4k\Omega$

$R_B = R_2 // R_3 = 3k\Omega$

$R_{EQ} = 3k\Omega$

$$V_o = 12\left[\frac{R_B}{R_B + R_1}\right] \Rightarrow \boxed{V_o = 3V}$$

2.83 Find I_o in the circuit in Fig. P2.83.

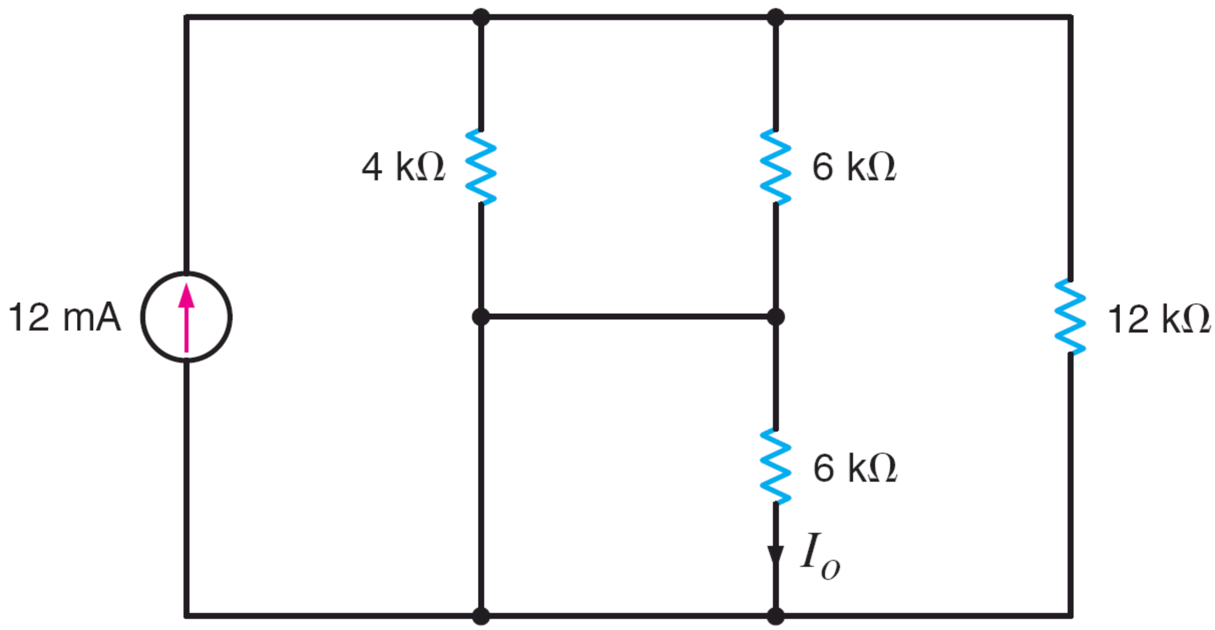
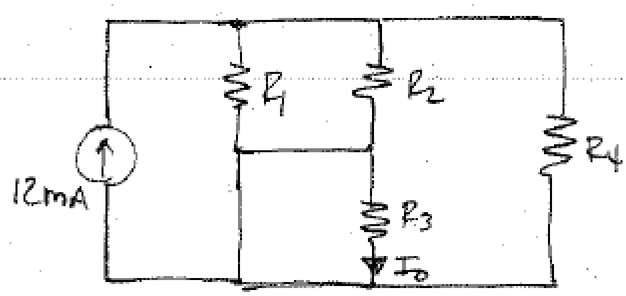


Figure P2.83

SOLUTION:

2.83 Find I_o.



R_3 is shorted, so

$$\boxed{I_o = 0\,A}$$

2.84 Determine the value of V_o in the circuit in Fig. P2.84.

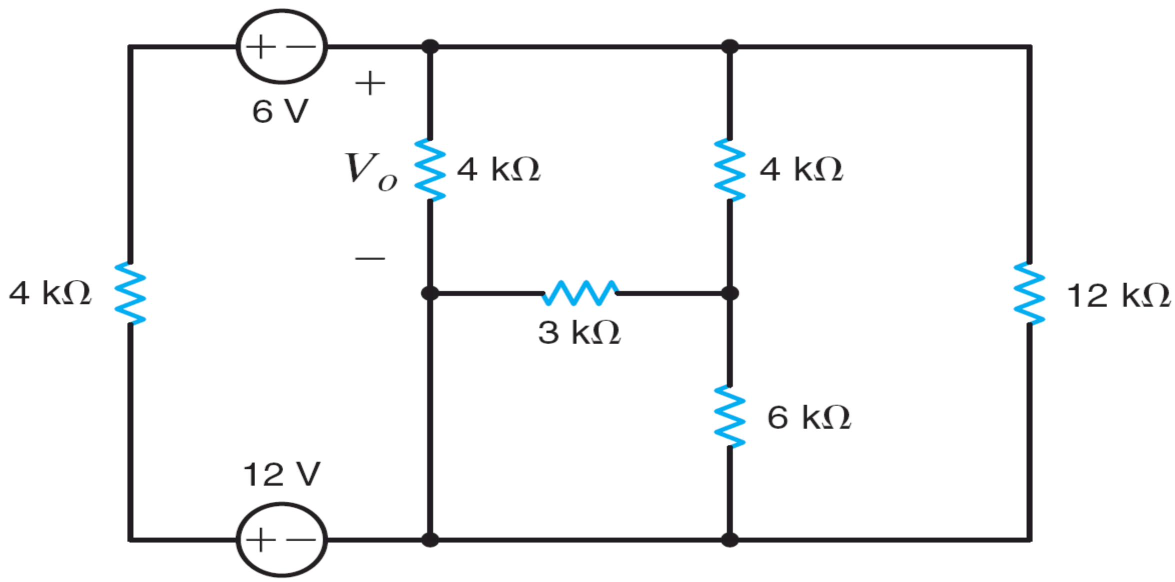
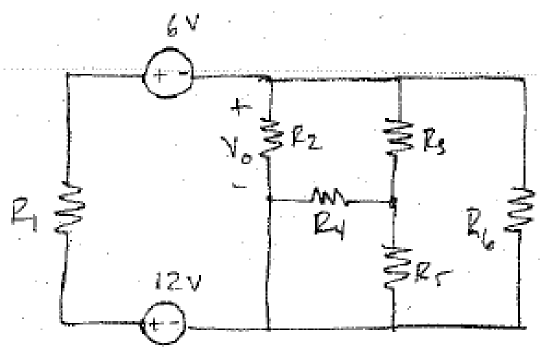


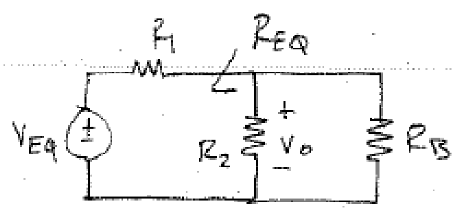
Figure P2.84

SOLUTION:

2.84



$\Rightarrow$



$R_1 = R_2 = R_3 = 4k\Omega \quad R_4 = 3k\Omega$

$R_5 = 6k\Omega \quad R_6 = 12k\Omega$

$V_{EQ} = 12 - 6 = 6V$

$R_B = R_6 \,//\, [R_3 + (R_4 \,//\, R_5)] = 4k\Omega$

$R_{EQ} = R_2 \,//\, R_B = 2k\Omega$

$V_o = V_{EQ}\left[\dfrac{R_{EQ}}{R_1 + R_{EQ}}\right] \Rightarrow \boxed{V_o = 2V}$

2.85 Find $P_{4\,\Omega}$ in the network in Fig. P2.85.

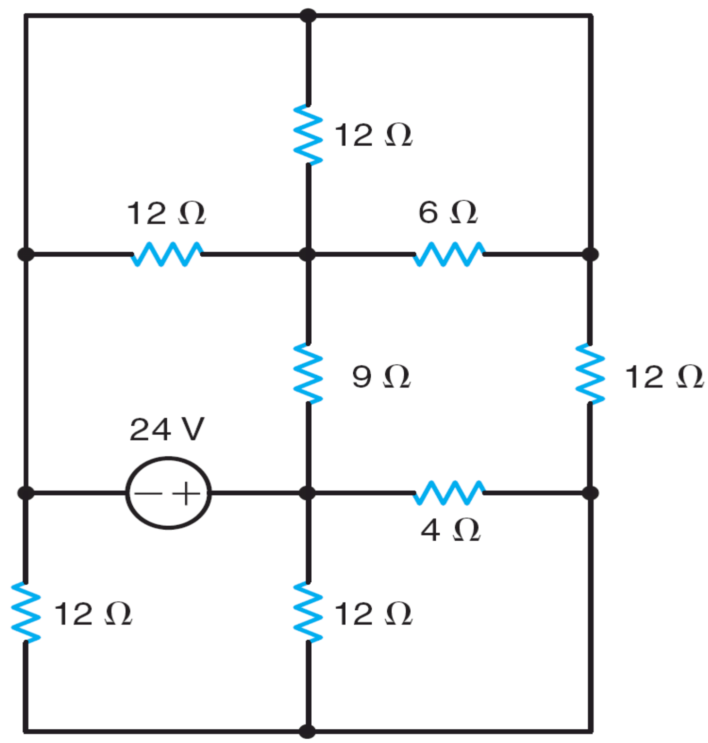
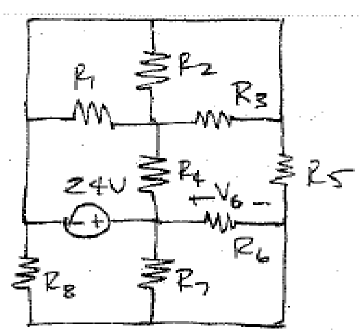


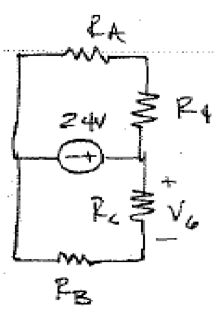
Figure P2.85

SOLUTION:

Find $P_{4\Omega}$ (P_{R_6})



$\Rightarrow$



$R_A = R_1 // R_2 // R_3$

$R_B = R_5 // R_8 = 6\Omega$

$R_C = R_6 // R_7 = 3\Omega$

$$V_6 = 24\left[\frac{R_C}{R_C + R_B}\right] = 8V$$

$$P_{R_6} = \frac{V_6^2}{R_6} \qquad \boxed{P_{R_6} = 16W}$$

$R_1 = R_2 = R_5 = R_7 = R_8 = 12\Omega$

$R_3 = 6\Omega \quad R_4 = 9\Omega \quad R_6 = 4\Omega$

2.86 Find I_o in the network in Fig. P2.86.

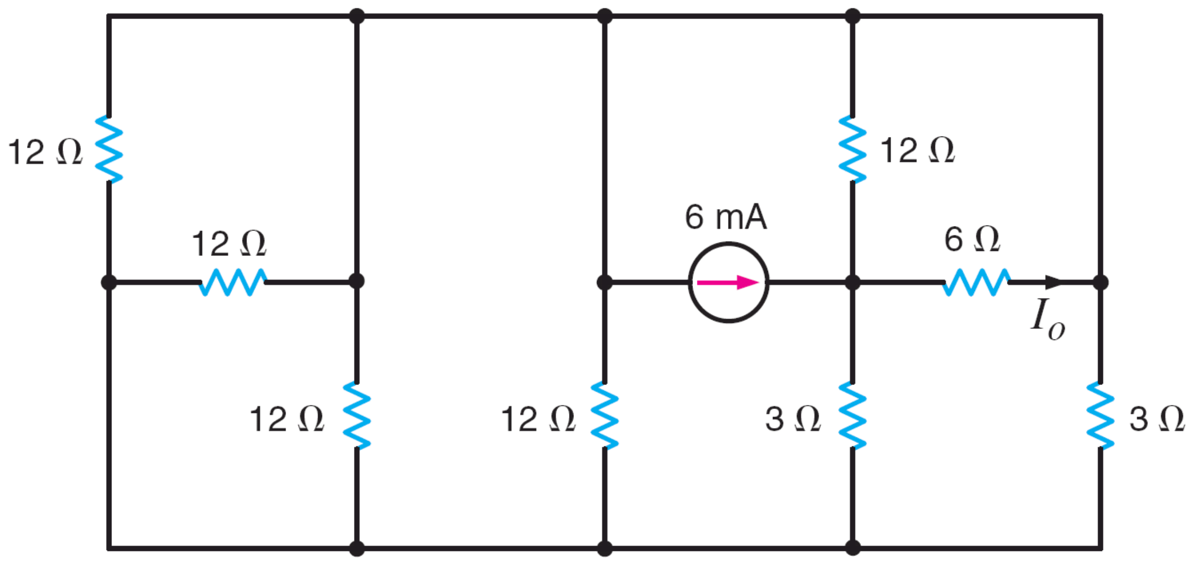
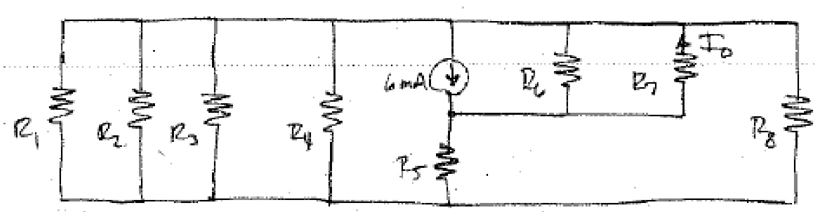


Figure P2.86

SOLUTION:

2.86 Find I_o.

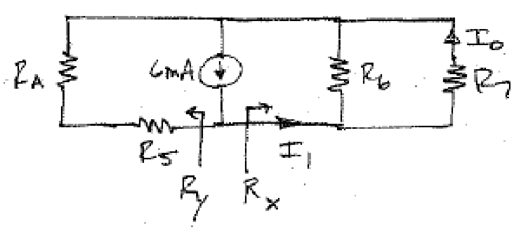


$R_1 = R_2 = R_3 = R_4 = R_6 = 12\Omega$

$R_5 = R_8 = 3\Omega$

$R_7 = 6\Omega$

$\Downarrow$



$R_A = R_1 // R_2 // R_3 // R_4 // R_8 = 1.5\ \Omega$

$R_x = R_6 // R_7 = 4\Omega$

$R_y = R_5 + R_A = 4.5\Omega$

$I_1 = 6 \times 10^{-3} \left[\frac{R_y}{R_y + R_x} \right] = 3.18\ mA$

$I_o = I_1 \left[\frac{R_6}{R_6 + R_7} \right]$ $\boxed{I_o = 2.12\ mA}$

2.87 In the network in Fig. P2.87, the power absorbed by the 4-Ω resistor is 100 W. Find V_S. **CS**

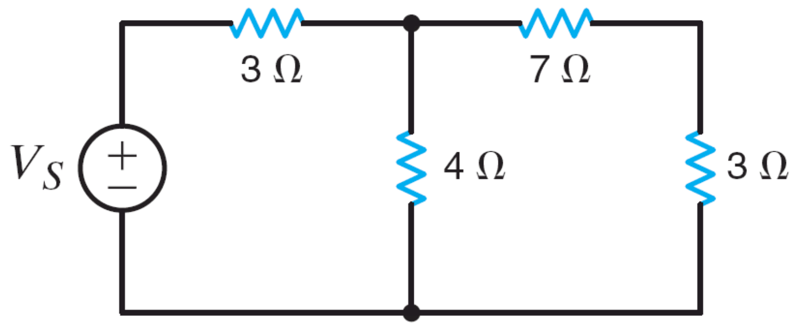
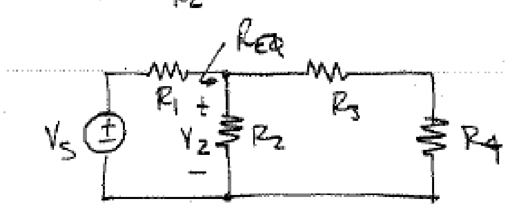


Figure P2.87

SOLUTION:

2.87 $P_{R2} = 100W$. Find V_S



$R_1 = 3\Omega \quad R_2 = 4\Omega \quad R_3 = 7\Omega \quad R_4 = 3\Omega$

$P_{R_2} = V_2^2 / R_2 \Rightarrow V_2 = 20V$

$V_2 = V_S \left[\frac{R_{EQ}}{R_1 + R_{EQ}} \right]$ but, $R_{EQ} = R_2 \,//\, (R_3 + R_4) = 2.86\,\Omega$

$\boxed{V_S = 41\,V}$

2.88 If $V_o = 2$ V in the circuit in Fig. P2.88, find V_S.

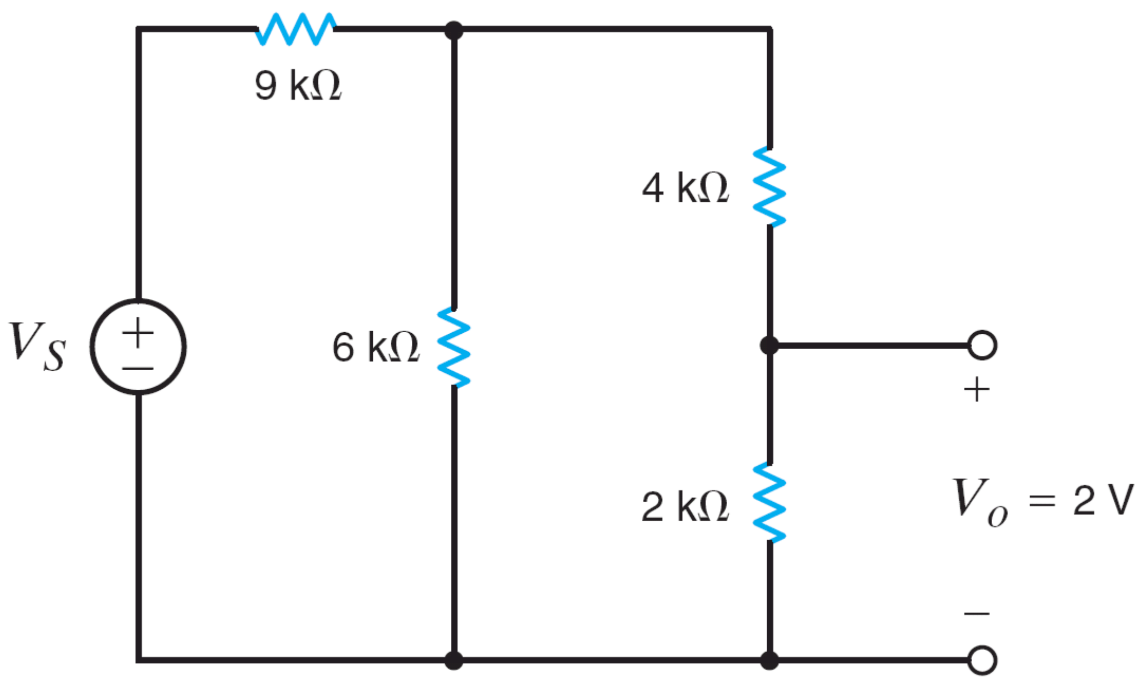
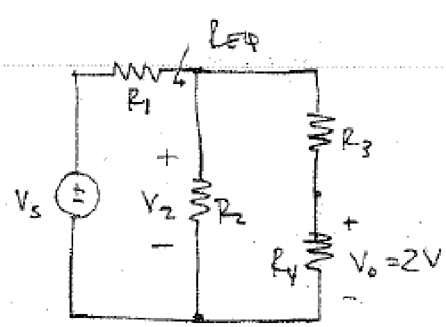


Figure P2.88

SOLUTION:

2.88 $V_o = 2V$, Find V_s



$R_1 = 9k\Omega \quad R_2 = 6k\Omega \quad R_3 = 4k\Omega \quad R_4 = 2k\Omega$

$$V_o = V_2 \left[\frac{R_4}{R_3 + R_4} \right] \Rightarrow \quad V_2 = 6\,V$$

$$R_{EQ} = R_2 \,//\, (R_3 + R_4) = 3k\Omega$$

$$V_2 = V_s \left[\frac{R_{EQ}}{R_1 + R_{EQ}} \right] \Rightarrow \boxed{V_s = 24V}$$

2.89 If $V_o = 6$ V in the circuit in Fig. P2.89, find I_S.

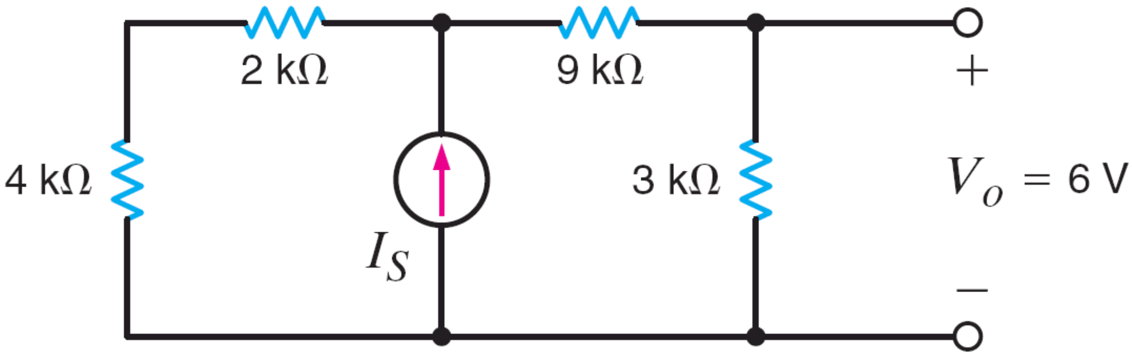
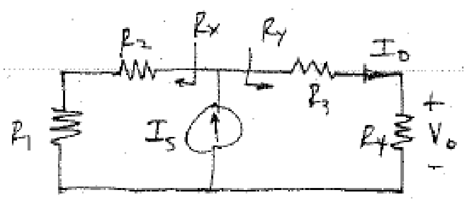


Figure P2.89

SOLUTION:

2.89 $V_o = 6V$. Find I_S.



$R_1 = 4k\Omega$ $R_2 = 2k\Omega$ $R_3 = 9k\Omega$

$R_4 = 3k\Omega$

$R_x = R_1 + R_2 = 6k\Omega$ $R_y = R_3 + R_4 = 12k\Omega$

$I_o = I_S \left[\frac{R_x}{R_x + R_y} \right]$ and $I_o = \frac{V_o}{R_4} = 2mA$

$\boxed{I_S = 6mA}$

2.90 If $I_o = 2$ mA in the circuit in Fig. P2.90, find V_S.

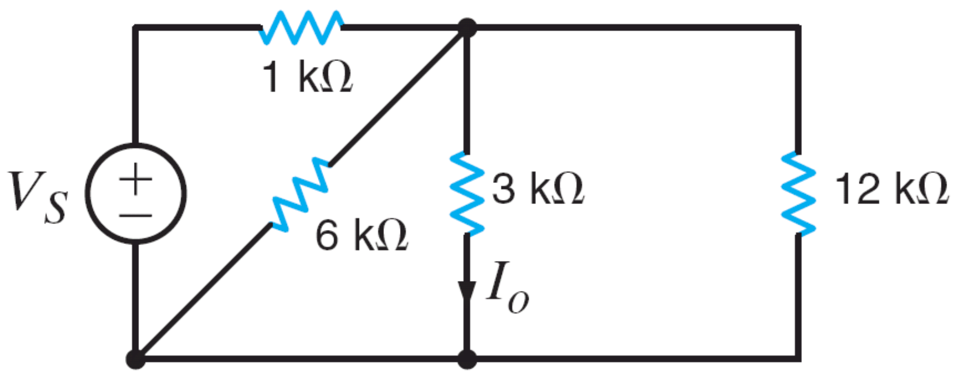
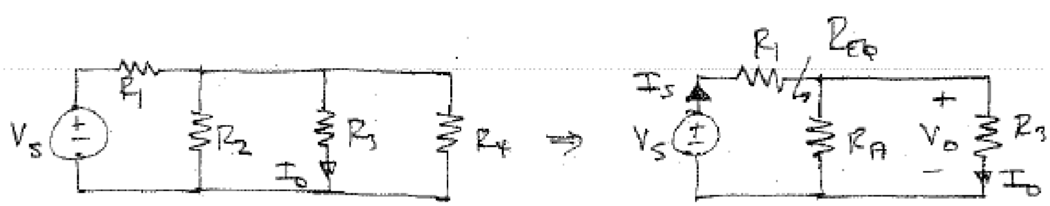


Figure P2.90

SOLUTION:

2.90 $I_o = 2mA$. Find V_s



$R_1 = 1k\Omega$ $R_2 = 6k\Omega$ $R_3 = 3k\Omega$

$R_4 = 12k\Omega$

$R_A = R_2 // R_4 = 4k\Omega$

$R_{EQ} = R_A // R_3 = 1.71k\Omega$

$V_o = V_s \left[\dfrac{R_{EQ}}{R_1 + R_{EQ}} \right]$ and $V_o = I_o R_3 = 6V$

$\boxed{V_s = 9.5V}$

2.91 If $V_1 = 5$ V in the circuit in Fig. P2.91, find I_S.

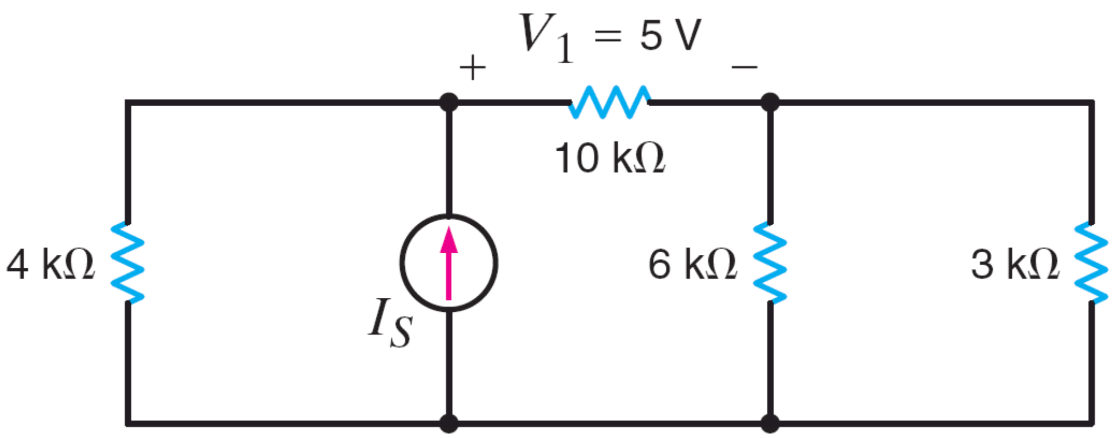
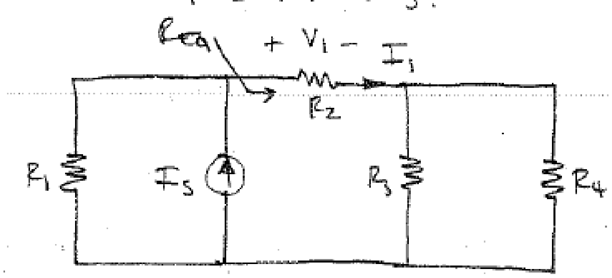


Figure P2.91

SOLUTION:

2.91 $V_1 = 5$V. Find I_S.



$R_1 = 4k\Omega$, $R_2 = 10k\Omega$, $R_3 = 6k\Omega$, $R_4 = 3k\Omega$

$R_{EQ} = R_2 + (R_3 // R_4) = 12k\Omega$

$I_1 = V_1/R_2 = \frac{1}{2}$ mA

$I_1 = I_S\left[\frac{R_1}{R_1 + R_{EQ}}\right] \Rightarrow \boxed{I_S = 2mA}$

2.92 In the network in Fig. P2.92, $V_1 = 12$ V. Find V_S. **CS**

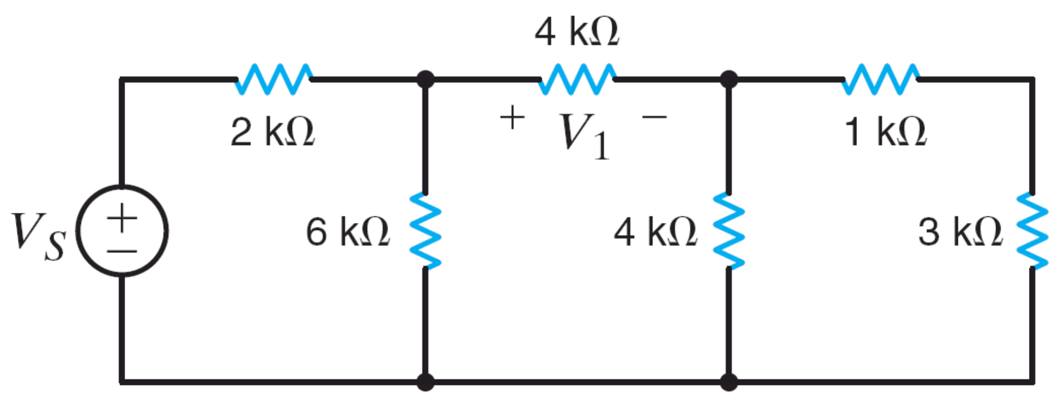
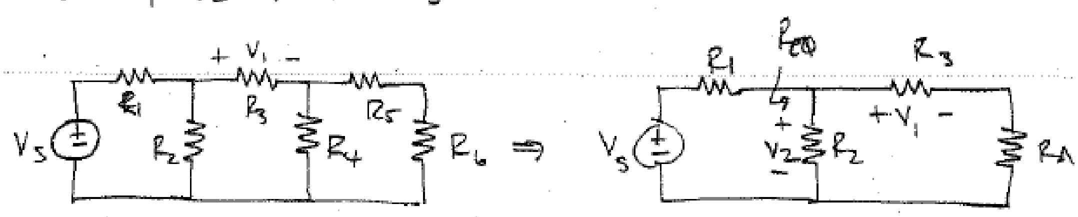


Figure P2.92

SOLUTION:

2.92 $V_1 = 12V$. Find V_s



$R_1 = 2k\Omega$ $R_2 = 6k\Omega$ $R_3 = 4k\Omega$

$R_4 = 4k\Omega$ $R_5 = 1k\Omega$ $R_6 = 3k\Omega$

$R_A = R_4 \,//\, (R_5 + R_6) = 2k\Omega$

$R_{EQ} = R_2 \,//\, (R_3 + R_A) = 3k\Omega$

$$V_1 = V_2 \left[\frac{R_3}{R_3 + R_A} \right] \Rightarrow V_2 = 18\text{ V}$$

$$V_2 = V_s \left[\frac{R_{EQ}}{R_{EQ} + R_1} \right] \Rightarrow \boxed{V_s = 30V}$$

2.93 In the circuit in Fig. P2.93, $V_o = 2$ V. Find I_S.

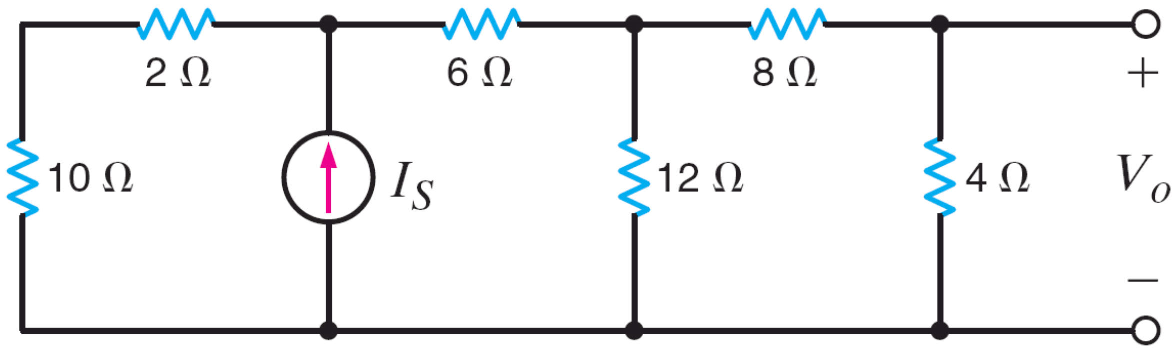
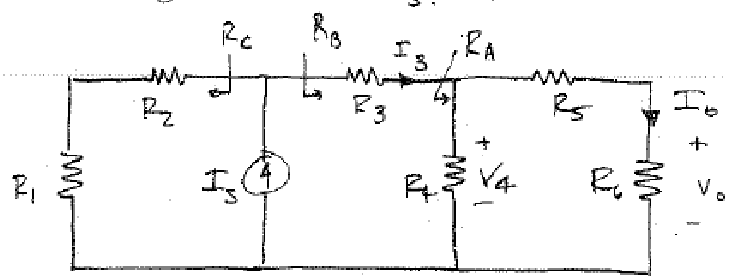


Figure P2.93

SOLUTION:

2.93 $V_o = 2V$. Find I_s.



$R_1 = 10\Omega$ $R_2 = 2\Omega$ $R_3 = 6\Omega$

$R_4 = 12\Omega$ $R_5 = 8\Omega$ $R_6 = 4\Omega$

$R_C = R_1 + R_2 = 12k\Omega$

$R_A = R_4 \,//\, (R_5 + R_6) = 6k\Omega$

$R_B = R_3 + R_A = 12k\Omega$

$$I_3 = I_S \left[\frac{R_C}{R_C + R_B}\right] = \frac{I_S}{2}$$

$$I_o = I_3 \left[\frac{R_4}{R_4 + (R_5 + R_6)}\right] = \frac{I_3}{2}$$

$I_o = V_o / R_6 = 0.5A$

$\boxed{I_S = 2A}$

2.94 In the network in Fig. P2.94, $V_o = 6$ V. Find I_S.

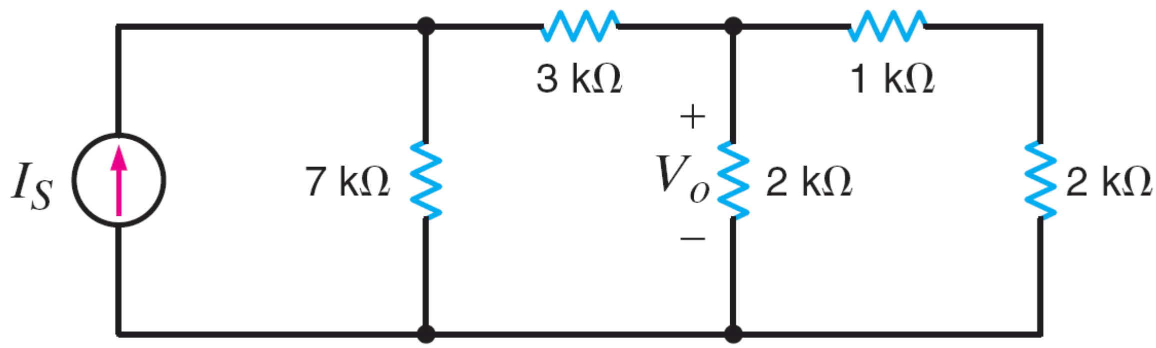
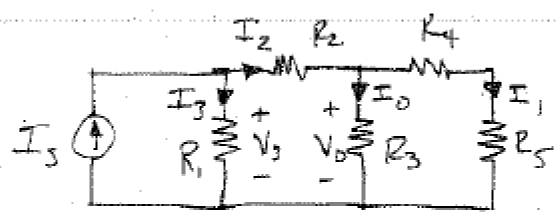


Figure P2.94

SOLUTION:

2.94 $V_o = 6V$. Find I_S.



$R_1 = 7k\Omega \quad R_2 = 3k\Omega$

$R_3 = R_5 = 2k\Omega \quad R_4 = 1k\Omega$

$I_o = V_o / R_3 = 3mA$

$I_1 = \dfrac{V_o}{R_4 + R_5} = 2mA$

$I_2 = I_o + I_1 = 5\,mA$

$V_3 = I_2 R_2 + V_o = 21V$

$I_3 = V_3 / R_1 = 3\,mA$

$I_S = I_2 + I_3 \Rightarrow \boxed{I_S = 8\,mA}$

2.95 In $I_o = 4$ mA in the circuit in Fig. P2.95, find I_S. **CS**

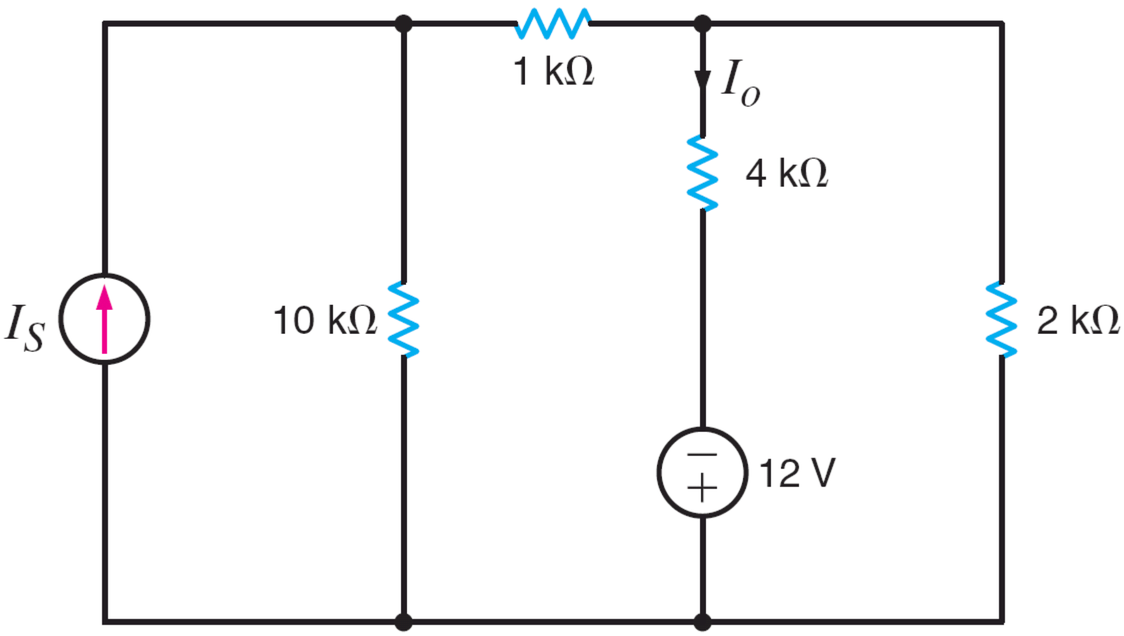
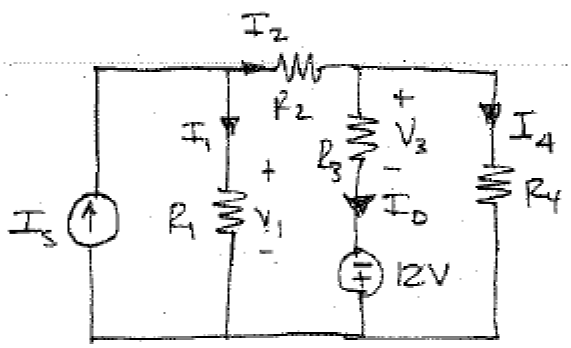


Figure P2.95

SOLUTION:

2.95 $I_o = 4mA$. Find I_S.



$R_1 = 10k\Omega$ $R_2 = 1k\Omega$ $R_3 = 4k\Omega$

$R_4 = 2k\Omega$

$V_3 = R_3 I_o = 16V$

$I_4 = \dfrac{V_3 - 12}{R_4} = 2mA$

$I_2 = I_o + I_4 = 6mA$

$V_1 = I_2 R_2 + I_4 R_4 = 10V$

$I_1 = V_1 / R_1 = 1mA$

$I_S = I_1 + I_2 \Rightarrow \boxed{I_S = 7mA}$

2.96 If $V_o = 6$ V in the circuit in Fig. P2.96, find I_S.

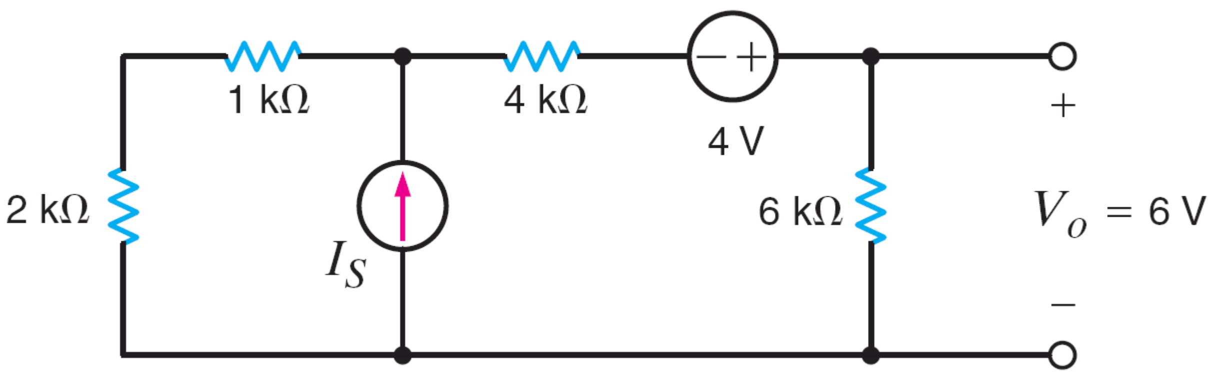
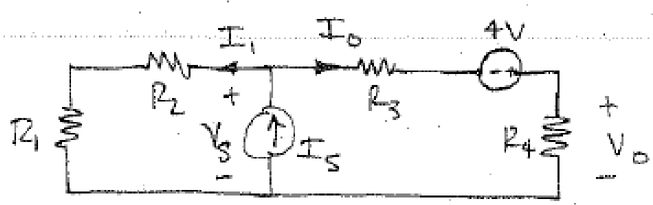


Figure P2.96

SOLUTION:

2.96 $V_o = 6V$. $I_s = ?$



$R_1 = 2k\Omega$ $R_2 = 1k\Omega$ $R_3 = 4k\Omega$ $R_4 = 6k\Omega$

$I_o = V_o / R_4 = 1mA$

$V_S = I_o R_3 - 4 + I_o R_4$

$V_S = 6V$

$I_1 = \dfrac{V_S}{R_1 + R_2} = 2mA$

$I_S = I_1 + I_o = 3mA$ $\boxed{I_S = 3mA}$

2.97 Given that $V_o = 4$ V in the network in Fig. P2.97, find V_S.

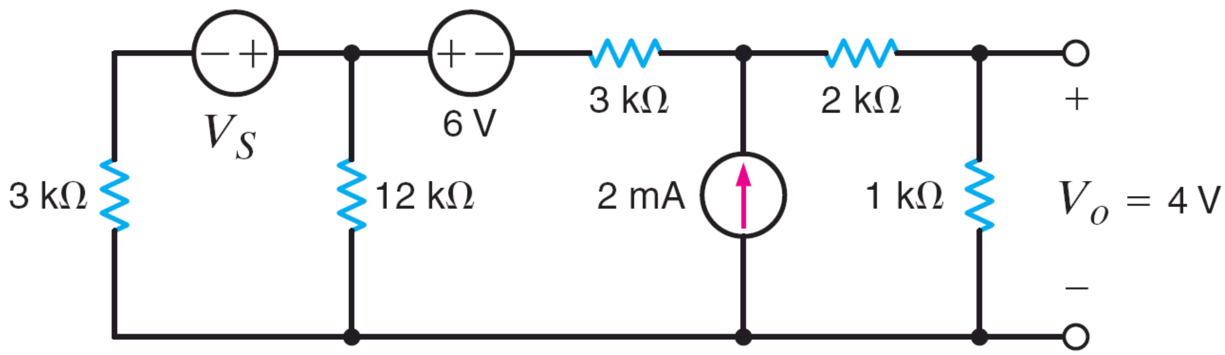
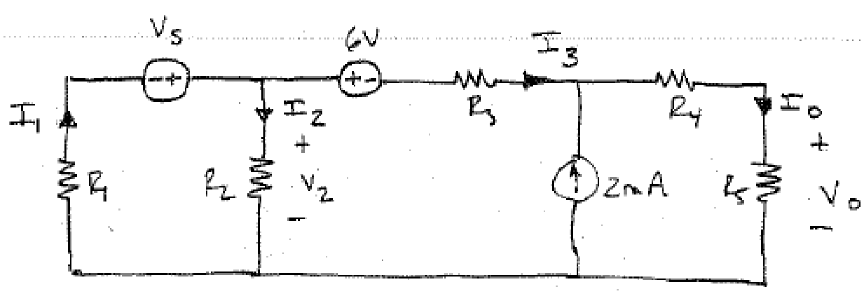


Figure P2.97

SOLUTION:

2.97 $V_o = 4V$. Find V_s.



$R_1 = 3k\Omega$ $R_2 = 12k\Omega$

$R_3 = 3k\Omega$ $R_4 = 2k\Omega$

$R_5 = 1k\Omega$

$I_o = V_o / R_5 = 4mA$ $\quad$ $I_3 = I_o - 2\times10^{-3} = 2mA$

$V_2 = 6 + R_3 I_3 + I_o R_4 + I_o R_5 = 24V$

$I_2 = V_2 / R_2 = 2mA$

$I_1 = I_2 + I_3 = 4mA$

$V_s = I_2 R_2 + I_1 R_1 \Rightarrow \boxed{V_s = 36V}$

2.98 Find I_o in the circuit in Fig. P2.98.

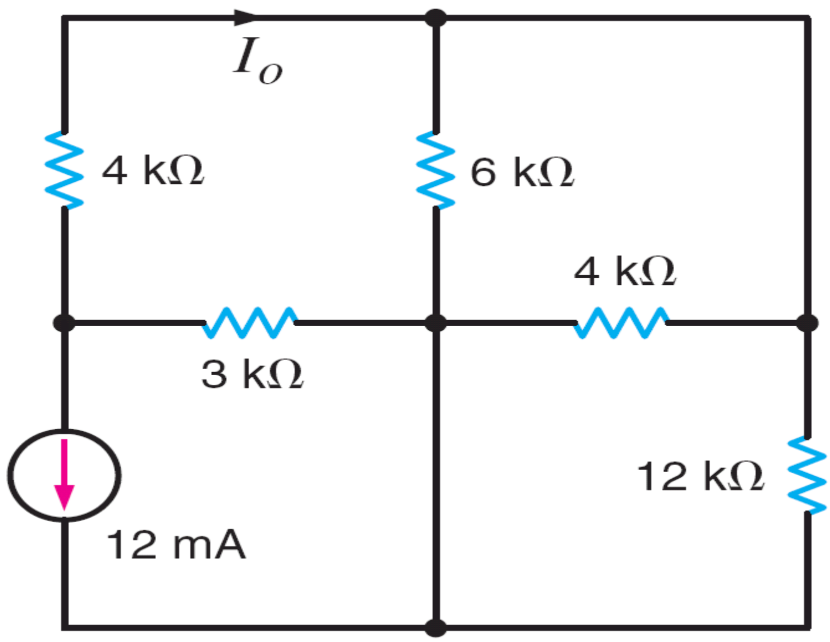
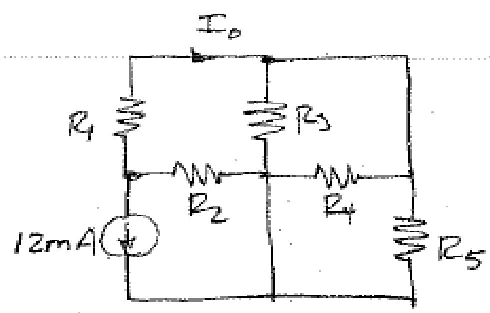
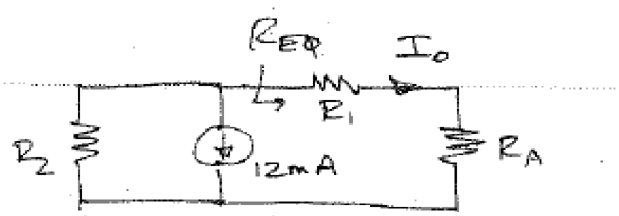


Figure P2.98

SOLUTION:

2.98 Find I_o





$R_1 = 4k\Omega \quad R_2 = 3k\Omega$

$R_3 = 6k\Omega \quad R_4 = 4k\Omega$

$R_5 = 12k\Omega$

$R_{EQ} = R_1 + R_A = 6k\Omega$

$R_A = R_3 // R_4 // R_5 = 2k\Omega$

$$I_o = -12\times10^{-3}\left[\frac{R_2}{R_2 + R_{EQ}}\right]$$

$$\boxed{I_o = -4mA}$$

2.99 Given V_o in the network in Fig. P2.99, find I_A.

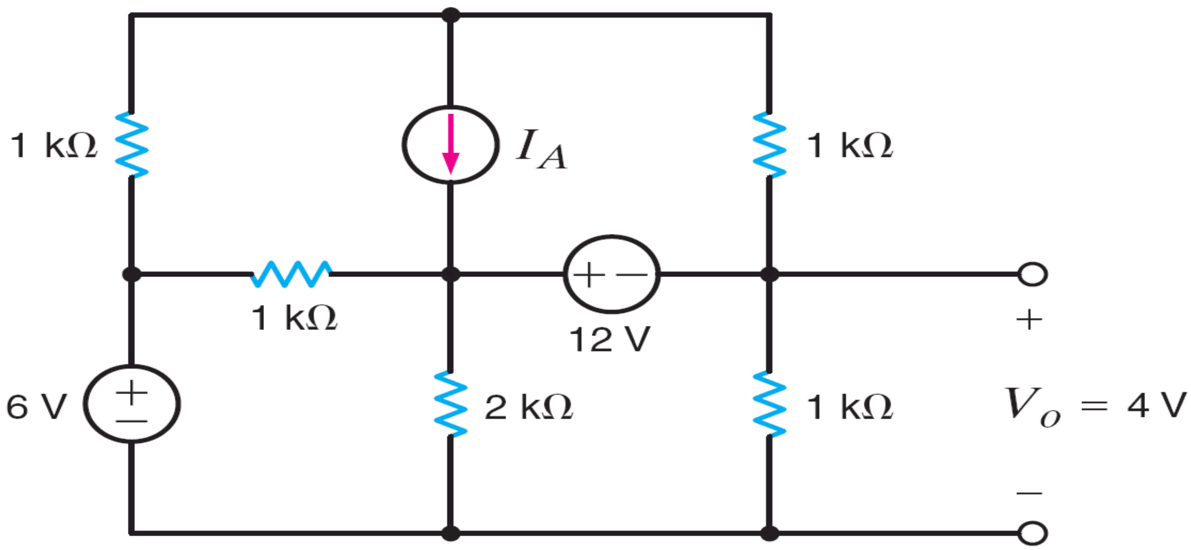
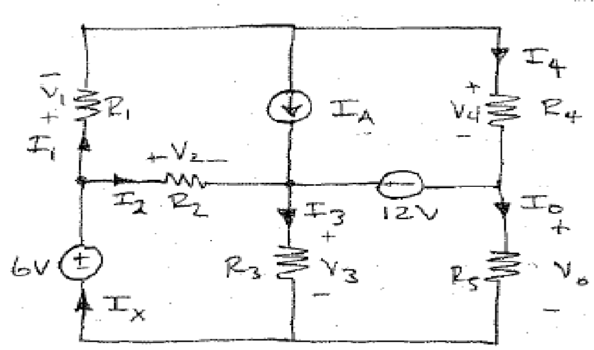


Figure P2.99

SOLUTION:

2.99 $V_o = 4V$. Find I_A



$R_1 = R_2 = R_4 = R_5 = 1k\Omega \quad R_3 = 2k\Omega$

$I_o = V_o / R_5 = 4mA$

$V_3 = 12 + I_o R_5 = 16V$

$I_3 = V_3 / R_3 = 8mA$

$I_x = I_3 + I_o = 12mA$

$V_2 = 6 - V_3 = -10V$

$I_2 = V_2 / R_2 = -10mA$

$I_1 = I_x - I_2 = 22mA$

$V_1 = I_1 R_1 = 22V$

$V_4 = -V_o + 6 - V_1 = -20V$

$I_4 = V_4 / R_4 = -20mA$

$I_A = I_1 - I_4 = 42mA \quad \boxed{I_A = 42mA}$

2.100 Given $I_o = 2$ mA in the circuit in Fig. P2.100, find I_A.

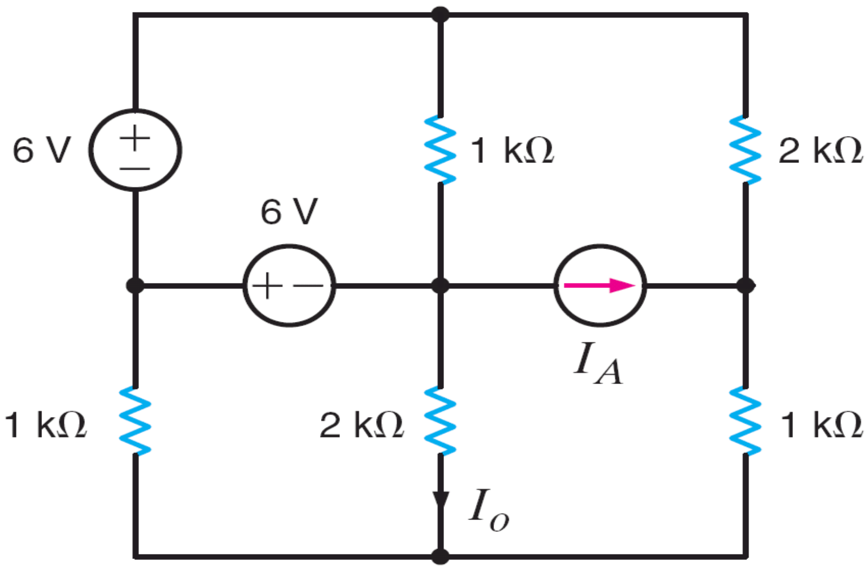


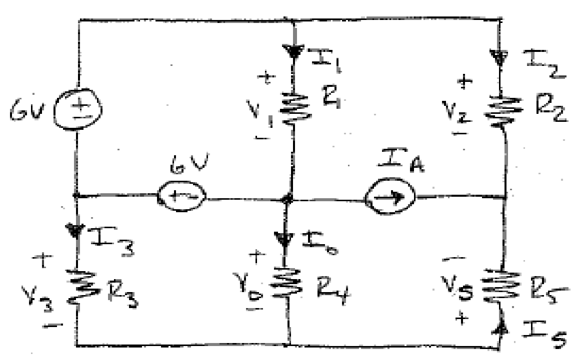
Figure P2.100

SOLUTION:

2.100 $I_o = 2mA$. Find I_A.

$R_4 I_o = V_o = 4V$

$V_3 = 6 + V_o = 10V$



$R_1 = R_3 = R_5 = 1k\Omega \quad R_2 = R_4 = 2k\Omega$

$I_3 = V_3/R_3 = 10mA$

$I_5 = I_3 + I_o = 12mA$

$V_1 = 6 + 6 = 12V$

$I_1 = V_1/R_1 = 12mA$

$V_2 = 6 + I_3R_3 + I_5R_5 = 28V$

$I_2 = V_2/R_2 = 14mA$

$I_A = -I_2 - I_5 \Rightarrow \boxed{I_A = -26mA}$

2.101 Given $I_o = 2$ mA in the network in Fig. P2.101, find V_A.

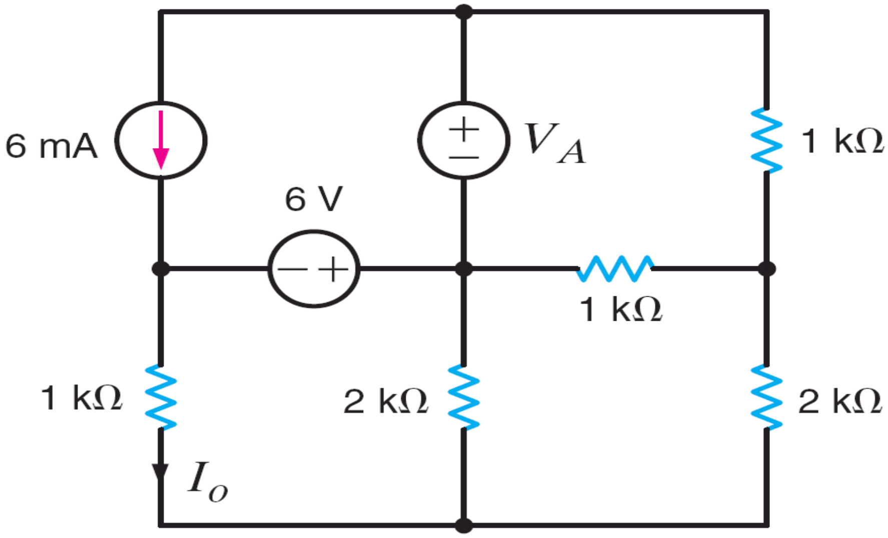
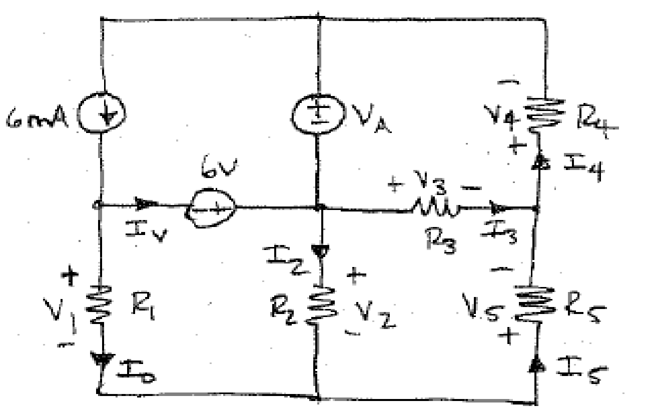


Figure P2.101

SOLUTION:

2.101 $I_o = 2mA$. Find V_A.



$R_1 = R_3 = R_4 = 1k\Omega \quad R_2 = R_5 = 2k\Omega$

$V_1 = I_o R_1 = 2V$

$V_2 = 6 + V_1 = 8V$

$I_2 = V_2 / R_2 = 4mA$

$I_5 = I_o + I_2 = 6mA$

$V_5 = I_5 R_5 = 12V$

$V_3 = 6 + V_1 + V_5 = 20V$

$I_3 = V_3 / R_3 = 20mA$

$I_4 = I_3 + I_5 = 26mA$

$V_4 = R_4 I_4 = 26V$

$V_A = -V_4 - V_3 \Rightarrow \boxed{V_A = -46V}$

2.102 Find the power absorbed by the network in Fig. P2.102.

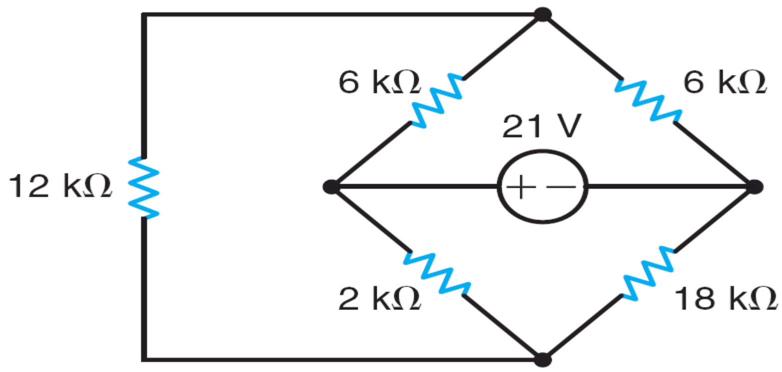
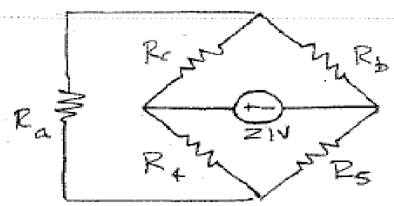


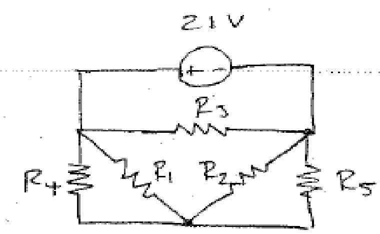
Figure P2.102

SOLUTION:

2.102 Find power absorbed.



R_a, R_b, R_c connected in wye config.



$R_a = 12k\Omega \quad R_b = 6k\Omega$

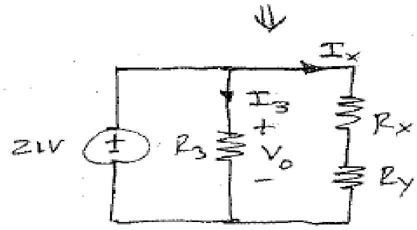
$R_c = 6k\Omega \quad R_4 = 2k\Omega$

$R_5 = 18k\Omega$

$$R_1 = \frac{R_aR_b + R_bR_c + R_aR_c}{R_b} = 30\,k\Omega$$

$R_2 = 30k\Omega \quad R_3 = 15\,k\Omega$

$\Downarrow$



$R_x = R_1 /\!/ R_4 = 1.875\,k\Omega$

$R_y = R_2 /\!/ R_5 = 11.25\,k\Omega$

$$P = \frac{V_o^2}{R_3} + \frac{V_o^2}{R_x + R_y}$$

$$\boxed{P = 63\,mW}$$

2.103 Find the value of g in the network in Fig. P2.103 such that the power supplied by the 3-A source is 20 W.

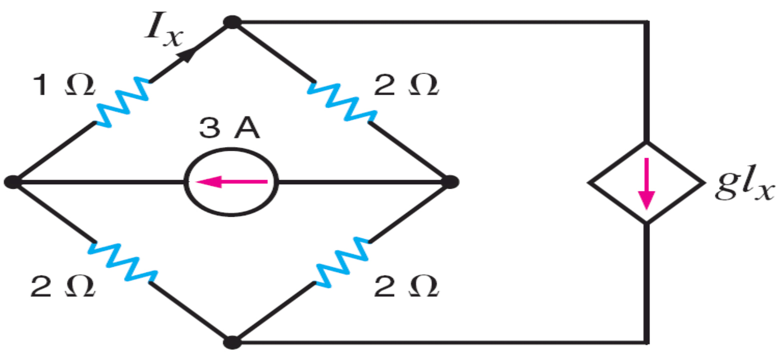
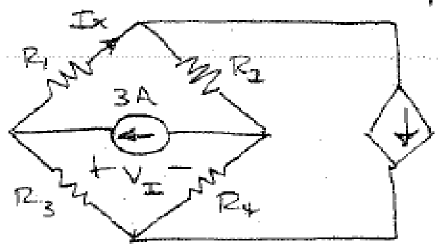


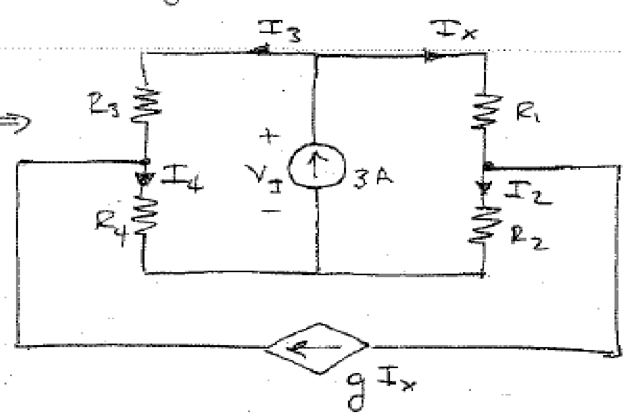
Figure P2.103

SOLUTION:

2.103 $P_{3A} = 20\,W$ supplied. Find g



$\Rightarrow$



$R_1 = 1\Omega \quad R_2 = R_3 = R_4 = 2\Omega$

$P_{3A} = 20 = V_I(3) \Rightarrow V_I = \frac{20}{3}V$

$V_I = I_x R_1 + I_2 R_2 \qquad I_2 = I_x(1-g)$

$V_I = I_3 R_3 + I_4 R_4 \qquad I_3 = 3 - I_x \qquad I_4 = I_3 + gI_x$

yields 2 equations

$$\left.\begin{aligned} \frac{20}{3} &= 3I_x - 2gI_x \\ \frac{20}{3} &= 12 - 4I_x + 2gI_x \end{aligned}\right\} \Rightarrow \boxed{g = 4}$$

2.104 Find the power supplied by the 24-V source in the circuit in Fig. P2.104.

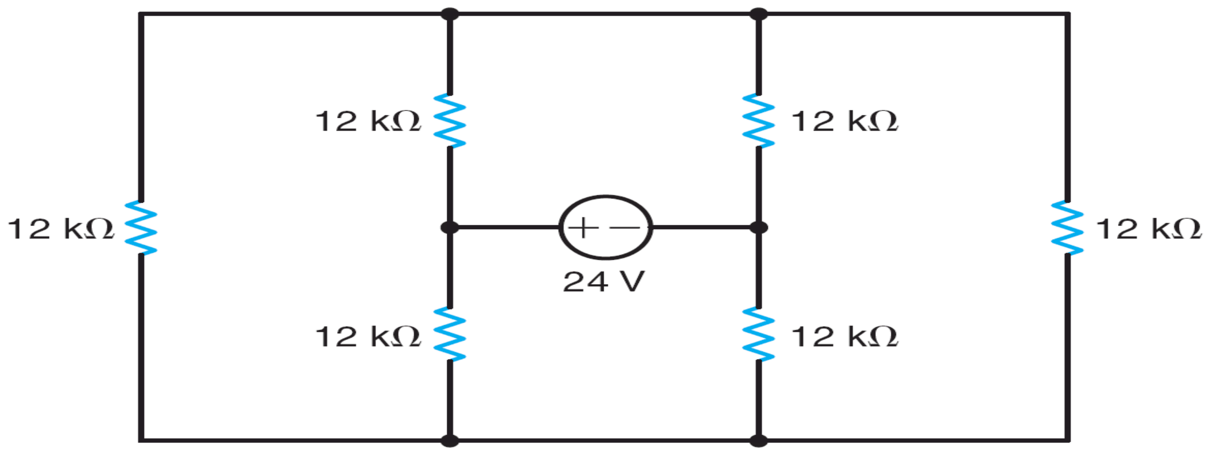
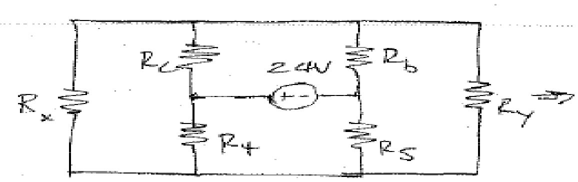


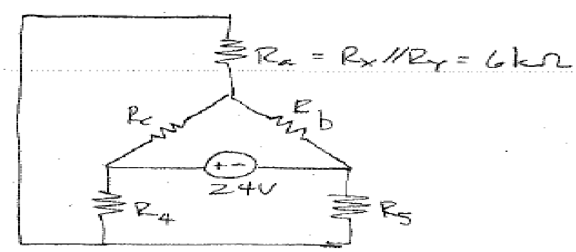
Figure P2.104

SOLUTION:

2.104 Find source power.



All $R = 12k\Omega$



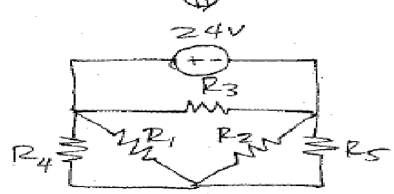
$R_a - R_b - R_c \Rightarrow$ wye connected

$$R_1 = \frac{R_aR_b + R_bR_c + R_aR_c}{R_b}$$

$R_1 = 24k\Omega$

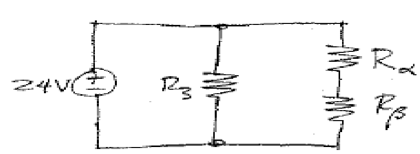
$R_2 = 24k\Omega$

$R_3 = 48k\Omega$



$R_\alpha = R_1 // R_4 = 8k\Omega$

$R_\beta = R_2 // R_5 = 8k\Omega$



$$P_{24V} = \frac{24^2}{R_3} + \frac{24^2}{R_\alpha + R_\beta}$$

$P_{24V} = 48$ mW

2.105 Find I_o in the circuit in Fig. P2.105. **PSV**

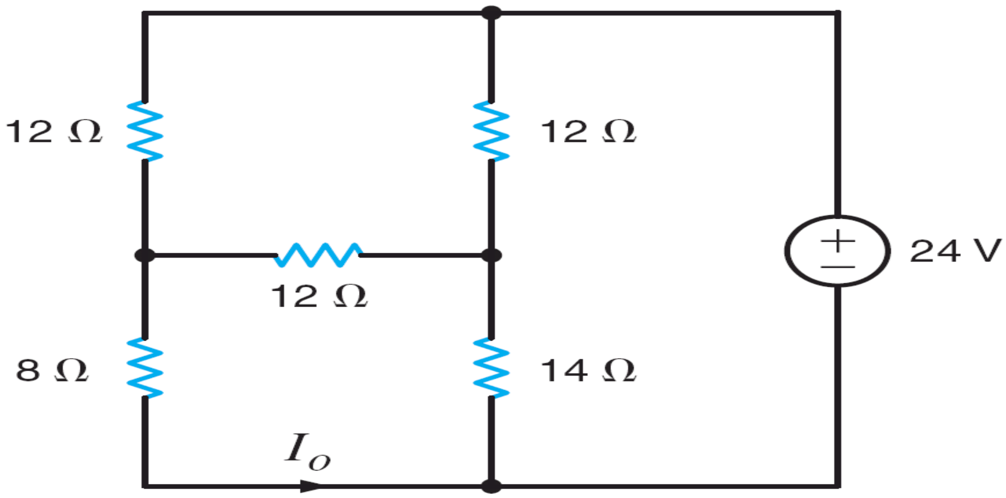
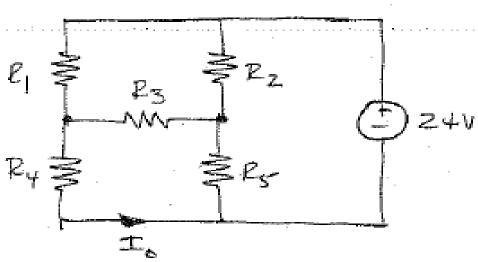


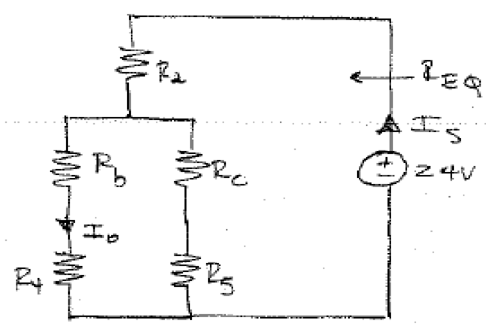
Figure P2.105

SOLUTION:

2.105 Find I_o



$R_1 - R_2 - R_3$ connected delta $\longrightarrow$



$R_1 = R_2 = R_3 = 12\Omega \quad R_4 = 8\Omega \quad R_5 = 14\Omega$

$R_a = R_b = R_c = 4\Omega$

$R_{EQ} = R_a + \left[(R_b + R_4) /\!/ (R_c + R_5)\right]$

$R_{EQ} = 11.2\ \Omega$

$I_S = \dfrac{24}{R_{EQ}} = 2.14\text{A} \qquad I_o = I_S \left[\dfrac{R_c + R_5}{R_c + R_5 + R_b + R_4}\right]$

$$\boxed{I_o = 1.29\text{A}}$$

2.106 Find I_o in the circuit in Fig. P2.106. CS

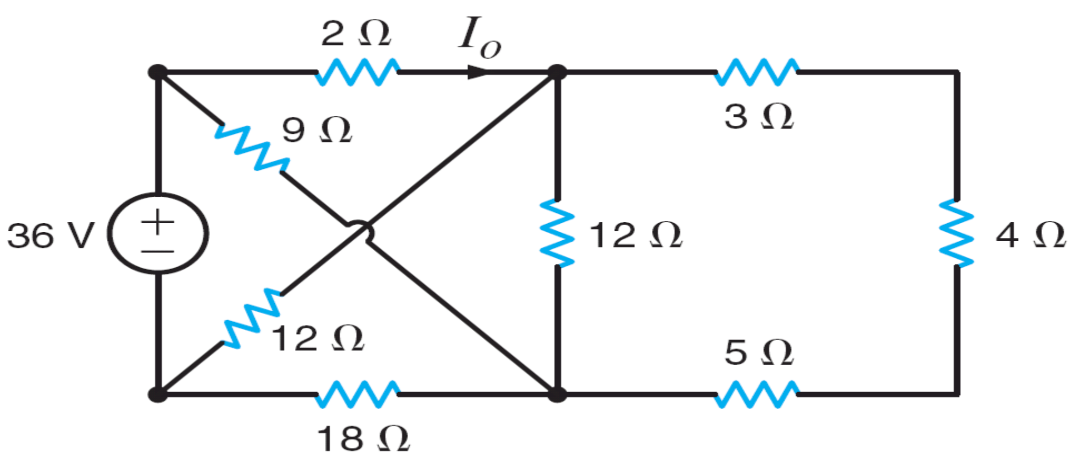
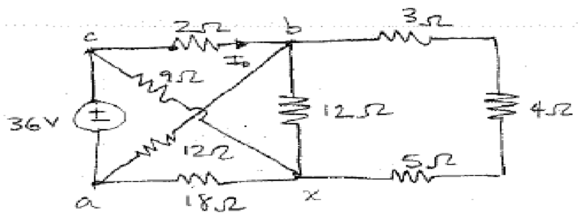


Figure P2.106

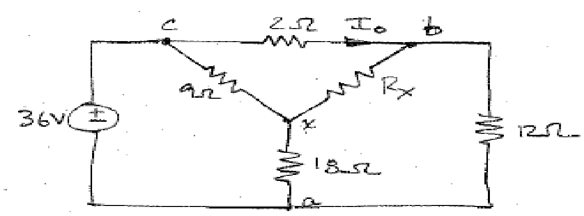
SOLUTION:

2.106 $I_o = ?$

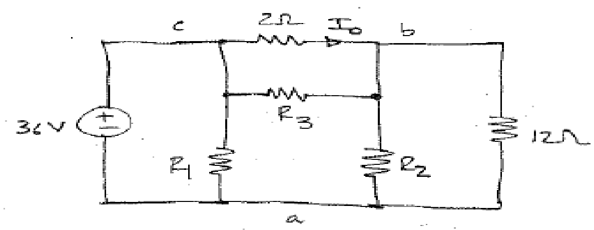


$R_x = 12 /\!/ (3+4+5) = 6\Omega$

⇓



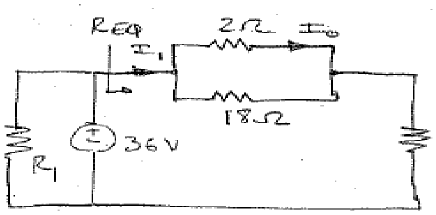
⇒



18Ω, 9Ω, R_x are wye

$R_1 = 54\Omega \quad R_2 = 36\Omega \quad R_3 = 18\Omega$

⇙



$R_Y = R_2 /\!/ 12 = 9\Omega$

$R_{EQ} = 2/\!/18 + R_Y = 10.8\Omega$

$I_1 = \frac{36}{R_{EQ}} = 3.33\text{ A}$

$I_o = I_1\left[\frac{18}{18+2}\right] \Rightarrow \boxed{I_o = 3\text{ A}}$

2.107 Find V_o in the network in Fig. P2.107.

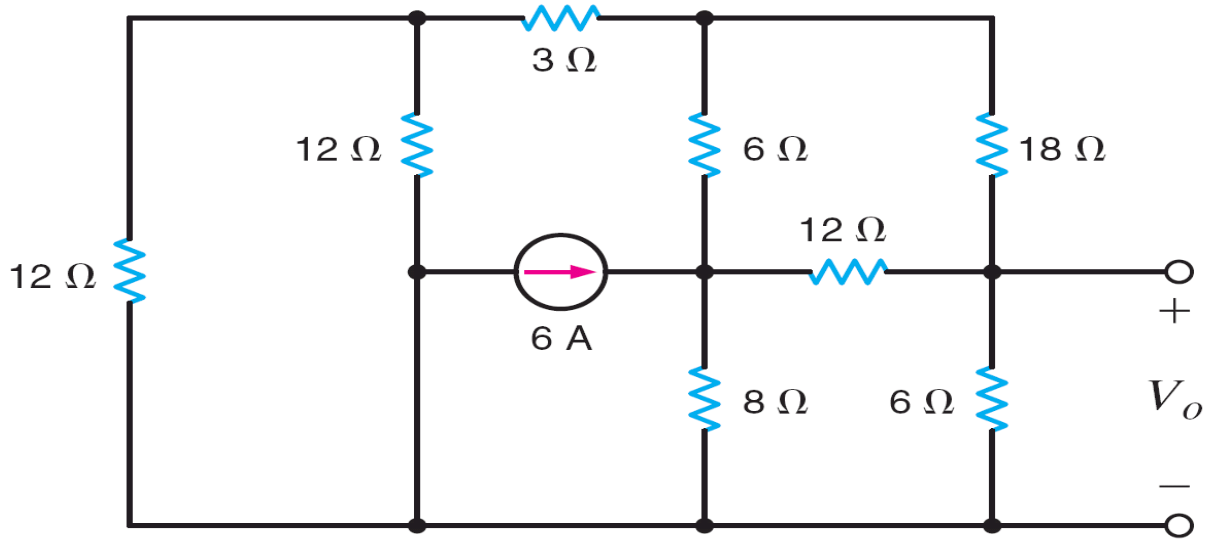
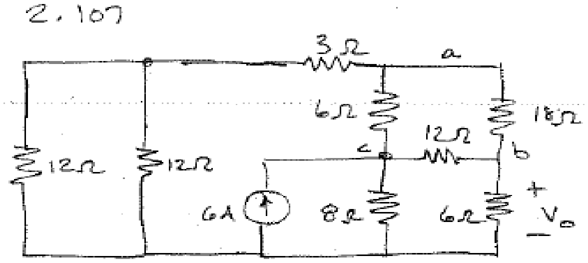


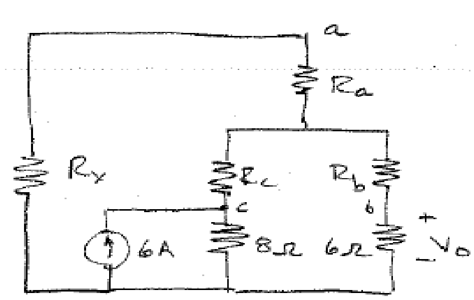
Figure P2.107

SOLUTION:

2.107



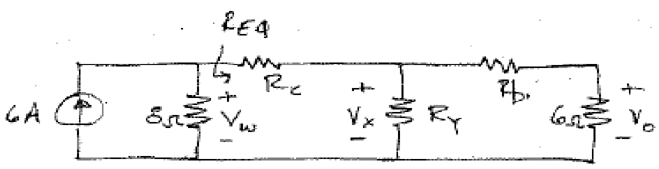
6Ω - 18Ω - 12Ω connected Δ



$R_x = 3 + (12 // 12) = 9\Omega$

$R_a = 3\Omega \quad R_b = 6\Omega \quad R_c = 2\Omega$

$R_Y = R_a + R_x = 12\Omega$



$R_{EQ} = R_c + [R_Y // (R_b + 6)] = 8\Omega$

$V_W = 6\,[8 // R_{EQ}] = 24V$

$V_x = V_W \left[\dfrac{R_{EQ} - R_c}{R_{EQ}}\right] = 18V$

$V_o = V_x \left[\dfrac{6}{6 + R_b}\right]$ $\boxed{V_o = 9V}$

2.108 Find I_x in the circuit in Fig. P2.108.

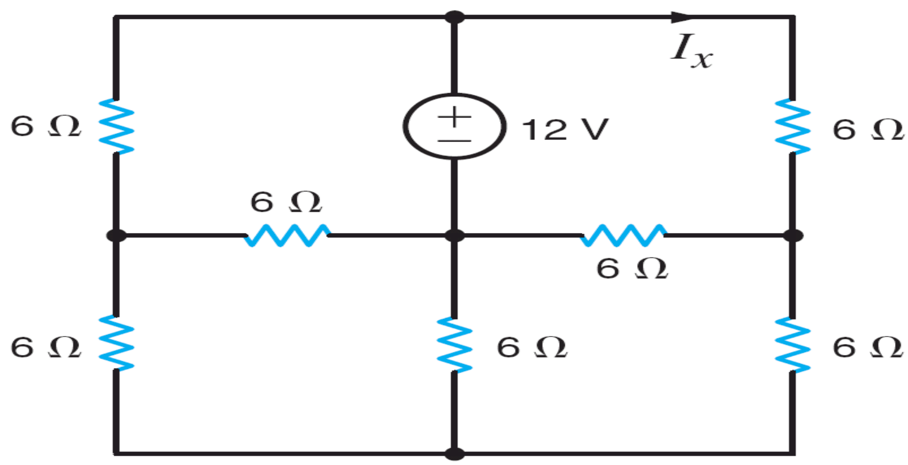
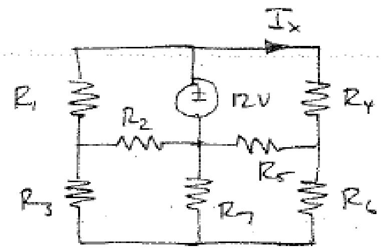


Figure P2.108

SOLUTION:

2. 108 Find I_x



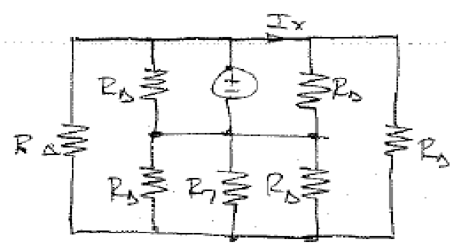
All $R = 6\Omega$

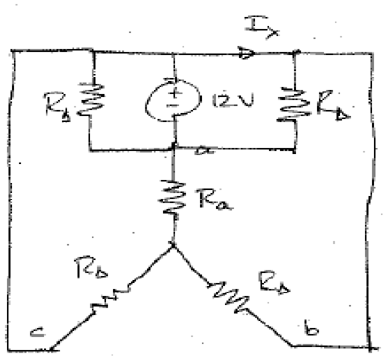
$R_1 - R_2 - R_3$ in wye

also

$R_4 - R_5 - R_6$ in wye

$R_Y = 6\Omega \Rightarrow R_\Delta = 18\Omega$

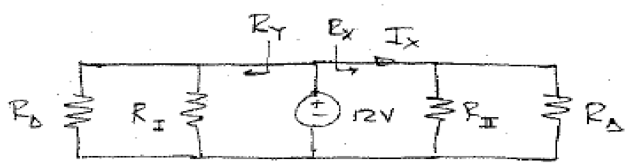




$R_a - R_\Delta - R_\Delta$ in wye!

$R_a = R_\Delta // R_\Delta // R_7 = 3.6\Omega$

$\Rightarrow$



$$R_I = \frac{R_a R_\Delta + R_\Delta^2 + R_a R_\Delta}{R_a} = 25.2\Omega$$

$$R_{II} = R_I = 25.2\Omega$$

$$R_x = R_{II} // R_\Delta = 10.5\Omega = R_Y$$

$$I_x = 12 / R_x = 1.14\ A$$

$$\boxed{I_x = 1.14A}$$

2.109 Find I_o in the circuit in Fig. P2.109. CS

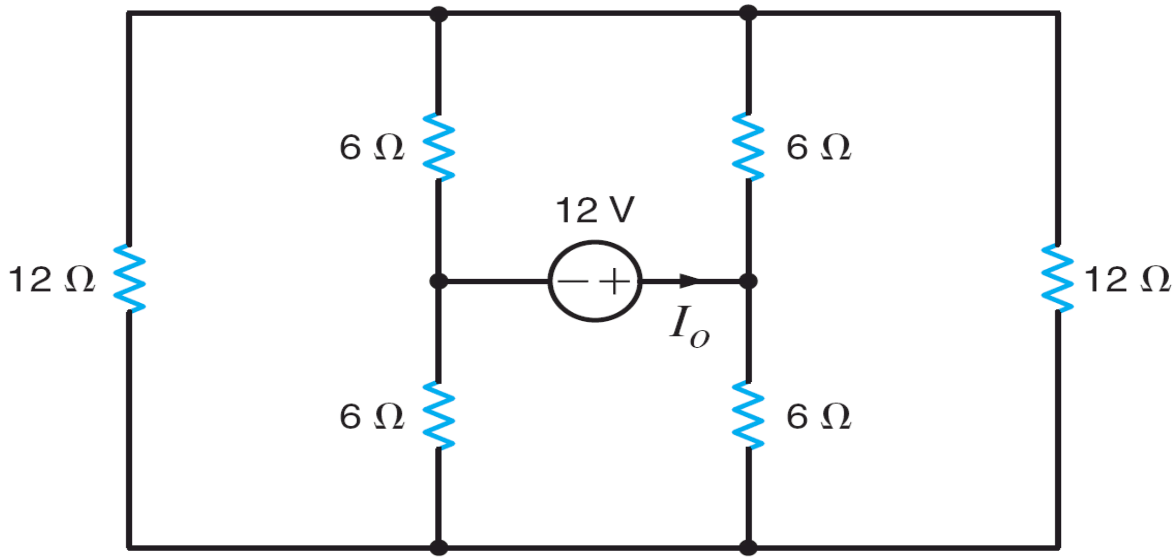
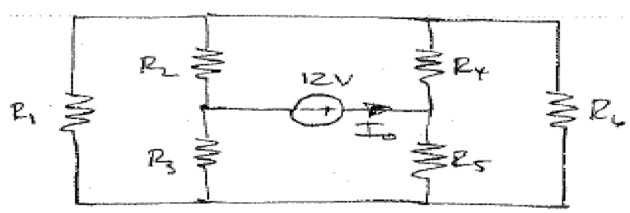
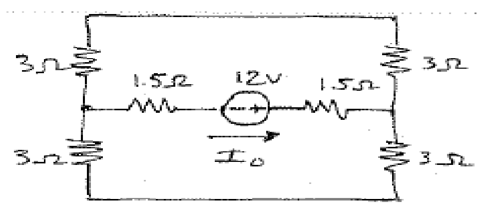


Figure P2.109

SOLUTION:

2.109 Find I_o.

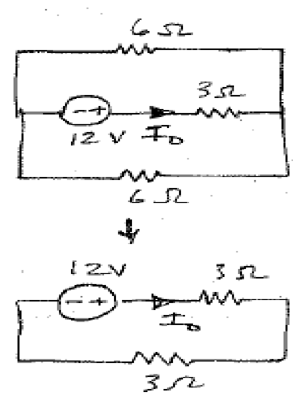




$R_1 = R_6 = 12\,\Omega$ $\quad R_2 = R_3 = R_4 = R_5 = 6\,\Omega$

$R_1 - R_2 - R_3$ connected Δ

$R_4 - R_5 - R_6$ connected Δ



$I_o = \frac{12}{6}$ $\quad \boxed{I_o = 2\,A}$

2.110 Find V_o in the circuit in Fig. P2.110.

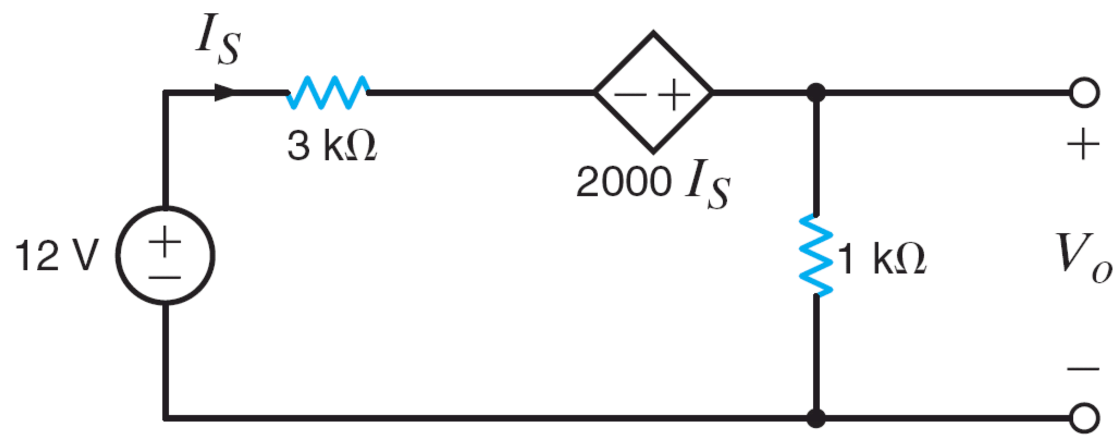
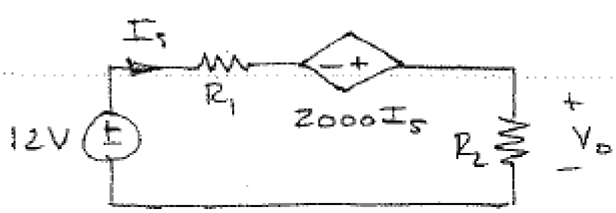


Figure P2.110

SOLUTION:

2.110 Find V_o



$R_1 = 3\,k\Omega \qquad R_2 = 1\,k\Omega$

$$12 = I_S R_1 - 2000 I_S + R_2 I_S$$

$$I_S = \frac{12}{2000} = 6\,mA$$

$$V_o = I_S R_2 \qquad \boxed{V_o = 6V}$$

2.111 Find V_o in the network in Fig. P2.111.

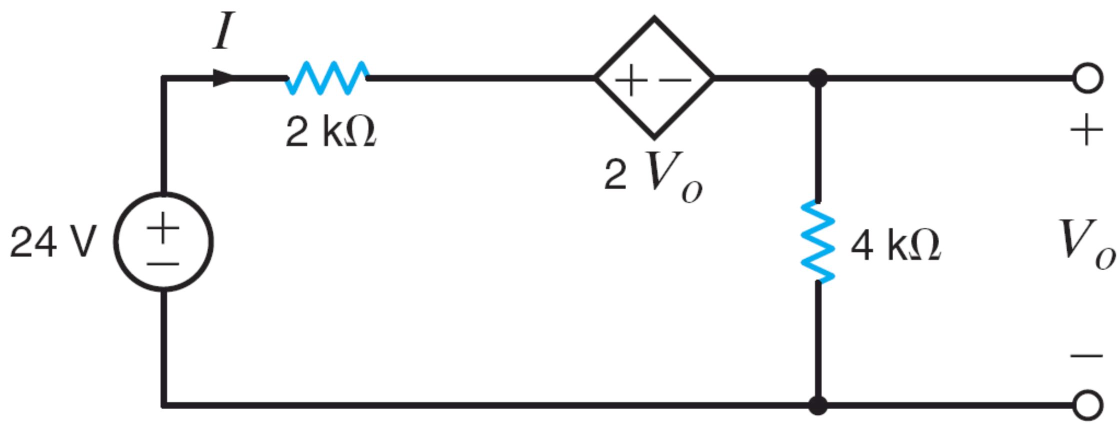


Figure P2.111

SOLUTION:

2.111 Find V_o.

$R_1 = 2k\Omega$ $R_2 = 4k\Omega$

$$24 = R_1 I + 2V_o + R_2 I$$

$$V_o = R_2 I = 4I$$

$$24 = I(14) \Rightarrow I = \frac{12}{7}$$

$$\boxed{V_o = 6.86\text{ V}}$$

2.112 Find V_o in the network in Fig. P2.112. PSV

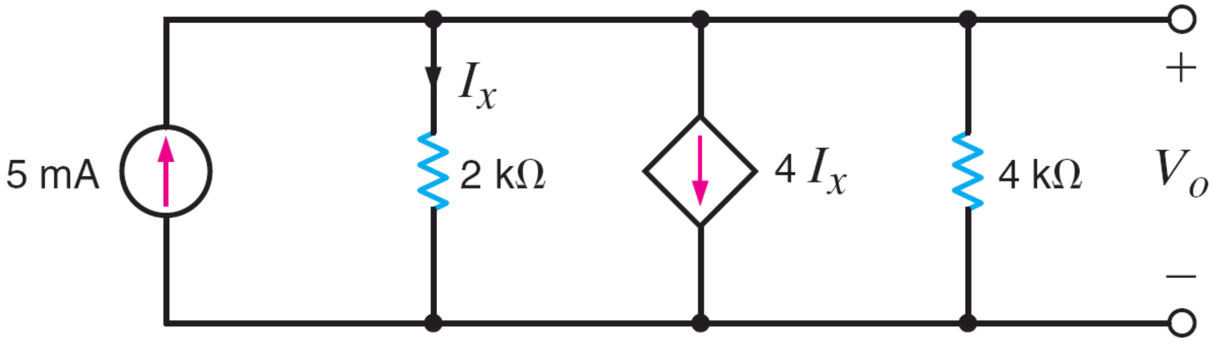
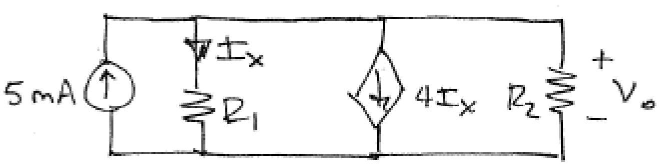


Figure P2.112

SOLUTION:

2.112 Find V_o



$R_1 = 2\,k\Omega$ $R_2 = 4\,k\Omega$

$$5\times10^{-3} = \frac{V_o}{R_1} + 4I_x + \frac{V_o}{R_2}$$

$$I_x = \frac{V_o}{R_1}$$

$$\boxed{V_o = 1.82\,V}$$

2.113 Find I_o in the network in Fig. P2.113. CS

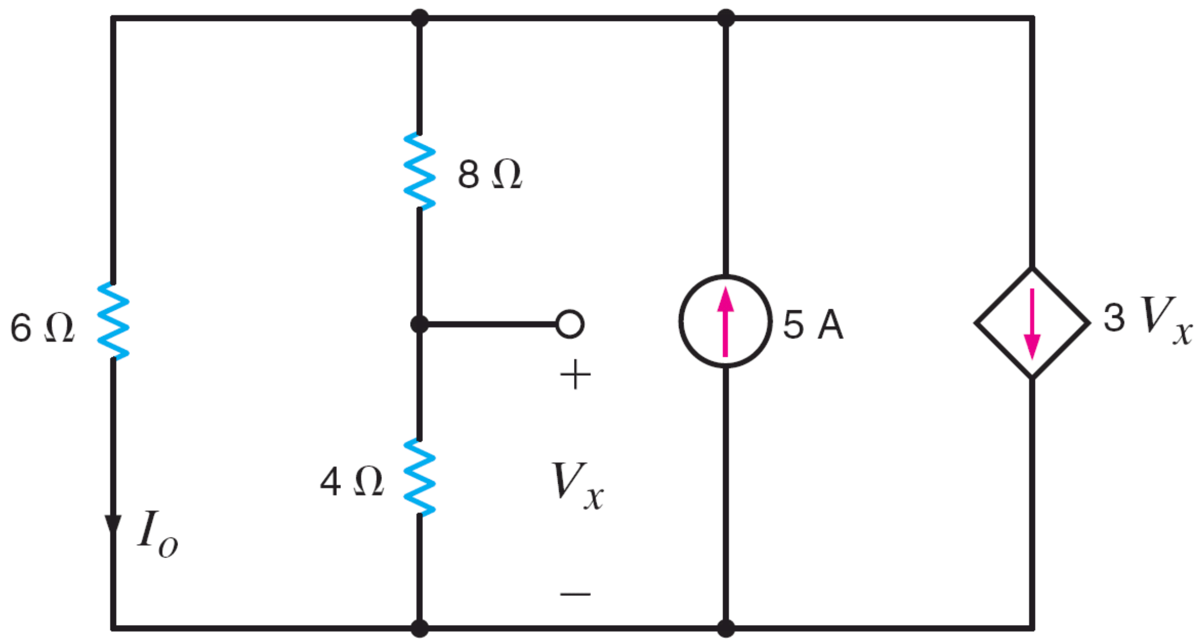
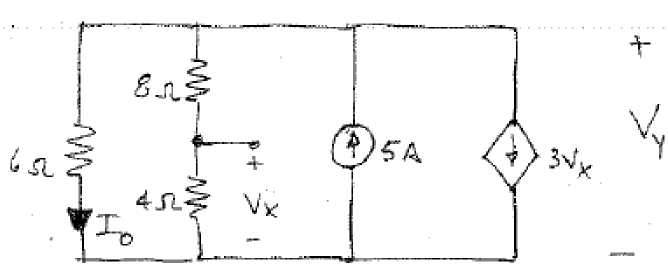


Figure P2.113

SOLUTION:

2.113 Find I_o



$$5 = 3V_x + \frac{V_Y}{6} + \frac{V_Y}{12}$$

$$V_x = V_Y\left[\frac{4}{4+8}\right] = \frac{V_Y}{3}$$

$$V_Y = 4V$$

$$I_o = \frac{V_Y}{6} \Rightarrow \boxed{I_o = 0.67A}$$

2.114 Find the power absorbed by the 10-kΩ resistor in the circuit in Fig. P2.114.

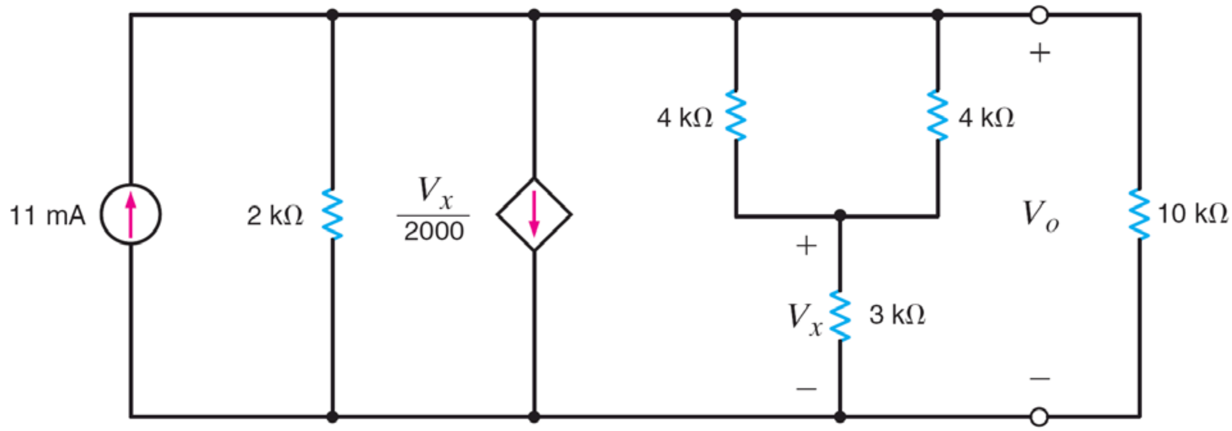
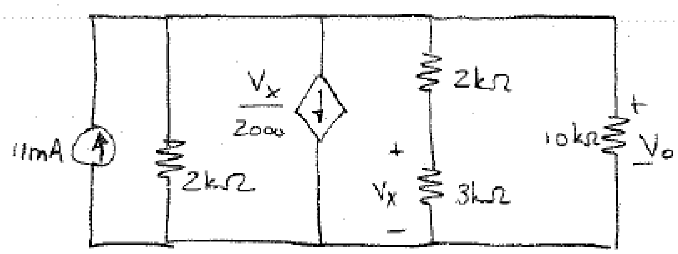


Figure P2.114

SOLUTION:

2.114 Find P_{10k}



$$11\times10^{-3} = \frac{V_o}{2000} + \frac{V_x}{2000} + \frac{V_o}{5000} + \frac{V_o}{10^4}$$

$$V_x = V_o\left[\frac{3000}{3000+2000}\right] = \frac{6V_o}{10}$$

$$V_o = 10V$$

$$P_{10k} = \frac{V_o^2}{10^4} \Rightarrow \boxed{P_{10k} = 10mW}$$

2.115 Find the value of k in the network in Fig. P2.115 such that the power supplied by the 6-A source is 108 W.

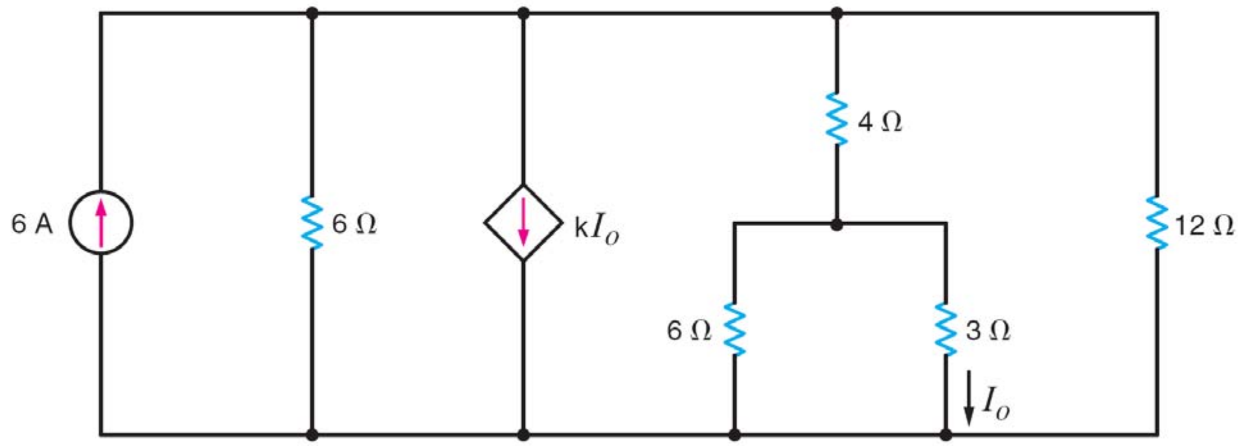
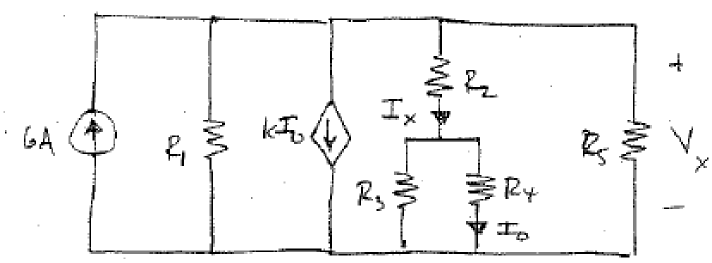


Figure P2.115

SOLUTION:

2.115 $P_{6A} = 108\text{ W}$ supplied. Find k.



$$6 = \frac{V_x}{R_1} + kI_o + \frac{V_x}{R_A} + \frac{V_x}{R_5}$$

$$I_o = I_x\left[\frac{R_3}{R_3+R_4}\right] = \frac{2}{3} I_x$$

$$I_x = \frac{V_x}{R_A} = \frac{V_x}{6}$$

$$R_A = R_2 + \left[R_3 /\!/ R_4\right] = 6\,\Omega$$

$R_1 = R_3 = 6\,\Omega \quad R_2 = 4\,\Omega$

$R_4 = 3\,\Omega \quad R_5 = 12\,\Omega$

$P_{6A} = 6V_x = 108 \Rightarrow V_x = 18\text{ V}$ So, $I_x = 3A$ $\quad I_o = 2A$

$$\boxed{k = -0.75}$$

2.116 For the network in Fig. P2.116, choose the values of R_{in} and R_o such that is V_o maximized. What is the resulting ratio, V_o/V_S? CS

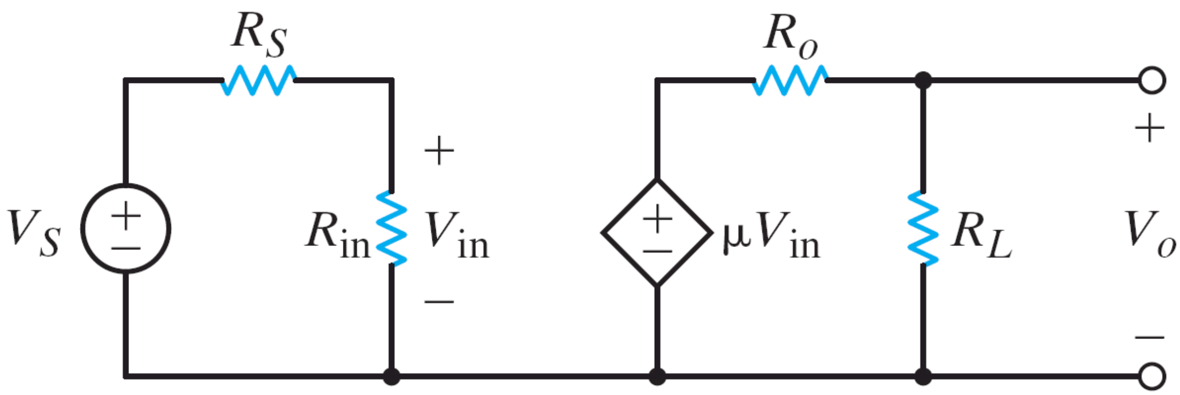
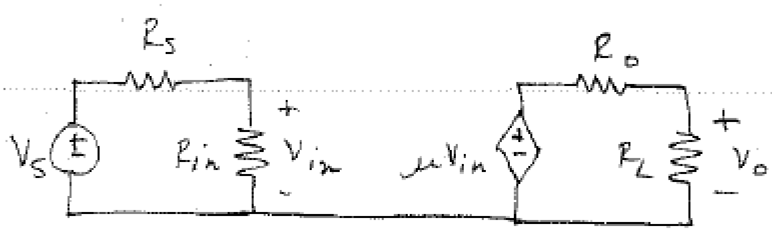


Figure P2.116

SOLUTION:

2.116 Find R_{in} and R_o to maximize V_o/V_s.



$$V_{in} = V_s \left[\frac{R_{in}}{R_{in}+R_s}\right] \qquad V_o = \mu V_{in}\left[\frac{R_L}{R_o+R_L}\right]$$

$$\frac{V_o}{V_s} = \mu\left[\frac{R_{in}}{R_{in}+R_s}\right]\left[\frac{R_L}{R_o+R_L}\right] \Rightarrow \mu \text{ at best.}$$

$$\boxed{R_{in} = \infty \;\; \& \;\; R_o = 0}$$

2.117 A typical transistor amplifier is shown in Fig. P2.117. Find the amplifier gain G (i.e., the ratio of the output voltage to the input voltage).

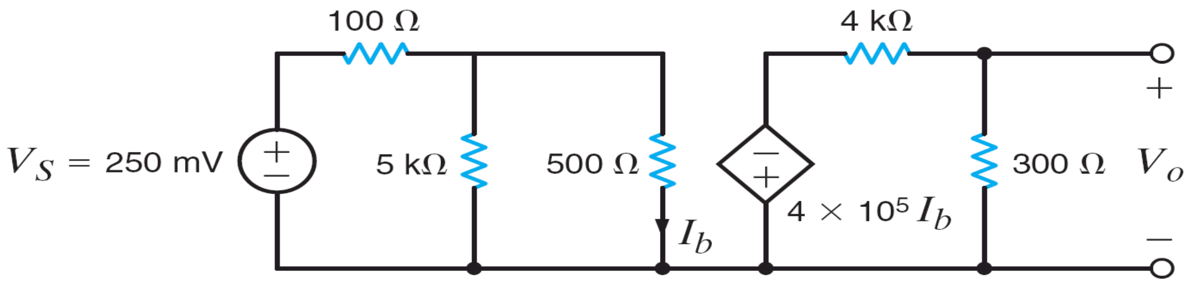
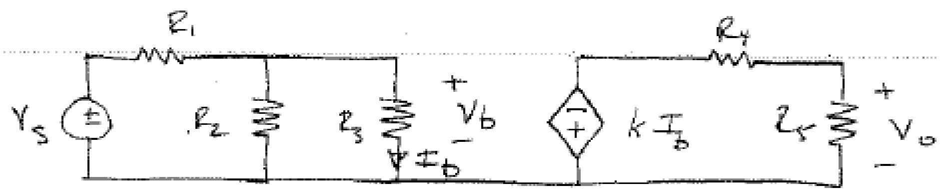


Figure P2.117

SOLUTION:

2.117 Find $G = V_o / V_S$



$V_S = \frac{1}{4}$ V $\quad R_1 = 100\Omega \quad R_2 = 5k\Omega \quad R_3 = 500\Omega$

$k = 4 \times 10^5 \quad R_4 = 4k\Omega \quad R_5 = 300\Omega$

$$V_b = V_S \left[\frac{R_2 // R_3}{R_1 + (R_2 // R_3)} \right] = 0.205 \text{ V}$$

$$I_b = V_b / R_3 = 410 \mu A$$

$$V_o = -k I_b \left[\frac{R_5}{R_4 + R_5} \right] \Rightarrow V_o = -11.4 \text{ V}$$

$$G = \frac{V_o}{V_S} \qquad \boxed{G = -45.8}$$

2.118 In many amplifier applications we are concerned not only with voltage gain, but also with power gain.

Power gain = A_p (power delivered to the load)/(power delivered to the input)

Find the power gain for the circuit in Fig. P2.118, where $R_L = 60\ \text{k}\Omega$.

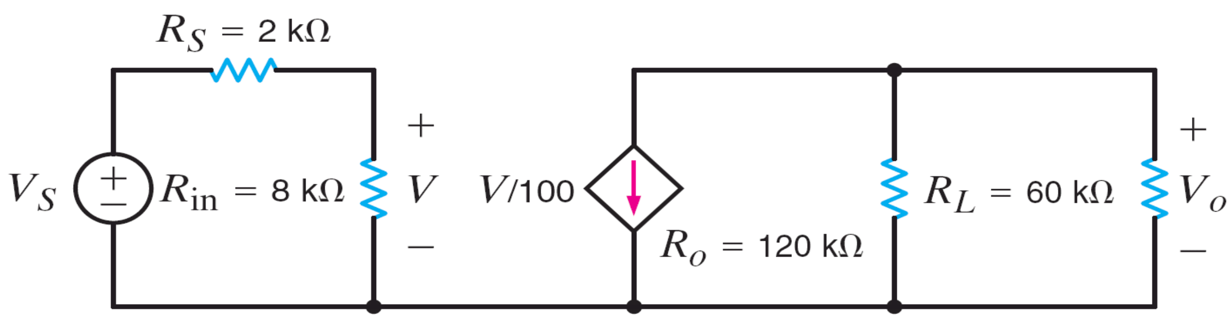


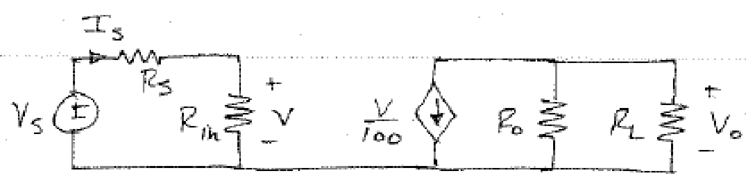
Figure P2.118

SOLUTION:

2.118 Find P_{OUT}/P_{in}

$R_S = 2k\Omega \quad R_{in} = 8k\Omega$

$R_o = 120k\Omega \quad R_L = 60k\Omega$



$P_{in} = V_S I_S \qquad I_S = \frac{V_S}{R_S + R_{in}} \qquad$ so, $\quad P_{in} = \frac{V_S^2}{R_S + R_{in}}$

$P_{OUT} = \frac{V_o^2}{R_L} \qquad V_o = -V_S\left[\frac{R_{in}}{R_{in}+R_S}\right]\left(\frac{1}{100}\right)\left[R_o // R_L\right]$

$V_o = -320 V_S$

$P_{OUT} = 1.71\ V_S^2$

$A_P = \frac{P_{OUT}}{P_{IN}} \qquad \boxed{A_P = 17.1 \times 10^3}$

2FE-1 Find the power generated by the source in the network in Fig. 2PFE-1. **CS**

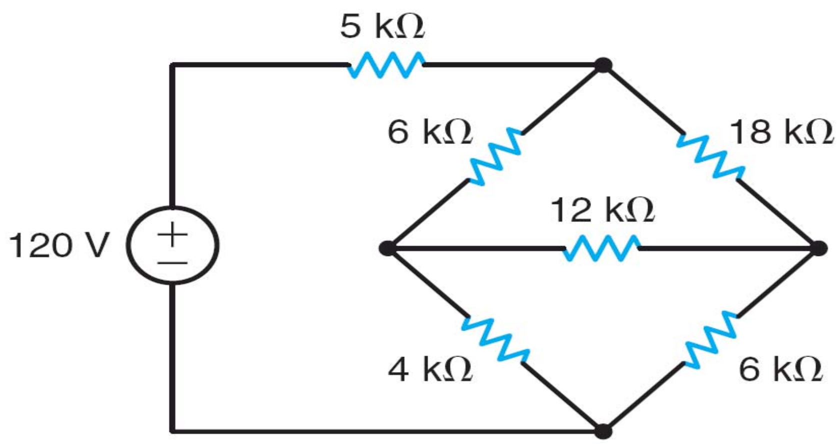
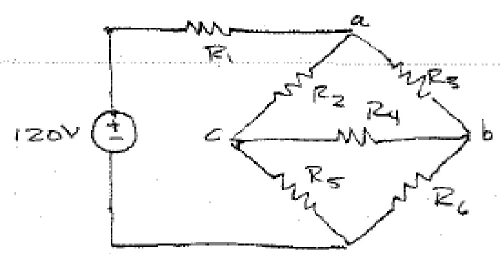


Fig. 2PFE-1

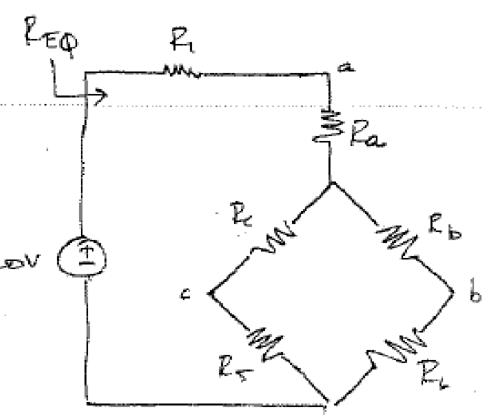
SOLUTION:

2FE-1 Find source Power, P_{VS}



$R_2 - R_3 - R_4$ are connected delta

$\Rightarrow$



$R_1 = 5k\Omega$ $R_2 = R_6 = 6k\Omega$

$R_3 = 18k\Omega$ $R_4 = 12k\Omega$

$R_5 = 4k\Omega$

$R_a = 3k\Omega$ $R_b = 6k\Omega$ $R_c = 2k\Omega$

$$R_{EQ} = R_1 + R_a + \left[(R_c + R_5) // (R_b + R_6) \right] = 12k\Omega$$

$$P_{VS} = \frac{V_s^2}{R_{EQ}} = \frac{120^2}{12 \times 10^3} \qquad \boxed{P_{VS} = 1.2\,W}$$

2FE-2 Find the equivalent resistance of the circuit in Fig. 2PFE-2 at the terminals *A*-*B*.

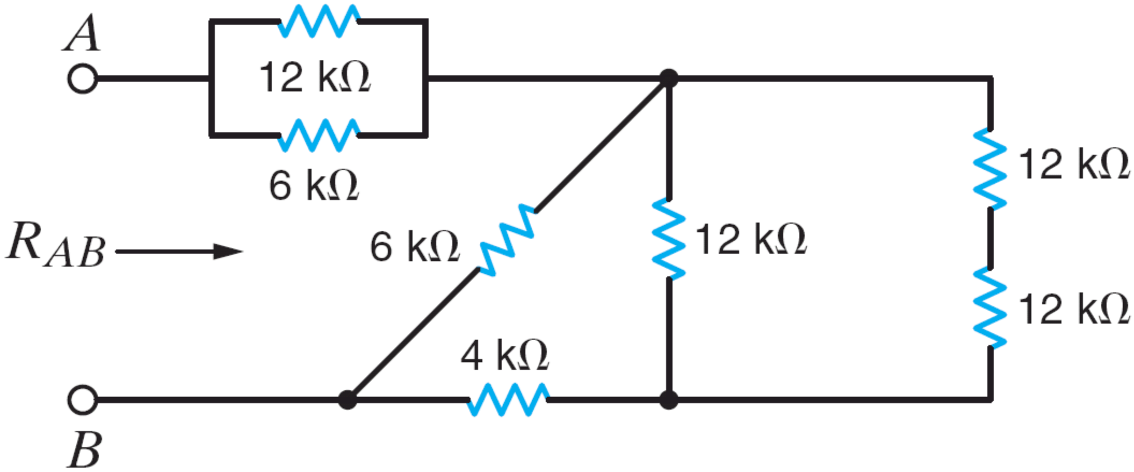
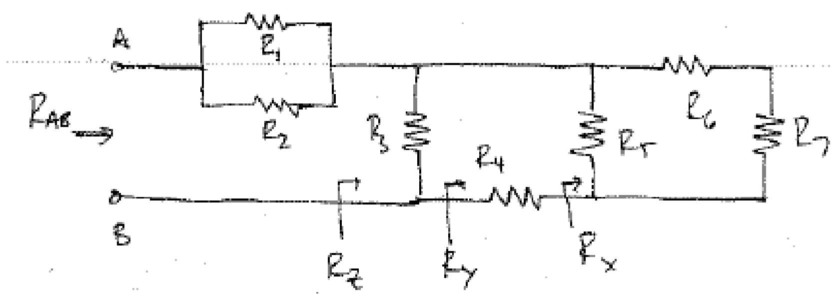


Fig. 2PFE-2

SOLUTION:

2FE-2 Find R_{AB}



$R_1 = R_5 = R_6 = R_7 = 12\,k\Omega$

$R_2 = R_3 = 6\,k\Omega$

$R_4 = 4\,k\Omega$

$R_X = R_5 \,//\, (R_6 + R_7) = 8\,k\Omega$ $\quad R_Y = R_4 + R_X = 12\,k\Omega$

$R_Z = R_3 \,//\, R_Y = 4\,k\Omega$ $\quad R_{AB} = (R_1 \,//\, R_2) + R_Z$ $\quad \boxed{R_{AB} = 8\,k\Omega}$

2FE-3 Find the voltage V_o in the network in Fig. 2PFE-3. **CS**

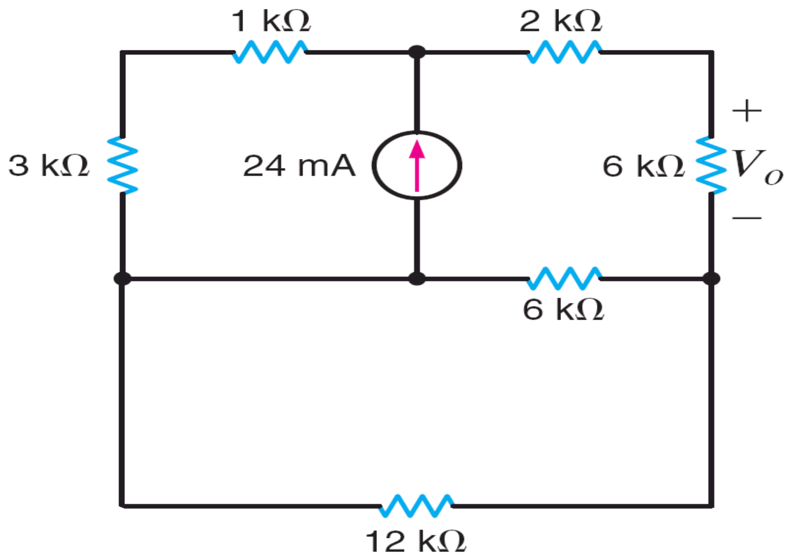
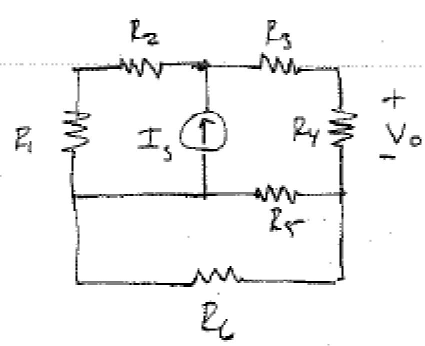


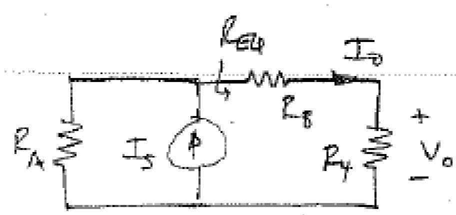
Fig. 2PFE-3

SOLUTION:

2FE-3 Find V_o.



$\Rightarrow$



$R_1 = 3k\Omega \quad R_2 = 1k\Omega$

$R_3 = 2k\Omega \quad R_4 = R_5 = 6k\Omega$

$R_6 = 12k\Omega \quad I_s = 24mA$

$R_A = R_1 + R_2 = 4k\Omega$

$R_B = R_3 + (R_5 // R_6) = 6k\Omega$

$R_{EQ} = R_B + R_4 = 12k\Omega$

$$I_o = I_s \left[\frac{R_A}{R_A + R_{EQ}} \right] = 6mA$$

$V_o = I_o R_4 \qquad \boxed{V_o = 24V}$

2FE-4 Find the current I_o in the circuit in Fig. 2PFE-4.

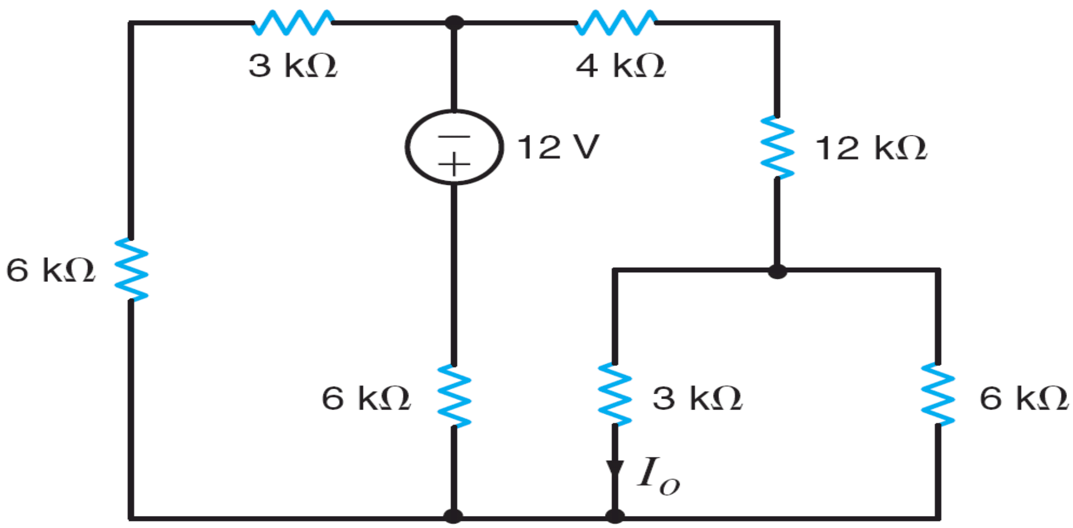
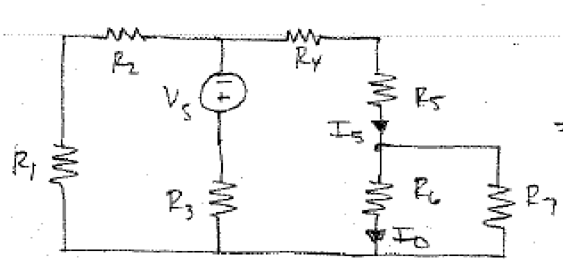


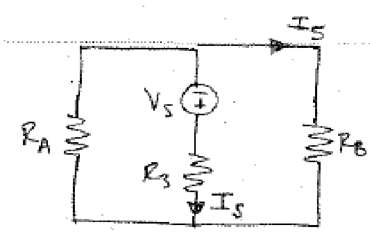
Fig. 2PFE-4

SOLUTION:

2FE-4 Find I_o.



$\Rightarrow$



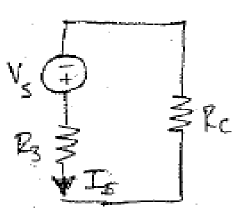
$R_1 = R_3 = R_7 = 6\ k\Omega$ $\quad R_4 = 4\ k\Omega$

$R_2 = R_6 = 3\ k\Omega$ $\quad R_5 = 12\ k\Omega$ $\quad V_s = 12V$

$R_A = R_1 + R_2 = 9\ k\Omega$

$R_B = R_4 + R_5 + (R_6 // R_7) = 18\ k\Omega$

$$I_o = I_5\left(\frac{R_7}{R_6 + R_7}\right) = \frac{2I_5}{3}$$



$R_C = R_A // R_B = 6\ k\Omega$

$$I_s = \frac{V_s}{R_3 + R_C} = 1\ mA$$

$$I_5 = -I_s\left[\frac{R_A}{R_A + R_B}\right] = -\frac{1}{3}\ mA$$

$$\boxed{I_o = -0.22\ mA}$$

3.1 Find I_o in the circuit in Fig. P3.1 using nodal analysis. **CS**

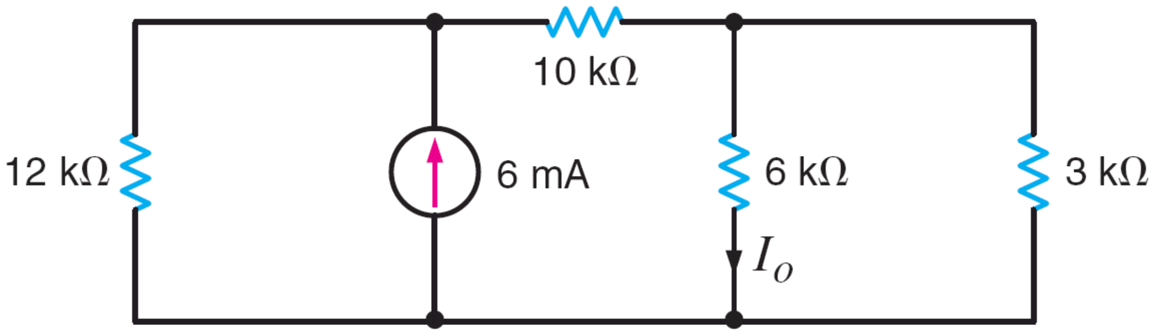
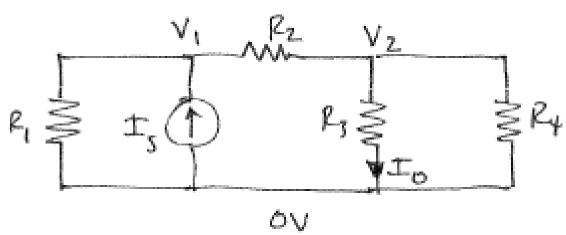


Figure P3.1

SOLUTION:

3.1 I_o via nodal.



$R_1 = 12k\Omega$ $R_2 = 10k\Omega$ $R_3 = 6k\Omega$

$R_4 = 3k\Omega$ $I_s = 6mA$

@ V_1: $\dfrac{V_1}{R_1} - I_S + \dfrac{V_1 - V_2}{R_2} = 0$

@ V_2: $\dfrac{V_2 - V_1}{R_2} + \dfrac{V_2}{R_3} + \dfrac{V_2}{R_4} = 0$

$\therefore$: $I_o = V_2 / R_3$

$$\boxed{I_o = 1\,mA}$$

3.2 Use nodal analysis to find V_1 in the circuit in Fig. P3.2.

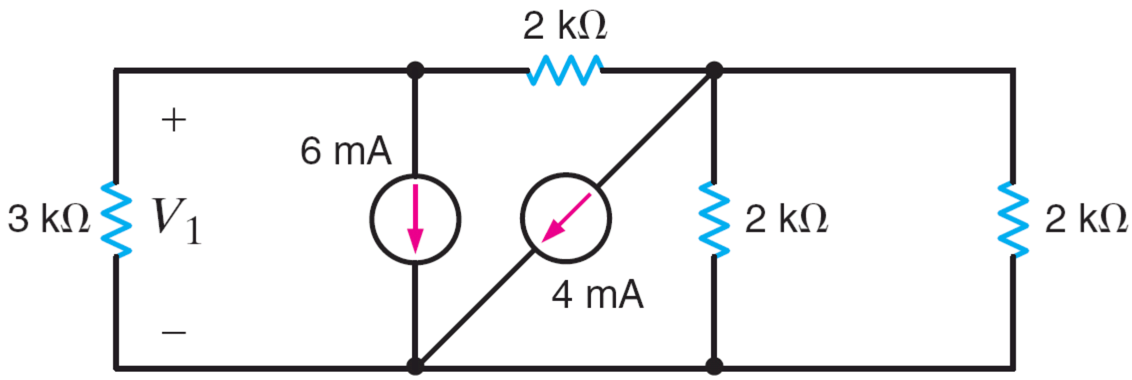
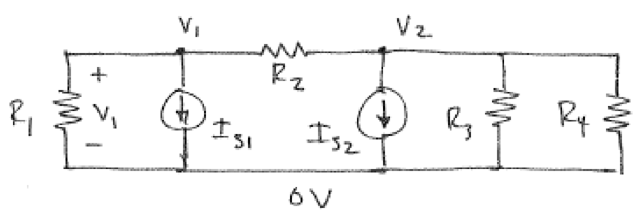


Figure P3.2

SOLUTION:

3.2 Find V_1 via nodal.



$R_1 = 3k\Omega \quad R_2 = R_3 = R_4 = 2k\Omega$

$I_{S1} = 6mA \quad I_{S2} = 4mA$

@ V_1: $\frac{V_1}{R_1} + I_{S1} + \frac{V_1 - V_2}{R_2} = 0$

@ V_2: $\frac{V_2 - V_1}{R_2} + \frac{V_2}{R_3} + \frac{V_2}{R_4} + I_{S2} = 0$

$\boxed{V_1 = -11V}$

3.3 Use nodal analysis to find both V_1 and V_o in the circuit in Fig. P3.3. **PSV**

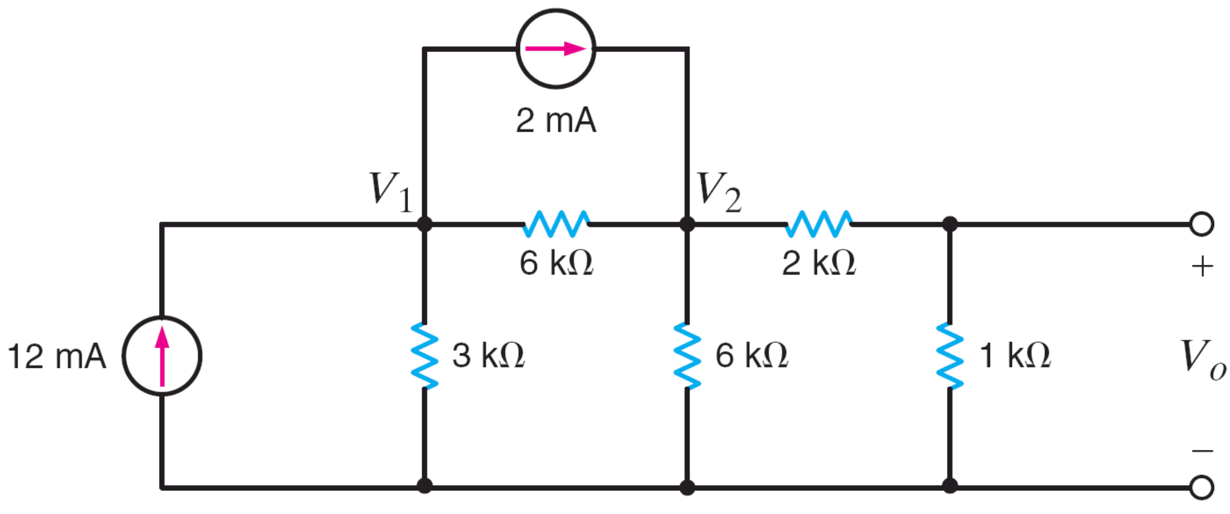
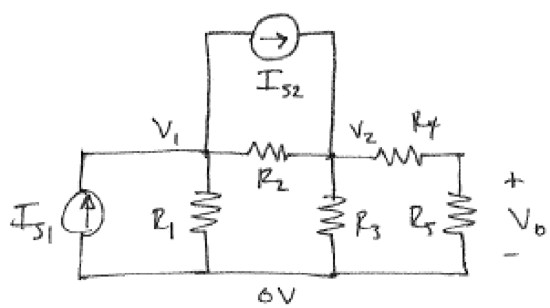


Figure P3.3

SOLUTION:

3.3 Find V_o & V_1 by nodal.



$R_1 = 3k\Omega$ $R_2 = R_3 = 6k\Omega$ $R_4 = 2k\Omega$

$R_5 = 1k\Omega$ $I_{s1} = 12mA$ $I_{s2} = 2mA$

$$@V_1: \quad I_{s2} - I_{s1} + \frac{V_1}{R_1} + \frac{V_1 - V_2}{R_2} = 0$$

$$@V_2: \quad -I_{s2} + \frac{V_2 - V_1}{R_2} + \frac{V_2}{R_3} + \frac{V_2 - V_o}{R_4} = 0$$

$$@V_o: \quad \frac{V_2 - V_o}{R_4} = \frac{V_o}{R_5}$$

$$\boxed{V_o = 2.91V \quad V_1 = 22.9V}$$

3.4 Find V_1 and V_2 in the circuit in Fig. P3.4 using nodal analysis. Then solve the problem using MATLAB and compare your answers.

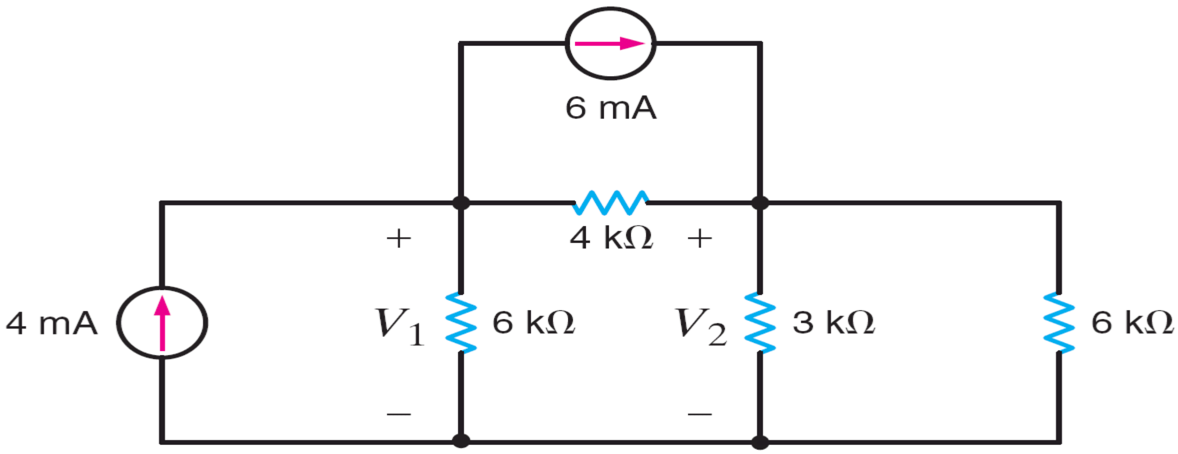
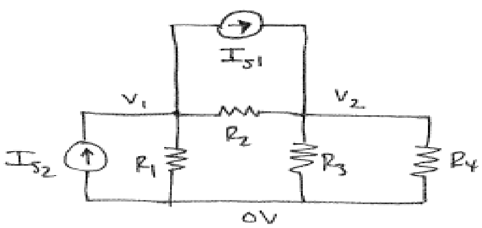


Figure P3.4

SOLUTION:

3.4 Find V_1 & V_2 by nodal



@ V_1: $I_{S1} - I_{S2} + \frac{V_1}{R_1} + \frac{V_1 - V_2}{R_2} = 0$

@ V_2: $-I_{S1} + \frac{V_2 - V_1}{R_2} + \frac{V_2}{R_3} + \frac{V_2}{R_4} = 0$

$\boxed{V_1 = 0V \qquad V_2 = 8V}$

$R_1 = R_4 = 6k\Omega \quad R_2 = 4k\Omega \quad R_3 = 3k\Omega$

$I_{S1} = 6mA \quad I_{S2} = 4mA$

Matrix Format

$$\begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} & -\frac{1}{R_2} \\ -\frac{1}{R_2} & \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I_{S2} - I_{S1} \\ I_{S1} \end{bmatrix}$$

Continued on the next page.

3_4.txt

```
MATLAB WORK

Factor 1/24000 out of conductance matrix.

EDU» g=[12,-6;-6,18]

g =

    12    -6
    -6    18

EDU» i=[-0.002;0.006]

i =

   -0.0020
    0.0060

EDU» 24000*inv(g)*i

ans =

     0
     8
```

3.5 Find I_o in the circuit in Fig. P3.5 using nodal analysis. **CS**

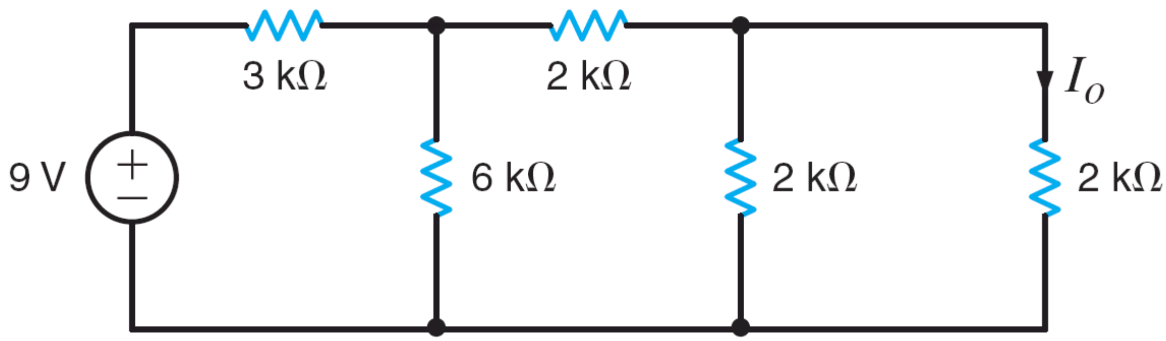
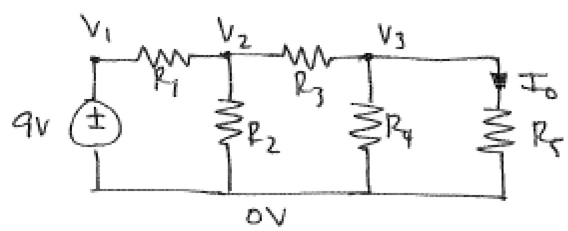


Figure P3.5

SOLUTION:

3.5 Find I_o by nodal.



$R_1 = 3k\Omega \quad R_2 = 6k\Omega \quad R_3 = R_4 = R_5 = 2k\Omega$

@ V_1: $V_1 = 9V$

@ V_2: $\dfrac{V_2 - V_1}{R_1} + \dfrac{V_2}{R_2} + \dfrac{V_2 - V_3}{R_3} = 0$

@ V_3: $\dfrac{V_3 - V_2}{R_3} + \dfrac{V_3}{R_4} + \dfrac{V_3}{R_5} = 0$

and $I_o = V_3 / R_5$

$\boxed{I_o = 0.6mA}$

3.6 Find I_o in the network in Fig. P3.6 using nodal analysis.

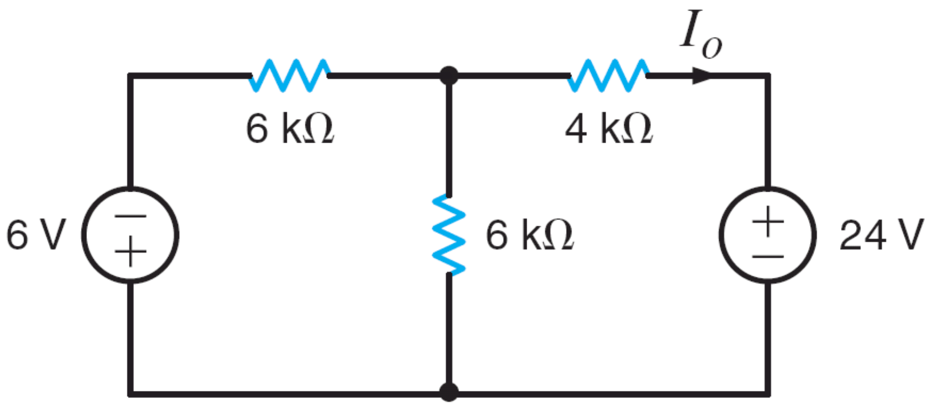
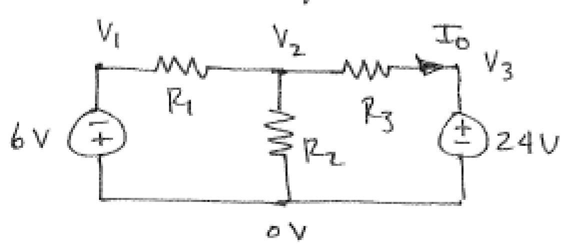


Figure P3.6

SOLUTION:

3.6 Find I_o by nodal.



$R_1 = R_2 = 6k\Omega \quad R_3 = 4k\Omega$

@ V_1: $\quad V_1 = -6V$

@ V_2: $\quad \dfrac{V_2 - V_1}{R_1} + \dfrac{V_2}{R_2} + \dfrac{V_2 - V_3}{R_3} = 0$

@ V_3: $\quad V_3 = 24V$

and $\quad I_o = (V_2 - V_3)/R_3$

$V_2 = 8.57\ V$

$$\boxed{I_o = -3.86mA}$$

3.7 Find V_o in the network in Fig. P3.7 using nodal analysis.
CS

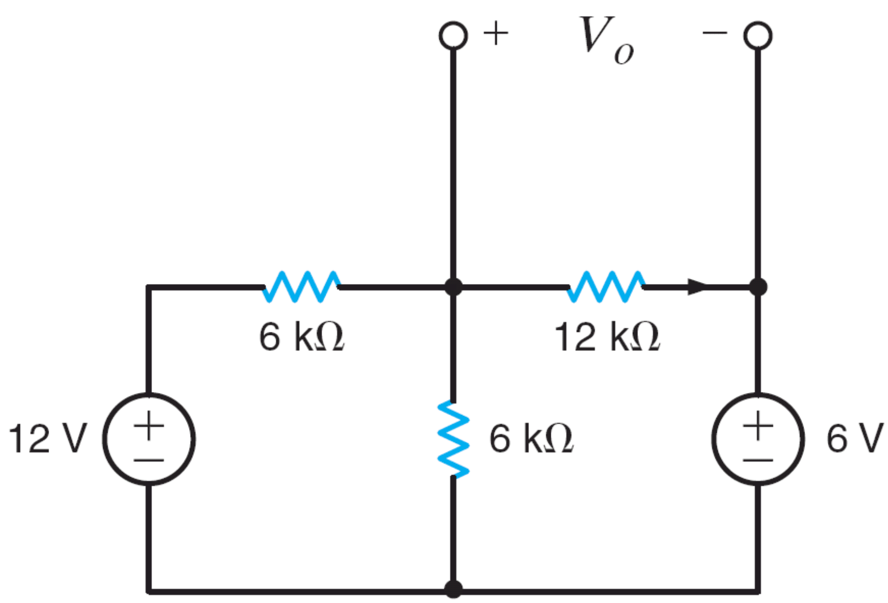
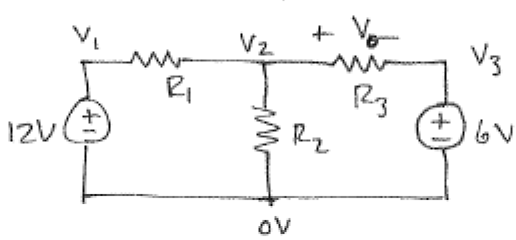


Figure P3.7

SOLUTION:

3.7 Find V_o by nodal.



$R_1 = R_2 = 6k\Omega$

$R_3 = 12k\Omega$

@ V_1: $V_1 = 12V$

@ V_2: $$\frac{V_2 - V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_2 - V_3}{R_3} = 0$$

@ V_3: $V_3 = 6V$

and $V_o = V_2 - V_3$

$V_2 = 6V$ $\boxed{V_o = 0V}$

3.8 Find V_o in the circuit in Fig. P3.8 using nodal analysis.
PSV

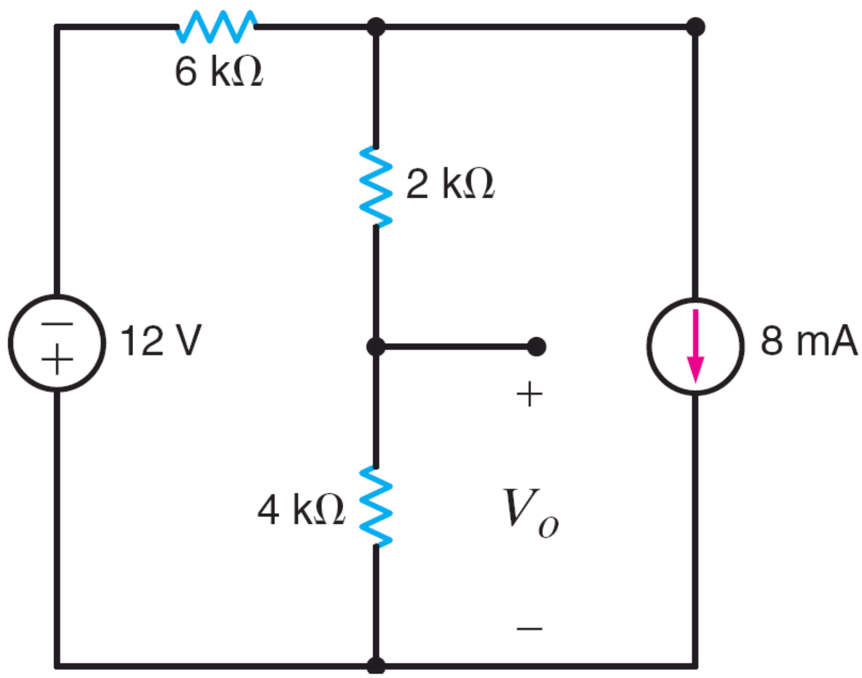
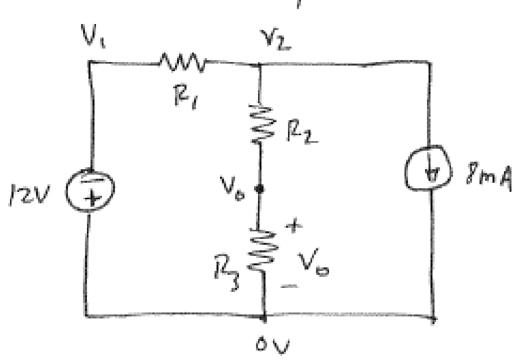


Figure P3.8

SOLUTION:

3.8 Find V_o by nodal.



$R_1 = 6k\Omega \quad R_2 = 2k\Omega \quad R_3 = 4k\Omega$

$V_1 = -12V$

@ V_2: $\dfrac{V_2 - V_1}{R_1} + \dfrac{V_2 - V_o}{R_2} + 8\times10^{-3} = 0$

@ V_o: $\dfrac{V_2 - V_o}{R_2} = \dfrac{V_o}{R_3}$

$\boxed{V_o = -20V}$

3.9 Use nodal analysis to find V_o in the circuit in Fig. P3.9.

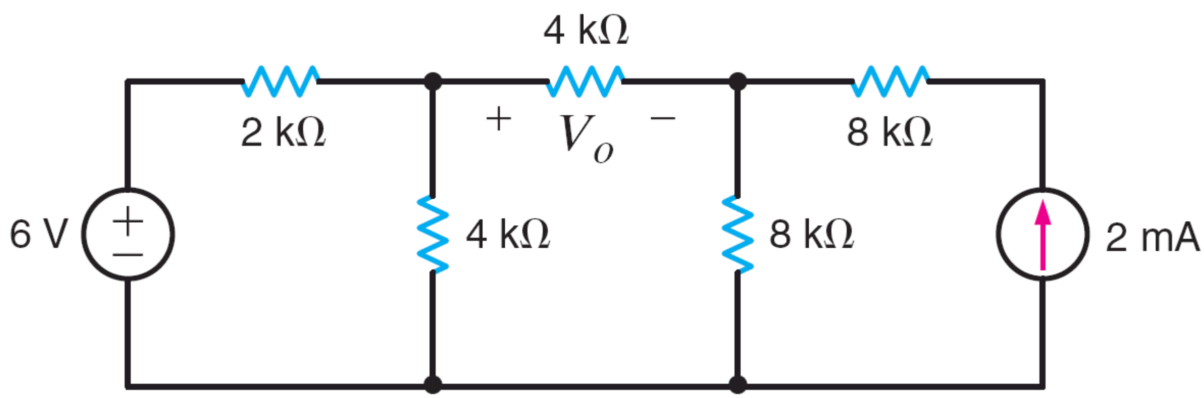
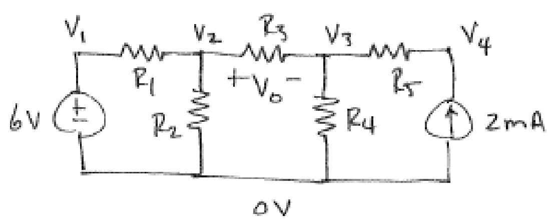


Figure P3.9

SOLUTION:

3.9 Find V_o by nodal.



$R_1 = 2k\Omega \quad R_2 = R_3 = 4k\Omega$

$R_4 = R_5 = 8k\Omega$

$V_1 = 6V$

@ V_2: $\dfrac{V_2 - V_1}{R_1} + \dfrac{V_2}{R_2} + \dfrac{V_2 - V_3}{R_3} = 0$

@ V_3: $\dfrac{V_3 - V_2}{R_3} + \dfrac{V_3}{R_4} + \dfrac{V_3 - V_4}{R_5} = 0$

@ V_4: $\dfrac{V_4 - V_3}{R_5} = 2 \times 10^{-3}$

and $V_o = V_2 - V_3$

$V_2 = 5.2V$, $V_3 = 8.8V$ $\boxed{V_o = -3.6V}$

3.10 Find I_o in the circuit in Fig. P3.10 using nodal analysis.

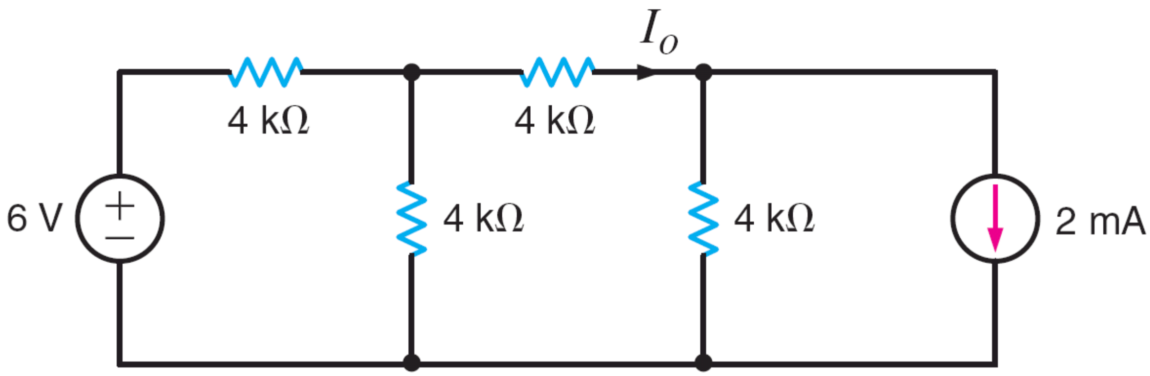
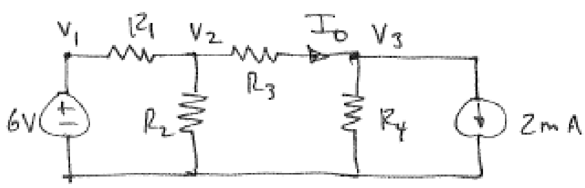


Figure P3.10

SOLUTION:

3.10 Find I_o by nodal.



all $R = 4k\Omega$

$V_1 = 6V$

@ V_2: $$\frac{V_2 - V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_2 - V_3}{R_3} = 0$$

@ V_3: $$\frac{V_3 - V_2}{R_3} + \frac{V_3}{R4} + 2\times10^{-3} = 0$$

and $$I_o = \frac{V_2 - V_3}{R_3}$$

$V_2 = 0.8V$ $V_3 = -3.6V$ $\boxed{I_o = 1.1mA}$

3.11 Use nodal analysis to find V_o in the network in Fig. P3.11. Then solve the problem using MATLAB and compare your answers.

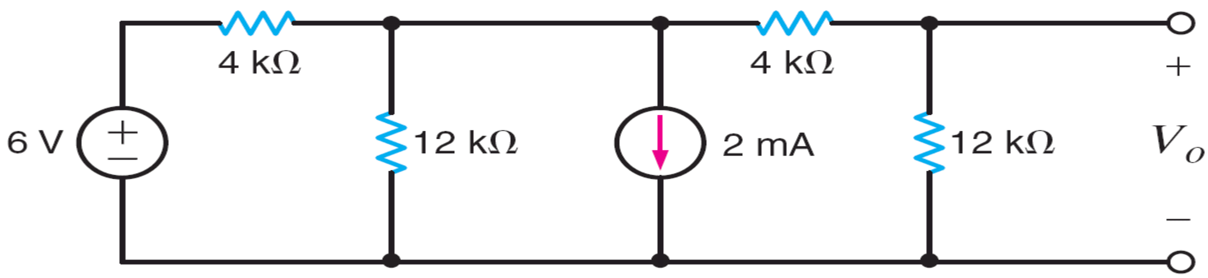
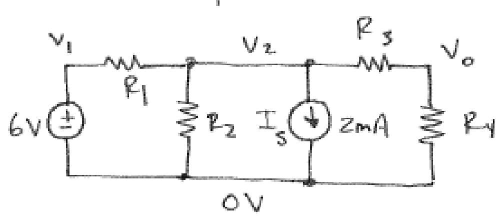


Figure P3.11

SOLUTION:

3.11 Find V_o by nodal & MATLAB



$R_1 = 4k\Omega$ $R_2 = 12k\Omega$ $R_3 = 4k\Omega$

$R_4 = 12k\Omega$

$V_1 = 6V$

@ V_2: $\frac{V_2 - V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_2 - V_o}{R_3} + I_s = 0$

@ V_o: $\frac{V_o - V_2}{R_3} + \frac{V_o}{R_4} = 0$

$$\boxed{V_o = -0.95V}$$

In matrix form:

$$\begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{R_1} & \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} & -\frac{1}{R_3} \\ 0 & -1/R_3 & \frac{1}{R_3} + \frac{1}{R_4} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_o \end{bmatrix} = \begin{bmatrix} 6 \\ -2\times10^{-3} \\ 0 \end{bmatrix}$$

Continued on the next page.

```
                                                   3_11.txt
MATLAB WORK

Factor out 1/12000 from the conductance matrix


EDU» g=[12000,0,0;-3,7,-3;0,-3,4]

g =

       12000            0            0
          -3            7           -3
           0           -3            4

EDU» i=[6;-0.002;0]

i =

    6.0000
   -0.0020
         0


EDU» v=12000*inv(g)*i

v =

    6.0000
   -1.2632
   -0.9474
```

3.12 Use nodal analysis to find V_o in the circuit in Fig. P3.12.

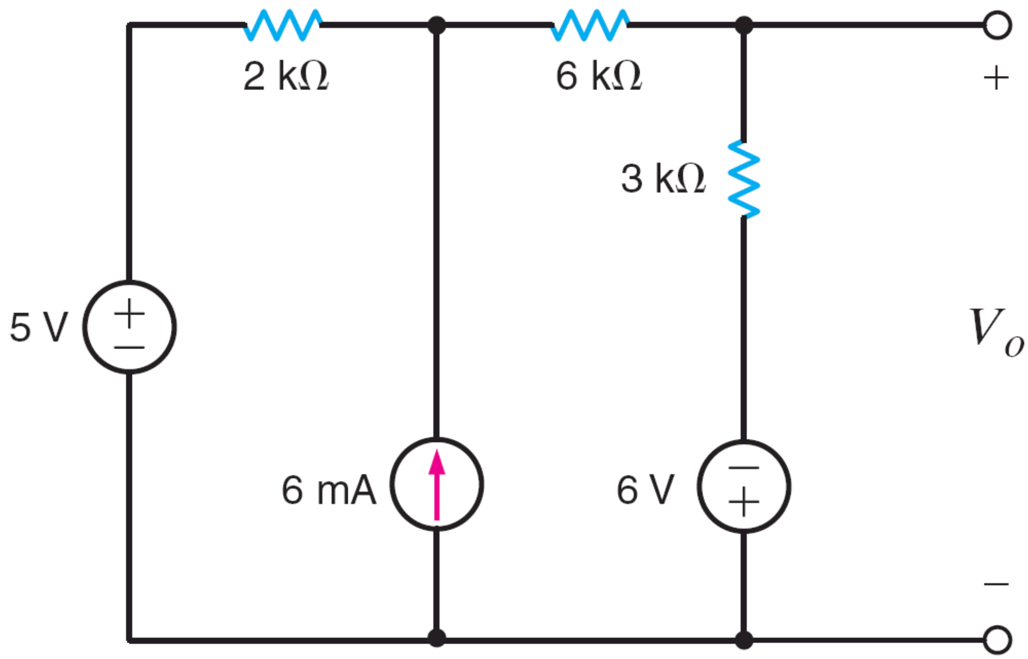
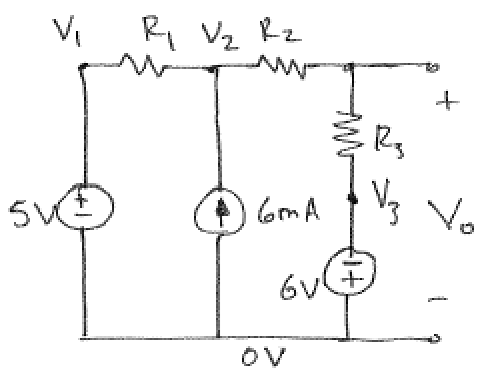


Figure P3.12

SOLUTION:

3.12 V_o by nodal.



$R_1 = 2k\Omega$ $R_2 = 6k\Omega$ $R_3 = 3k\Omega$

$V_1 = 5V$ & $V_3 = -6V$

@ V_2: $$\frac{V_2 - V_1}{R_1} + \frac{V_2 - V_o}{R_2} - 6\times10^{-3} = 0$$

@ V_o: $$\frac{V_o - V_2}{R_2} + \frac{V_o - V_3}{R_3} = 0$$

$$\boxed{V_o = 0.27\ V}$$

3.13 Use nodal analysis to find V_o in the circuit in Fig. P3.13.

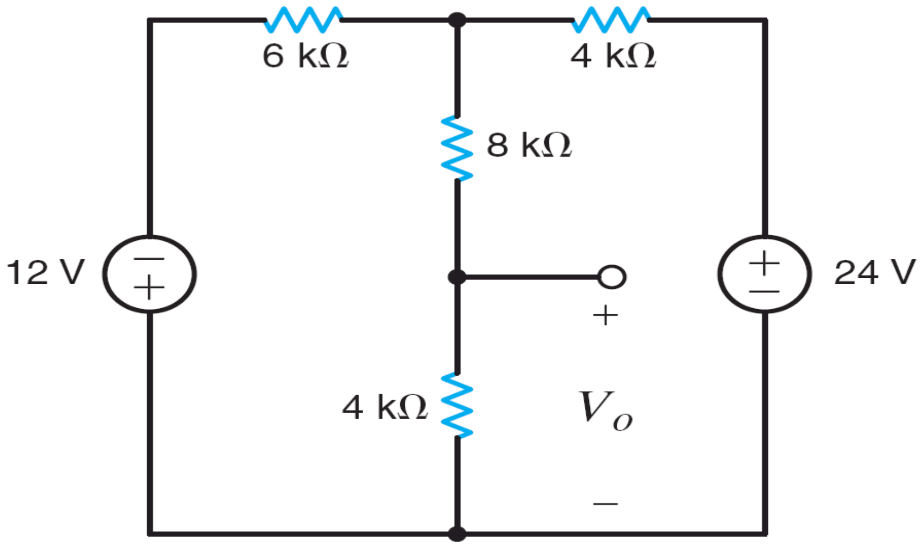
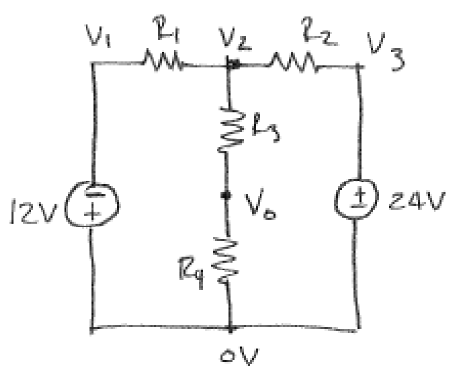


Figure P3.13

SOLUTION:

3.13 Find V_o by nodal.



$V_1 = -12V$ $\quad V_3 = 24V$

@ V_2: $\dfrac{V_2 - V_1}{R_1} + \dfrac{V_2 - V_3}{R_2} + \dfrac{V_2 - V_o}{R_3} = 0$

@ V_o: $\dfrac{V_o - V_2}{R_3} + \dfrac{V_o}{R_4} = 0$

$$\boxed{V_o = 2.67V}$$

$R_1 = 6k\Omega$ $\quad R_2 = R_4 = 4\,k\Omega$

$R_3 = 8\,k\Omega$

3.14 Find I_o in the network in Fig. P3.14. CS

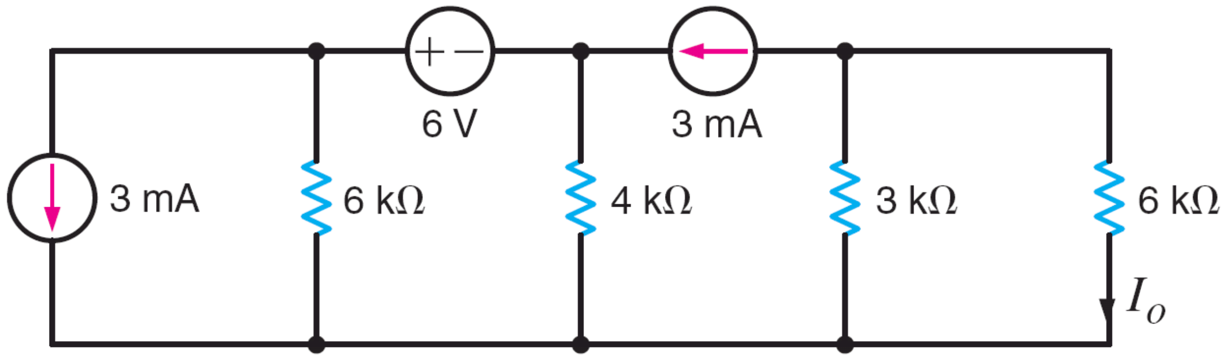
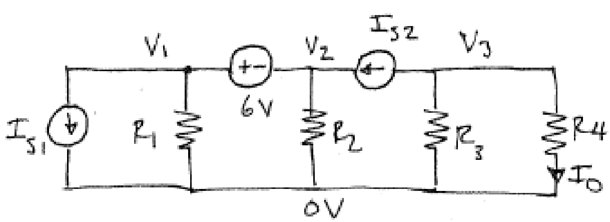


Figure P3.14

SOLUTION:

3.14 Find I_o.



$R_1 = R_4 = 6k\Omega \quad R_2 = 4k\Omega \quad R_3 = 3k\Omega$

$I_{S1} = 3mA \quad I_{S2} = 3mA$

$V_1 - V_2 = 6V$

@ V_3: $I_{S2} + \frac{V_3}{R_3} + \frac{V_3}{R_4} = 0$

@ ref: $I_{S1} + \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} + \frac{V_3}{R_4} = 0$

also, $I_o = V_3/R_4$

$\boxed{I_o = -1mA}$

3.15 Find I_1 in the network in Fig. P3.15. CS

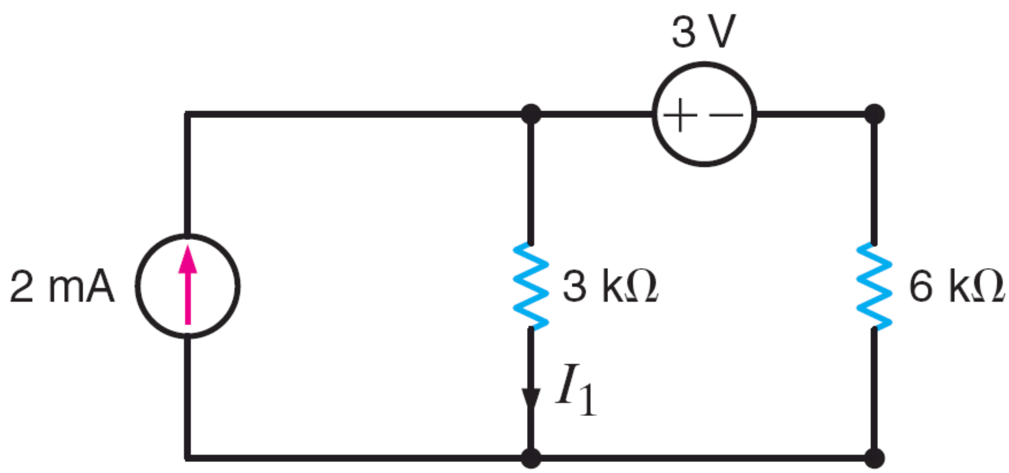
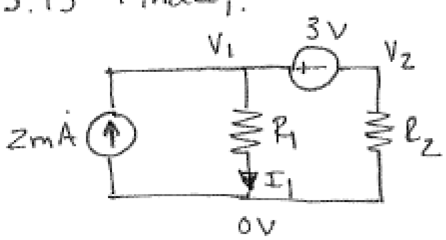


Figure P3.15

SOLUTION:

3.15 Find I_1.



$R_1 = 3k\Omega \quad R_2 = 6k\Omega$

$V_1 - V_2 = 3V$

@ ref: $\frac{V_1}{R_1} + \frac{V_2}{R_2} - 2\times10^{-3} = 0$

and, $I_1 = V_1/R_1$

$\boxed{I_1 = 1.67 \text{ mA}}$

3.16 Find I_o in the network in Fig. P3.16.

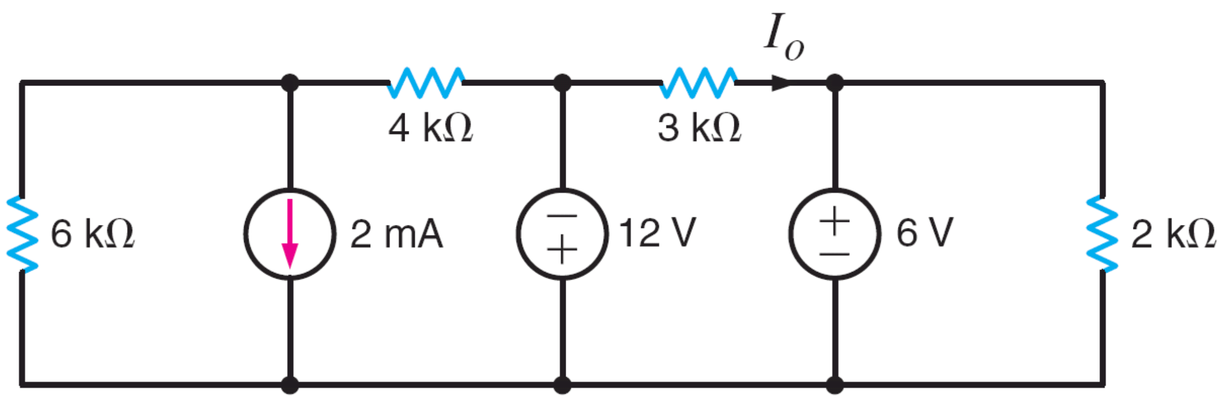
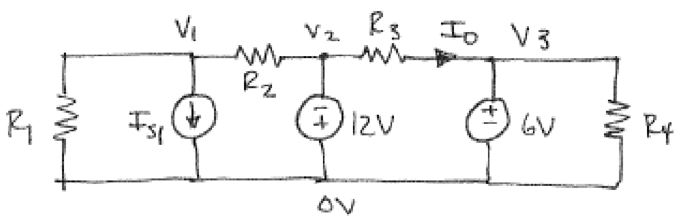


Figure P3.16

SOLUTION:

3.16 Find I_o.



$R_1 = 6k\Omega$ $R_2 = 4k\Omega$ $R_3 = 3k\Omega$ $R_4 = 2k\Omega$

$I_{s1} = 2mA$

$V_2 = -12V$ $V_3 = 6V$

$@V_1: \quad \frac{V_1}{R_1} + \frac{V_1 - V_2}{R_2} + I_{s1} = 0$

and $I_o = \frac{V_2 - V_3}{R_3}$

$$\boxed{I_o = -6mA}$$

3.17 Use nodal analysis to find V_x and V_y in the circuit in Fig. P3.17.

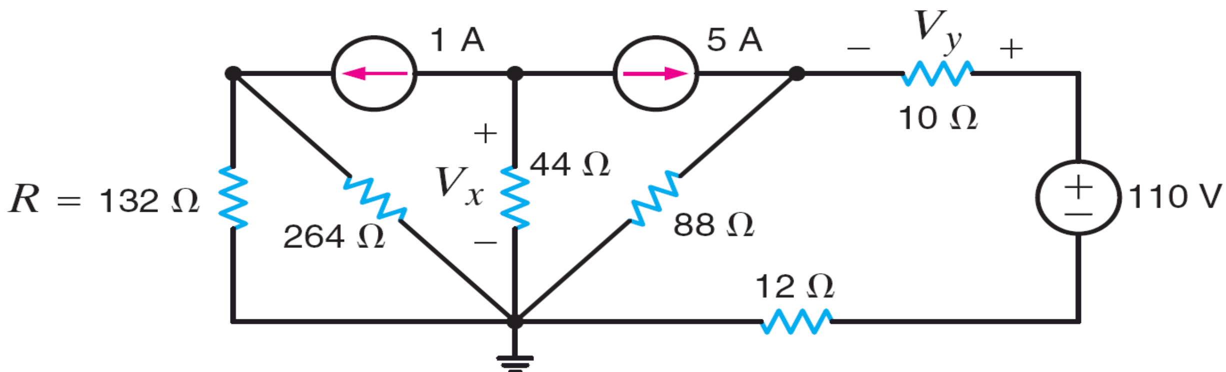
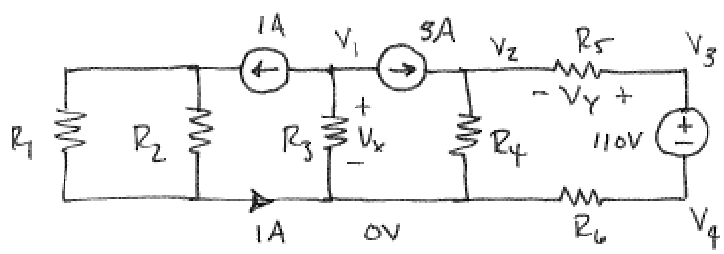


Figure P3.17

SOLUTION:

3.17 Find V_x and V_y by nodal.



$R_1 = 132\,\Omega$ $R_2 = 264\,\Omega$

$R_3 = 44\,\Omega$ $R_4 = 88\,\Omega$

$R_5 = 10\,\Omega$ $R_6 = 12\,\Omega$

@ V_1: $1 + 5 + \frac{V_x}{R_3} = 0 \Rightarrow \boxed{V_x = -264\text{V}}$

@ V_2: $5 = \frac{V_2}{R_4} + \frac{V_2 - V_3}{R_5}$

@ ref: $1 + \frac{V_1}{R_3} + \frac{V_2}{R_4} + \frac{V_4}{R_6}$

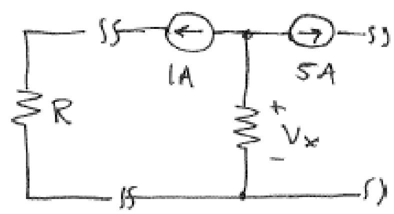
$V_3 - V_4 = 110\text{V}$

$V_Y = V_3 - V_2$ $\boxed{V_Y = 30\text{V}}$

3.18 For the network in Fig P3.17, explain why the resistor R plays no role in determining V_x and V_y.

SOLUTION:

3.18 As shown in the simple circuit here the 1-A current source fixes the current in its branch, independent of the value of R.



Since V_x & V_y depend only on the branch's current, the value of R does not impact V_x or V_y.

3.19 Use nodal analysis to find V_o in the network in Fig P3.19.

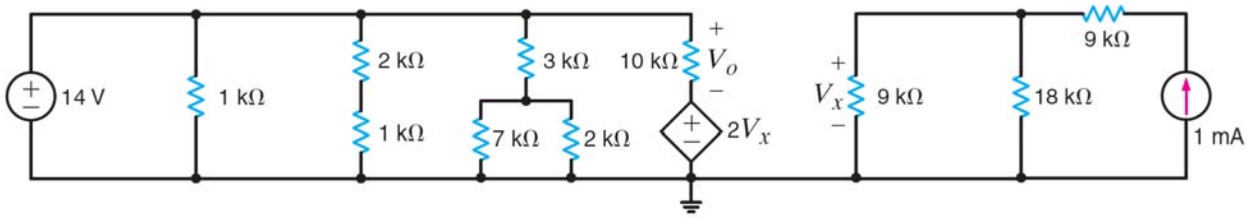
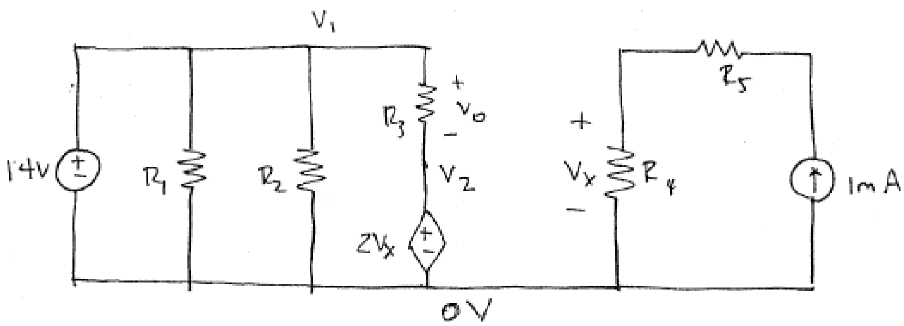


Figure P3.19

SOLUTION:

3.19 Find V_o by nodal



$R_1 = 1000 // (2000 + 1000) = 750\Omega$ $\quad R_3 = 10k\Omega \quad R_5 = 9k\Omega$

$R_4 = 9000 // 18000 = 6k\Omega$ $\quad R_2 = 3000 + (7000 // 2000) = 4.55k\Omega$

$V_1 = 14V \quad V_2 = 2V_x \quad V_x = 1\times10^{-3}\,(R_4) = 6V$

$V_o = V_1 - V_2 \quad \boxed{V_o = 2V}$

3.20 Use nodal analysis to find V_A and V_B in the network in Fig. P3.20. Simplify the analysis by making an insightful choice for the reference node.

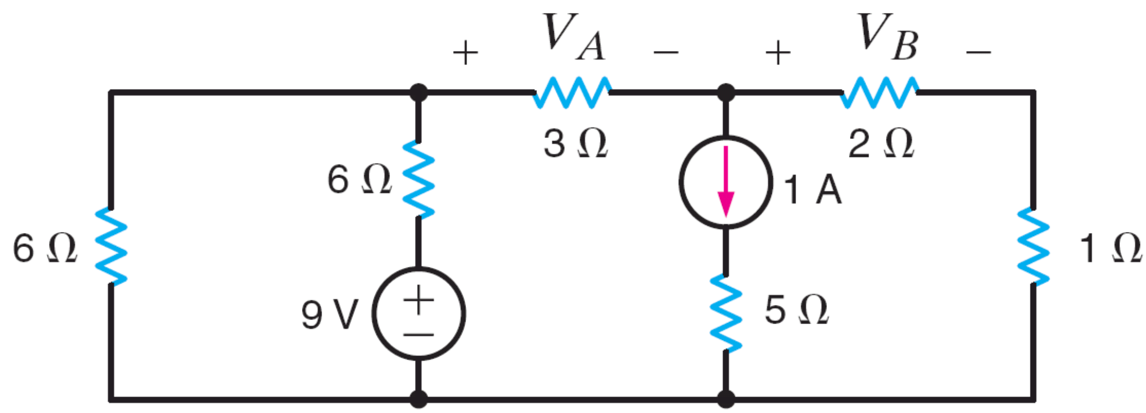
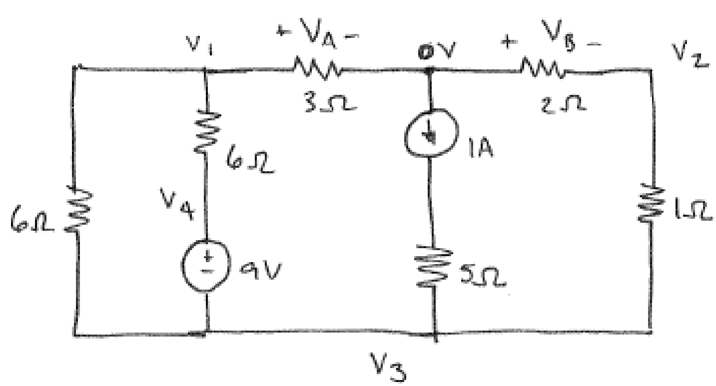


Figure P3.20

SOLUTION:

3.20 Find V_A & V_B by nodal



$$V_4 - V_3 = 9$$

$$\frac{V_1 - V_4}{6} + \frac{V_1}{3} + \frac{V_1 - V_3}{6} = 0$$

$$\frac{V_1}{3} + \frac{V_2}{2} = 1$$

$$\frac{V_2}{2} + \frac{V_2 - V_3}{1} = 0$$

$V_A = V_1$

$V_B = -V_2$

$$\boxed{\begin{aligned} V_A &= 2.5\,V \\ V_B &= -0.33\,V \end{aligned}}$$

3.21 Find I_o in the circuit in Fig. P3.21.

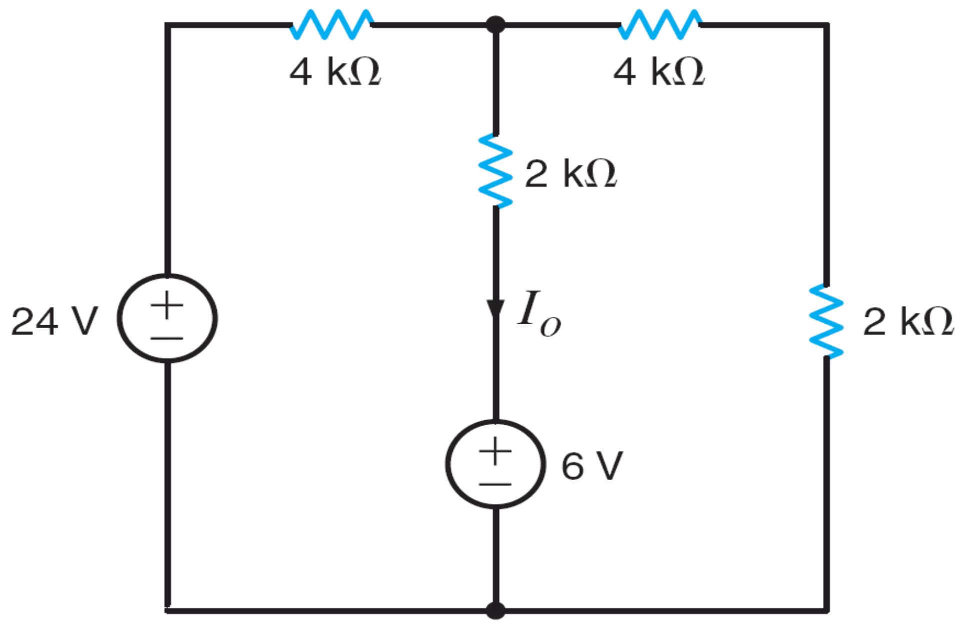
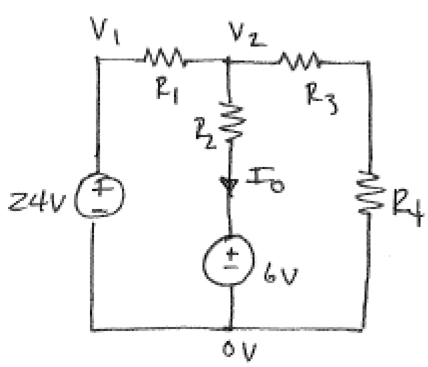


Figure P3.21

SOLUTION:

3.21 Find I_o.



$V_1 = 24V$

$$\frac{V_2 - V_1}{R_1} + \frac{V_2 - 6}{R_2} + \frac{V_2}{R_3 + R_4} = 0$$

$$I_o = \frac{V_2 - 6}{R_2}$$

$\boxed{I_o = 1.91\ mA}$

$R_1 = R_3 = 4k\Omega$

$R_2 = R_4 = 2k\Omega$

3.22 Use nodal analysis to find I_o and I_S in the circuit in Fig. P3.22.

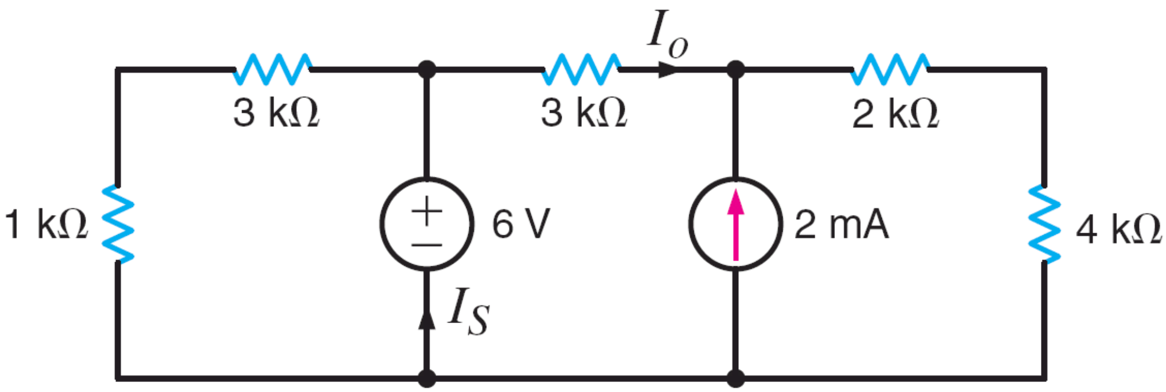
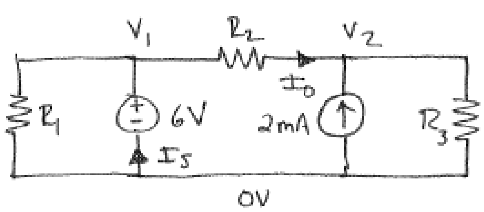


Figure P3.22

SOLUTION:

3.22 Find I_o and I_s by nodal.



$R_1 = 1000 + 3000 = 4000\,\Omega$

$R_3 = 2000 + 4000 = 6\,k\Omega$

$V_1 = 6\,V$

$I_o = (V_1 - V_2)/R_2$

$$\frac{V_1 - V_2}{R_2} + 2\times10^{-3} = V_2/R_3$$

$\boxed{I_o = -0.67\,mA}$

$I_S = \frac{V_1}{R_1} + I_o \Rightarrow \boxed{I_S = 0.83\,mA}$

3.23 Use nodal analysis to find V_o in the network in Fig. P3.23.

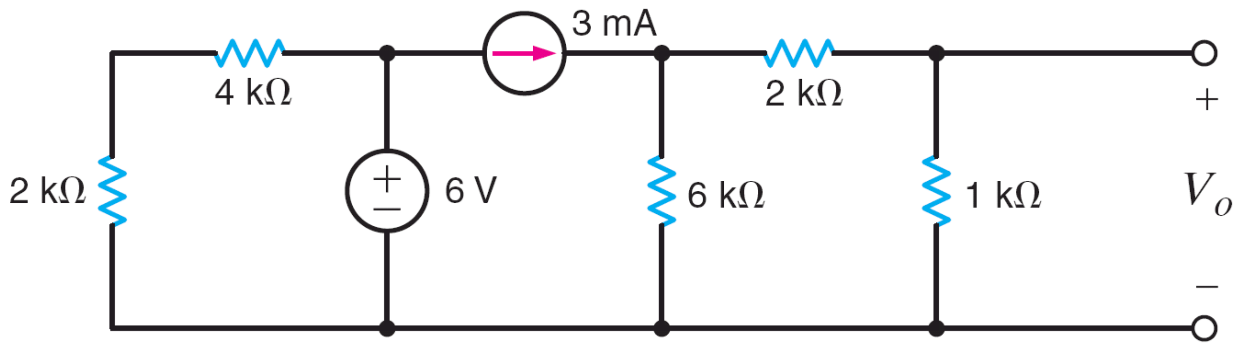
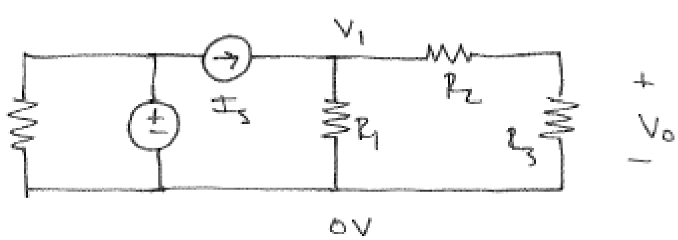


Figure P3.23

SOLUTION:

3.23 Find V_o by nodal.



$R_1 = 6k\Omega$ $R_2 = 2k\Omega$ $R_3 = 1k\Omega$

$I_s = 3mA$

$$I_s = \frac{V_1}{R_1} + \frac{V_1}{R_2 + R_3}$$

$$V_o = V_1 \left(\frac{R_3}{R_2 + R_3} \right)$$

$$\boxed{V_o = 2V}$$

3.24 Use nodal analysis to find I_o in the circuit in Fig. P3.24.

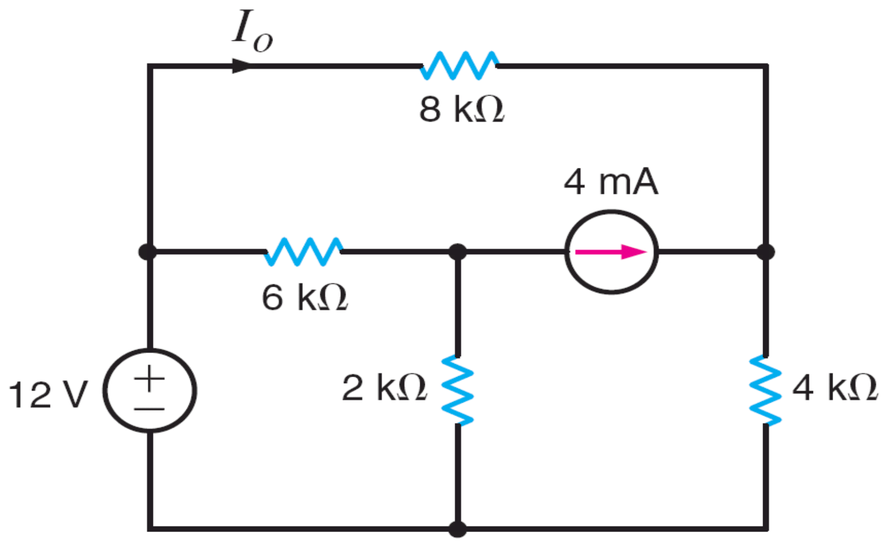
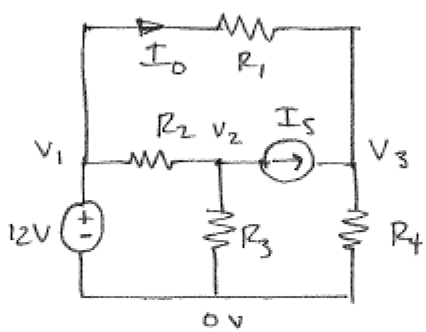


Figure P3.24

SOLUTION:

3.24 Find I_o by nodal.



$R_1 = 8k\Omega \quad R_2 = 6k\Omega$

$R_3 = 2k\Omega \quad R_4 = 4k\Omega$

$I_S = 4mA$

$V_1 = 12V$

$$\frac{V_2 - V_1}{R_2} + \frac{V_2}{R_3} + I_S = 0$$

$$\frac{V_3 - V_1}{R_1} + \frac{V_3}{R_4} = I_S$$

and, $I_o = \dfrac{V_1 - V_3}{R_1}$

$$\boxed{I_o = -0.33mA}$$

3.25 Find I_o in the network in Fig. P3.25 using nodal analysis.

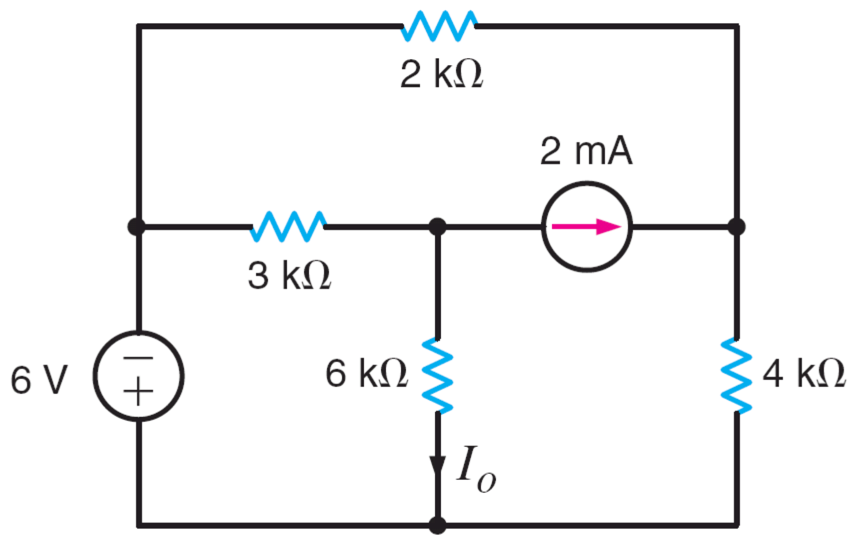
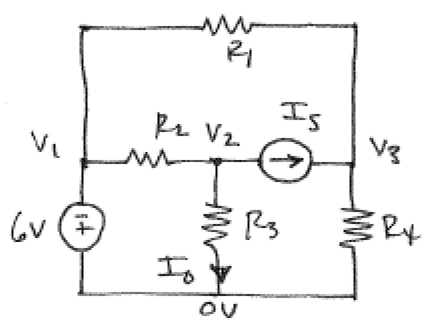


Figure P3.25

SOLUTION:

3.25 Find I_o by nodal.



$R_1 = 2k\Omega$ $R_2 = 3k\Omega$

$R_3 = 6k\Omega$ $R_4 = 4k\Omega$

$I_S = 2mA$

$V_1 = -6V$

$$\frac{V_2 - V_1}{R_2} + \frac{V_2}{R_3} + I_S = 0$$

$$I_o = V_2 / R_3$$

$$\boxed{I_o = 0\ A}$$

3.26 Use nodal analysis to find I_o in the network in Fig. P3.26.

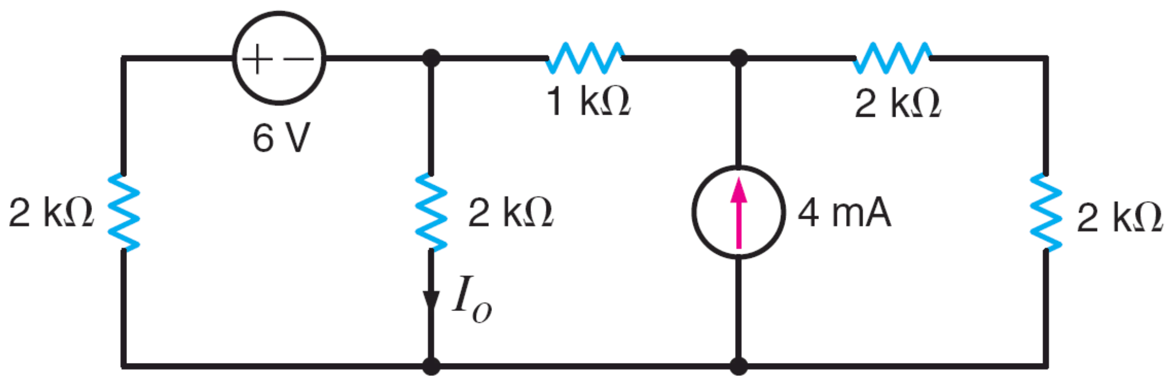
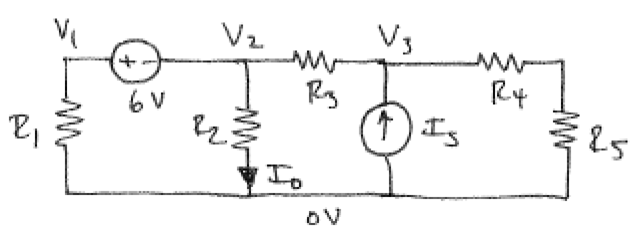


Figure P3.26

SOLUTION:

3.26 Find I_o by nodal.



$R_1 = R_2 = R_4 = R_5 = 2k\Omega$

$R_3 = 1k\Omega \quad I_S = 4mA$

$V_1 - V_2 = 6V$

@ V_3: $\dfrac{V_3 - V_2}{R_3} + \dfrac{V_3}{R_4 + R_5} = I_S$

@ ref: $\dfrac{V_1}{R_1} + \dfrac{V_2}{R_2} + \dfrac{V_3}{R_4 + R_5} = I_S$

and $I_o = V_2 / R_2$

$\boxed{I_o = 83.3\mu A}$

3.27 Use nodal analysis to find V_o in the network in Fig. P3.27. Then solve this problem using MATLAB and compare your answers. **CS**

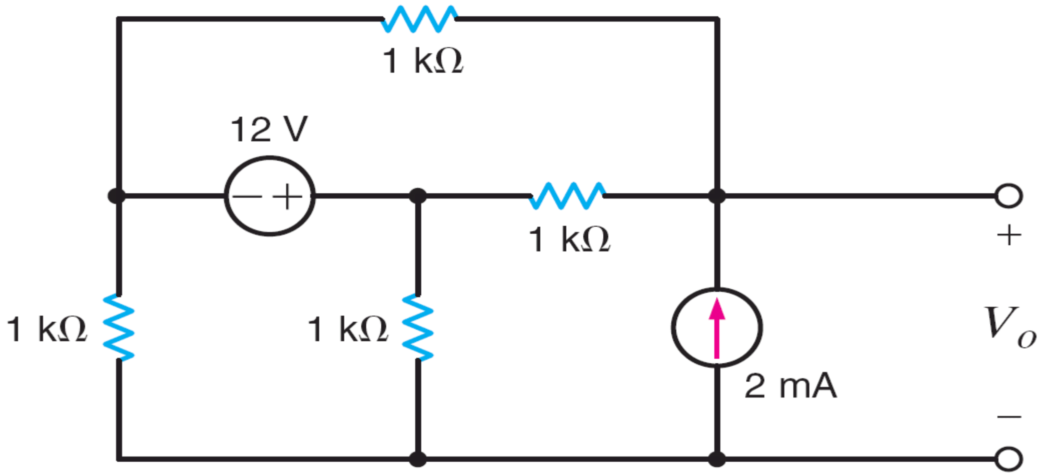
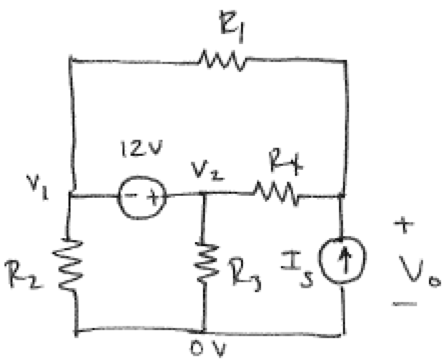


Figure P3.27

SOLUTION:

3.27 Find V_o by nodal and MATLAB.



$R_1 = R_2 = R_3 = R_4 = 1k\Omega$

$I_S = 2mA$

$V_2 - V_1 = 12V$

at V_o: $\dfrac{V_o - V_2}{R_4} + \dfrac{V_o - V_1}{R_1} = I_S$

@ ref: $\dfrac{V_1}{R_2} + \dfrac{V_2}{R_3} = I_S$

$\boxed{V_o = 2V}$

3.28 Find V_o in the circuit in Fig. P3.28 using nodal analysis.
PSV

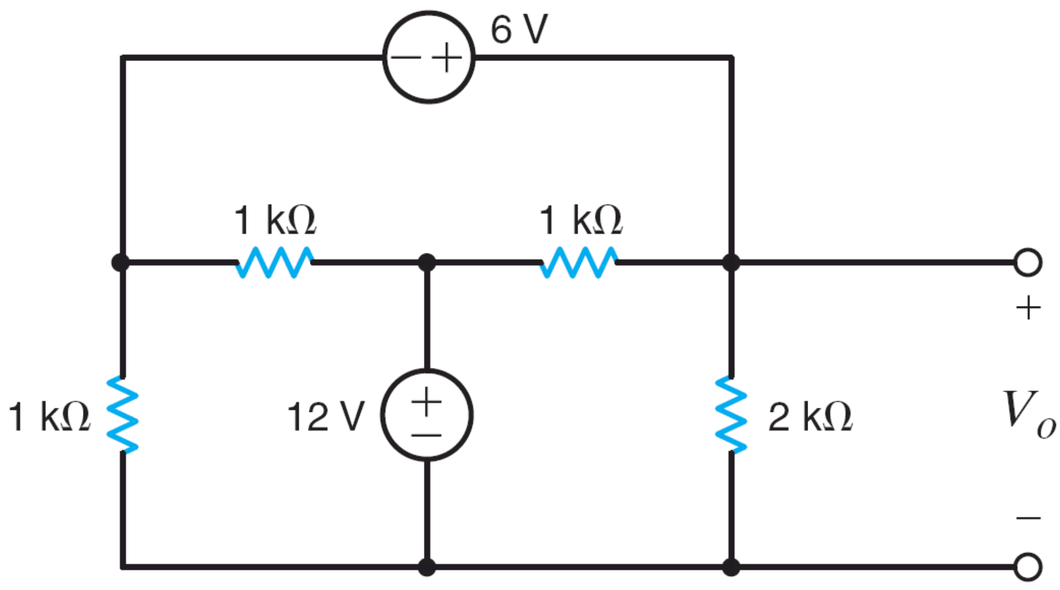
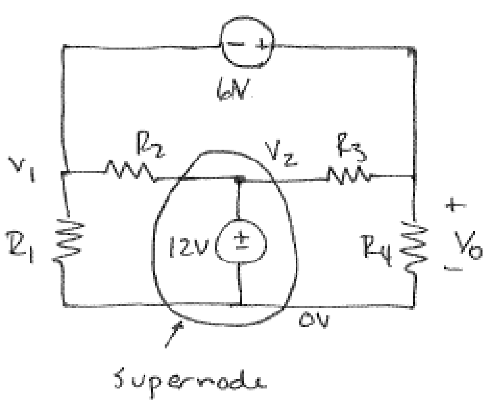


Figure P3.28

SOLUTION:

3.28 Find V_o by nodal.



$R_1 = R_2 = R_3 = 1k\Omega \quad R_4 = 2k\Omega$

$V_o - V_1 = 6V \qquad V_2 = 12V$

at supernode,

$$\frac{V_1 - V_2}{R_2} + \frac{V_1}{R_1} + \frac{V_o - V_2}{R_3} + \frac{V_o}{R_4} = 0$$

$$\boxed{V_o = 10.3V}$$

3.29 Use nodal analysis to find V_o in the circuit in Fig. P3.29.
CS

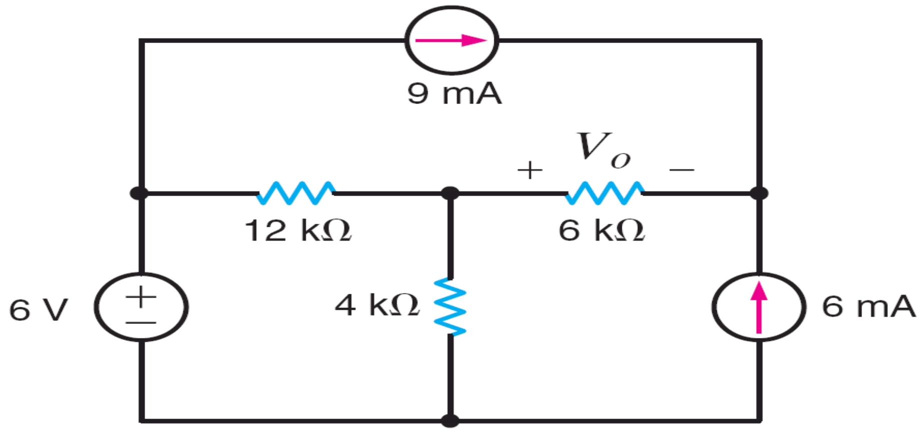
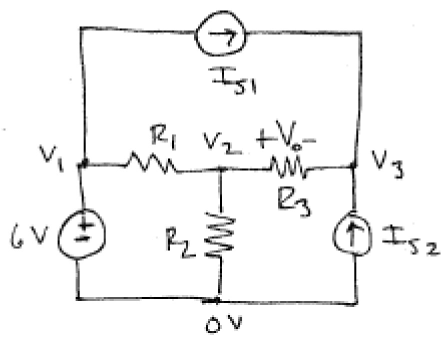


Figure P3.29

SOLUTION:

3.29 Find V_o by nodal



$V_1 = 6V$

@ V_2: $\dfrac{V_2 - V_1}{R_1} + \dfrac{V_2}{R_2} + \dfrac{V_2 - V_3}{R_3} = 0$

@ V_3: $\dfrac{V_3 - V_2}{R_3} = I_{S1} + I_{S2}$

and $V_o = V_2 - V_3$

$R_1 = 12k\Omega$ $R_2 = 4k\Omega$

$R_3 = 6k\Omega$ $I_{S1} = 9mA$

$I_{S2} = 6mA$

$$\boxed{V_o = -90V}$$

3.30 Use nodal analysis to find V_o in the circuit in Fig. P3.30.

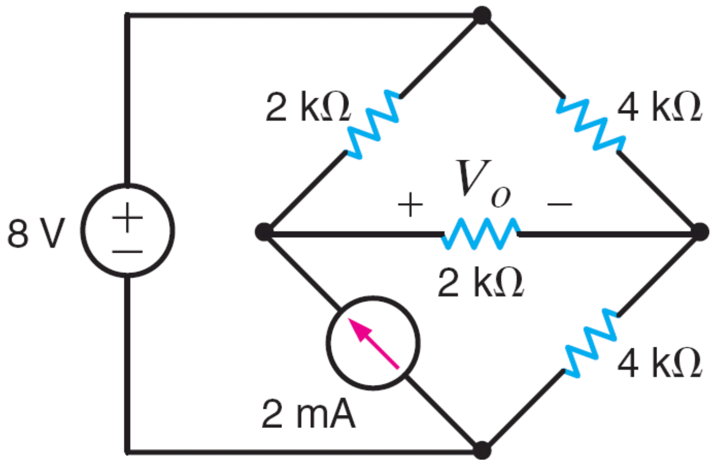
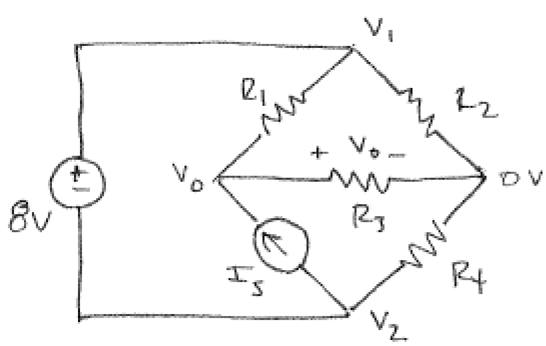


Figure P3.30

SOLUTION:

3.30 Find V_o by nodal



$R_1 = 2k\Omega \quad R_2 = 4k\Omega \quad R_3 = 2k\Omega$

$R_4 = 4k\Omega \quad I_S = 2mA$

$V_1 - V_2 = 8V$

@ V_o: $\dfrac{V_o - V_1}{R_1} + \dfrac{V_o}{R_3} = I_S$

@ ref: $\dfrac{V_1}{R_2} + \dfrac{V_o}{R_3} + \dfrac{V_2}{R_4} = 0$

$\boxed{V_o = 2.67V}$

3.31 Use nodal analysis to find V_o in the circuit in Fig. P3.31.

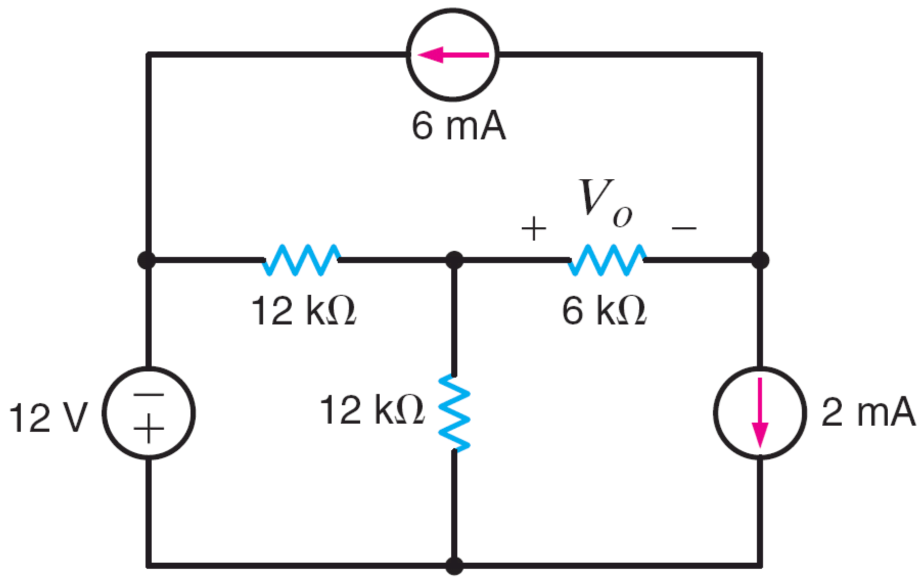
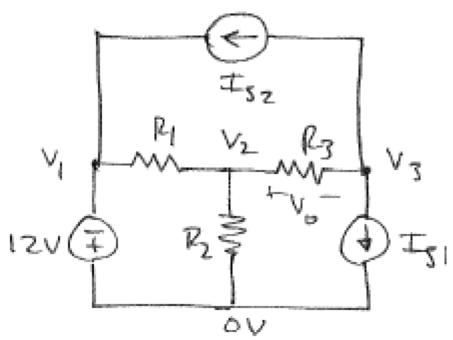


Figure P3.31

SOLUTION:

3.31 Find V_o by nodal.



$R_1 = R_2 = 12k\Omega \quad R_3 = 6k\Omega$

$I_{S1} = 2mA \quad I_{S2} = 6mA$

$V_1 = -12V$

@ V_2: $\dfrac{V_2 - V_1}{R_1} + \dfrac{V_2}{R_2} + \dfrac{V_2 - V_3}{R_3} = 0$

@ V_3: $\dfrac{V_3 - V_2}{R_3} + I_{S1} + I_{S2} = 0$

$V_o = V_2 - V_3$

$V_o = 48V$

3.32 Find V_o in the network in Fig. P3.32 using nodal analysis.
CS

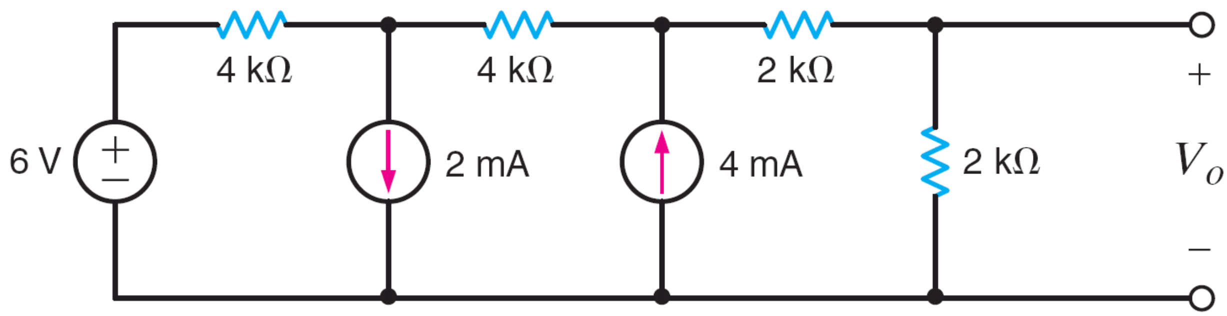
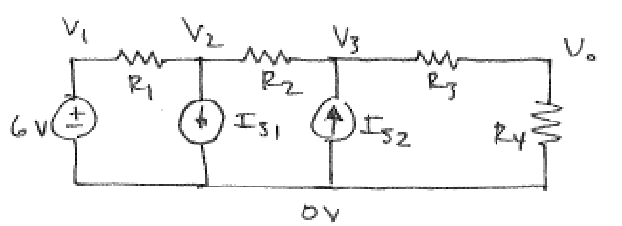


Figure P3.32

SOLUTION:

3.32 Find V_o by nodal.



$R_1 = R_2 = 4k\Omega \qquad R_3 = R_4 = 2k\Omega$

$I_{S1} = 2mA \qquad I_{S2} = 4mA$

$V_1 = 6V$

@ V_2: $\dfrac{V_2 - V_1}{R_1} + \dfrac{V_2 - V_3}{R_2} + I_{S1} = 0$

@ V_3: $\dfrac{V_3 - V_2}{R_2} + \dfrac{V_3 - V_o}{R_3} = I_{S2}$

@ V_o: $\dfrac{V_o - V_3}{R_3} + \dfrac{V_o}{R_4} = 0$

$\boxed{V_o = 5V}$

3.33 Use nodal analysis to find V_o in the network in Fig. P3.33.

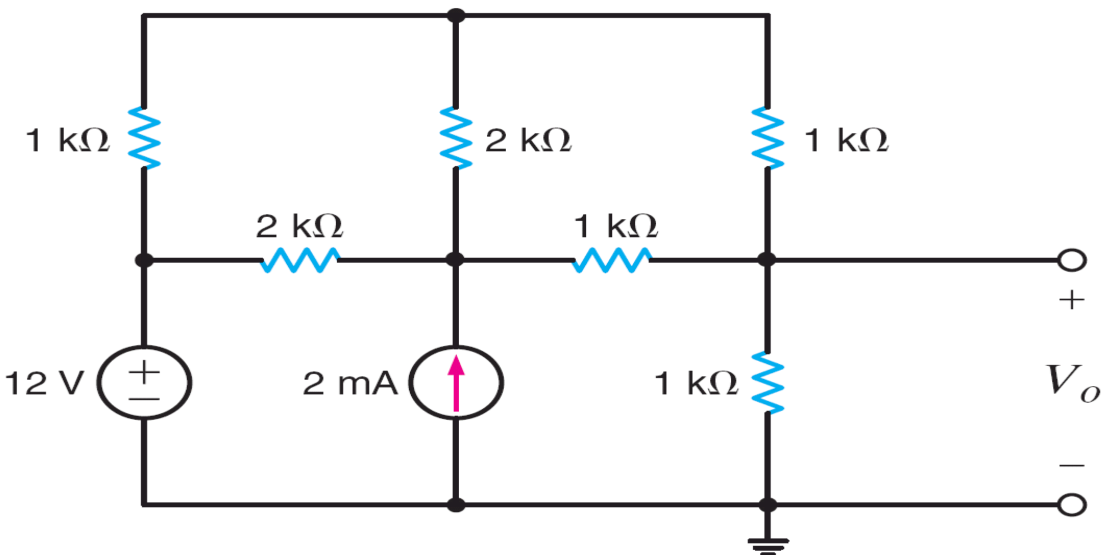
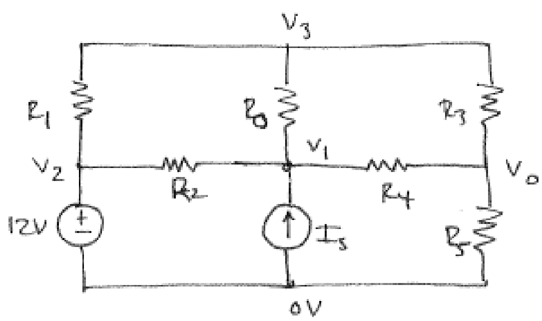


Figure P3.33

SOLUTION:

3.33 Find V_o by nodal.



$V_2 = 12\,V$

@ V_3: $\dfrac{V_3 - V_2}{R_1} + \dfrac{V_3 - V_1}{R_0} + \dfrac{V_3 - V_o}{R_3} = 0$

@ V_1: $I_S = \dfrac{V_1 - V_3}{R_0} + \dfrac{V_1 - V_0}{R_4} + \dfrac{V_1 - V_2}{R_2}$

$R_1 = R_3 = R_4 = R_5 = 1k\Omega \quad R_2 = R_0 = 2k\Omega$

$I_S = 2mA$

@ V_o: $\dfrac{V_o - V_1}{R_4} + \dfrac{V_o}{R_5} + \dfrac{V_o - V_3}{R_3} = 0$

$\boxed{V_o = 6.17V}$

3.34 Find V_o in the circuit in Fig. P3.34 using nodal analysis.

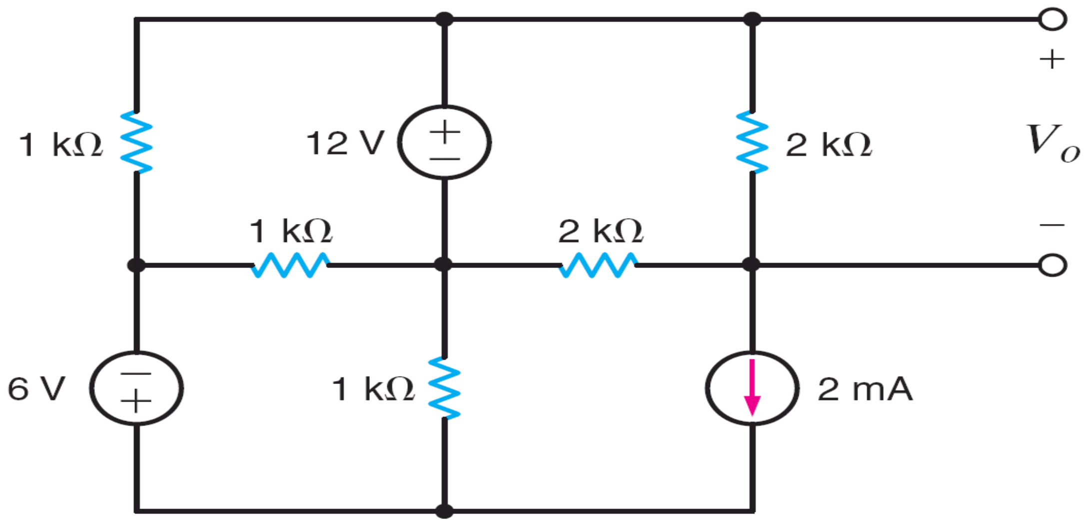
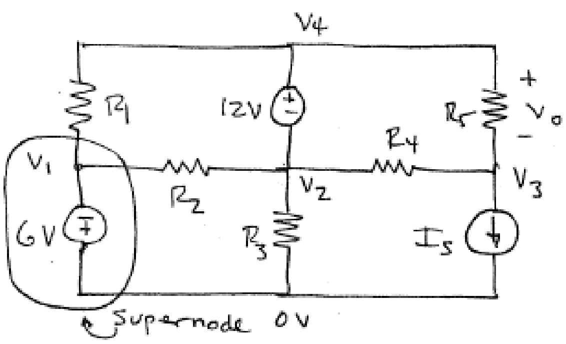


Figure P3.34

SOLUTION:

3.34 Find V_o by nodal.



$R_1 = R_2 = R_3 = 1k\Omega \quad I_S = 2mA$

$R_4 = R_5 = 2k\Omega$

$V_1 = -6\,V \qquad V_4 - V_2 = 12\,V$

@ V_3: $\frac{V_3 - V_2}{R_4} + \frac{V_3 - V_4}{R_5} + I_S = 0$

@ supernode:

$$\frac{V_4 - V_1}{R_1} + \frac{V_2 - V_1}{R_2} + \frac{V_2}{R_3} + I_S = 0$$

$V_o = V_4 - V_3$

$\boxed{V_o = 8V}$

3.35 Use nodal analysis to find V_o in the network in Fig. P3.35.

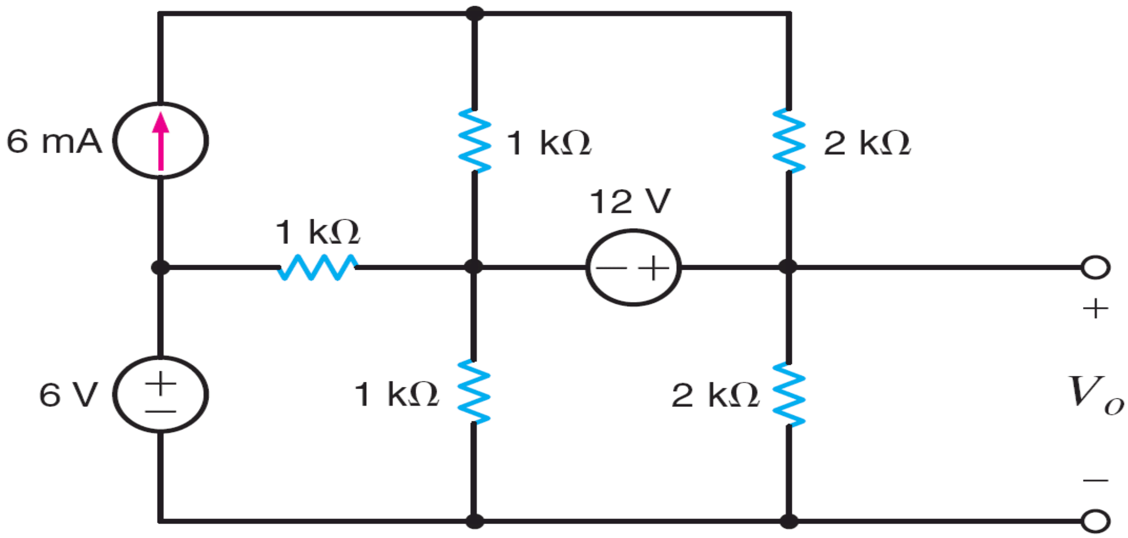
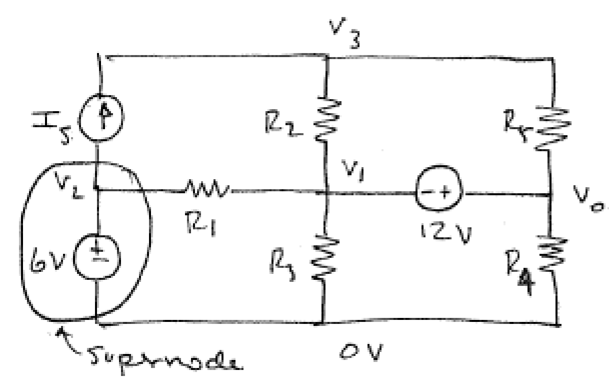


Figure P3.35

SOLUTION:

3.35 Use nodal to find V_o



$R_1 = R_2 = R_3 = 1\,k\Omega \qquad I_S = 6\,mA$

$R_4 = R_5 = 2\,k\Omega$

$V_2 = 6\,V \qquad V_o - V_1 = 12\,V$

@ V_3: $I_S = \dfrac{V_3 - V_1}{R_2} + \dfrac{V_3 - V_o}{R_5}$

@ supernode:

$$\frac{V_1 - V_2}{R_1} + \frac{V_1}{R_3} + \frac{V_o}{R_4} = I_S$$

$$\boxed{V_o = 14.4\,V}$$

3.36 Use MATLAB to find the node voltages in the network in Fig. P3.36. CS

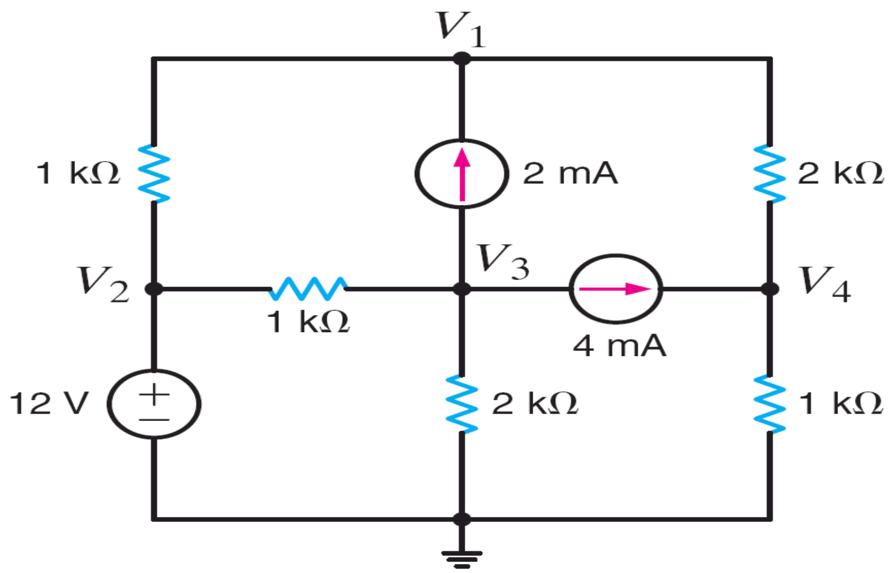
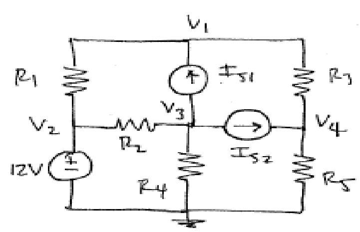


Figure P3.36

SOLUTION:

3.36 Use MATLAB to find node voltages.



$V_2 = 12V$

@V_1: $\frac{V_1 - V_2}{R_1} + \frac{V_1 - V_4}{R_3} = I_{S1}$

@V_3: $\frac{V_3 - V_2}{R_2} + \frac{V_3}{R_4} + I_{S1} + I_{S2} = 0$

@V_4: $\frac{V_4 - V_1}{R_3} + \frac{V_4}{R_5} = I_{S2}$

$R_1 = R_2 = R_5 = 1k\Omega$

$R_4 = R_3 = 2k\Omega$

$I_{S1} = 2mA \quad I_{S2} = 4mA$

matrix form:

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{1}{R_1} + \frac{1}{R_3} & -\frac{1}{R_1} & 0 & -\frac{1}{R_3} \\ 0 & -\frac{1}{R_2} & \frac{1}{R_2} + \frac{1}{R_4} & 0 \\ -\frac{1}{R_3} & 0 & 0 & \frac{1}{R_3} + \frac{1}{R_5} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 12 \\ I_{S1} \\ -I_{S1} - I_{S2} \\ I_{S2} \end{bmatrix}$$

MATLAB:

Continued on the next page.

```
MATLAB WORK

Factor 1/1000 out of the conductance matrix.


EDU» g=[0,1000,0,0;1.5,-1,0,-0.5;0,-1,1.5,0;-0.5,0,0,1.5]

g =

  1.0e+003 *

         0    1.0000         0         0
    0.0015   -0.0010         0   -0.0005
         0   -0.0010    0.0015         0
   -0.0005         0         0    0.0015


EDU» i=[12;0.002;-0.006;0.004]

i =

   12.0000
    0.0020
   -0.0060
    0.0040


EDU» v=1000*inv(g)*i

v =

   11.5000
   12.0000
    4.0000
    6.5000
```

3.37 Determine V_o in the network in Fig. P3.37 using nodal analysis.

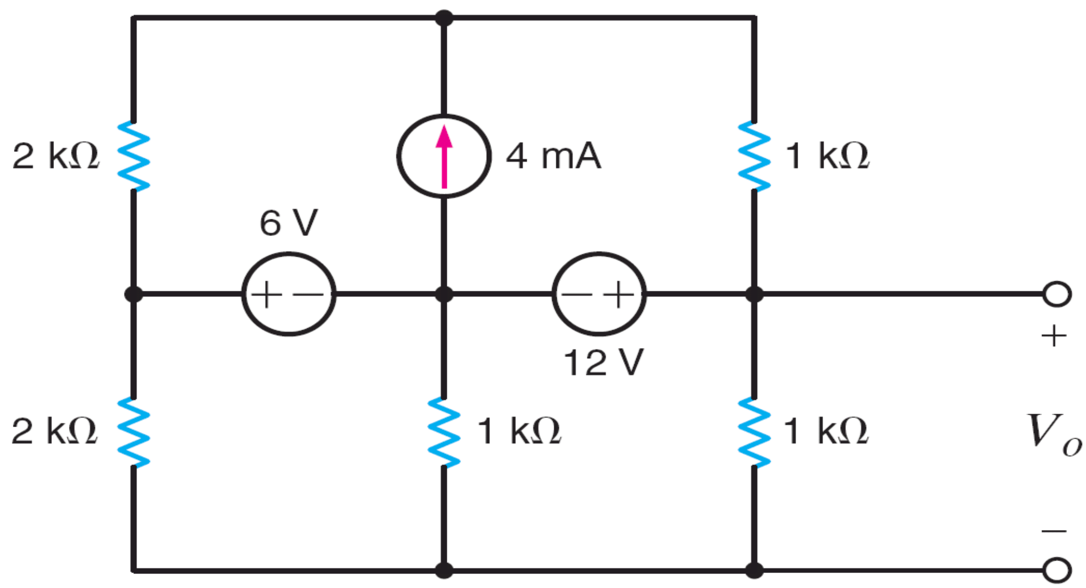
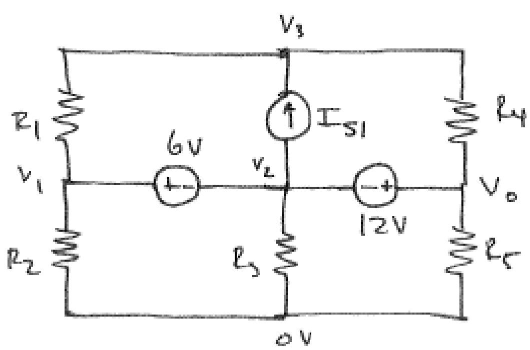


Figure P3.37

SOLUTION:

3.37 Find V_o by nodal.



$V_1 - V_2 = 6V$ $\quad V_o - V_2 = 12V$

@ V_3: $\dfrac{V_3 - V_1}{R_1} + \dfrac{V_3 - V_o}{R_4} = I_{S1}$

@ ref: $\dfrac{V_1}{R_2} + \dfrac{V_2}{R_3} + \dfrac{V_o}{R_5} = 0$

$R_1 = R_2 = 2k\Omega$ $\quad I_{S1} = 4mA$

$R_3 = R_4 = R_5 = 1k\Omega$

$\boxed{V_o = 6V}$

3.38 Find V_o in the circuit in Fig. P3.38.

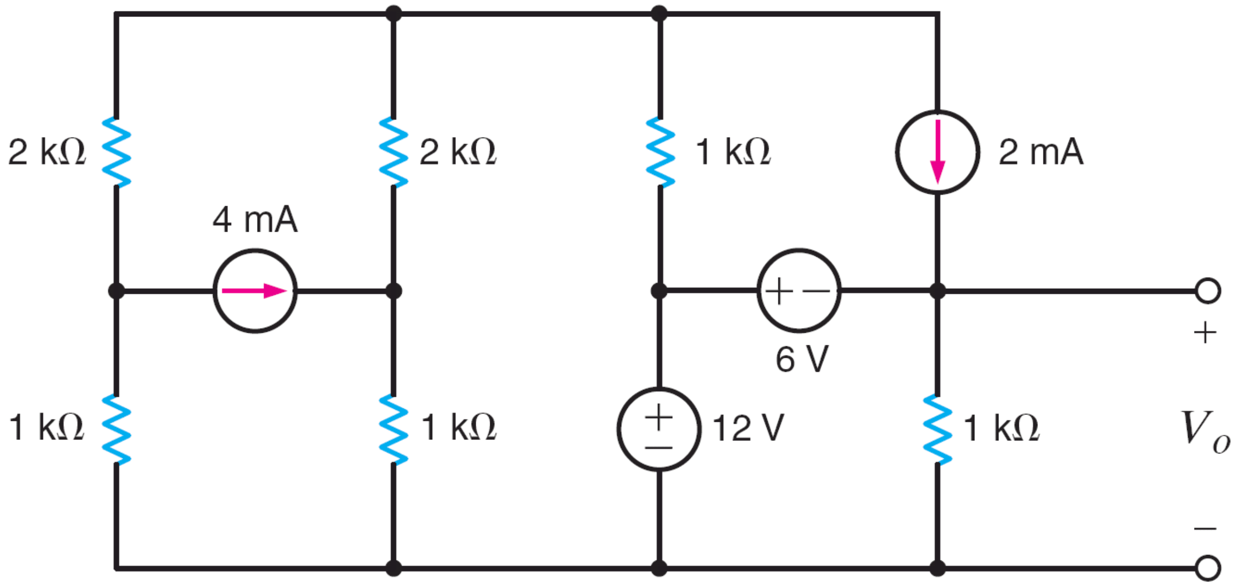
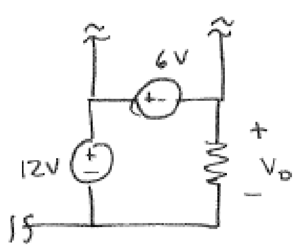


Figure P3.38

SOLUTION:

3.38 Find V_o



Rest of circuit is of no consequence to V_o

$V_o = 12-6$ $\boxed{V_o = 6V}$

3.39 Find V_o in the network in Fig. P3.39.

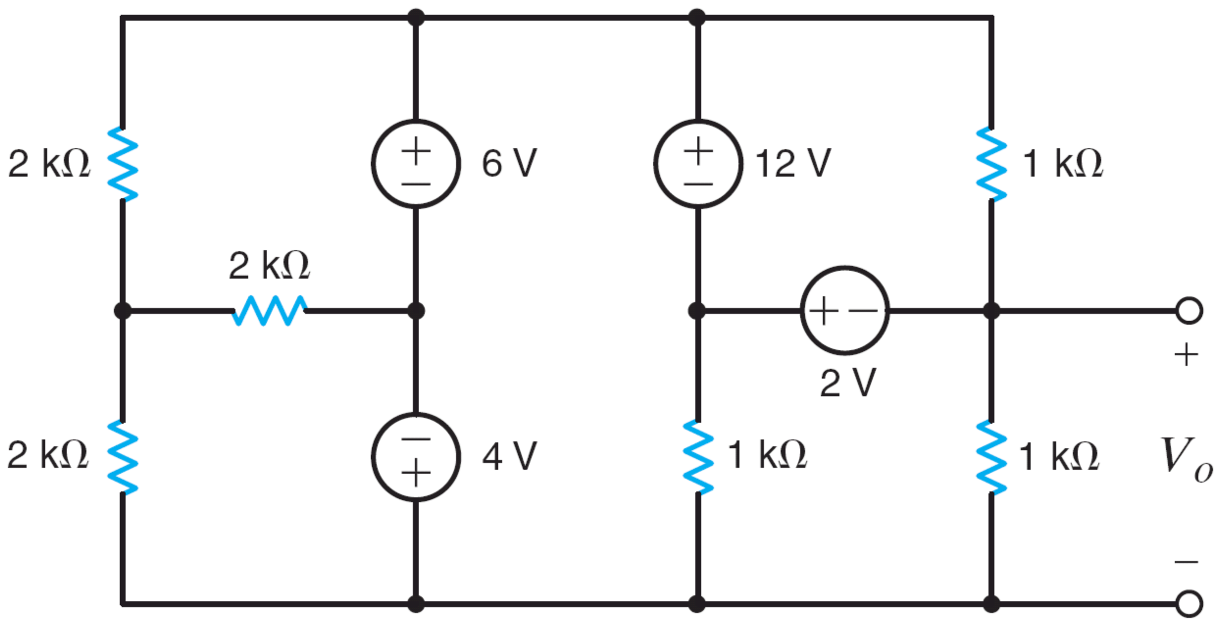
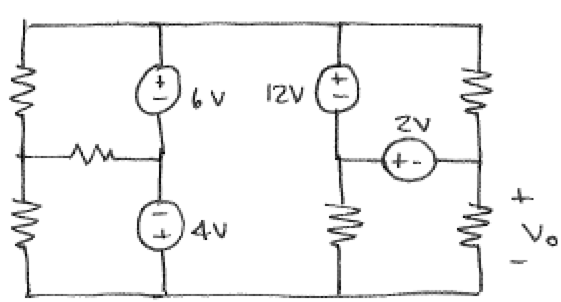


Figure P3.39

SOLUTION:

3.39 Find V_o



By KVL:

$$4 - 6 + 12 + 2 + V_o = 0$$

$$\boxed{V_o = -12}$$

3.40 Use nodal analysis to find V_o in the circuit in Fig. P3.40.

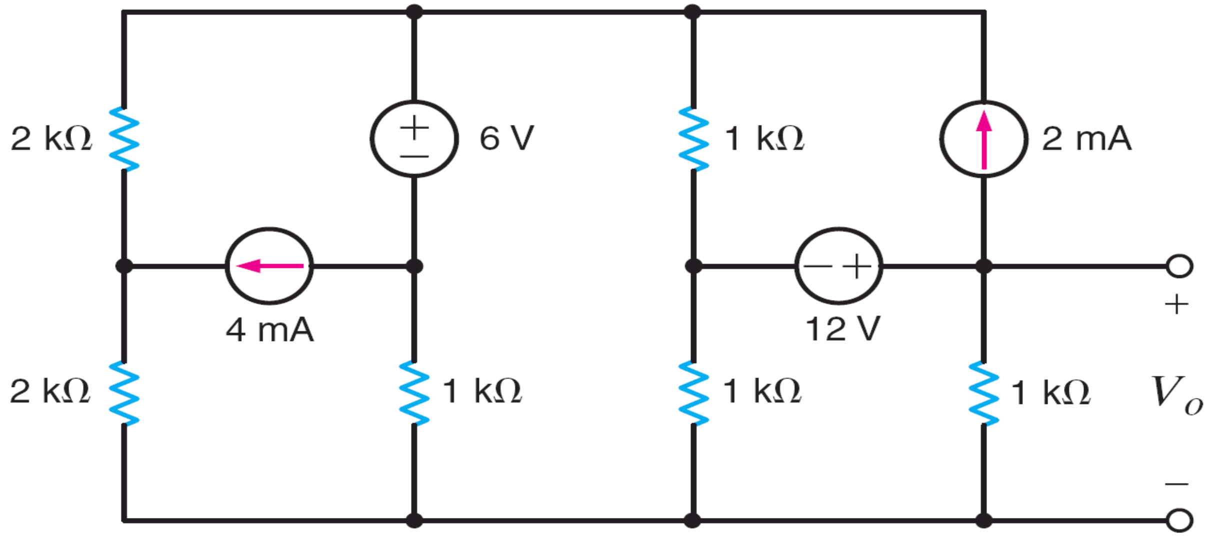
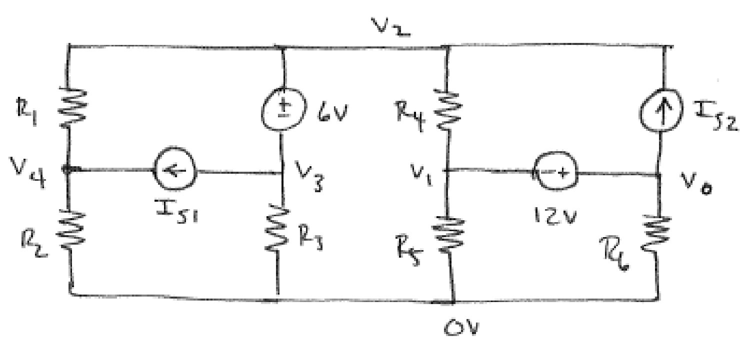


Figure P3.40

SOLUTION:

3.40 Find V_o by nodal.



$R_1 = R_2 = 2k\Omega$

$R_3 = R_4 = R_5 = R_6 = 1k\Omega$

$I_{S1} = 4mA$

$I_{S2} = 2mA$

$V_2 - V_3 = 6V \qquad V_o - V_1 = 12V$

@ V_4: $\dfrac{V_4 - V_2}{R_1} + \dfrac{V_4}{R_2} = I_{S1}$

$\boxed{V_o = 7.57\,V}$

@ ref: $\dfrac{V_4}{R_2} + \dfrac{V_3}{R_3} + \dfrac{V_1}{R_5} + \dfrac{V_o}{R_6} = 0$

3.41 Determine V_o in the network in Fig. P3.41.

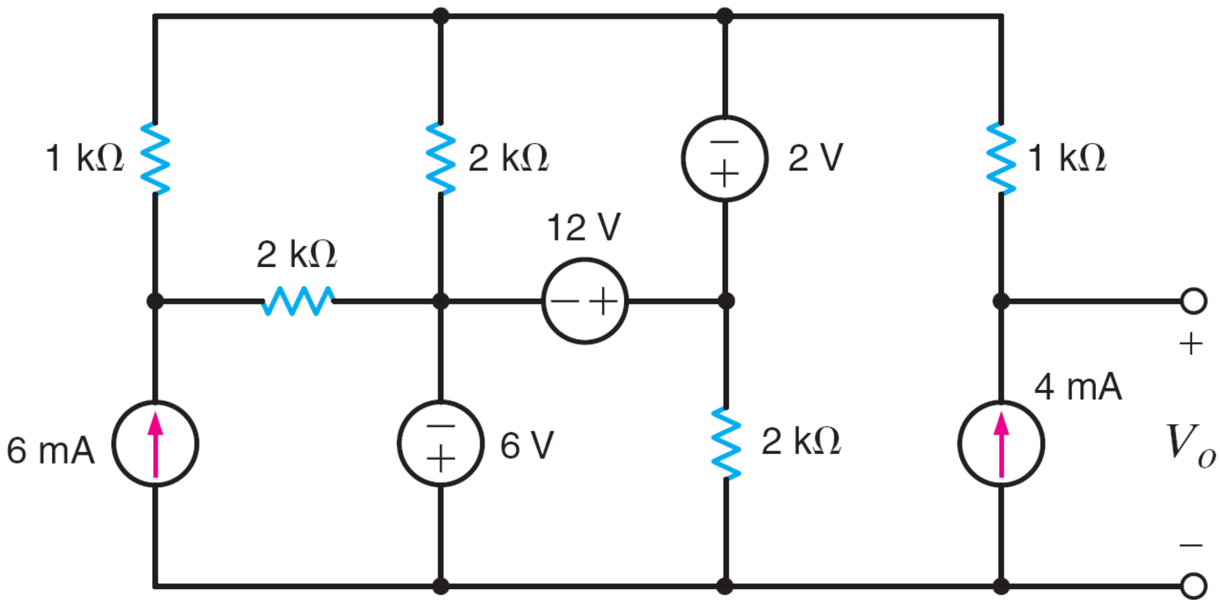
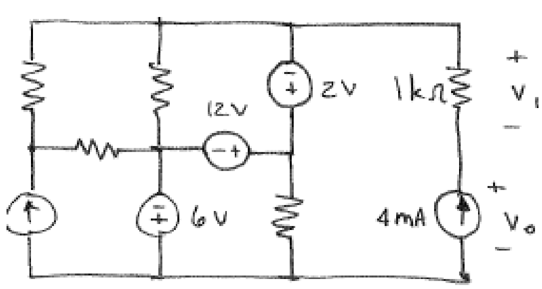


Figure P3.41

SOLUTION:

3.41 Find V_o



KVL: $6 - 12 + 2 + V_1 + V_o = 0$

$V_1 = (-4\times10^{-3})(10^3) = -4V$

$\boxed{V_o = 8V}$

3.42 Find V_o in the circuit in Fig. P3.42.

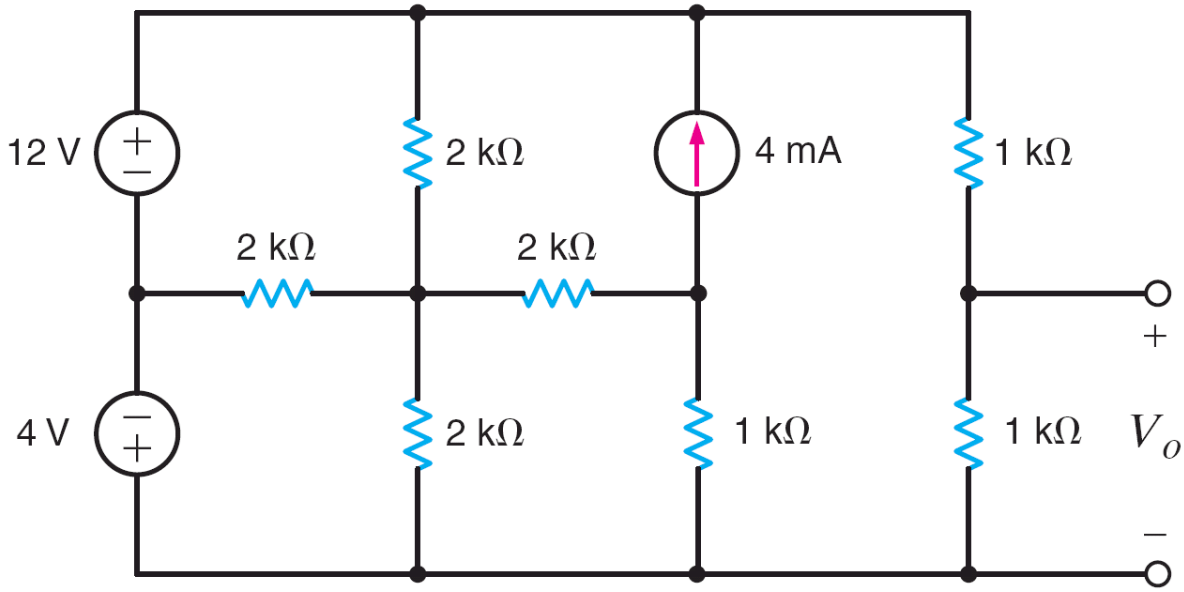
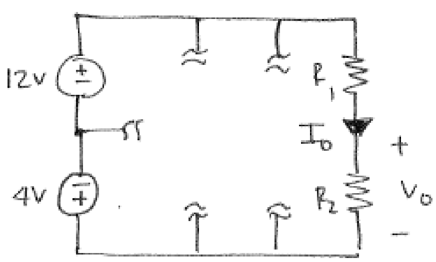


Figure P3.42

SOLUTION:

3.42 Find V_o



$R_1 = R_2 = 1k\Omega$

Rest of circuit has no effect on V_o!

KVL: $4 - 12 + I_o R_1 + I_o R_2 = 0$

$I_o = 4mA$

$V_o = R_2 I_o$ $\boxed{V_o = 4V}$

3.43 Find I_o in the circuit in Fig. P3.43 using nodal analysis.

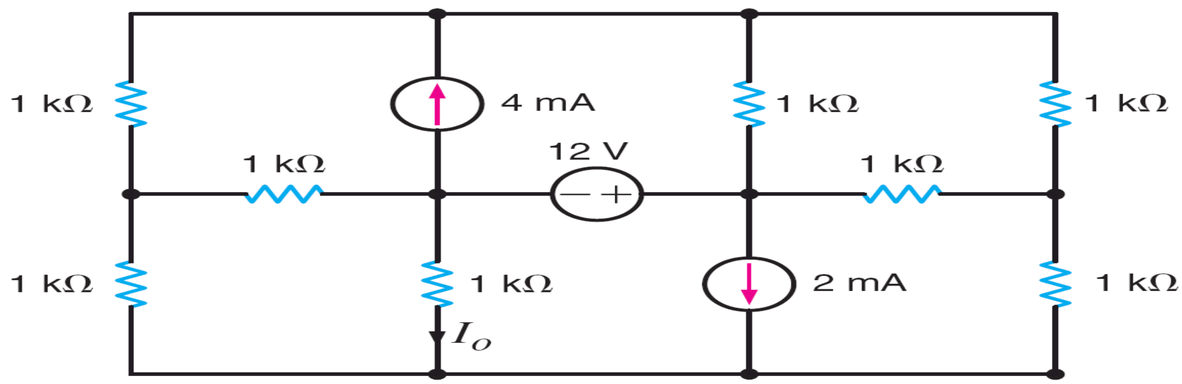
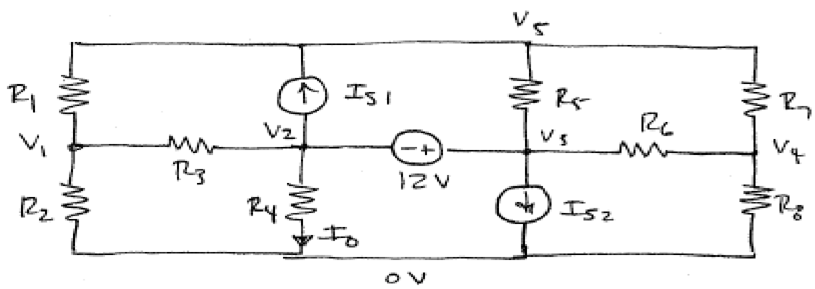


Figure P3.43

SOLUTION:

3.43 Find I_o by nodal.



all $R = 1k\Omega$

$I_{S1} = 4mA$

$I_{S2} = 2mA$

$V_3 - V_2 = 12$

@ V_1: $\quad \frac{V_1 - V_5}{R_1} + \frac{V_1}{R_2} + \frac{V_1 - V_2}{R_3} = 0$

@ V_4: $\quad \frac{V_4 - V_3}{R_6} + \frac{V_4 - V_5}{R_7} + \frac{V_4}{R_8} = 0$

@ V_5: $\quad \frac{V_5 - V_1}{R_1} + \frac{V_5 - V_3}{R_5} + \frac{V_5 - V_4}{R_7} = I_{S1}$

@ ref: $\quad \frac{V_1}{R_2} + \frac{V_2}{R_4} + \frac{V_4}{R_8} + I_{S2} = 0$

$I_o = V_2 / R_4 \qquad \boxed{I_o = -5.47mA}$

3.44 Use nodal analysis to find V_o in Fig. P3.44.

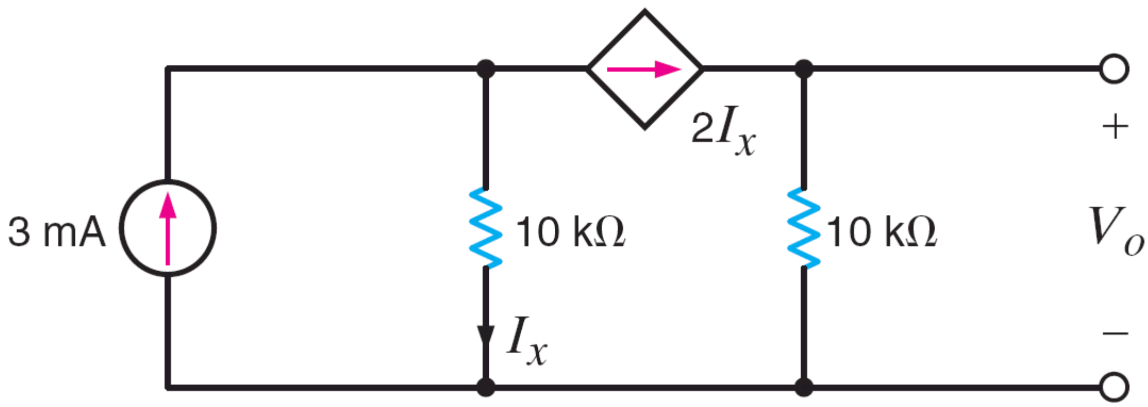
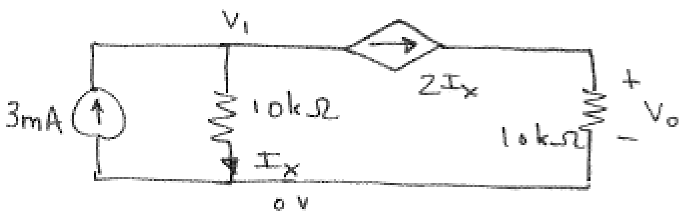


Figure P3.44

SOLUTION:

3.44 Find V_o by nodal.



@V_1: $3 \times 10^{-3} = \frac{V_1}{10^4} + 2I_x$ $\qquad I_x = \frac{V_1}{10^4}$

@V_o: $2I_x = \frac{V_o}{10^4}$ $\qquad \boxed{V_o = 20\text{V}}$

3.45 Find V_o in the circuit in Fig. P3.45 using nodal analysis.

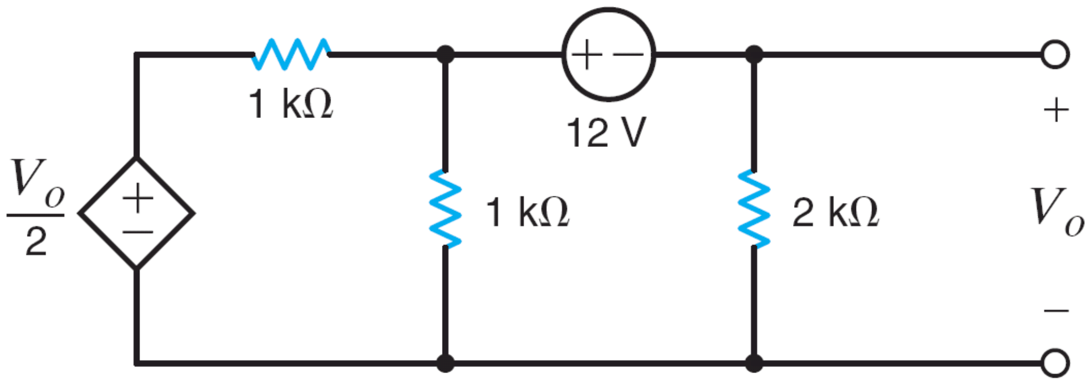
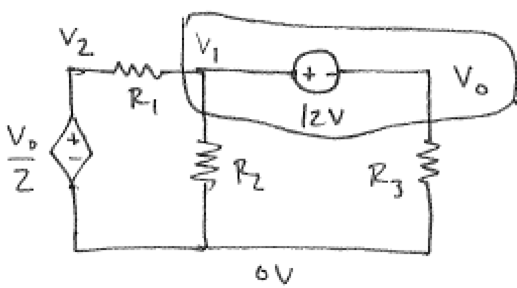


Figure P3.45

SOLUTION:

3.45 Find V_o by nodal.

$V_1 - V_0 = 12V$ $\qquad$ $V_2 = V_o/2$



supernode: $\frac{V_1 - V_2}{R_1} + \frac{V_1}{R_2} + \frac{V_o}{R_3} = 0$

$R_1 = 1\ k\Omega$ $\quad R_2 = 1k\Omega$ $\quad R_3 = 2k\Omega$

$\boxed{V_0 = -12V}$

3.46 Find V_o in the circuit in Fig. P3.46 using nodal analysis. Then solve the problem using MATLAB and compare your answers. **CS**

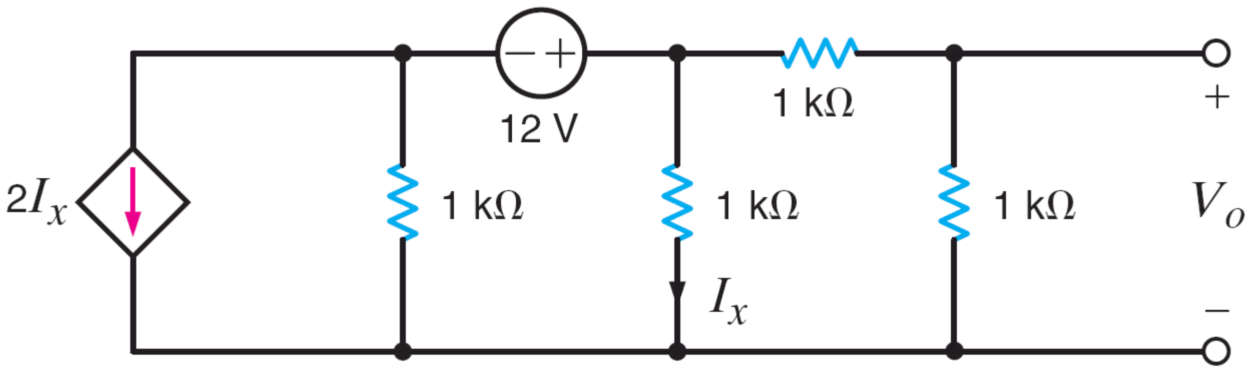
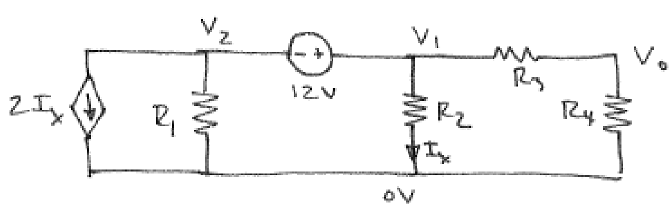


Figure P3.46

SOLUTION:

3.46 Find V_o by nodal & MATLAB



All $R = 1k\Omega$

$V_1 - V_2 = 12$

@ V_o: $\frac{V_o - V_1}{R_3} + \frac{V_o}{R_4} = 0$

@ ref: $2I_x + \frac{V_2}{R_1} + \frac{V_1}{R_2} + \frac{V_o}{R_4} = 0$

$I_x = V_1 / R_2$

$\boxed{V_o = 1.33V}$

3.47 Find I_o in the network in Fig. P3.47. **PSV**

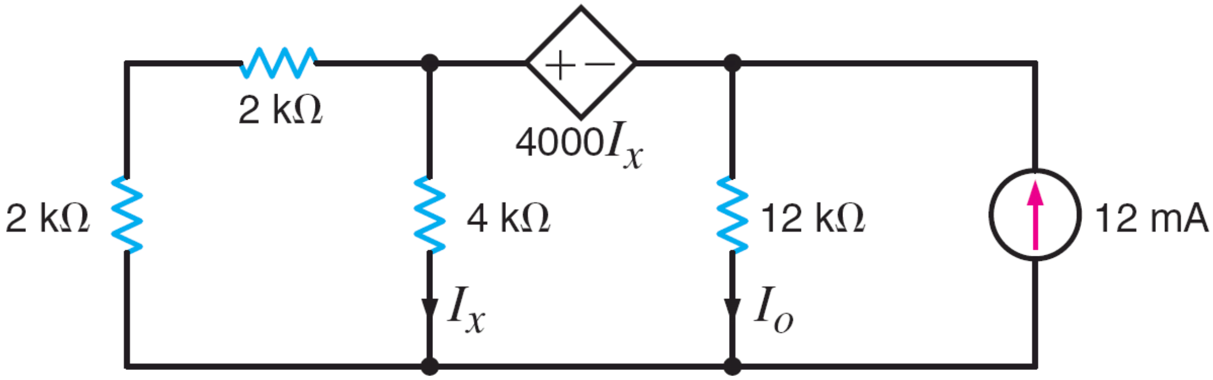
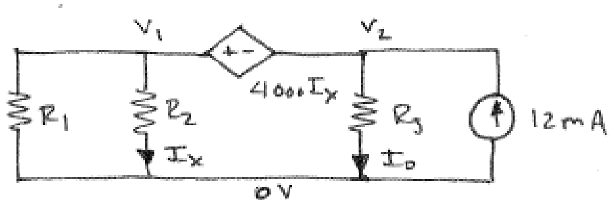


Figure P3.47

SOLUTION:

3.47 Find I_o.



$R_1 = R_2 = 4k\Omega$ $R_3 = 12k\Omega$

$V_1 - V_2 = 4000\,I_x$

$I_x = V_1 / R_2$

$\frac{V_1}{R_1} + \frac{V_1}{R_2} + \frac{V_2}{R_3} = 12 \times 10^{-3}$

$I_o = V_2 / R_3$

$\boxed{I_o = 0A}$

3.48 Find I_o in the circuit in Fig. P3.48 using nodal analysis.

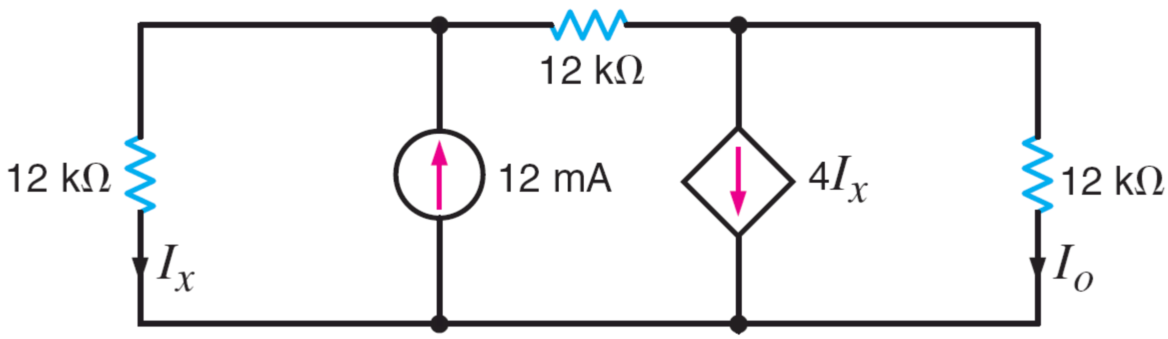
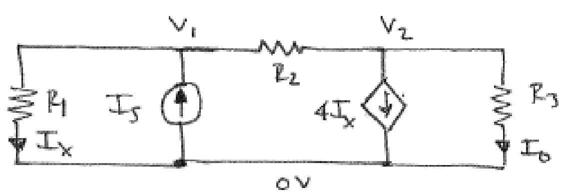


Figure P3.48

SOLUTION:

3.48 Find I_o by nodal.



All $R = 12k\Omega$ $\quad I_S = 12mA$

@ V_1: $\quad I_S = \frac{V_1}{R_1} + \frac{V_1 - V_2}{R_2}$

@ V_2: $\quad \frac{V_2 - V_1}{R_2} + \frac{V_2}{R_3} + 4I_x = 0$

$I_x = V_1 / R_1$

$I_o = V_2 / R_3$

$\boxed{I_o = 5.14mA}$

3.49 Find V_o in the network in Fig. P3.49 using nodal analysis.

CS

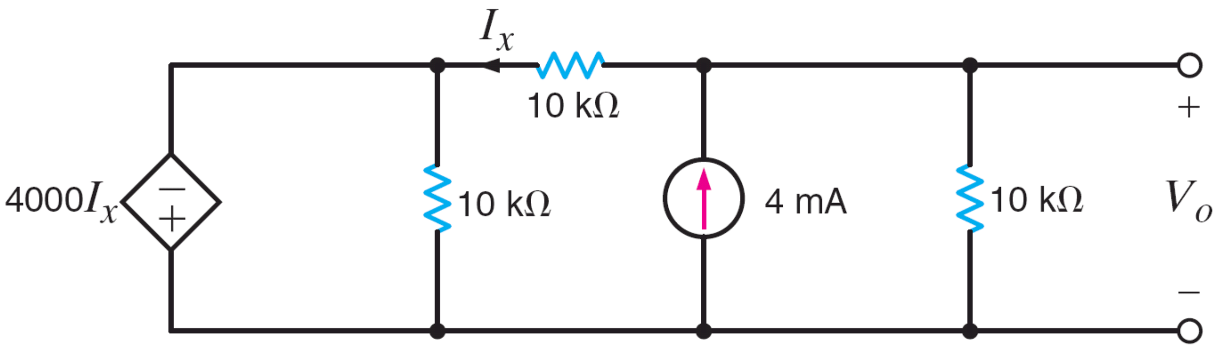
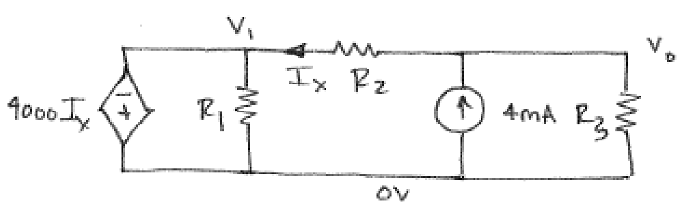


Figure P3.49

SOLUTION:

3.49 Find V_o by nodal.



$R_1 = R_2 = R_3 = 10k\Omega$

$V_1 = -4000I_x$

$I_x = (V_o - V_1)/R_2$

@ V_o: $\dfrac{V_o}{R_3} + \dfrac{V_o - V_1}{R_2} = 4\times10^{-3}$

$\boxed{V_o = 15V}$

3.50 Find V_o in the circuit in Fig. P3.50. **PSV**

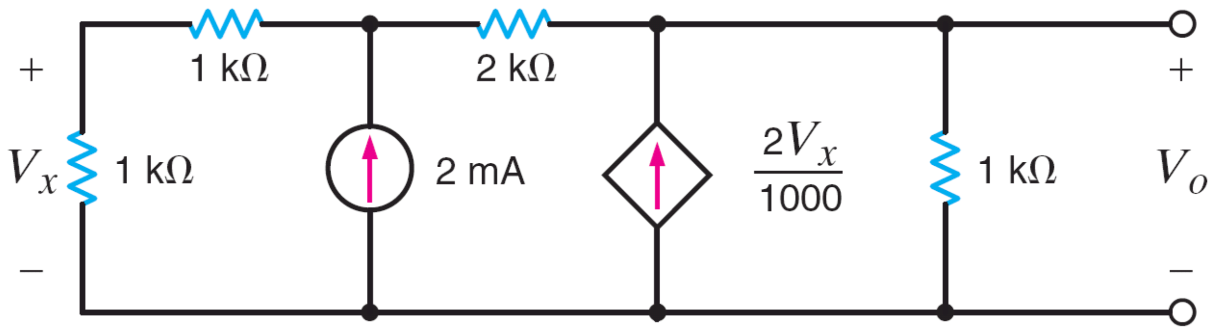
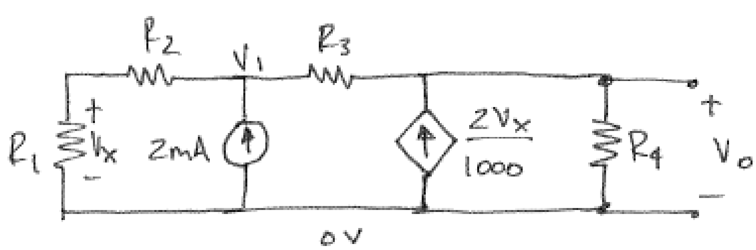


Figure P3.50

SOLUTION:

3.50 Find V_o



$R_1 = R_2 = R_4 = 1k\Omega$

$R_3 = 2k\Omega$

Nodal Analysis:

$$\left\{\begin{aligned} 2\times10^{-3} &= \frac{V_1 - V_o}{R_3} + \frac{V_1}{R_1 + R_2} \\ \frac{2V_x}{1000} &= \frac{V_o - V_1}{R_3} + \frac{V_o}{R_4} \end{aligned}\right.$$

Voltage Division

$$V_x = V_1\left[\frac{R_1}{R_1+R_2}\right]$$

$$V_x = V_1/2$$

$$\boxed{V_o = 4V}$$

3.51 Use nodal analysis to find V_o in the circuit in Fig. P3.51. In addition, find all branch currents and check your answers using KCL at every node.

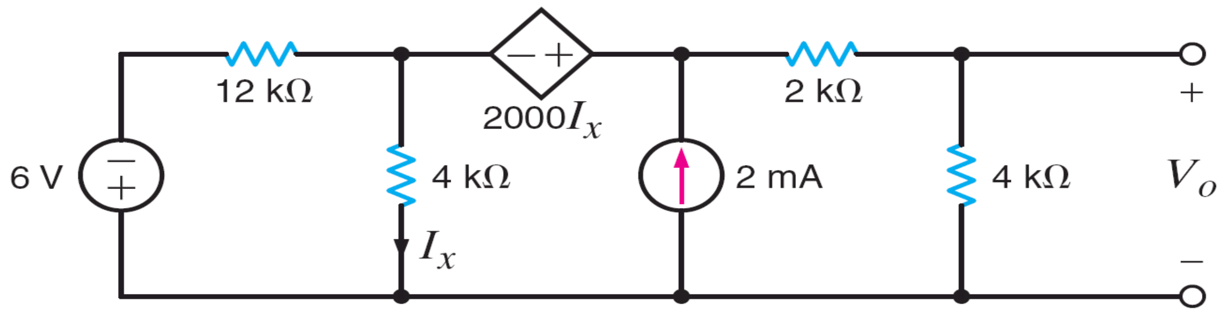
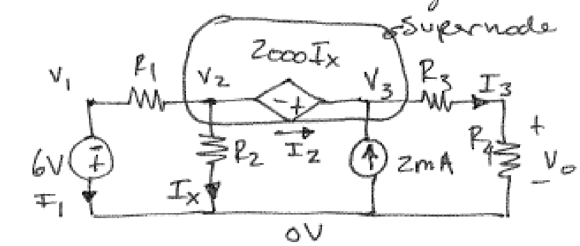


Figure P3.51

SOLUTION:

3.51 Use nodal analysis to find V_o & check via KCL.



$R_1 = 12\,k\Omega \quad R_2 = 4\,k\Omega$

$R_3 = 2\,k\Omega \quad R_4 = 4\,k\Omega$

$V_1 = -6V \qquad V_2 - V_3 = -2000 I_x \qquad I_x = V_2 / R_2$

$$\frac{V_3 - V_o}{R_3} = \frac{V_o}{R_4} \qquad \frac{V_2 - V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3 - V_o}{R_3} = 2\times10^{-3}$$

Solve for V_o : $\boxed{V_o = 2.57V}$

$I_1 = 714\mu A \qquad I_x = 643\mu A \qquad I_2 = -1357\mu A \qquad I_3 = 643\mu A$

@ V_2 $\quad I_1 + I_2 + I_x = 0\ ? \quad 714 + 643 - 1357 = 0$ ✓

@ V_3 $\quad I_2 + 2\times10^{-3} = I_3 \quad -1357 + 2000 = 643$ ✓

@ ref: $\quad I_1 + I_x + I_3 = 2\times10^{-3} \quad 714 + 643 + 643 - 2000 = 0$ ✓

3.52 Find the power supplied by the 2-A current source in the network in Fig. P3.52 using nodal analysis.

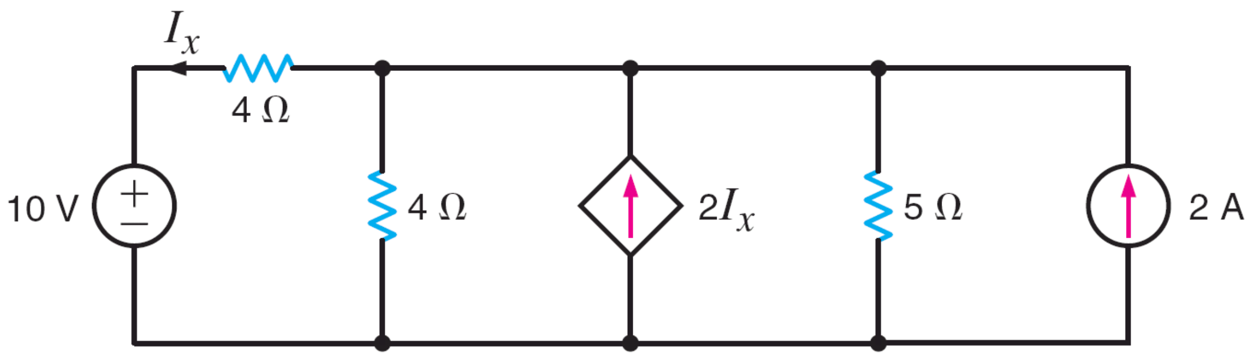
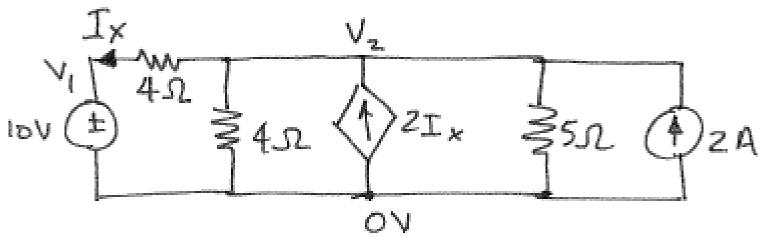


Figure P3.52

SOLUTION:

3.52 Find P_{2A} by nodal analysis.



$V_1 = 10V$

$I_x = \frac{V_2 - V_1}{4}$

$$2 + 2I_x = \frac{V_2}{4} + \frac{V_2}{5} + \frac{V_2 - V_1}{4} \Rightarrow V_2 =$$

$P_{2A} = (2)(V_2) = -5$ W supplied

$\boxed{\text{2A -source absorbs 5W}}$

3.53 Use nodal equations for the circuit in Fig. P3.53 to determine V_o .

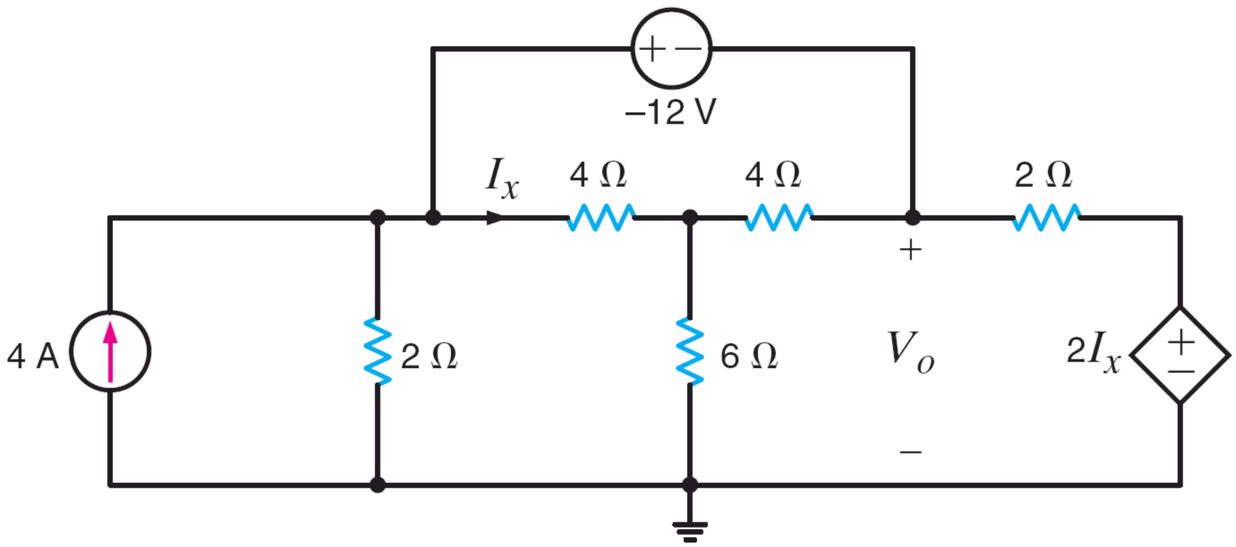
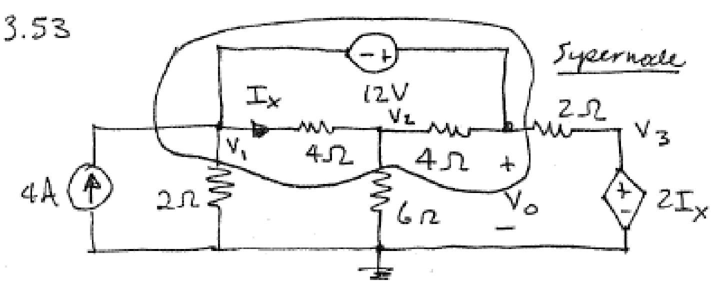


Figure P3.53

SOLUTION:

3.53



Find V_o by nodal analysis.

$12 = V_o - V_1$

$2I_x = V_3$

$I_x = (V_1 - V_2)/4$

at V_2: $\dfrac{V_1 - V_2}{4} + \dfrac{V_o - V_2}{4} = \dfrac{V_2}{6}$

at supernode: $4 = \dfrac{V_1}{2} + \dfrac{V_2}{6} + \dfrac{V_o - V_3}{2}$

Solve for V_o: $\boxed{V_o = 8.35\text{ V}}$

3.54 Determine V_o in the network in Fig. P3.54 using nodal analysis.

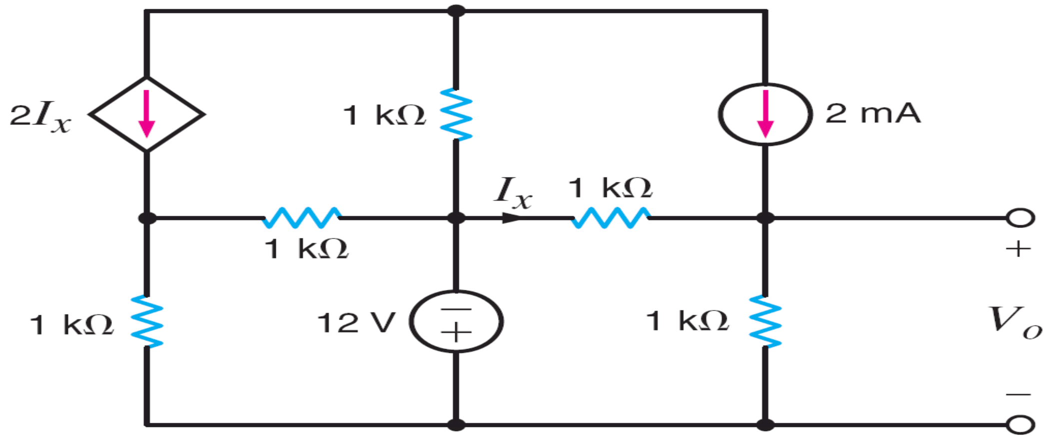
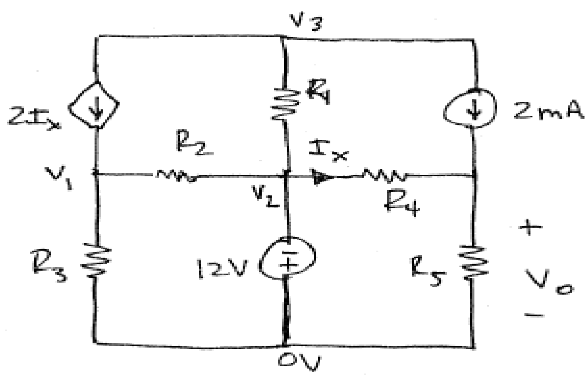


Figure P3.54

SOLUTION:

3.54 Find V_o by nodal analysis.



All R's = 1kΩ

$V_2 = -12$

$I_x = \dfrac{V_2 - V_o}{R_4}$

at V_3: $\quad 2I_x + 2\times10^{-3} + \dfrac{V_3 - V_2}{R_1} = 0$

at V_1: $\quad 2I_x + \dfrac{V_2 - V_1}{R_2} = \dfrac{V_1}{R_3}$

at V_o: $\quad 2\times10^{-3} + \dfrac{V_2 - V_o}{R_4} = \dfrac{V_o}{R_5}$

Solve for V_o: $\quad \boxed{V_o = -5V}$

3.55 Calculate V_o in the circuit in Fig. P3.55 using nodal analysis.

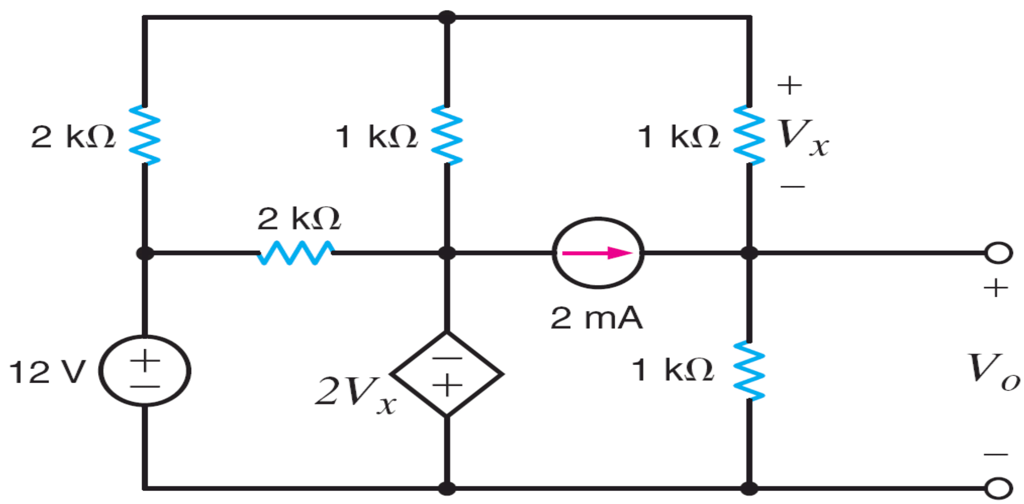
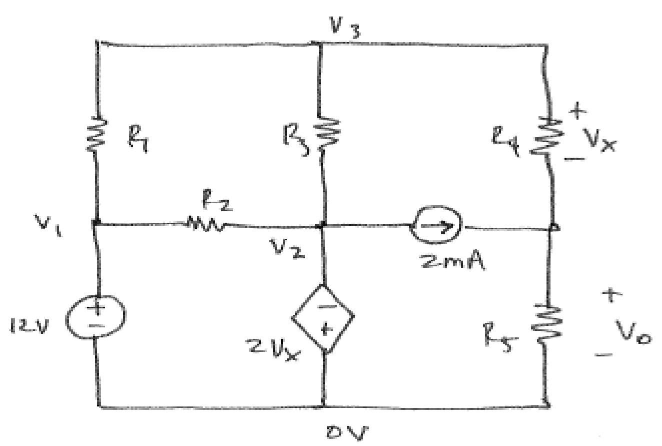


Figure P3.55

SOLUTION:

3.55 Find V_o by nodal analysis.



$R_1 = R_2 = 2k\Omega$

$R_3 = R_4 = R_5 = 1k\Omega$

$V_x = V_3 - V_o$

$V_1 = 12V$

$V_2 = -2V_x$

at V_3: $$\frac{V_3 - V_1}{R_1} + \frac{V_3 - V_2}{R_3} + \frac{V_3 - V_o}{R_4} = 0$$

at V_o: $$\frac{V_3 - V_o}{R_4} + 2\times10^{-3} = \frac{V_o}{R_5}$$

Solve for V_o: $\boxed{V_o = 2.5V}$

3.56 Using nodal analysis, find V_o in the network in Fig. P3.56.

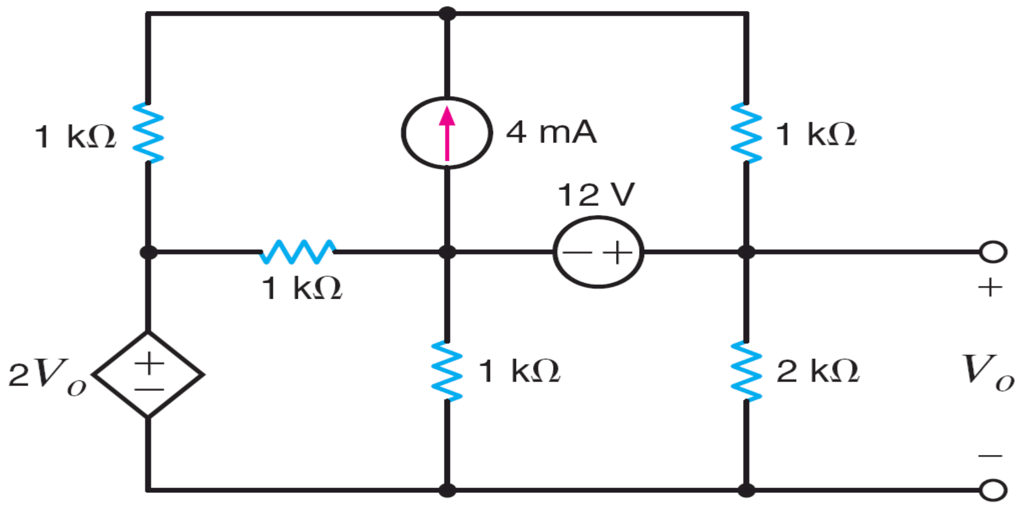
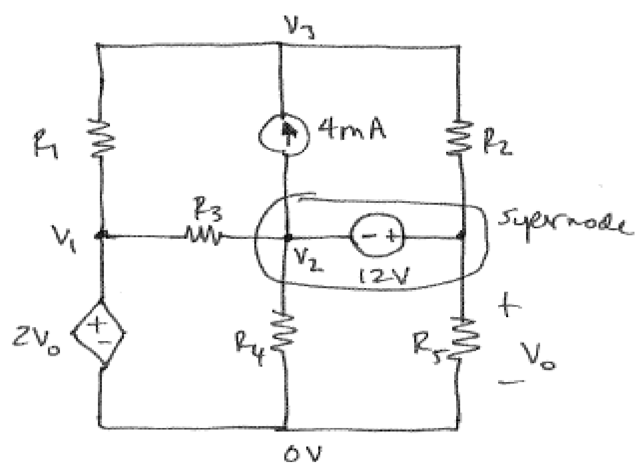


Figure P3.56

SOLUTION:

3.56 Find V_o by nodal analysis.



$R_1 = R_2 = R_3 = R_4 = 1k\Omega$

$R_5 = 2k\Omega$

$V_1 = 2V_o$

$V_o - V_2 = 12V$

at V_3: $\quad 4\times10^{-3} = \frac{V_3 - V_1}{R_1} + \frac{V_3 - V_o}{R_2}$

at supernode: $\quad \frac{V_o}{R_5} + \frac{V_2}{R_4} + \frac{V_o - V_3}{R_2} + \frac{V_2 - V_1}{R_3} + 4\times10^{-3} = 0$

Solve for V_o: $\quad \boxed{V_o = \infty \ V}$

3.57 Use nodal analysis to find V_o in the circuit in Fig. P3.57.

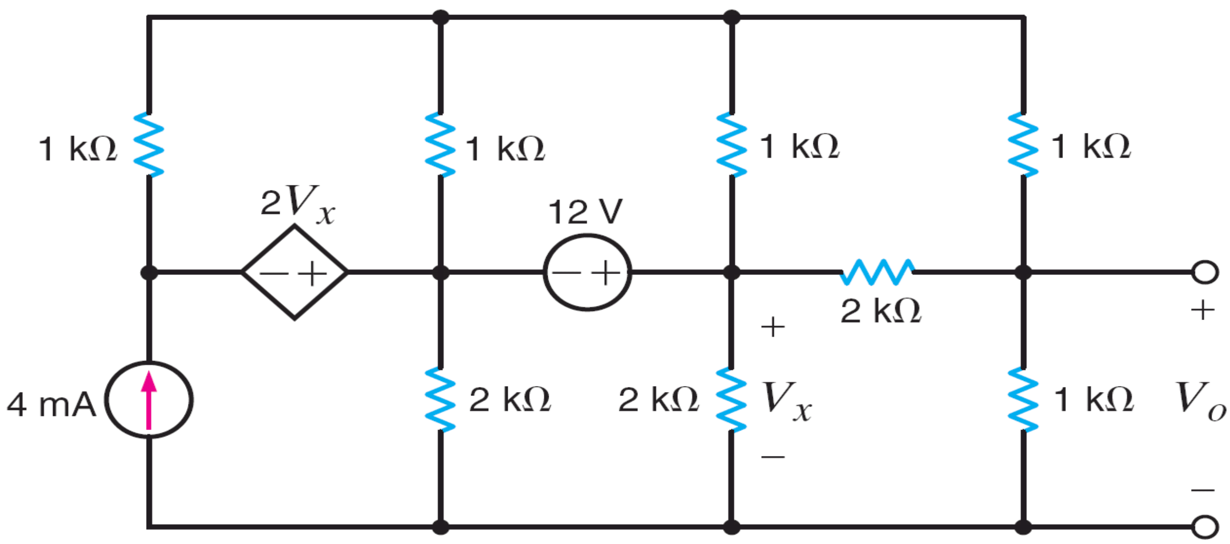
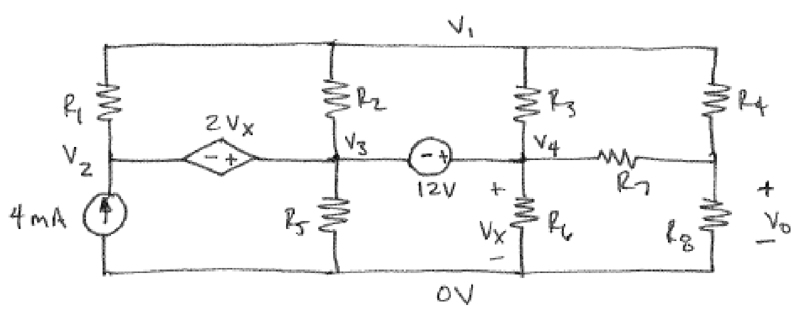


Figure P3.57

SOLUTION:

3.57 Find V_o by nodal analysis.



$R_1 = R_2 = R_3 = R_4 = R_8 = 1k\Omega$

$R_5 = R_6 = R_7 = 2k\Omega$

$V_4 - V_3 = 12$

$V_3 - V_2 = 2V_x$

$V_x = V_4$

at V_1 $\quad \frac{V_1 - V_2}{R_1} + \frac{V_1 - V_3}{R_2} + \frac{V_1 - V_4}{R_3} + \frac{V_1 - V_o}{R_4} = 0$

at ref $\quad \frac{V_3}{R_5} + \frac{V_4}{R_6} + \frac{V_o}{R_8} = 4\times10^{-3}$

Solve for V_o: $\boxed{V_o = 0.5V}$

3.58 Use nodal analysis to determine I_o in the circuit in Fig. P3.58.

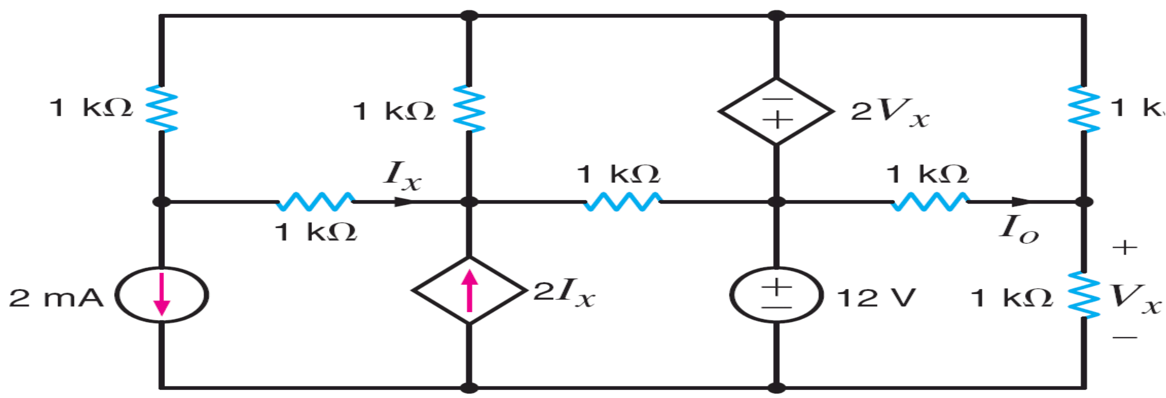
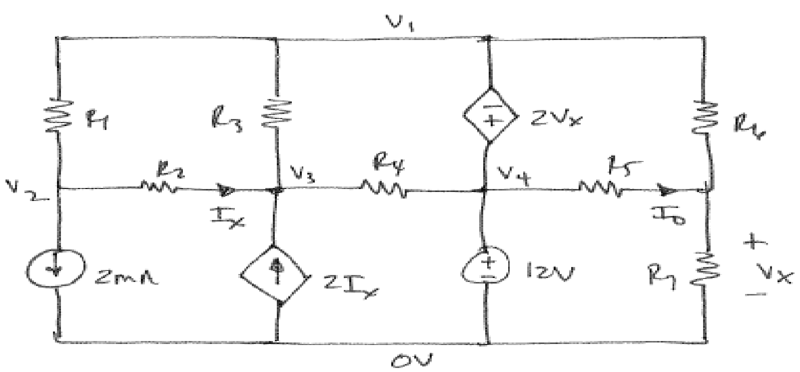


Figure P3.58

SOLUTION:

3.58 Use nodal to find I_o



All R's = 1kΩ

$V_4 - V_1 = 2V_x$

$V_4 = 12V$

$I_x = (V_2 - V_3)/R_2$

at V_2 $\quad \dfrac{V_2 - V_1}{R_1} + \dfrac{V_2 - V_3}{R_2} + 2\times10^{-3} = 0$

at V_3 $\quad \dfrac{V_3 - V_2}{R_2} + \dfrac{V_3 - V_1}{R_3} + \dfrac{V_3 - V_4}{R_4} = 2I_x$

at V_x $\quad \dfrac{V_x}{R_7} + \dfrac{V_x - V_4}{R_5} + \dfrac{V_x - V_1}{R_6} = 0$

Finally, $I_o = (V_4 - V_x)/R_5$

$V_4 = 12V$ $\qquad V_x = 4.8V$ $\qquad \boxed{I_o = 7.2\,mA}$

3.59 Find I_o in the network in Fig. P3.59 using nodal analysis.

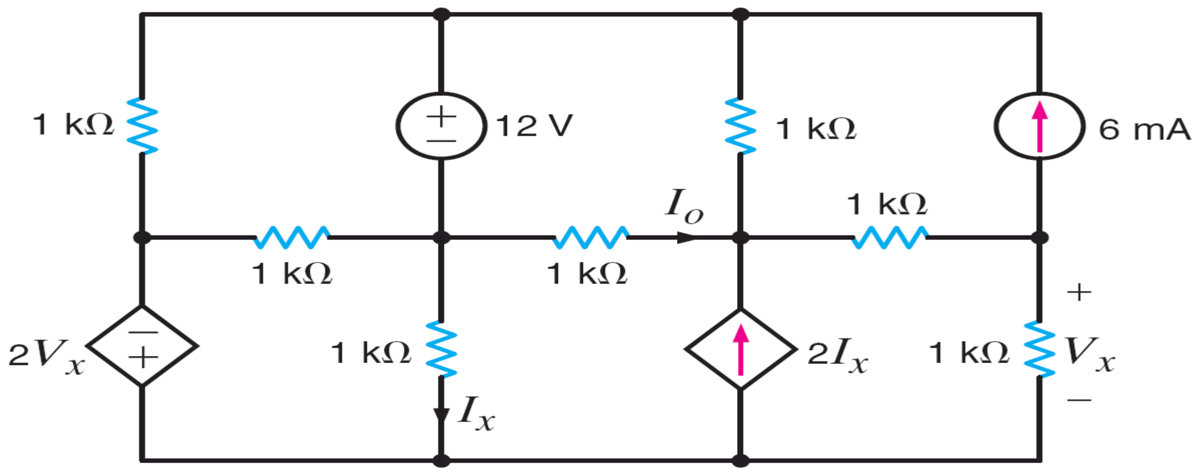
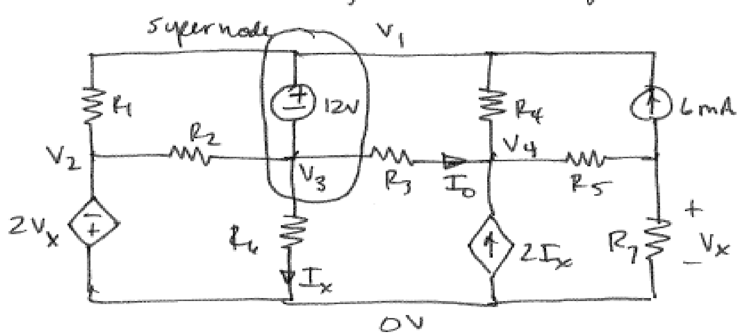


Figure P3.59

SOLUTION:

3.59 Find I_o by nodal analysis



All R's = 1kΩ

$12 = V_1 - V_3$

$2V_x + V_2 = 0$

$I_x = V_3 / R_6$

At V_4

$$\frac{V_4 - V_1}{R_4} + \frac{V_4 - V_x}{R_5} + \frac{V_4 - V_3}{R_3} = 2I_x$$

At supernode:

$$\frac{V_1 - V_2}{R_1} + \frac{V_1 - V_4}{R_4} - 6\times 10^{-3} + \frac{V_3 - V_2}{R_2} + \frac{V_3 - V_4}{R_3} + \frac{V_3}{R_6} = 0$$

And $I_o = \dfrac{V_3 - V_4}{R_3}$

$V_3 = -1.2V$ $\quad$ $V_4 = 1.68V$ $\quad$ $\boxed{I_o = 2.88mA}$

3.60 Given the network in Fig. P3.60, we wish to determine the power dissipated in the resistor R_3.

(a) Is mesh or nodal analysis the most efficient approach? Why?

(b) For a nodal analysis, comment on the advantages of selecting node 1 as the reference node. Repeat for nodes 2, 3, and 4.

(c) Based on your results in (b), write the node equations.

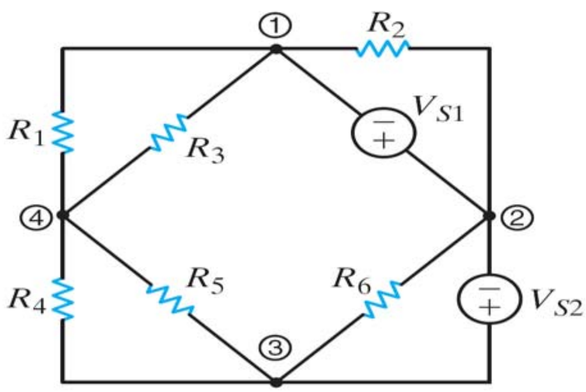


Figure P3.60

SOLUTION:

3.60 Find power absorbed by R_3.

a) Mesh or nodal?

5 mesh vs. 3 non-ref nodes ⇒ nodal.

b) Justify ref node choice

Node 1 - Great choice. V_{R_3} will be V_4.

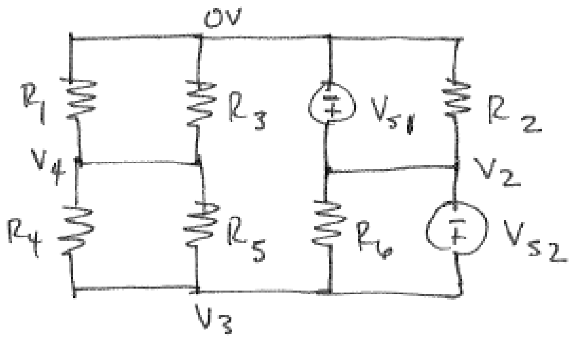
Node 4 - Just as good as choosing node 1. V_{R_3} will be V_1.

Node 2 - Poor choice. $V_{R_3} = V_1 - V_4$ requiring 2 calculations. But V_{S1} & V_{S2} will be simple to write in terms of node voltages

Node 3 - Worst choice of all.

Continued on the next page.

c) Choose V_1 as reference.



$$V_{S1} = V_2$$

$$V_{S2} = V_3 - V_2$$

$$\frac{V_4}{R_1} + \frac{V_4}{R_3} + \frac{V_4 - V_3}{R_4} + \frac{V_4 - V_3}{R_5} = 0$$

3.61 In the circuit in Fig. P3.61, use Gaussian elimination to determine V_o.

(a) Would mesh or nodal analysis be the most efficient approach? Why?

(b) If mesh analysis is used, are any supermeshes required? Write the mesh equations. If nodal analysis is used, are any supernodes required? If so, how many? What is the best location for the reference node and why? Write the node equations.

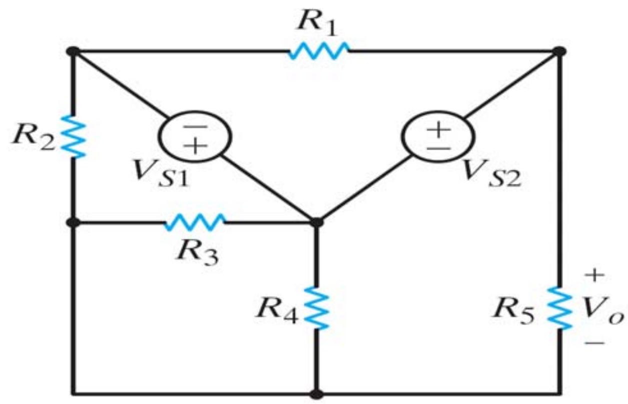


Figure P3.61

SOLUTION:

3.61 Find V_o

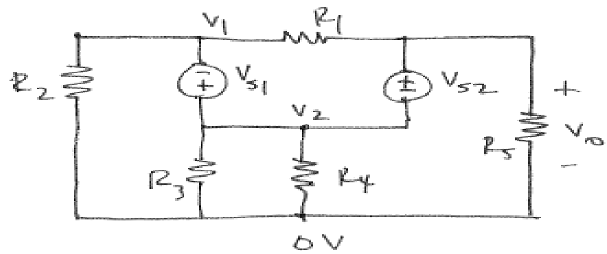
a) Mesh or nodal?

4 meshes, 3 non-ref. nodes, 2 voltage sources ⇒ Nodal

b) No supermeshes required because there are no current sources

No supernodes are needed because the 2 voltage sources connect at only 3 of the nodes, leaving the 4th node free for application of KCL.

Best location for ref node is at bottom of schematic.



$$V_1 - V_2 = -V_{S1}$$

$$V_o - V_2 = V_{S2}$$

$$\frac{V_1}{R_2} + \frac{V_2}{R_3} + \frac{V_2}{R_4} + \frac{V_o}{R_5} = 0$$

3.62 Use mesh equations to find V_o in the circuit in Fig. P3.62.

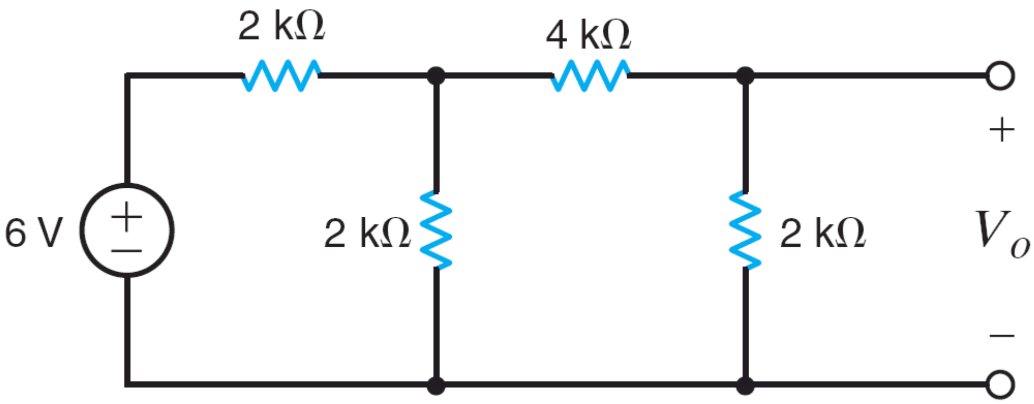
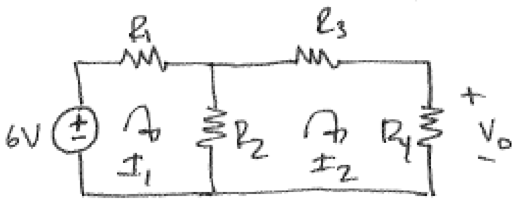


Figure P3.62

SOLUTION:

3.62 Use mesh analysis to find V_o.



$R_1 = R_2 = R_4 = 2k\Omega \quad R_3 = 4k\Omega$

$6 = I_1 R_1 + (I_1 - I_2) R_2$

$0 = -R_2 I_1 + I_2 (R_2 + R_3 + R_4)$

$V_o = I_2 R_4$

$I_2 = 429 \mu A \qquad \boxed{V_o = 858 mV}$

3.63 Find V_o in the network in Fig. P3.63 using mesh equations. **PSV**

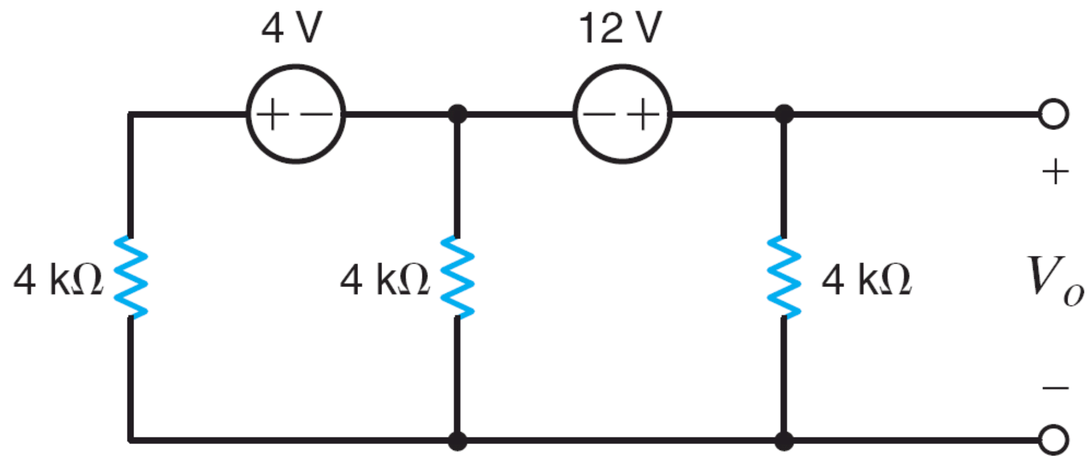
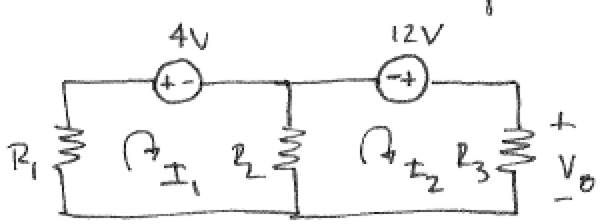


Figure P3.63

SOLUTION:

3.63 Use mesh analysis to find V_o.



all R's = 4kΩ

$I_1R_1 + (I_1 - I_2)R_2 + 4 = 0$

$I_2R_3 + (I_2 - I_1)R_2 = 12$

$V_o = I_2R_3$

$I_2 = 1.67 \text{ mA}$

$\boxed{V_o = 6.67\text{V}}$

3.64 Use mesh analysis to find V_o in the circuit in Fig. P3.64.

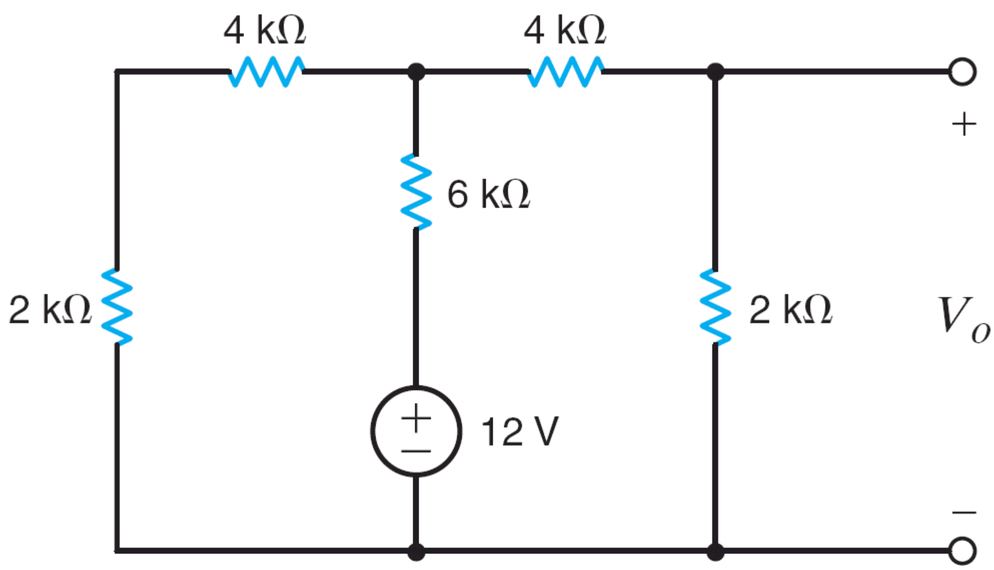
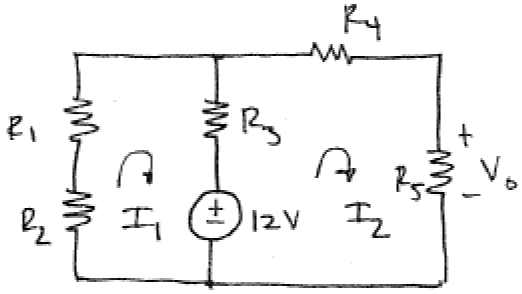


Figure P3.64

SOLUTION:

3.64 Use mesh to find V_o



$R_1 = 4k\Omega$ $R_2 = 2k\Omega$ $R_3 = 6k\Omega$

$R_4 = 4k\Omega$ $R_5 = 2k\Omega$

$V_o = I_2 R_5$

$$I_1 R_1 + I_1 R_2 + (I_1 - I_2) R_3 + 12 = 0$$

$$I_2 R_4 + I_2 R_5 + (I_2 - I_1) R_3 = 12$$

$I_2 = 0.67\ mA$ $\boxed{V_o = 1.33V}$

3.65 Use mesh analysis to find V_o in the circuit in Fig. P3.65.

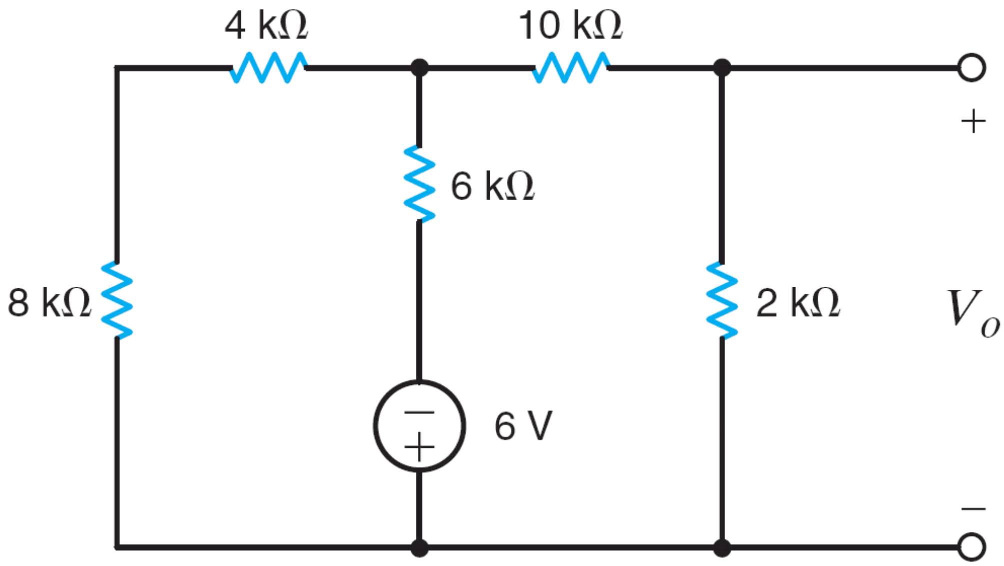
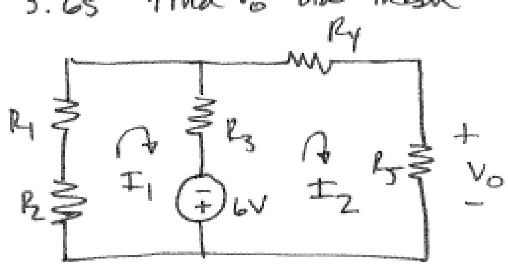


Figure P3.65

SOLUTION:

3.65 find V_o via mesh



$R_1 = 4k\Omega \quad R_2 = 8k\Omega \quad R_3 = 6k\Omega$

$R_4 = 10k\Omega \quad R_5 = 2k\Omega$

$V_o = I_2 R_5$

$$I_1 R_1 + I_1 R_2 + (I_1 - I_2) R_3 = 6$$

$$I_2 R_4 + I_2 R_5 + (I_2 - I_1) R_3 = -6$$

$$I_2 = -250\mu A \qquad \boxed{V_o = -0.5V}$$

3.66 Use mesh analysis to find V_o in the network in Fig. P3.66. **CS**

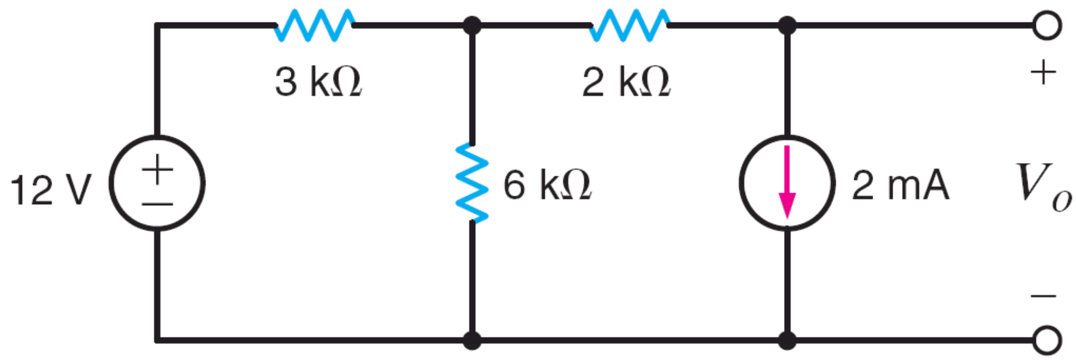
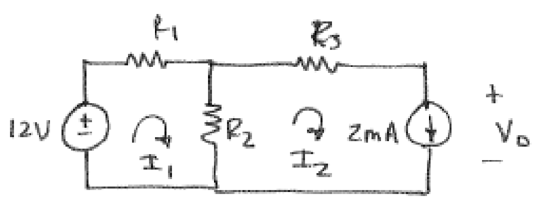


Figure P3.66

SOLUTION:

3.66 Find V_o using mesh analysis.



$R_1 = 3k\Omega \quad R_2 = 6k\Omega \quad R_3 = 2k\Omega$

$I_2 = 2mA$

$12 = I_1R_1 + (I_1 - I_2)R_2$

$V_o = 12 - I_1R_1 - I_2R_3$

$I_1 = 2.67\ mA$

$\boxed{V_o = 0V}$

3.67 Use loop analysis to find V_o in the circuit in Fig. P3.67.

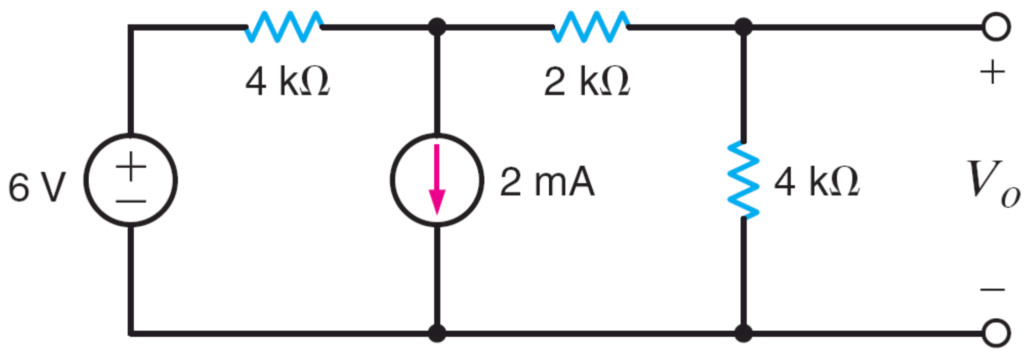
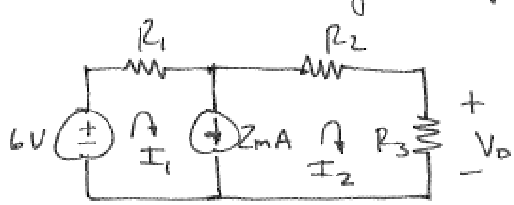


Figure P3.67

SOLUTION:

3.67 Find V_o using loop analysis.



$R_1 = 4k\Omega \quad R_2 = 2k\Omega \quad R_3 = 4k\Omega$

$2mA = I_1 - I_2$

$6 = I_1 R_1 + I_2 R_2 + I_2 R_3$

$V_o = I_2 R_3$

$I_2 = -200\mu A$ $\quad \boxed{V_o = -0.8V}$

3.68 Use loop analysis to find V_o in the network in Fig. P3.68.
PSV

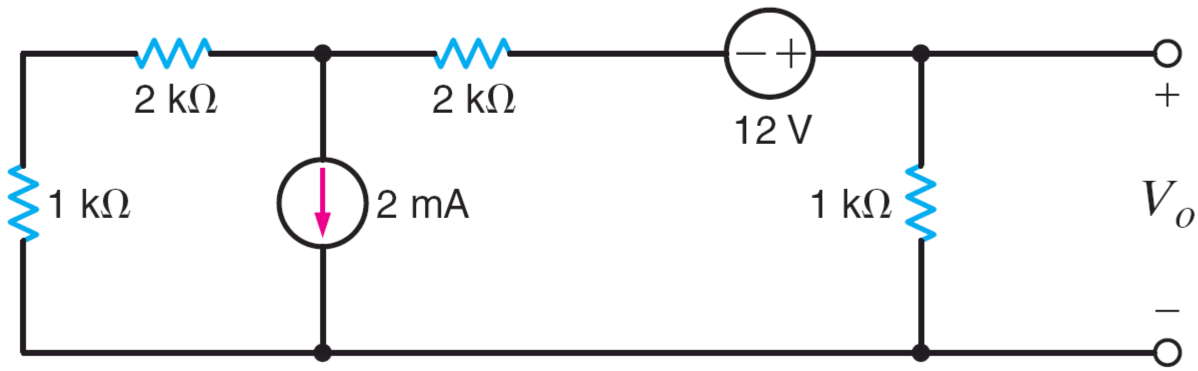
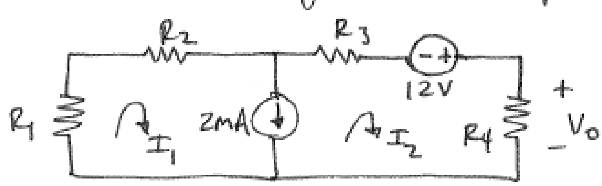


Figure P3.68

SOLUTION:

3.68 Find V_o using loop analysis.



$R_1 = R_4 = 1k\Omega$ $R_2 = R_3 = 2k\Omega$

$V_o = I_2 R_4$

$I_1 - I_2 = 2mA$ $I_1 R_1 + I_1 R_2 + I_2 R_3 + I_2 R_4 = 12$

$I_2 = 1mA$ $\boxed{V_o = 1V}$

3.69 Find I_o in the network in Fig. P3.69 using mesh analysis.
CS

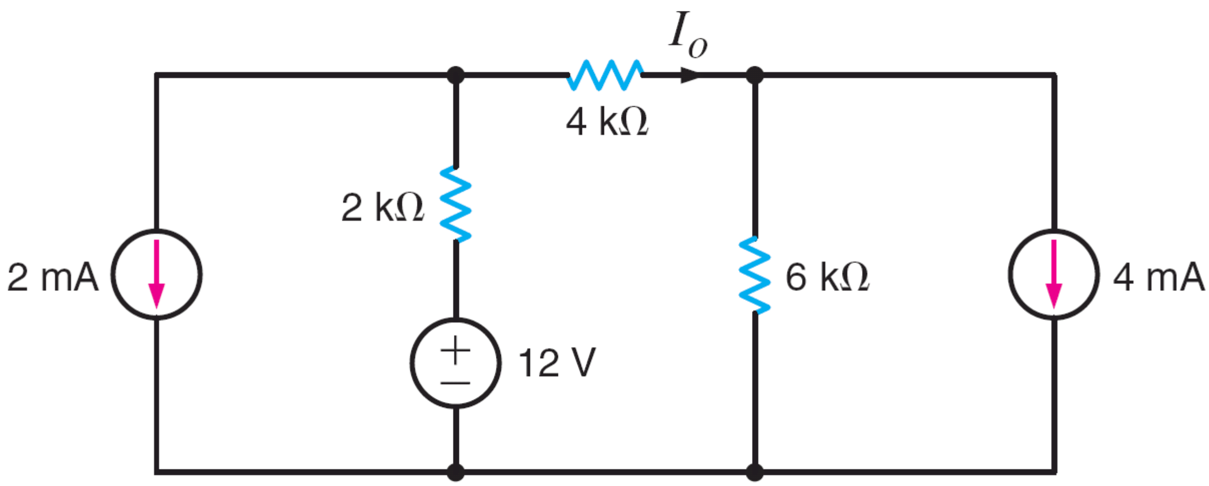
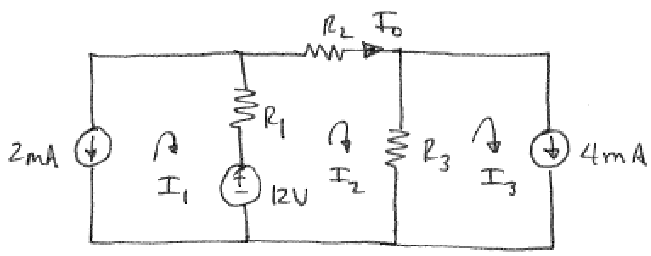


Figure P3.69

SOLUTION:

3.69 Use mesh analysis to find I_o.



$R_1 = 2k\Omega \quad R_2 = 4k\Omega \quad R_3 = 6k\Omega$

$I_o = I_2$

$I_1 = -2mA$

$I_3 = 4mA$

$12 = (I_2 - I_1)R_1 + I_2R_2 + (I_2 - I_3)R_3 \Rightarrow I_2 = 2.67mA$

$\boxed{I_o = 2.67mA}$

3.70 Use both nodal analysis and mesh analysis to find I_o in the circuit in Fig. P3.70.

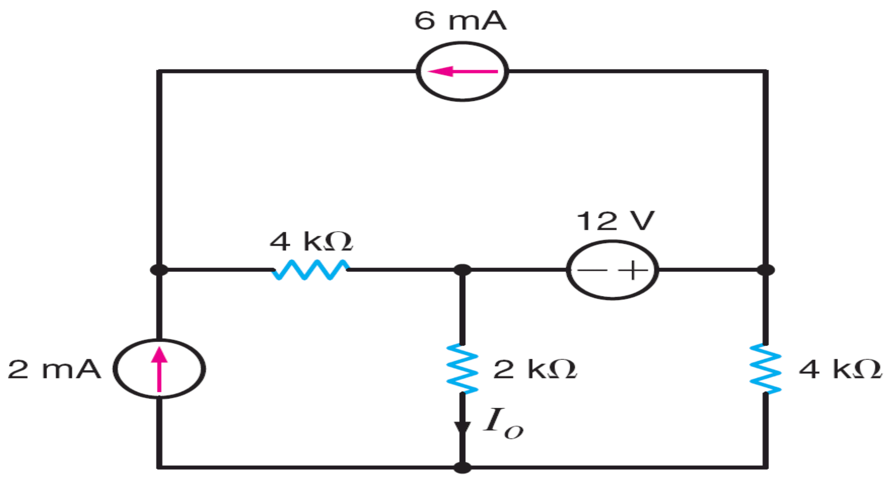
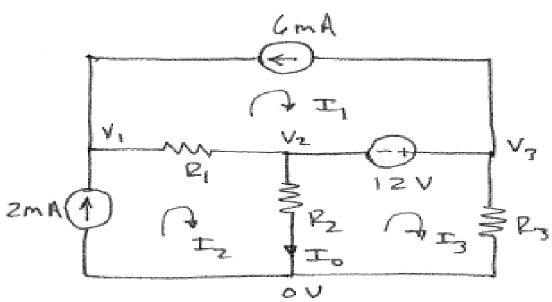


Figure P3.70

SOLUTION:

3.70 Use nodal & mesh to find I_o.



$R_1 = 4k\Omega$ $R_2 = 2k\Omega$ $R_3 = 4k\Omega$

Mesh

$I_1 = -6mA$

$I_2 = 2mA$

$12 = I_3 R_3 + (I_3 - I_2) R_2$

$I_o = I_2 - I_3$

$I_2 = 2mA$

$I_3 = 2.67mA$

$\boxed{I_o = -0.67mA}$

Nodal

$V_3 - V_2 = 12$

$6\times10^{-3} + 2\times10^{-3} + \frac{V_2 - V_1}{R_1} = 0$

$V_2/R_2 + V_3/R_3 = 2\times10^{-3}$

$I_o = V_2/R_2$

$V_2 = -1.33V$

$\boxed{I_o = -0.67mA}$

3.71 Find I_o in the network in Fig. P3.71 using loop analysis. Then solve the problem using MATLAB and compare your answers. **CS**

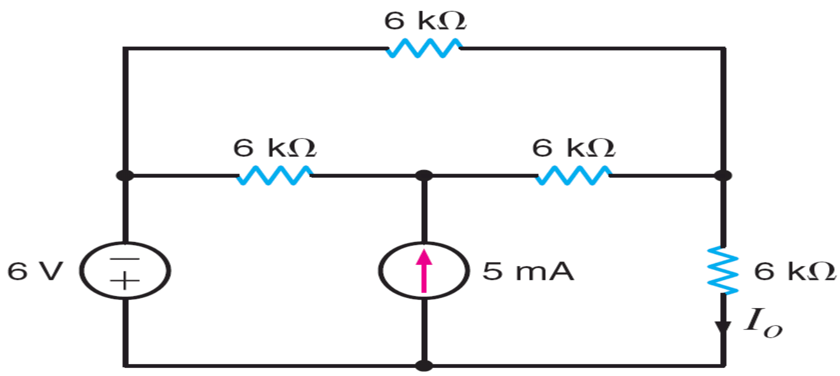
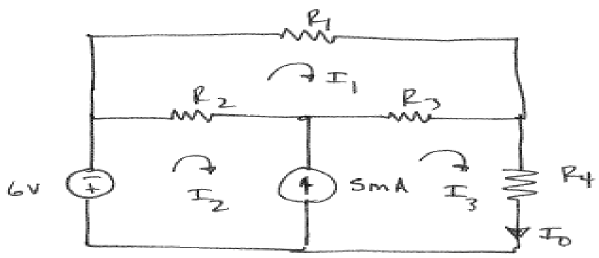


Figure P3.71

SOLUTION:

3.71 Find I_o using loop analysis. Verify with MATLAB.



All R's = 6kΩ

$I_3 - I_2 = 5mA$

$I_o = I_3$

$$I_1 R_1 + (I_1 - I_3) R_3 + (I_1 - I_2) R_2 = 0$$

$$6 + (I_2 - I_1) R_2 + (I_3 - I_1) R_3 + I_3 R_4 = 0$$

$I_3 = 0.4 mA$ $\boxed{I_o = 0.4 mA}$

Format for MATLAB

$$I_1 (R_1 + R_2 + R_3) - R_2 I_2 - R_3 I_3 = 0$$

$$-I_1 (R_2 + R_3) + R_2 I_2 + I_3 (R_3 + R_4) = -6$$

$$-I_2 + I_3 = 5 \times 10^{-3}$$

$$\begin{bmatrix} 18 \times 10^3 & -6 \times 10^3 & -6 \times 10^3 \\ -12 \times 10^3 & +6 \times 10^3 & 12 \times 10^3 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -6 \\ 5 \times 10^{-3} \end{bmatrix}$$

Continued on the next page.

3_71.txt

```
MATLAB WORK

Factor 1000 out of the resistance matrix.


EDU» r=[18,-6,-6;-12,6,12;0,-0.001,0.001]

r =

   18.0000   -6.0000   -6.0000
  -12.0000    6.0000   12.0000
         0   -0.0010    0.0010

EDU» v=[0;-6;0.005]

v =

         0
   -6.0000
    0.0050


EDU» 0.001*inv(r)*v

ans =

   -0.0014
   -0.0046
    0.0004
```

3.72 Find V_o in the network in Fig. P3.72 using both mesh and nodal analysis. **PSV**

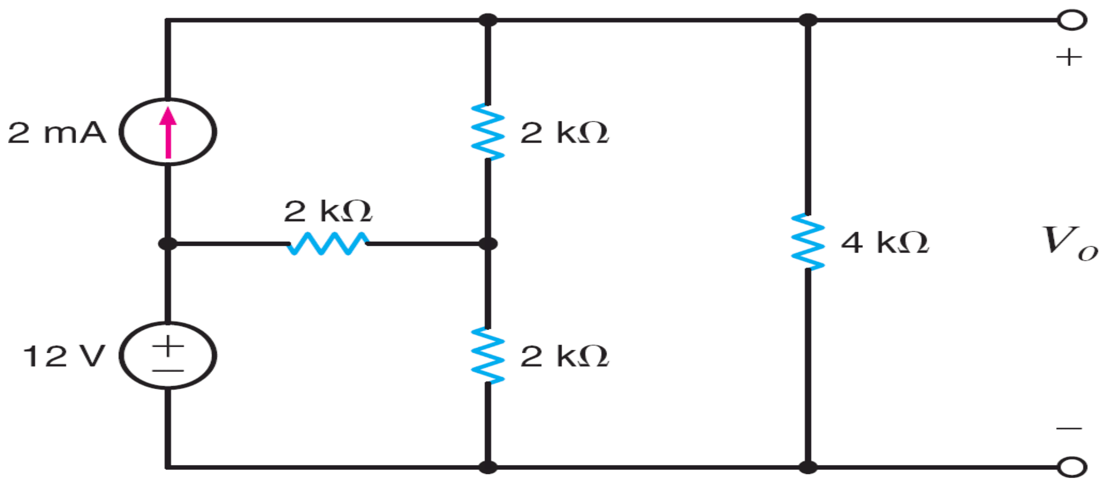
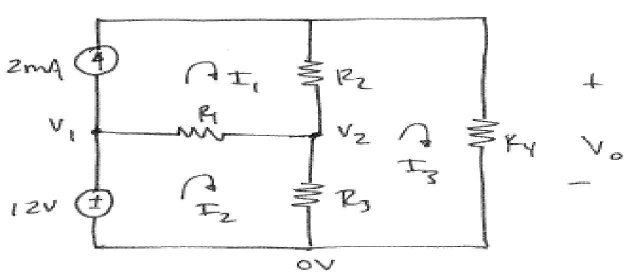


Figure P3.72

SOLUTION:

3.72 Find V_o using mesh & nodal.



$R_1 = R_2 = R_3 = 2\,k\Omega$

$R_4 = 4\,k\Omega$

Mesh

$I_1 = 2mA$

$12 = (I_2 - I_1)R_1 + (I_2 - I_3)R_3$

$0 = (I_3 - I_2)R_3 + (I_3 - I_1)R_2 + I_3 R_4$

$V_o = R_4 I_3$

$I_3 = 1.71\ mA$

$\boxed{V_o = 6.86\ V}$

Nodal

$V_1 = 12V$

$2\times10^{-3} = \frac{V_o - V_2}{R_2} + \frac{V_o}{R_4}$

$\frac{V_o - V_2}{R_2} + \frac{V_1 - V_2}{R_1} + \frac{0 - V_2}{R_3} = 0$

$\boxed{V_o = 6.86\ V}$

3.73 Use loop analysis to find I_o in the network in Fig. P3.73.
cs

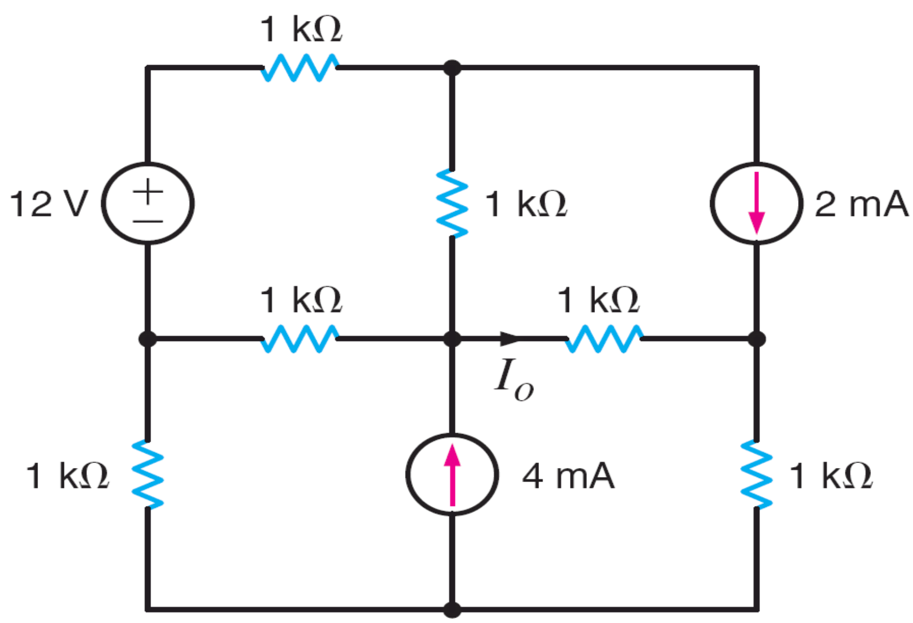
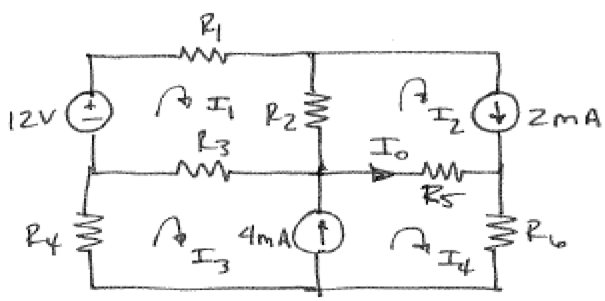


Figure P3.73

SOLUTION:

3.73 Use loop to find I_o.



all R's = 1kΩ

$I_2 = 2mA$

$I_4 - I_3 = 4mA$

$I_o = I_4 - I_2$

$$12 = I_1 R_1 + R_2(I_1 - I_2) + R_3(I_1 - I_3)$$

$$0 = I_3 R_4 + R_3(I_3 - I_1) + R_5(I_4 - I_2) + I_4 R_6$$

$I_4 = 3.64mA$ $\boxed{I_o = 1.64mA}$

3.74 Find I_o in the circuit in Fig. P3.74. **CS**

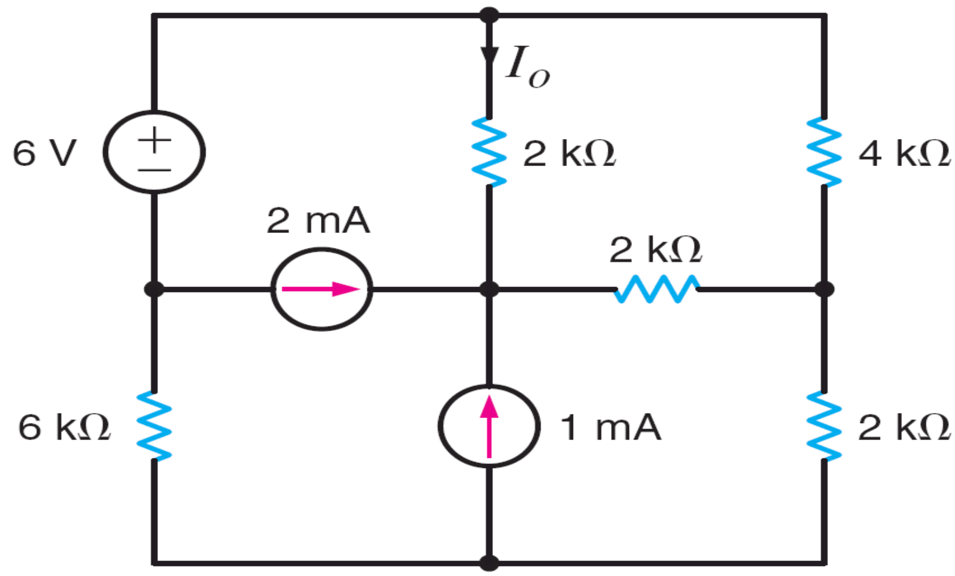
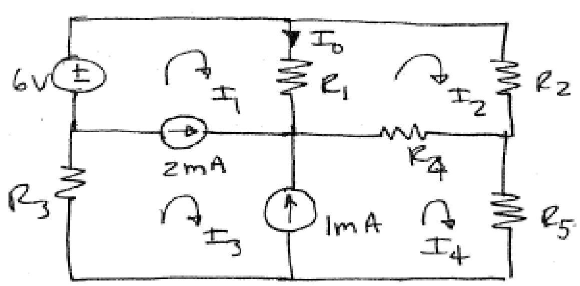


Figure P3.74

SOLUTION:

3.74 Find I_o



$R_1 = R_5 = R_4 = 2k\Omega$

$R_2 = 4k\Omega \quad R_3 = 6k\Omega$

$I_3 - I_1 = 2mA$

$I_4 - I_3 = 1mA$

$$I_2 R_2 + (I_2 - I_4) R_4 + (I_2 - I_1) R_1 = 0$$

$$6 = I_2 R_2 + I_4 R_5 + I_3 R_3$$

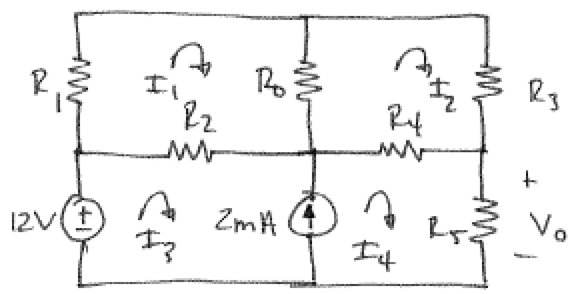
$$I_o = I_1 - I_2$$

$I_1 = -1.5mA \qquad I_2 = 0A \qquad \boxed{I_o = -1.5mA}$

3.75 Solve Problem 3.33 using loop analysis.

SOLUTION:

3.75 Find V_o by loop analysis.



$R_1 = R_3 = R_4 = R_5 = 1k\Omega$

$R_2 = R_o = 2k\Omega$

$V_o = I_4 R_5$

$I_4 - I_3 = 2mA$

$I_1 R_1 + (I_1 - I_2) R_o + (I_1 - I_3) R_2 = 0$

$I_2 R_3 + (I_2 - I_4) R_4 + (I_2 - I_1) R_o = 0$

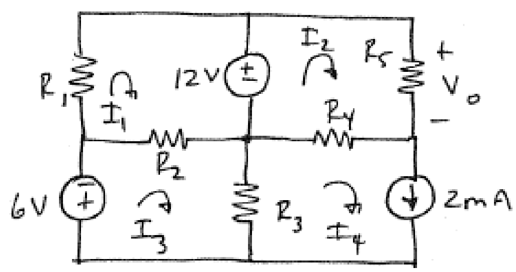
$12 = (I_3 - I_1) R_2 + (I_4 - I_2) R_4 + I_4 R_5$

$I_4 = 6.17mA$ $\boxed{V_o = 6.17V}$

3.76 Solve Problem 3.34 using loop analysis.

SOLUTION:

3.76 Find V_o by loop analysis.



$R_1 = R_2 = R_3 = 1k\Omega \quad R_4 = R_5 = 2k\Omega$

$I_4 = 2mA \quad V_o = I_2 R_5$

$$6 + (I_3 - I_1) R_2 + (I_3 - I_4) R_3 = 0$$

$$I_1 R_1 + (I_1 - I_3) R_2 + 12 = 0$$

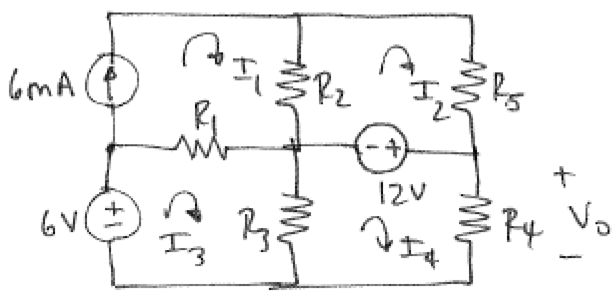
$$I_2 R_5 + (I_2 - I_4) R_4 = 12$$

$I_2 = 4mA \qquad \boxed{V_o = 8V}$

3.77 Solve Problem 3.35 using loop analysis.

SOLUTION:

3.77 Find V_o using loop analysis.



$R_1 = R_2 = R_3 = 1k\Omega$

$R_4 = R_5 = 2k\Omega$

$I_1 = 6mA$

$I_4 R_4 = V_o$

$6 = (I_3 - I_1)R_1 + (I_3 - I_4)R_3$

$-12 = (I_2 - I_1)R_2 + I_2 R_5$

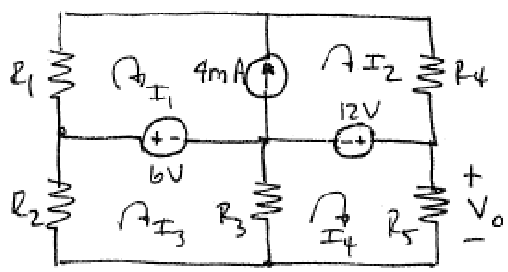
$12 = I_4 R_4 + (I_4 - I_3)R_3$

$I_4 = 7.2mA$ $\boxed{V_o = 14.4V}$

3.78 Solve Problem 3.37 using loop analysis.

SOLUTION:

3.78 Find V_o by loop analysis.



$R_1 = R_2 = 2k\Omega \qquad R_3 = R_4 = R_5 = 1k\Omega$

$V_o = I_4 R_5$

$I_2 - I_1 = 4\,mA$

$-6 = R_3(I_3 - I_4) + R_2(I_3)$

$6 = I_1 R_1 + I_2 R_4 + 12$

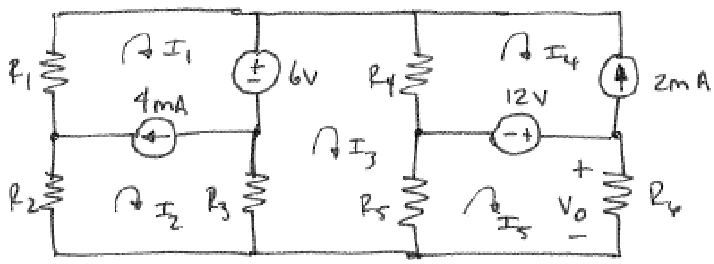
$12 = I_4 R_5 + (I_4 - I_3) R_3$

$I_4 = 6mA \qquad \boxed{V_o = 6V}$

3.79 Solve Problem 3.40 using loop analysis.

SOLUTION:

3.79 Find V_o by loop analysis.



$R_1 = R_2 = 2k\Omega$

$R_3 = R_4 = R_5 = R_6 = 1k\Omega$

$I_4 = -2mA$

$I_1 - I_2 = 4mA$

$$-6 = R_3(I_2 - I_3) + I_2R_2 + I_1R_1$$

$$6 = R_4(I_3 - I_4) + R_5(I_3 - I_5) + R_3(I_3 - I_2)$$

$$12 = R_6 I_5 + R_5(I_5 - I_3)$$

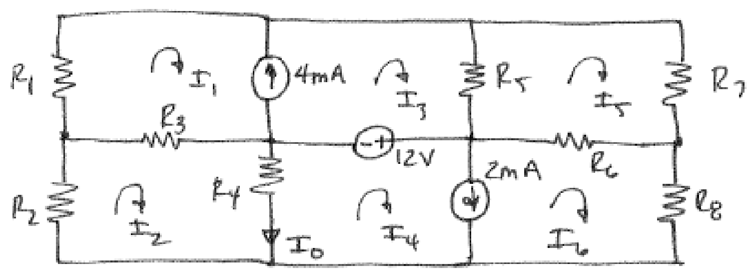
$$V_0 = I_5 R_6$$

$I_5 = 7.57\,mA$ $\boxed{V_0 = 7.57V}$

3.80 Solve Problem 3.43 using loop analysis.

SOLUTION:

3.80 find I_o using loop analysis.



all R's = 1kΩ

$I_3 - I_1 = 4mA$

$I_4 - I_6 = 2mA$

$I_o = I_2 - I_4$

$$0 = I_2 R_2 + (I_2 - I_1) R_3 + (I_2 - I_4) R_4$$

$$0 = I_5 R_7 + (I_5 - I_6) R_6 + (I_5 - I_3) R_5$$

$$-12 = I_1 R_1 + (I_3 - I_5) R_5 + (I_1 - I_2) R_3$$

$$12 = (I_6 - I_5) R_6 + I_6 R_8 + I_2 R_2 + (I_2 - I_1) R_3$$

$I_2 = 267 \mu A$ $\qquad$ $I_4 = 5.74 mA$ $\qquad$ $\boxed{I_o = -5.47 mA}$

3.81 Use MATLAB to find the mesh currents in the network in Fig. P3.81. CS

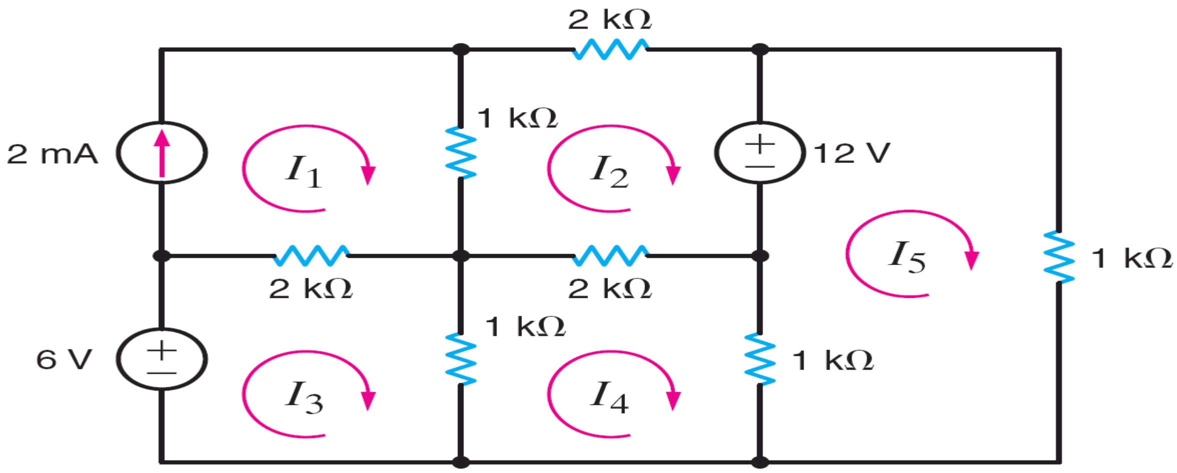
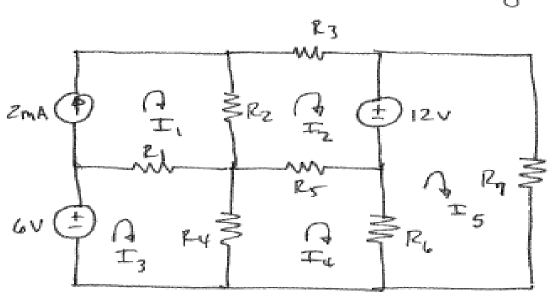


Figure P3.81

SOLUTION:

3.81 Find mesh currents using MATLAB.



$R_1 = R_3 = R_5 = 2k\Omega$

$R_2 = R_4 = R_6 = R_7 = 1k\Omega$

$I_1 = 2mA$

$$6 = (I_3 - I_1)R_1 + (I_3 - I_4)R_4$$

$$0 = (I_4 - I_3)R_4 + (I_4 - I_2)R_5 + (I_4 - I_5)R_6$$

$$-12 = I_2 R_3 + (I_2 - I_4)R_5 + (I_2 - I_1)R_2$$

$$12 = I_5 R_7 + (I_5 - I_4)R_6$$

In matrix form,

$$\begin{bmatrix} -2000 & 0 & 3000 & -1000 & 0 \\ 0 & -2000 & -1000 & 4000 & -1000 \\ -1000 & 5000 & 0 & -2000 & 0 \\ 0 & 0 & 0 & -1000 & 2000 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \\ -12 \\ 12 \\ 2\times10^{-3} \end{bmatrix}$$

Continued on the next page.

3_81.txt

```
MATLAB WORK

r*i = v

EDU»
r=[-2000,0,3000,-1000,0;0,-2000,-1000,4000,-1000;-1000,5000,0,-2000,0;0,0,0,-1000,20
00;1,0,0,0,0]

r =

       -2000           0        3000       -1000           0
           0       -2000       -1000        4000       -1000
       -1000        5000           0       -2000           0
           0           0           0       -1000        2000
           1           0           0           0           0

EDU» v=[6;0;-12;12;0.002]

v =

    6.0000
         0
  -12.0000
   12.0000
    0.0020


EDU» i=1000*inv(r)*v

i =

    2.0000
   -1.0986
    4.0845
    2.2535
    7.1268
```

3.82 Write mesh equations for the circuit in Fig. P3.82 using the assigned currents.

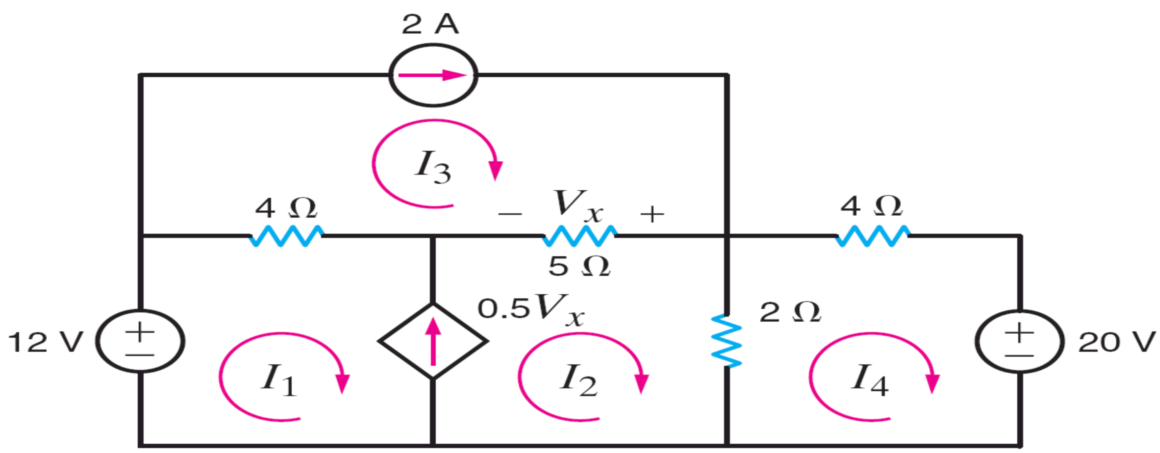
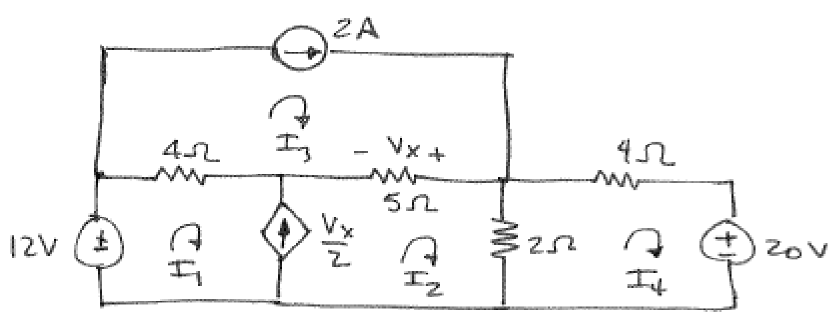


Figure P3.82

SOLUTION:

3.82 Write mesh equations.



$V_x = (I_3 - I_2)\,5$

① $I_3 = 2\text{A}$

$$I_2 - I_1 = \frac{V_x}{2} = \frac{(I_3 - I_2)\,5}{2}$$

② $3.5 I_2 - I_1 - 2.5 I_3 = 0$

③ $4 I_4 + 2(I_4 - I_2) = -20$

④ $(I_1 - I_3)4 + (I_2 - I_3)\,5 + 2 I_2 = 12$

3.83 Use mesh analysis to find V_o in the circuit in Fig. P3.83.
PSV

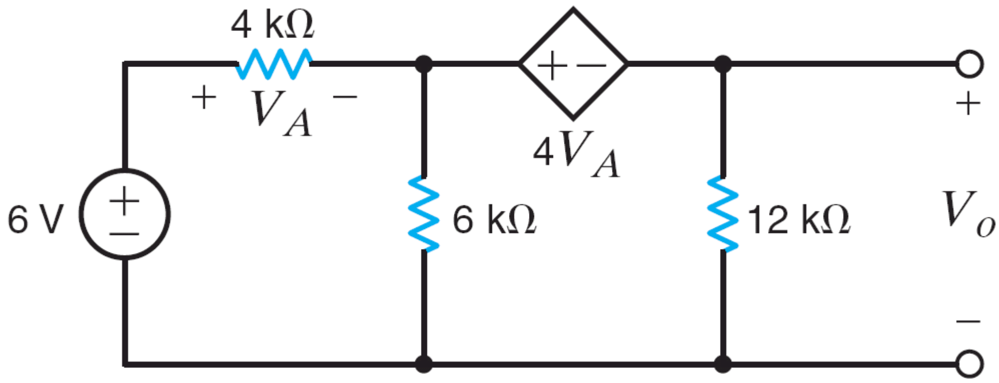
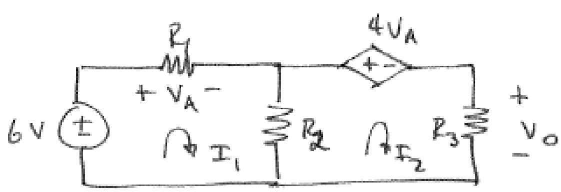


Figure P3.83

SOLUTION:

3.83 Find V_o using mesh analysis.



$R_1 = 4k\Omega \quad R_2 = 6k\Omega \quad R_3 = 12k\Omega$

$V_A = I_1 R_1 \qquad V_o = I_2 R_3$

$6 = I_1 R_1 + (I_1 - I_2) R_2 \qquad I_2 R_3 + (I_2 - I_1) R_2 + 4V_A = 0$

$6 = I_1 (R_1 + R_2) - I_2 R_2 \qquad I_2 (R_2 + R_3) + I_1 (4R_1 - R_2) = 0$

$I_2 = -0.25\text{mA} \qquad \boxed{V_o = -3V}$

3.84 Find V_o in the circuit in Fig. P3.84 using mesh analysis.
cs

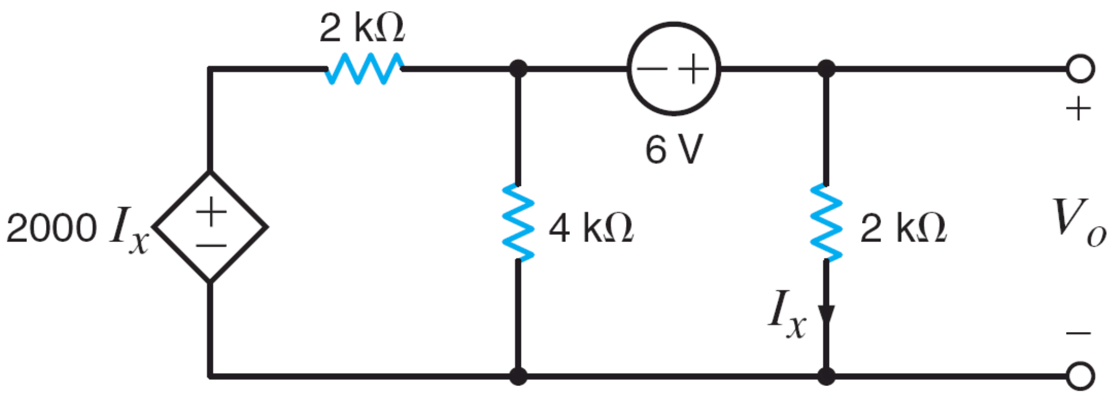
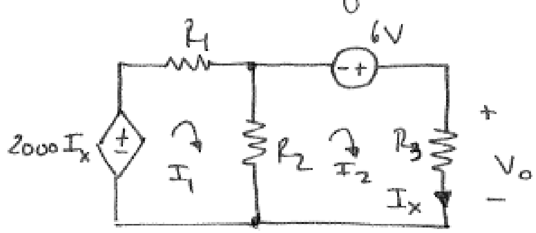


Figure P3.84

SOLUTION:

3.84 Find V_o using mesh analysis.



$R_1 = 2k\Omega$ $R_2 = 4k\Omega$ $R_3 = 2k\Omega$

$I_x = I_2$ $V_o = R_3 I_2$

$$2000 I_x = I_1 R_1 + (I_1 - I_2) R_2$$

$$6 = I_2 R_3 + (I_2 - I_1) R_2$$

$I_2 = 3 \text{ mA}$ $\boxed{V_o = 6 V}$

3.85 Use loop analysis to find V_o in the network in Fig. P3.85.

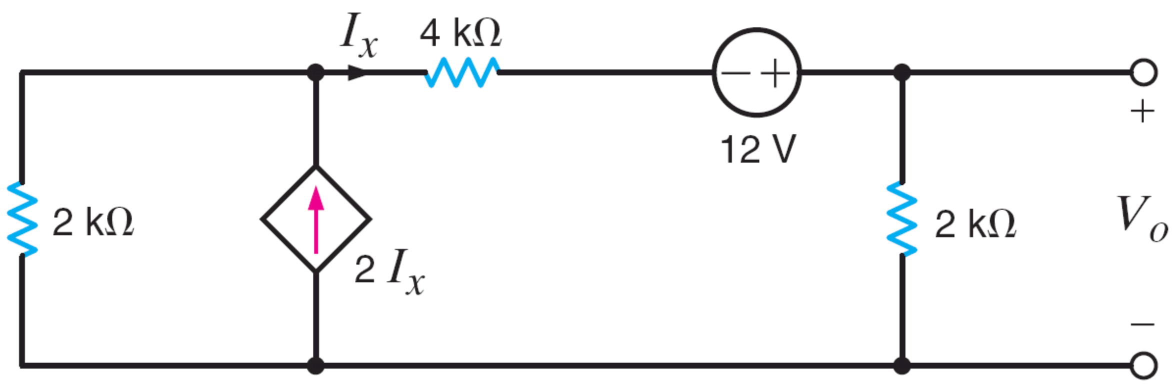
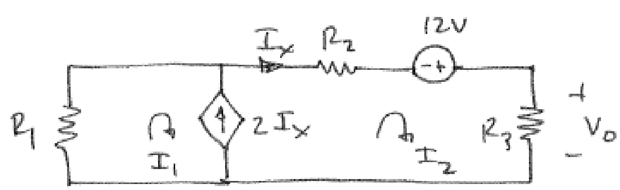


Figure P3.85

SOLUTION:

3.85 Find V_o using loop analysis.



$R_1 = 2k\Omega \quad R_2 = 4k\Omega \quad R_3 = 2k\Omega$

$I_x = I_2 \qquad V_o = R_3 I_2$

$I_1 R_1 + I_2 R_2 + I_2 R_3 = 12 \qquad 2I_x = I_2 - I_1$

$I_2 = 3mA \qquad \boxed{V_o = 6V}$

3.86 Use loop analysis to find V_o in the circuit in Fig. P3.86.

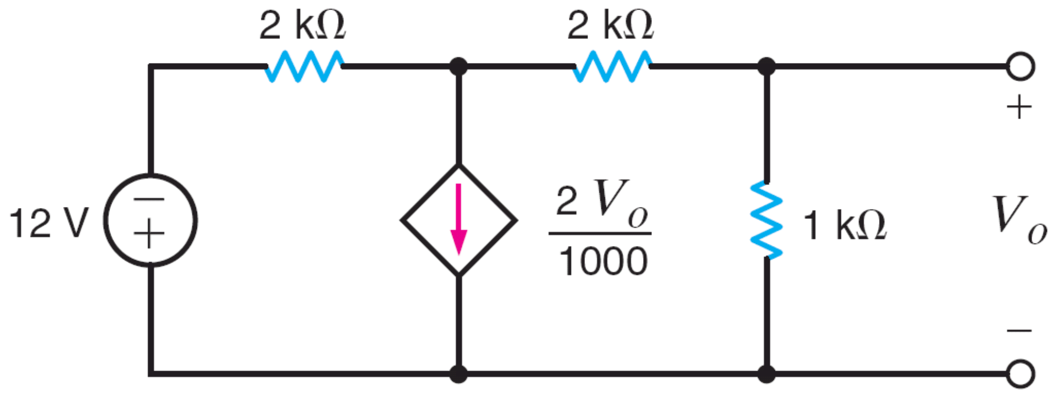
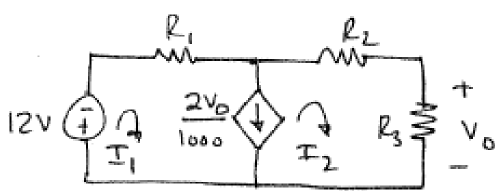


Figure P3.86

SOLUTION:

3.86 find V_o using loop analysis.



$R_1 = 2k\Omega \quad R_2 = 2k\Omega \quad R_3 = 1k\Omega$

$V_o = R_3 I_2$

$$\frac{2V_o}{1000} = I_1 - I_2$$

$$-12 = I_1 R_1 + I_2 R_2 + I_2 R_3$$

$I_2 = -1.33\ mA$ $\boxed{V_o = -1.33\ V}$

3.87 Use both nodal analysis and mesh analysis to find V_o in the circuit in Fig. P3.87.

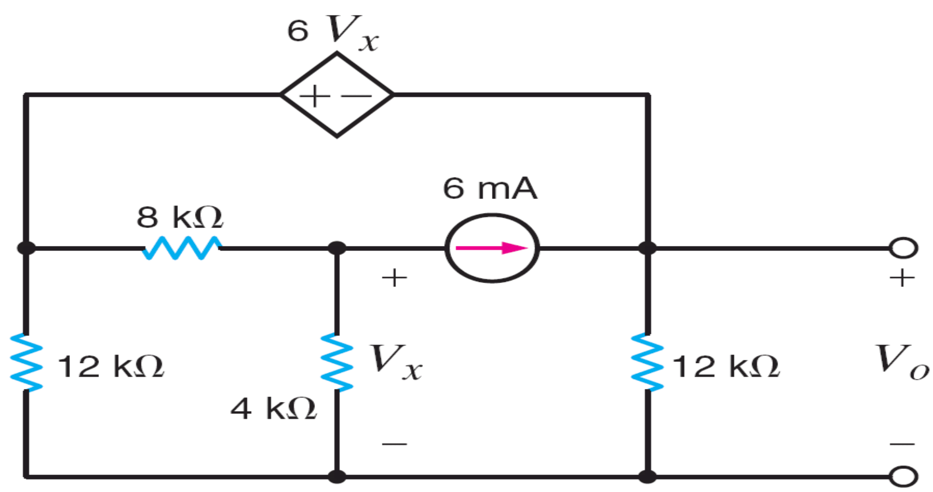
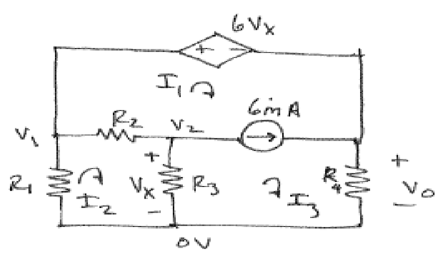


Figure P3.87

SOLUTION:

3.87 Use both nodal & mesh to find V_o.



$R_1 = 12k\Omega$ $R_2 = 8k\Omega$ $R_3 = 4k\Omega$ $R_4 = 12k\Omega$

Mesh

$I_3 - I_1 = 6mA$

$6V_x + R_4 I_3 + R_1 I_2 = 0$

$0 = I_2 R_2 + (I_2 - I_1)R_2 + (I_2 - I_3)R_3$

$V_x = (I_2 - I_3)R_3$

$V_o = I_3 R_4$

$I_3 = 12mA$

$\boxed{V_o = 144V}$

Nodal

$V_1 - V_o = 6V_x$

$V_x = V_2$

$\frac{V_2 - V_1}{R_2} + \frac{V_2}{R_3} + 6\times10^{-3} = 0$

$\frac{V_1}{R_1} + \frac{V_2}{R_3} + \frac{V_o}{R_4} = 0$

$\boxed{V_o = 144V}$

3.88 Using mesh analysis, find V_o in the circuit in Fig. P3.88.

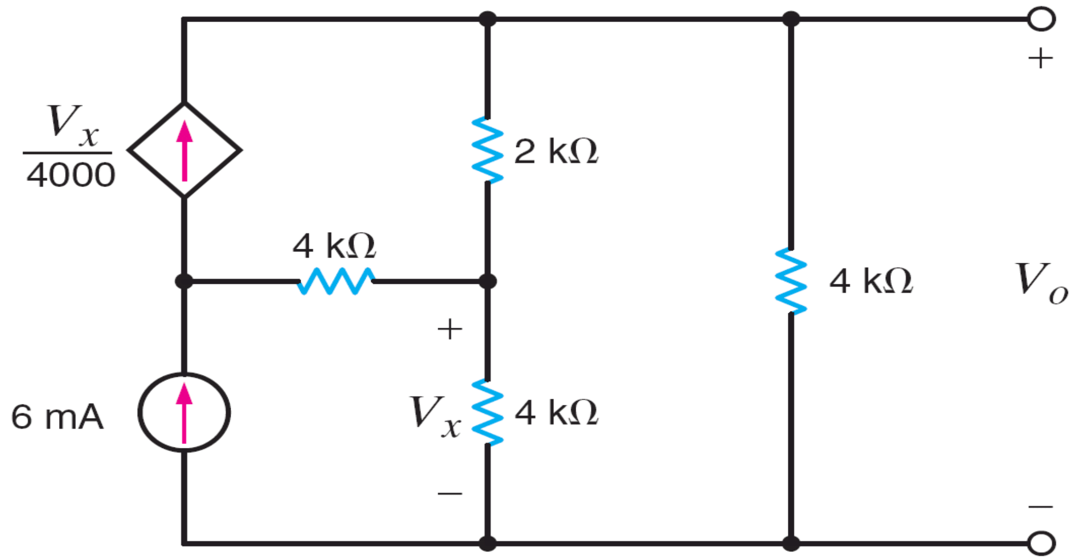
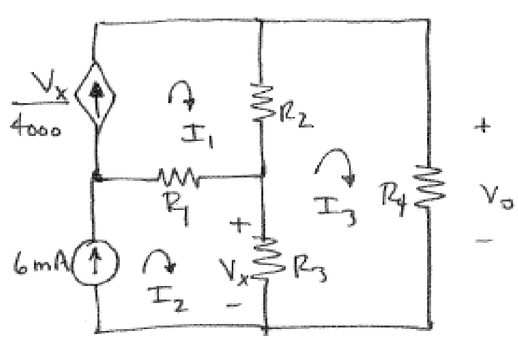


Figure P3.88

SOLUTION:

3.88 find V_o using mesh analysis.



$R_1 = R_3 = R_4 = 4k\Omega \qquad R_2 = 2k\Omega$

$V_x = R_3(I_2 - I_3)$

$V_o = R_4 I_3$

$I_2 = 6mA$

$\frac{V_x}{4000} = I_1$

$0 = I_3 R_4 + (I_3 - I_1)R_2 + (I_3 - I_2)R_3$

$I_3 = 3mA$

$\boxed{V_o = 12V}$

3.89 Find V_o in the network in Fig. P3.89. **PSV**

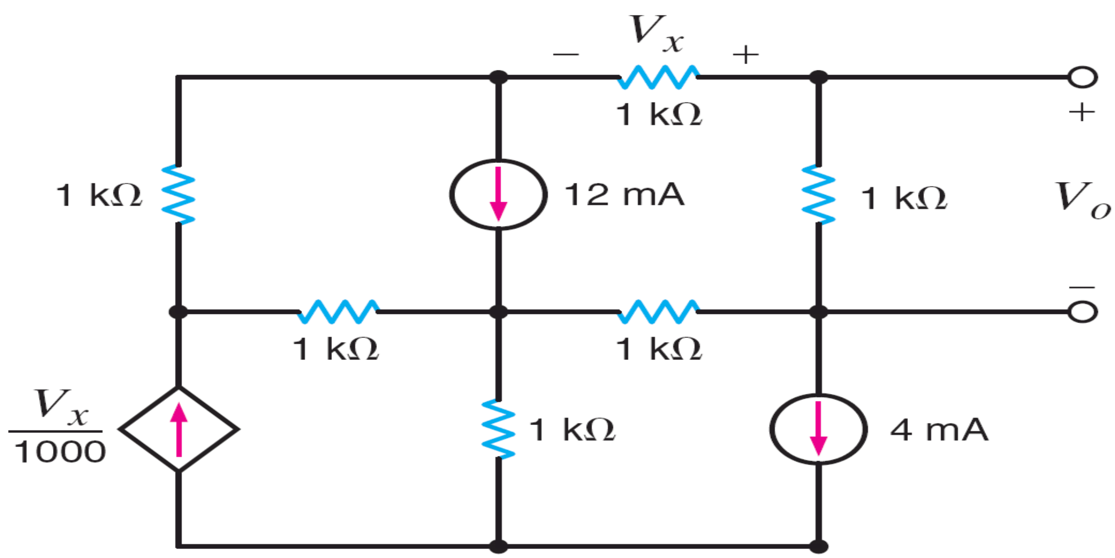
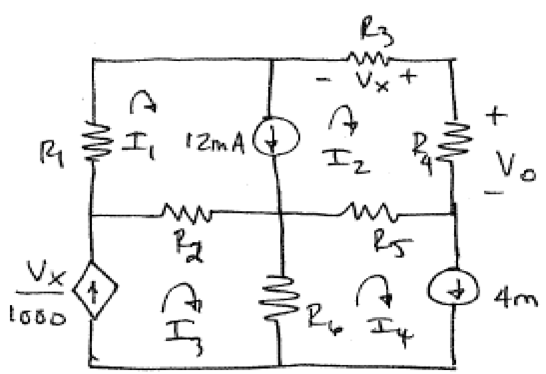


Figure P3.89

SOLUTION:

3.89 Find V_o.



All R's = 1kΩ

$I_4 = 4mA$

$I_1 - I_2 = 12mA$

$\frac{V_x}{1000} = I_3 \qquad V_x = -I_2 R_3$

$$0 = I_1 R_1 + I_2 R_3 + I_2 R_4 + (I_2 - I_4) R_5 + (I_1 - I_3) R_2$$

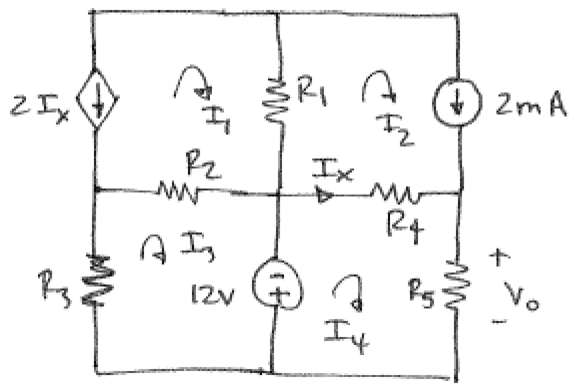
$$V_o = I_2 R_4$$

$I_2 = -3.33mA \qquad \boxed{V_o = -3.33\ V}$

3.90 Solve Problem 3.54 using loop analysis.

SOLUTION:

3.90 find V_o by loop analysis.



all R's = 1kΩ

$I_2 = 2mA$

$2I_x = -I_1$

$I_x = I_4 - I_2$

$V_o = I_4 R_5$

$12 = I_3 R_3 + (I_3 - I_1) R_2$

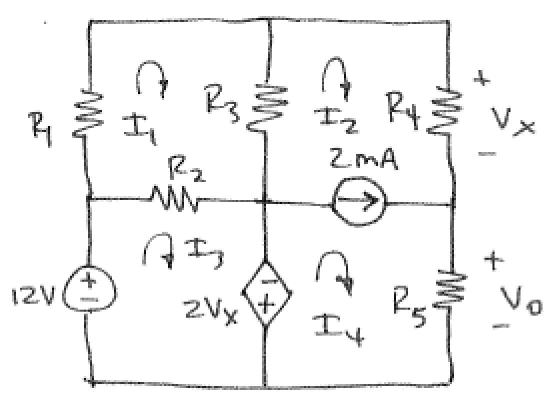
$-12 = I_4 R_5 + (I_4 - I_2) R_4$

$I_4 = -5mA$ $\boxed{V_o = -5V}$

3.91 Solve Problem 3.55 using loop analysis.

SOLUTION:

3.91 Find V_o using loop analysis.



$R_1 = R_2 = 2k\Omega$

$R_3 = R_4 = R_5 = 1k\Omega$

$I_4 - I_2 = 2mA$

$V_o = I_4 R_5$

$V_x = R_4 I_2$

$12 = I_3 R_2 - 2V_x$

$0 = I_1 R_1 + (I_1 - I_2)R_3 + (I_1 - I_3) R_2$

$12 = I_1 R_1 + I_2 R_4 + I_4 R_5$

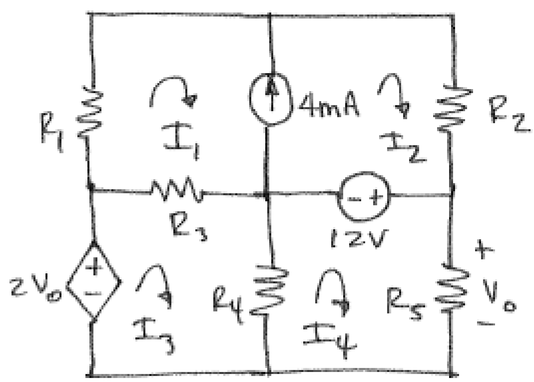
$I_4 = 2.5mA$

$\boxed{V_o = 2.5V}$

3.92 Solve Problem 3.56 using loop analysis.

SOLUTION:

3.92 Find V_o using loop analysis.



$R_1 = R_2 = R_3 = R_4 = 1k\Omega$

$R_5 = 2k\Omega$

$I_2 - I_1 = 4mA$

$V_o = R_5 I_4$

$2V_o = (I_3 - I_1)R_3 + (I_3 - I_4)R_4$

$12 = I_4 R_5 + (I_4 - I_3) R_4$

$+2V_o = I_1 R_1 + I_2 R_2 + I_4 R_5$

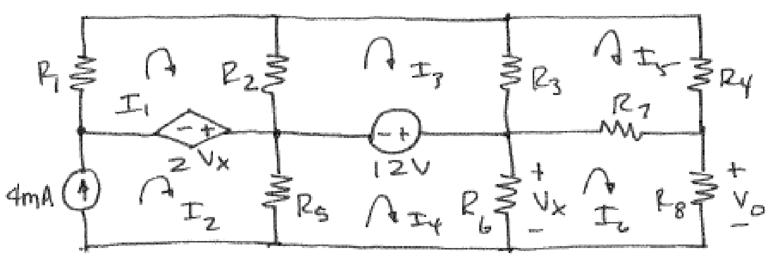
$I_4 = \infty$

$\boxed{V_o = \infty\ V}$

3.93 Solve Problem 3.57 using loop analysis.

SOLUTION:

3.93 Find V_o using loop analysis.



$R_1 = R_2 = R_3 = R_4 = R_8 = 1k\Omega$

$R_5 = R_6 = R_7 = 2k\Omega$

$V_o = R_8 I_6$

$I_2 = 4mA$

$I_1 R_1 + (I_1 - I_3) R_2 + 2V_x = 0 \qquad V_x = (I_4 - I_6) R_6$

$12 = (I_4 - I_6) R_6 + (I_4 - I_2) R_5$

$-12 = (I_3 - I_1) R_2 + (I_3 - I_5) R_3$

$0 = I_5 R_4 + (I_5 - I_6) R_7 + (I_5 - I_3) R_3$

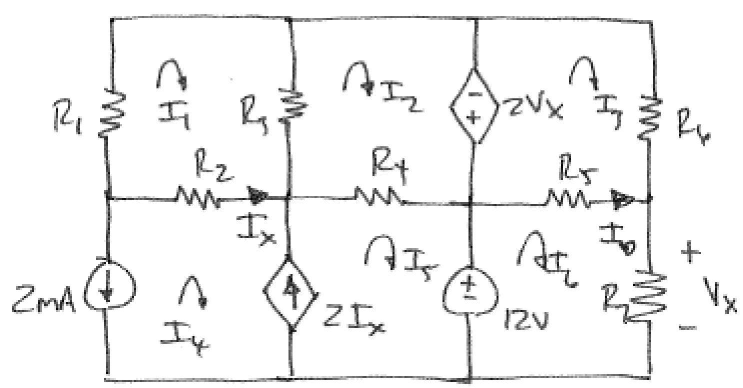
$0 = I_6 R_8 + (I_6 - I_4) R_6 + (I_6 - I_5) R_7$

$I_6 = 500 \mu A \qquad \boxed{V_o = 0.5V}$

3.94 Solve Problem 3.58 using loop analysis.

SOLUTION:

3.94 find I_o using loop analysis



All R's = 1kΩ

$I_4 = -2mA$

$I_5 - I_4 = 2I_x$

$I_x = I_4 - I_1$

$I_o = I_6 - I_3$

$$0 = I_1 R_1 + (I_1 - I_2) R_3 + R_2 (I_1 - I_4)$$

$$2V_x = (I_2 - I_5) R_4 + (I_2 - I_1) R_3$$

$$V_x = I_6 R_7$$

$$-2V_x = I_3 R_6 + (I_3 - I_6) R_5$$

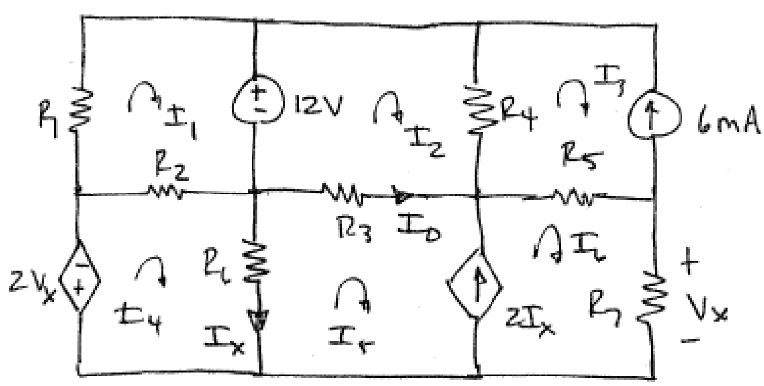
$$12 = (I_6 - I_3) R_5 + I_6 R_7$$

$$\boxed{I_o = 7.2\ mA}$$

3.95 Solve Problem 3.59 using loop analysis.

SOLUTION:

3.95 Find I_o using loop analysis



All R's = 1kΩ

$I_3 = -6\,mA$

$2I_x = I_6 - I_5$

$I_x = I_4 - I_5$

$I_o = I_5 - I_2$

$V_x = I_6 R_7$

$$-2V_x = (I_4 - I_1)R_2 + (I_4 - I_5)R_6$$

$$-12 = I_1 R_1 + (I_1 - I_4)R_2$$

$$12 = (I_2 - I_3)R_4 + (I_2 - I_5)R_3$$

$$0 = (I_5 - I_4)R_6 + (I_5 - I_2)R_3 + (I_6 - I_3)R_5 + I_6 R_7$$

$$\boxed{I_o = -2.88\,mA}$$

3.96 Use mesh analysis to determine the power delivered by the independent 3-V source in the network in Fig. P3.96.

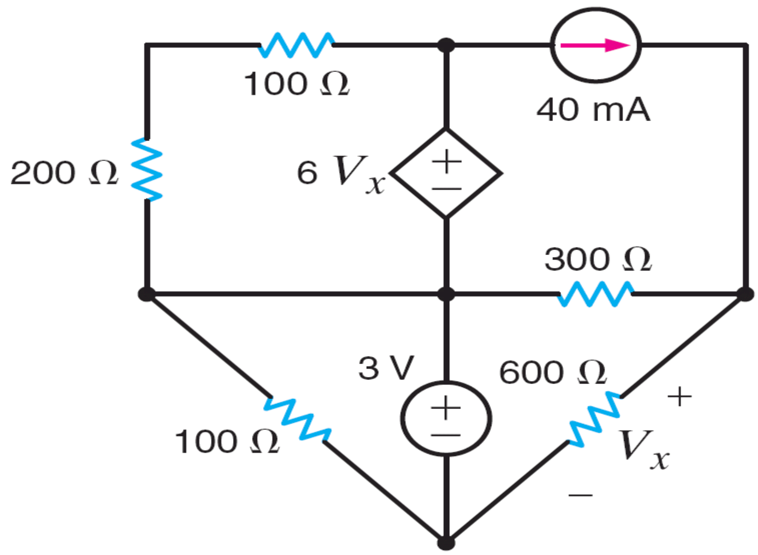
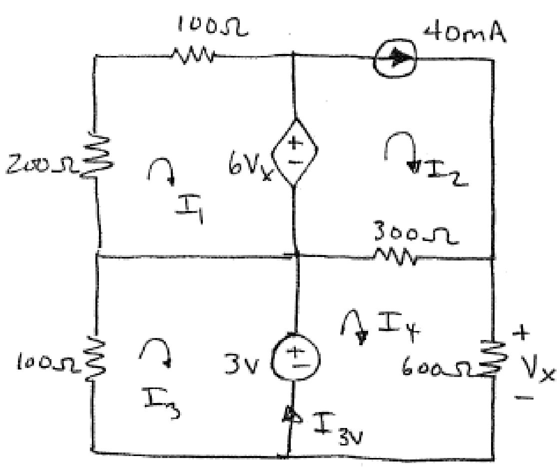


Figure P3.96

SOLUTION:

3.96 Find power delivered by 3-V source.



$I_2 = 40\,mA$

$-6V_x = 300 I_1$

$V_x = 600 I_4$

$3 = 300(I_4 - I_2) + 600 I_4$

$-3 = 100 I_3$

$I_{3V} = I_4 - I_3 = 46.67\,mA$

$P_{3V} = 3 I_{3V}$ $\boxed{P_{3V} = 140\,mW}$

3.97 Use mesh analysis to find the power delivered by the current-controlled voltage source in the circuit in Fig. P3.97.

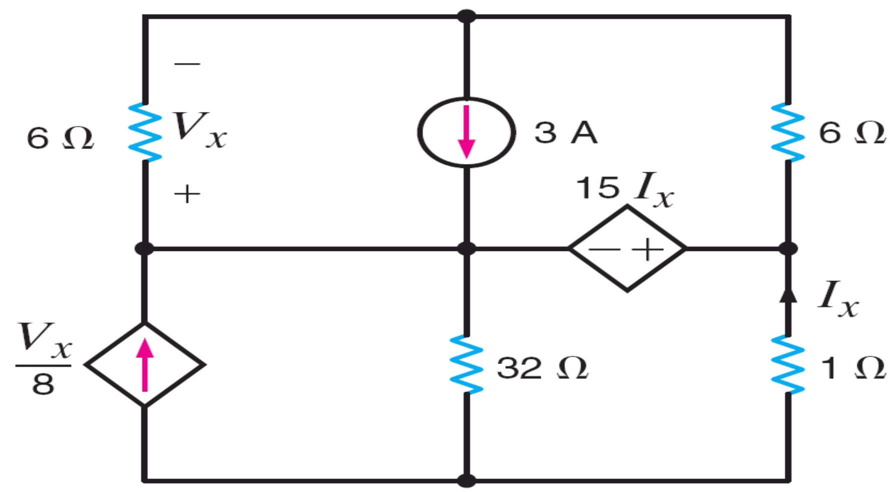
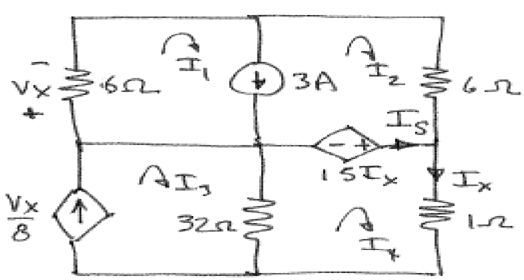


Figure P3.97

SOLUTION:

3.97 find power delivered by the voltage source.



$I_1 - I_2 = 3$

$I_3 = V_x/8$

$V_x = 6I_1$

$I_x = I_4$

$I_S = I_4 - I_2$

$15I_x = I_4 + 32(I_4 - I_3)$

$-15I_x = 6I_1 + 6I_2$

$I_4 = 2A \qquad I_2 = 1A \qquad I_S = 1A$

$P_{ccvs} = (15I_x)I_S = 15I_4I_S$

$\boxed{P_{ccvs} = 30W}$

3FE-1 Find V_o in the circuit in Fig. 3PFE-1. **cs**

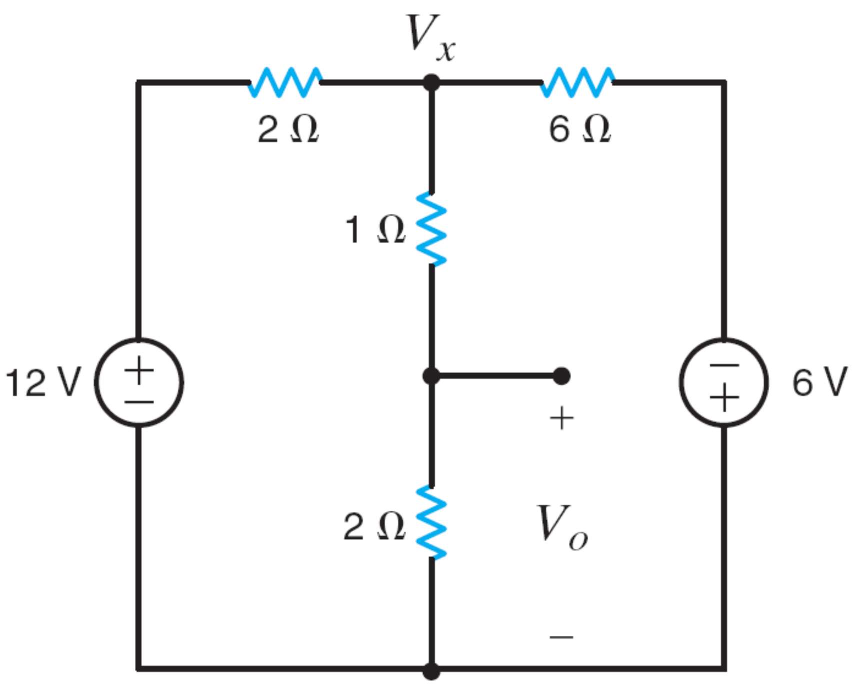
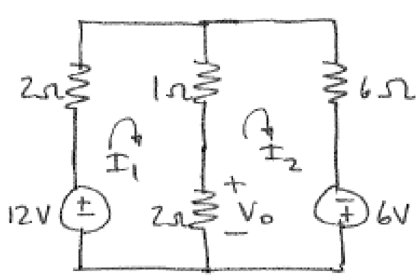


Figure 3PFE-1

SOLUTION

3FE-1 Find V_o.



$$12 = 2I_1 + (I_1 - I_2) + 2(I_1 - I_2)$$

$$6 = 2(I_2 - I_1) + (I_2 - I_1) + 6I_2$$

$$V_o = 2(I_1 - I_2)$$

Results: $I_1 = 3.5A$, $I_2 = 1.83A$

$$\boxed{V_o = 3.33\text{ V}}$$

3FE-2 Determine the power dissipated in the 6-ohm resistor in the network in Fig. 3PFE-2.

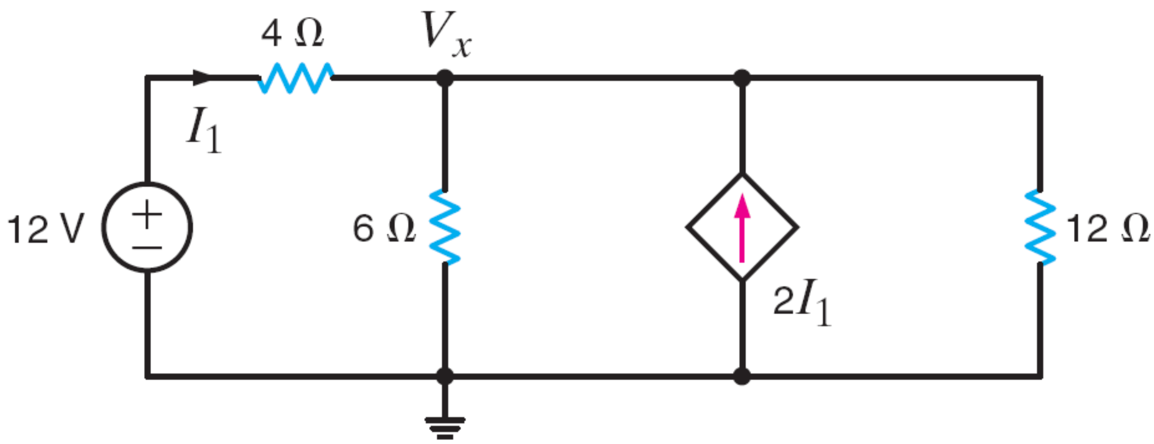
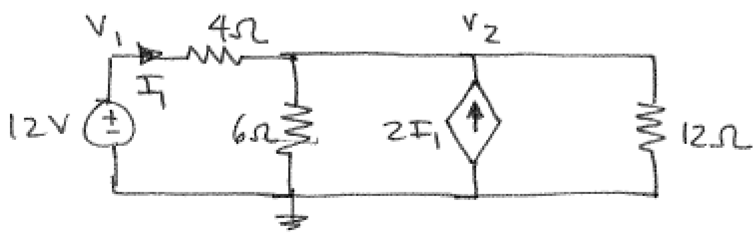


Figure 3PFE-2

SOLUTION

3FE-2 find power absorbed by 6-Ω resistor.



$$I_1 = (V_1 - V_2)/4$$

$$V_1 = 12\text{V}$$

$$\frac{V_2}{6} + \frac{V_2}{12} + \frac{V_2 - V_1}{4} = 2I_1$$

Results: $V_2 = 9\text{V}$

$$P_{6\Omega} = \frac{V_2^2}{6} \qquad \boxed{P_{6\Omega} = 13.5\text{W}}$$

3FE-3 Find the current I_x in the 4-ohm resistor in the circuit in Fig. 3PFE-3. **CS**

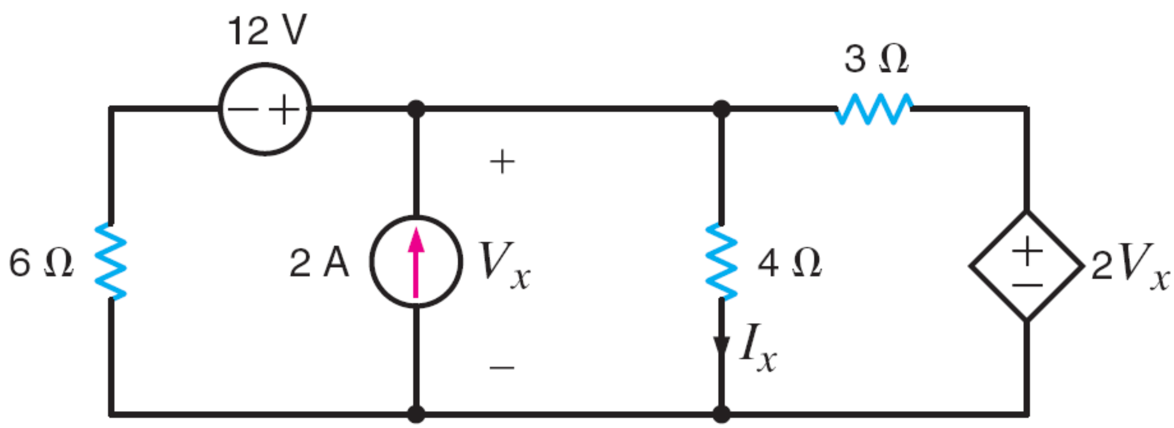
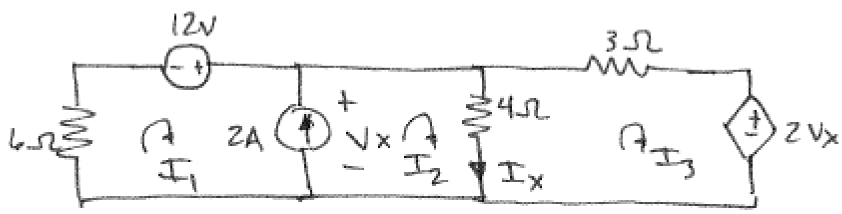


Figure 3PFE-3

SOLUTION

3FE-3 Find I_x.



$V_x = 4 I_x$

$I_x = I_2 - I_3$

$I_2 - I_1 = 2$

$12 = 4(I_2 - I_3) + 6I_1$

$-2V_x = 4(I_3 - I_2) + 3I_3$

Results: $I_2 = -4A$, $I_3 = -16A$ $\boxed{I_x = 12A}$

3FE-4 Determine the voltage V_o in the circuit in Fig. 3PFE-4.

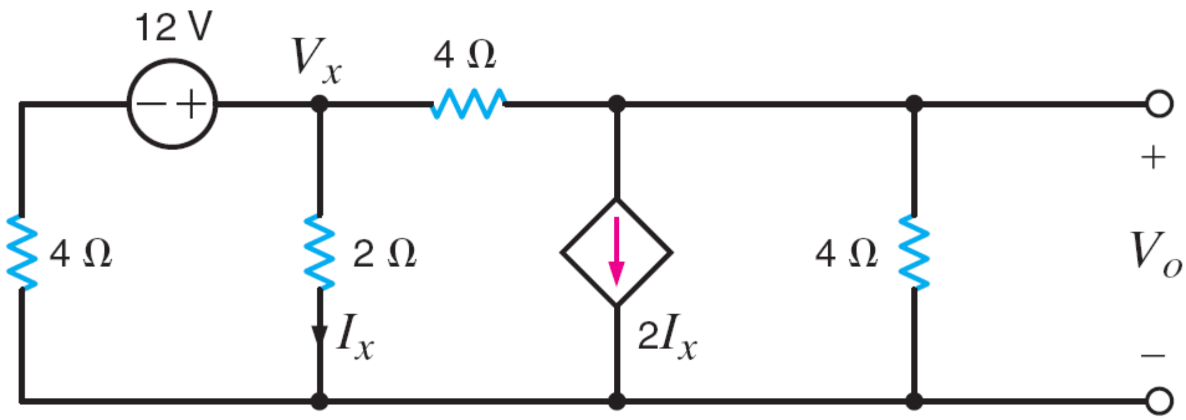
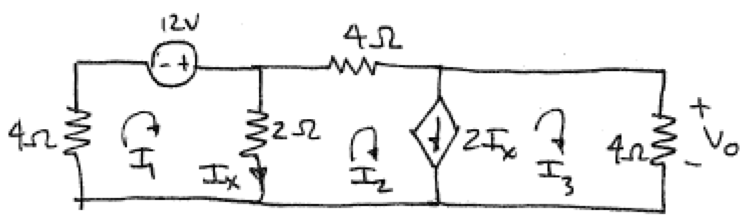


Figure 3PFE-4

SOLUTION

3FE-4 Find V_o.



$I_x = I_1 - I_2$

$V_o = 4I_3$

$2I_x = I_2 - I_3$

$12 = 4I_1 + 2(I_1 - I_2)$

$4I_1 + 4I_2 + 4I_3 = 12$

Results: $I_3 = -0.818A$ $\boxed{V_o = -3.27V}$

Chapter Four:

Operational Amplifiers

4.1 An amplifier has a gain of 15 and the input waveform shown in Fig. P4.1. Draw the output waveform.

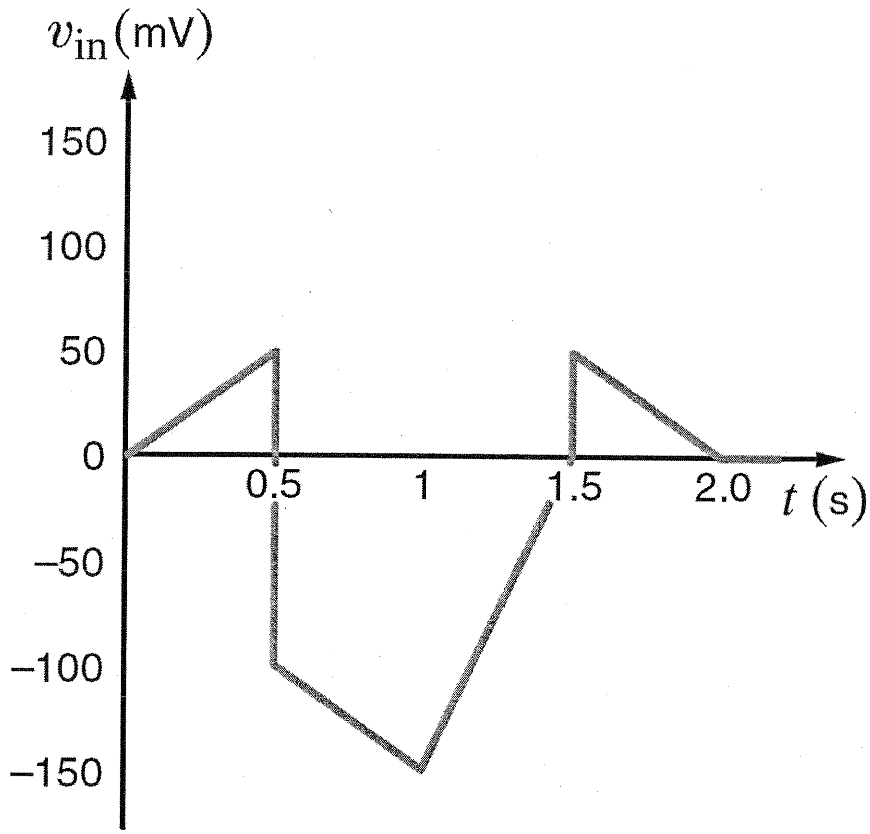
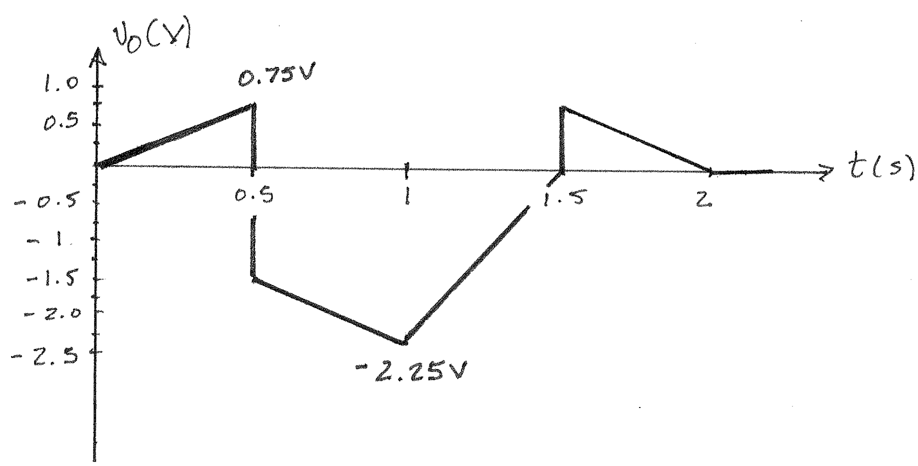


Figure P4.1

SOLUTION:



4.2 An amplifier has a gain of −5 and the output waveform shown in Fig. P4.2. Sketch the input waveform.

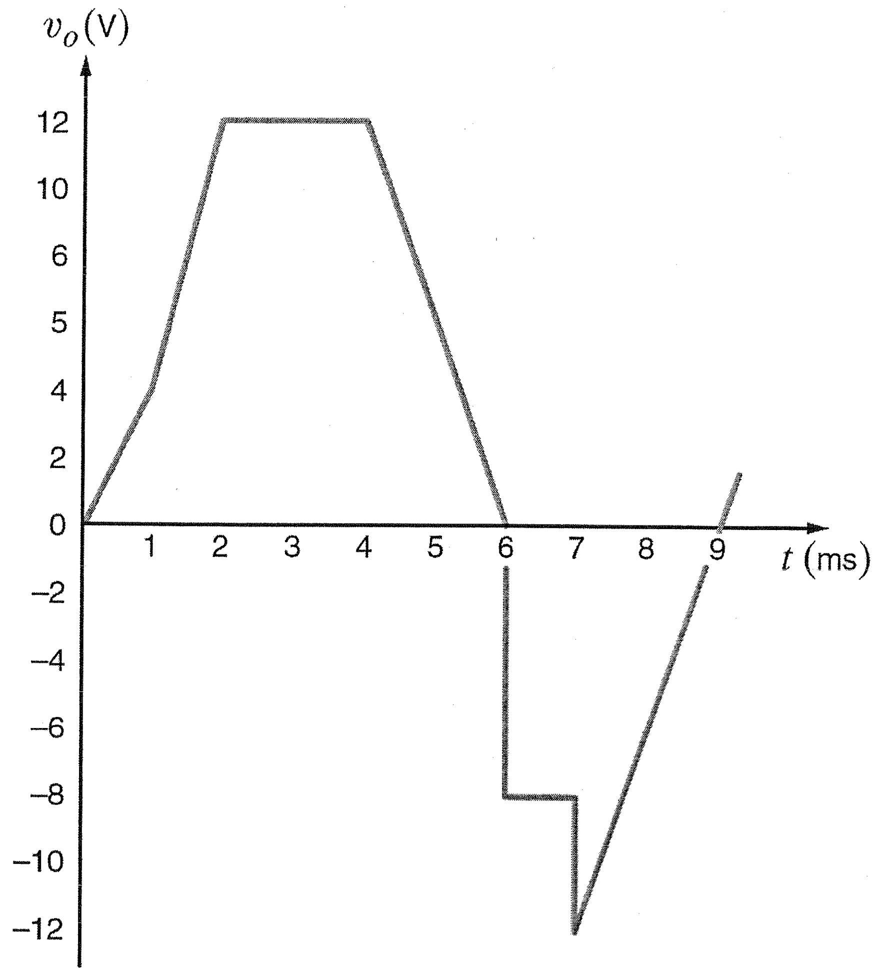
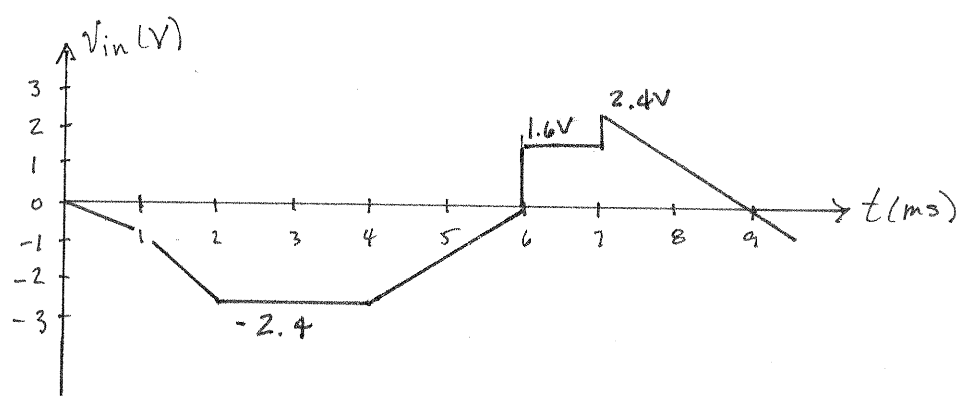


Figure P4.2

SOLUTION:



4.3 An op-amp base amplifier has supply voltages of ±5 V and a gain of 20.

(a) Sketch the input waveform from the output waveform in Fig. P4.3.

(b) Double the amplitude of your results in (a) and sketch the new output waveform.

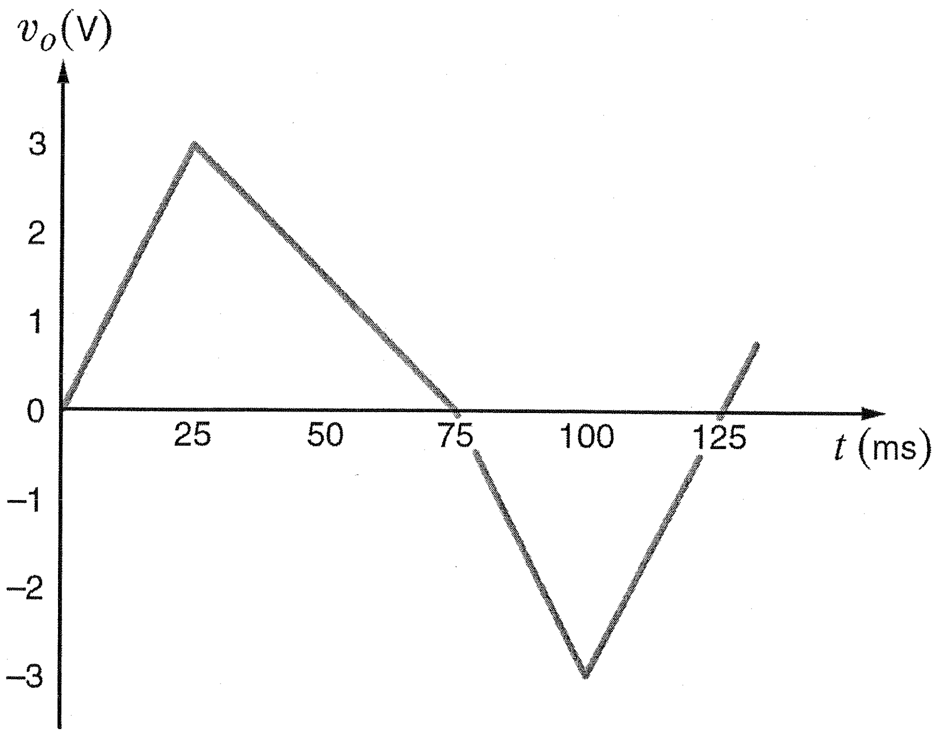
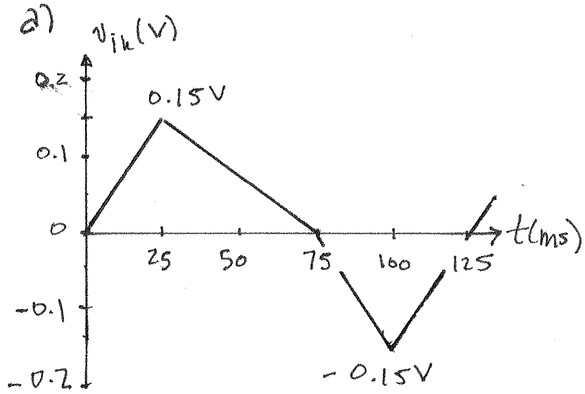


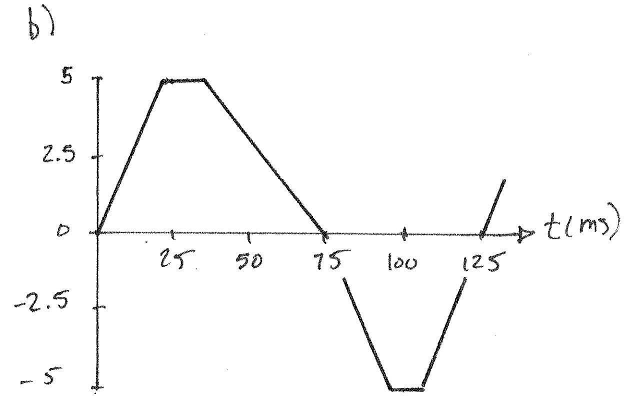
Figure P4.3

SOLUTION:

a)



b)



4.4 For an ideal op-amp, the voltage gain and input resistance are infinite while the output resistance is zero. What are the consequences for

(a) the op-amp's input voltage?

(b) the op-amp's input currents?

(c) the op-amp's output current?

SOLUTION:

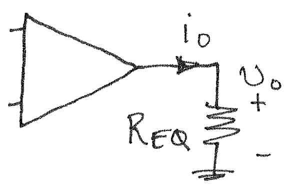
a) Since gain is infinite, an input voltage of zero can produce a finite output voltage.

$v_{in} = 0$

b) Since $R_{in} = \infty$, no input current flows.

$i_{in} = 0$

c) Since $R_{OUT} = 0$, the output current is limited only by external circuitry



$$i_o = \frac{v_o}{R_{EQ}}$$

4.5 Revisit your answers in Problem 4.4 under the following nonideal scenarios. **CS**

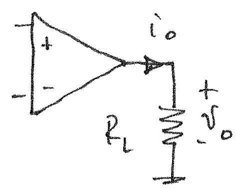
(a) $R_{\text{in}} = \infty, R_{\text{out}} = 0, A_o \neq \infty.$

(b) $R_{\text{in}} = \infty, R_{\text{out}} > 0, A_o = \infty.$

(c) $R_{\text{in}} \neq \infty, R_{\text{out}} = 0, A_o = \infty.$

SOLUTION:

a) Since $R_{in} = \infty$, $i_{in} = 0$
Since $R_{OUT} = 0$, $i_{OUT} = v_{OUT}/R_L$
Since $A_o \neq \infty$, $v_{in} \neq 0$



b) Since $R_{in} = \infty$, $i_{in} = 0$
Since $A_o = \infty$, $v_{in} = 0$
Since $R_{OUT} > 0$, i_{OUT} is limited by both R_{OUT} & R_L

c) Since $A_o = \infty$, $v_{in} = 0$
Since $R_{in} \neq \infty$, $i_{in} = v_{in}/R_{in}$
$i_{in} = 0$ only because $v_{in} = 0$

Since $R_o = 0$, i_{OUT} limited only by R_L

4.6 Revisit the exact analysis of the inverting configuration in Section 4.3.

(a) Find an expression for the gain if $R_{in} = \infty$, $R_{out} = 0$, $A_o \neq \infty$.

(b) Plot the ratio of the gain in (a) to the ideal gain versus A_o for $1 \leq A_o \leq 1000$ for an ideal gain of -10.

(c) From your plot, does the actual gain approach the ideal value as A_o increases or decreases?

(d) From your plot, what is the minimum value of A_o if the actual gain is within 5% of the ideal case?

SOLUTION:

a) From section 4.3 $$\frac{v_o}{v_s} = \frac{-R_2/R_1}{1 - \dfrac{\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_{in}}\right)\left(\frac{1}{R_2}+\frac{1}{R_o}\right)}{\frac{1}{R_2}\left(\frac{1}{R_2}-\frac{A_o}{R_o}\right)}}$$

For $R_{in} = \infty$, $R_{out} = 0$, $A_o \neq \infty$ we have

$$\frac{1}{R_{in}} << \frac{1}{R_1} \text{ and } \frac{1}{R_2} \quad \& \quad \frac{1}{R_o} >> \frac{1}{R_2}$$

Yields,

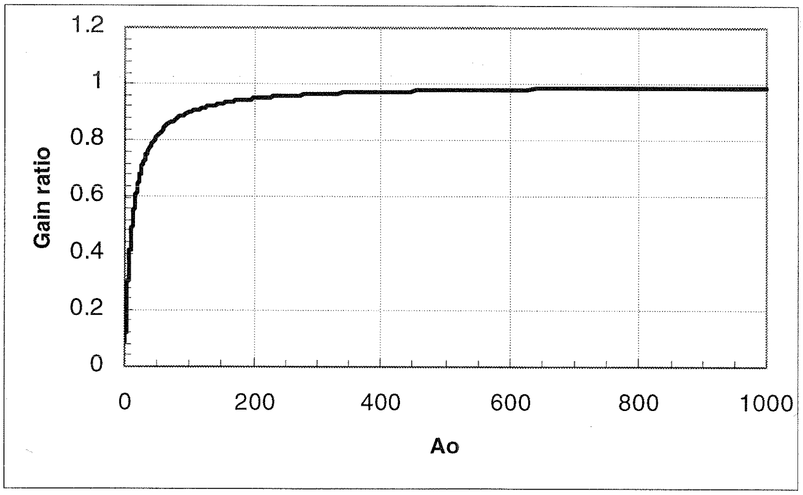
$$\frac{v_o}{v_s} = \frac{-R_2/R_1}{1 + \dfrac{\left(\frac{1}{R_1}+\frac{1}{R_2}\right)\frac{1}{R_o}}{\frac{1}{R_2}\left(A_o/R_o\right)}} = \frac{-R_2/R_1}{1 + \left(\frac{R_1+R_2}{R_1}\right)\frac{1}{A_o}}$$

or,

$$\boxed{\frac{v_o}{v_s} = \frac{-R_2/R_1}{1 + \frac{1}{A_o}\left(1 + R_2/R_1\right)}}$$

b) $A_{ideal} = -\frac{R_2}{R_1} = -10$

$A_{actual} = \frac{v_o}{v_s} = \frac{-10}{1 + \frac{11}{A_o}}$



c) As A_o increases, A_{actual} approaches A_{ideal}

d) from b, $\frac{A_{actual}}{A_{ideal}} = \frac{\frac{-10}{1+\frac{11}{A_o}}}{-10} \geq 0.95$

$$\boxed{A_o \geq 209}$$

4.7 Revisit the exact analysis of the inverting amplifier in Section 4.3.

(a) Find an expression for the voltage gain if $R_{in} \neq \infty$, $R_{out} = 0$, $A_o \neq \infty$.

(b) For $R_2 = 27\ \text{k}\Omega$ and $R_1 = 3\ \text{k}\Omega$, plot the ratio of the actual gain to the ideal gain for $A_o = 1000$ and $1\ \text{k}\Omega \leq R_{in} \leq 100\ \text{k}\Omega$.

(c) From your plot, does the ratio approach unity as R_{in} increases or decreases?

(d) From your plot in (b), what is the minimum value of R_{in} if the gain ratio is to be at least 0.98?

SOLUTION:

a)

$$\frac{v_o}{v_s} = A_{actual} = \frac{-R_2/R_1}{1 - \dfrac{\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_i}\right)\left(\frac{1}{R_2}+\frac{1}{R_o}\right)}{\frac{1}{R_2}\left(\frac{1}{R_2}-\frac{A_o}{R_o}\right)}}$$

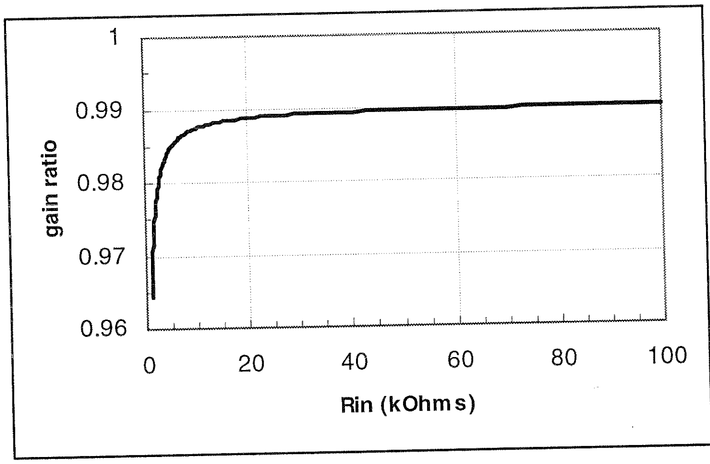
For $R_o = 0$, $\frac{1}{R_o} \gg \frac{1}{R_2}$ and $\frac{A_o}{R_o} \gg \frac{1}{R_2}$

$$A_{actual} = -\frac{R_2/R_1}{1 + \dfrac{\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_i}\right)}{A_o/R_2}}$$

$$\boxed{A_{actual} = \frac{-R_2/R_1}{1 + \frac{1}{A_o}\left(1 + \frac{R_2}{R_1} + \frac{R_2}{R_i}\right)}}$$

b) $R_2 = 27k\Omega$ $R_1 = 3k\Omega$ $A_0 = 1000$ $A_{ideal} = -9$

$$\frac{A_{actual}}{A_{ideal}} = \frac{-9/-9}{1 + \dfrac{10 + \dfrac{27000}{R_i}}{1000}}$$



c) The ratio approaches unity as R_i increases.

d) $$0.98 \leq \frac{1}{1 + 0.01 + \dfrac{27}{R_i}}$$ $\boxed{R_i \geq 2.59k\Omega}$

4.8 An op-amp based amplifier has ±18 V supplies and a gain of −80. Over what input range is the amplifier linear?

SOLUTION:

For linear operation

$$\frac{v_o}{v_{in}} = -80$$

Due to output limits, $|v_o| \leq 18\text{V}$

Linear region limited to

$$v_{in} \leq \frac{v_o}{-80}$$

$$\boxed{|v_{in}| \leq 0.225\text{V}}$$

4.9 Determine the gain of the amplifier in Fig. P4.9. What is the value of I_o? CS

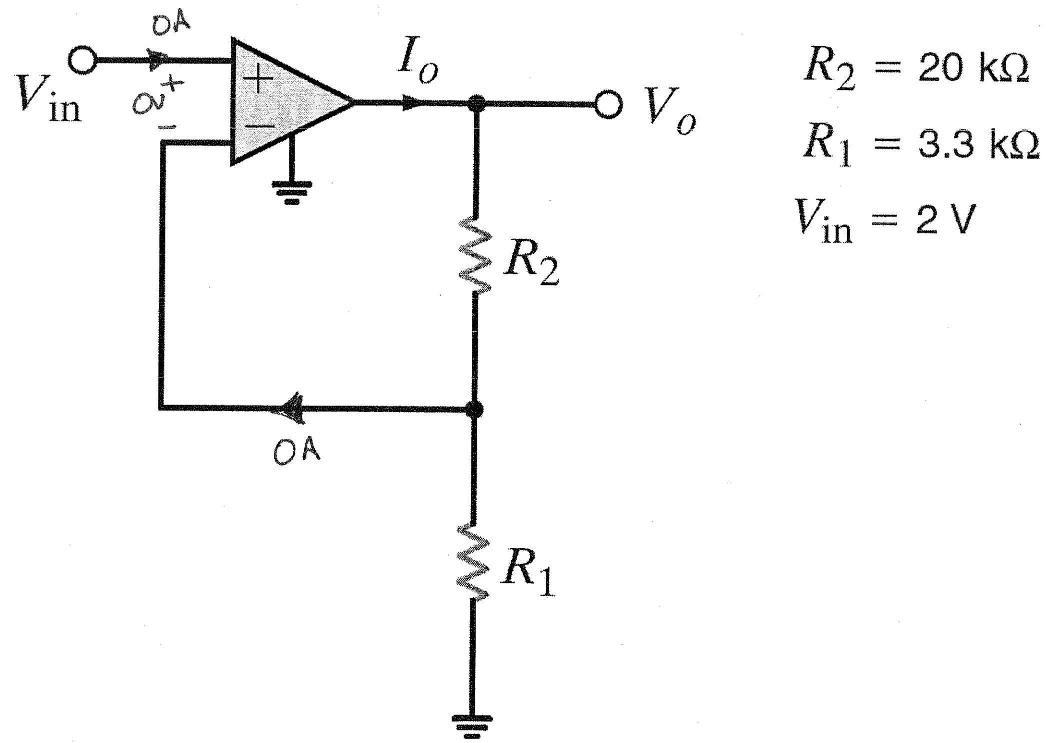


Figure P4.9

SOLUTION:

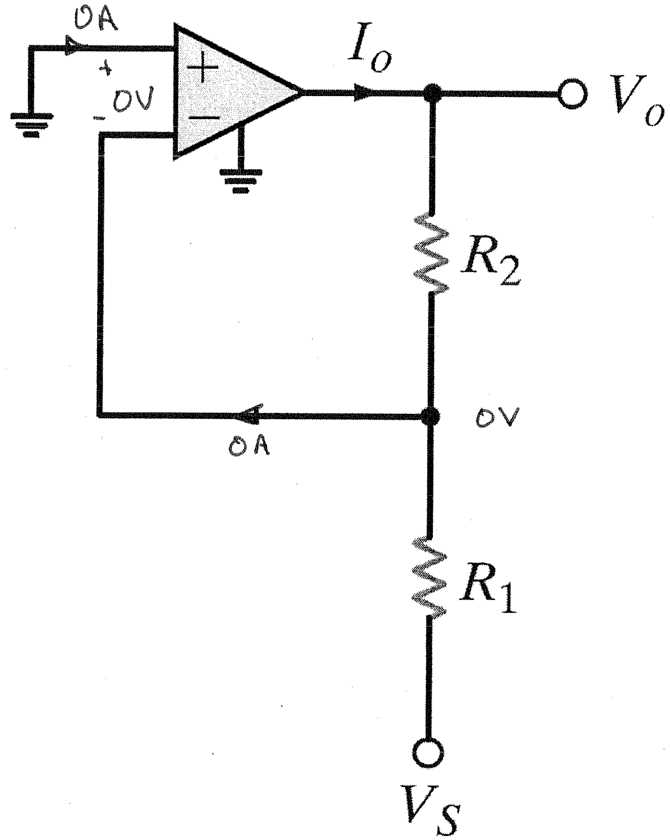
Basic noninverting configuration

$$\frac{V_o}{V_{in}} = 1 + \frac{R_2}{R_1} \Rightarrow \boxed{\frac{V_o}{V_{in}} = 7.06}$$

If $V_{in} = 2V$, $V_o = 14.12V$

$$I_o = \frac{V_o}{R_1 + R_2} = 606\,\mu A \qquad \boxed{I_o = 606\,\mu A}$$

4.10 For the amplifier in Fig. P4.10, find the gain and I_o?



$R_2 = 20\ \text{k}\Omega$
$R_1 = 3.3\ \text{k}\Omega$
$V_S = 2\ \text{V}$

Figure P4.10

SOLUTION:

Basic inverting configuration, $\frac{V_o}{V_S} = -\frac{R_2}{R_1} \Rightarrow \boxed{-6.06 = \frac{V_o}{V_S}}$

$I_o = V_o / R_2$ $\quad V_o = (-6.06)V_S = -12.12$

$\boxed{I_o = -606\,\mu A}$

4.11 Using the ideal op-amp assumptions, determine the values of V_o and I_1 in Fig. P4.11.

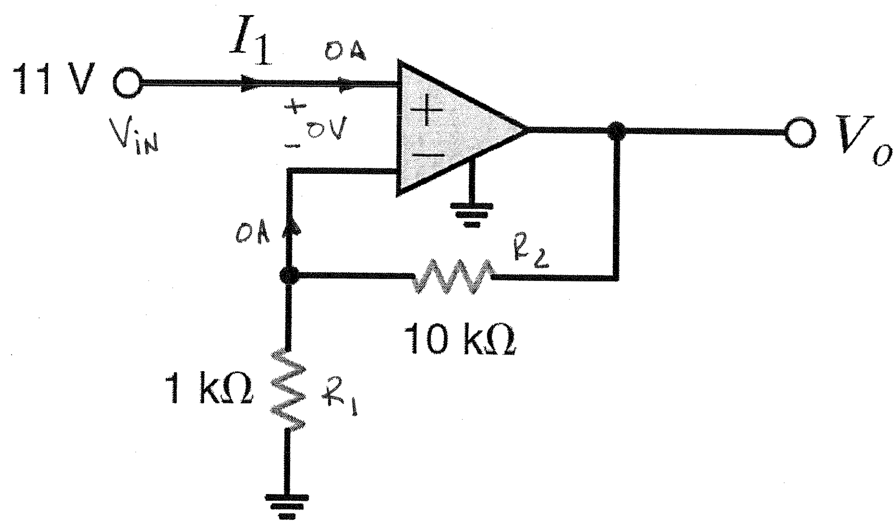


Figure P4.11

SOLUTION:

Basic non-inverting configuration,

$$\frac{V_O}{V_{IN}} = 1 + \frac{R_2}{R_1} = 11 \quad \Rightarrow \quad V_O = 11 V_{IN} \qquad \boxed{V_o = 121V}$$

Since $R_{in} = \infty$, $I_{in} = 0$ $\qquad \boxed{I_1 = 0 A}$

4.12 Using the ideal op-amp assumptions, determine I_1, I_2, and I_3 in Fig. P4.12. **PSV**

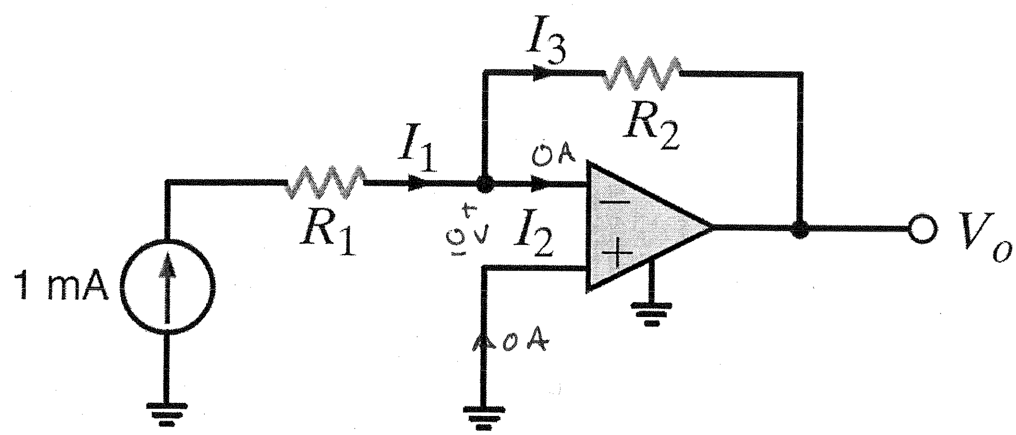


Figure P4.12

SOLUTION:

$\boxed{I_1 = 1\text{mA}}$ $\boxed{I_2 = 0\text{A}}$ (ideal op-amp)

By KCL,

$I_3 = I_1 - I_2$ $\boxed{I_3 = 1\text{mA}}$

4.13 In a useful application, the amplifier drives a load. The circuit in Fig. P4.13 models this scenario. **CS**

(a) Sketch the gain V_o/V_S for $10\ \Omega \le R_L \le \infty$.

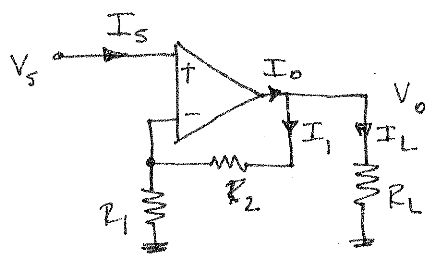
(b) Sketch I_o for $10\ \Omega \le R_L \le \infty$ if $V_S = 0.1$ V.

(c) Repeat (b) if $V_S = 1.0$ V.

(d) What is the minimum value of R_L if $|I_o|$ must be less than 100 mA for $|V_S| < 0.5$ V?

(e) What is the current I_S if R_L is $100\ \Omega$? Repeat for $R_L = 10\ \mathrm{k}\Omega$.

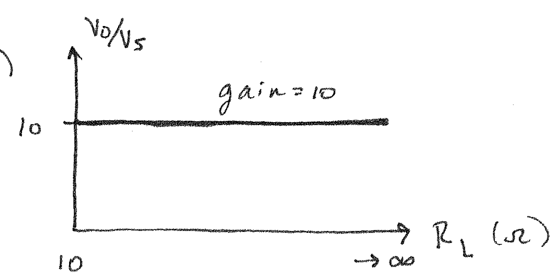
SOLUTION:



$R_2 = 27\text{k}\Omega \qquad R_1 = 3\text{k}\Omega$

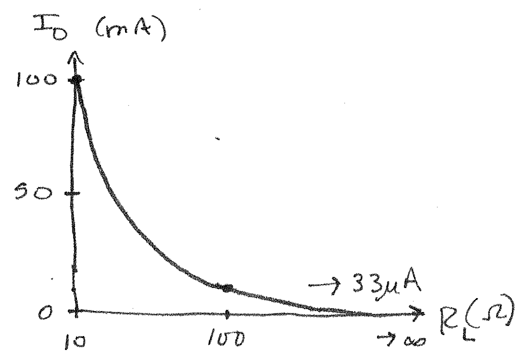
$$\frac{V_o}{V_S} = 1 + \frac{R_2}{R_1} = 10$$

a)

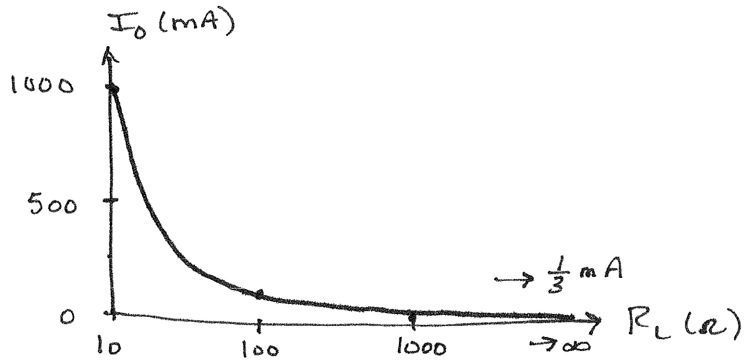


b) $V_S = 0.1\text{V}$, $V_O = 1\text{V}$

$$I_O = I_L + I_1 = \frac{V_o}{30\times10^3} + \frac{V_o}{R_L}$$



c. $V_S = 1V$, $V_o = 10V$, $I_o = \frac{10}{30\times10^3} + \frac{10}{R_L}$



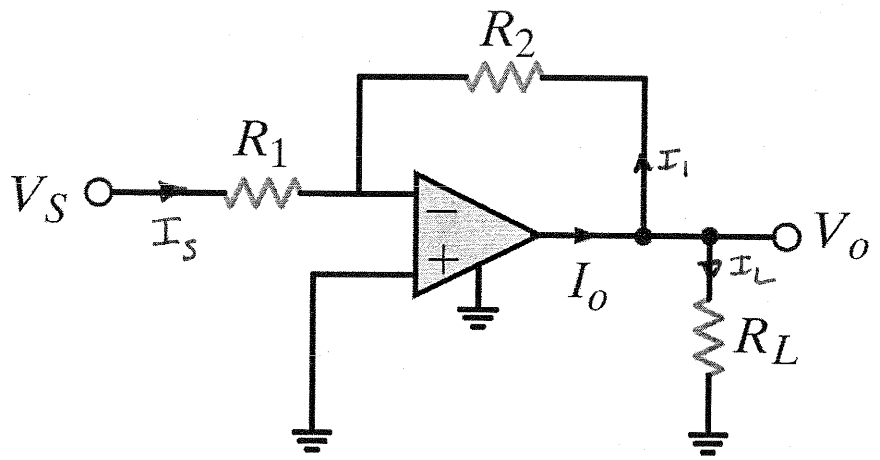
d) $V_S = 0.5V$, $V_o = 5V$, $I_o = \frac{5}{30\times10^3} + \frac{5}{R_L} < 100mA$

$$\boxed{R_L > 50.1\Omega}$$

e) I_S flows directly into the opamp's non-inverting input. I_S is zero regardless of R_L

$$\boxed{I_S = 0}$$

4.14 Repeat Problem 4.13 for the circuit in Fig. P4.14.



$R_2 = 27\text{ k}\Omega$

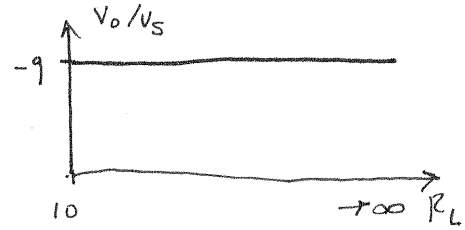
$R_1 = 3\text{ k}\Omega$

Figure P4.14

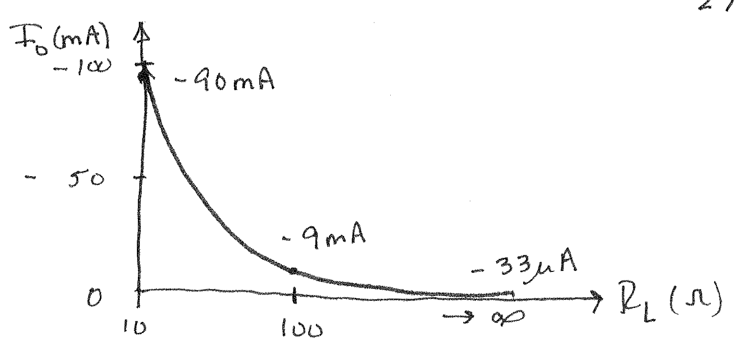
SOLUTION:

a) $V_o = -R_2/R_1\, V_s = -9\, V_s$

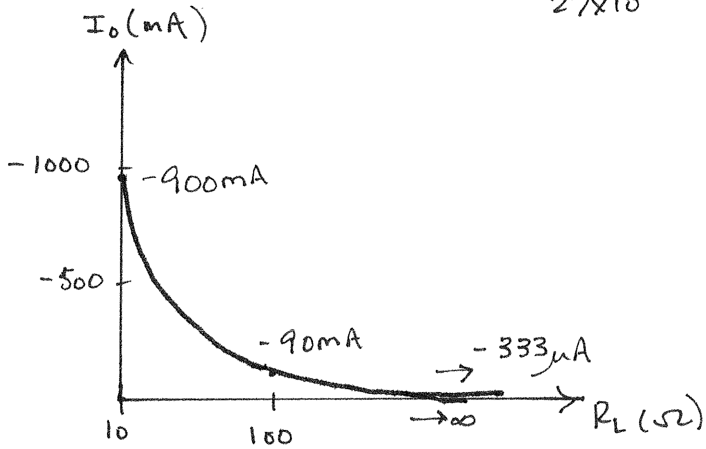
$I_o = I_1 + I_L = \dfrac{V_o}{R_2} + \dfrac{V_o}{R_L}$



b) $V_s = 0.1\text{V}$, $V_o = -0.9\text{V}$ $\quad I_o = \dfrac{-.9}{27\times10^{+3}} - \dfrac{.9}{R_L}$



c) $V_S = 1V$, $V_O = -9V$ $\quad I_o = \dfrac{-9}{27\times10^3} - \dfrac{9}{R_L}$



d) $$I_O = \frac{-9V_s}{27\times10^3} - \frac{9V_s}{R_L}$$

at $V_s = 0.5V$, $|I_o| = +\dfrac{4.5}{27\times10^3} + \dfrac{4.5}{R_L} < 100mA$

$$\boxed{R_L > 45.1\Omega}$$

e) $I_S = \dfrac{V_s}{R_1} = \dfrac{V_s}{3000}$ $\quad I_S$ is independent of R_L

at $V_s = 0.5V$, $\boxed{I_S = 167\mu A}$

4.15 The op-amp in the amplifier in Fig. P4.15 operates with ±15 V supplies and can output no more than 200 mA. What is the maximum gain allowable for the amplifier if the maximum value of V_S is 1 V?

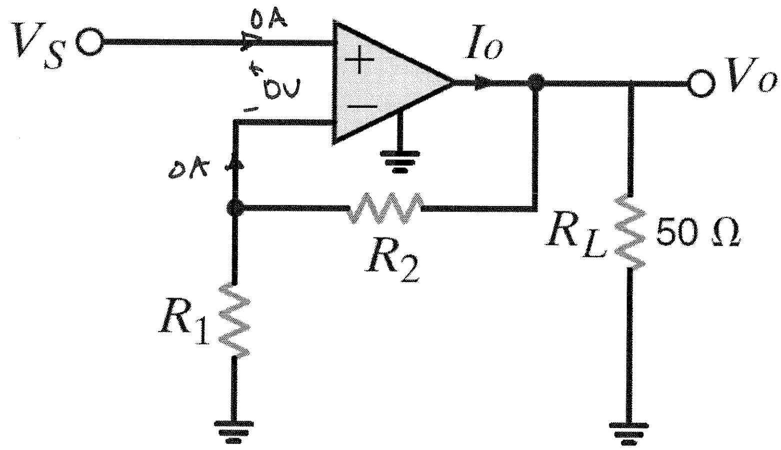


Figure P4.15

SOLUTION:

Basic non-inverting configuration: $\frac{V_o}{V_s} = 1 + \frac{R_2}{R_1} = \frac{R_1+R_2}{R_1} = \frac{10^4}{R_1}$

$|V_o| = |V_s| \left(\frac{10^4}{R_1}\right) \le 15$ since $V_s = 1\,V$, $\frac{10^4}{R_1} = V_o$

Also, $I_o = \frac{V_o}{R_1+R_2} + \frac{V_o}{50} = \frac{1}{R_1} + \frac{10^4}{50R_1} = \frac{1}{R_1}\left[201\right] \le 200\text{mA}$

$R_1 \ge 1005\,\Omega$ for $I_o \le 200\text{mA}$

$R_2 \le 8995\,\Omega$

Check gain limit: $1 + R_2/R_1 = 9.95$ $I_s < 15$!

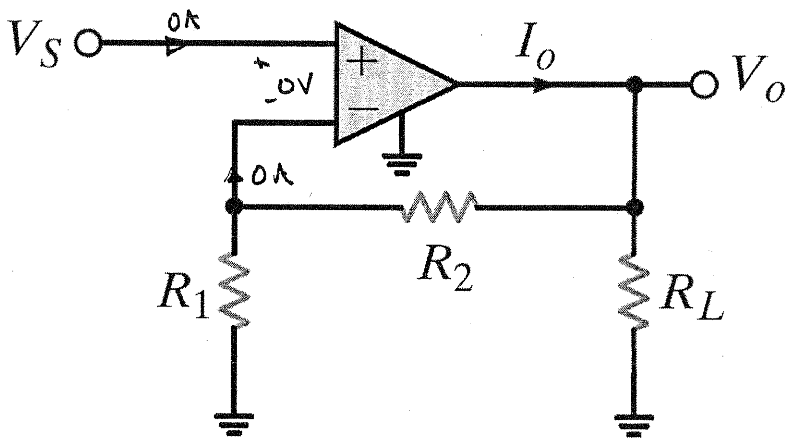
Final answer $\boxed{R_1 = 1005\,\Omega \quad R_2 = 8995\,\Omega \quad A_v = 9.95}$

4.16 For the amplifier in Fig. P4.16, the maximum value of V_S is 2 V and the op-amp can deliver no more than 100 mA.

(a) If ±10 V supplies are used, what is the maximum allowable value of R_2?

(b) Repeat for ±3 V supplies.

(c) Discuss the impact of the supplies on the maximum allowable gain.



R_L = 10 kΩ

R_1 = 100 kΩ

Figure P4.16

SOLUTION:

a) Basic non-inverting configuration: $V_o = V_S\left[1+\frac{R_2}{R_1}\right]$ & $I_o = \frac{V_o}{R_L // (R_1+R_2)}$

For $V_S = 2V$, $V_o = 2\left(1+\frac{R_2}{R_1}\right) \leq 10V$

$\boxed{R_2 \leq 400k\Omega}$

Check I_o limit: $\frac{10}{10^4 // (10^5 + 4\times10^5)} \leq 100mA$? yes!

b) $V_o = 2\left(1+R_2/R_1\right) \leq 3V$ $\boxed{R_2 \leq 50k\Omega}$ $I_o = \frac{3}{9.375\times10^3} = 320\mu A$ ✓

c) For a given Vs value, Av max is linearly related to supply voltage until I_o limit becomes an issue.

4.17 For the circuit in Fig. P4.17, **PSV**

(a) find V_o in terms of V_1 and V_2.

(b) If $V_1 = 2$ V and $V_2 = 6$ V, find V_o.

(c) If the op-amp supplies are ± 12 V, and $V_1 = 4$ V, what is the allowable range of V_2?

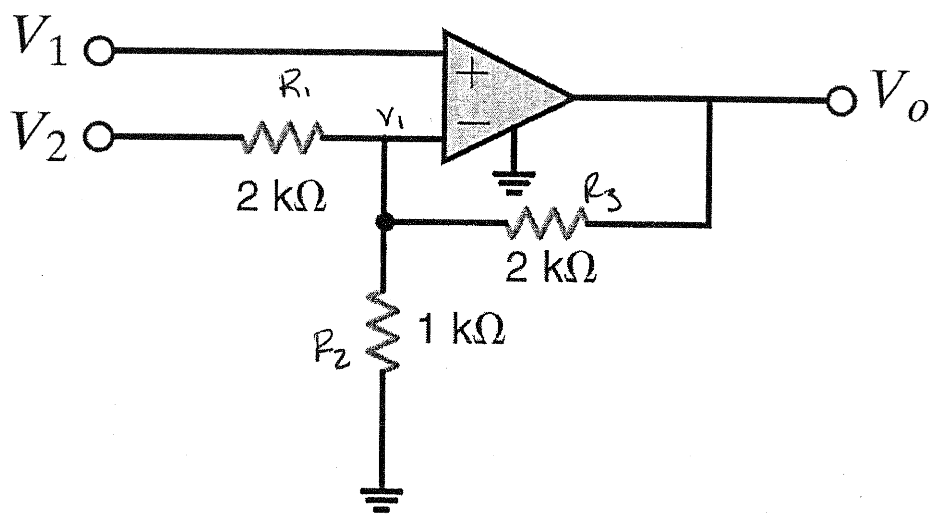


Figure P4.17

SOLUTION:

a) KCL at v_- input: (remember $v_+ = v_- = V_1$)

$$\frac{V_2 - V_1}{R_1} = \frac{V_1}{R_2} + \frac{V_1 - V_o}{R_3} \Rightarrow V_o = V_1\left(1 + \frac{R_3}{R_1} + \frac{R_3}{R_2}\right) - v_2\left(\frac{R_3}{R_1}\right)$$

$$\boxed{v_o = 4V_1 - V_2}$$

b) $V_o = 4(2) - 6$ $\boxed{V_o = 2\text{ V}}$

c) $V_o = |4(4) - V_2| \leq 12\text{ V}$ $\boxed{4\text{V} \leq V_2 \leq 28\text{ V}}$

4.18 Find V_o in the circuit in Fig. P4.18 assuming the op-amp is ideal.

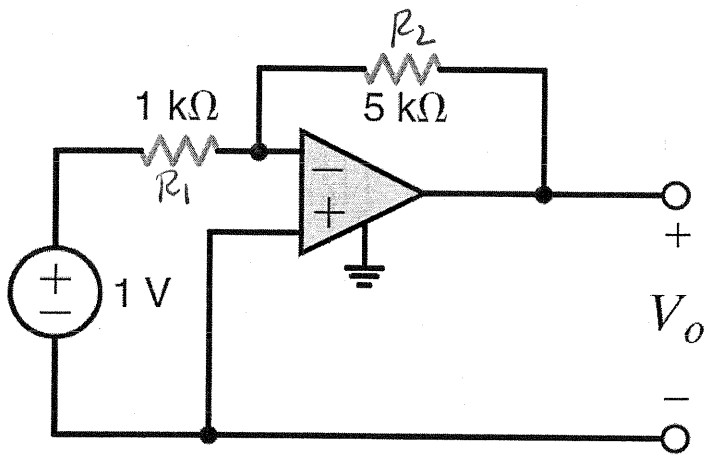


Figure P4.18

SOLUTION:

Basic inverting configuration.

$$V_o = V_s\left[-R_2/R_1\right] \qquad V_o = 1\left(-\frac{5000}{1000}\right) \qquad \boxed{V_o = -5V}$$

4.19 The network in Fig. P4.19 is a current-to-voltage converter or transconductance amplifier. Find v_o/i_S for this network.

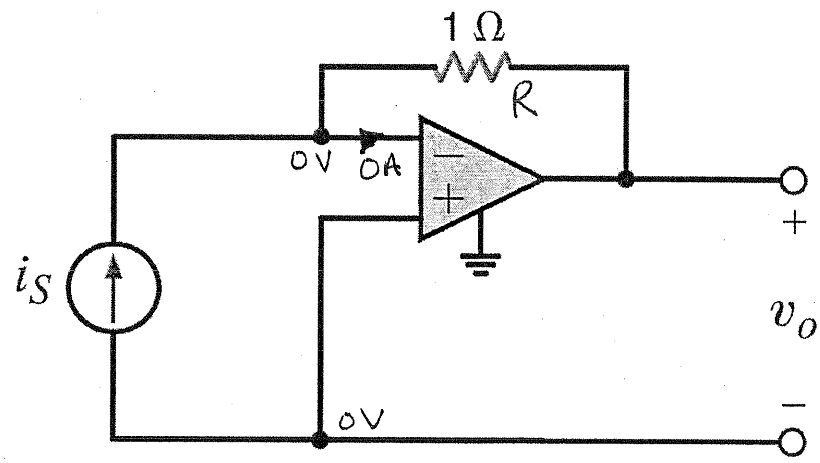


Figure P4.19

SOLUTION:

KCL at v_- input: $i_S = \dfrac{0 - v_o}{R}$

$\dfrac{v_o}{i_S} = -R$ $\boxed{\dfrac{v_o}{i_S} = -1}$

4.20 Calculate the transfer function i_o/v_1 for the network shown in Fig. P4.20.

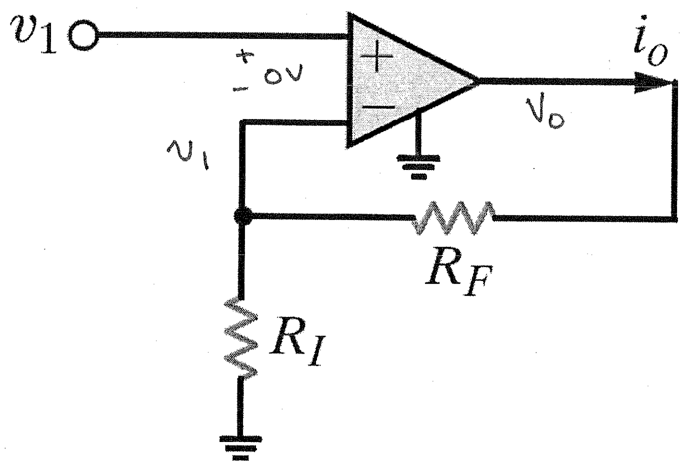


Figure P4.20

SOLUTION:

KCL at v_- input: $\dfrac{v_1}{R_I} = \dfrac{V_o - V_1}{R_F}$

$$\frac{V_o}{V_1} = 1 + \frac{R_F}{R_I} = \frac{R_I + R_F}{R_I}$$

$$i_o = \frac{V_o}{R_I + R_F} = V_1/R_I \qquad \boxed{\frac{i_o}{V_1} = \frac{1}{R_I}}$$

4.21 Determine the relationship between v_1 and i_o in the circuit shown in Fig. P4.21. CS

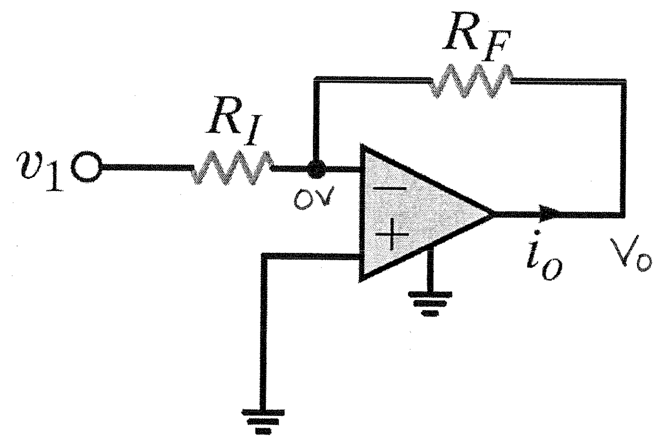


Figure P4.21

SOLUTION:

Basic inverting configuration:

$$\frac{v_o}{v_1} = -\frac{R_F}{R_I} \qquad i_o = \frac{v_o}{R_F} = -v_1/R_I$$

$$\boxed{\frac{i_o}{v_1} = -\frac{1}{R_I}}$$

4.22 Find V_o in the network in Fig. P4.22 and explain what effect R_1 has on the output.

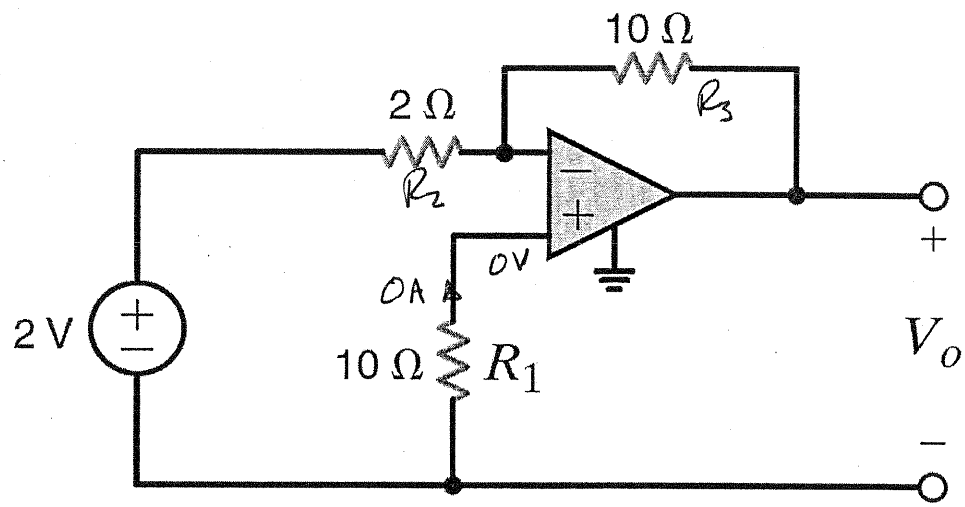


Figure P4.22

SOLUTION:

Since $i_{in} = 0$ for ideal op amp, voltage across $R_1 = 0$ and v_+ input is at 0V as well. Result is a basic inverting configuration.

$$V_o = -2\left(R_3/R_2\right) \Rightarrow \boxed{V_o = -10\text{V}}$$

<u>R_1 has no impact on the circuit at all!</u>

4.23 Determine the expression for v_o in the network in Fig. P4.23.

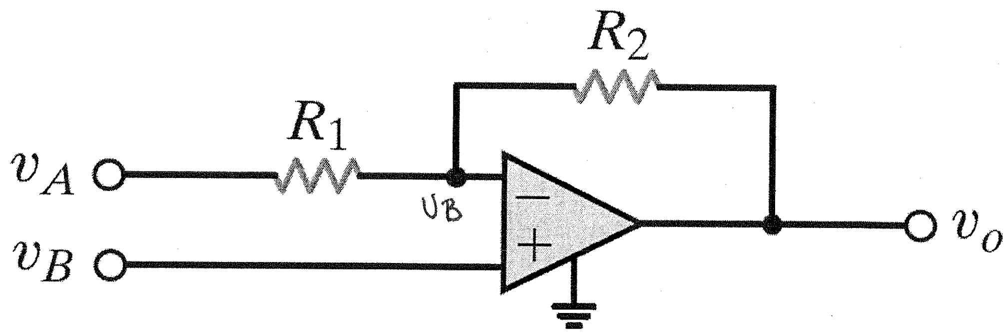


Figure P4.23

SOLUTION:

KCL at v_- node:

$$\frac{v_A - v_B}{R_1} + \frac{v_o - v_B}{R_2} = 0$$

$$\boxed{v_o = v_B\left(1 + \frac{R_2}{R_1}\right) - v_A\left(R_2/R_1\right)}$$

4.24 Show that the output of the circuit in Fig. P4.24 is

$$V_o = \left[1 + \frac{R_2}{R_1}\right]V_1 - \frac{R_2}{R_1}V_2$$

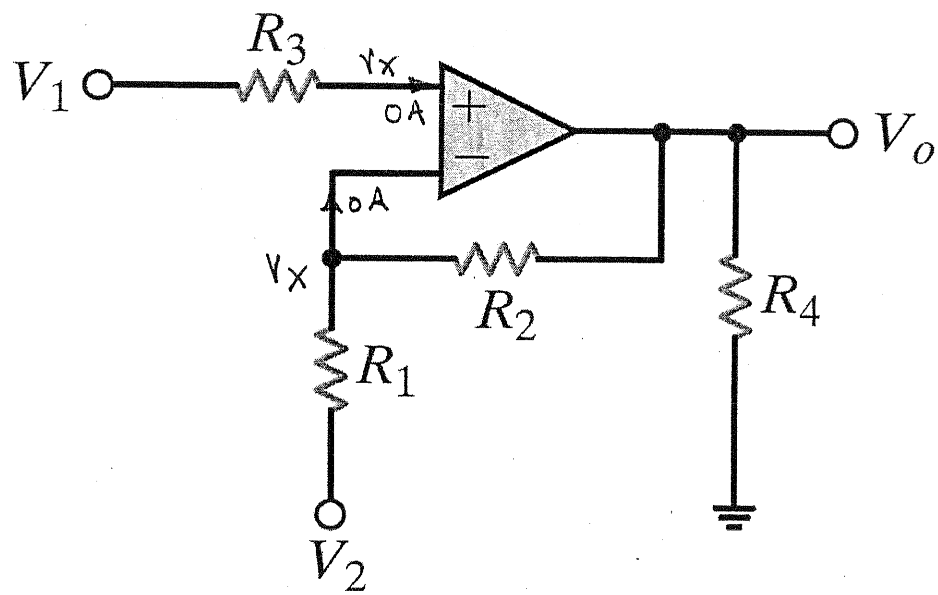


Figure P4.24

SOLUTION:

KCL at v_+ input: $\Rightarrow \frac{V_1 - V_x}{R_3} = 0 \Rightarrow V_1 = V_x$

$$\frac{V_2 - V_x}{R_1} + \frac{V_o - V_x}{R_2} = 0 \qquad V_o = V_x\left(1 + \frac{R_2}{R_1}\right) - V_2\left(R_2/R_1\right)$$

$$\boxed{V_o = \left[1 + \frac{R_2}{R_1}\right]V_1 - \frac{R_2}{R_1}V_2}$$

4.25 Find V_o in the network in Fig. P4.25. CS

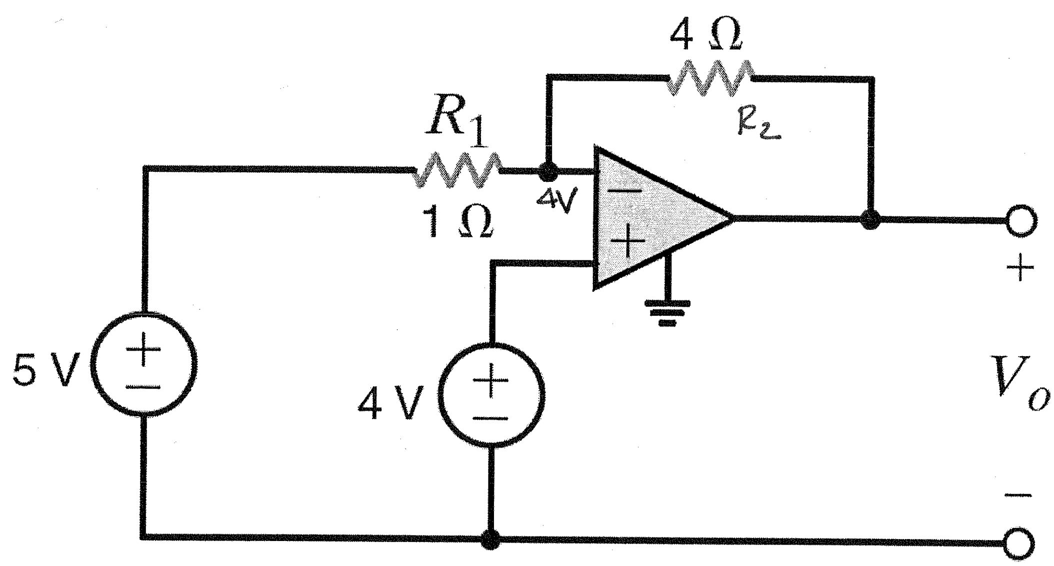


Figure P4.25

SOLUTION:

KCL at V_- input: $\frac{5-4}{R_1} + \frac{V_o - 4}{R_2} = 0$

$\boxed{V_o = 0\,V}$

4.26 Find the voltage gain of the op-amp circuit shown in Fig. P4.26.

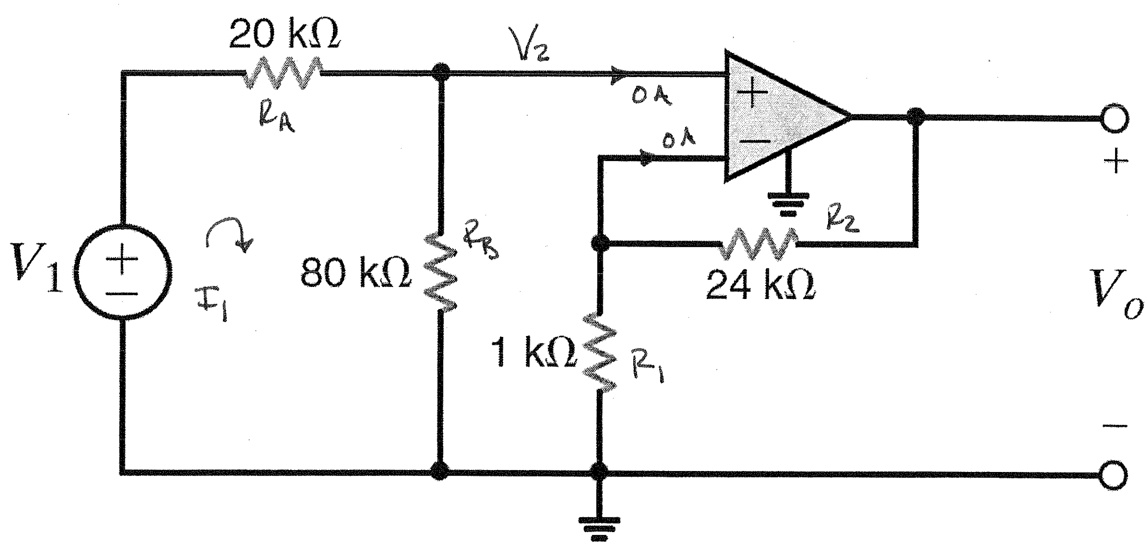


Figure P4.26

SOLUTION:

Two step solution: 1) Find V_2/V_1
2) Find V_o/V_2

1) Loop analysis:

$$V_1 = I_1 R_A + I_1 R_B \quad \& \quad V_2 = I_1 R_B$$

$$I_1 = V_2/R_B \qquad \frac{V_2}{V_1} = \frac{R_B}{R_A+R_B} = 0.8$$

2) Op-amp is in basic non-inverting configuration.

$$\frac{V_o}{V_2} = 1 + \frac{R_2}{R_1} = 25$$

Overall gain is $v_o/v_1 = (v_2/v_1)(v_o/v_2)$ $\boxed{\frac{v_o}{v_1} = 20}$

4.27 For the circuit in Fig. 4.27 find the value of R_1 that produces a voltage gain of 10.

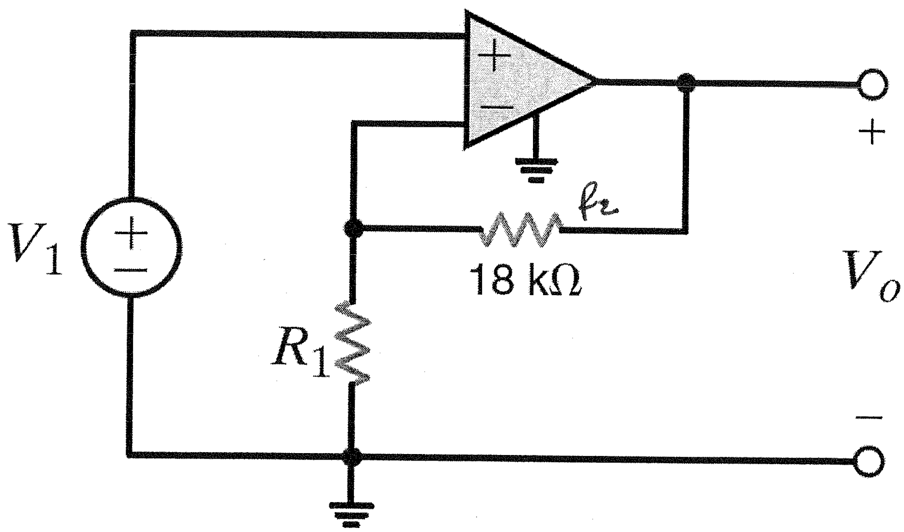


Figure P4.27

SOLUTION:

Basic non-inverting configuration:

$$\frac{V_o}{V_1} = 1 + \frac{R_2}{R_1} = 1 + \frac{18 \times 10^3}{R_1} = 10$$

$$\boxed{R_1 = 2k\Omega}$$

4.28 Determine the relationship between v_o and v_{in} in the circuit in Fig. P4.28.

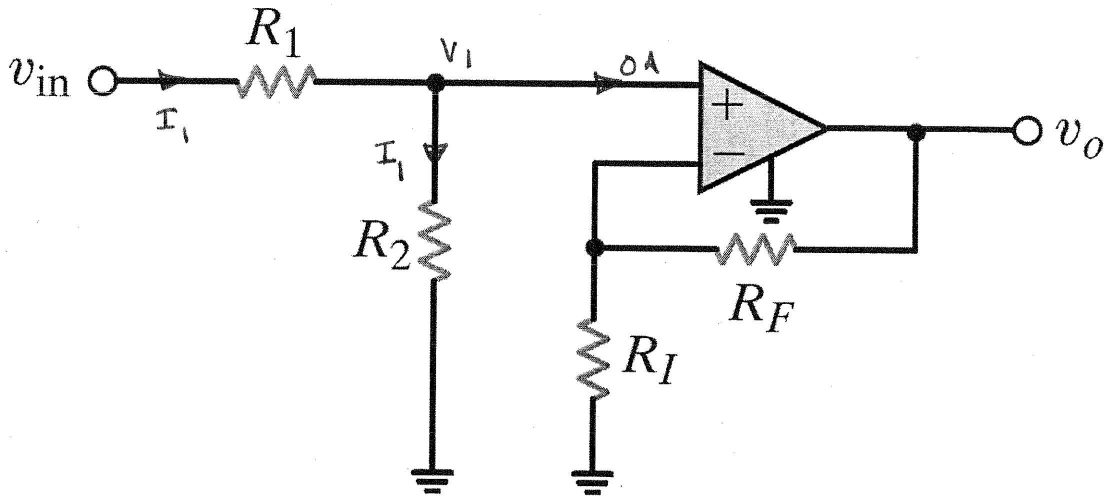


Figure P4.28

SOLUTION: Two step solution: 1) find V_1/V_{in}
2) find V_o/V_1

1) $V_{in} = I_1 R_1 + I_1 R_2 \qquad V_1 = I_1 R_2 \quad \Rightarrow \quad \dfrac{V_1}{V_{in}} = \dfrac{R_2}{R_1+R_2}$

2) Opamp is in basic non-inverting configuration

$$\frac{V_o}{V_1} = 1 + \frac{R_F}{R_I}$$

Overall gain $\dfrac{V_o}{V_{in}} = \left(\dfrac{V_o}{V_1}\right)\left(\dfrac{V_1}{V_{in}}\right)$. $\boxed{\dfrac{V_o}{V_{in}} = \left(\dfrac{R_2}{R_1+R_2}\right)\left(\dfrac{R_I+R_F}{R_I}\right)}$

4.29 In the network in Fig. P4.29 derive the expression for v_o in terms of the inputs v_1 and v_2. **CS**

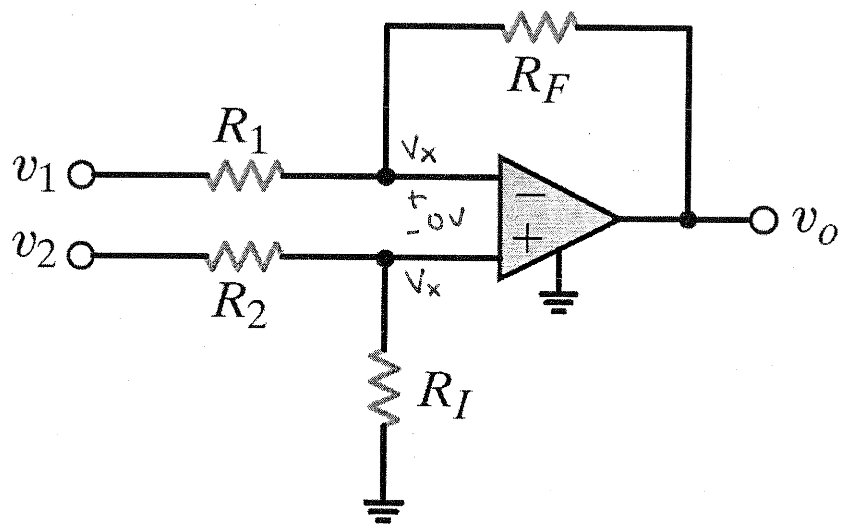


Figure P4.29

SOLUTION:

KCL at V_+ input: $\dfrac{V_2 - V_x}{R_2} = \dfrac{V_x}{R_I} \Rightarrow V_x = V_2\left(\dfrac{R_I}{R_I + R_2}\right)$

KCL at V_- input: $\dfrac{V_1 - V_x}{R_1} = \dfrac{V_x - V_o}{R_F} \Rightarrow V_o = V_x\left(1 + \dfrac{R_F}{R_1}\right) - \dfrac{R_F}{R_1} V_1$

$$\boxed{V_o = V_2\left(\frac{R_I}{R_I + R_2}\right)\left(\frac{R_1 + R_F}{R_1}\right) - \frac{R_F}{R_1} V_1}$$

4.30 Find V_o in the circuit in Fig. P4.30.

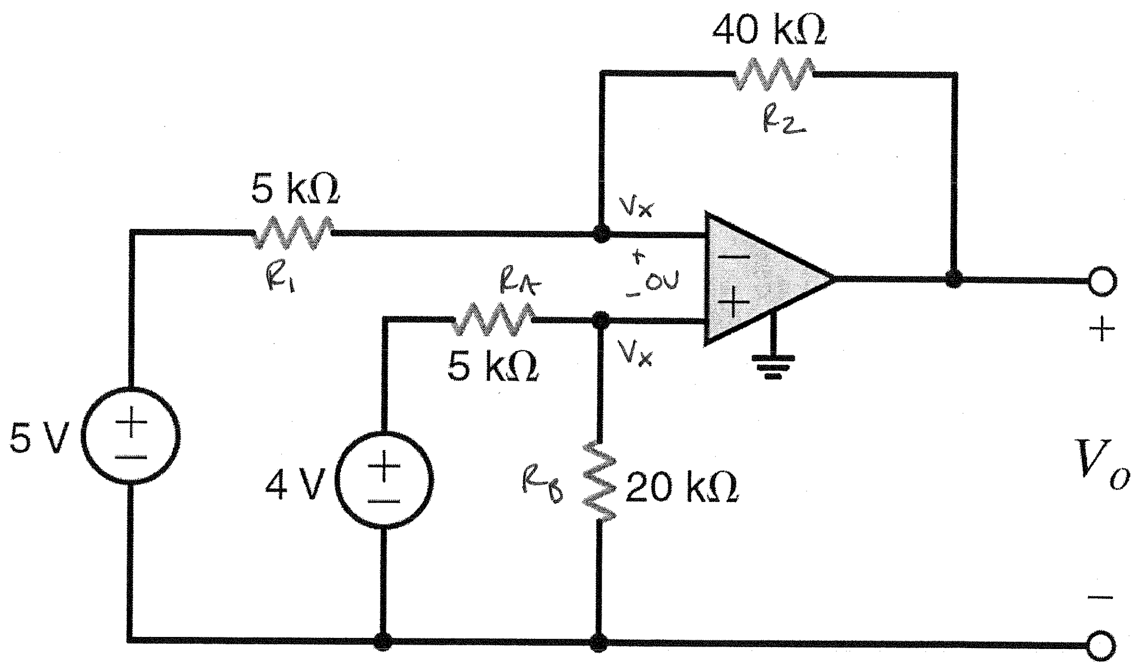


Figure P4.30

SOLUTION:

KCL at V_- input: $\frac{5-V_x}{R_1} = \frac{V_x - V_o}{R_2} \Rightarrow V_x = 5\left(\frac{R_2}{R_1+R_2}\right) + V_o\left(\frac{R_1}{R_1+R_2}\right)$

KCL @ V_+ input: $\frac{4-V_x}{R_A} = \frac{V_x}{R_B} \Rightarrow V_x = \frac{4R_B}{R_A+R_B}$

$$5\left(\frac{R_2}{R_1+R_2}\right) + V_o\left(\frac{R_1}{R_1+R_2}\right) = \frac{4R_B}{R_A+R_B}$$

$$V_o = \left(\frac{4R_B}{R_A+R_B} - \frac{5R_2}{R_1+R_2}\right)\frac{R_1+R_2}{R_1}$$

$\boxed{V_o = -11.2\text{V}}$

4.31 Find V_o in the circuit in Fig. P4.31.

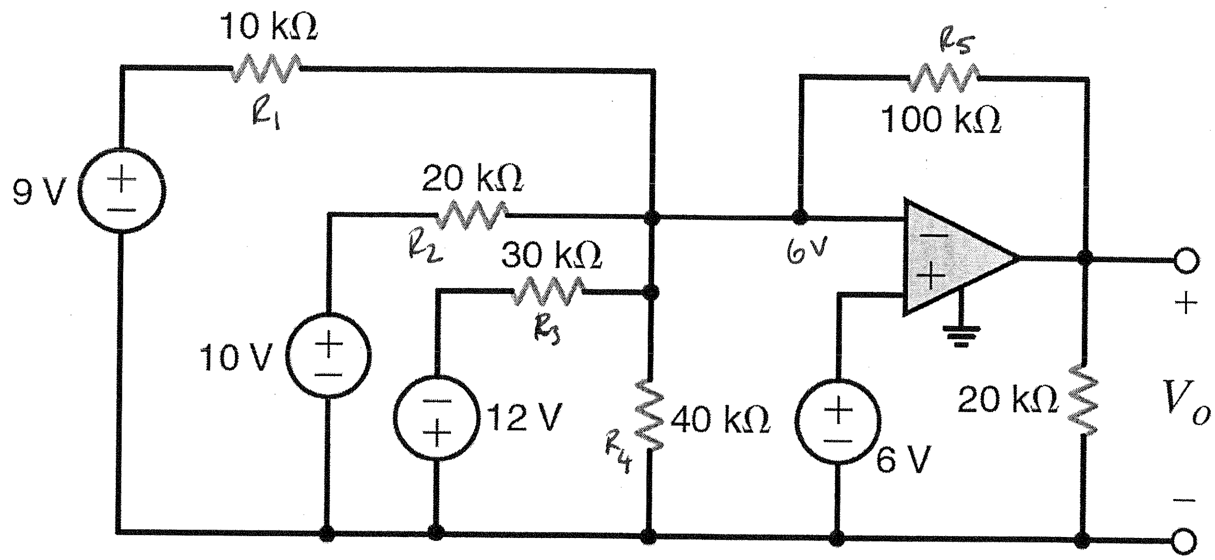


Figure P4.31

SOLUTION:

KCL at v- input: $\frac{9-6}{R_1} + \frac{10-6}{R_2} + \frac{-12-6}{R_3} = \frac{6}{R_4} + \frac{6-V_o}{R_5}$

$$\frac{3}{10^4} + \frac{4}{2\times10^4} - \frac{18}{3\times10^4} = \frac{6}{4\times10^4} + \frac{6}{10^5} - \frac{V_o}{10^5}$$

$$\boxed{V_o = 31V}$$

4.32 Determine the expression for the output voltage, v_o, of the inverting summer circuit shown in Fig. P4.32.

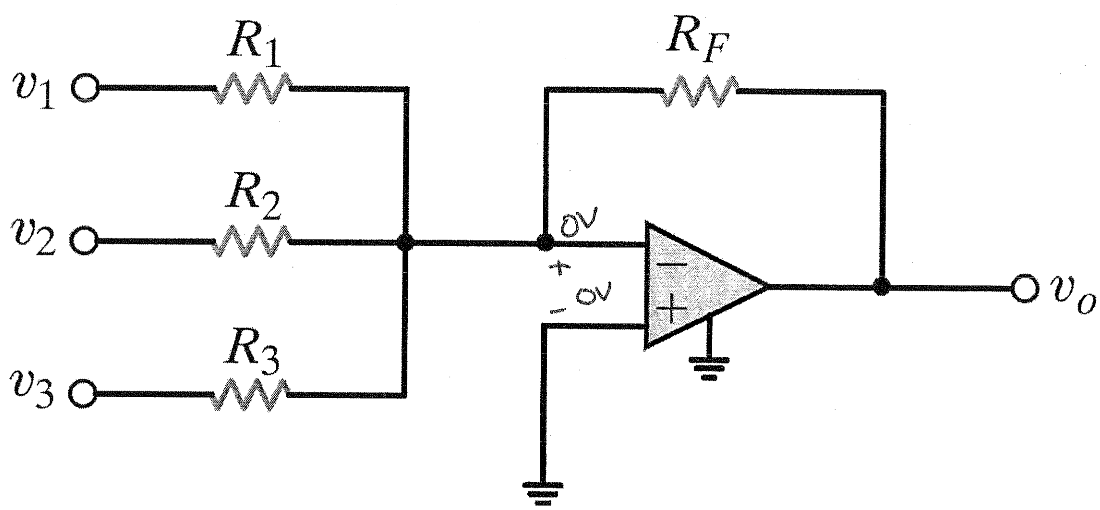


Figure P4.32

SOLUTION:

KCL at V₋ input: $\frac{v_1 - 0}{R_1} + \frac{v_2 - 0}{R_2} + \frac{v_3 - 0}{R_3} = \frac{0 - v_o}{R_F}$

$$\boxed{v_o = -\frac{R_F}{R_1}v_1 - \frac{R_F}{R_2}v_2 - \frac{R_F}{R_3}v_3}$$

4.33 Determine the output voltage, v_o, of the noninverting averaging circuit shown in Fig. P4.33. CS

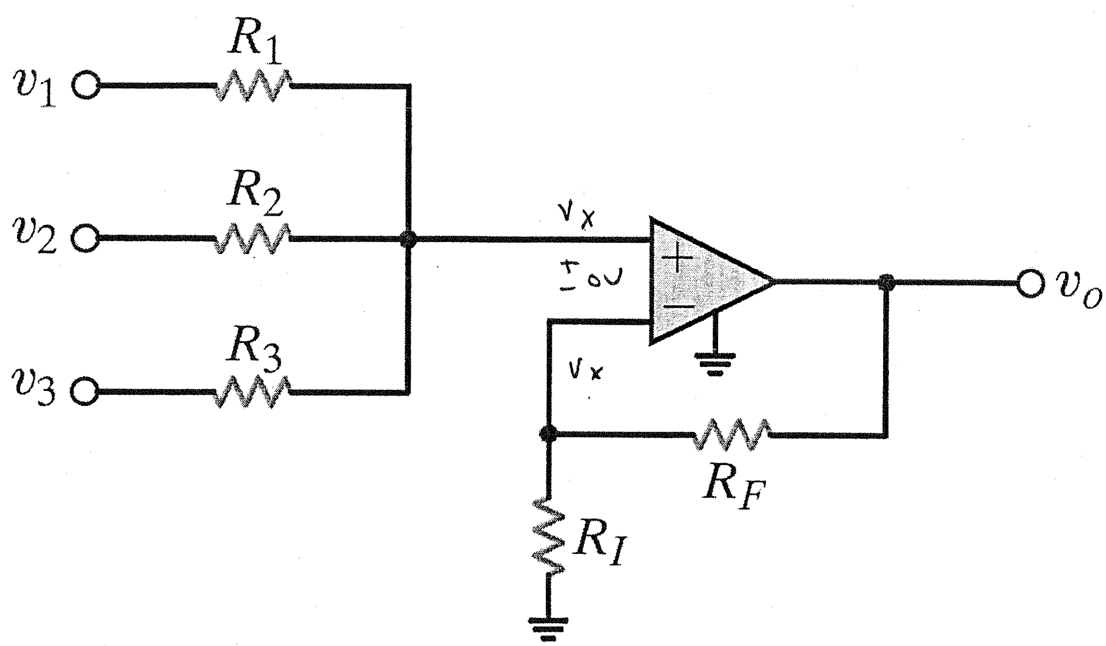


Figure P4.33

SOLUTION:

KCL at v_+ input: $$\frac{v_1 - v_x}{R_1} + \frac{v_2 - v_x}{R_2} + \frac{v_3 - v_x}{R_3} = 0$$

KCL at v_- input: $$\frac{v_o - v_x}{R_F} = \frac{v_x}{R_I} \qquad v_x = v_o\left(\frac{R_I}{R_I + R_F}\right)$$

Eliminate v_x,

$$\boxed{v_o = \left(\frac{R_I + R_F}{R_I}\right)\left[\frac{R_2R_3v_1 + R_1R_3v_2 + R_1R_2v_3}{R_1R_2 + R_2R_3 + R_1R_3}\right]}$$

4.34 Find the input/output relationship for the current amplifier shown in Fig. P4.34.

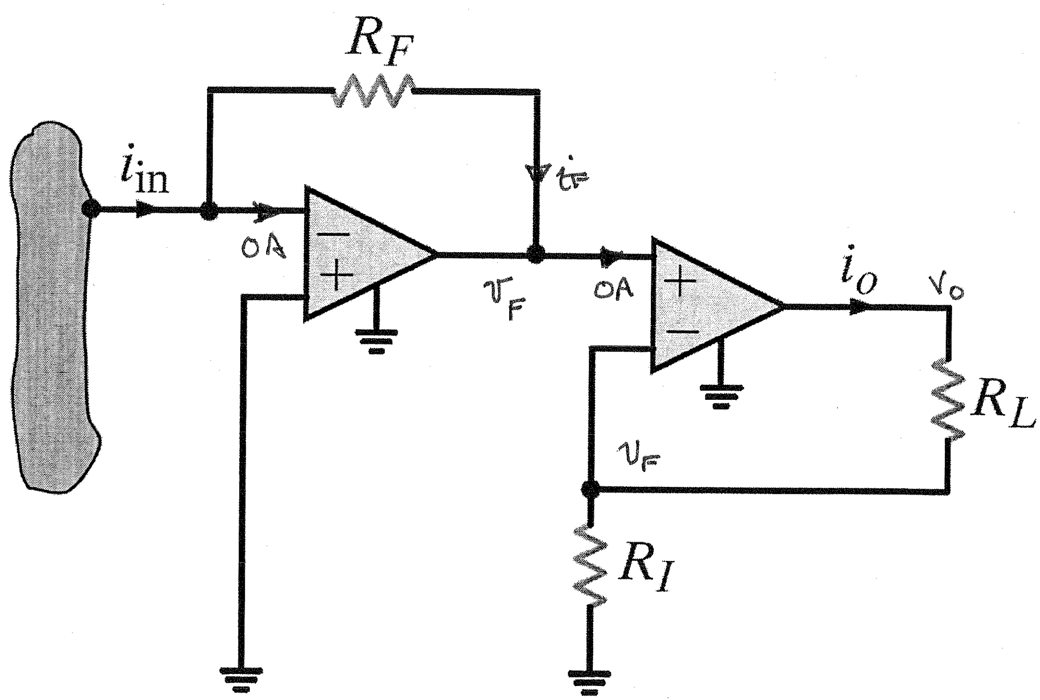


Figure P4.34

SOLUTION: KCL at v_- input of 1st op amp.

$$i_{in} = \frac{0 - v_F}{R_F} \qquad v_F = -R_F\, i_{in}$$

2nd op-amp in classic inverting configuration

$$v_o = v_F\left(1 + \frac{R_L}{R_I}\right) \qquad i_o = \frac{v_o - v_F}{R_L} = \frac{v_F}{R_I}$$

$$i_o/i_{in} = (v_F/i_{in})(i_o/v_F) \qquad \frac{i_o}{i_{in}} = -\frac{R_F}{R_I}$$

4.35 Find V_o in the circuit in Fig. P4.35. **PSV**

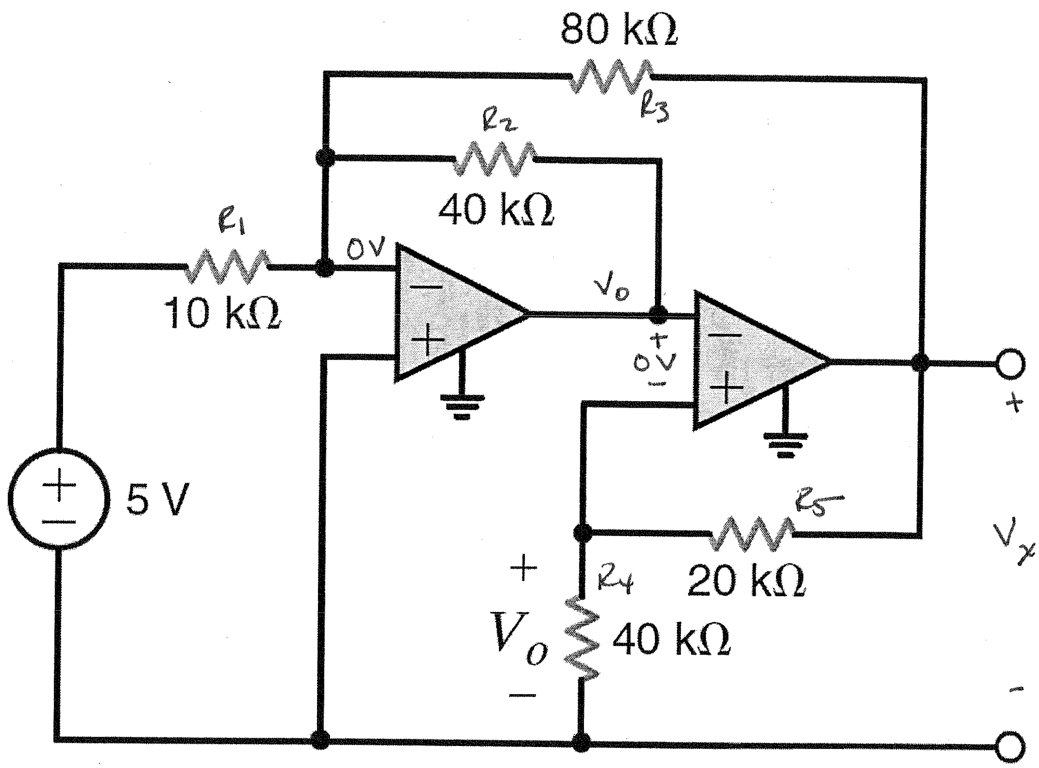


Figure P4.35

SOLUTION:

KCL at v_- of 1st op amp: $\frac{5}{R_1} + \frac{V_o}{R_2} + \frac{V_x}{R_3} = 0 \Rightarrow V_x = -\frac{R_3}{R_1}(5) - \frac{R_3}{R_2}V_o$

KCL at v_+ of 2nd op amp: $\frac{v_o}{R_4} + \frac{v_o - v_x}{R_5} = 0 \Rightarrow V_x = v_o\left(1 + R_5/R_4\right)$

Put in numbers,

$v_x = -40 - 2V_o$ & $v_x = 1.5v_o$

Eliminate V_x, $\boxed{V_o = -11.43\text{ V}}$

4.36 Find v_o in the circuit in Fig. P4.36.

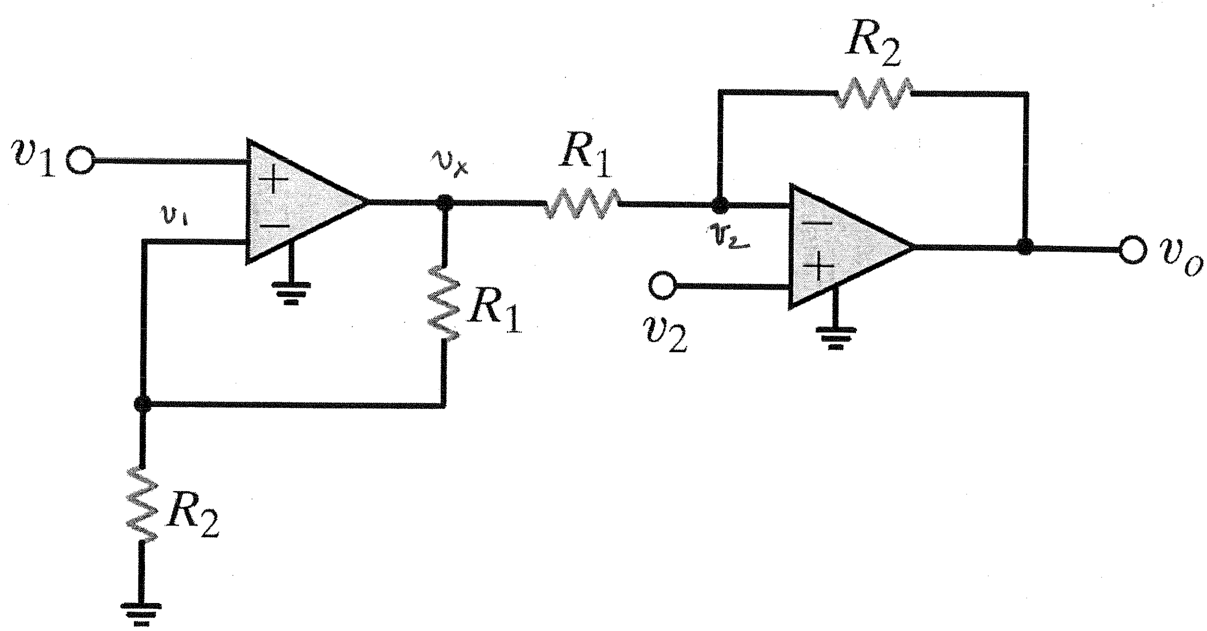


Figure P4.36

SOLUTION:

1st Op amp in basic non-inverting configuration:

$$\frac{v_x}{v_1} = 1 + \frac{R_1}{R_2} \Rightarrow v_x = v_1\left(\frac{R_2+R_1}{R_2}\right)$$

KCL at v_- of 2nd op amp: $\frac{v_x - v_2}{R_1} + \frac{v_o - v_2}{R_2} = 0$

$$v_o = v_2\left(1+\frac{R_2}{R_1}\right) - \frac{R_2}{R_1}v_x$$

$$\boxed{v_o = \left(1+\frac{R_2}{R_1}\right)(v_2 - v_1)}$$

4.37 Find the expression for v_o in the differential amplifier circuit shown in Fig. P4.37. **CS**

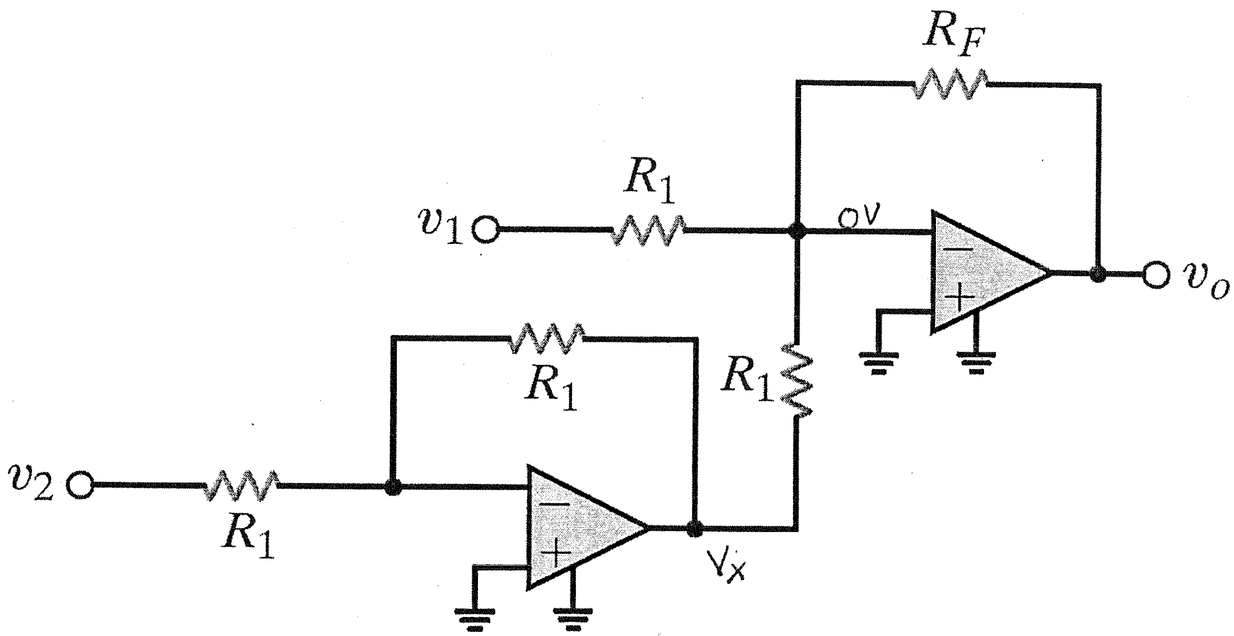


Figure P4.37

SOLUTION:

1st Op-amp in classic inverting configuration:

$v_x = -\frac{R_1}{R_1} v_2 \qquad v_x = -v_2$

KCL at v_- of 2nd opamp: $\frac{v_1}{R_1} + \frac{v_x}{R_1} + \frac{v_o}{R_F} = 0$

$$\boxed{v_o = \frac{R_F}{R_1}\left[v_2 - v_1\right]}$$

4.38 Find v_o in the circuit in Fig. P4.38. **PSV**

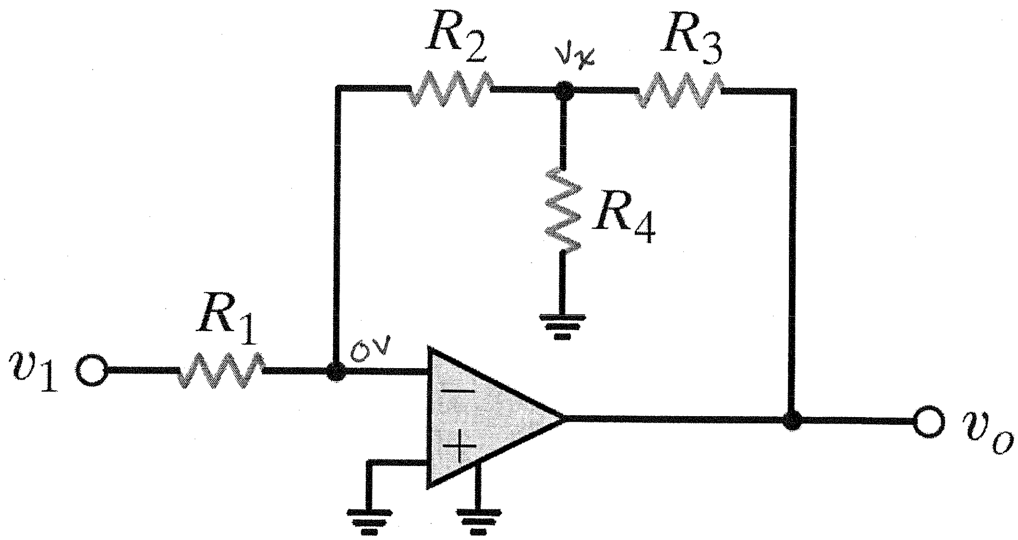


Figure P4.38

SOLUTION:

KCL at v_- input: $\frac{v_1}{R_1} + \frac{v_x}{R_2} = 0$ $\quad v_x = -\frac{R_2}{R_1} v_1$

KCL at v_x node: $\frac{v_x}{R_2} + \frac{v_x}{R_4} + \frac{v_x - v_o}{R_3} = 0$ $\quad v_o = v_x\left(\frac{R_3}{R_2} + \frac{R_3}{R_4} + 1\right)$

$$\boxed{v_o = v_1\left[1 + \frac{R_3}{R_2} + \frac{R_3}{R_4}\right]\left(-\frac{R_2}{R_1}\right)}$$

4.39 Find the output voltage, v_o, in the circuit in Fig. P4.39.

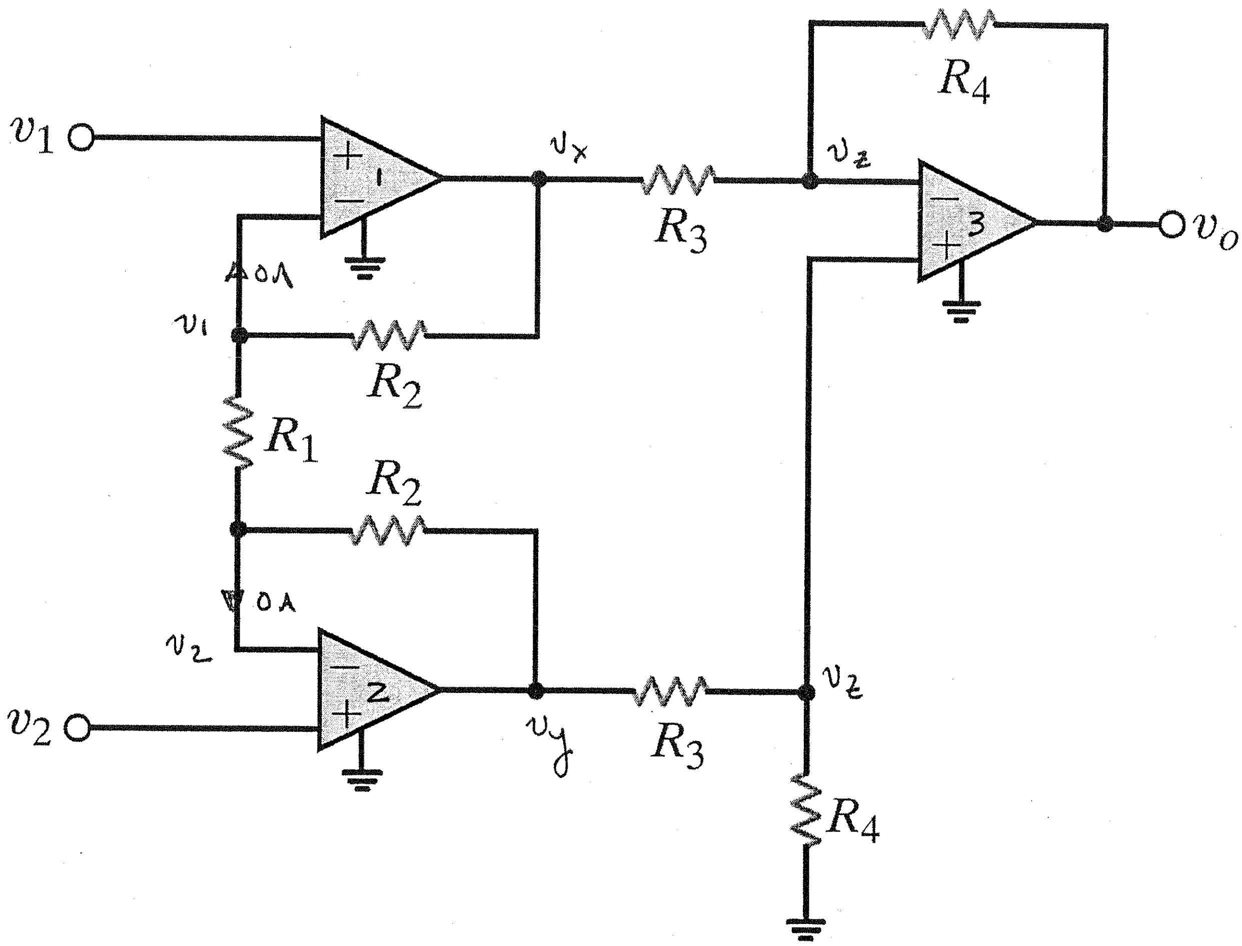


Figure P4.39

SOLUTION:

KCL at v_- of op amp 1: $\dfrac{v_x - v_1}{R_2} = \dfrac{v_1 - v_2}{R_1} \Rightarrow v_x = v_1\left(1 + \dfrac{R_2}{R_1}\right) - \dfrac{R_2}{R_1} v_2$

KCL at v_- of op amp 2: $\dfrac{v_y - v_2}{R_2} = \dfrac{v_2 - v_1}{R_1} \Rightarrow v_y = v_2\left(1 + \dfrac{R_2}{R_1}\right) - \dfrac{R_2}{R_1} v_1$

KCL at v_+ of op amp 3: $\dfrac{v_y - v_z}{R_3} = \dfrac{v_z}{R_4} \Rightarrow v_z = v_y\left(\dfrac{R_4}{R_3 + R_4}\right)$

KCL at v_- of op amp 3: $\dfrac{v_x - v_z}{R_3} + \dfrac{v_o - v_z}{R_4} = 0 \Rightarrow v_o = v_z\left(1 + R_4/R_3\right) - v_x \dfrac{R_4}{R_3}$

$$\boxed{v_o = \frac{R_4}{R_3}\left(1 + 2\frac{R_2}{R_1}\right)(v_2 - v_1)}$$

4.40 The electronic ammeter in Example 4.9 has been modified and is shown in Fig. P4.40. The selector switch allows the user to change the range of the meter. Using values for R_1 and R_2 from Example 4.9, find the values of R_A and R_B that will yield a 10-V output when the current being measured is 100 mA and 10 mA, respectively.

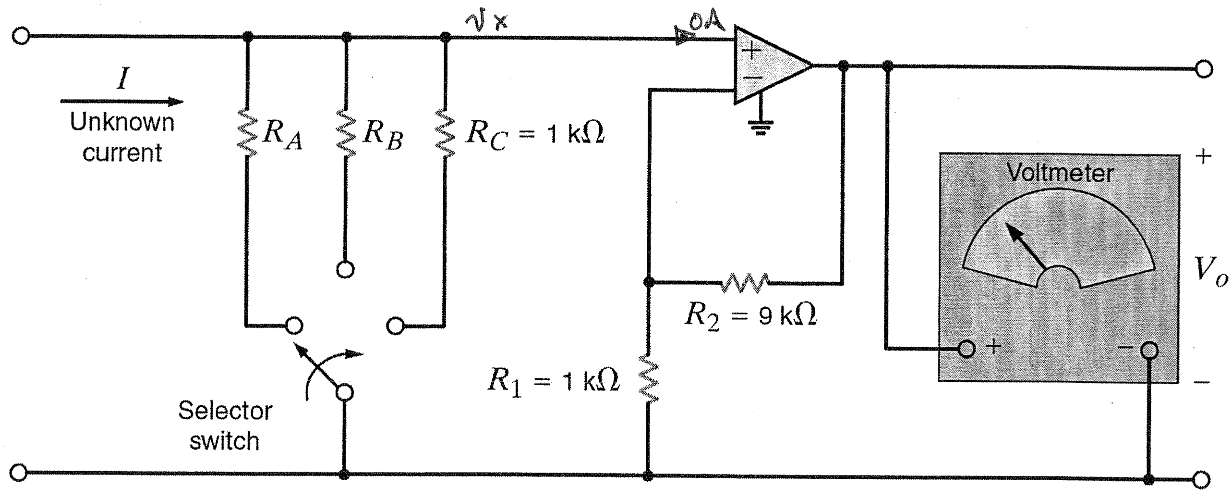


Figure P4.40

SOLUTION:

Op amp in non-inverting configuration: $V_o = V_x \left(1 + \frac{R_2}{R_1}\right) = 10V_x$

$v_x = I R_A = 0.1 R_A$

$V_o = 10V_x = 10\,(0.1 R_A) = R_A = 10$ $\boxed{R_A = 10\,\Omega}$

$v_x = I R_B = 0.01 R_B$

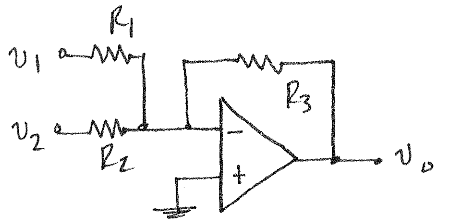
$V_o = 10V_x = 10\,(0.01 R_B) = \frac{R_B}{10} = 10$ $\boxed{R_B = 100\,\Omega}$

4.41 Given a box of 10-kΩ resistors and an op-amp, design a circuit that will have an output voltage of

$$V_o = -2V_1 - 4V_2$$ CS

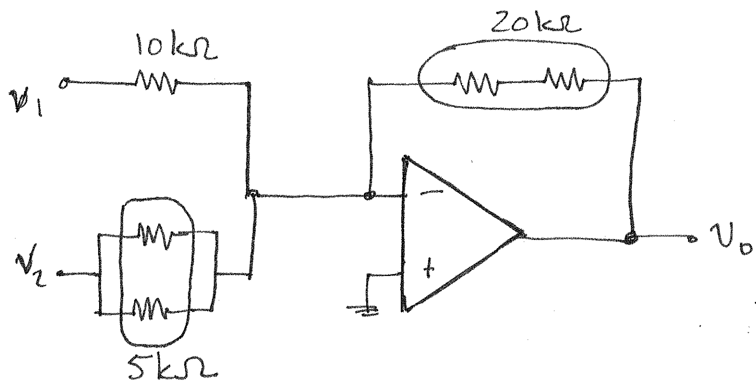
SOLUTION:

Since signs on gains associated with V_1 & V_2 are both negative, a simple summer will suffice.



$$V_o = -\frac{R_3}{R_1}V_1 - \frac{R_3}{R_2}V_2 = -2V_1 - 4V_2$$

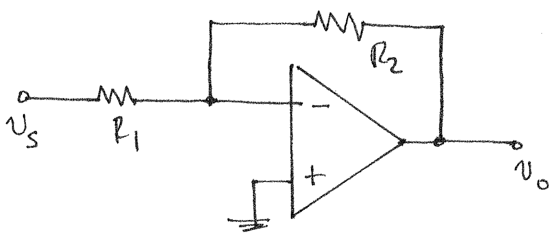
the following circuit with all resistors = 10kΩ works.



4.42 Design an op-amp circuit that has a gain of −50 using resistors no smaller than 1 kΩ.

SOLUTION:

Since gain is negative, use inverting configuration:



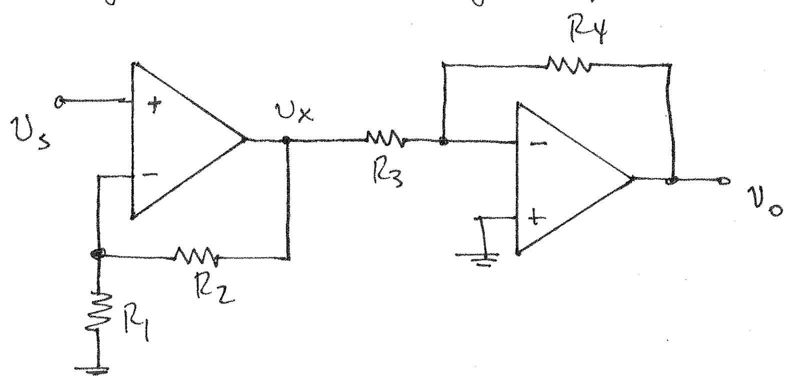
$$\frac{v_o}{v_s} = -\frac{R_2}{R_1} = -50$$

$$\boxed{\text{Choose } R_1 = 2\,k\Omega \Rightarrow R_2 = 100\,k\Omega}$$

4.43 Design a two-stage op-amp network that has a gain of −50,000 while drawing no current into its input terminal. Use no resistors smaller than 1 kΩ.

SOLUTION:

for no input current, a non-inverting configuration is needed
for negative, one inverting stage is needed.



$$\frac{v_x}{v_s} = 1 + \frac{R_2}{R_1} \qquad \frac{v_o}{v_x} = -\frac{R_4}{R_3} \qquad \frac{v_o}{v_s} = -\frac{R_4}{R_3}\left(1 + \frac{R_2}{R_1}\right)$$

Choose $\frac{v_x}{v_s} = 250$ and $\frac{v_o}{v_x} = -200$ & $\boxed{R_1 = R_3 = 2k\Omega}$

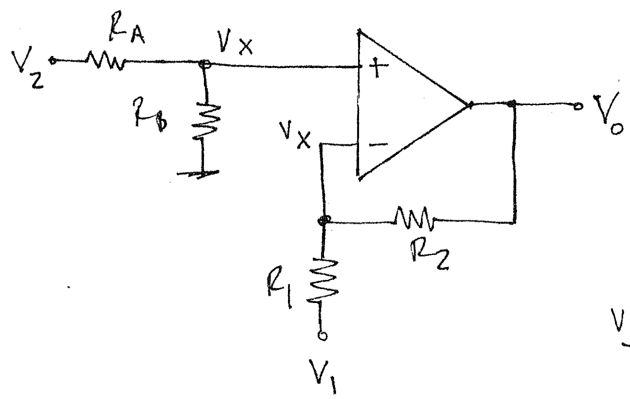
$\boxed{R_2 = 498\ k\Omega}$ $\boxed{R_4 = 400\ k\Omega}$

4.44 Design an op-amp circuit that has the following input/output relationship:

$$V_o = -5\,V_1 + 0.5\,V_2$$

SOLUTION:

A single op-amp will do if we use both + & − inputs.



KCL at V_+,

$$\frac{V_2 - V_X}{R_A} = \frac{V_X}{R_B} \Rightarrow \frac{V_X}{V_2} = \frac{R_B}{R_A + R_B}$$

KCL at V_-,

$$\frac{V_o - V_X}{R_2} = \frac{V_X - V_1}{R_1}$$

$$V_o = V_X\left(1 + \frac{R_2}{R_1}\right) - \frac{R_2}{R_1} V_1$$

$$V_o = \left(1 + \frac{R_2}{R_1}\right)\left(\frac{R_B}{R_A + R_B}\right) V_2 - \frac{R_2}{R_1} V_1$$

So, $R_2/R_1 = 5$

Now, $\dfrac{6 R_B}{R_A + R_B} = \dfrac{1}{2}$

Choose $R_1 = 1\,k\Omega \Rightarrow R_2 = 5\,k\Omega$

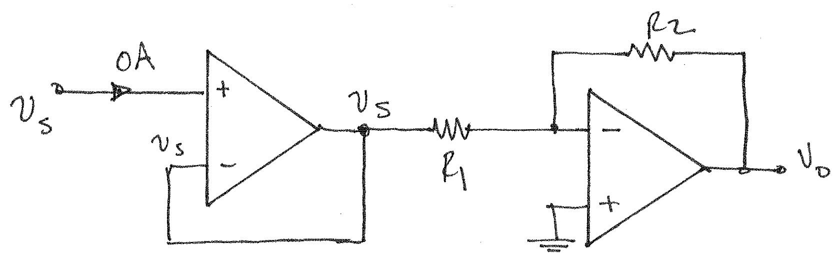
Choose $R_B = 1\,k\Omega \Rightarrow R_A = 11\,k\Omega$

4.45 A voltage waveform with a maximum value of 200 mV must be amplified to a maximum of 10 V and inverted. However, the circuit that produces the waveform can provide no more than 100 μA. Design the required amplifier. **CS**

SOLUTION:

Best to use 2 stages: non-inverting followed by inverting.

Required gain $= -\frac{10}{0.2} = -50$

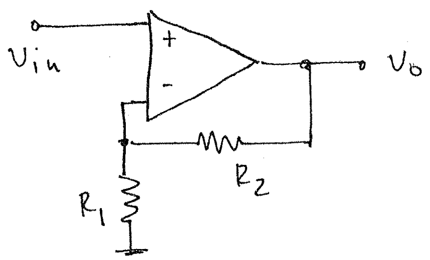


$$\frac{v_o}{v_s} = -\frac{R_2}{R_1}$$

$\boxed{\text{Choose } R_1 = 1\,k\Omega \Rightarrow R_2 = 50\,k\Omega}$

4.46 An amplifier with a gain of $\pi \pm 1\%$ is needed. Using resistor values from Table 2.1, design the amplifier. Use as few resistors as possible.

SOLUTION: For positive gain, use non-inverting config.



$$\frac{v_o}{v_{in}} = 1 + \frac{R_2}{R_1} = A$$

for $A = \pi \pm 1\%$, $2.110 \le \frac{R_2}{R_1} \le 2.173$

Best choices are

$$\boxed{R_1 = 20\,k\Omega \qquad R_2 = 43\,k\Omega}$$

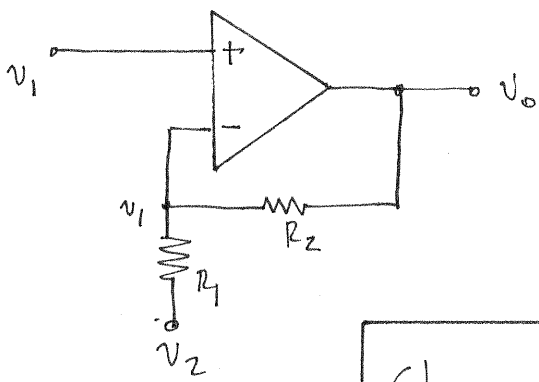
$A = 3.15$ within 0.27% of π.

4.47 Design an op-amp-based circuit to produce the function

$$V_o = 5\,V_1 - 4\,V_2$$

SOLUTION:

To get + & - gains, we can use both + & - inputs.



KCL @ V- input,

$$\frac{v_o - v_1}{R_2} = \frac{v_1 - v_2}{R_1}$$

$$v_o = v_1\left(1 + \frac{R_2}{R_1}\right) - \frac{R_2}{R_1} v_2$$

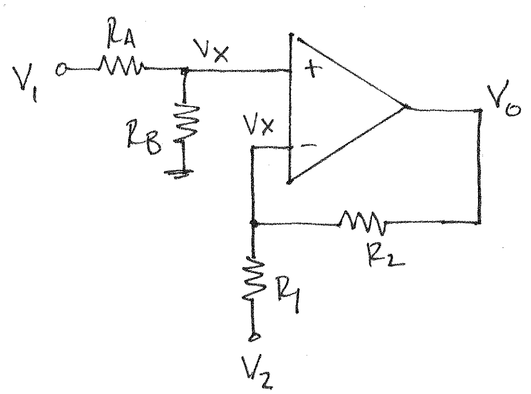
$$\boxed{\text{Choose } R_1 = 5\,k\Omega \Rightarrow R_2 = 20\,k\Omega}$$

4.48 Design an op-amp-based circuit to produce the function

$$V_o = 5\,V_1 - 7\,V_2$$

SOLUTION:

Use both + & - inputs to get + & - gains.



KCL at V_+ input,

$$\frac{V_1 - V_x}{R_A} = \frac{V_x}{R_B} \Rightarrow \frac{V_x}{V_1} = \frac{R_B}{R_A + R_B}$$

KCL at V_- input

$$\frac{V_o - V_x}{R_2} = \frac{V_x - V_2}{R_1}$$

$$V_o = V_x\left(1 + \frac{R_2}{R_1}\right) - \frac{R_2}{R_1} V_2$$

$$V_o = V_1\left(\frac{R_B}{R_A + R_B}\right)\left(1 + \frac{R_2}{R_1}\right) - \frac{R_2}{R_1} V_2$$

$$R_2/R_1 = 7$$

$$\left(\frac{R_B}{R_A + R_B}\right) 8 = 5$$

$$\boxed{\begin{array}{l} \text{Choose } R_1 = 1k\Omega \Rightarrow R_2 = 7k\Omega \\ \text{Choose } R_B = 5k\Omega \Rightarrow R_A = 3k\Omega \end{array}}$$

4.49 Show that the circuit in Fig. P4.49 can produce the output

$$V_o = K_1 V_1 - K_2 V_2$$

only for $0 \leq K_1 \leq K_2 + 1$. **CS**

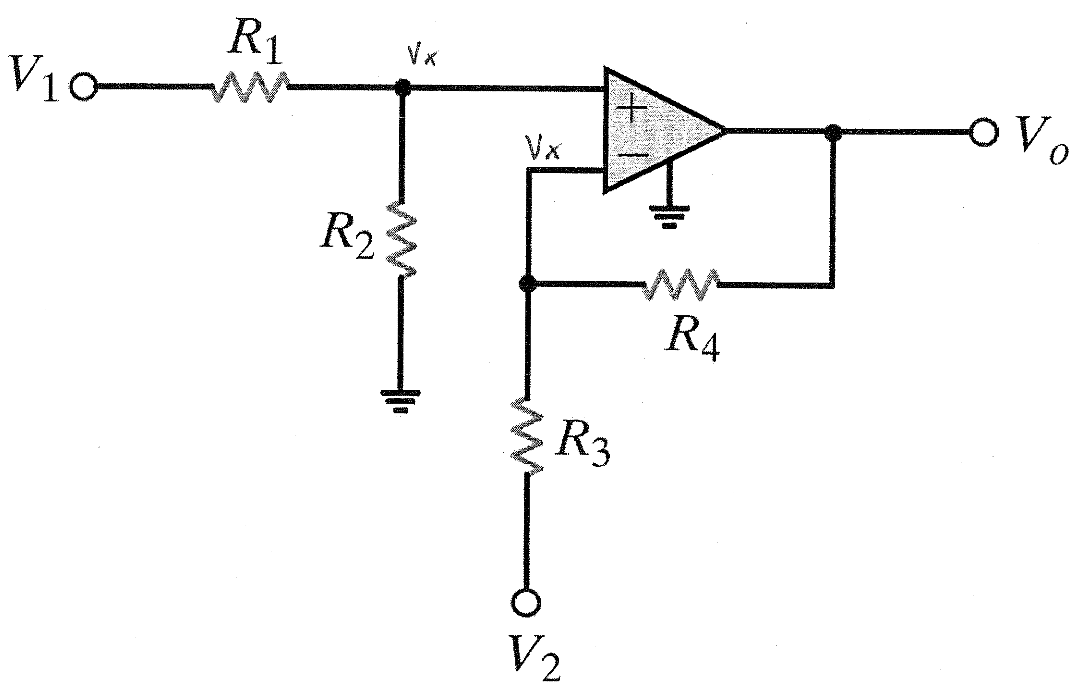


Figure P4.49

SOLUTION:

KCL at v_+ input: $\frac{V_1 - V_x}{R_1} = \frac{V_x}{R_2} \Rightarrow V_x = \frac{V_1 R_2}{R_1 + R_2}$

KCL at v_- input: $\frac{V_o - V_x}{R_4} = \frac{V_x - V_2}{R_3} \Rightarrow V_o = V_x\left(1 + \frac{R_4}{R_3}\right) - \frac{R_4}{R_3} V_2$

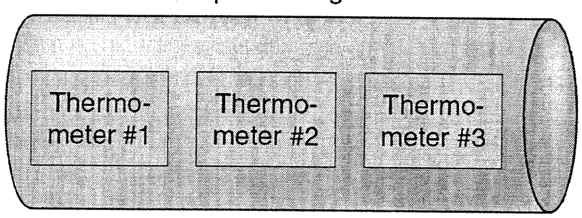
$V_o = V_1\left(\frac{R_2}{R_1 + R_2}\right)\left(1 + \frac{R_4}{R_3}\right) - \frac{R_4}{R_3} V_2 = K_1 V_1 - K_2 V_2 \Rightarrow K_2 = R_4/R_3$

If $R_2 = 0$, $K_1 = 0$

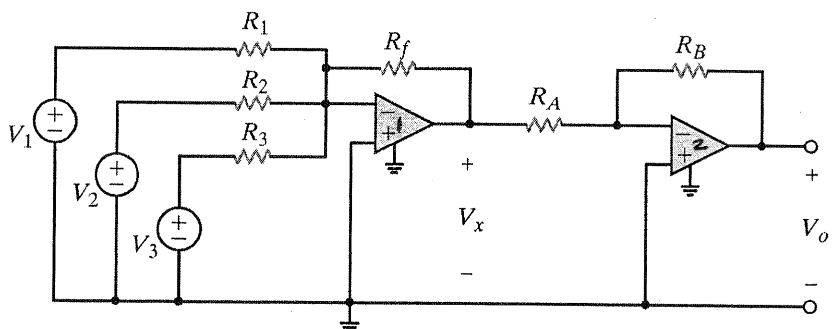
If $R_2 \neq 0$ but $R_1 = 0$, $K_1 = 1 + R_4/R_3 = 1 + K_2$

$\boxed{0 \leq K_1 \leq (K_2 + 1)}$

4.50 A 170°C maximum temperature digester is used in a paper mill to process wood chips that will eventually become paper. As shown in Fig. P4.50a, three electronic thermometers are placed along its length. Each thermometer outputs 0 V at 0°C, and the voltage changes 25 mV/°C. We will use the average of the three thermometer voltages to find an aggregate digester temperature. Furthermore, 1 volt should appear at V_o for every 10°C of average temperature. Design such an averaging circuit using the op-amp configuration shown in Fig. 4.50b if the final output voltage must be positive.



(a)



(b)

SOLUTION: Op amp 1 is a summer. Op amp 2 is a inverting configuration.

$$V_x = -\left[\frac{R_f}{R_1}V_1 + \frac{R_f}{R_2}V_2 + \frac{R_f}{R_3}V_3\right] \quad \& \quad V_o = -\frac{R_B}{R_A}V_x$$

$$V_o = \frac{R_B}{R_A}\left[\frac{R_fV_1}{R_1} + \frac{R_fV_2}{R_2} + \frac{R_fV_3}{R_3}\right] \qquad \text{Choose } \frac{R_f}{R_1} = \frac{R_f}{R_2} = \frac{R_f}{R_3} = \frac{1}{3}$$

$$V_o = \frac{R_B}{R_A}\left(\frac{V_1+V_2+V_3}{3}\right) \qquad \text{If } \Delta T_1 = 10°C,\ \Delta V_1 = 0.25V \ \& \ \Delta V_o = 1V$$

$$\frac{\Delta V_o}{\Delta V_1} = \frac{R_B}{R_A}\left(\frac{1}{3}\right) = \frac{1}{0.25} = 4$$

$$\boxed{\begin{array}{l}\text{Choose } R_A = R_f = 1k\Omega \Rightarrow R_B = 12k\Omega \\ R_1 = R_2 = R_3 = 3k\Omega\end{array}}$$

4.51 A 0.1-Ω shunt resistor is used to measure current in a fuel-cell circuit. The voltage drop across the shunt resistor is to be used to measure the current in the circuit. The maximum current is 20 A. Design the network shown in Fig. P4.51 so that a voltmeter attached to the output will read 0 volts when the current is 0 A and 20 V when the current is 20 A. Be careful not to load the shunt resistor, since loading will cause an inaccurate reading.

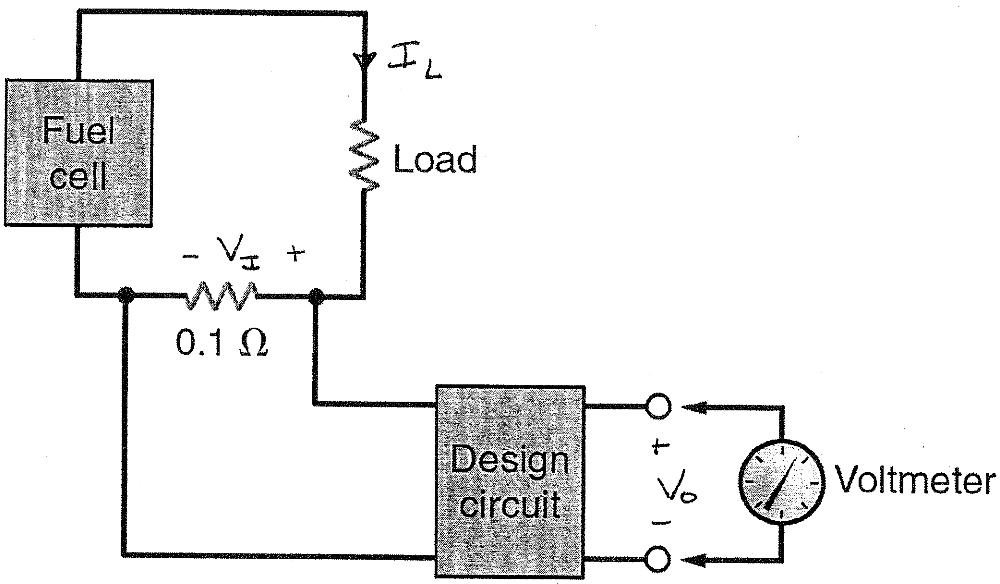
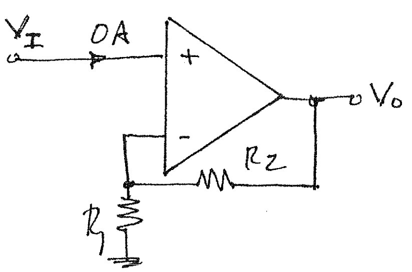


Figure P4.51

SOLUTION:

When $I_L = 20A$, $V_I = (0.1) I_L = 2V$ and $V_o = 20V$.

Need gain of 10 with a buffered input ⇒ non-inverting conf!



$$\frac{V_o}{V_I} = 1 + \frac{R_2}{R_1} = 10$$

$$\boxed{\text{Chose } R_1 = 1k\Omega \text{ \& } R_2 = 9k\Omega}$$

4.52 Wood pulp is used to make paper in a paper mill. The amount of lignin present in pulp is called the kappa number. A very sophisticated instrument is used to measure kappa, and the output of this instrument ranges from 1 to 5 volts, where 1 volt represents a kappa number of 12 and 5 volts represents a kappa number of 20. The pulp mill operator has asked to have a kappa meter installed on his console. Design a circuit that will employ as input the 1- to 5-volt signal and output the kappa number. An electronics engineer in the plant has suggested the circuit shown in Fig. P4.52.

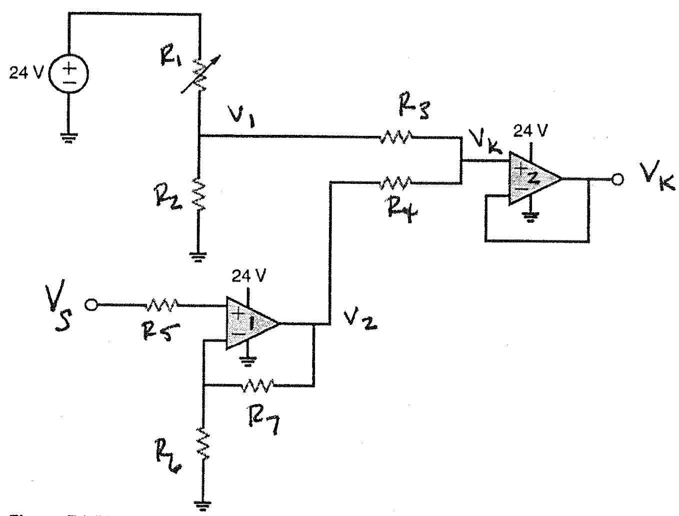


Figure P4.52

SOLUTION:

KCL at V_1: $\dfrac{24-V_1}{R_1} = \dfrac{V_1}{R_2} + \dfrac{V_1 - V_K}{R_3}$

KCL at V_K: $\dfrac{V_1 - V_K}{R_3} = \dfrac{V_K - V_2}{R_4}$

Op amp 1 is in non-inverting configuration : $V_2 = V_s\left(1+\dfrac{R_7}{R_6}\right) = g\,V_s$

Let $R_2 = R_3 = R_4 = R_5 = R$ and $R_1 = 1k\Omega$. Eliminate V_1 in KCL eqs.

@ V_K: $V_1 = 2V_K - V_2$

@ V_1: $24R = V_1(2R_1 + R) - V_K R_1$

Yields $V_K = \frac{24R}{3R_1 + 2R} + V_S \frac{g(2R_1 + R)}{3R_1 + 2R}$

Also,

$$V_K = b + mV_S$$

and at $V_S = 1V$, $V_K = 12V$ and at $V_S = 5V$, $V_K = 20V$.

So, $m = \frac{\Delta V_K}{\Delta V_S} = \frac{20-12}{5-1} = 2$ and $b = 10$

$$V_K = 10 + 2V_S = \frac{24R}{3R_1 + 2R} + g\frac{(2R_1 + R)}{3R_1 + 2R} V_S$$

Yields,

$$R = 7.5k\Omega \text{ \& } g = 3.79$$

Let $R_6 = 1k\Omega \Rightarrow R_7 = 2.79k\Omega$.

Results:	$R_1 = 1k\Omega$	$R_7 = 2.79k\Omega$	$R_6 = 1k\Omega$
	Chose: $R_2 = R_3 = R_4 = R_5 =$		$7.5k\Omega$

4.53 An operator in a chemical plant would like to have a set of indicator lights that indicate when a certain chemical flow is between certain specific values. The operator wants a RED light to indicate a flow of at least 10 GPM (gallons per minute), RED and YELLOW lights to indicate a flow of 60 GPM, and RED, YELLOW, and GREEN lights to indicate a flow rate of 80 GPM. The 4–20 mA flow meter instrument outputs 4 mA when the flow is zero and 20 mA when the flow rate is 100 GPM.

An experienced engineer has suggested the circuit shown in Fig. P4.53. The 4–20 mA flow meter and 250 Ω resistor provide a 1–5 V signal, which serves as one input for the three comparators. The light bulbs will turn on when the negative input to a comparator is higher than the positive input. Using this network, design a circuit that will satisfy the operator's requirements. **CS**

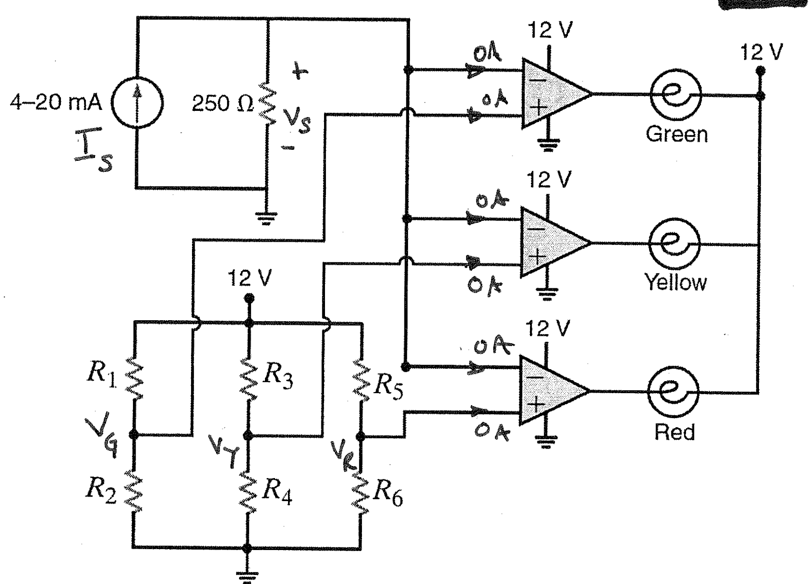


Figure P4.53

SOLUTION:

Desire $V_G = V_s$ when flow = 80 GPM

$I_s = m(\text{flow}) + b$ $\quad$ $4\text{mA} = m(0) + b \Rightarrow b = 4\text{mA}$

$20\text{mA} = m(100) + b \Rightarrow m = 0.16\ \text{mA/GPM}$

At flow = 80 GPM, $I_s = 16.8\text{mA}$ & $V_s = 250 I_s = 4.2\text{V}$

$V_G = \left(\frac{R_2}{R_1 + R_2}\right) 12 \Rightarrow$ Choose $R_2 = 1\text{k}\Omega \Rightarrow R_1 = 1.86\text{k}\Omega$

Similarly, at flow = 60 GPm, $I_S = 13.6mA$ and $V_S = 3.4V$

Need $V_Y = 3.4V = \frac{12 R_4}{R_4 + R_3}$ Choose $R_4 = 1k\Omega \Rightarrow R_3 = 2.53k\Omega$

Finally, at flow = 10 GPM, $I_S = 5.6mA$ and $V_S = 1.4V$

For $V_R = 1.4V = \frac{12 R_6}{R_6 + R_5}$ choose $R_6 = 1k\Omega \Rightarrow R_5 = 7.57k\Omega$

$$\boxed{\begin{array}{ll} \text{Choosing} & R_2 = R_4 = R_6 = 1k\Omega \\ \text{yields} & R_1 = 1.86k\Omega \\ & R_3 = 2.53k\Omega \\ & R_5 = 7.57k\Omega \end{array}}$$

4.54 An industrial plant has a requirement for a circuit that uses as input the temperature of a vessel and outputs a voltage proportional to the vessel's temperature. The vessel's temperature ranges from 0°C to 500°C, and the corresponding output of the circuit should range from 0 to 12 V. A RTD (resistive thermal device), which is a linear device whose resistance changes with temperature according to the plot in Fig. P4.54a, is available. The problem then is to use this RTD to design a circuit that employs this device as an input and produces a 0- to 12-V signal at the output, where 0 V corresponds to 0°C and 12 V corresponds to 500°C. An engineer familiar with this problem suggests the use of the circuit shown in Fig. P4.54b in which the RTD bridge circuit provides the input to a standard instrumentation amplifier. Determine the component values in this network needed to satisfy the design requirements.

SOLUTION: Need relationships for v_1, v_2, v_o and T.

KCL at RTD: $\dfrac{12-v_1}{R_1} = \dfrac{v_1}{R_{RTD}} \Rightarrow 12R_{RTD} = v_1(R_1 + R_{RTD})$

$v_1 = 12R_{RTD}/(R_1 + R_{RTD})$

KCL at v_{2+}: $v_2 = 12R_3/(R_2+R_3)$

$R_{RTD}(T)$: $R_{RTD} = K_1 + K_2 T$

At $T = 0°C$, $R_{RTD} = K_1 = 100\ \Omega$

At $T = 500°C$, $R_{RTD} = 600\Omega = 100 + K_2(500)$

$K_2 = 1\ \Omega/°C \Rightarrow R_{RTD} = 100 + T$

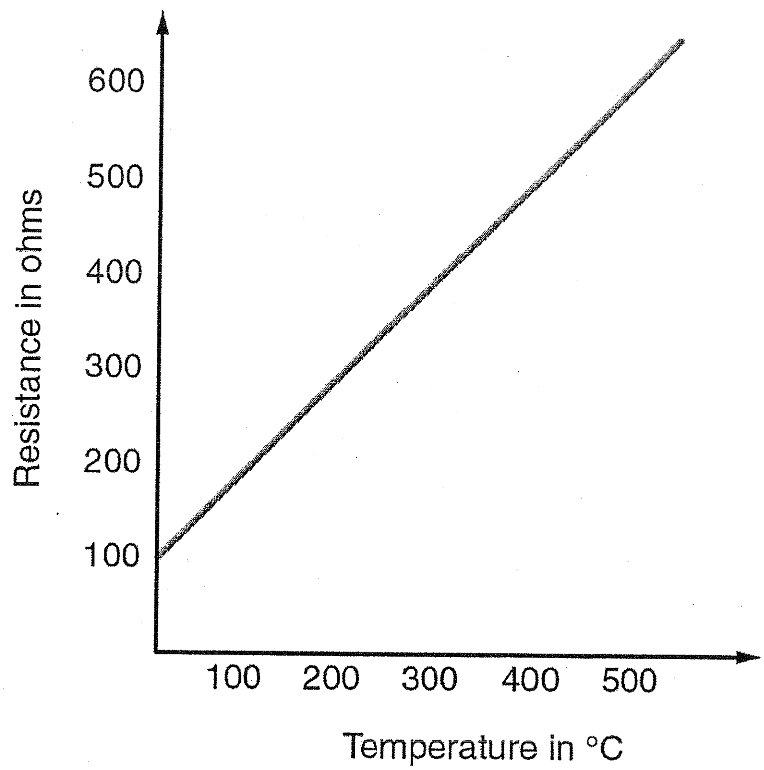


Figure P4.54a

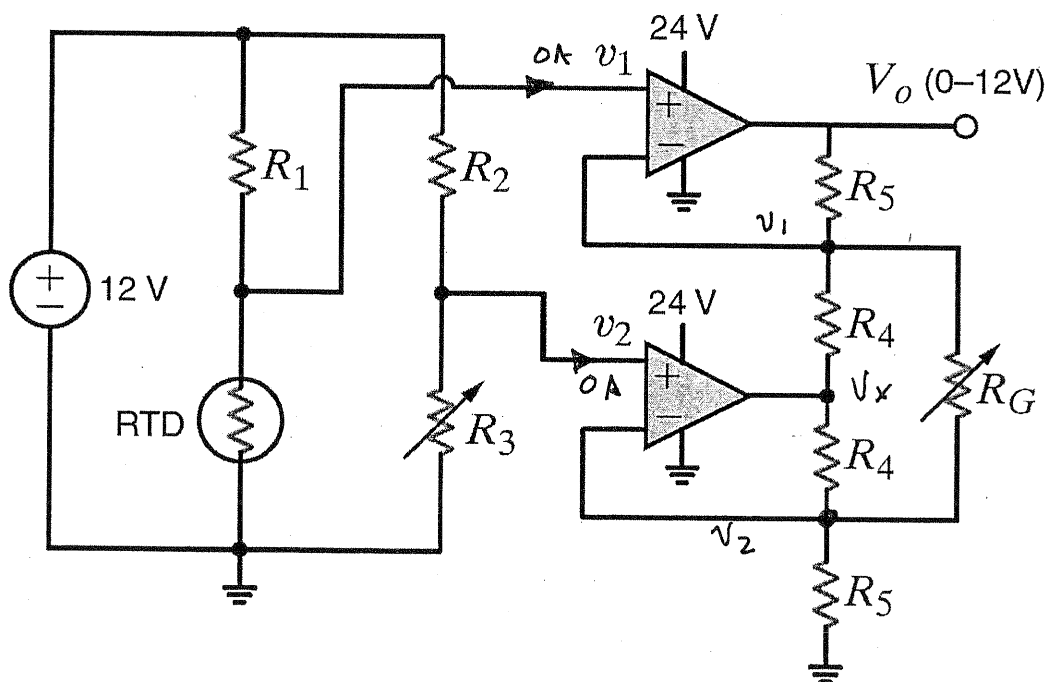


Figure P4.54b

Let $R_1 = R_2 = R_3 = 100\Omega$ (An arbitrary but good choice.

Now,

$$12 - v_1 = \frac{v_1}{1 + T/100} \quad \& \quad v_2 = 6V \quad \Leftarrow \text{eq. 1}$$

KCL at v_1 at R_4-R_5 node:

$$\frac{v_0 - v_1}{R_5} = \frac{v_1 - v_2}{R_G} + \frac{v_1 - v_x}{R_4}$$

KCL at v_2 at R_5-R_4 node:

$$\frac{v_x - v_2}{R_4} + \frac{v_1 - v_2}{R_G} = \frac{v_2}{R_5}$$

Let $R_4 = R_5 = 10k\Omega$ (Again, arbitrary)

$$v_0 = \left(2 + \frac{R_4}{R_G}\right)v_1 - v_2\frac{R_4}{R_G} - v_x \quad \& \quad v_x = v_2\left(2 + \frac{R_4}{R_G}\right) - v_1\frac{R_4}{R_G}$$

Eliminate $v_x \Rightarrow \quad v_0 = 2(v_1 - v_2)\left(1 + \frac{10^4}{R_G}\right)$

But, $v_1 = 12\left(\frac{100+T}{200+T}\right)$ (solve eq 1. for v_1)

Now,

$$v_0 = 24\left(\frac{100+T}{200+T} - \frac{1}{2}\right)\left(1 + \frac{10^4}{R_G}\right)$$

At $T = 0°C$, $v_0 = 24\left(\frac{1}{2} - \frac{1}{2}\right)\left(1 + \frac{10^4}{R_G}\right) = 0V$ good!

At $T = 500°C$, $v_0 = 24\left(\frac{600}{700} - \frac{1}{2}\right)\left(1 + \frac{10^4}{R_G}\right) = 12V$

$R_G = 25\ k\Omega$

$R_1 = R_2 = R_3 = 100\Omega \qquad R_4 = R_5 = 10\ k\Omega$

4FE-1 Given the summing amplifier shown in Fig. 4PFE-1 select the values of R_2 that will produce an output voltage of −3 V. CS

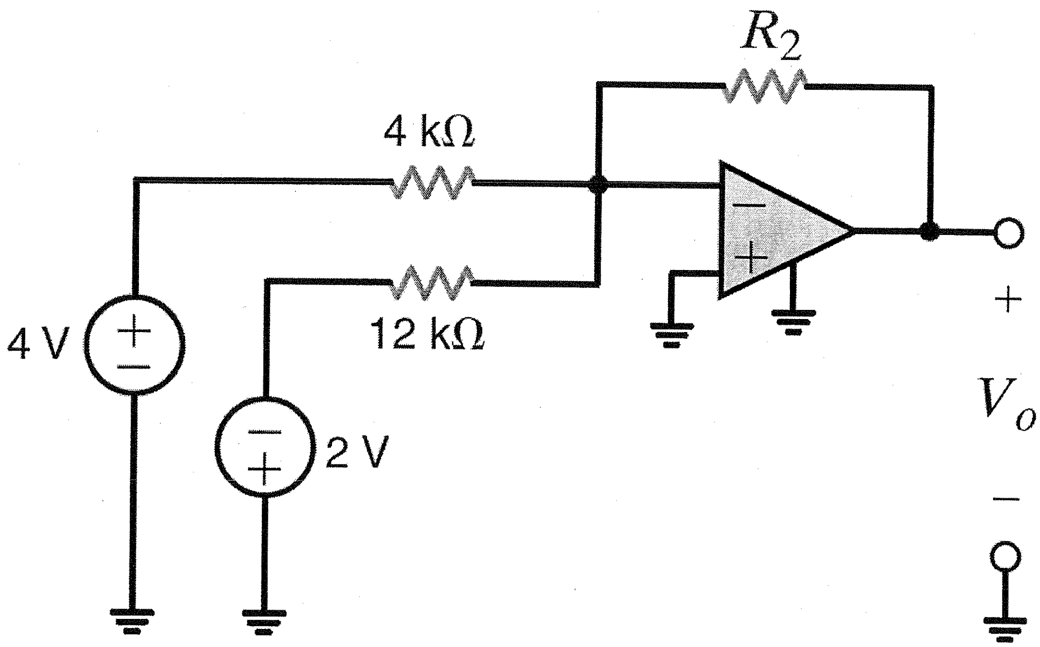


Figure 4PFE-1

SOLUTION: For summing amp:

$$v_o = -\left(\frac{R_2}{4000}\right)4 - \left(\frac{R_2}{12000}\right)(2) = -3$$

$$\boxed{R_2 = 2.57 k\Omega}$$

4FE-2 Determine the output voltage V_o of the summing op-amp circuit shown in Fig. 4PFE-2.

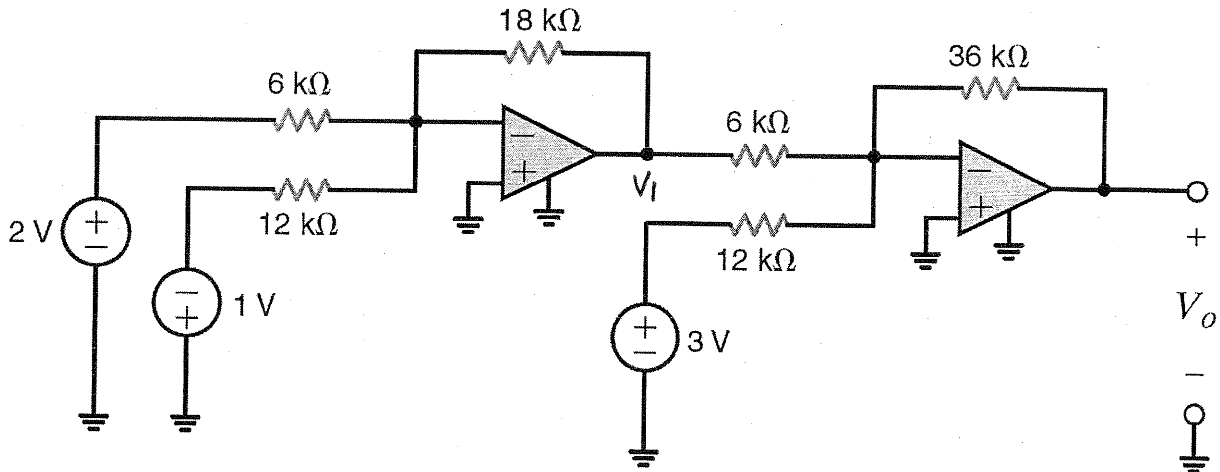


Figure 4PFE-2

SOLUTION:

$$V_1 = -2\left(\frac{18\times10^3}{6\times10^3}\right) + 1\left(\frac{18\times10^3}{12\times10^3}\right) = -4.5\text{V}$$

$$V_o = -V_1\left(\frac{36\times10^3}{6\times10^3}\right) - 3\left(\frac{36\times10^3}{12\times10^3}\right) \Rightarrow \boxed{V_o = 18\text{V}}$$

Chapter Five:

Additional Analysis Techniques

5.1 Find I_o in the circuit in Fig. P5.1 using linearity and the assumption that $I_o = 1$ mA. CS

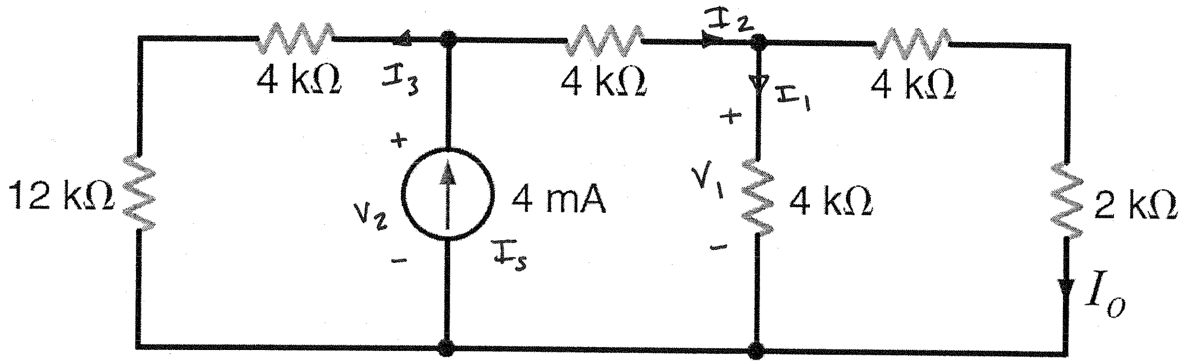


Figure P5.1

SOLUTION: If $I_o = 1\text{mA}$

$$V_1 = I_o(4000 + 2000) = 6\text{V}$$

$$I_2 = I_1 + I_o = \frac{V_1}{4000} + I_o = 2.5\text{mA}$$

$$V_2 = 4000\, I_2 + V_1 = 16\text{V}$$

$$I_s = \frac{V_2}{16\times10^3} + I_2 = 3.5\text{mA}$$

But I_s is actually 4mA. So

$$I_o = 10^{-3}\left(\frac{4\times10^{-3}}{3.5\times10^{-3}}\right) = 1.14\text{mA} \qquad \boxed{I_o = 1.14\text{mA}}$$

5.2 Find V_o in the network in Fig. P5.2 using linearity and the assumption that $V_o = 1$ V.

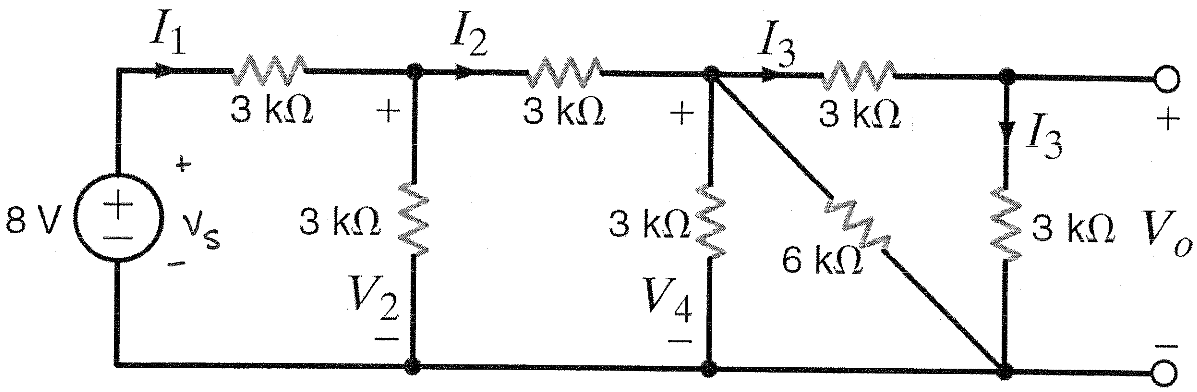


Figure P5.2

SOLUTION:

If $V_o = 1V$, $\quad I_3 = \frac{V_o}{3000} = \frac{1}{3}mA$

$V_4 = I_3(6000) = 2V \quad I_2 = \frac{V_4}{3000} + \frac{V_4}{6000} + I_3 = \frac{4}{3}mA$

$V_2 = 3000 I_2 + V_4 = 6V$

$I_1 = \frac{V_2}{3000} + I_2 = \frac{10}{3}mA \quad V_S = 3000 I_1 + V_2 = 16V$

But, V_S actually $= 8V$. So

$V_o = (1)\left(\frac{8}{16}\right) = 0.5V \quad \boxed{V_o = 0.5V}$

5.3 Find I_o in the network in Fig. P5.3 using linearity and the assumption that $I_o = 1$ mA. **PSV**

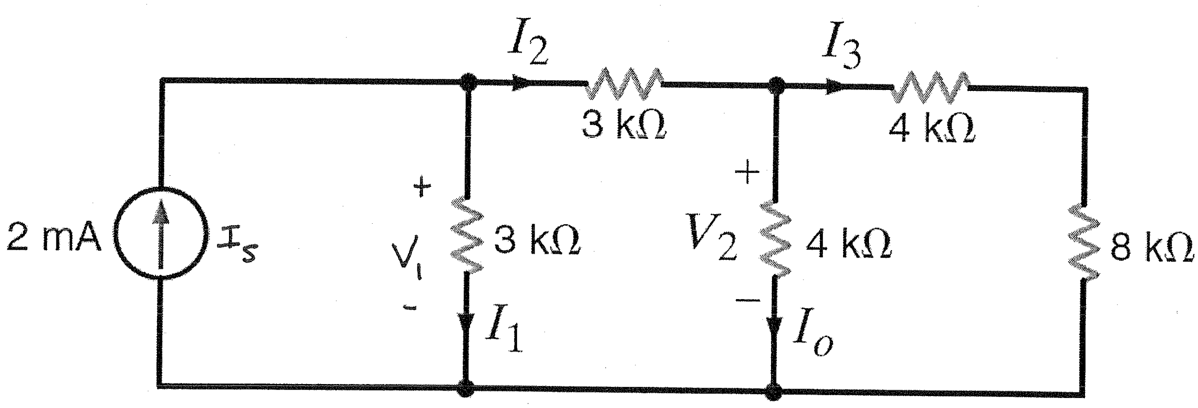


Figure P5.3

SOLUTION: If $I_o = 1mA$,

$V_2 = 4000\, I_o = 4V \qquad I_2 = I_o + \frac{V_2}{12\times10^3} = \frac{4}{3} mA$

$V_1 = 3000\, I_2 + V_2 = 8V$

$I_S = \frac{V_1}{3000} + I_2 = 4mA$

But, I_s actually equals 2mA. So

$I_o = 10^{-3}\left(\frac{2\times10^{-3}}{4\times10^{-3}}\right) = 0.5mA \qquad \boxed{I_o = 0.5mA}$

5.4 Find I_o in the network in Fig. P5.4 using linearity and the assumption that $I_o = 1$ mA.

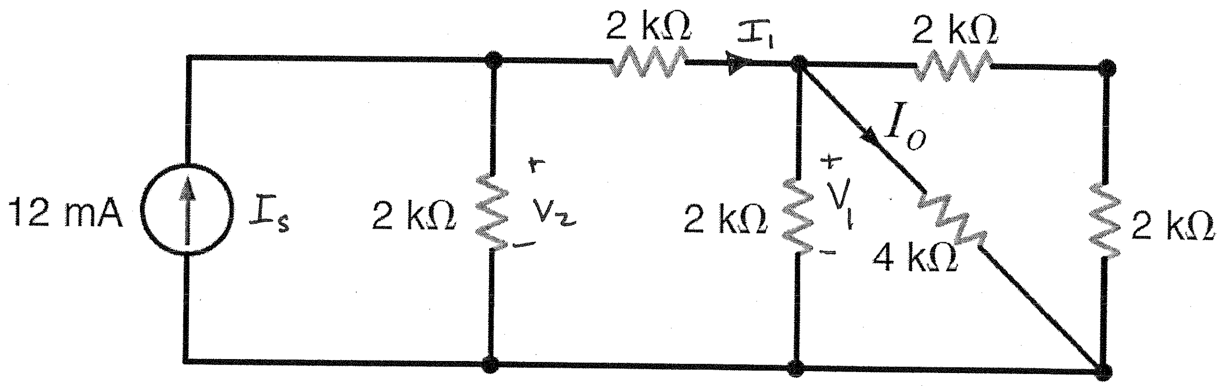


Figure P5.4

SOLUTION: If $I_o = 1\text{mA}$, $V_1 = 4000\, I_o = 4V$

$$I_1 = \frac{V_1}{2000} + \frac{V_1}{4000} + I_o = 4\text{mA}$$

$$V_2 = 2000\, I_1 + V_1 = 12V \qquad I_S = \frac{V_2}{2000} + I_1 = 10\text{mA}$$

But, I_s actually equals 12mA. So, I_o is

$$I_o = 10^{-3}\left(\frac{12\times10^{-3}}{10\times10^{-3}}\right) \qquad \boxed{I_o = 1.2\text{mA}}$$

5.5 In the network in Fig. P5.5, find I_o using superposition. CS

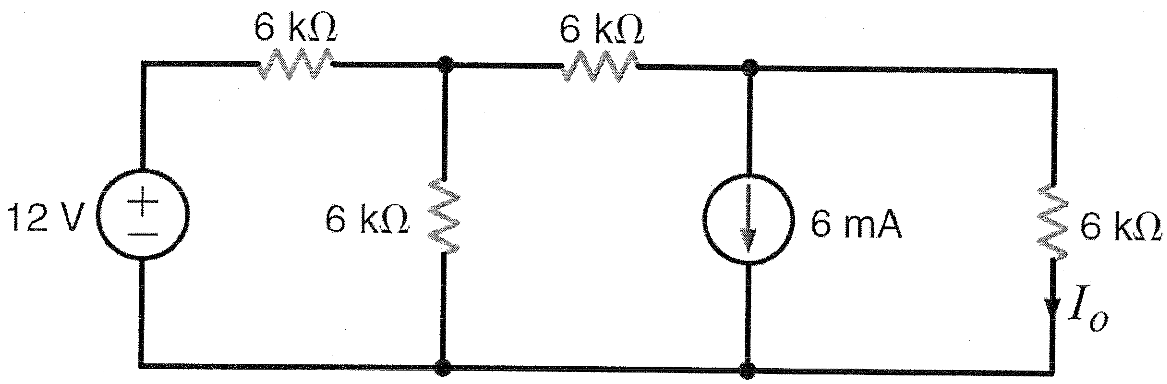
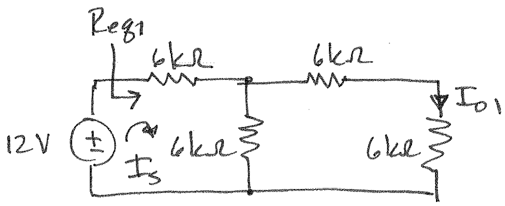
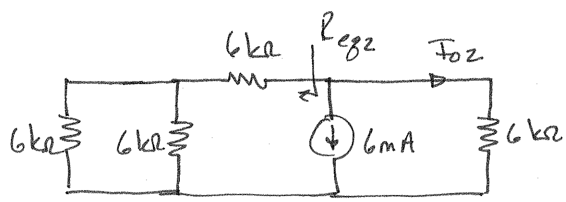


Figure P5.5

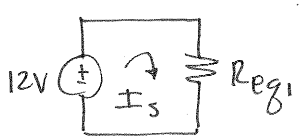
SOLUTION:



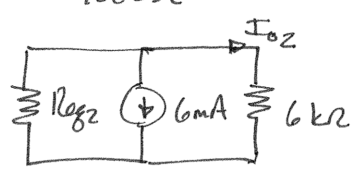


$$R_{eq1} = 6000 + [6000 // 12{,}000]$$
$$= 10k\Omega$$

$$R_{eq2} = (6000 // 6000) + 6000$$
$$= 9000\Omega$$



$$I_s = \frac{12}{R_{eq1}} = 1.2mA$$



$$I_{o1} = I_s\left[\frac{6000}{18{,}000}\right] = 0.4mA$$

$$I_{o2} = -\frac{6\times10^{-3}(R_{eq2})}{R_{eq2}+6000}$$

$$I_{o2} = -3.6mA$$

$$\boxed{I_o = I_{o1} + I_{o2} = -3.2mA}$$

5.6 Find V_o in the network in Fig. P5.6 using superposition.

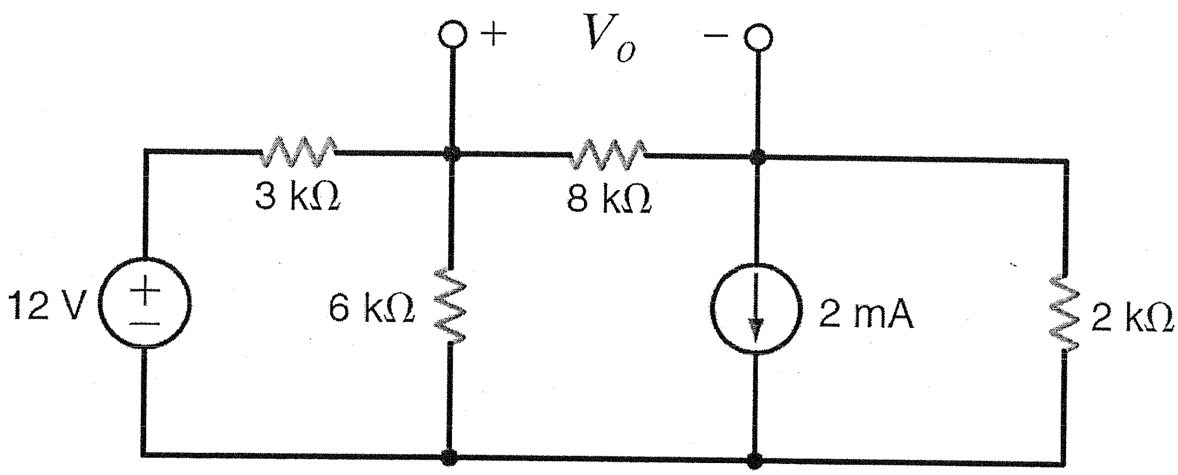
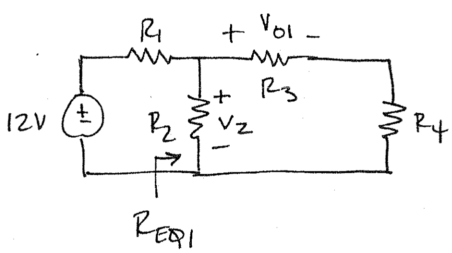
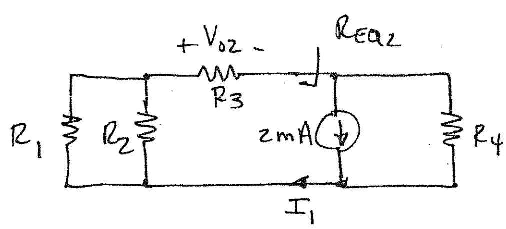


Figure P5.6

SOLUTION:



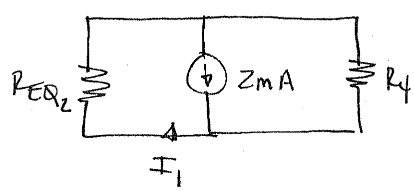


$$R_{EQ1} = R_2 // (R_3 + R_4) = 3.75\,k\Omega$$

$$V_2 = 12\left(R_{EQ1} / (R_{EQ1} + R_1)\right) = 6.67V$$

$$V_{01} = V_2\left(\frac{R_3}{R_3 + R_4}\right) = 5.33V$$

$$R_{EQ2} = (R_1 // R_2) + R_3 = 10\,k\Omega$$



$$I_1 = 2\times10^{-3}\left(\frac{R_4}{R_4 + R_{EQ2}}\right) = \frac{1}{3}\,mA$$

$$V_{02} = I_1 R_3 = \frac{8}{3}V$$

$$V_o = V_{01} + V_{02}$$

$$\boxed{V_o = 8.0\ V}$$

5.7 Find I_o in the network in Fig. P5.7 using superposition. CS

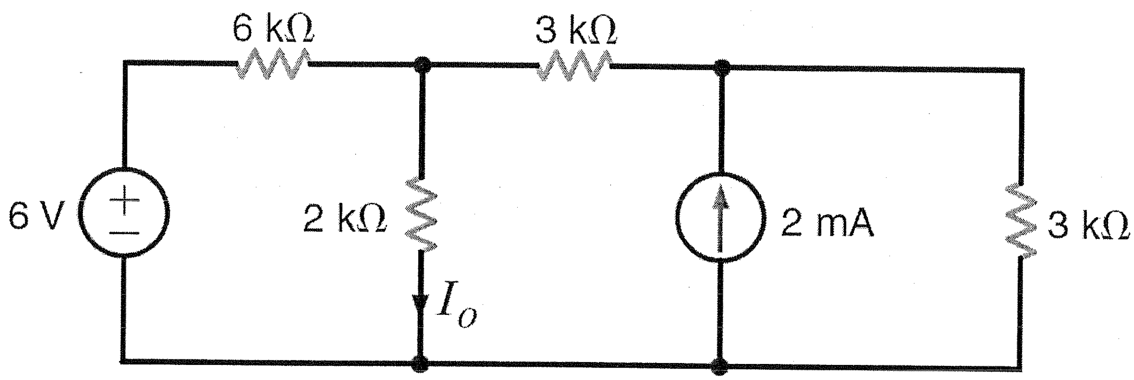
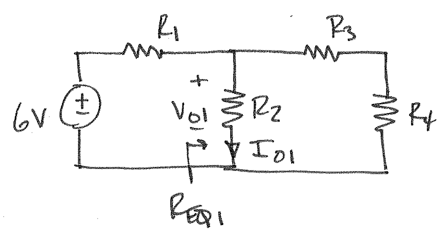
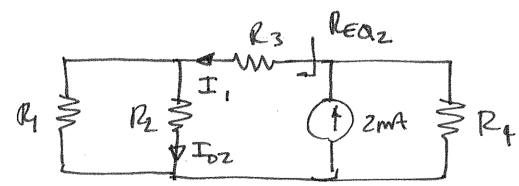


Figure P5.7

SOLUTION:





$$R_{EQ2} = (R_1 // R_2) + R_3 = 4.5k\Omega$$

$$I_1 = 2\times10^{-3}\left(\frac{R_4}{R_4 + R_{EQ2}}\right) = 0.8mA$$

$$I_{o2} = I_1\left(\frac{R_1}{R_1 + R_2}\right) = 0.6mA$$

$$R_{EQ1} = R_2 // (R_3 + R_4)$$

$$= 1.5k\Omega$$

$$V_{o1} = 6\left(\frac{R_{EQ1}}{R_{EQ1} + R_1}\right) = 1.2V$$

$$I_{o1} = V_{o1}/R_2 = 0.6mA$$

$$I_o = I_{o1} + I_{o2}$$

$$\boxed{I_o = 1.2mA}$$

5.8 Use superposition to find V_o in the circuit in Fig. P5.8. **PSV**

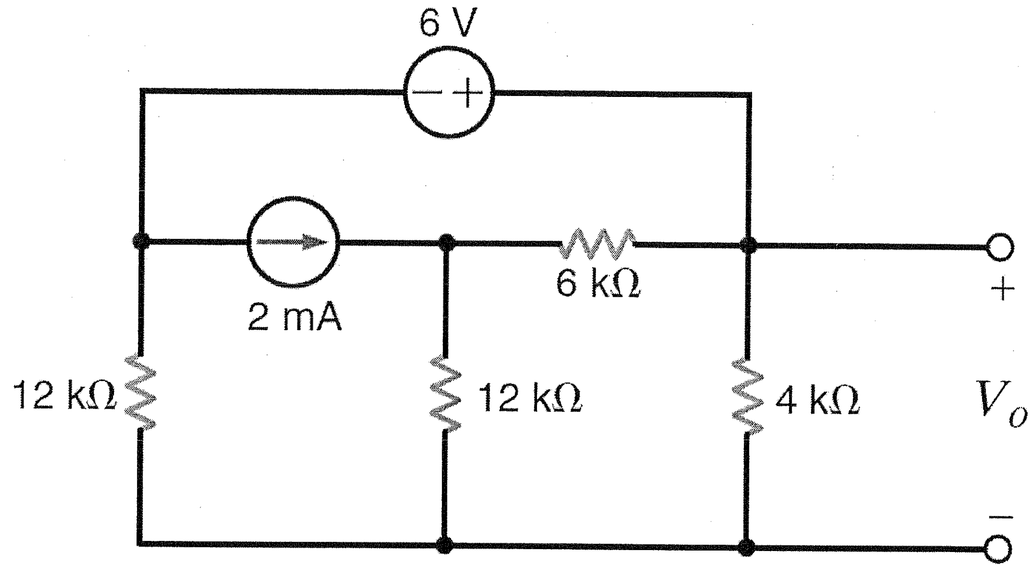
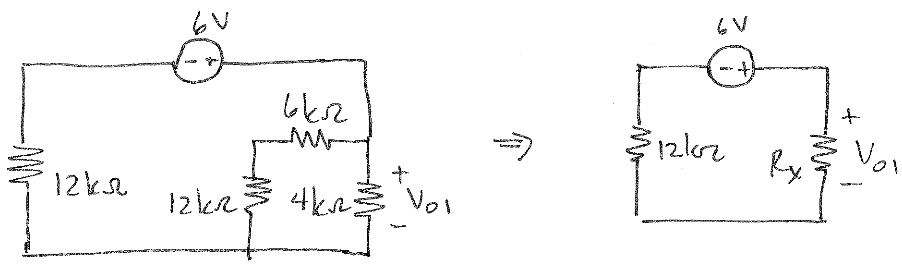


Figure P5.8

SOLUTION:

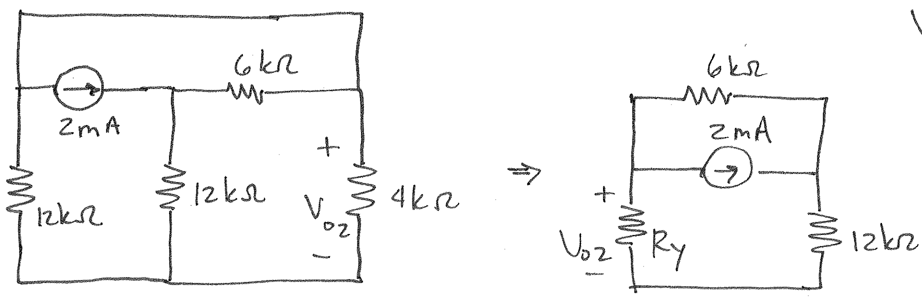


$R_x = 18{,}000 // 4000$

$R_x = 3.27k\Omega$

$V_{01} = 6\left(\frac{R_x}{R_x + 12000}\right)$

$V_{01} = 1.29V$



$R_Y = 4000 // 12000$

$R_Y = 3000\Omega$

$$V_{02} = -R_Y\left\{2\times10^{-3}\left[\frac{6000}{18000 + R_Y}\right]\right\}$$

$V_{02} = -1.71V$

$V_o = V_{01} + V_{02} = -0.42V$

$\boxed{V_o = -0.42V}$

5.9 Find I_o in the circuit in Fig. P5.9 using superposition.
CS

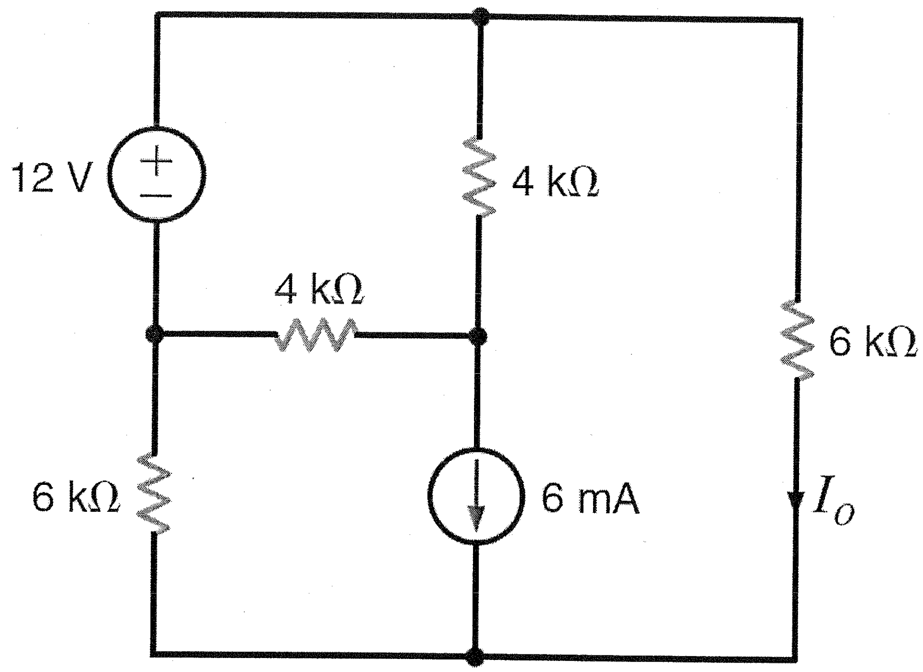
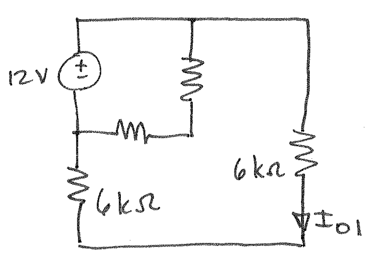
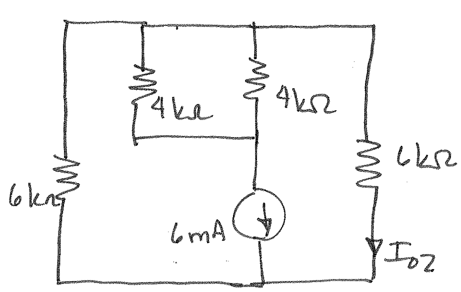


Figure P5.9

SOLUTION:



$12 = I_{o1}\,(6k + 6k) \qquad I_{o1} = 1mA$



By current division, $I_{o2} = -6\times10^{-3}\left(\dfrac{6000}{6000+6000}\right)$

$I_{o2} = -3mA$

$I_{o1} + I_{o2} = I_o \Rightarrow \boxed{I_o = -2mA}$

5.10 Use superposition to find I_o in the circuit in Fig. P5.10.

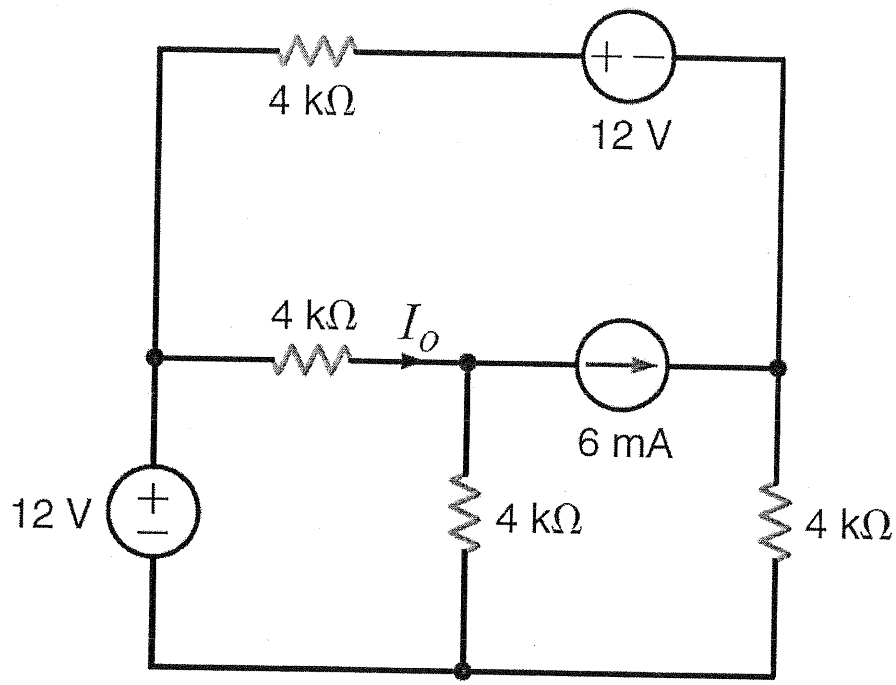
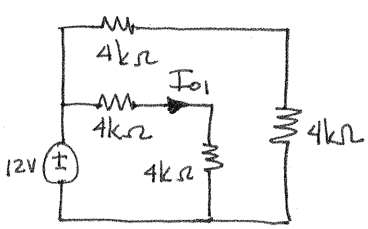
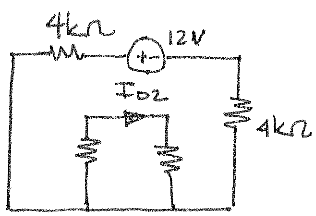


Figure P5.10

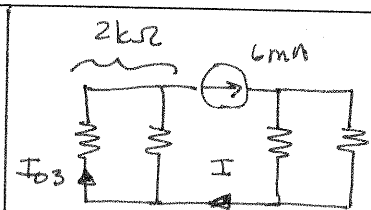
SOLUTION:



$I_{o1} = \frac{12}{8000} = 1.5mA$



$I_{o2} = 0$



All R = 4kΩ

I = 6mA

$I_{o3} = I\left(\frac{R}{R+R}\right) = 3mA$

$I_o = I_{o1} + I_{o2} + I_{o3} \Rightarrow$ $\boxed{I_o = 4.5mA}$

5.11 Find I_o in the network in Fig. P5.11 using superposition.
CS

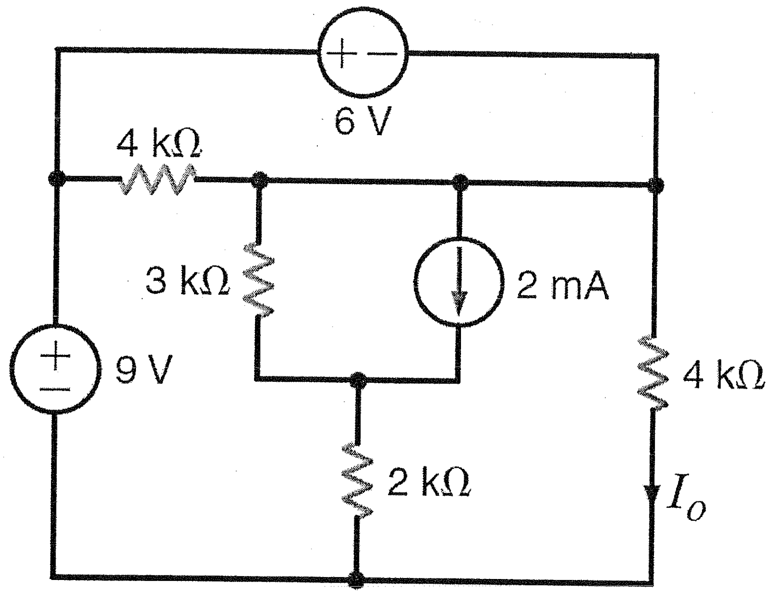
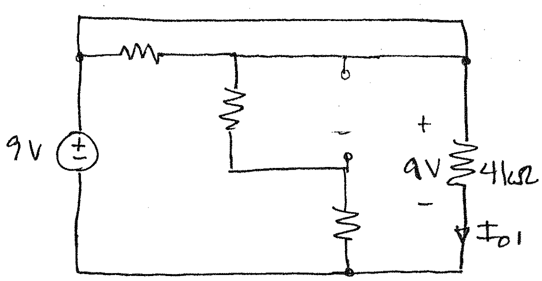
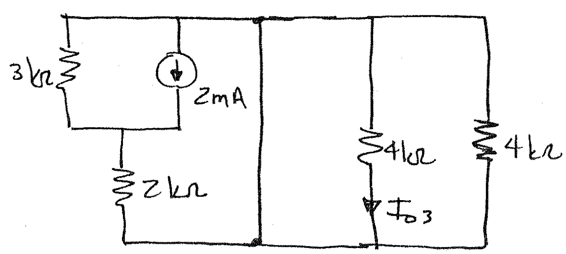


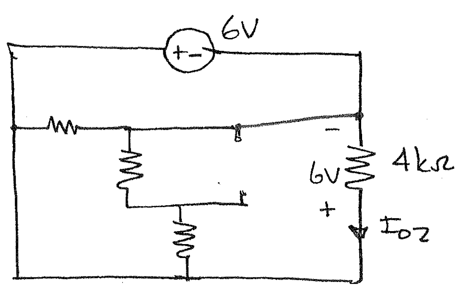
Figure P5.11

SOLUTION:



$$I_{o1} = \frac{9}{4000} = 2.25\,mA$$





$$I_{o3} = 0$$

$$I_o = I_{o1} + I_{o2} + I_{o3}$$

$$I_{o2} = -\frac{6}{4000} = -1.5\,mA$$

$$\boxed{I_o = 0.75\,mA}$$

5.12 Find V_o in the circuit in Fig. P5.12 using superposition.

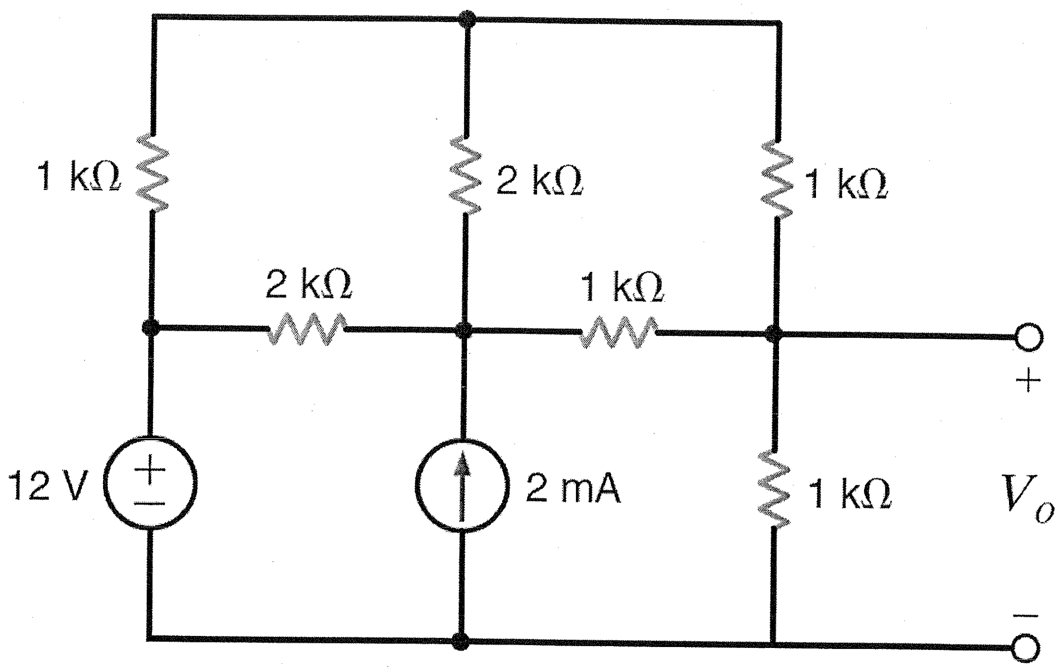
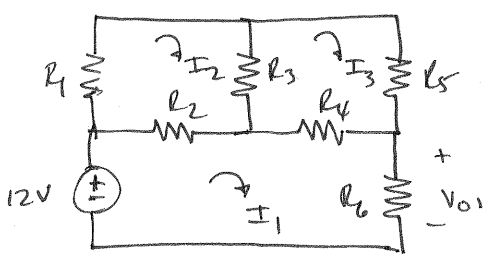


Figure P5.12

SOLUTION:



$R_1 = R_4 = R_5 = R_6 = 1k\Omega$

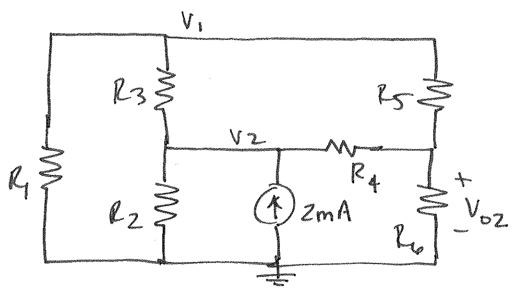
$R_2 = R_3 = 2k\Omega$

$5000 I_2 - 2000 I_1 - 2000 I_3 = 0$

$-1000 I_1 - 2000 I_2 + 4000 I_3 = 0$

$I_1 = 5.49mA$ $\Leftarrow$ $12 = 4000 I_1 - 2000 I_2 - 1000 I_3$

$V_{o1} = R_6 I_1 = 5.49V$



$$\frac{V_1}{R_1} + \frac{V_1 - V_2}{R_3} + \frac{V_1 - V_{o2}}{R_5} = 0$$

$$\frac{V_2 - V_1}{R_3} + \frac{V_2}{R_2} + \frac{V_2 - V_{o2}}{R_4} = 2\times10^{-3}$$

$$\frac{V_2 - V_{o2}}{R_4} = \frac{V_{o2}}{R_6} + \frac{V_{o2} - V_1}{R_5} \Rightarrow V_{o2} = 0.686V$$

$V_o = V_{o1} + V_{o2}$ $\boxed{V_o = 6.18V}$

5.13 Given the network in Fig. P5.13, use superposition to find V_o.

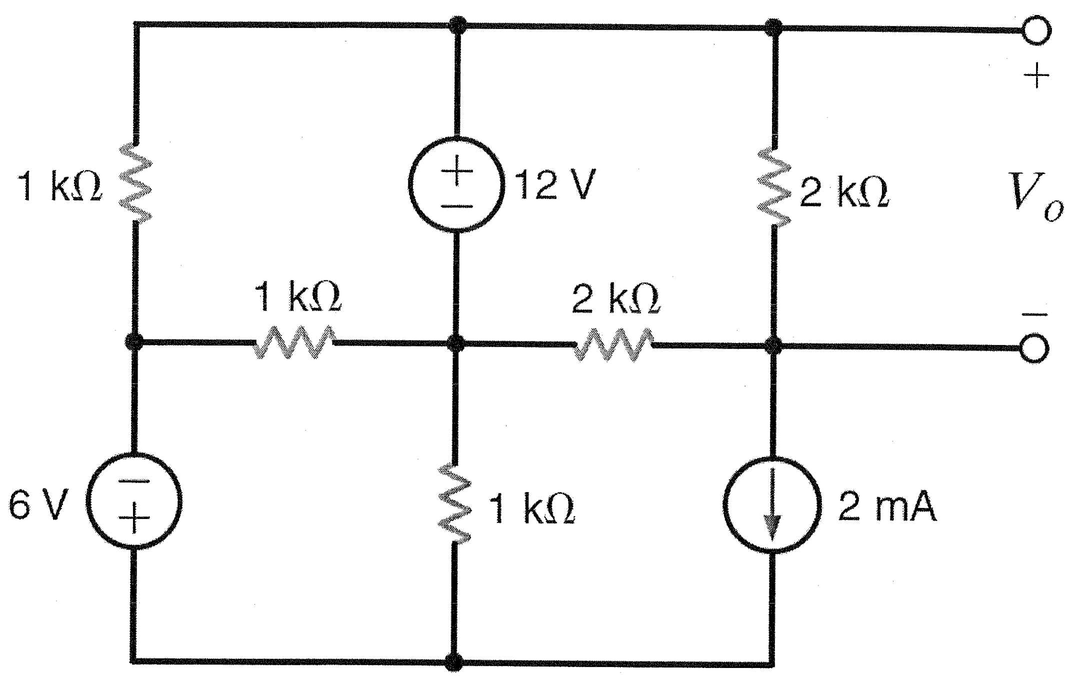
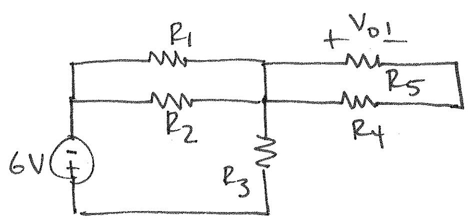


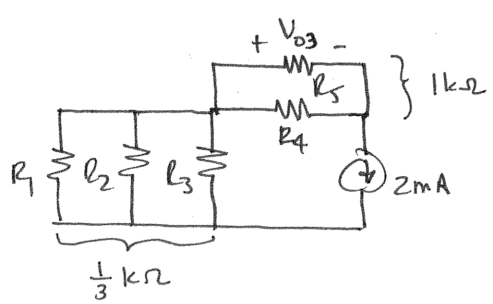
Figure P5.13

SOLUTION:



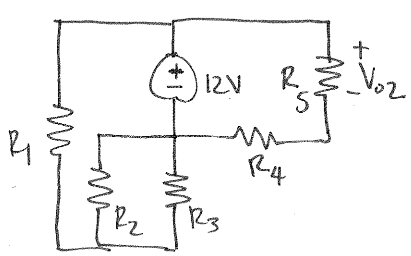
$R_1 = R_2 = R_3 = 1\,k\Omega \quad R_4 = R_5 = 2\,k\Omega$

$V_{o1} = 0\,V$



$V_{o3} = 1000\left(2\times10^{-3}\right) = 2V$

$V_{o3} = 2V$



$V_{o2} = 12\left[\dfrac{R_5}{R_4+R_5}\right]$

$V_{o2} = 6V$

$V_o = V_{o1} + V_{o2} + V_{o3}$ $\boxed{V_o = 8\,V}$

5.14 Use superposition to find V_o in the circuit in Fig. P5.14.

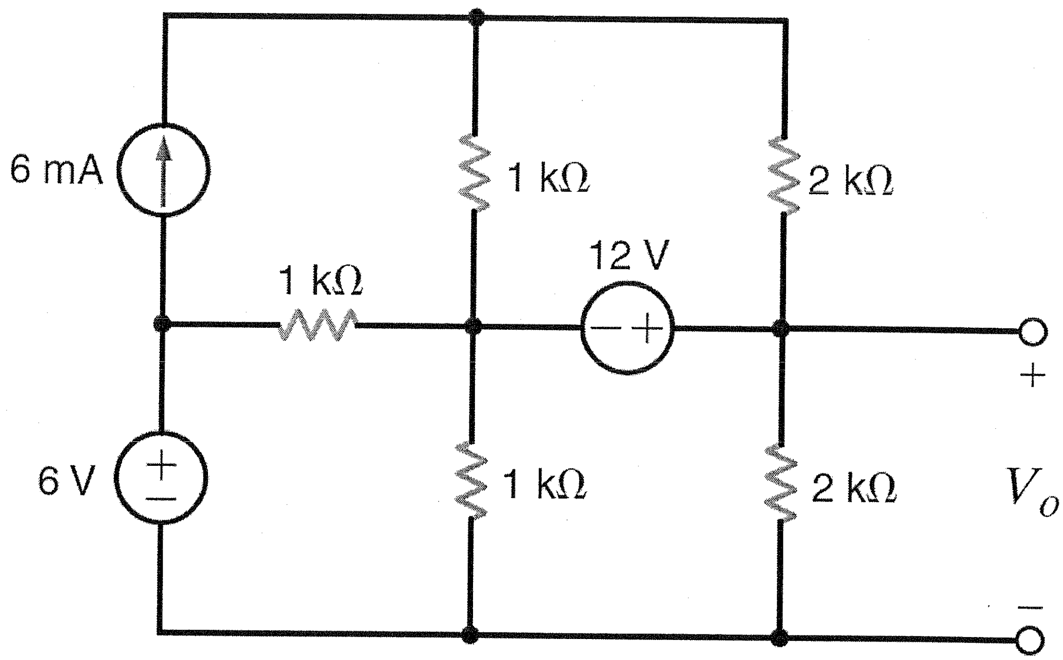
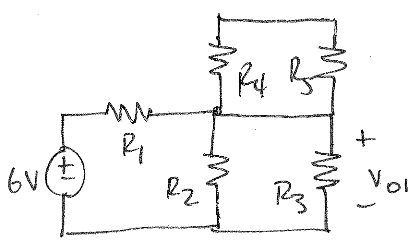


Figure P5.14

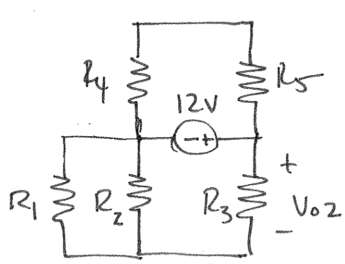
SOLUTION:



$R_1 = R_2 = R_4 = 1k\Omega \qquad R_3 = R_5 = 2k\Omega$

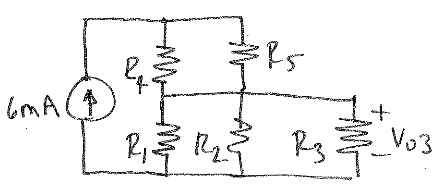
$R_x = R_2 // R_3$

$V_{o1} = 6\left[\dfrac{R_x}{R_x + R_1}\right] = 2.4V$



$R_y = R_1 // R_2 = 500\Omega$

$V_{o2} = 12\left[\dfrac{R_3}{R_3 + R_y}\right] = 9.6V$



$R_A = R_4 // R_5 = 667\Omega \quad R_B = R_1 // R_2 // R_3 = 400\Omega$

$V_{o3} = 6\times10^{-3} R_B = 2.4V$

$V_o = V_{o1} + V_{o2} + V_{o3}$ $\boxed{V_o = 14.4V}$

5.15 Find V_o in the circuit in Fig. P5.15 using superposition.

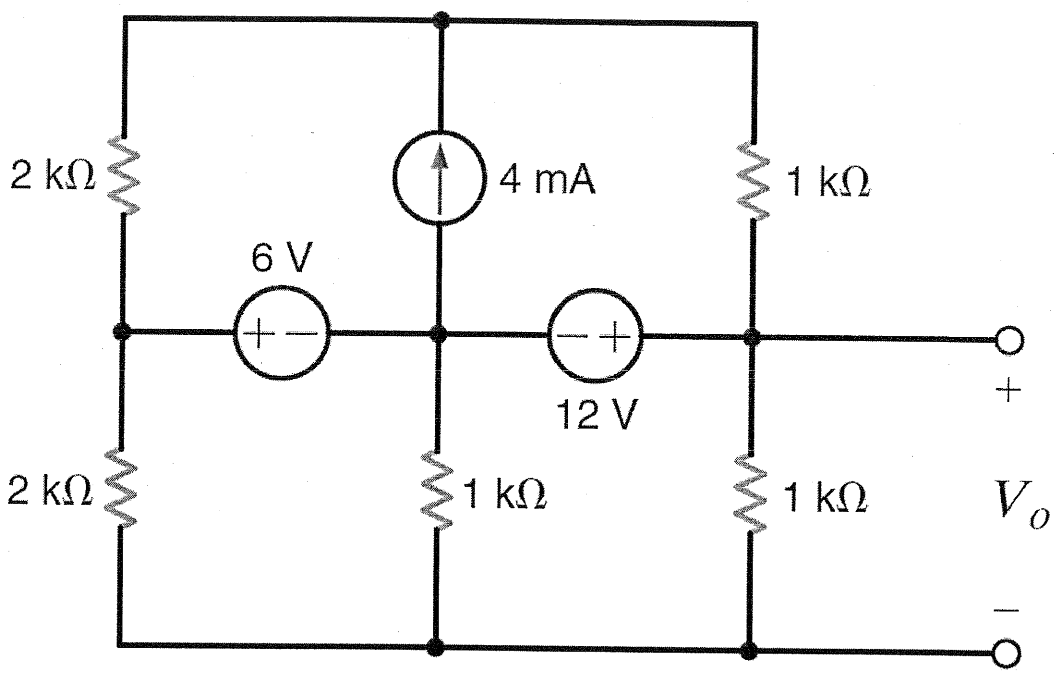
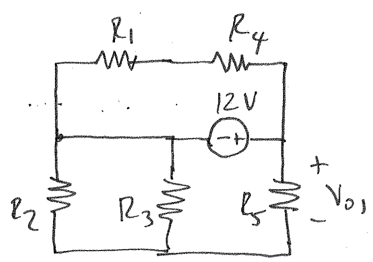


Figure P5.15

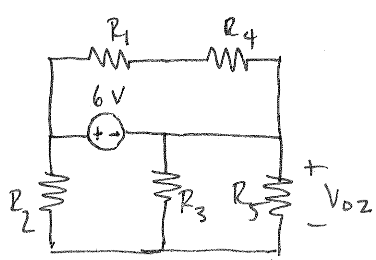
SOLUTION:



$R_1 = R_2 = 2k\Omega \qquad R_3 = R_4 = R_5 = 1k\Omega$

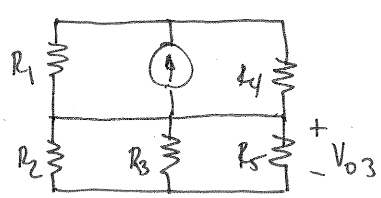
$R_A = R_2 // R_3 = 667\Omega$

$V_{01} = \dfrac{12\, R_5}{R_5 + R_A} \qquad V_{01} = 7.2V$



$R_B = R_3 // R_5 = 500\Omega$

$V_{02} = -\dfrac{6\, R_B}{R_B + R_2} \qquad V_{02} = -1.2V$



$V_{03} = 0V \qquad V_o = V_{01} + V_{02} + V_{03}$

$\boxed{V_o = 5.2V}$

5.16 Find I_o in the circuit in Fig. P5.16 using superposition.

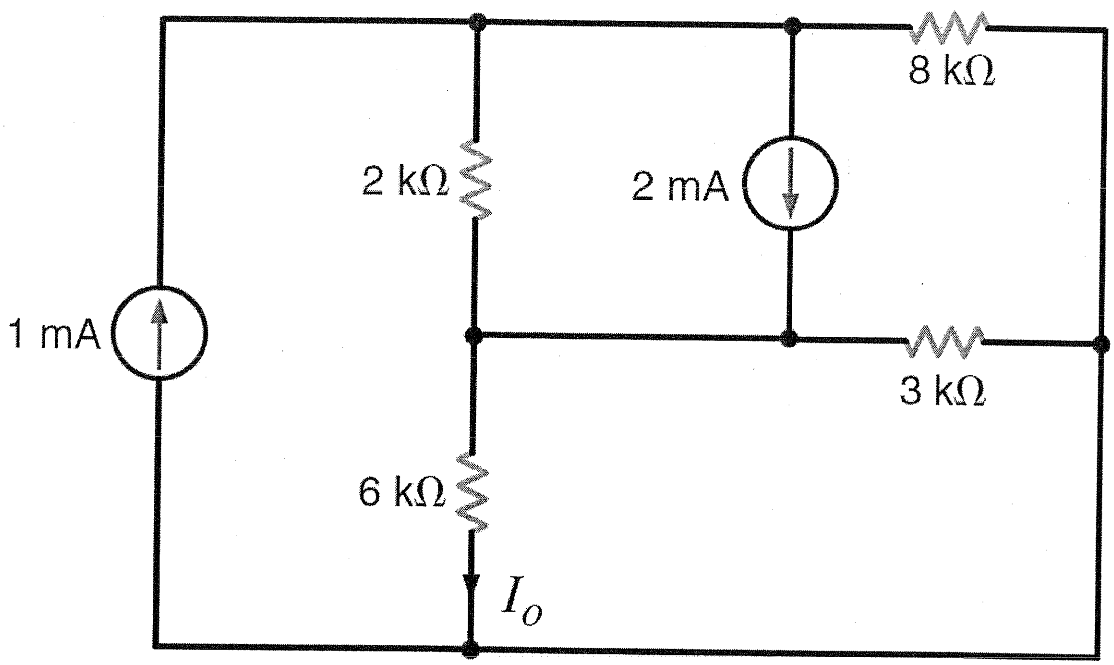
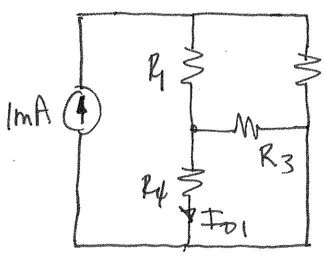
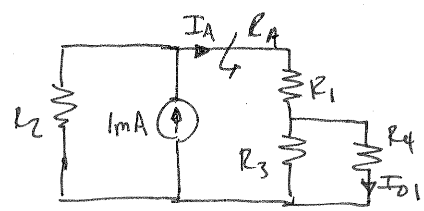


Figure P5.16

SOLUTION:



$\Rightarrow$



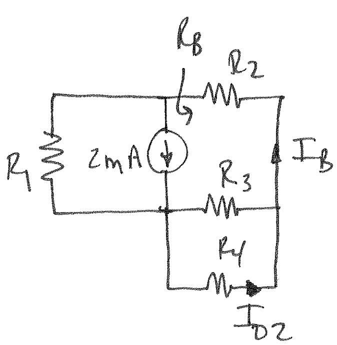
$R_1 = 2k\Omega \quad R_2 = 8k\Omega \quad R_3 = 3k\Omega \quad R_4 = 6k\Omega$

$R_A = R_1 + (R_3 // R_4) = 4k\Omega$

$I_A = 10^{-3}\left[\dfrac{R_2}{R_2 + R_A}\right]$

$I_A = 0.667\,mA$

$I_{o1} = I_A\left[\dfrac{R_3}{R_3 + R_4}\right] = 0.222\,mA$



$R_C = R_3 // R_4 = 2k\Omega \quad R_B = R_2 + R_C = 10k\Omega$

$I_B = 2\times10^{-3}\left[\dfrac{R_1}{R_1 + R_B}\right] = 0.333\,mA$

$I_{o2} = I_B\left[\dfrac{R_3}{R_3 + R_4}\right] = 0.111\,mA$

$\boxed{I_o = 0.333\,mA}$

5.17 Use superposition to find I_o in the network in Fig. P5.17.

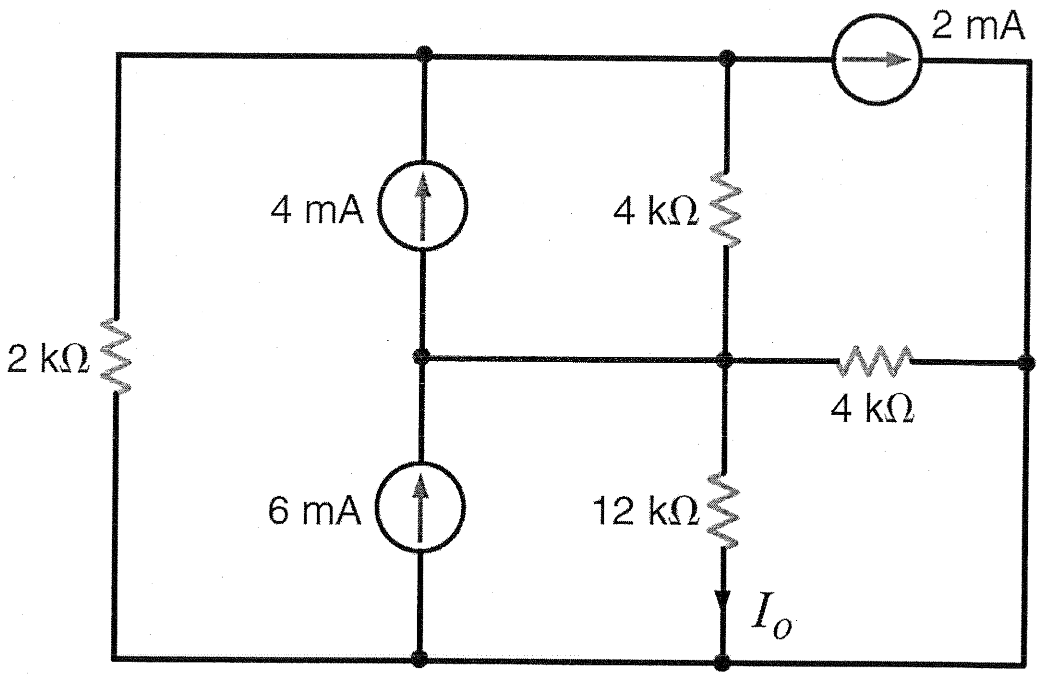
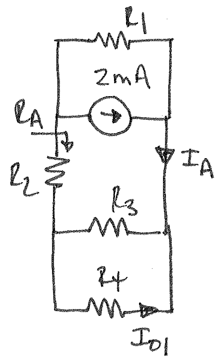


Figure P5.17

SOLUTION:

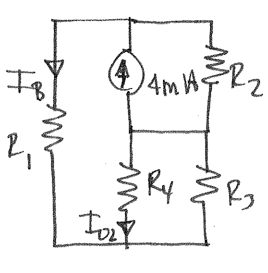


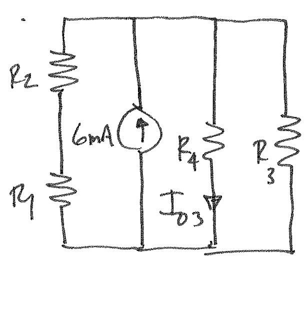
$R_1 = 2k\Omega \quad R_2 = 4k\Omega \quad R_3 = 4k\Omega \quad R_4 = 12k\Omega$

$R_A = R_2 + (R_3 // R_4) = 7k\Omega \qquad I_A = 2\times10^{-3}\left[\dfrac{R_1}{R_1+R_A}\right]$

$I_A = 0.444mA$

$I_{o1} = -I_A\, R_3 / (R_3 + R_4) = -0.111mA$





$R_C = R_1 + R_2 = 6k\Omega \qquad G_C = 1/R_C$

$G_4 = 1/R_4 \qquad G_3 = 1/R_3$

$I_{o3} = 6\times10^{-3}\, G_4 / (G_C + G_3 + G_4) = 1mA$

$\boxed{I_o = 0.444mA}$

$R_B = R_1 + (R_3 // R_4) = 5k\Omega \qquad I_B = 4\times10^{-3}\, R_2 / (R_2 + R_B) = 1.78mA$

$I_{o2} = -I_B\, R_3 / (R_3 + R_4) = -0.444mA$

5.18 Use superposition to find I_o in the circuit in Fig. P5.18.

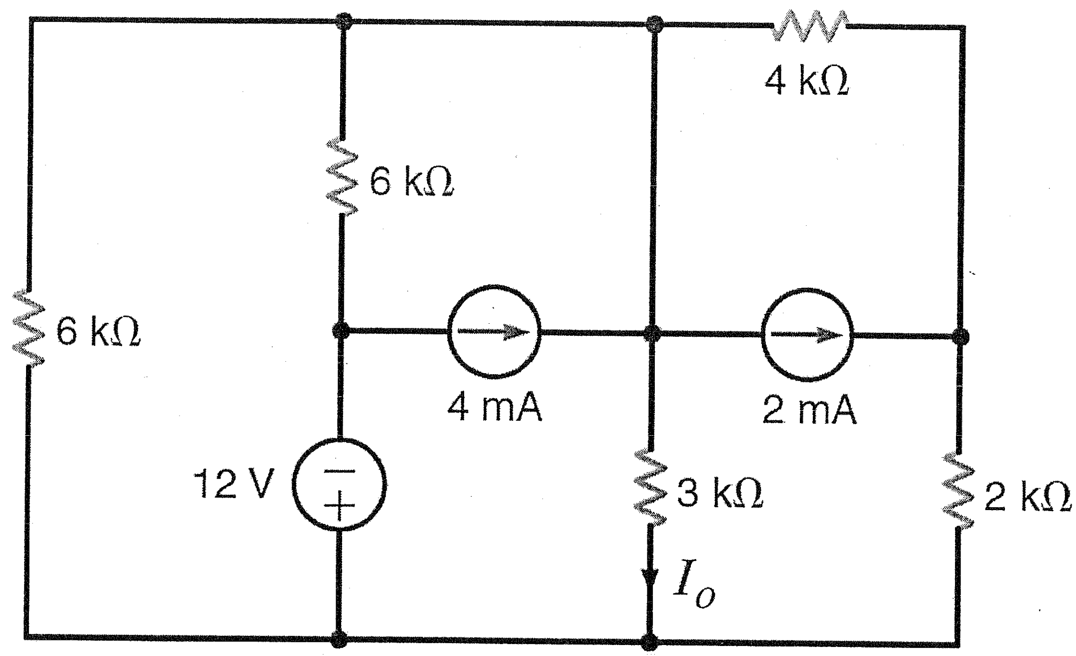
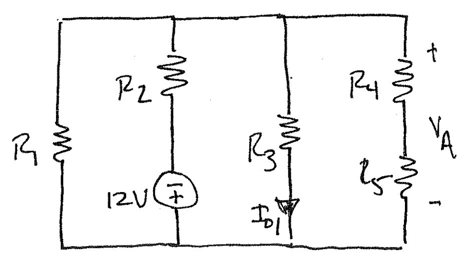


Figure P5.18

SOLUTION:



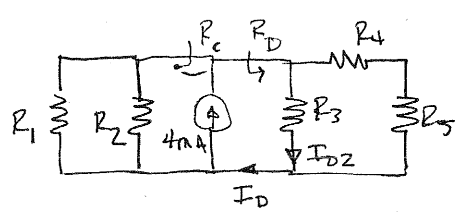
$R_1 = R_2 = 6k\Omega \quad R_3 = 3k\Omega \quad R_4 = 4k\Omega \quad R_5 = 2k\Omega$

$R_A = R_4 + R_5 = 6k\Omega$

$R_B = R_1 // R_3 // R_A = 1.5k\Omega$

$V_A = -12\, R_B / (R_B + R_2) = -2.4V$

$I_{01} = V_A / R_3 = -0.80mA$

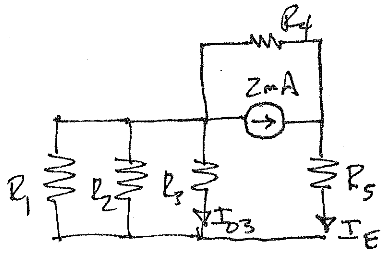


$R_C = R_1 // R_2 = 3k\Omega$

$R_D = R_3 // (R_4 + R_5) = 2k\Omega$

$I_D = 4\times10^{-3}\, R_C / (R_C + R_D) = 2.4 \ mA$

$$I_{D2} = \frac{I_D\,(R_4 + R_5)}{R_3 + R_4 + R_5} \qquad I_{02} = 1.6\ mA$$



$R_E = R_1 // R_2 // R_3 = 1.5k\Omega$

$R_F = R_5 + R_E = 3.5k\Omega$

$I_E = 2\times10^{-3}\, R4 / (R4 + R_F) = 1.07mA$

$$I_{o3} = -\frac{I_E G_3}{G_1 + G_2 + G_3}$$

$G_1 = 1/R_1 \quad G_2 = 1/R_2 \quad G_3 = 1/R_3$

$I_{o3} = -0.53 \text{ mA}$

$I_o = I_{o1} + I_{o2} + I_{o3}$

$\boxed{I_o = 0.267 \text{ mA}}$

5.19 Find I_o in the circuit in Fig. P5.19 using superposition.

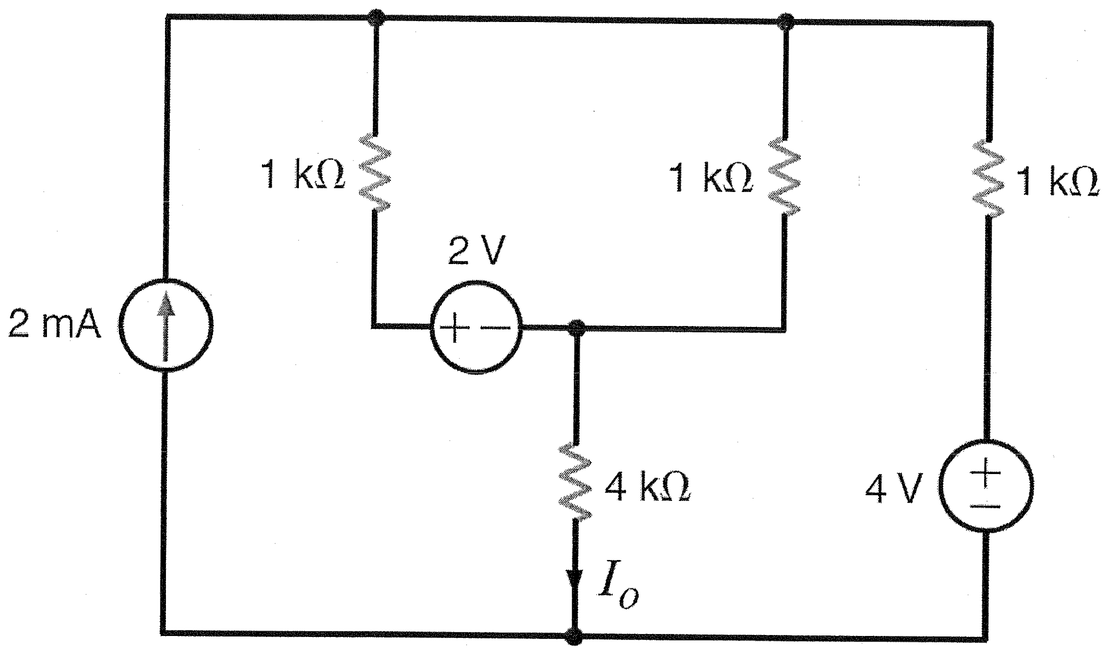
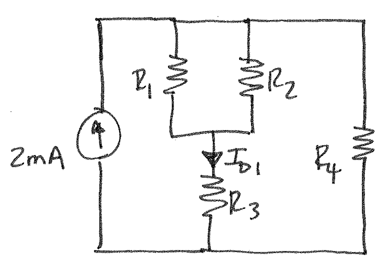


Figure P5.19

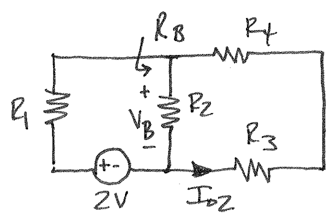
SOLUTION:



$R_1 = R_2 = R_4 = 1k\Omega \quad R_3 = 4k\Omega$

$R_A = (R_1 // R_2) + R_3 = 4.5k\Omega$

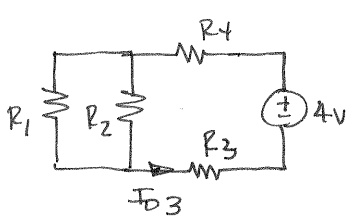
$I_{O1} = 2\times10^{-3} R_4 / (R_4 + R_A) \quad I_{O1} = 0.364mA$



$R_B = R_2 // (R_3 + R_4) = 833\Omega$

$V_B = 2 R_B / (R_1 + R_B) = 0.910V$

$I_{O2} = -V_B / (R_3 + R_4) = -0.182mA$



$R_C = R_1 // R_2 = 500\Omega$

$I_{O3} = 4 / (R_4 + R_C + R_3) = 0.727mA$

$I_O = I_{O1} + I_{O2} + I_{O3} \quad \boxed{I_O = 0.909mA}$

5.20 Use superposition to find I_o in the circuit in Fig. P5.20.

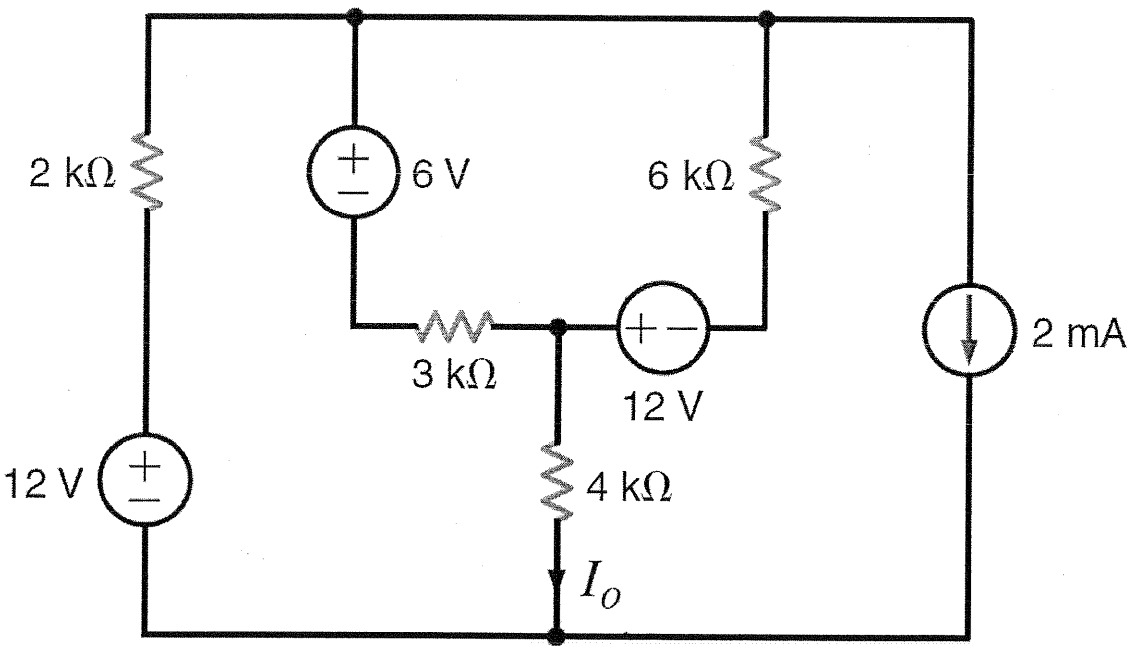
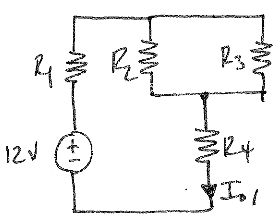


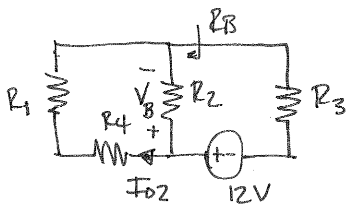
Figure P5.20

SOLUTION:

$R_1 = 2k\Omega \quad R_2 = 3k\Omega \quad R_3 = 6k\Omega \quad R_4 = 4k\Omega$



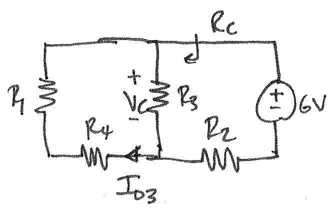
$\Leftarrow R_A = R_2 // R_3 = 2k\Omega \qquad I_{o1} = 12 / (R_1 + R_A + R_4) = 1.5mA$



$R_B = (R_1 + R_4) // R_2 = 2k\Omega$

$V_B = 12 R_B / (R_B + R_3) = 3V$

$I_{o2} = V_B / (R_1 + R_4) = 0.5mA$



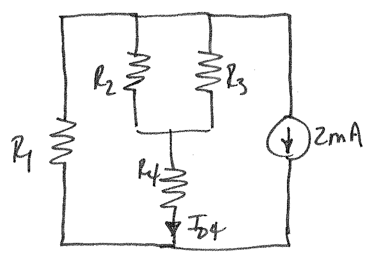
$R_c = (R_1 + R_4) // R_3 = 3k\Omega$

$V_c = 6 R_c / (R_c + R_2)$

$V_c = 3V$

$I_{o3} = -V_c / (R_1 + R_4)$

$I_{o3} = -0.5mA$



$R_D = R_4 + (R_2 // R_3) = 6k\Omega$

$I_{o4} = -2 \times 10^{-3} R_1 / (R_1 + R_D)$

$I_{o4} = -0.5mA$

$\boxed{I_o = 1mA}$

5.21 Find I_o in the circuit in Fig. P5.21 using superposition.

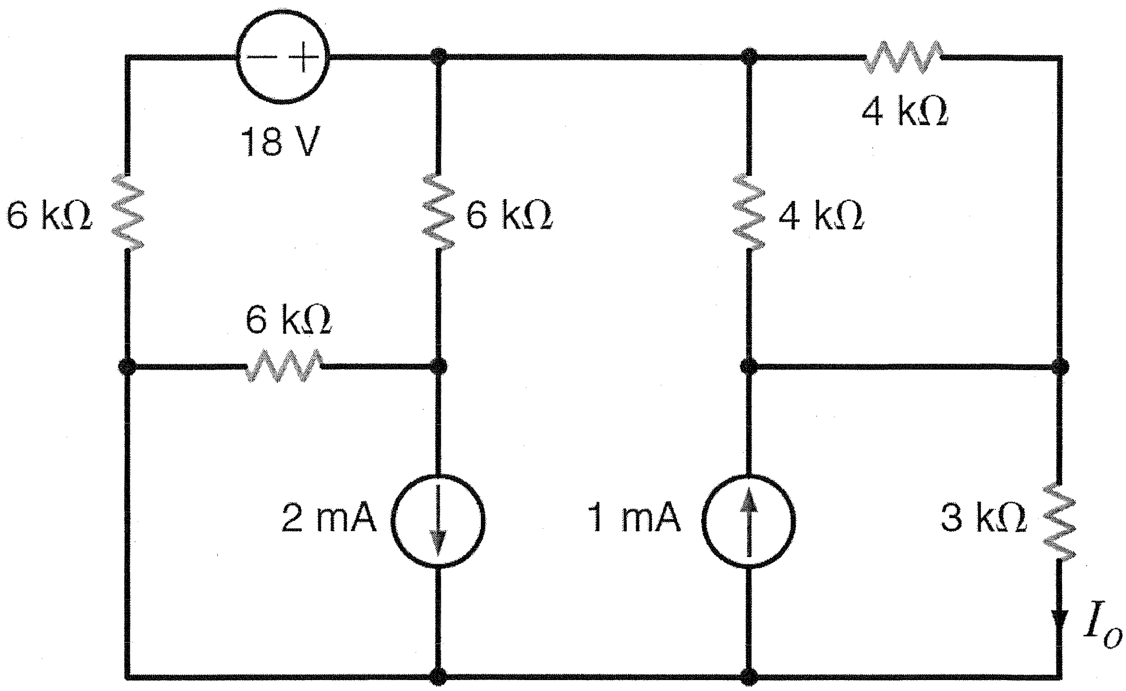
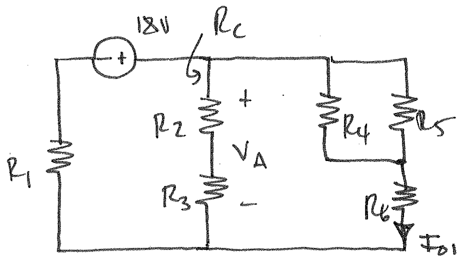


Figure P5.21

SOLUTION:

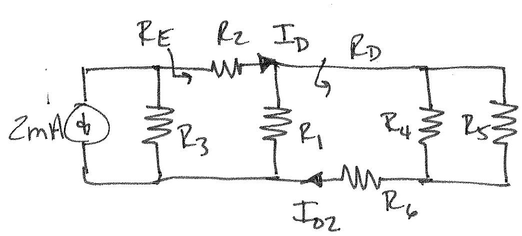


$R_1 = R_2 = R_3 = 6k\Omega \quad R_4 = R_5 = 4k\Omega \quad R_6 = 3\,k\Omega$

$R_A = R_2 + R_3 = 12k\Omega \quad R_B = R_4 // R_5 = 2k\Omega$

$R_C = R_A // (R_B + R_6) = 3.53\,k\Omega$

$V_A = 18 R_C / (R_1 + R_C) = 6.67\,V \quad I_{01} = V_A / (R_B + R_6) = 1.33\,mA$

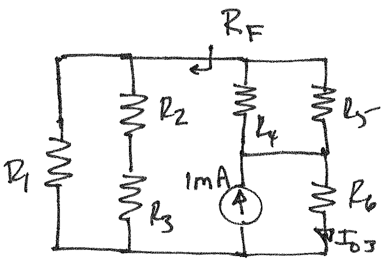


$R_D = R_6 + (R_4 // R_5) = 5k\Omega$

$R_E = R_2 + (R_1 // R_D) = 8.73\,k\Omega$

$I_D = -2\times10^{-3} R_3 / (R_3 + R_E) = -0.815\,mA$

$I_{02} = \dfrac{I_D R_1}{R_1 + R_D} \quad I_{02} = -0.444\,mA$



$R_F = R_1 // (R_2 + R_3) = 4k\Omega$

$R_G = R_F + (R_4 // R_5) = 6k\Omega$

$I_{o3} = 10^{-3}\ R_G / (R_G + R_6) = 0.667mA$

$I_o = I_{o1} + I_{o2} + I_{o3} \Rightarrow$ $\boxed{I_o = 1.55\ mA}$

5.22 Use superposition to find I_o in the network in Fig. P5.22.
PSV

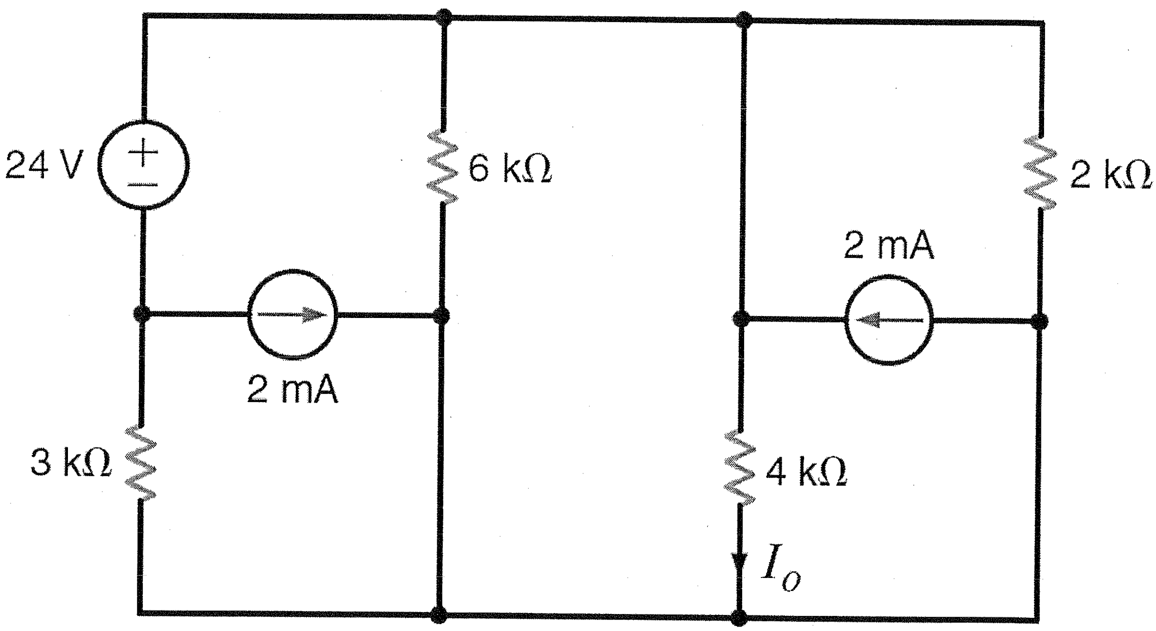
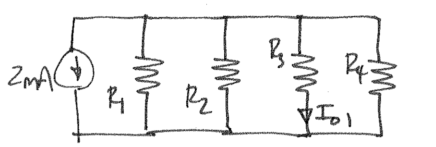


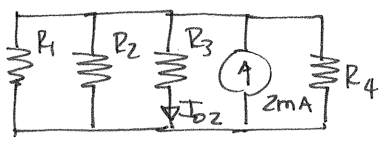
Figure P5.22

SOLUTION:

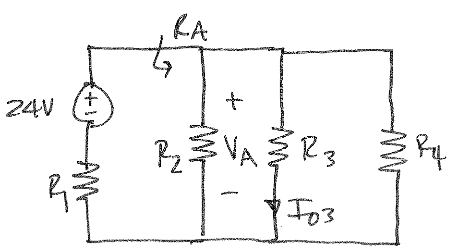
$R_1 = 3k\Omega \quad R_2 = 6k\Omega \quad R_3 = 4k\Omega \quad R_4 = 2k\Omega$



$I_{o1} = -2\times10^{-3}\, G_3 \,/\,(G_1 + G_2 + G_3 + G_4) = -0.4mA$



$I_{o2} = 2\times10^{-3}\, G_3 \,/\,(G_1 + G_2 + G_3 + G_4) = 0.4mA$



$R_A = R_2 // R_3 // R_4 = 1.091k\Omega$

$V_A = 24\, R_A \,/\,(R_A + R_1) = 6.4V$

$I_{o3} = V_A / R_3 = 1.6mA$

$I_o = I_{o1} + I_{o2} + I_{o3}$ $\quad \boxed{I_o = 1.6mA}$

5.23 The loop equations for a two-loop network are

$$I_1 R_{11} + I_2 R_{12} = V_1$$
$$I_1 R_{21} + I_2 R_{22} = V_2$$

What is the relationship among V_1, V_2, and R_{ij} for $I_1 = 0$.

SOLUTION:

$I_2 R_{12} = V_1 \quad \& \quad I_2 R_{22} = V_2$

$\dfrac{I_2 R_{12}}{V_1} = 1 \quad \& \quad \dfrac{I_2 R_{22}}{V_2} = 1$

$\dfrac{I_2 R_{12}}{V_1} = \dfrac{I_2 R_{22}}{V_2} \Rightarrow \boxed{\dfrac{V_2}{V_1} = \dfrac{R_{22}}{R_{12}}}$

5.24 Use the results of Problem 5.23 to find the value of I_x that yields a $V_1 = 0$ V in the network in Fig. P5.24.

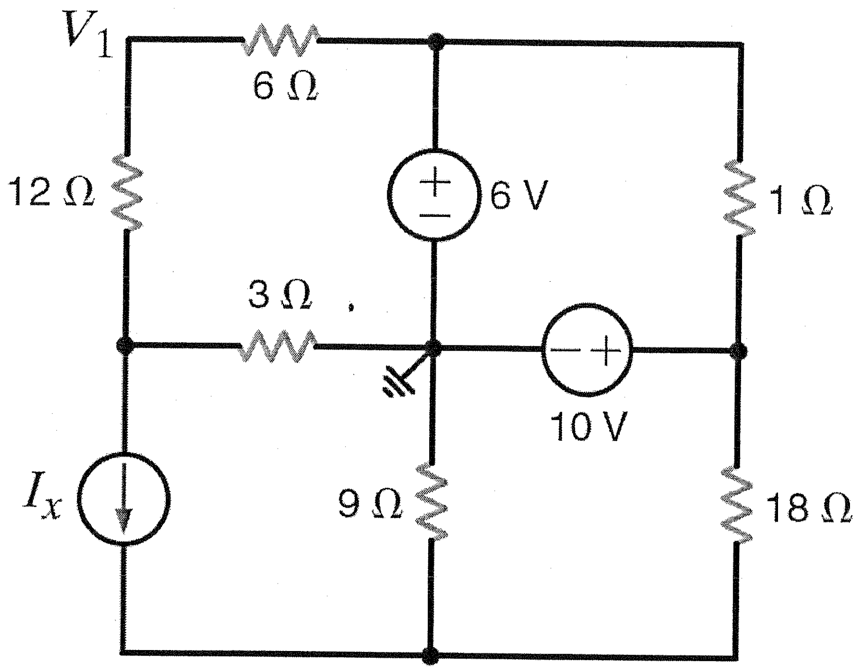


Figure P5.24

SOLUTION:

$V_1 = 6 + 6I_1 = 0 \Rightarrow I_1 = -1A$

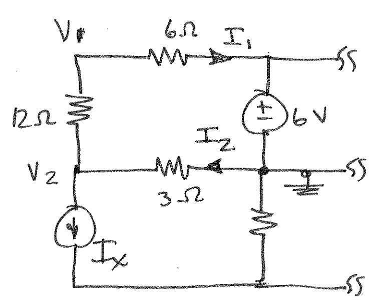
$V_1 - V_2 = -I_1(12) = 12V$

$V_2 = -12V$

$I_x = -I_1 + I_2$

$I_2 = (0 - V_2)/3 = 4A$

$\boxed{I_x = 5A}$



5.25 A three-node circuit is described by the following equations:

$$G_{11}V_1 + G_{12}V_2 + G_{13}V_3 = I_1$$
$$G_{21}V_1 + G_{22}V_2 + G_{23}V_3 = I_2$$
$$G_{31}V_1 + G_{32}V_2 + G_{33}V_3 = I_3$$

Show that for $V_2 = 0$,

$$I_1\begin{vmatrix} G_{21} & G_{23} \\ G_{31} & G_{33} \end{vmatrix} - I_2\begin{vmatrix} G_{11} & G_{13} \\ G_{31} & G_{33} \end{vmatrix} + I_3\begin{vmatrix} G_{11} & G_{13} \\ G_{21} & G_{23} \end{vmatrix} = 0$$

SOLUTION:

By Cramer's Rule,

$$V_2 = \frac{\begin{vmatrix} G_{11} & I_1 & G_{13} \\ G_{21} & I_2 & G_{23} \\ G_{31} & I_3 & G_{33} \end{vmatrix}}{\begin{vmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \\ G_{31} & G_{32} & G_{33} \end{vmatrix}} = \frac{-I_1\begin{vmatrix} G_{21} & G_{23} \\ G_{31} & G_{33} \end{vmatrix} + I_2\begin{vmatrix} G_{11} & G_{13} \\ G_{31} & G_{33} \end{vmatrix} - I_3\begin{vmatrix} G_{11} & G_{13} \\ G_{21} & G_{23} \end{vmatrix}}{|G|} = 0$$

So,

$$I_1\begin{vmatrix} G_{21} & G_{23} \\ G_{31} & G_{33} \end{vmatrix} - I_2\begin{vmatrix} G_{11} & G_{13} \\ G_{31} & G_{33} \end{vmatrix} + I_3\begin{vmatrix} G_{11} & G_{13} \\ G_{21} & G_{23} \end{vmatrix} = 0$$

5.26 Use the results of Problem 5.25 to determine the value of V_B such that V_o is zero in the network in Fig. P5.26.

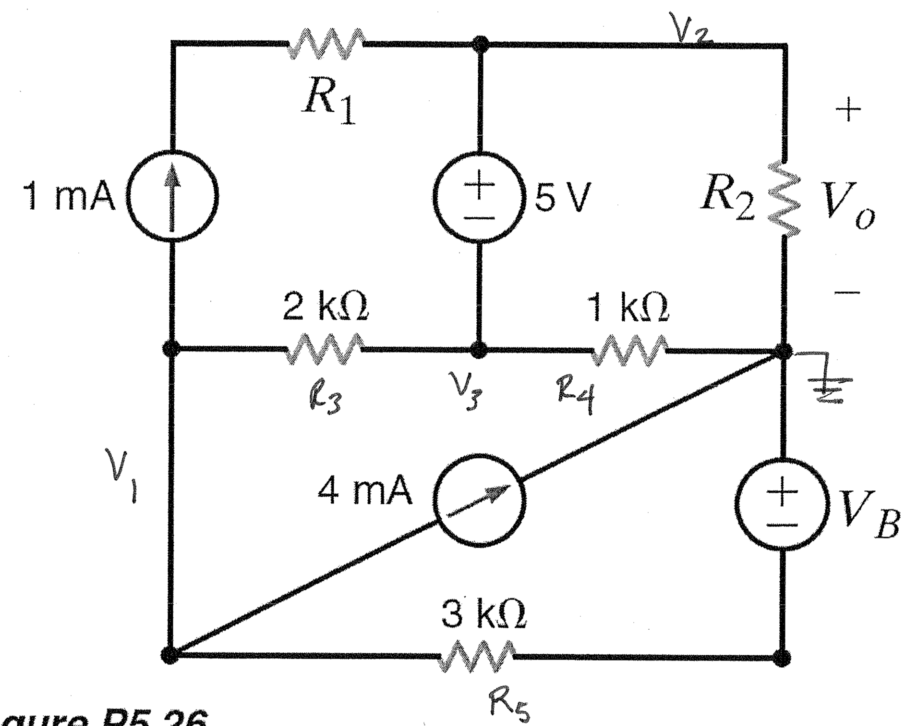


Figure P5.26

Explain why the values of R_1 and R_2 have no impact on your analysis.

SOLUTION:

$$\frac{V_2}{R_2} + \frac{V_3}{R_4} + 4\times10^{-3} + \frac{V_1+V_B}{R_5} = 0$$

$$\frac{V_1-V_3}{R_3} + \frac{V_1+V_B}{R_5} + 10^{-3} + 4\times10^{-3} = 0$$

$$V_2 - V_3 = 5$$

$$10^{-3}\begin{bmatrix} 1/3 & \frac{1000}{R_2} & 1 \\ 5/6 & 0 & -1/2 \\ 0 & 1 & -1 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} -4 - V_B/R_5 \\ -5 - V_B/R_5 \\ 5 \end{bmatrix}$$

$$-\left(4+\frac{V_B}{R_5}\right)\begin{vmatrix} 5/6 & -1/2 \\ 0 & -1 \end{vmatrix} + \left(5+\frac{V_B}{R_5}\right)\begin{vmatrix} 1/3 & 1 \\ 0 & -1 \end{vmatrix} + 5\begin{vmatrix} 1/3 & 1 \\ 5/6 & -1/2 \end{vmatrix} = V_2 = 0$$

$$-(4+V_B/R_5)(-5/6) + (5+V_B/R_5)(-1/3) + 5(-1) = 0$$

$$\boxed{V_B = 20\text{V}}$$

The current through R_1 is fixed by the 1-mA current source. Thus, the value of R_1 has no impact in our nodal analysis.

Also, when $V_2 = 0$, the current through R_2 is zero regardless of the value of R_2.

5.27 **(a)** Given the network in Fig. P5.27, find the value of R_2 such that $V_o = 0$ V. (b) Then find the Thévenin and Norton equivalent circuits at A-B as seen by R_3 using the results of (a).

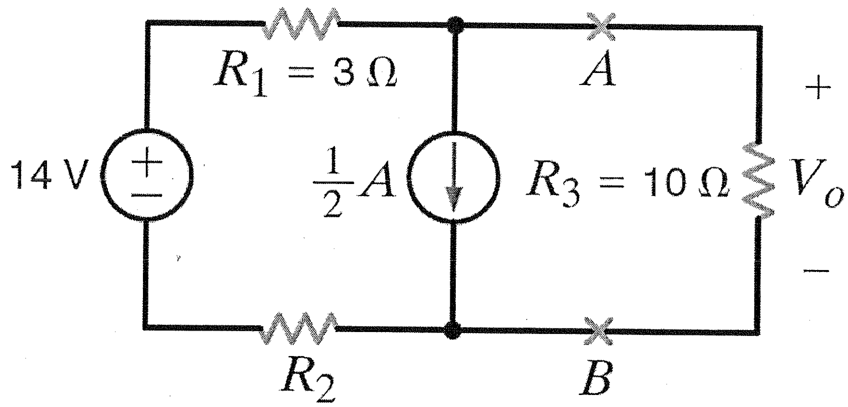
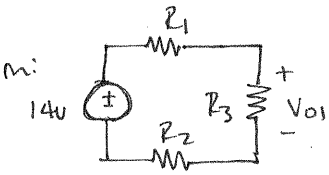
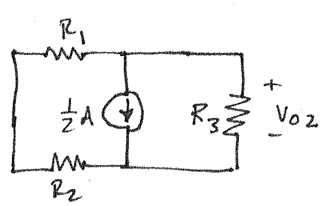


Figure P5.27

SOLUTION:

a) Superposition:





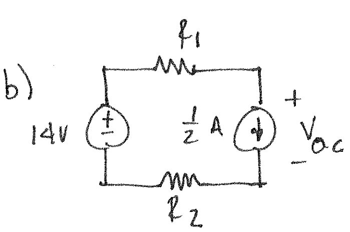
$$V_{o1} = 14R_3/(R_1+R_2+R_3)$$

$$V_{o2} = -\frac{\frac{1}{2}(R_1+R_2)}{R_1+R_2+R_3}R_3$$

$$V_{o1}+V_{o2} = V_o = 0 = \frac{14R_3}{R_1+R_2+R_3} - \frac{R_1+R_2}{2}\,\frac{R_3}{R_1+R_2+R_3}$$

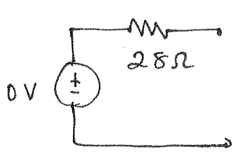
$$\boxed{R_2 = 25\,\Omega}$$

b)

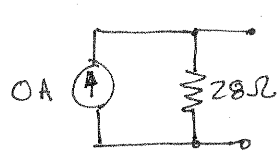


$$V_{oc} = 14 - \tfrac{1}{2}R_1 - \tfrac{1}{2}R_2 = 0\text{ V}$$

$$R_{TH} = R_1 + R_2 = 28\,\Omega$$



Thevenin Eq.



Norton Eq

5.28 Use Thévenin's theorem to find V_o in the network in Fig. P5.28. CS

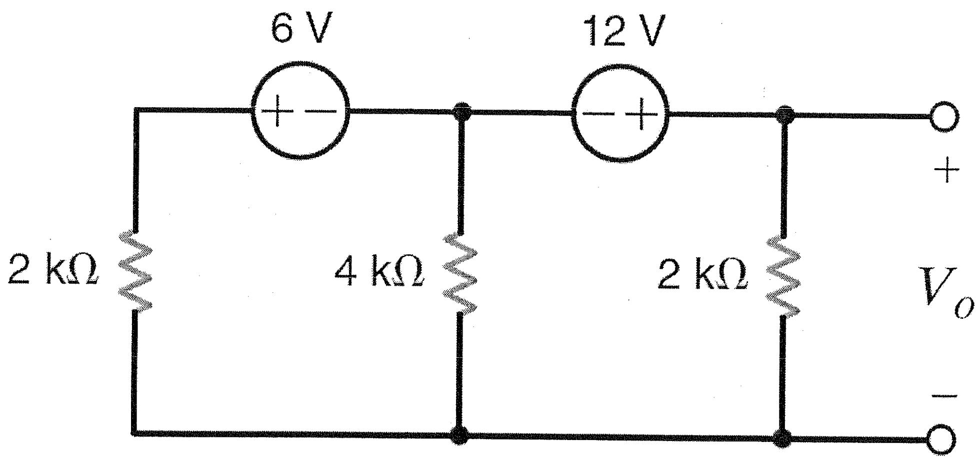
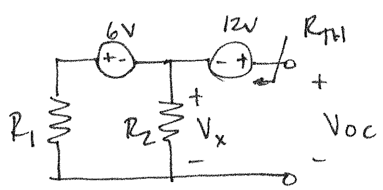


Figure P5.28

SOLUTION:

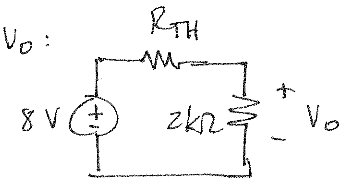


$R_1 = 2k\Omega \quad R_2 = 4k\Omega$

$V_x = -6 R_2 / (R_1 + R_2) = -4V$

$V_{oc} = 12 + V_x = 8V \qquad R_{TH} = R_1 // R_2 = 1.33k\Omega$

Find V_o:



$V_o = \dfrac{8(2000)}{2000 + R_{TH}}$ $\boxed{V_o = 4.8V}$

5.29 Find I_o in the network in Fig. P5.29 using Thévenin's theorem.

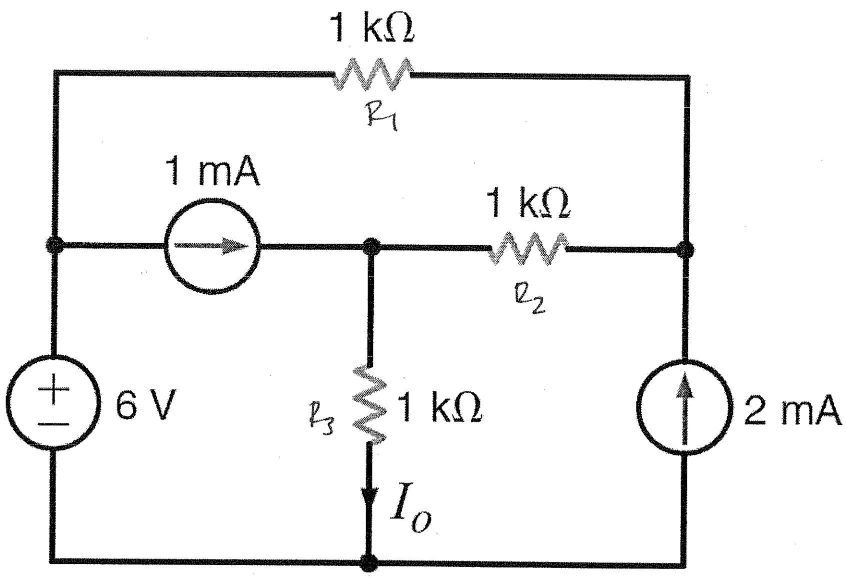
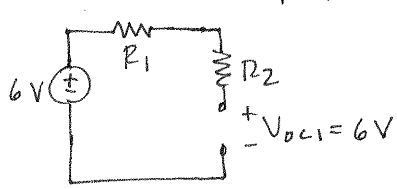
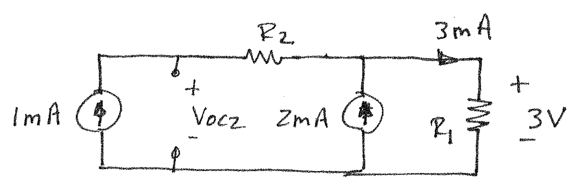


Figure P5.29

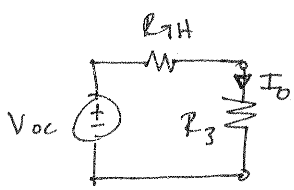
SOLUTION: Superposition!

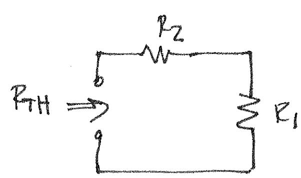




$V_{oc2} = 10^{-3} R_2 + 3\times10^{-3} R_1 = 4V$

$V_{oc} = V_{oc1} + V_{oc2} = 10V$





$R_{TH} = R_2 + R_1$

$R_{TH} = 2k\Omega$

$I_o = V_{oc}/(R_3 + R_{TH})$ $\boxed{I_o = 3.33mA}$

5.30 Find I_o in the circuit in Fig. P5.30 using Thévenin's theorem. CS

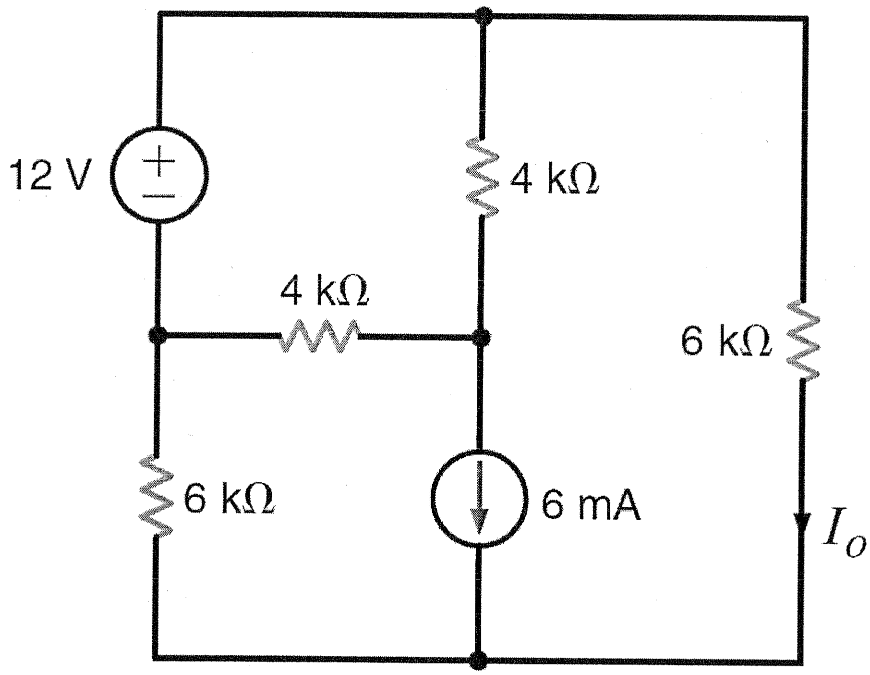
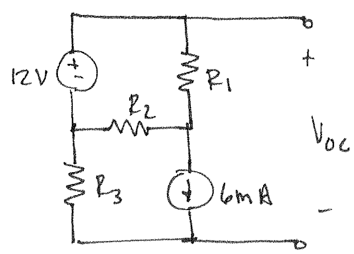
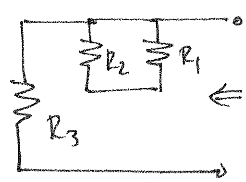


Figure P5.30

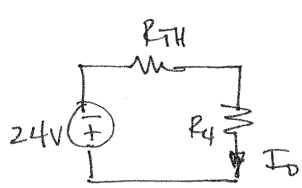
SOLUTION: $R_1 = R_2 = 4k\Omega$ $R_3 = 6k\Omega$ $R_4 = 6k\Omega$



$12 = V_{oc} + 6\times10^{-3} R_3 \Rightarrow V_{oc} = -24V$



$\Leftarrow R_{TH} = R_3 = 6k\Omega$



$I_o = \frac{-24}{R_{TH} + R_4}$ $\boxed{I_o = -2mA}$

5.31 Find V_o in the network in Fig. P5.31 using Thévenin's theorem. **PSV**

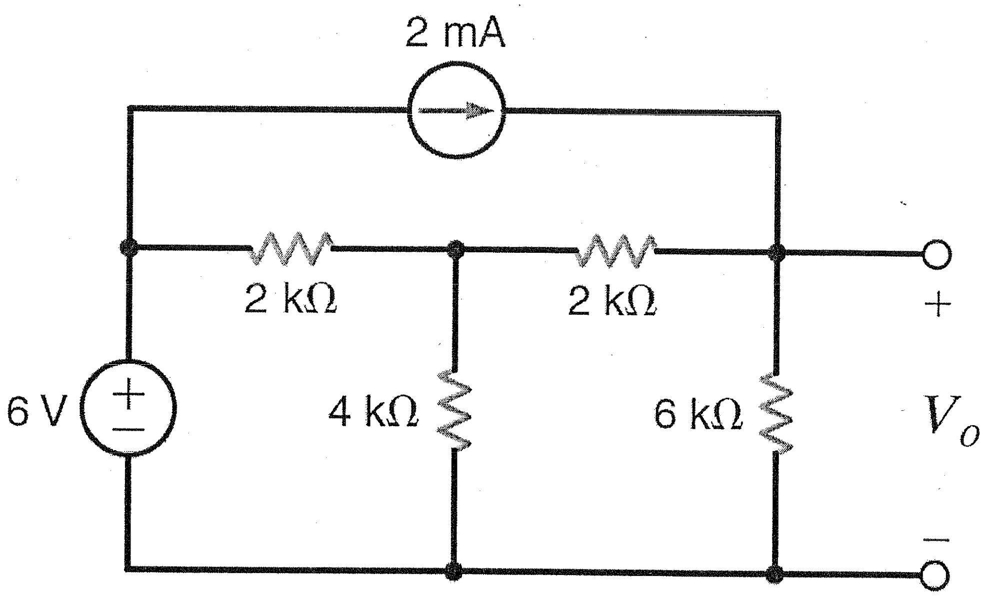
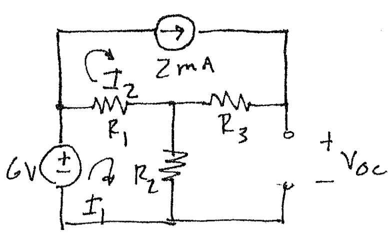


Figure P5.31

SOLUTION: $R_1 = R_3 = 2k\Omega \quad R_2 = 4k\Omega \quad R_4 = 6k\Omega$

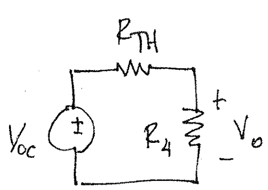


$6 = 6000I_1 - 2000I_2 \qquad I_2 = 2mA$

$I_1 = 1.67\,mA$

$6 = R_1\,(I_1 - I_2) + R_3\,(-I_2) + V_{oc} \Rightarrow V_{oc} = 10.67V$

$R_{TH} = R_3 + (R_1 // R_2) = 3.33k\Omega$



$V_o = V_{oc}R_4 \,/ (R_{TH} + R_4) \qquad \boxed{V_o = 6.86\,V}$

5.32 Find V_o in the circuit in Fig. P5.32 using Thévenin's theorem.

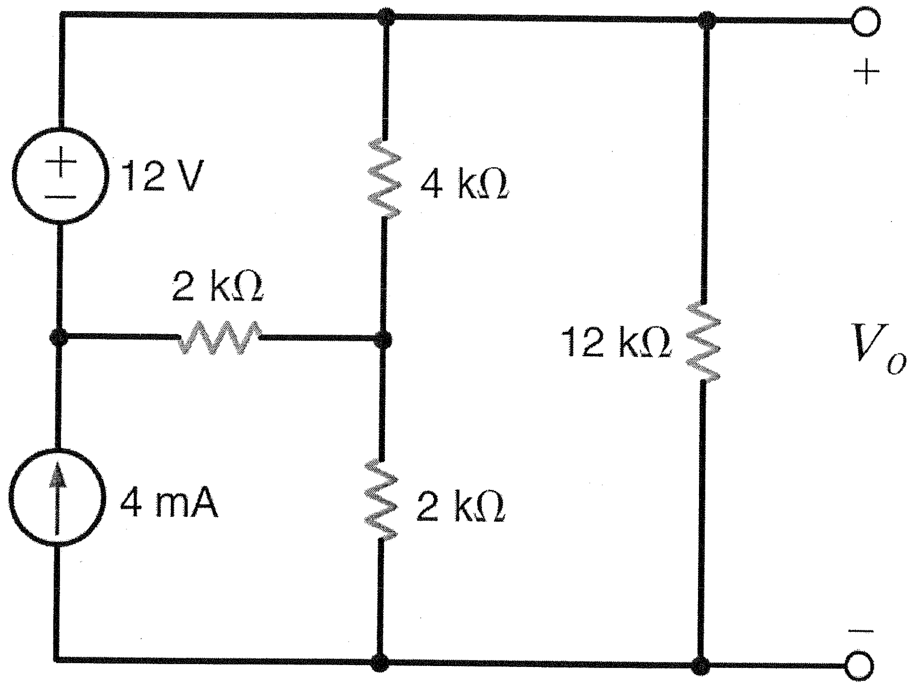
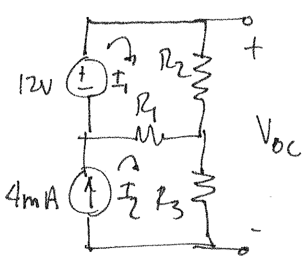


Figure P5.32

SOLUTION:



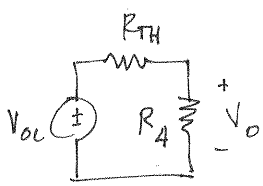
$R_1 = R_3 = 2k\Omega \quad R_3 = 4k\Omega \quad R_4 = 12k\Omega$

$12 = I_1 \, (R_1 + R_2) - R_1 I_2 \qquad I_2 = 4mA$

$I_1 = 3.33mA$

$R_2 I_1 + R_3 I_2 = V_{oc} = 21.33V$

$R_{Th} = (R_1 // R_2) + R_3 = 3.33k\Omega$



$$V_o = \frac{V_{oc} R_4}{R_4 + R_{Th}} \qquad \boxed{V_o = 16.70V}$$

5.33 Find I_o in the network in Fig. P5.33 using Thévenin's theorem. CS

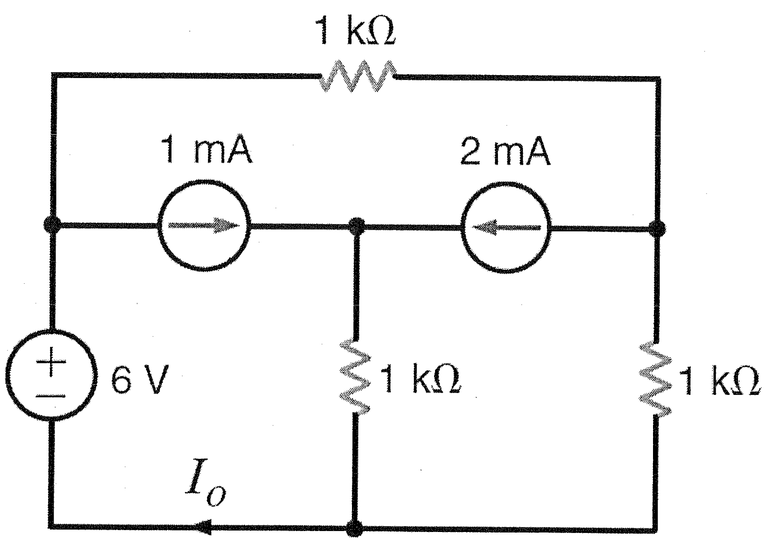
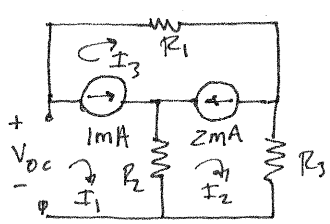


Figure P5.33

SOLUTION:

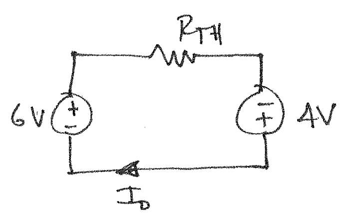


$I_1 = 0 \qquad I_1 - I_3 = 1mA \qquad I_3 - I_2 = 2mA$

$I_3 = -1mA$ & $I_2 = -3mA$

$V_{OC} = R_1 I_3 + R_3 I_2 = -4V$

$R_{TH} = R_1 + R_3 = 2k\Omega$



$6 = I_o R_{TH} - 4 \qquad I_o = \frac{10}{R_{TH}} \qquad \boxed{I_o = 5mA}$

5.34 Find I_o in the network in Fig. P5.34 using Thévenin's theorem.

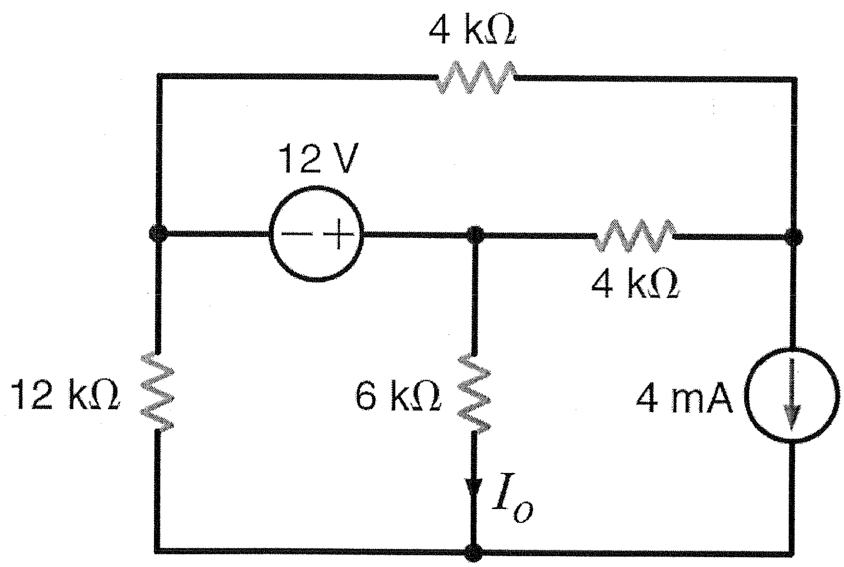
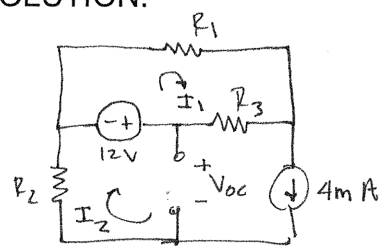


Figure P5.34

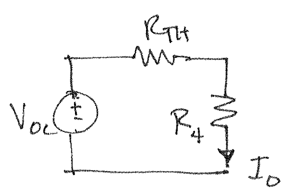
SOLUTION:



$12 = V_{oc} + I_2 R_2 \qquad I_2 = 4mA$

$V_{oc} = -36V$

$R_{TH} = R_2 = 12k\Omega$



$I_o = \dfrac{V_{oc}}{R_{TH} + R_4} \qquad R_4 = 6k\Omega$

$\boxed{I_o = -2mA}$

5.35 Find V_o in the circuit in Fig. P5.35 using Thévenin's theorem.

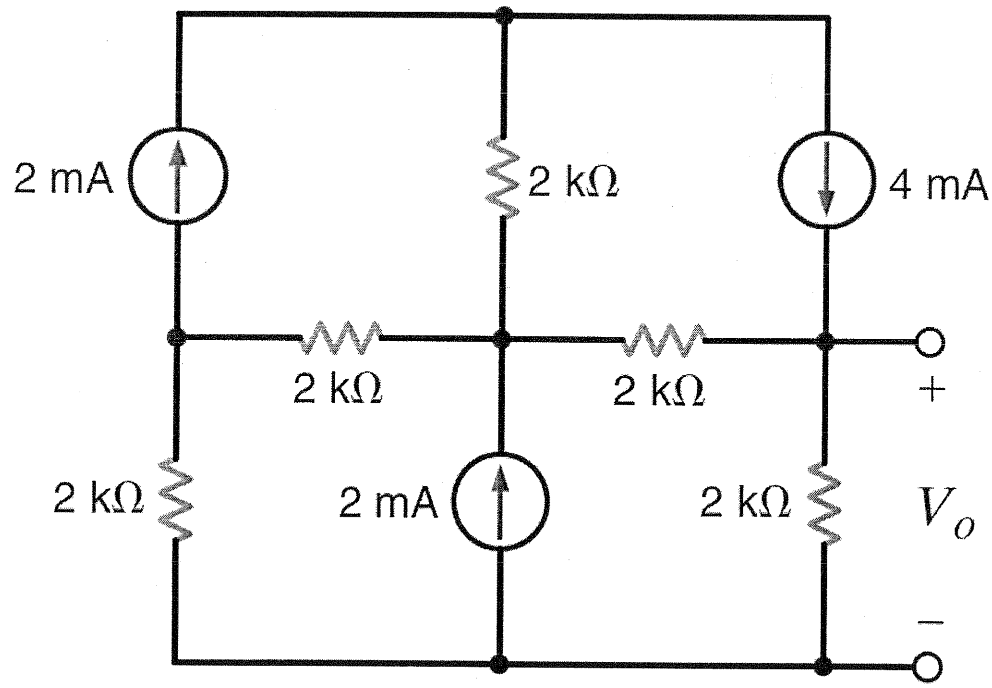
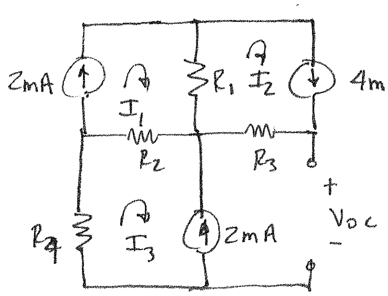


Figure P5.35

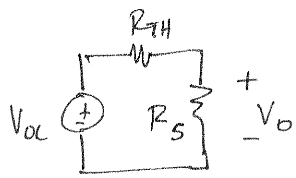
SOLUTION:



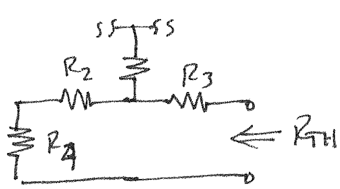
$I_1 = 2mA \qquad I_2 = 4mA \qquad I_3 = -2mA$

$I_3 R_4 + R_2(I_3 - I_1) + R_3(0 - I_2) + V_{oc} = 0$

$V_{oc} = 20V$



$V_o = V_{oc} \dfrac{R_5}{R_5 + R_{TH}}$



$\boxed{V_o = 5V}$

$R_{TH} = R_4 + R_2 + R_3 = 6k\Omega$

5.36 Find V_o in the network in Fig. P5.36 using Thévenin's theorem. CS

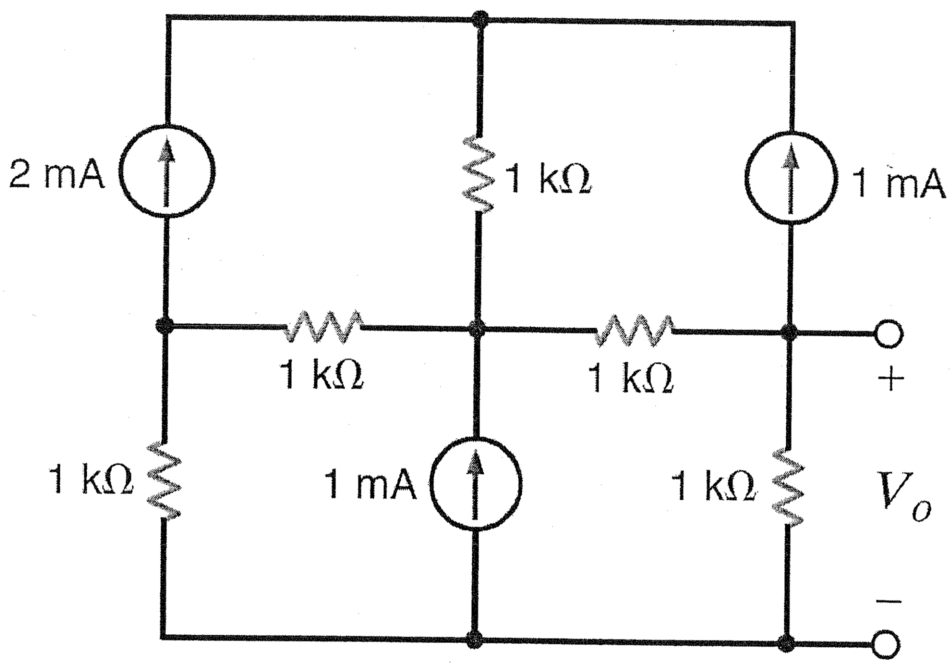
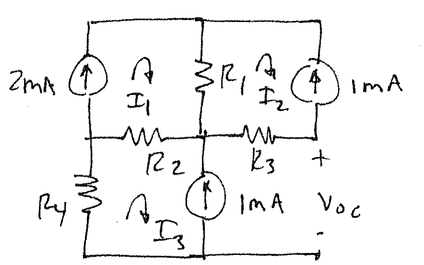


Figure P5.36

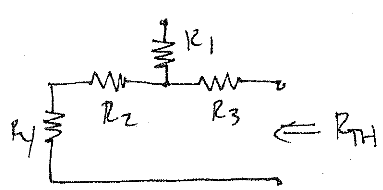
SOLUTION: All $R = 1k\Omega$



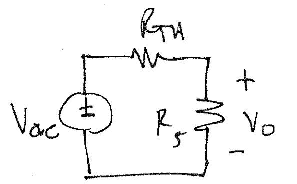
$I_1 = 2mA \quad I_2 = -1mA \quad I_3 = -1mA$

$R_4 I_3 + R_2 (I_3 - I_1) - R_3 I_2 + V_{OC} = 0$

$V_{OC} = 3V$



$R_{TH} = R_4 + R_2 + R_3 = 3k\Omega$



$V_o = V_{oc} \dfrac{R_5}{R_{TH} + R_5}$

$\boxed{V_{oc} = 0.75V}$

5.37 Find V_o in the network in Fig. P5.37 using Thévenin's theorem.

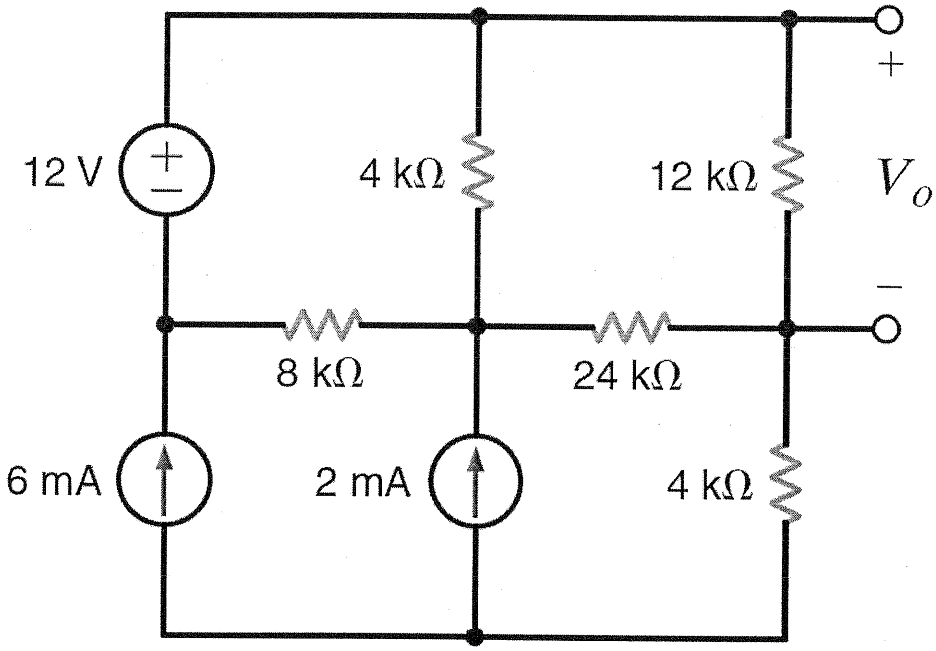
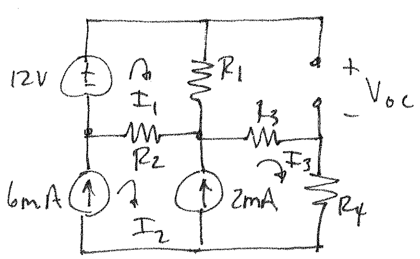


Figure P5.37

SOLUTION:

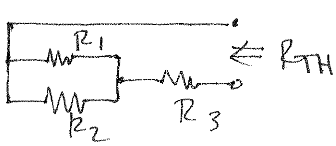


$I_2 = 6mA \quad I_3 - I_2 = 2mA \Rightarrow I_3 = 8mA$

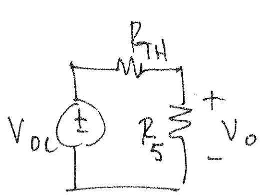
$12 = I_1 R_1 + (I_1 - I_2) R_2 \Rightarrow I_1 = 5mA$

$12 = V_{oc} - I_3 R_3 + R_2 (I_1 - I_2)$

$V_{oc} = 212V$



$R_{TH} = (R_1 // R_2) + R_3 = 26.67 k\Omega$



$V_o = \frac{V_{oc} R_5}{R_5 + R_{TH}} \quad R_5 = 12k\Omega \quad \boxed{V_o = 65.79V}$

5.38 Use a combination of Thévenin's theorem and superposition to find V_o in the circuit in Fig. P5.38.

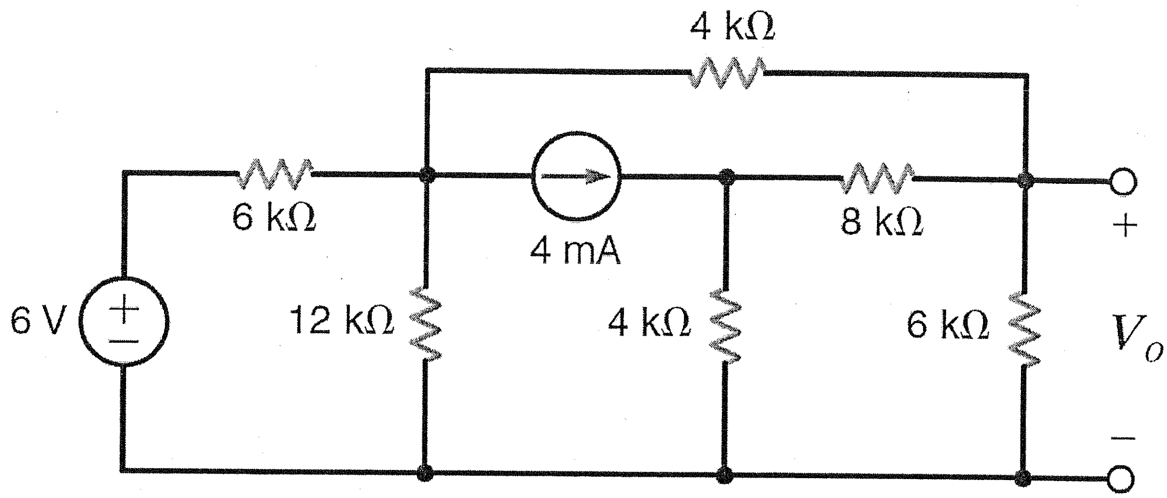
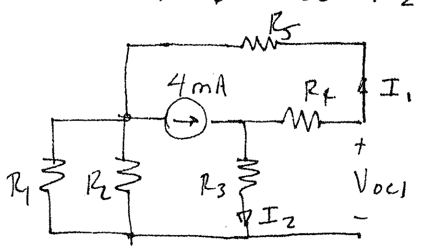


Figure P5.38

SOLUTION: $R_1 = R_6 = 6k\Omega \quad R_2 = 12k\Omega \quad R_3 = R_5 = 4k\Omega \quad R_4 = 8k\Omega$

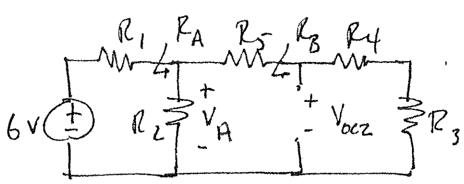


$R_A = R_1 // R_2 = 4k\Omega$

$$I_1 = \frac{4\times10^{-3}(R_3 + R_A)}{R_3 + R_A + R_4 + R_5} = 1.6mA$$

$$I_2 = \frac{4\times10^{-3}(R_4 + R_5)}{R_3 + R_A + R_4 + R_5} = 2.4mA$$

$-I_2 R_3 + I_1 R_4 + V_{oc1} = 0 \qquad V_{oc1} = -3.2V$



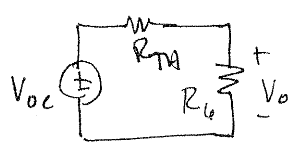
$R_B = R_3 + R_4 = 12k\Omega$

$R_A = R_2 // (R_5 + R_B) = 6.86k\Omega$

$V_A = 6 R_A / (R_1 + R_A) = 3.2V \qquad V_{oc2} = \dfrac{V_A R_B}{R_B + R_5} = 2.4V$

$R_{TH} = (R_3 + R_4) // [(R_1 // R_2) + R_5] = 4.8k\Omega$

$V_{oc} = -0.8V$



$V_o = \dfrac{V_{oc} R_6}{R_6 + R_{TH}} \qquad \boxed{V_o = -0.444V}$

5.39 Find V_o in the network in Fig. P5.39 using Thévenin's theorem.

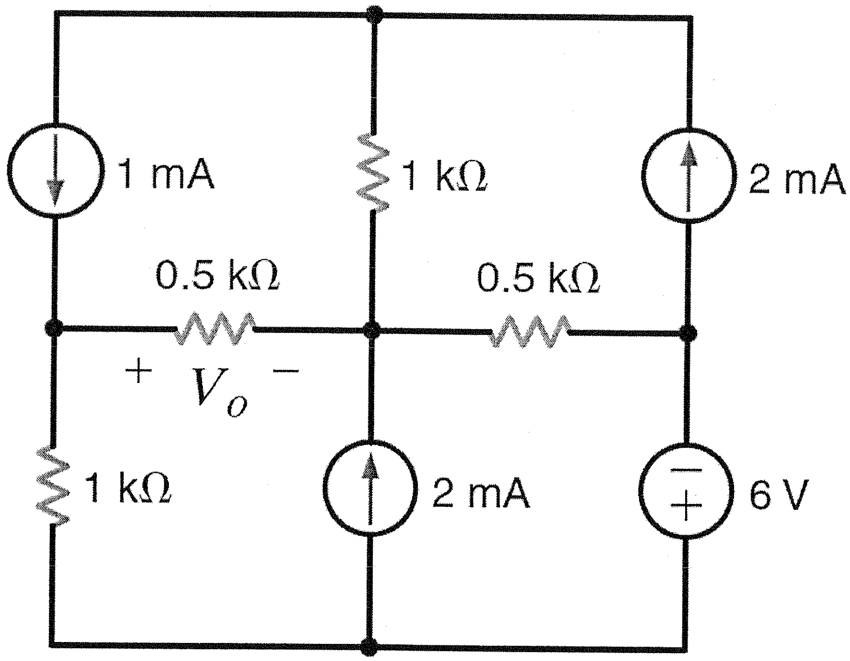
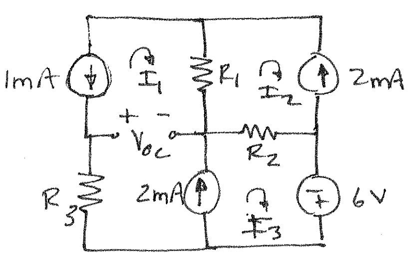


Figure P5.39

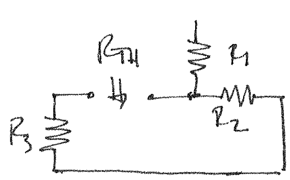
SOLUTION: $R_1 = R_3 = 1k\Omega$ $\quad R_2 = R_4 = 500\Omega$



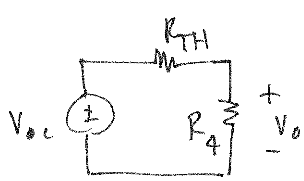
$I_1 = -1mA$ $\quad I_2 = -2mA$ $\quad I_3 - I_1 = 2mA \Rightarrow I_3 = 1mA$

$I_1 R_3 + V_{oc} + R_2(I_3 - I_2) = 6$

$V_{oc} = 5.5V$



$R_{TH} = R_3 + R_2 = 1.5k\Omega$



$$V_o = \frac{V_{oc} R_4}{R_4 + R_{TH}}$$

$\boxed{V_o = 1.375\,V}$

5.40 Find V_o in the network in Fig. P5.40 using Thévenin's theorem.

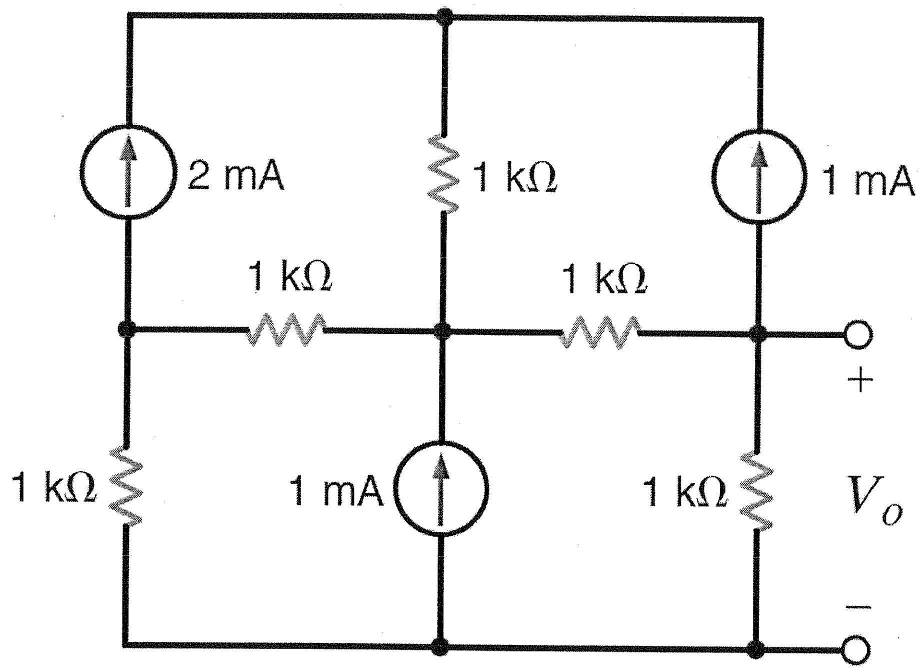
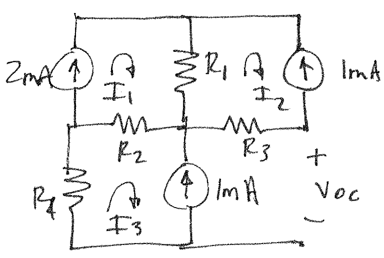


Figure P5.40

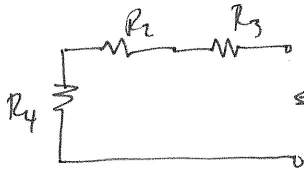
SOLUTION: All $R = 1k\Omega$



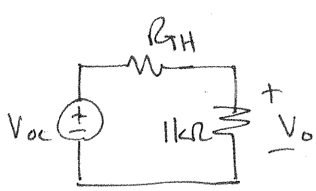
$I_1 = 2mA \quad I_2 = -1mA \quad I_3 = -1mA$

$R_4 I_3 + R_2(I_3 - I_1) + R_3(0 - I_2) + V_{oc} = 0$

$V_{oc} = 3V$



$\leftarrow R_{TH} = R_2 + R_3 + R_4 = 3k\Omega$

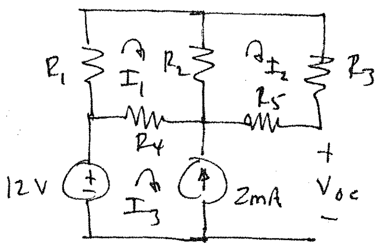


$$V_o = V_{oc}\frac{(1000)}{1000 + R_{TH}}$$

$$\boxed{V_o = 0.75V}$$

5.41 Solve Problem 5.12 using Thévenin's theorem.

SOLUTION: $R_1 = R_3 = R_5 = R_6 = 1k\Omega \qquad R_2 = R_4 = 2k\Omega$

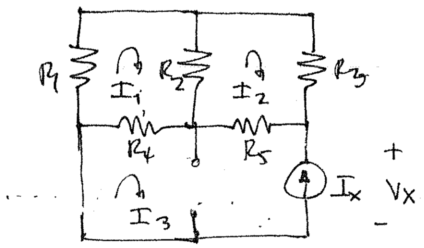


$$-R_2 I_1 + I_2(R_2 + R_3 + R_5) = 0$$

$$I_1(R_1 + R_2 + R_4) - I_2 R_2 - I_3 R_4 = 0$$

$$I_3 = -2mA$$

$$12 = (I_3 - I_1)R_4 - R_5 I_2 + V_{oc} \qquad V_{oc} = 13.5V$$



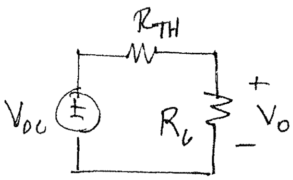
$$I_1(R_1 + R_2 + R_4) - R_2 I_2 - R_4 I_3 = 0$$

$$-R_2 I_1 + I_2(R_2 + R_3 + R_5) - R_5 I_3 = 0$$

$$I_3 = -I_x \qquad \text{Let } I_x = 1mA$$

$$R_{TH} = V_x / I_x \qquad R_{TH} = 1.19k\Omega$$

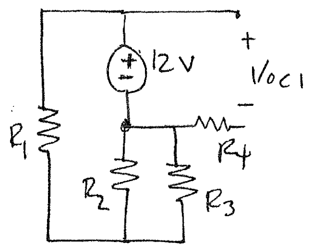
$$V_x + R_4(I_3 - I_1) + R_5(I_3 - I_2) = 0$$



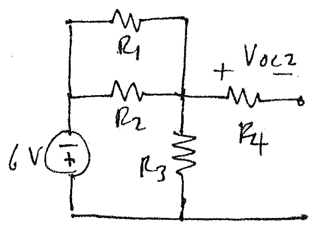
$$V_o = V_{oc}\frac{R_6}{R_6 + R_{TH}} \qquad \boxed{V_o = 6.18V}$$

5.42 Solve Problem 5.13 using Thévenin's theorem.

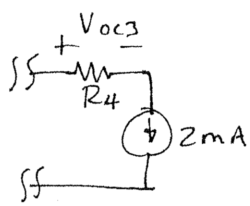
SOLUTION: $R_1 = R_2 = R_3 = 1k\Omega \qquad R_4 = R_5 = 2k\Omega$



$V_{oc1} = 12V$

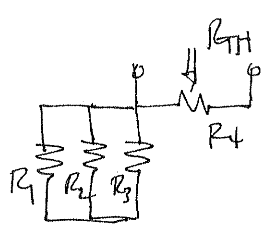


$V_{oc2} = 0V$

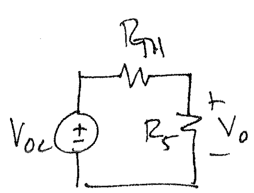


$V_{oc3} = 2\times10^{-3} R_4 = 4V$

$V_{oc} = 16V$



$R_{TH} = R_4 = 2k\Omega$

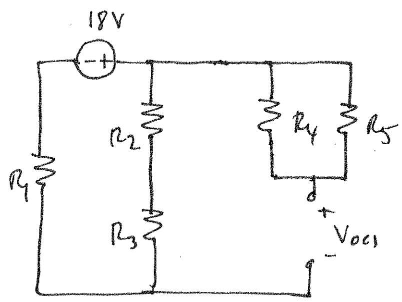


$$V_o = \frac{V_{oc} R_5}{R_5 + R_{TH}}$$

$\boxed{V_o = 8V}$

5.43 Use Thévenin's theorem to solve Problem 5.21.

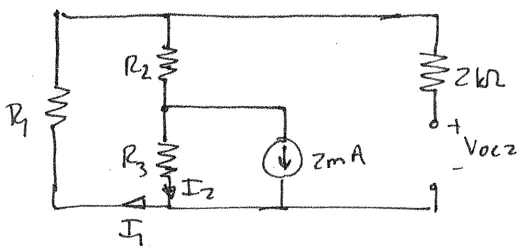
SOLUTION: $R_1 = R_2 = R_3 = 6k\Omega \quad R_4 = R_5 = 4k\Omega \quad R_6 = 3k\Omega$



$$V_{oc1} = 18\left[\frac{R_2+R_3}{R_1+R_2+R_3}\right] = 12V$$

$$R_4 // R_5 = 2k\Omega$$

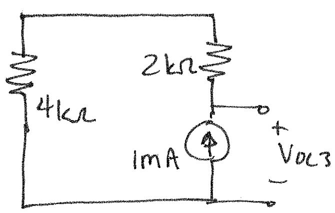
$$R_1 // (R_2+R_3) = 4k\Omega$$



$$I_1 = 2\times10^{-3} R_3 / (R_1+R_2+R_3) = \frac{2}{3} mA$$

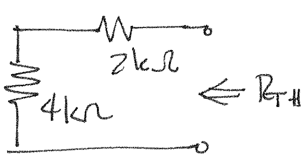
$$I_2 = -2\times10^{-3}(R_1+R_2)/(R_1+R_2+R_3) = -\frac{4}{3} mA$$

$$V_{oc2} = I_1 R_2 + I_2 R_3 = -4V$$

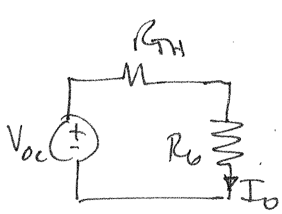


$$V_{oc3} = 10^{-3}(2000 + 4000) = 6V$$

$$V_{oc} = V_{oc1} + V_{oc2} + V_{oc3} = 14V$$



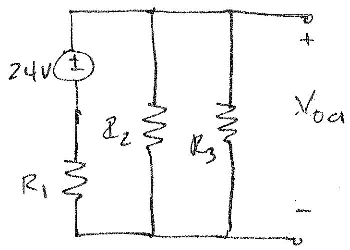
$$R_{TH} = 6k\Omega.$$



$$I_o = \frac{V_{oc}}{R_6 + R_{TH}} \qquad \boxed{I_o = 1.56mA}$$

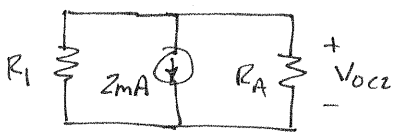
5.44 Use Thévenin's theorem to solve Problem 5.22.

SOLUTION: $R_1 = 3\,k\Omega$ $R_2 = 6\,k\Omega$ $R_3 = 2\,k\Omega$ $R_4 = 4\,k\Omega$

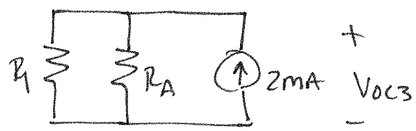


$$V_{oc1} = \frac{24\,R_A}{R_A + R_1} \qquad R_A = R_2 // R_3 = 1.5\,k\Omega$$

$$V_{oc1} = 8\,V$$



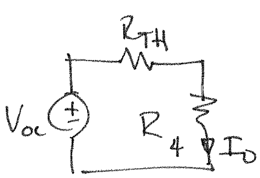
$$V_{oc2} = -2\times10^{-3}\,(R_1 // R_A) = -2V$$



$$V_{oc3} = 2\times10^{-3}\,(R_1 // R_A) = 2V$$

$$V_{oc} = V_{oc1} + V_{oc2} + V_{oc3} = 8\,V$$

$$R_{TH} = R_1 // R_A = 1\,k\Omega$$



$$I_o = \frac{V_{oc}}{R_{TH} + R_4} \qquad \boxed{I_o = 1.6\,mA}$$

5.45 Given the linear circuit in Fig. P5.45, it is known that when a 2-kΩ load is connected to the terminals A-B, the load current is 10 mA. If a 10-kΩ load is connected to the terminals, the load current is 6 mA. Find the current in a 20-kΩ load.

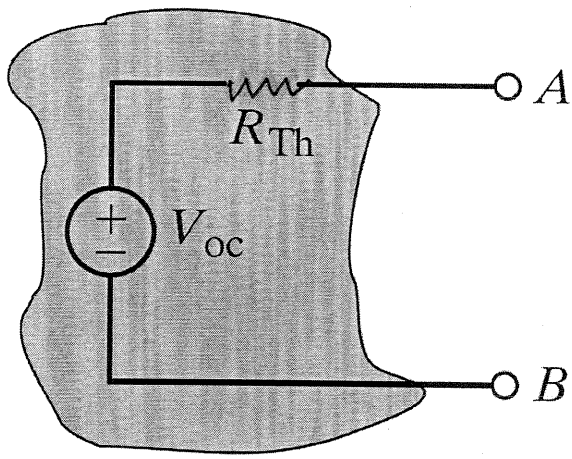
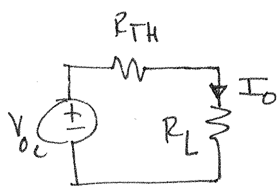


Figure P5.45

SOLUTION:



$$I_o = \frac{V_{oc}}{R_{TH} + R_L}$$

$$\frac{V_{oc}}{R_{TH} + 2000} = 10^{-2} \qquad \frac{V_{oc}}{R_{TH} + 10^4} = 6 \times 10^{-3}$$

$$\frac{R_{TH} + 2000}{R_{TH} + 10^4} = 0.6 \Rightarrow R_{TH} = 10k\Omega \;\; \& \;\; V_{oc} = 120\,V$$

$$\frac{V_{oc}}{10^4 + 2 \times 10^4} = I_o = 4mA \qquad \boxed{I_o = 4mA}$$

5.46 If an 8-kΩ load is connected to the terminals of the network in Fig. P5.46, $V_{AB} = 16$ V. If a 2-kΩ load is connected to the terminals, $V_{AB} = 8$ V. Find V_{AB} if a 20-kΩ load is connected to the terminals.

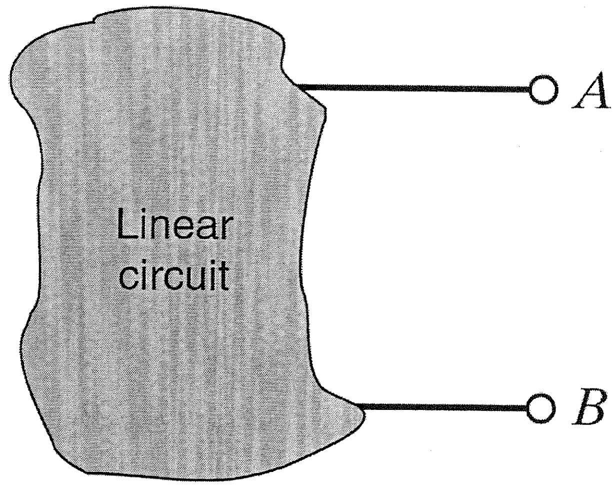
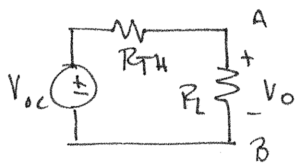


Figure P5.46

SOLUTION:



$$V_o = \frac{V_{oc} R_L}{R_L + R_{TH}}$$

$$\frac{V_{oc}(8000)}{8000 + R_{TH}} = 16 \quad \& \quad \frac{V_{oc}(2000)}{2000 + R_{TH}} = 8 \Rightarrow \frac{8000}{2000}\left(\frac{2000 + R_{TH}}{8000 + R_{TH}}\right) = \frac{16}{8} = 2$$

$R_{TH} = 4\,k\Omega$ & $V_{oc} = 24\,V$

$$V_o = 24\left(\frac{20 \times 10^3}{20 \times 10^3 + 4 \times 10^3}\right) \qquad \boxed{V_o = 20\,V}$$

5.47 Find I_o in the network in Fig. P5.47 using Norton's theorem.

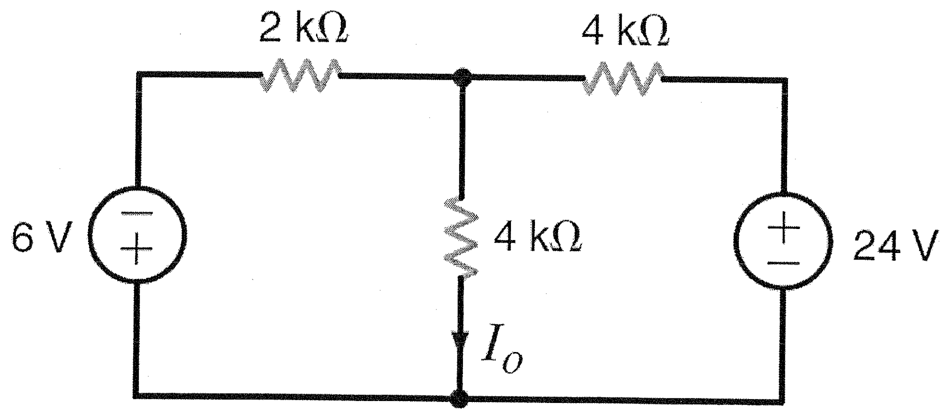
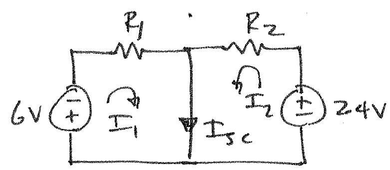


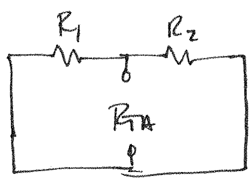
Figure P5.47

SOLUTION: $R_1 = 2k\Omega \quad R_2 = 4k\Omega \quad R_3 = 4k\Omega$

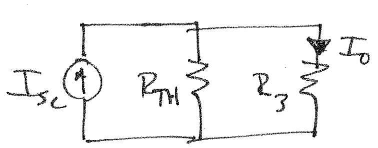


$I_{SC} = I_1 + I_2 \qquad I_1 = -\frac{6}{R_1} = -3\,mA \qquad I_2 = \frac{24}{R_2} = 6mA$

$I_{SC} = 3mA$



$R_{TH} = R_1 // R_2 = \frac{4}{3}\,k\Omega$



$I_o = \dfrac{I_{SC} R_{TH}}{R_{TH} + R_3}$

$\boxed{I_o = 0.75\,mA}$

5.48 Find I_o in the network in Fig. P5.48 using Norton's theorem.

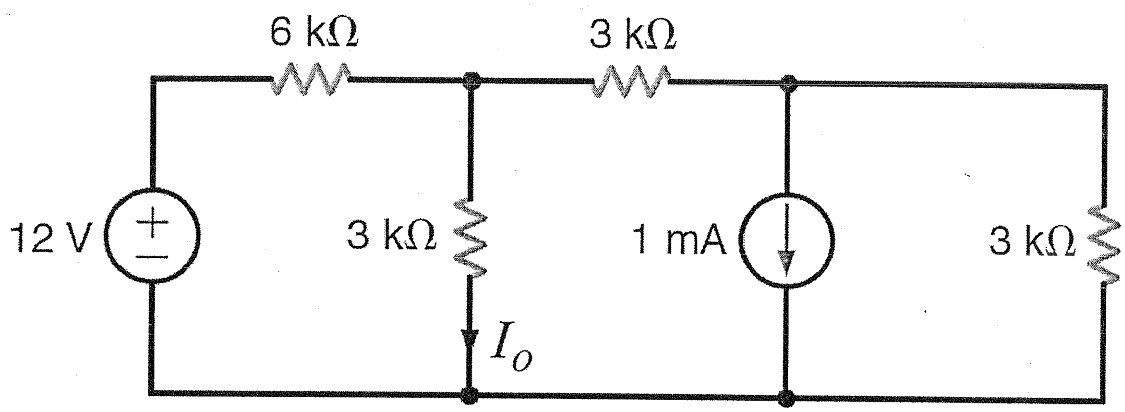
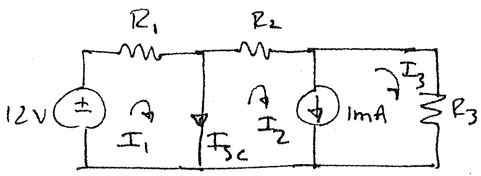


Figure P5.48

SOLUTION: $R_1 = 6k\Omega \quad R_2 = R_3 = R_4 = 3k\Omega$

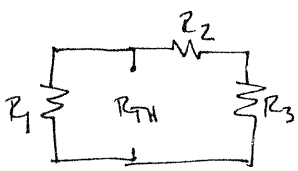


$I_{SC} = I_1 - I_2 \qquad I_2 - I_3 = 1mA$

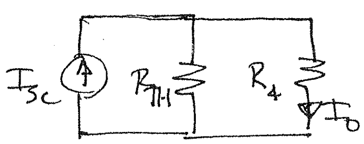
$I_1 = 12/R_1 = 2mA$

$12 = R_1I_1 + R_2I_2 + R_3I_3 \Rightarrow I_2 = \tfrac{1}{2}\ mA$

$I_{SC} = I_1 - I_2 = \frac{3}{2}\ mA$



$R_{TH} = R_1 // (R_2 + R_3) = 3k\Omega$



$$I_o = \frac{I_{SC}\, R_{TH}}{R_4 + R_{TH}}$$

$\boxed{I_o = 0.75mA}$

5.49 Use Norton's theorem to find I_o in the circuit in Fig. P5.49.

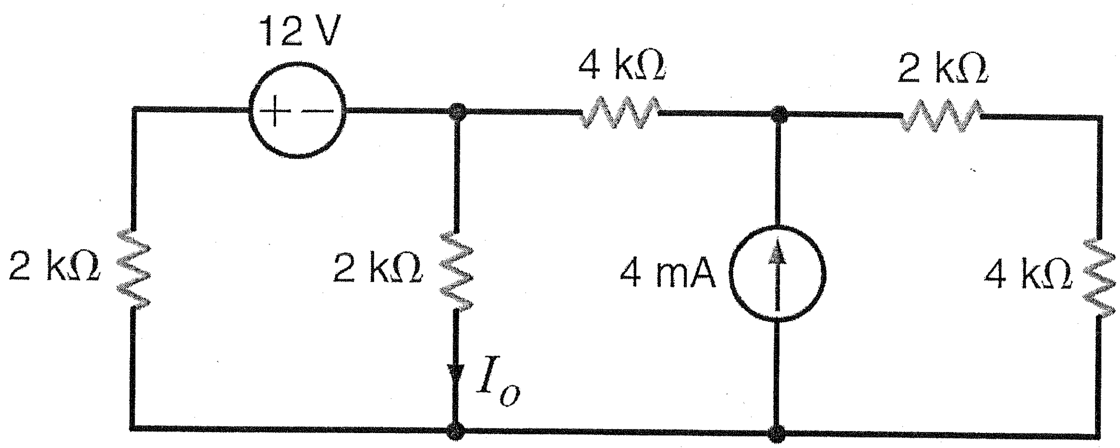
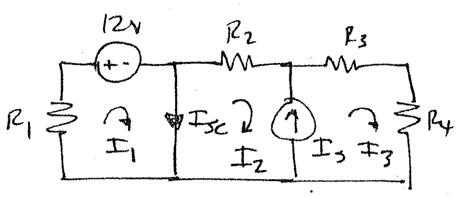


Figure P5.49

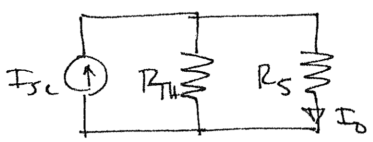
SOLUTION: $R_1 = R_3 = 2k\Omega$ $\quad R_2 = R_4 = 4k\Omega$ $\quad R_5 = 2k\Omega$ $\quad I_S = 4mA$



$I_1 = -12/R_1 = -6\,mA$

$$\left.\begin{aligned} R_2 I_2 + (R_3 + R_4) I_3 &= 0 \\ I_3 - I_2 &= 4mA \end{aligned}\right\} \quad I_2 = -2.4mA$$

$I_{SC} = I_1 - I_2 = -3.6mA$ $\quad R_{TH} = R_1 \,//\, (R_2 + R_3 + R_4) = 5/3\ k\Omega$



$I_o = \dfrac{I_{SC} R_{TH}}{R_{TH} + R_5}$ $\quad \boxed{I_o = -1.64mA}$

5.50 Find I_o in the network in Fig. P5.50 using Norton's theorem. **CS**

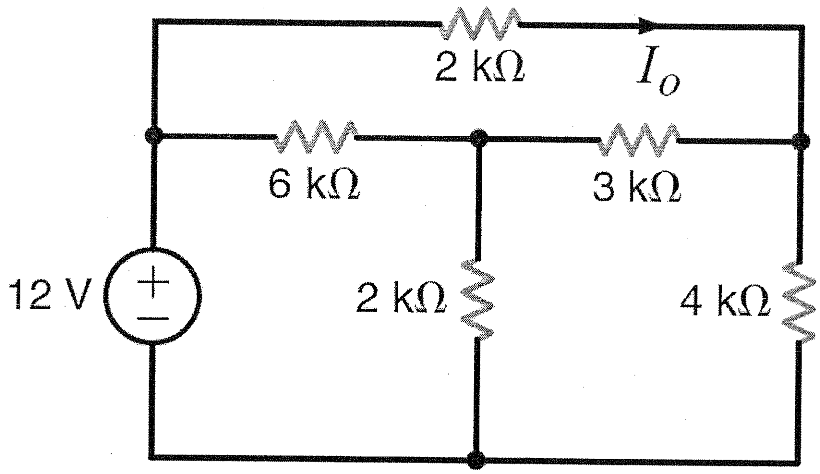
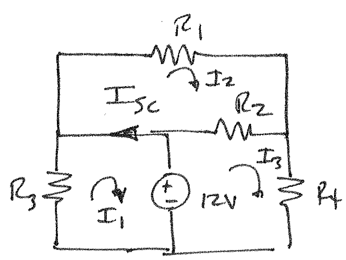


Figure P5.50

SOLUTION: $R_1 = 3\,k\Omega$ $R_2 = 6\,k\Omega$ $R_3 = 4\,k\Omega$ $R_4 = 2\,k\Omega$ $R_5 = 2\,k\Omega$

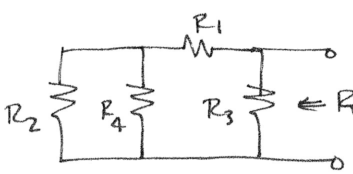
Redraw



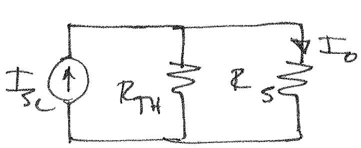
$$I_1 = -12/R_2 = -3\,mA$$

$$\left.\begin{aligned}(R_1+R_2)\,I_2 - R_2 I_3 &= 0\\ 12 &= (R_2+R_4)\,I_3 - R_2 I_2\end{aligned}\right\}\quad I_2 = 2\,mA$$

$$I_{SC} = I_2 - I_1 = 5\,mA$$



$$\leftarrow R_{TH} = R_3 \,//\, [R_1 + (R_2 \,//\, R_4)] = 2.12\,k\Omega$$



$$I_o = \frac{I_{SC}\,R_{TH}}{R_{TH}+R_5}$$

$$\boxed{I_o = 2.57\,mA}$$

5.51 Use Norton's theorem to find V_o in the network in Fig. P5.51.

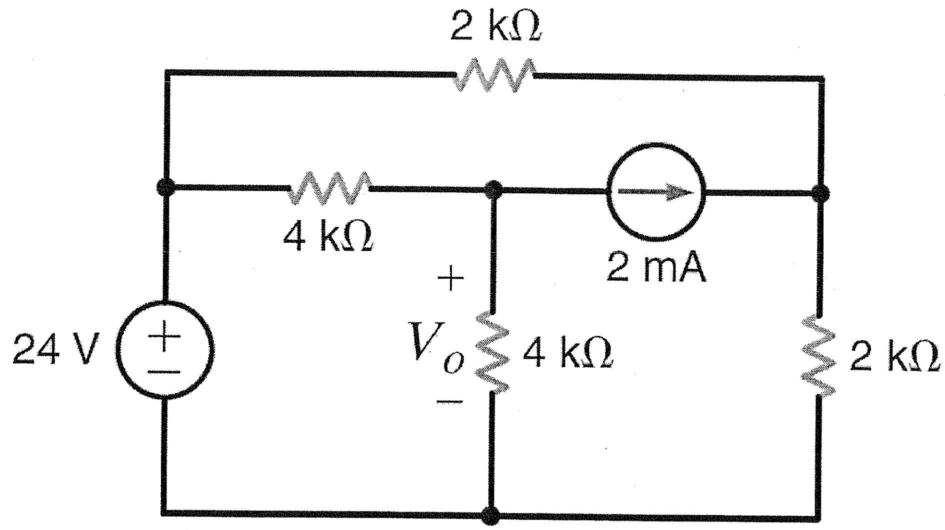
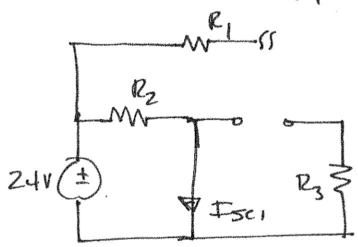
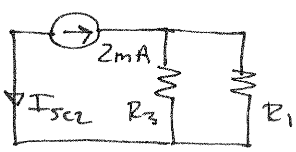


Figure P5.51

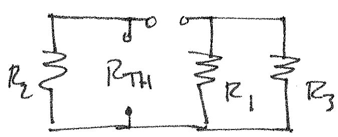
SOLUTION: $R_1 = R_3 = 2k\Omega \qquad R_2 = R_4 = 4k\Omega$

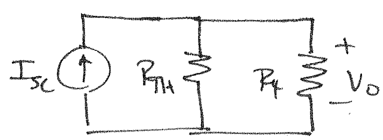


$I_{SC1} = 24/R_2 = 6mA$



$I_{SC2} = -2mA \qquad I_{SC} = I_{SC1} + I_{SC2} = 4mA$





$R_{TH} = R_2 = 4k\Omega$

$V_o = I_{SC}[R_{TH} // R_4] \Rightarrow \boxed{V_o = 8V}$

5.52 Find V_o in the network in Fig. P5.52 using Norton's theorem.

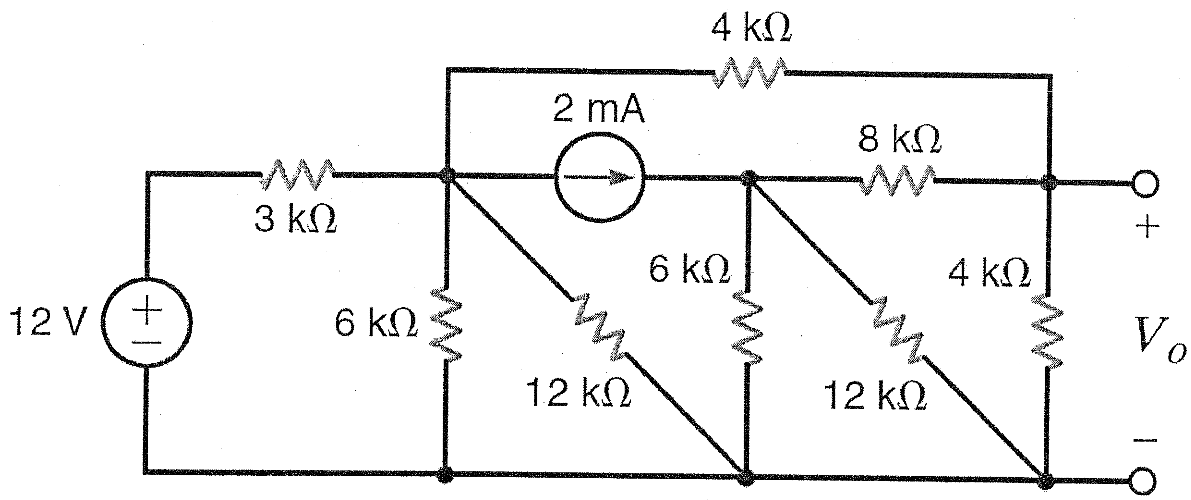
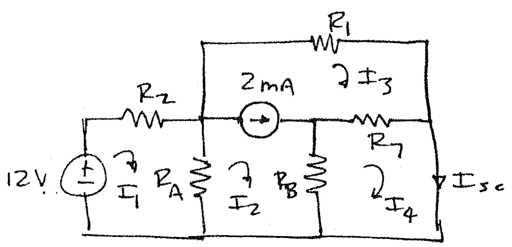


Figure P5.52

SOLUTION:



$R_2 = 3k\Omega$ $\quad R_3 = R_5 = 6k\Omega$ $\quad R_4 = R_6 = 12k\Omega$

$R_1 = 4k\Omega$ $\quad R_7 = 8k\Omega$ $\quad R_8 = 4k\Omega$

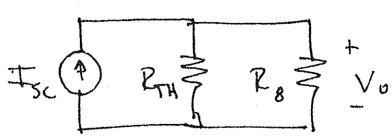
$R_A = R_3 // R_4 = 4k\Omega$ $\quad R_B = R_5 // R_6 = 4k\Omega$

$I_{sc} = I_4$

$$12 = I_1 (R_2 + R_A) - R_A I_2$$
$$I_2 - I_3 = 2mA$$
$$I_4 (R_B + R_7) - R_B I_2 - R_7 I_3 = 0$$
$$12 = I_1 R_2 + I_3 R_1$$

$I_4 = 1.27 mA = I_{sc}$

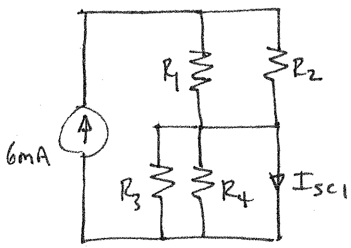
$$R_{TH} = [(R_2 // R_A) + R_1] // (R_B + R_7) = 3.87k\Omega$$



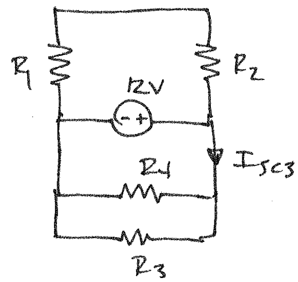
$V_o = I_{sc} (R_{TH} // R_8)$ $\quad \boxed{V_o = 2.49V}$

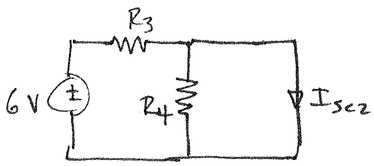
5.53 Solve Problem 5.14 using Norton's theorem.

SOLUTION: $R_1 = R_3 = R_4 = 1k\Omega \qquad R_2 = R_5 = 2k\Omega$



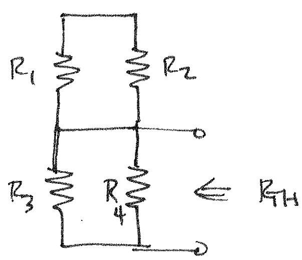
$I_{sc1} = 6mA$



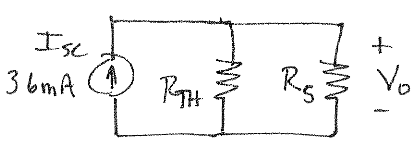


$$I_{sc2} = \frac{6}{R_3} = 6mA \qquad I_{sc3} = \frac{12}{R_3 /\!/ R_4} = 24mA$$

$$I_{sc} = I_{s1} + I_{sc2} + I_{sc3} = 36mA$$



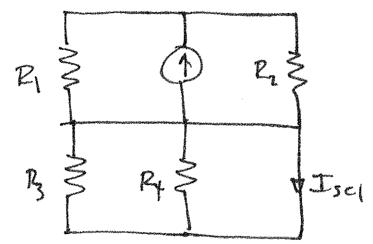
$$R_{TH} = R_3 /\!/ R_4 = \tfrac{1}{2}\ k\Omega$$



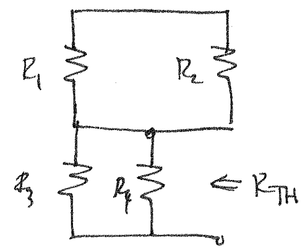
$$V_0 = I_{sc}\,(R_{TH} /\!/ R_5) \qquad \boxed{V_0 = 14.4V}$$

5.54 Solve Problem 5.15 using Norton's theorem.

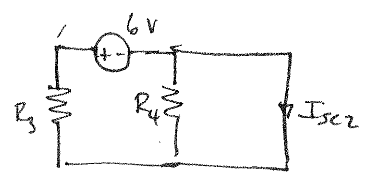
SOLUTION: $R_1 = R_3 = 2\,k\Omega \qquad R_2 = R_4 = R_5 = 1\,k\Omega$



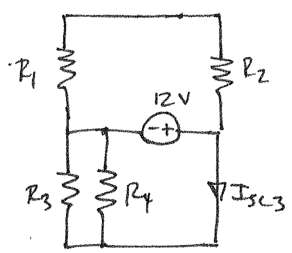
$I_{sc1} = 0\,A$



$R_{TH} = R_3 // R_4$
$= \frac{2}{3}\,k\Omega$

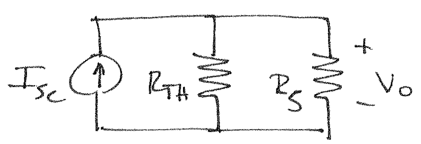


$I_{sc2} = \frac{6}{R_3} = -3\,mA$



$I_{sc3} = \frac{12}{R_3 // R_4} = 18\,mA$

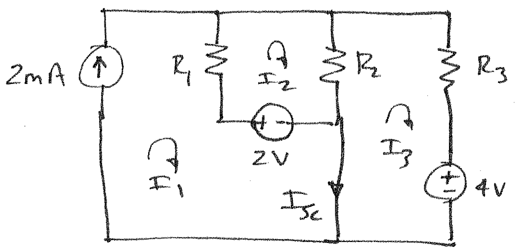
$I_{sc} = I_{sc1} + I_{sc2} + I_{sc3} = 15\,mA$



$V_o = I_{sc}\,(R_{TH} // R_5)$ $\boxed{V_o = 6\,V}$

5.55 Use Norton's theorem to solve Problem 5.19.

SOLUTION: $R_1 = R_2 = R_3 = 1k\Omega$ $\quad R_4 = 4k\Omega$



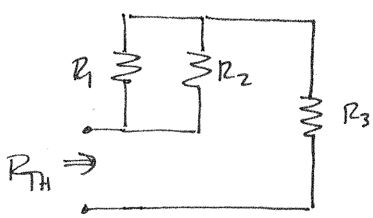
$I_1 = 2mA$

$2 = I_2(R_1 + R_2) - I_1 R_1 - I_3 R_2$

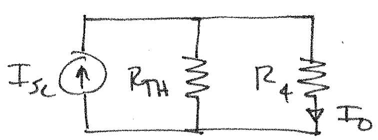
$-4 = -I_2 R_2 + I_3 (R_2 + R_3)$

$I_{SC} = I_1 - I_3 \qquad I_3 = -1.33mA$

$I_{SC} = 3.33mA$



$R_{TH} = (R_1 // R_2) + R_3 = 1.5k\Omega$



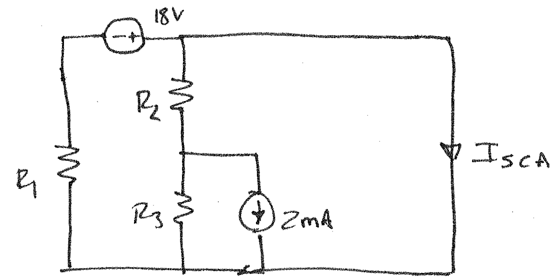
$$I_o = \frac{I_{SC} R_{TH}}{R_{TH} + R_4}$$

$$\boxed{I_o = 0.909mA}$$

5.56 Use Norton's theorem to solve Problem 5.21.

SOLUTION: $R_1 = R_2 = R_3 = 6k\Omega$ $\quad R_4 = R_5 = 4k\Omega$ $\quad R_6 = 3k\Omega$

Perform 2 Norton operations.

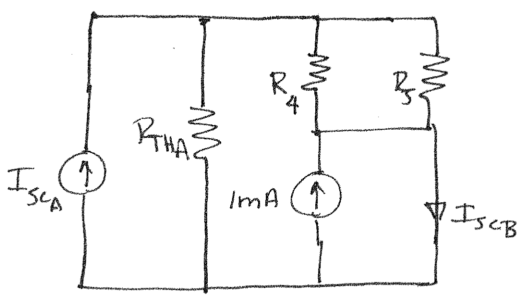


Superposition,

$$I_{SCA} = \frac{18}{R_1} - \frac{2\times10^{-3} R_3}{R_3 + R_2}$$

$$I_{SCA} = 2mA$$

$$R_{THA} = R_1 // (R_2 + R_3) = 4k\Omega$$



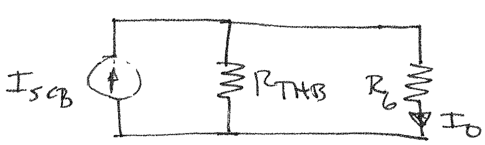
$$R_A = R_4 // R_5 = 2k\Omega$$

Superposition,

$$I_{SCB} = \frac{I_{SCA} R_{THA}}{R_{THA} + R_B} + 10^{-3}$$

$$I_{SCB} = 2.33mA$$

$$R_{THB} = R_A + R_{THA} = 6k\Omega$$



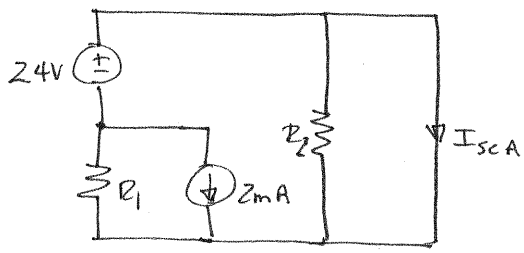
$$I_o = \frac{I_{SCB} R_{THB}}{R_{THB} + R_6}$$

$$\boxed{I_o = 1.55mA}$$

5.57 Use Norton's theorem to solve Problem 5.22.

SOLUTION: $R_1 = 3k\Omega \quad R_2 = 6k\Omega \quad R_3 = 2k\Omega \quad R_4 = 4k\Omega$

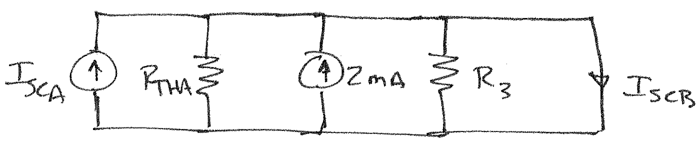
2 Norton operations



Superposition,

$$I_{SCA} = \frac{24}{R_1} - 2\times10^{-3} = 6mA$$

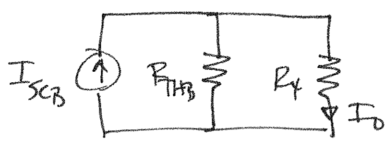
$$R_{THA} = R_1 // R_2 = 2k\Omega$$



$$I_{SCB} = I_{SCA} + 2\times10^{-3}$$

$$I_{SCB} = 8mA$$

$$R_{THB} = R_{THA}//R_3 = 1k\Omega$$



$$I_o = \frac{I_{SCB}\,R_{THB}}{R_{THB} + R_4}$$

$$\boxed{I_o = 1.6mA}$$

5.58 Find I_o in the circuit in Fig. P5.58 using Norton's theorem. **PSV**

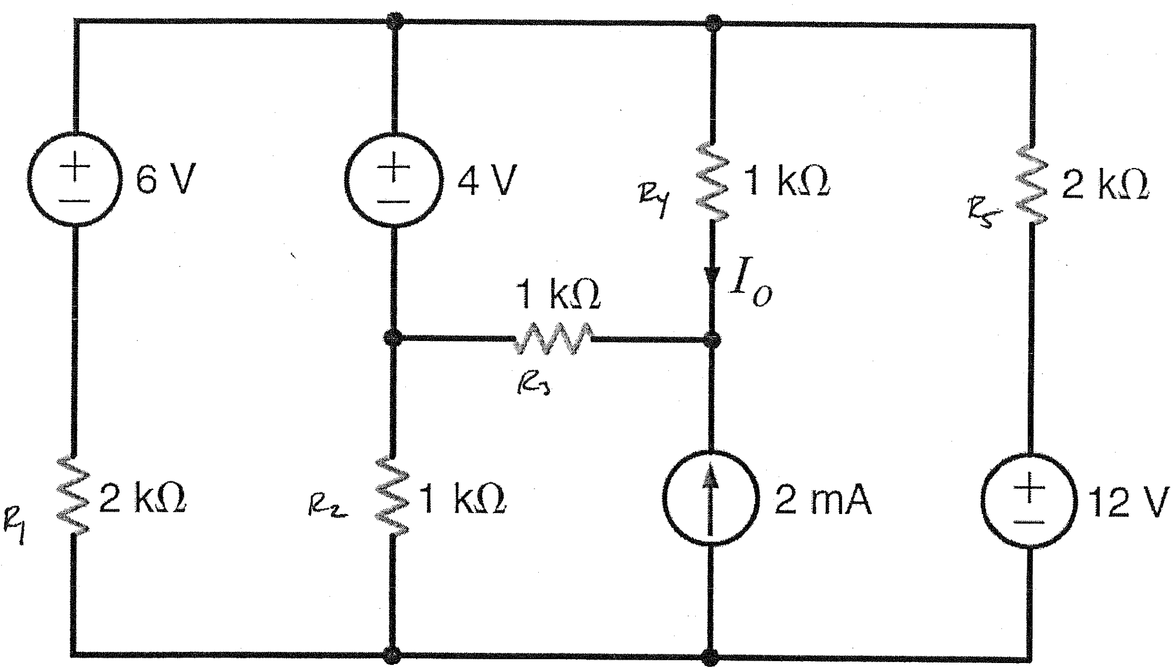
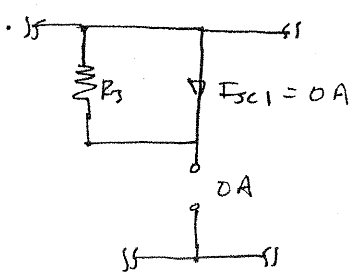
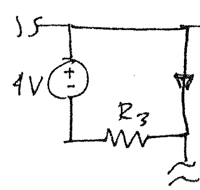


Figure P5.58

SOLUTION: Superposition. Consider effect of 6-V & 12-V sources

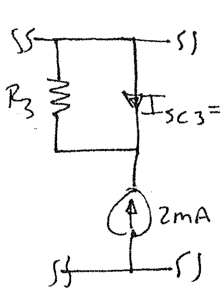


For 4-V source:



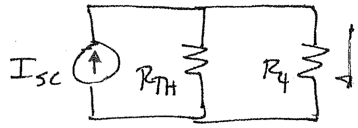
$I_{sc2} = \frac{4}{R_3} = 4\,mA$

for 2-mA Source



$I_{sc} = I_{sc1} + I_{sc2} + I_{sc3} = 2\,mA$

$R_{TH} = R_3 = 1\,k\Omega$



$I_o = \frac{I_{sc}\,R_{TH}}{R_4 + R_{TH}}$

$\boxed{I_o = 1\,mA}$

5.59 Find the Thévenin equivalent of the network in Fig. P5.59 at the terminals A-B. CS

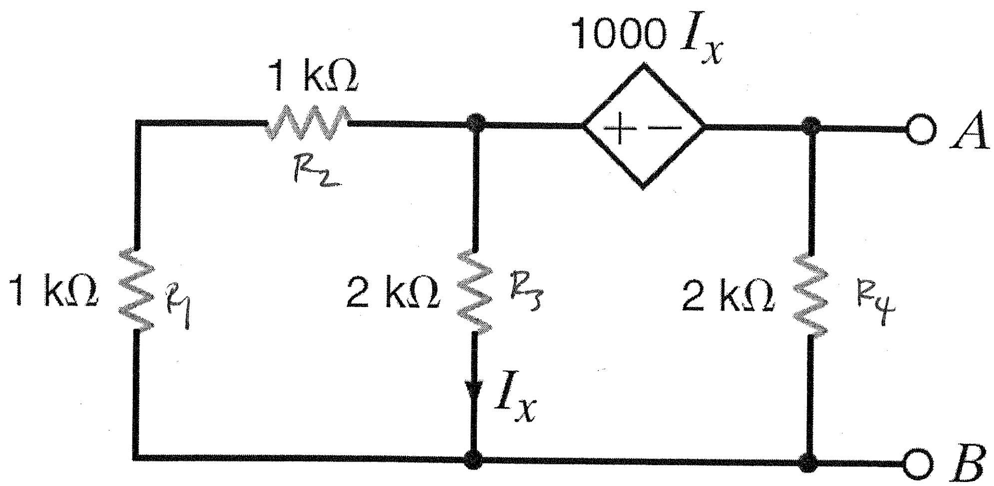
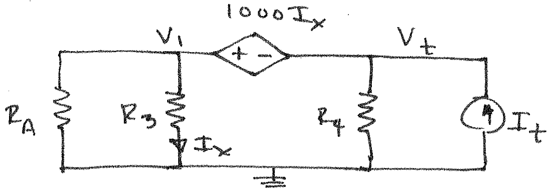


Figure P5.59

SOLUTION:

$V_{oc} = 0V$



$R_A = R_1 + R_2 = 2k\Omega$

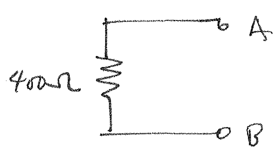
$R_{TH} = V_t / I_t$

$$\left.\begin{array}{l} V_1 - V_t = 1000 I_x \\ I_x = V_1 / R_3 \end{array}\right\} V_1 = 2V_t$$

$$\frac{V_1}{R_A} + \frac{V_1}{R_3} + \frac{V_t}{R_4} = I_t$$

$$R_{TH} = \frac{V_t}{I_t} = 400\Omega$$

Thevenin Eq.



5.60 Find the Thévenin equivalent of the network in Fig. P5.60 at the terminals *A*-*B*. **PSV**

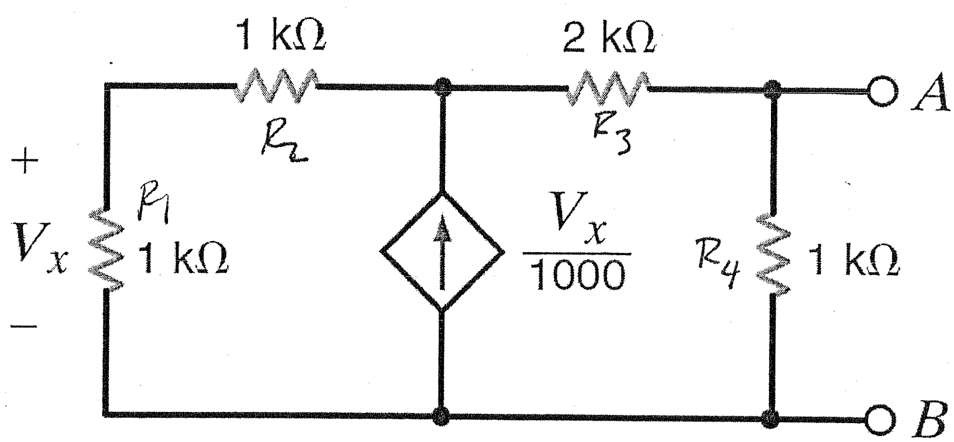
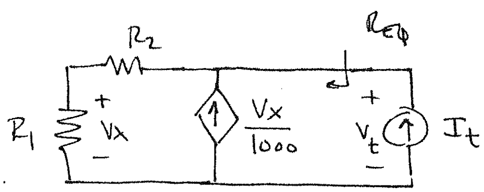


Figure P5.60

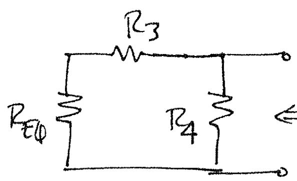
SOLUTION:



$$R_{EQ} = V_t / I_t$$

$$I_t + \frac{V_x}{1000} - \frac{V_x}{R_1} = 0 \Rightarrow I_t = 0$$

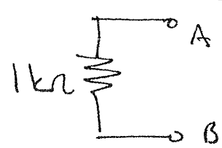
$$R_{EQ} = V_t / 0 = \infty.$$



$$\Leftarrow R_{TH} = R_4 \,//\, (R_3 + R_{EQ}) = R_4 = 1\text{k}\Omega$$

Note: With no internal independent sources, V_{oc} for this network is 0V!

<u>Thevenin Eq.</u>



5.61 Find V_o in the network in Fig. P5.61 using Thévenin's theorem.

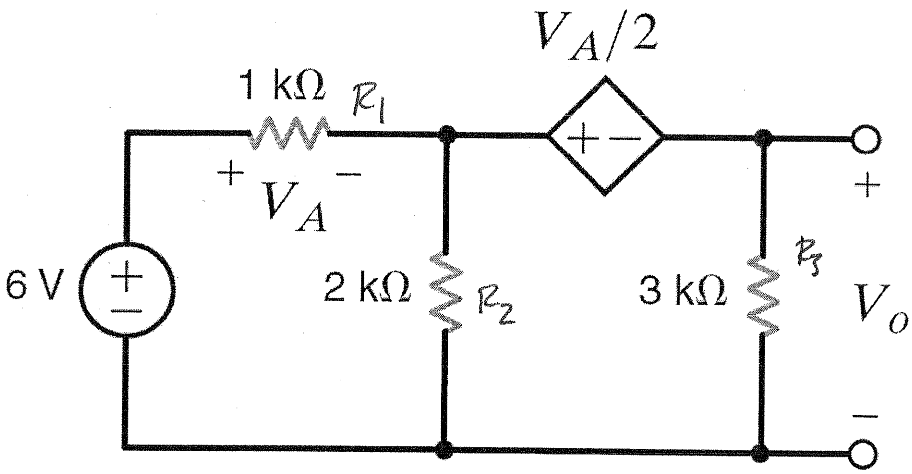
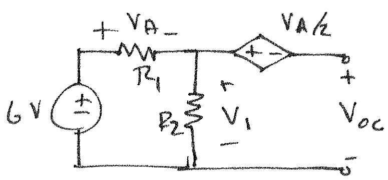


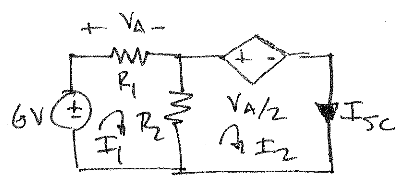
Figure P5.61

SOLUTION:



$$V_1 = \frac{6R_2}{R_1+R_2} = 4V \qquad V_A = \frac{6R_1}{R_1+R_2} = 2V$$

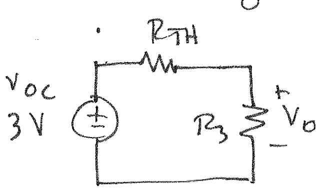
$$V_{oc} = V_1 - \frac{V_A}{2} = 3V$$



$$\left.\begin{aligned} 6 &= I_1(R_1+R_2) - I_2R_2 \\ 0 &= I_2R_2 - I_1R_2 + V_A/2 \\ V_A &= I_1R_1 \end{aligned}\right\} \; I_{SC} = I_2 = 3mA$$

$$R_{TH} = \frac{V_{oc}}{I_{SC}} = 1k\Omega$$

<u>Thevenin Eq.</u>



$$V_o = \frac{V_{oc}R_3}{R_{TH}+R_3} \qquad \boxed{V_o = 2.25V}$$

5.62 Use Thévenin's theorem to find V_o in the network in Fig. P5.62. CS

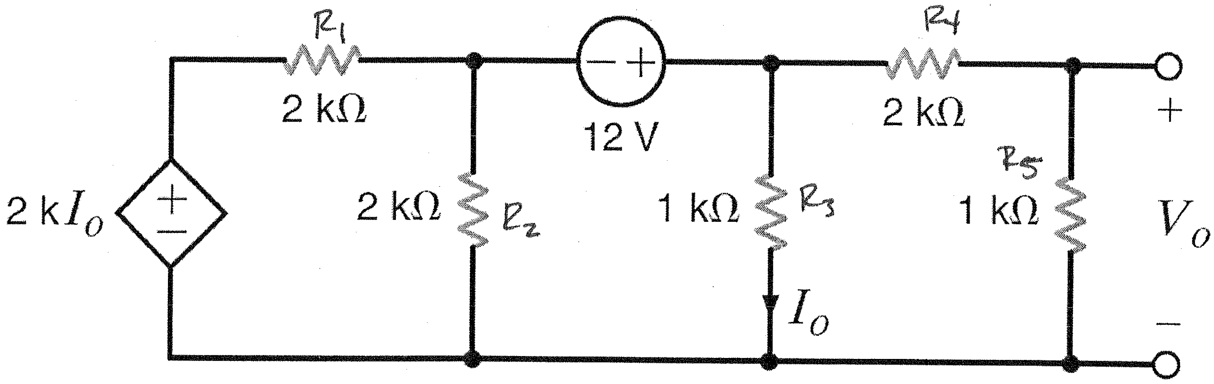
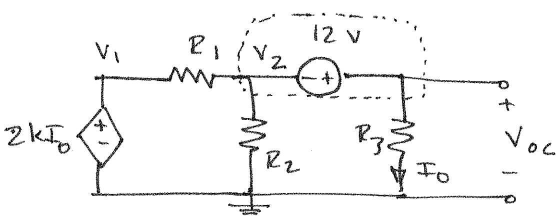


Figure P5.62

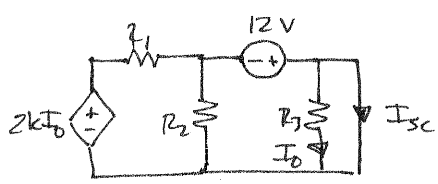
SOLUTION:



$$V_1 = 2kI_o = \frac{2kV_{oc}}{R_3} \quad \& \quad V_{oc} - V_2 = 12$$

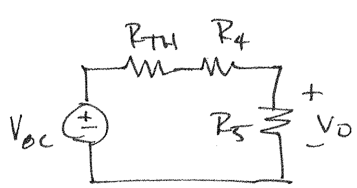
$$\frac{V_2 - V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_{oc}}{R_3} = 0$$

yields $V_{oc} = 12V$



$$I_o = 0 \Rightarrow I_{sc} = \frac{12}{R_1 // R_2} = 12mA$$

$R_{TH} = V_{oc} / I_{sc} = 1k\Omega$



$$V_o = \frac{V_{oc} R_5}{R_{TH} + R_4 + R_5}$$

$$\boxed{V_o = 3\,V}$$

5.63 Use Thévenin's theorem to find I_o in the circuit in Fig. P5.63. **PSV**

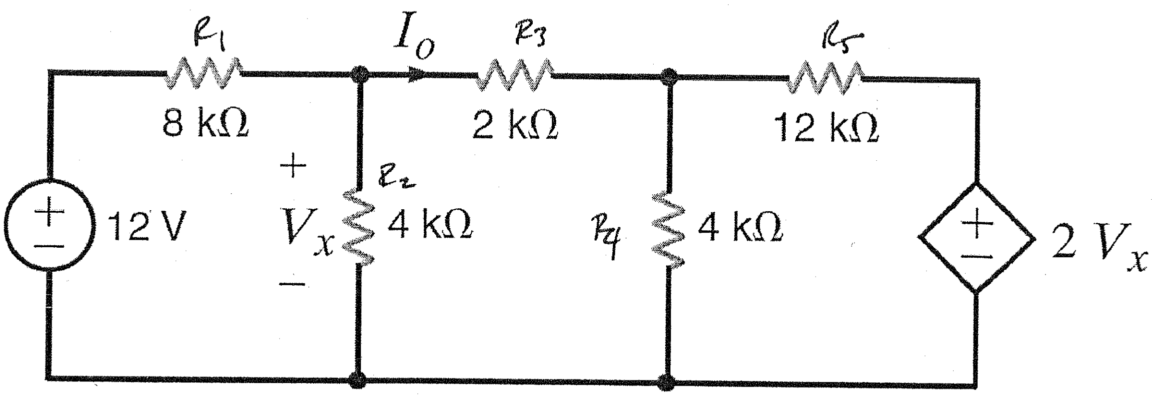
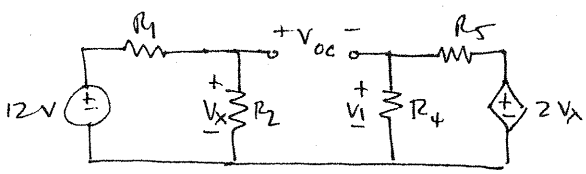


Figure P5.63

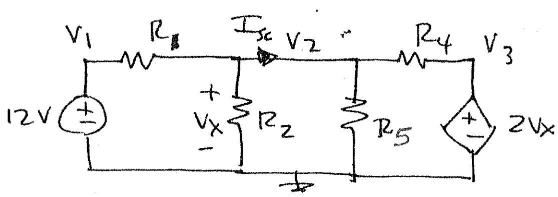
SOLUTION:



$$V_x = \frac{12R_2}{R_1+R_2} = 4V$$

$$V_1 = \frac{2V_x R_4}{R_4+R_5} = 2V$$

$$V_{oc} = V_x - V_1 = 2V$$



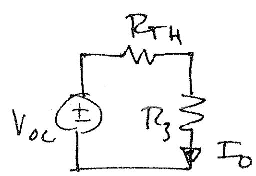
$V_1 = 12V$ $\quad V_3 = 2V_x = 2V_2$

$$\frac{V_2-V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_2}{R_5} + \frac{V_2-V_3}{R_4} = 0$$

$$V_2 = 2.77\ V$$

$$I_{sc} = \frac{V_1-V_2}{R_1} - \frac{V_2}{R_2} = 0.461mA$$

$$R_{TH} = V_{oc}/I_{sc} = 4.33k\Omega$$



$$I_o = \frac{V_{oc}}{R_{TH}+R_3}$$

$$\boxed{I_o = 0.316mA}$$

5.64 Find V_o in the network in Fig. P5.64 using Thévenin's theorem.

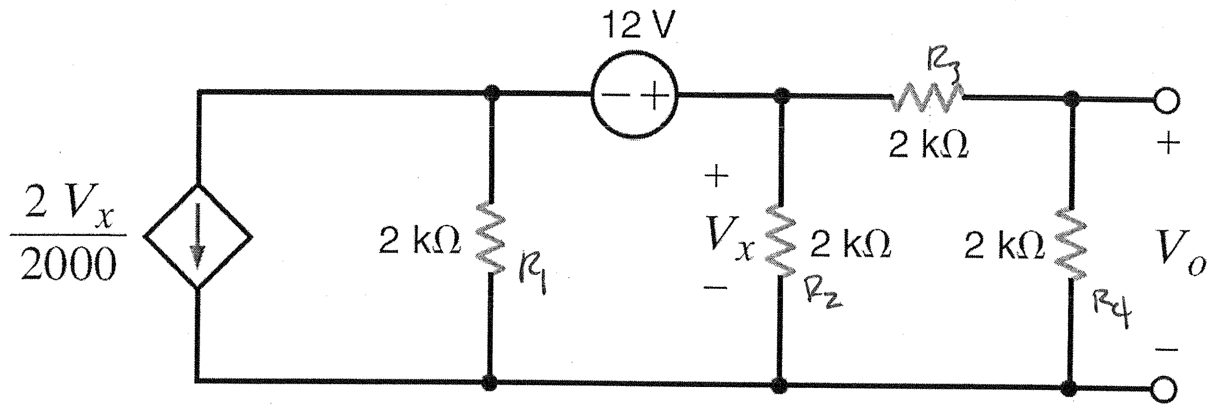
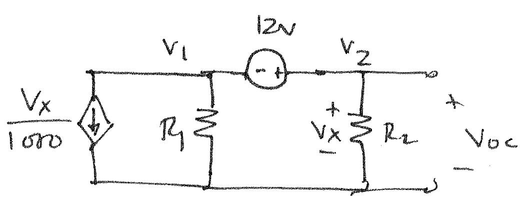


Figure P5.64

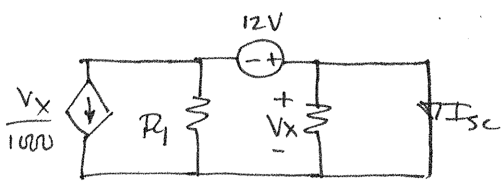
SOLUTION:



$V_2 - V_1 = 12 \qquad V_2 = V_x = V_{oc}$

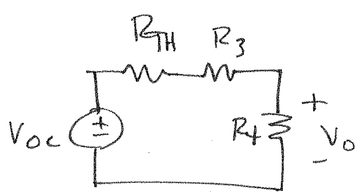
$\frac{V_x}{1000} + \frac{V_1}{R_1} + \frac{V_2}{R_2} = 0$

yields $V_2 = 3V$



$V_x = 0 \Rightarrow I_{sc} = 12/R_1 = 6\,mA$

$R_{TH} = \frac{V_{oc}}{I_{sc}} = 500\Omega$



$V_o = \frac{V_{oc} R_4}{R_{TH} + R_3 + R_4}$

$\boxed{V_o = 1.33\,V}$

5.65 Find V_o in the circuit in Fig. P5.65 using Thévenin's theorem. CS

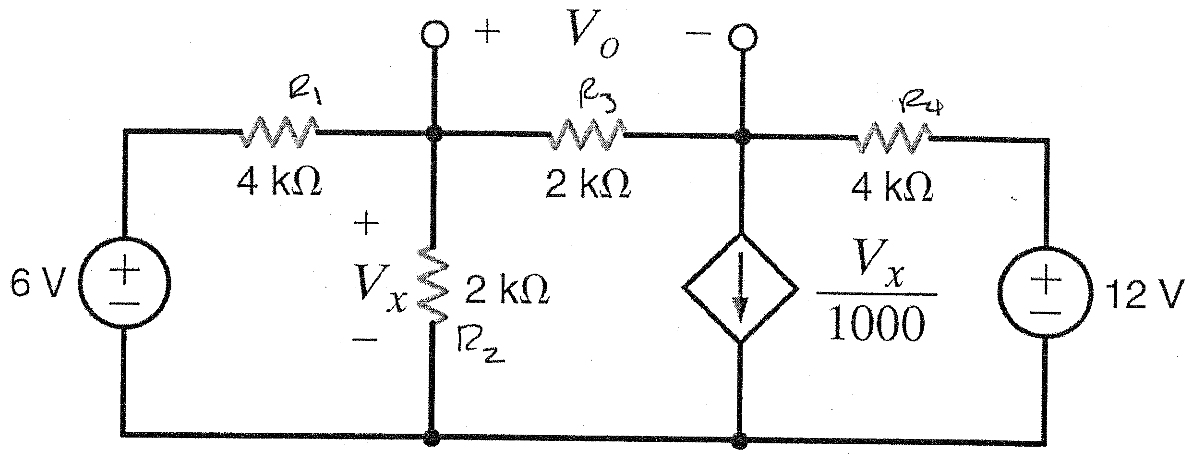
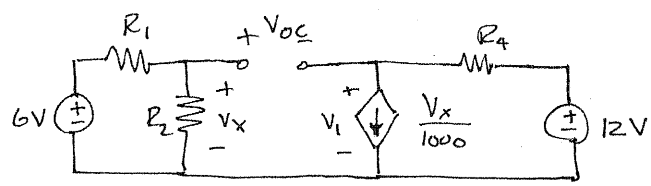


Figure P5.65

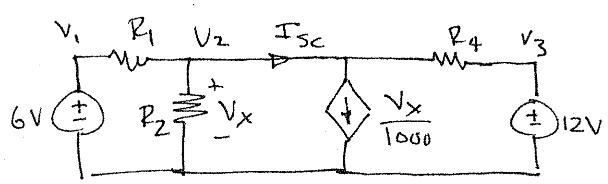
SOLUTION:



$V_x = 6R_2/(R_1+R_2) = 2V$

$V_1 = 12 - \frac{V_x R_4}{1000} = 4V$

$V_{OC} = V_x - V_1 = -2V$



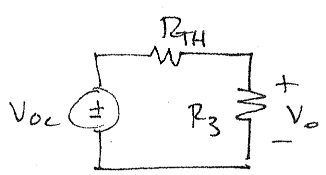
$V_1 = 6V \quad V_3 = 12V \quad V_x = V_2$

$$\frac{V_2 - V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_2}{1000} + \frac{V_2 - V_3}{R_4} = 0$$

$V_2 = 2.25V$

$$I_{SC} = \frac{V_x}{1000} + \frac{V_2 - V_3}{R_4} = -0.1875\,mA$$

$R_{TH} = V_{OC}/I_{SC} = 10.67\,k\Omega$



$$V_o = \frac{V_{OC} R_3}{R_3 + R_{TH}}$$

$$\boxed{V_o = -0.316\,V}$$

5.66 Find V_o in the network in Fig. P5.66 using Thévenin's theorem.

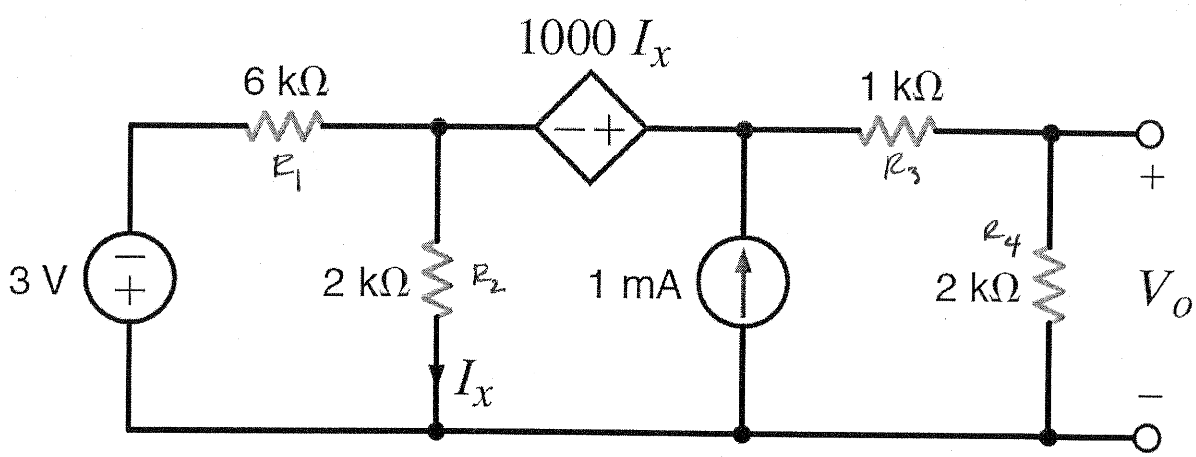
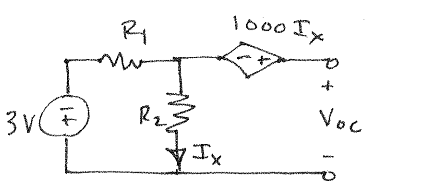


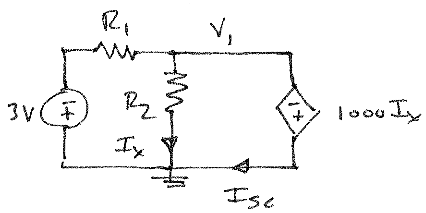
Figure P5.66

SOLUTION:



$$I_x = \frac{-3}{R_1+R_2} = -\frac{3}{8}\ mA$$

$$-I_x R_2 - 1000 I_x + V_{oc} = 0 \Rightarrow V_{oc} = -1.125V$$

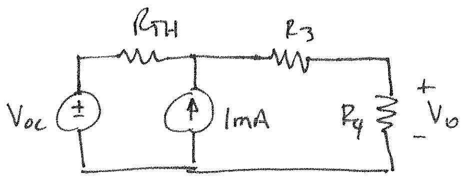


$I_x = V_1 / R_2$ & $V_1 = -1000 I_x$

only solution is $I_x = 0A$ & $V_1 = 0V$

$$I_{sc} = -3/R_1 = -0.5mA$$

$$R_{TH} = V_{oc}/I_{sc} = 2.25k\Omega$$



Superposition:

$$V_o = \frac{V_{oc} R_4}{R_{TH}+R_3+R_4} + \left(\frac{10^{-3} R_{TH}}{R_{TH}+R_3+R_4}\right) R_4$$

$$\boxed{V_o = 0.429V}$$

5.67 Use Thévenin's theorem to find V_o in the circuit in Fig. P5.67.

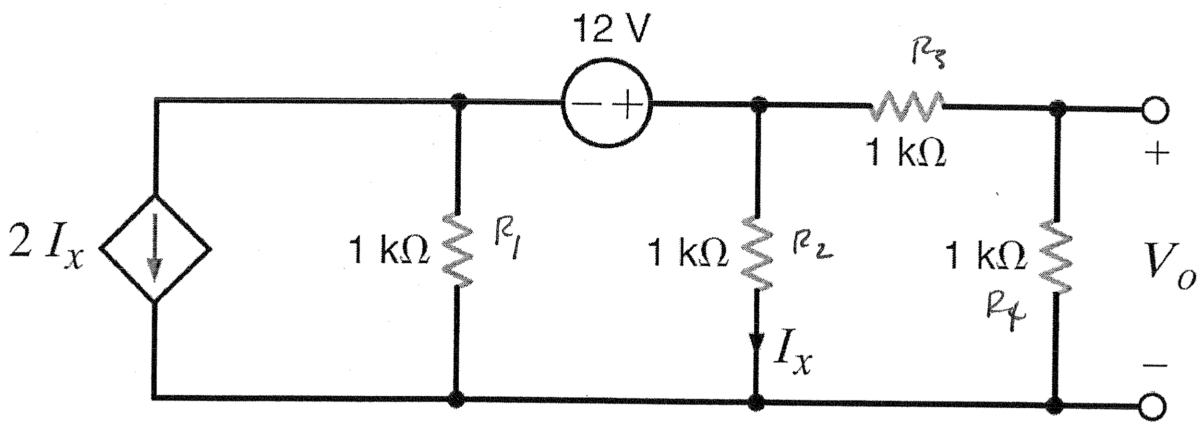
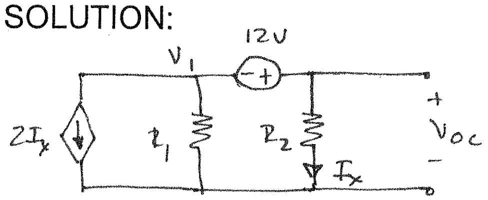


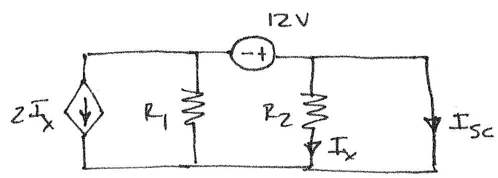
Figure P5.67

SOLUTION:



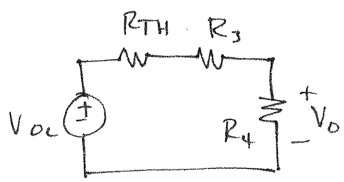
$V_{oc} - V_1 = 12 \qquad I_x = V_{oc}/R_2$

$\frac{V_1}{R_1} + \frac{V_{oc}}{R_2} + 2I_x = 0 \quad \Rightarrow \quad V_{oc} = 3\,V$



$I_x = 0 \qquad I_{sc} = 12/R_1 = 12\,mA$

$R_{TH} = V_{oc}/I_{sc} = 250\,\Omega$



$V_o = \frac{R_4 V_{oc}}{R_{TH} + R_3 + R_4}$

$\boxed{V_o = 1.33\,V}$

5.68 Use Norton's theorem to find V_o in the network in Fig. P5.68. CS

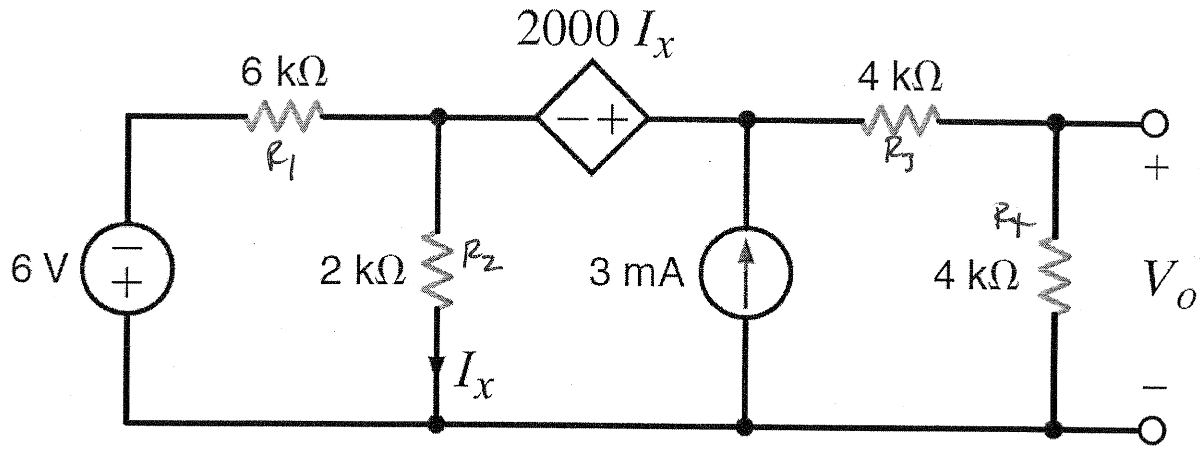
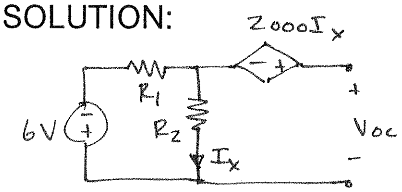


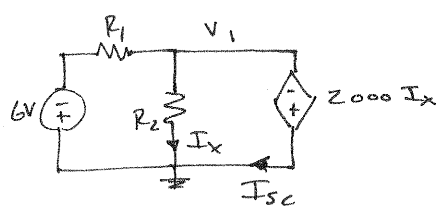
Figure P5.68

SOLUTION:



$I_x = -6/(R_1 + R_2) = -0.75\text{ mA}$

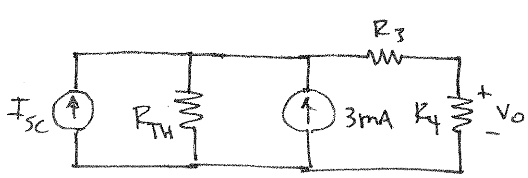
$-R_2 I_x - 2000 I_x + V_{oc} = 0 \qquad V_{oc} = -3\text{ V}$



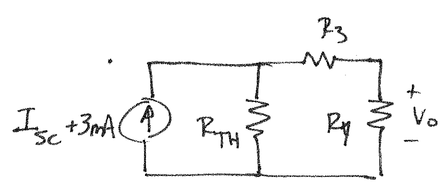
$V_1 = R_2 I_x$ & $V_1 = -2000 I_x \Rightarrow I_x = 0$ & $V_1 = 0$

$I_{sc} = -6/R_1 = -1\text{ mA}$

$R_{TH} = V_{oc}/I_{sc} = 3\text{ k}\Omega$



$\Rightarrow$



$$V_o = \frac{(I_{sc} + 3\times10^{-3})\, R_{TH}}{R_{TH} + R_3 + R_4}\, R_4$$

$$\boxed{V_o = 2.18\text{ V}}$$

5.69 Find V_o in the network in Fig. P5.69 using Thévenin's theorem.

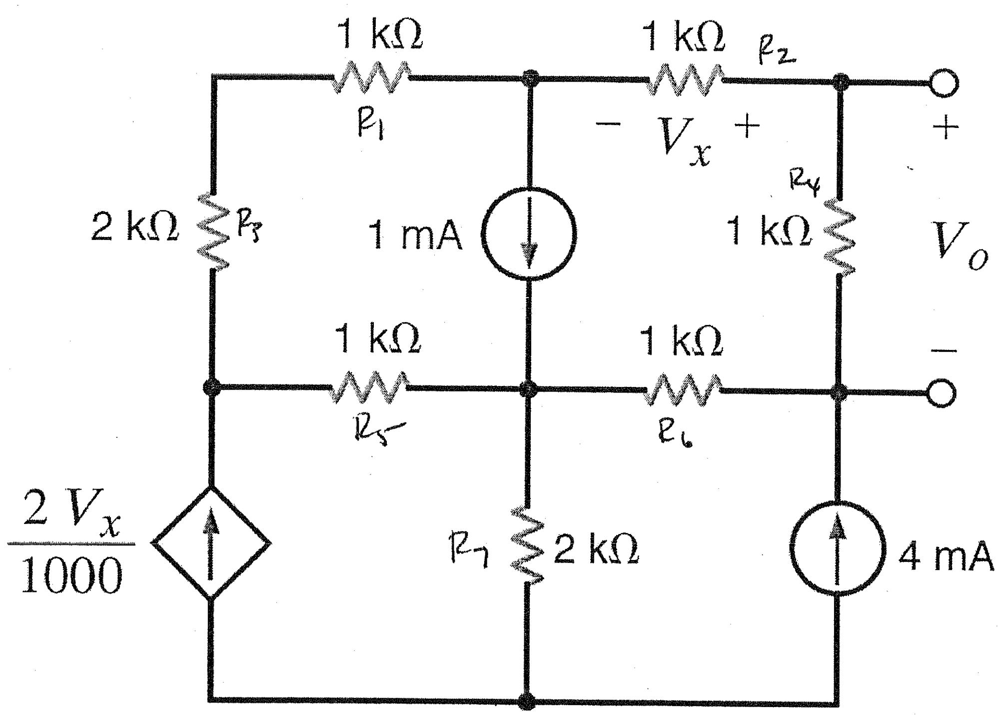
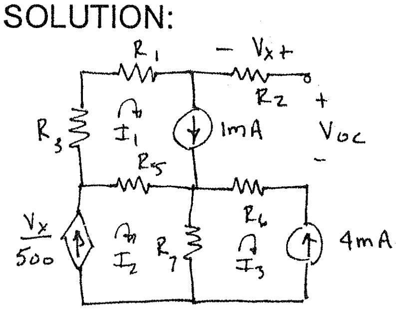
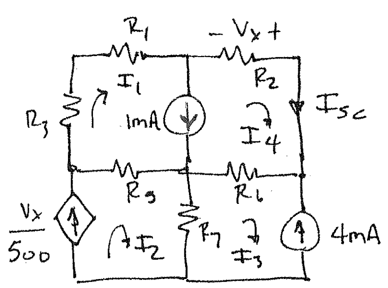


Figure P5.69

SOLUTION:



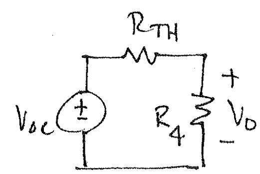
$$\left\{\begin{array}{l} I_1 = 1\,mA \quad I_2 = 2V_x \quad V_x = 0 \quad I_3 = -4mA \\ R_3 I_1 + R_1 I_1 + V_{oc} + R_6(0 - I_3) + R_5(I_1 - I_2) = 0 \\ V_{oc} = -8\,V \end{array}\right.$$



$I_1 - I_4 = 1mA \quad I_2 = 2V_x/1000 \quad V_x = -R_2 I_4 \quad I_3 = -4mA$

$(R_3 + R_1)I_1 + R_2 I_4 + R_6(I_4 - I_3) + R_5(I_1 - I_2) = 0$

$I_{sc} = -1\,mA \qquad R_{TH} = V_{oc}/I_{sc} = 8\,k\Omega$



$V_o = V_{oc} R_4/(R_{TH} + R_4)$

$$\boxed{V_o = -0.889\,V}$$

5.70 Find V_o in the circuit in Fig. P5.70 using Thévenin's theorem.

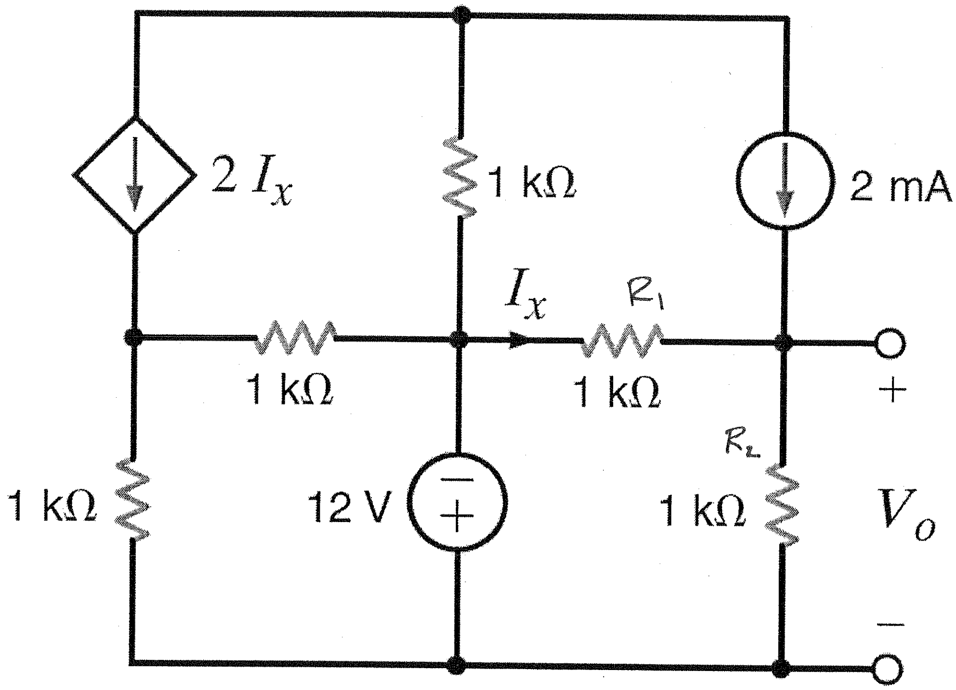
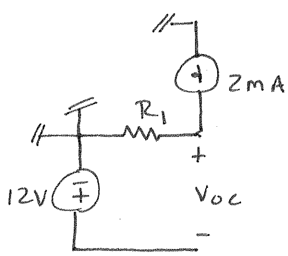


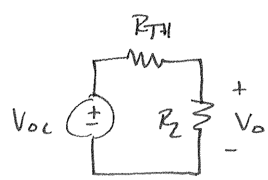
Figure P5.70

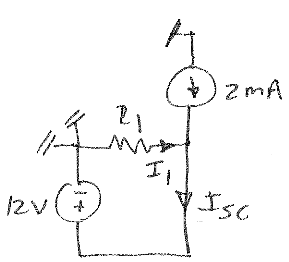
SOLUTION:



$12 - 2\times10^{-3} R_1 + V_{oc} = 0$

$V_{oc} = -10V$





$I_1 = -12/R_1 = -12mA$

$I_{SC} = 2\times10^{-3} + I_1 = -10mA$

$R_{TH} = \dfrac{V_{oc}}{I_{sc}} = 1k\Omega$

$V_o = \dfrac{V_{oc} R_2}{R_{TH} + R_2}$

$\boxed{V_o = -5V}$

5.71 Find V_o in the network in Fig. P5.71 using Thévenin's theorem.

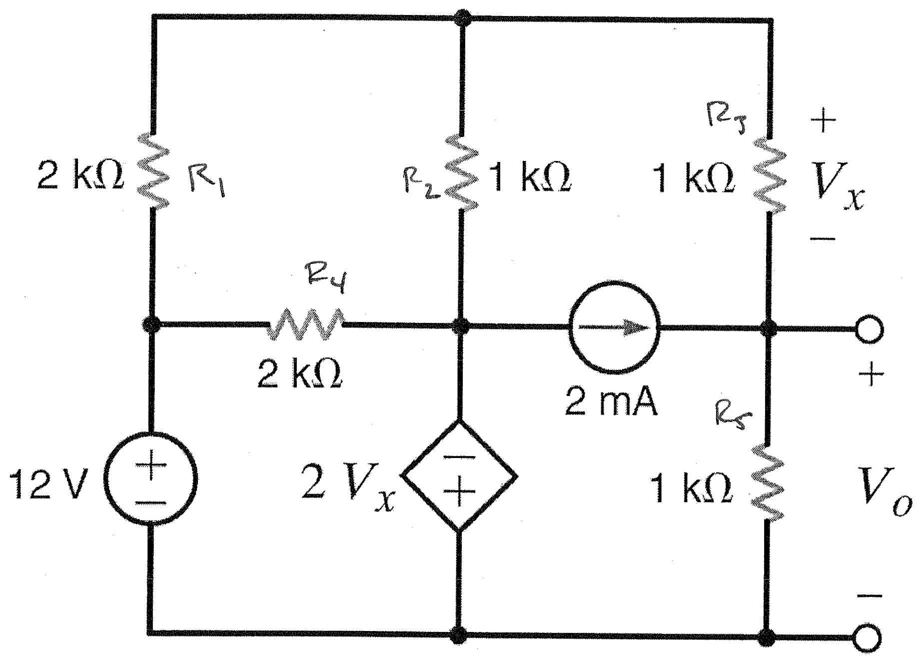
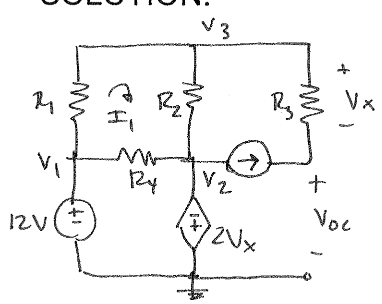


Figure P5.71

SOLUTION:

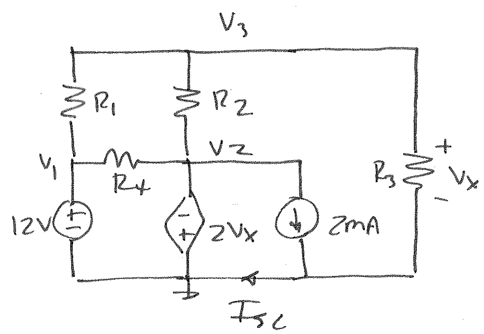


$V_1 = 12V \quad V_2 = -2V_x \quad V_x = V_3 - V_{oc}$

$(V_3 - V_{oc})/R_3 + 2\times10^{-3} = 0$

$$\frac{V_3 - V_1}{R_1} + \frac{V_3 - V_2}{R_2} + \frac{V_3 - V_{oc}}{R_3} = 0$$

yields $V_{oc} = 10V$



$V_1 = 12V \quad V_2 = -2V_x \quad V_x = V_3$

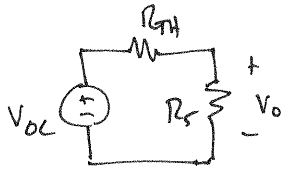
$$\frac{V_3 - V_1}{R_1} + \frac{V_3 - V_2}{R_2} + \frac{V_3}{R_3} = 0$$

yields $V_3 = 1.33V$

$I_{sc} = 2\times10^{-3} + V_3/R_3 \quad I_{sc} = 3.33mA$

cont

$R_{TH} = V_{oc}/I_{sc} = 3\,k\Omega$



$$V_o = \frac{V_{oc} R_5}{R_{TH}+R_5}$$

$$\boxed{V_o = 2.5V}$$

5.72 Use Thévenin's theorem to find V_o in the network in Fig. P5.72.

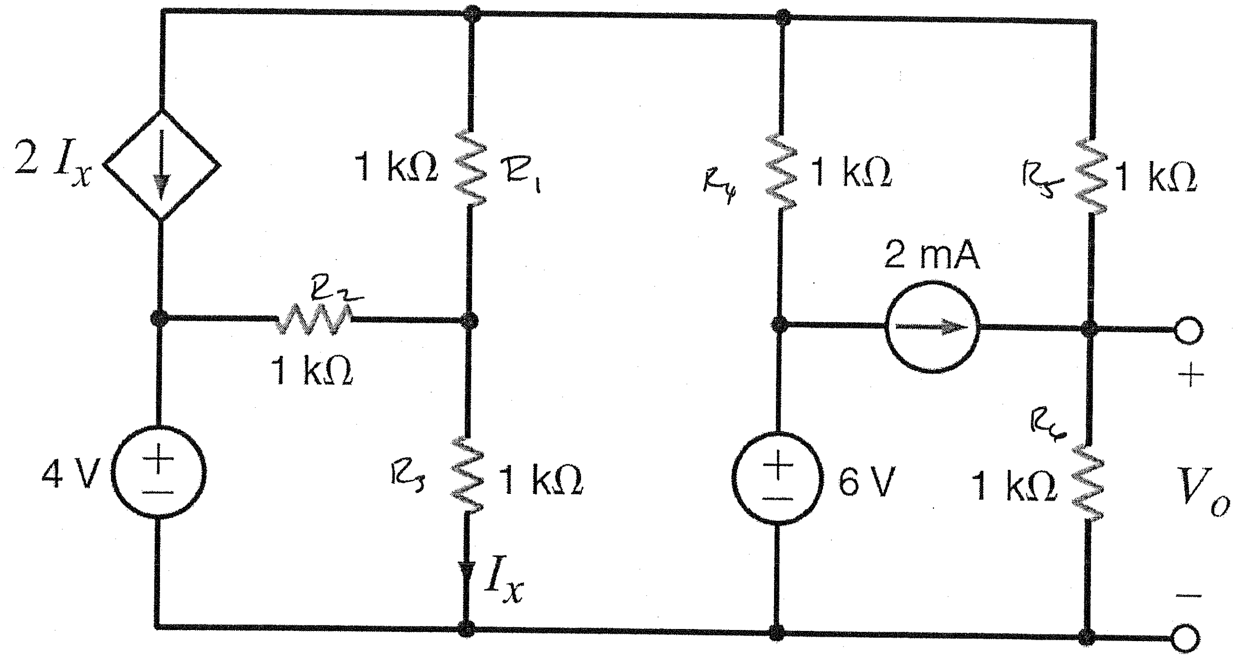
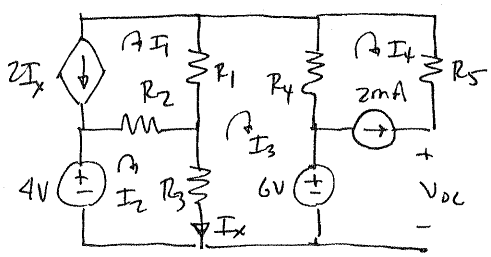


Figure P5.72

SOLUTION:



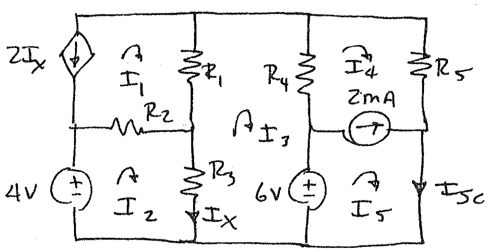
$I_1 = -2I_x \qquad I_x = I_2 - I_3 \qquad I_4 = -2mA$

$4 = (R_2 + R_3)I_2 - R_2 I_1 - R_3 I_3$

$-6 = (R_1 + R_4 + R_3)I_3 - R_3 I_2 - R_1 I_1 - I_4 R_4$

$6 = R_4(I_4 - I_3) + R_5 I_4 + V_{oc}$

yields $\quad V_{oc} = 4.86V$



$I_1 = -2I_x \qquad I_x = I_2 - I_3 \qquad I_5 - I_4 = 2mA$

$4 = (R_2 + R_3)I_2 - R_2 I_1 - R_3 I_3$

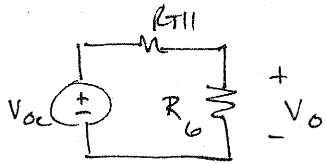
$-6 = (R_1 + R_3 + R_4)I_3 - R_1 I_1 - R_3 I_2 - R_4 I_4$

$6 = R_4(I_4 - I_3) + R_5 I_4$

yields $\quad I_{sc} = 3.4mA$

cont

$R_{TH} = V_{oc} / I_{sc} = 1.43\,k\Omega$



$$\boxed{V_o = 2.0\,V}$$

5.73 Use Thévenin's theorem to find I_o in the network in Fig. P5.73.

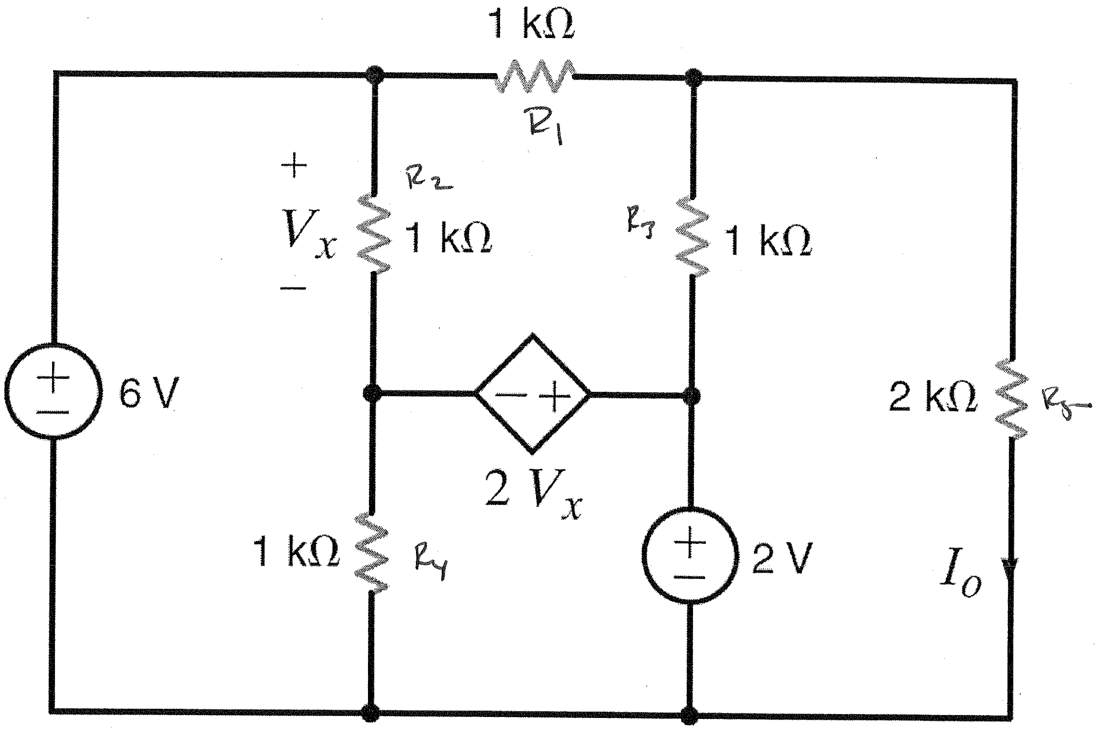
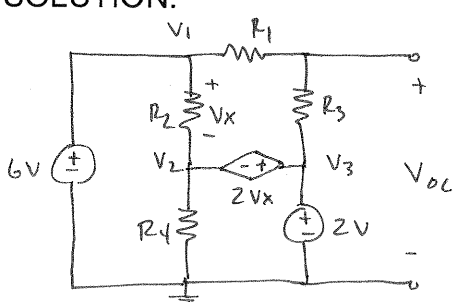


Figure P5.73

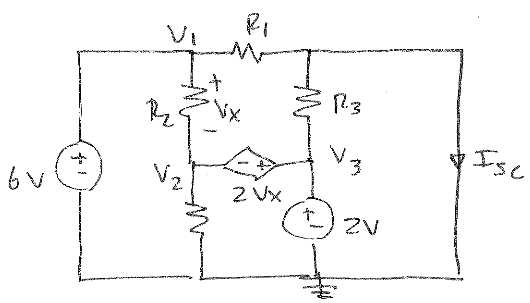
SOLUTION:



$V_1 = 6V \quad V_3 = 2V \quad V_3 - V_2 = 2V_x \quad V_x = V_1 - V_2$

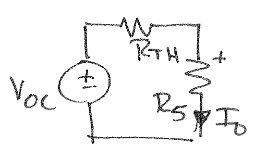
$$\frac{V_{oc} - V_1}{R_1} + \frac{V_{oc} - V_3}{R_3} = 0$$

yield $V_{oc} = 4V$.



$V_1 = 6V \quad V_3 = 2V \quad V_3 - V_2 = 2V_x \quad V_x = V_1 - V_2$

$$\frac{V_1}{R_1} + \frac{V_3}{R_3} = I_{sc}$$ yields $I_{sc} = 8mA$



$R_{TH} = V_{oc}/I_{sc} = 500\Omega$

$I_o = V_{oc}/(R_{TH} + R_5)$

$\boxed{I_o = 1.6mA}$

5.74 Using Thévenin's theorem find I_o in the circuit in Fig. P5.74.

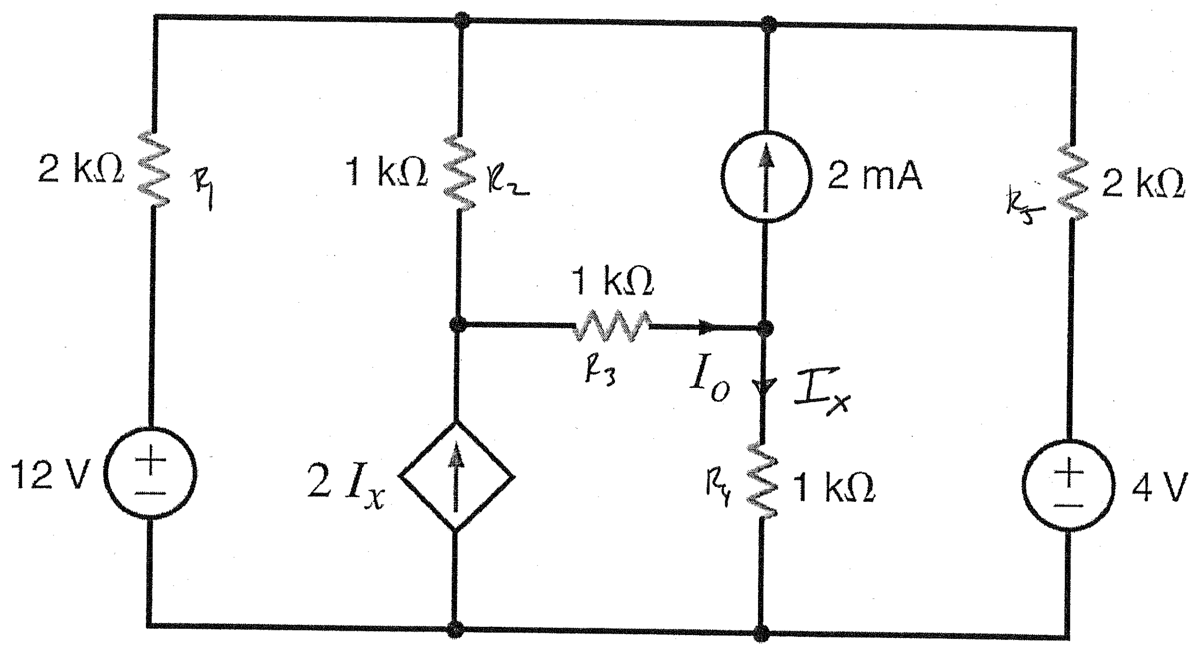
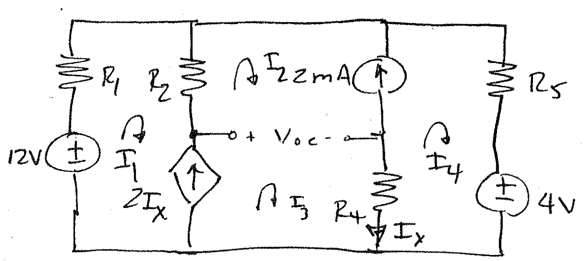


Figure P5.74

SOLUTION: Find Thevenin eq.

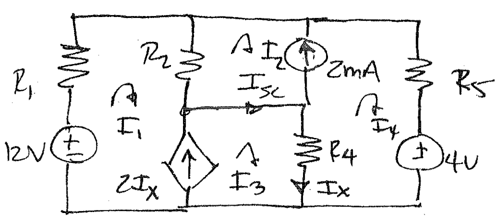


$I_3 - I_1 = 2I_x \quad I_x = I_3 - I_4 \quad I_4 - I_2 = 2mA$

$12 = R_1 I_1 + R_5 I_4 + 4$

$12 = R_1 I_1 + R_2(I_1 - I_2) + V_{oc} + R_4(I_3 - I_4)$

yields $V_{oc} = 12V$

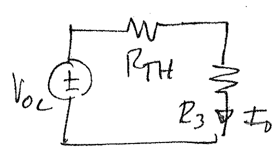


$I_3 - I_1 = 2I_x \quad I_x = I_3 - I_4 \quad I_4 - I_2 = 2mA$

$12 = I_1 R_1 + I_4 R_5 + 4$

$12 = I_1(R_1 + R_2) - I_2 R_2 + I_3 R_4 - I_4 R_4$

yields $I_{SC} = I_3 - I_2 = -4mA \Rightarrow R_{TH} = -3k\Omega$



$I_o = V_{oc} / (R_{TH} + R_3)$ $\boxed{I_o = -6mA}$

5.75 Find I_o in the network in Fig. P5.75 using Thévenin's theorem.

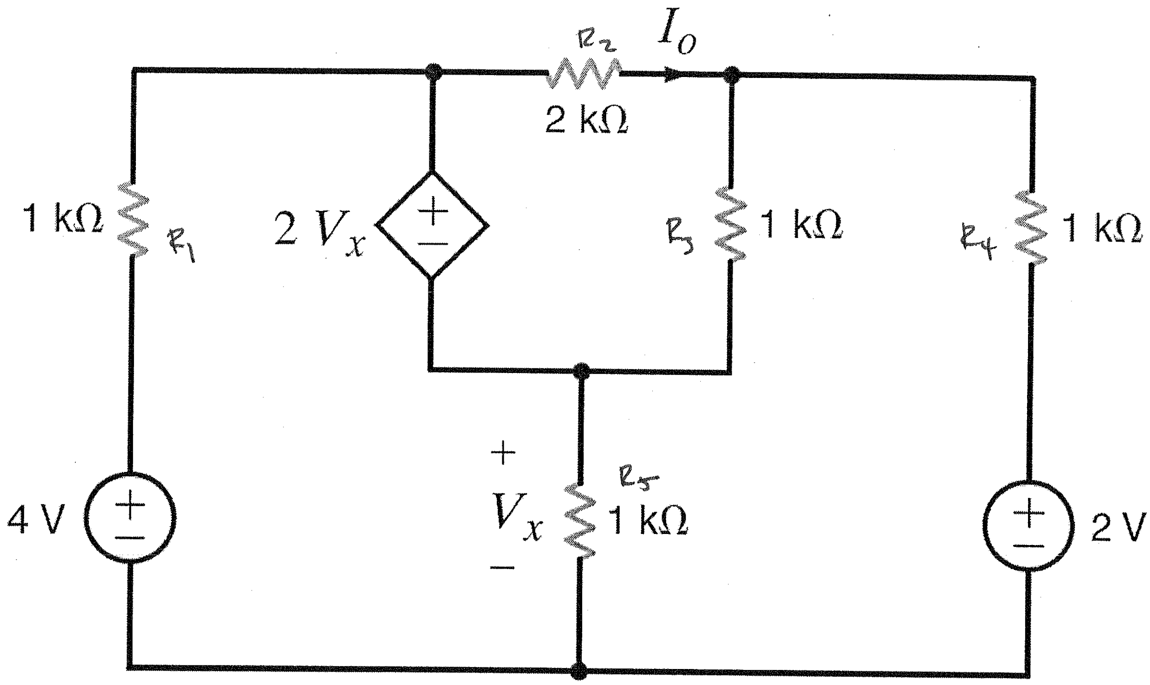
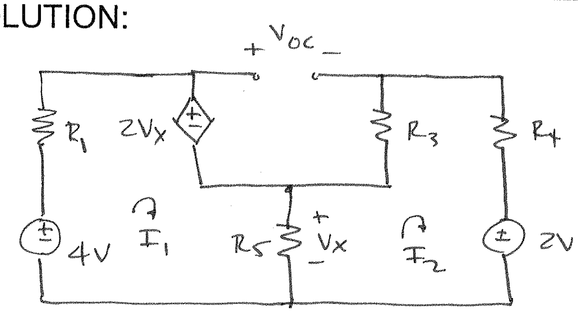


Figure P5.75

SOLUTION:

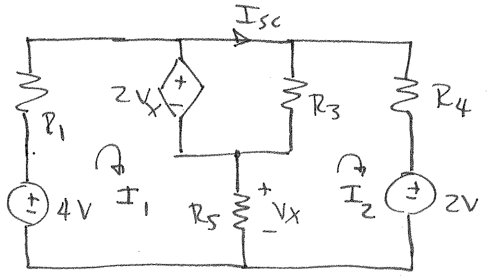


$4 = I_1(R_1 + R_5) - R_5 I_2 + 2V_x$

$V_x = R_5(I_1 - I_2)$

$-2 = I_2(R_5 + R_3 + R_4) - R_5 I_1$

$V_{oc} = 2V_x + R_3 I_2 = 1.78\text{V}$



$4 = (R_1 + R_5)I_1 - R_5 I_2 + 2V_x \qquad V_x = R_5(I_1 - I_2)$

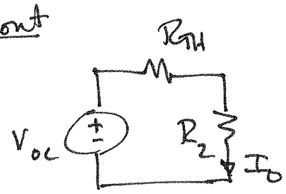
$-2 = I_2(R_5 + R_3 + R_4) - R_5 I_1 - R_3 I_{sc}$

$2V_x = R_3(I_{sc} - I_2)$

yields $I_{sc} = 2.29\,\text{mA}$

$R_{TH} = V_{oc}/I_{sc} = 777\,\Omega$

Cont

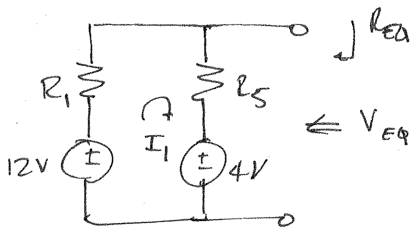


$$I_o = \frac{V_{oc}}{R_{TH} + R_2}$$

$$\boxed{I_o = 0.641\,mA}$$

5.76 Use Thévenin's theorem to find the power supplied by the 2-mA source in the network in Fig. P5.74. **PSV**

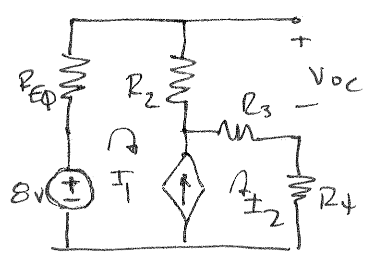
SOLUTION: $R_1 = R_5 = 2k\Omega$ $R_2 = R_3 = R_4 = 1k\Omega$



$12 = (R_1 + R_5) I_1 + 4$ $I_1 = 2mA$

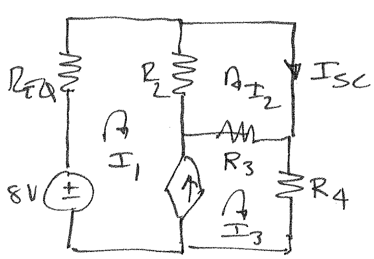
$V_{EQ} = 4 + R_5 I_1 = 8V$

$R_{EQ} = R_1 // R_5 = 1k\Omega$



$8 = I_1 (R_{EQ} + R_2) + I_2 (R_3 + R_4) \Rightarrow I_1 + I_2 = 4mA$

$8 = I_1 (R_{EQ}) + V_{OC} + I_2 R_4 \Rightarrow V_{OC} = 4V$



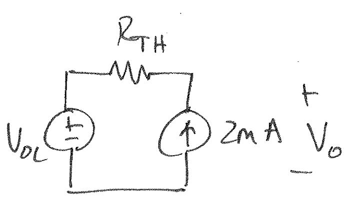
$8 = I_1 (R_{EQ} + R_2) - I_2 (R_2 + R_3) + I_3 (R_3 + R_4)$

$0 = I_2 (R_2 + R_3) - R_2 I_1 - R_3 I_3$

$I_{SC} = I_2$

$4mA = I_1 - I_2 + I_3$ $I_1 + I_3 = 2I_2$

$I_2 = 4mA = I_{SC}$



$R_{TH} = V_{OC} / I_{SC} = 1k\Omega$

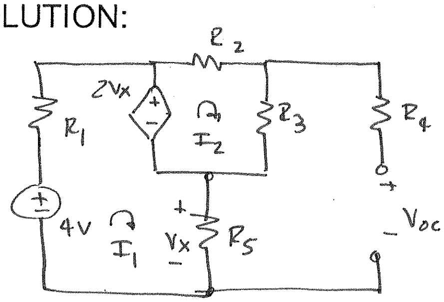
$P_{2mA} = (2 \times 10^{-3}) V_o$

$V_{OC} = -2 \times 10^{-3} R_{TH} + V_o \Rightarrow V_o = 6V$

$$\boxed{P_{2mA} = 12mW}$$

5.77 Use Thévenin's theorem to find the power supplied by the 2-V source in the circuit in Fig. P5.75.

SOLUTION:



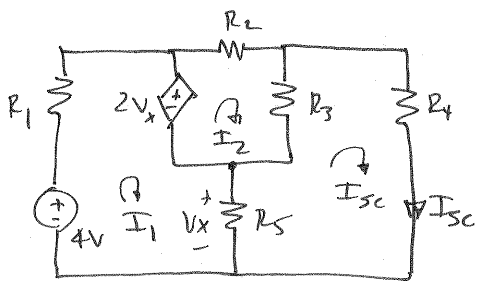
$R_2 = 2k\Omega$ else, $R = 1k\Omega$

$4 = I_1 R_1 + 2V_x + V_x \qquad V_x = I_1 R_5$

$2V_x = I_2 (R_2 + R_3)$

$V_{oc} = R_3 I_2 + I_5 I_1$

yields $V_{oc} = 1.67V$



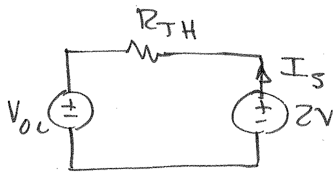
$4 = I_1 (R_1) + 3V_x \qquad V_x = R_5 (I_1 - I_{sc})$

$2V_x = I_2 (R_2 + R_3) - R_3 I_{sc}$

$I_{sc} (R_5 + R_3 + R_4) - R_3 I_2 - R_5 I_1 = 0$

yields $I_{sc} = 0.8mA$

$R_{TH} = V_{oc} / I_{sc} = 2.083k\Omega$



$I_S = \frac{2 - V_{oc}}{R_{TH}} = 0.16 mA$

$P_{2V} = 2 I_S \qquad \boxed{P_{2V} = 0.32mW}$

5.78 Use source transformation to find V_o in the network in Fig. P5.78. CS

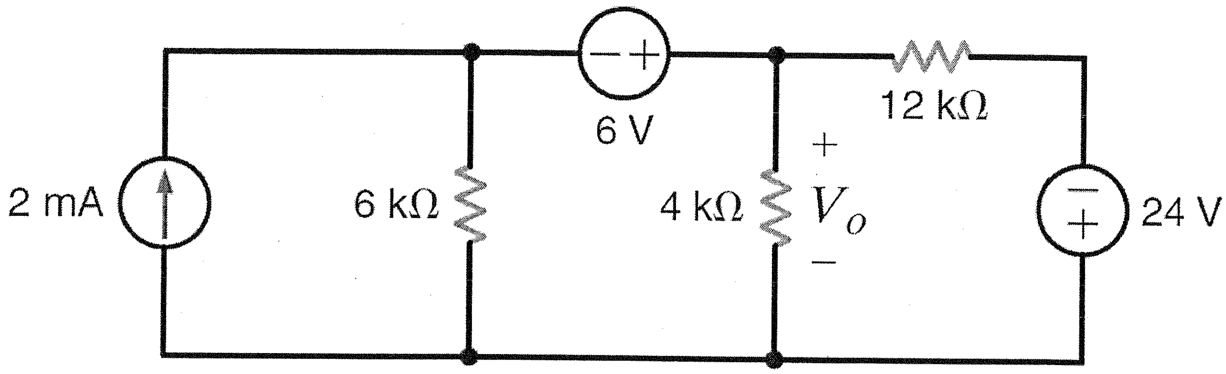
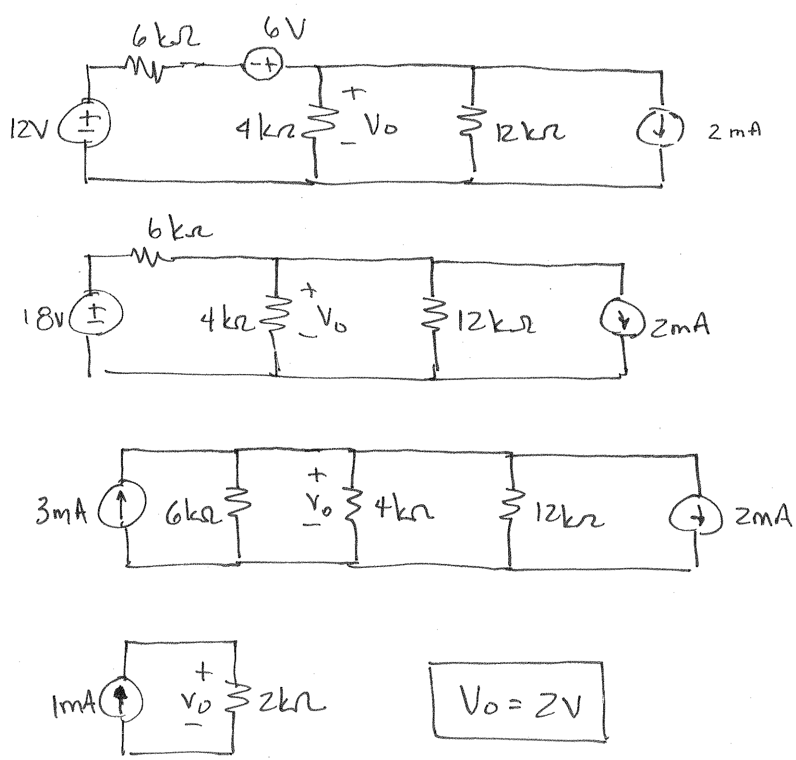


Figure P5.78

SOLUTION:



$\boxed{V_o = 2V}$

5.79 Find V_o in the network in Fig. P5.79 using source transformation.

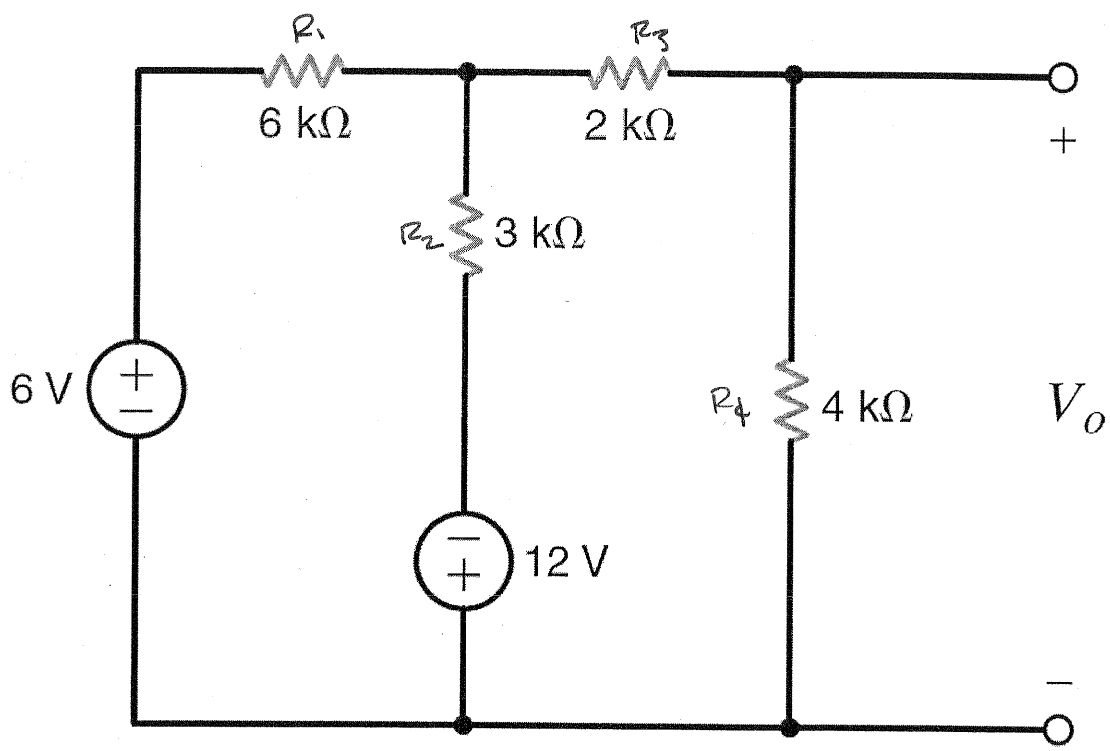
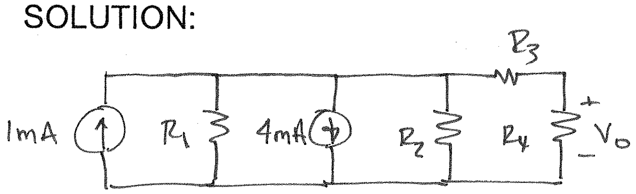
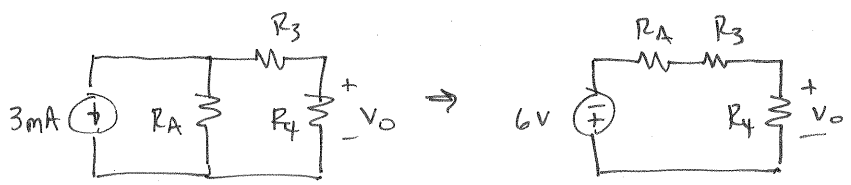


Figure P5.79

SOLUTION:



$R_A = R_1 // R_2 = 2k\Omega$



$$V_o = -6\frac{R_4}{R_A + R_3 + R_4}$$

$$\boxed{V_o = -3V}$$

5.80 Use source transformation to find I_o in the network in Fig. P5.80. **PSV**

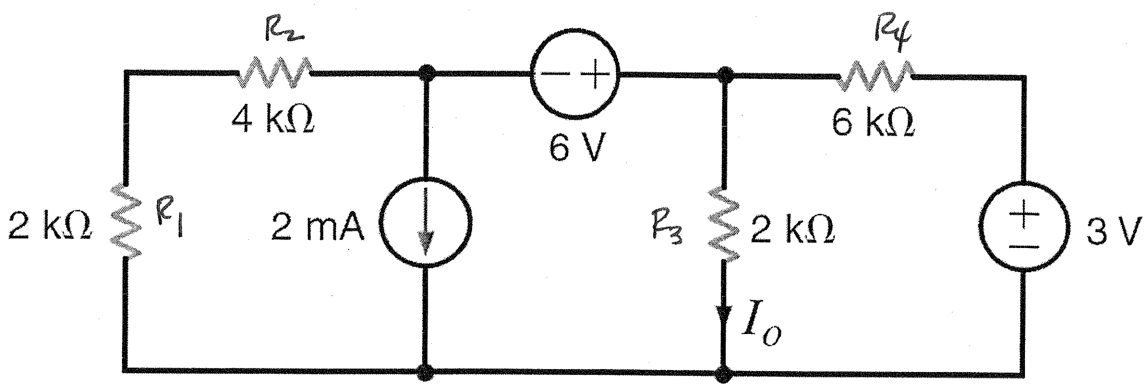
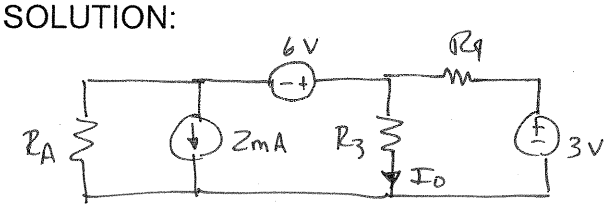
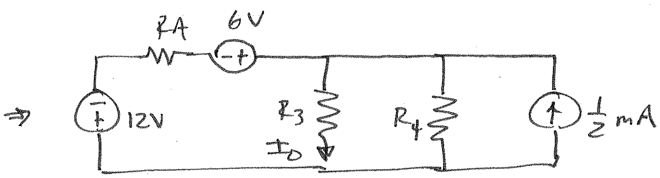


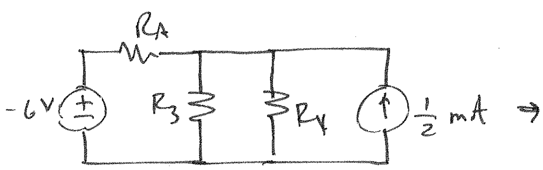
Figure P5.80

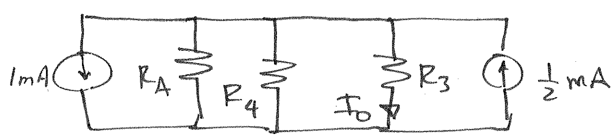
SOLUTION:

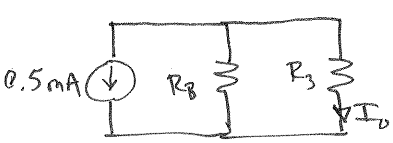




$R_A = R_1 + R_2 = 6\,k\Omega$







$R_B = R_A // R_4 = 3\,k\Omega$

$$I_o = \frac{-0.5\times10^{-3} R_B}{R_B + R_3}$$

$$\boxed{I_o = -0.3\,mA}$$

5.81 Use source transformation to find V_o in the network in Fig. P5.81.

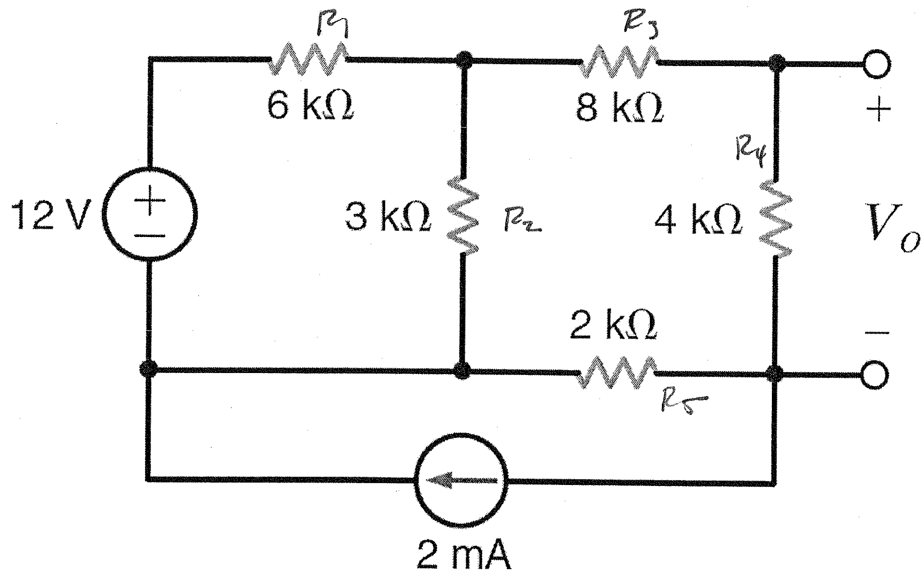
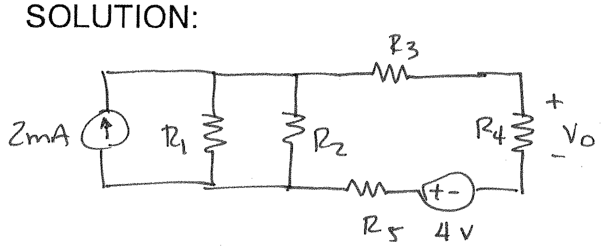
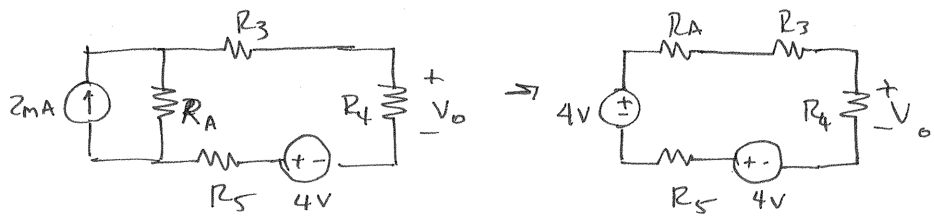


Figure P5.81

SOLUTION:



$R_A = R_1 // R_2 = 2k\Omega$



$$V_o = \frac{8 R_4}{R_A + R_3 + R_4 + R_5}$$

$\boxed{V_o = 2V}$

5.82 Find I_o in the network in Fig. P5.82 using source transformation.

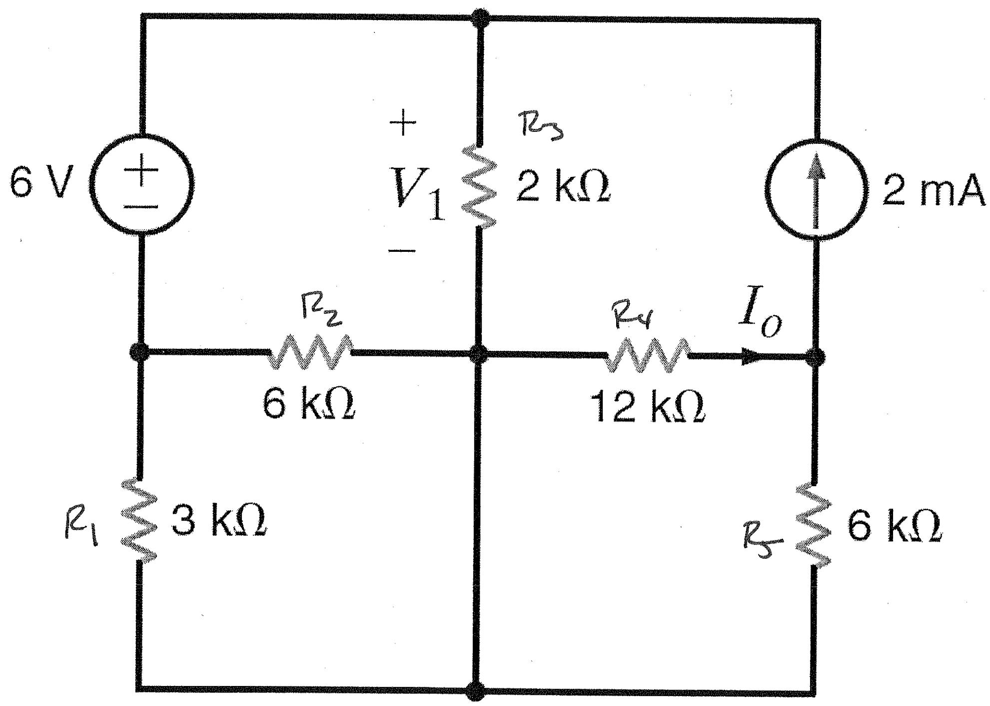
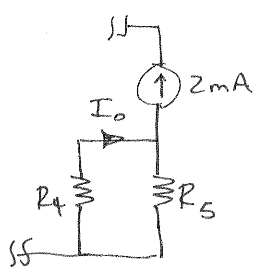


Figure P5.82

SOLUTION:



By current division:

$$I_o = \frac{2\times10^{-3} R_5}{R_4 + R_5} \Rightarrow \boxed{I_o = 0.67mA}$$

5.83 Find I_o in the network in Fig. P5.83.

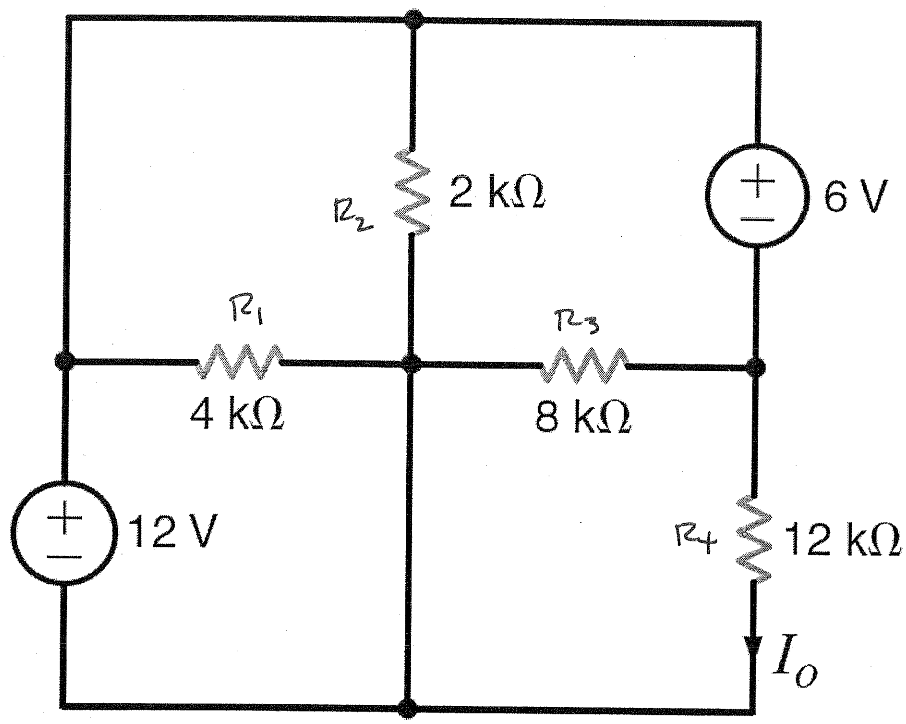


Figure P5.83

SOLUTION:

By KVL: $12 = 6 + R_4 I_o$ $\boxed{I_o = 0.5 mA}$

5.84 Find I_o in the network in Fig. P5.84 using source transformation.

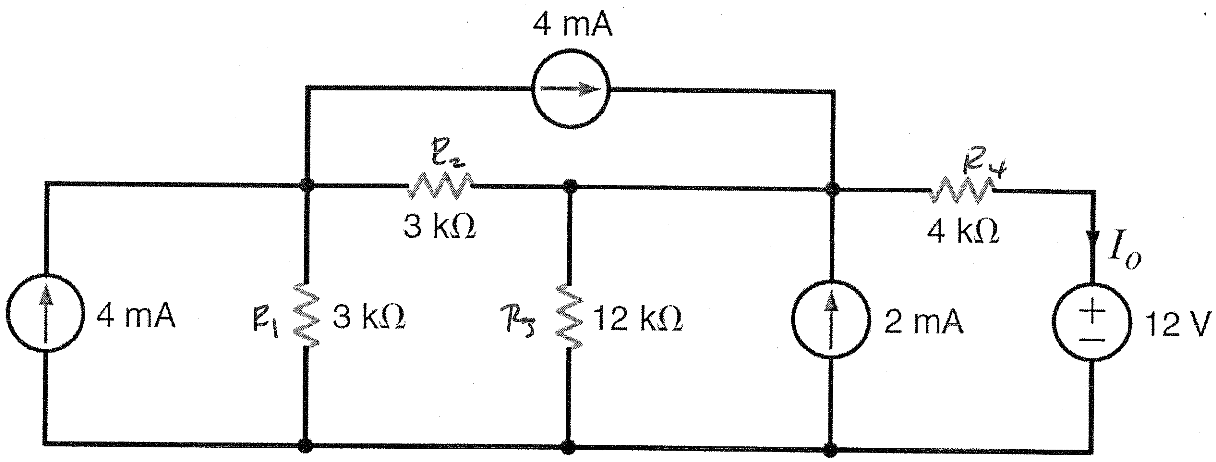
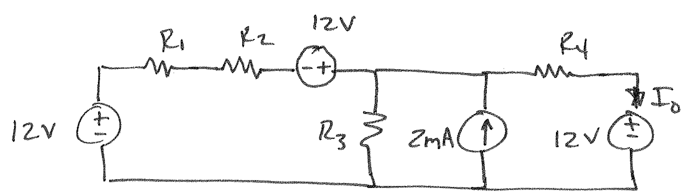


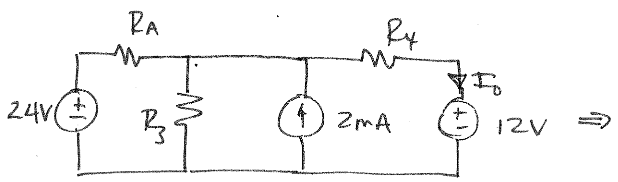
Figure P5.84

SOLUTION:

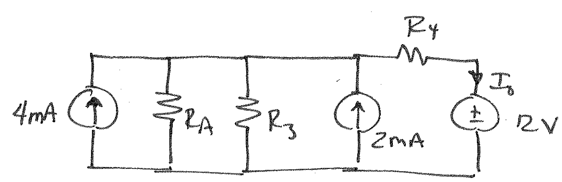


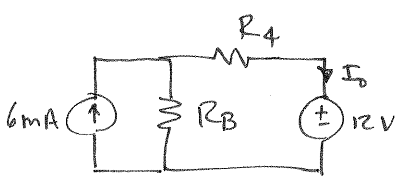
$R_A = R_1 + R_2 = 6\,k\Omega$

$R_B = R_A \,//\, R_3 = 4\,k\Omega$

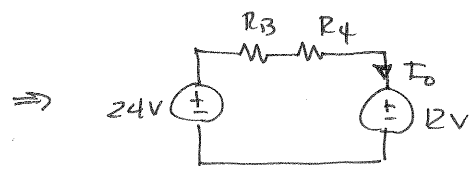


$\Rightarrow$





$\Rightarrow$



$$I_o = \frac{24-12}{R_B + R_4}$$

$$\boxed{I_o = 1.5\,mA}$$

5.85 Use source transformation to find I_o in the circuit in Fig. P5.85. **CS**

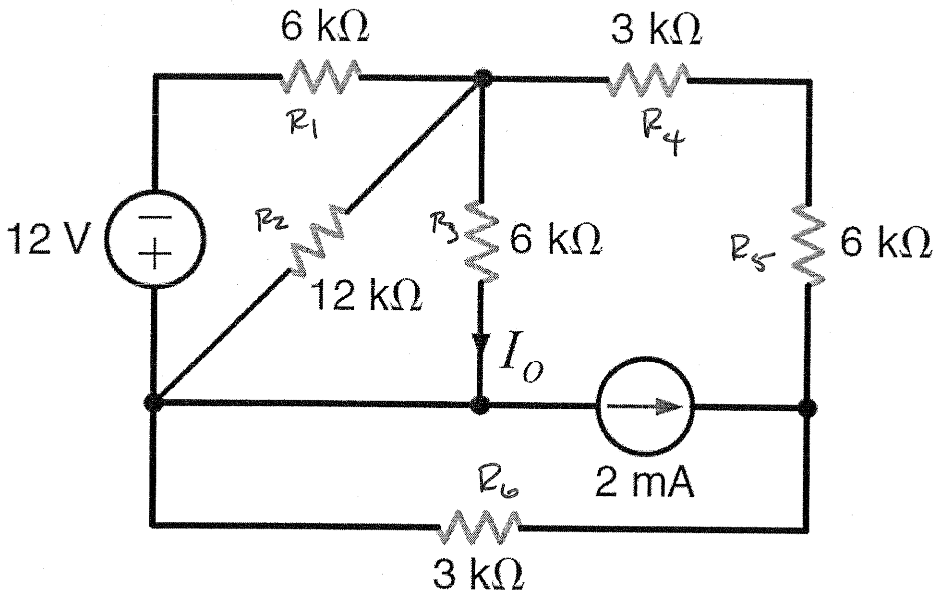
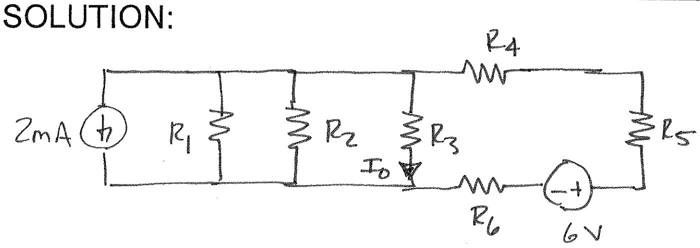


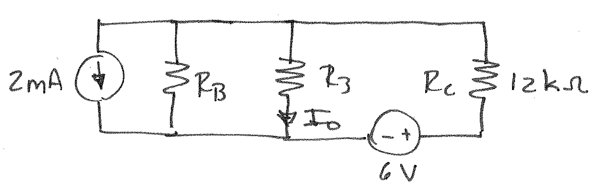
Figure P5.85

SOLUTION:

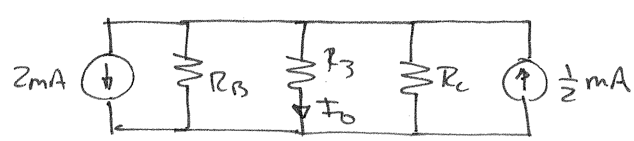


$R_B = R_1 // R_2 = 4k\Omega$

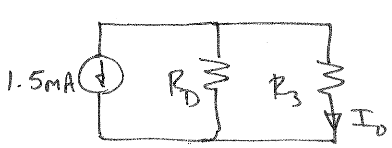
$R_C = R_4 + R_5 + R_6 = 12 k\Omega$



$\Rightarrow$



$R_D = R_B // R_C = 3k\Omega$



$I_o = -1.5\times10^{-3}\dfrac{R_D}{R_D + R_3}$

$\boxed{I_o = -0.5mA}$

5.86 Find V_o in the network in Fig. P5.86 using source transformation.

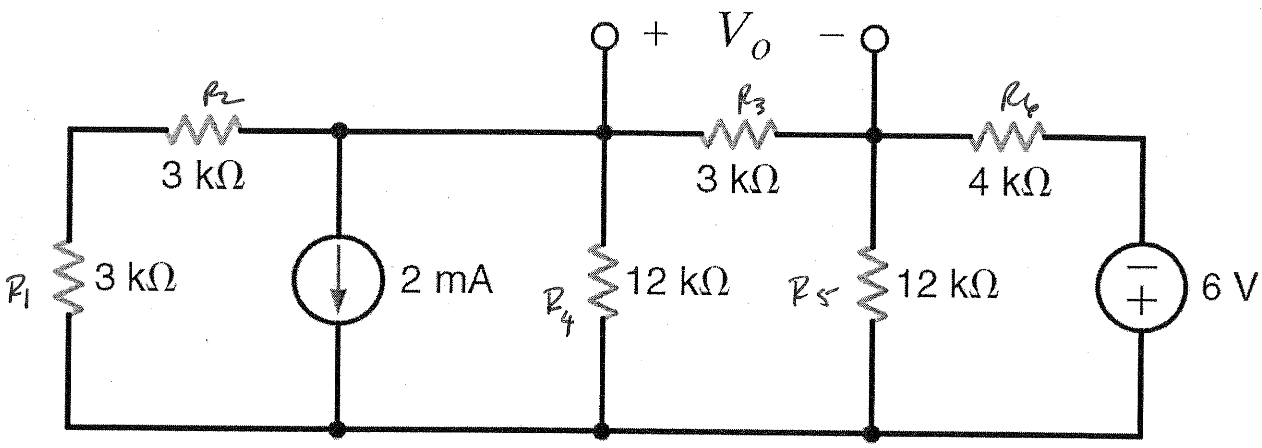
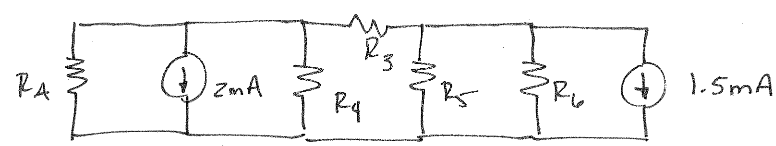


Figure P5.86

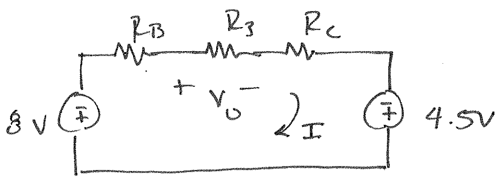
SOLUTION:



$R_A = R_1 + R_2 = 6\,k\Omega$

$R_B = R_A // R_4 = 4\,k\Omega$

$R_C = R_5 // R_6 = 3\,k\Omega$



$8 + I\,(R_B + R_3 + R_C) - 4.5 = 0 \Rightarrow I = -0.35\,mA$

$\boxed{V_o = I R_3 = -1.05\,V}$

5.87 Find I_o in the circuit in Fig. P5.87 using source transformation.

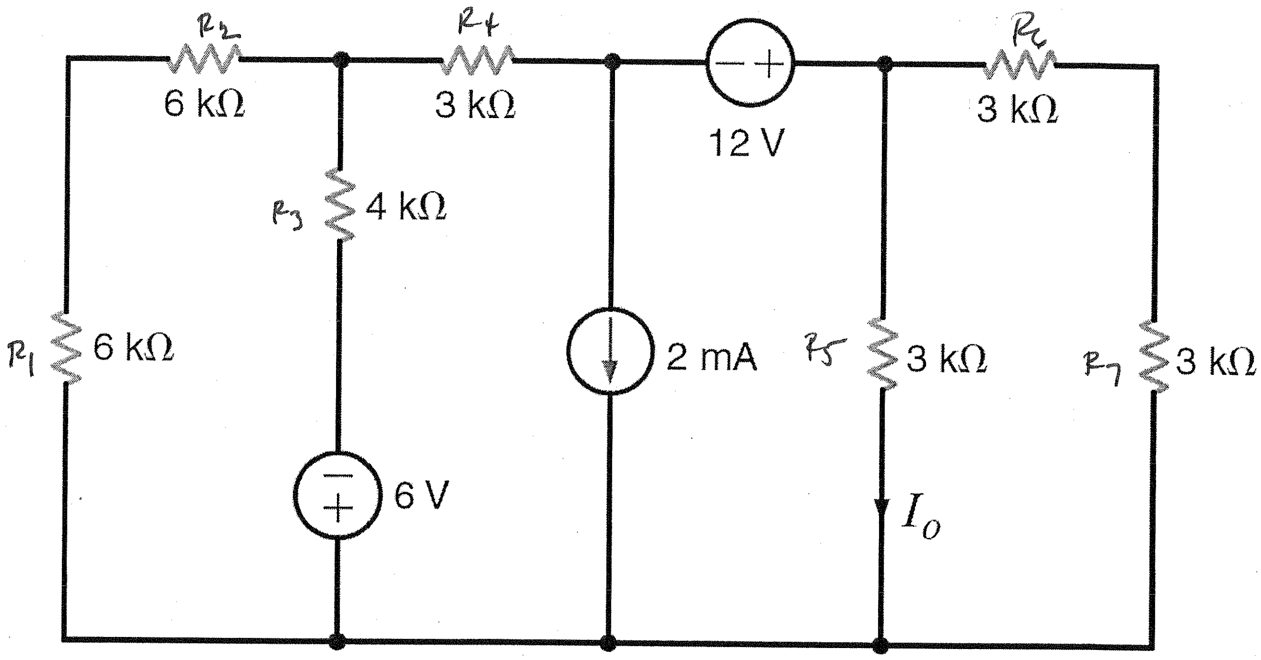
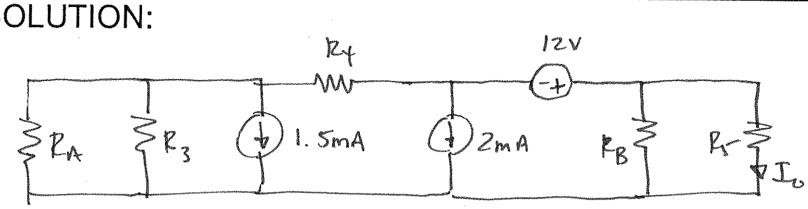


Figure P5.87

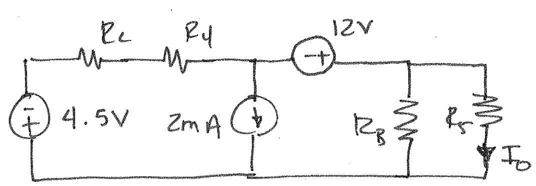
SOLUTION:



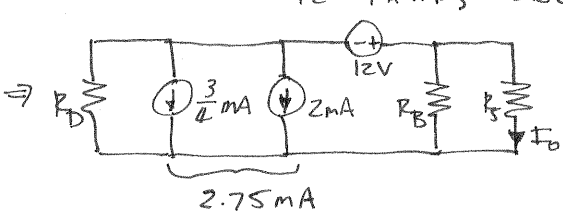
$R_A = R_1 + R_2 = 12k\Omega$

$R_B = R_6 + R_7 = 6k\Omega$

$R_C = R_A // R_3 = 3k\Omega$

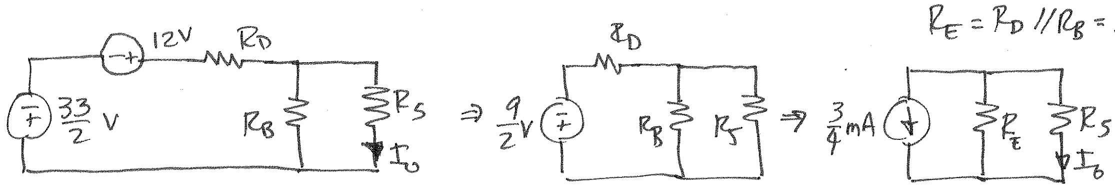


$\Rightarrow$



$R_D = R_C + R_4 = 6k\Omega$

$R_E = R_D // R_B = 3k\Omega$



$I_o = -0.375\,mA$

5.88 Find I_o in the network in Fig. P5.88 using source transformation. CS

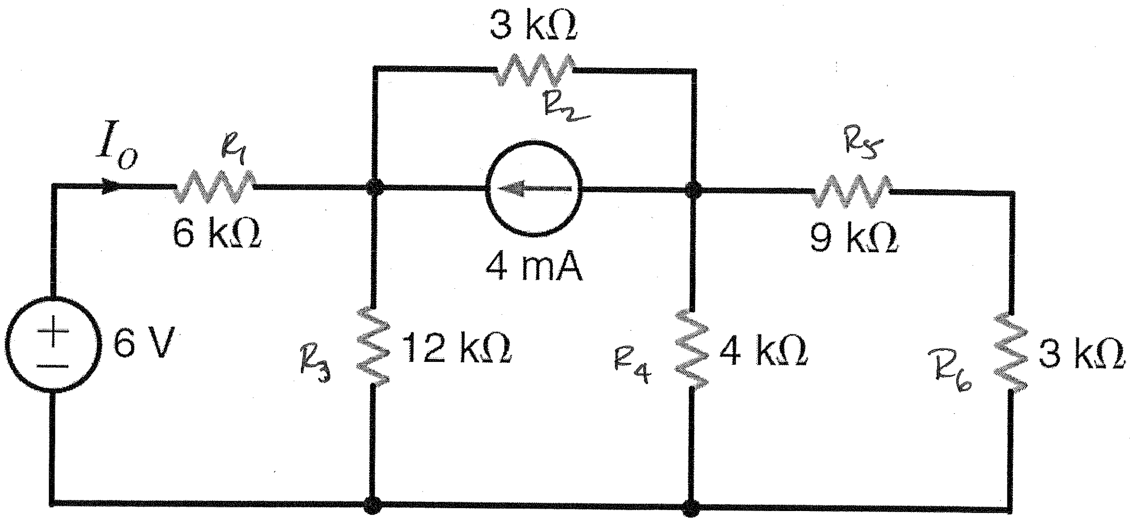
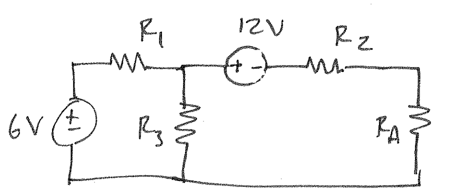


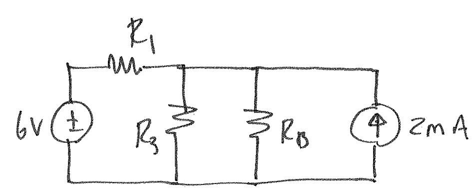
Figure P5.88

SOLUTION:

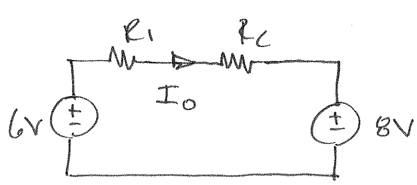


$R_A = R_4 // (R_5 + R_6) = 3k\Omega$

$R_B = R_2 + R_A = 6k\Omega$



$R_C = R_3 // R_B = 4k\Omega$

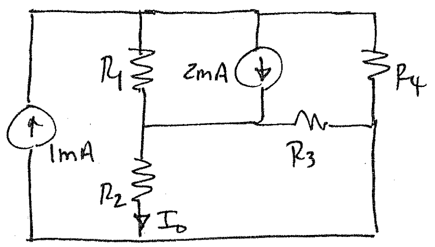


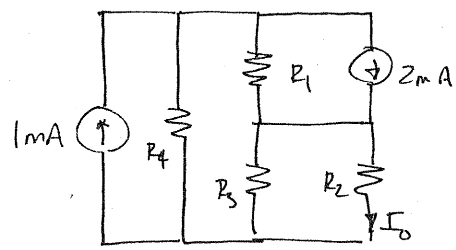
$6 = I_o(R_1 + R_C) + 8$

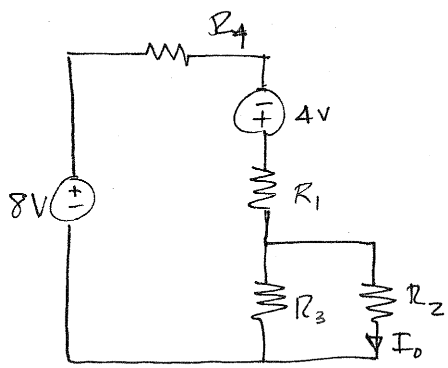
$\boxed{I_o = -0.2mA}$

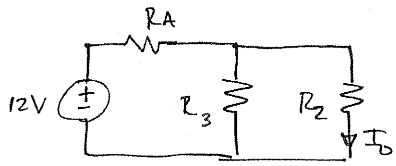
5.89 Solve Problem 5.16 using source transformation.

SOLUTION: $R_1 = 2k\Omega$ $R_2 = 6k\Omega$ $R_3 = 3k\Omega$ $R_4 = 8k\Omega$

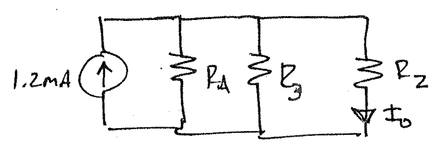




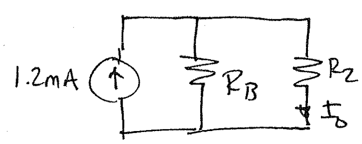




$R_A = R_1 + R_4 = 10k\Omega$



$R_B = R_A // R_3 = \frac{30}{13}\ k\Omega$

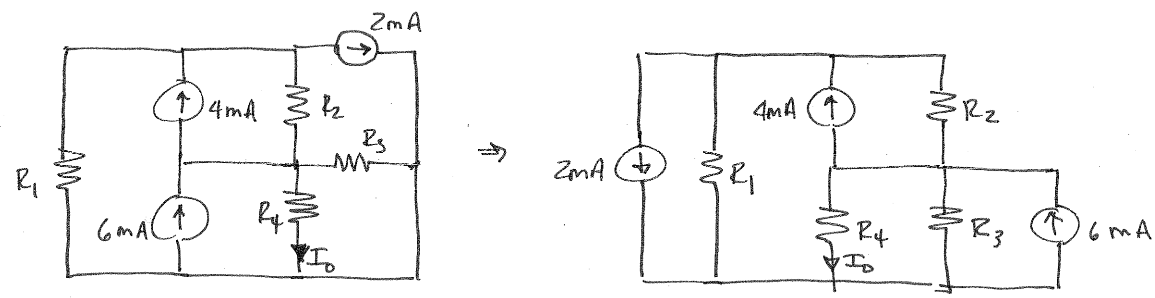


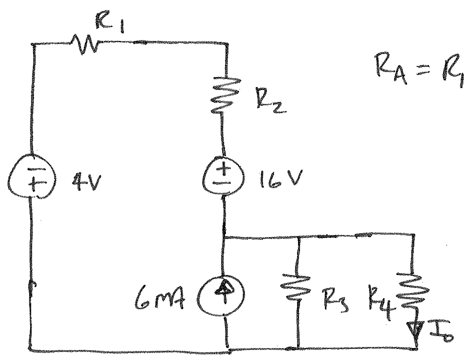
$$I_o = \frac{1.2\times10^{-3} R_B}{R_B + R_2}$$

$$\boxed{I_o = 0.33\ mA}$$

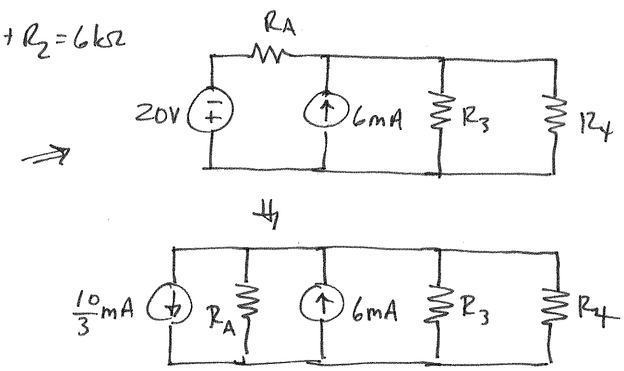
5.90 Solve Problem 5.17 using source transformation.

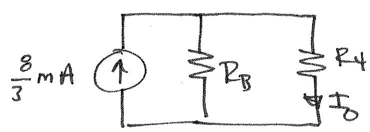
SOLUTION: $R_1 = 2k\Omega$ $R_2 = 4k\Omega$ $R_3 = 4k\Omega$ $R_4 = 12k\Omega$





$R_A = R_1 + R_2 = 6k\Omega$





$R_B = R_A // R_3 = 2.4k\Omega$

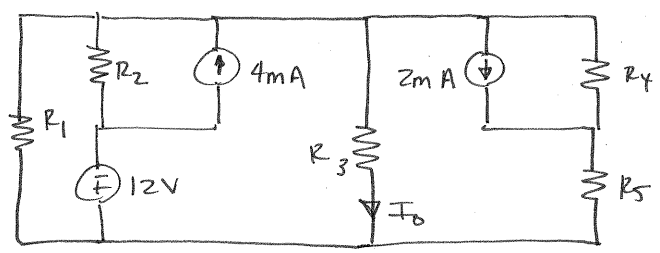
$$I_o = \frac{2.67 \times 10^{-3} R_B}{R_B + R_4}$$

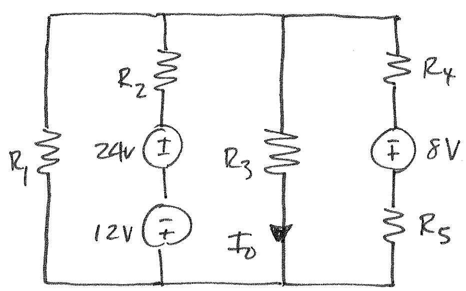
$$\boxed{I_o = 0.444 mA}$$

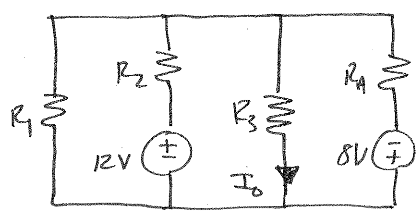
5.91 Use source transformation to solve Problem 5.18.

SOLUTION: $R_1 = R_2 = 6k\Omega \quad R_3 = 3k\Omega \quad R_4 = 4k\Omega \quad R_5 = 2k\Omega$

Circuit rearranged.

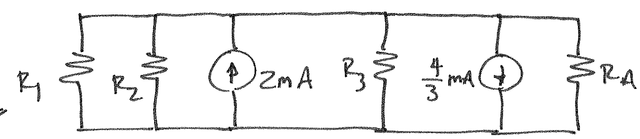


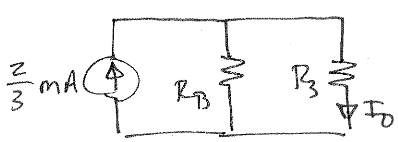




$R_A = R_4 + R_5$

$R_A = 6k\Omega$





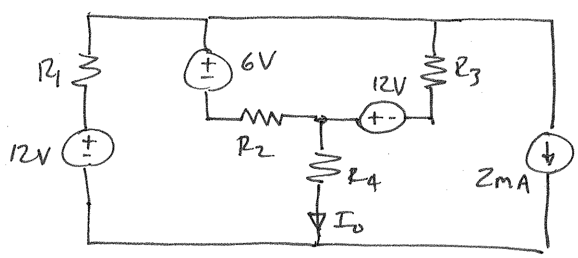
$R_B = R_1 // R_2 // R_A = 2k\Omega$

$$I_o = \frac{0.67\times10^{-3}\,R_B}{R_B + R_3}$$

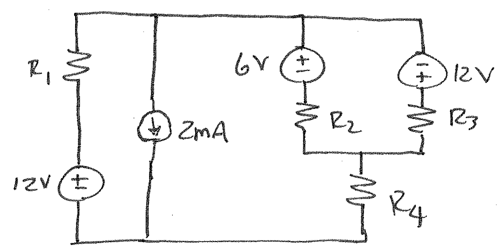
$$\boxed{I_o = 0.267\,mA}$$

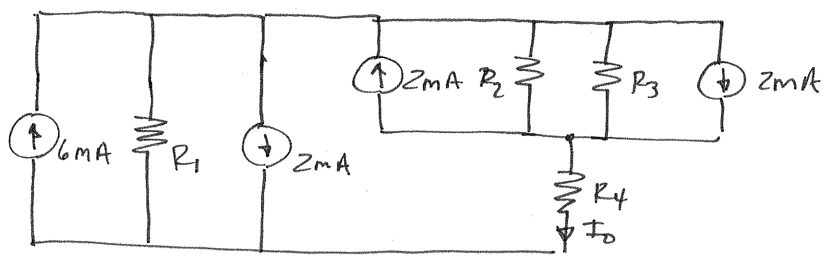
5.92 Use source transformation to solve Problem 5.20.

SOLUTION: $R_1 = 2k\Omega$ $R_2 = 3k\Omega$ $R_3 = 6k\Omega$ $R_4 = 4k\Omega$

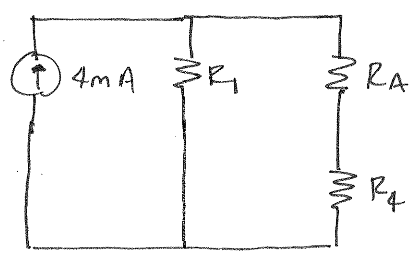


⇒

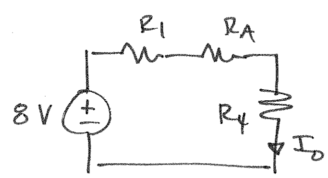




$R_A = R_2 // R_3 = 2k\Omega$



⇒

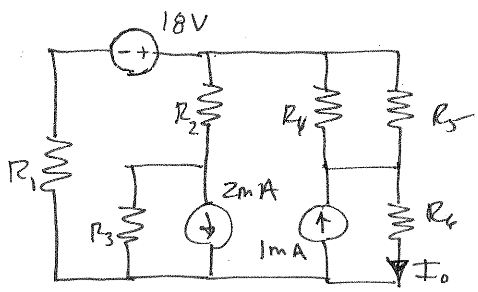


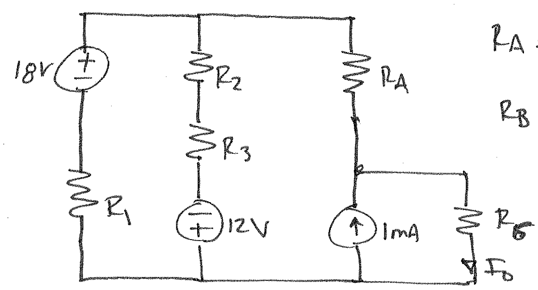
$I_o = \dfrac{8}{R_1 + R_A + R_4}$ $\boxed{I_o = 1mA}$

5.93 Use source transformation to solve Problem 5.21.

SOLUTION: $R_1 = R_2 = R_3 = 6\,k\Omega \quad R_4 = R_5 = 4\,k\Omega \quad R_6 = 3\,k\Omega$

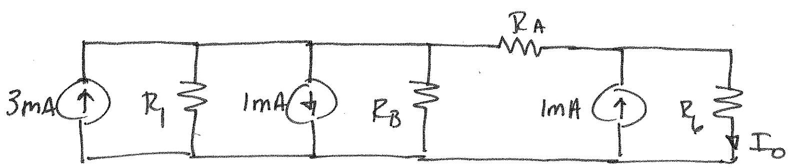
Circuit is rearranged.



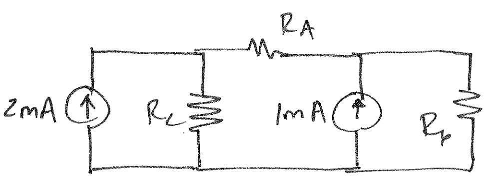


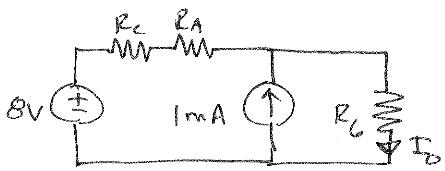
$R_A = R_4 // R_5 = 2\,k\Omega$

$R_B = R_2 + R_3 = 12\,k\Omega$



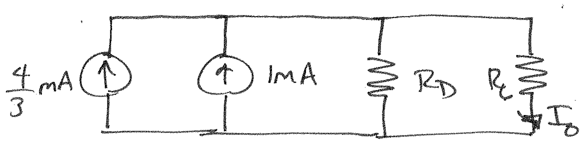
$R_C = R_1 // R_B = 4\,k\Omega$

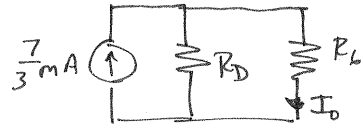




$R_D = R_C + R_A$

$R_D = 6\,k\Omega$





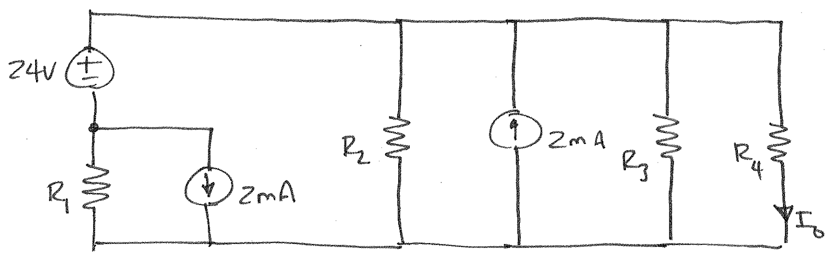
$$I_o = \frac{2.67 \times 10^{-3} R_D}{R_D + R_6}$$

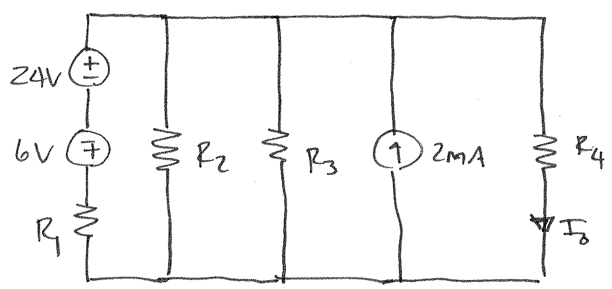
$$\boxed{I_o = 1.55\ mA}$$

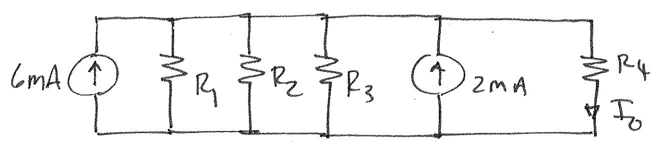
5.94 Solve Problem 5.22 using source transformation.

SOLUTION: $R_1 = 3k\Omega$ $R_2 = 6k\Omega$ $R_3 = 2k\Omega$ $R_4 = 4k\Omega$

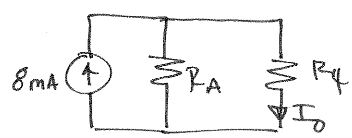
Circuit is rearranged.







⇒



$R_A = R_1 // R_2 // R_3 = 1k\Omega$

$$I_o = \frac{8\times10^{-3} R_A}{R_A + R_4}$$

$$\boxed{I_o = 1.6mA}$$

5.95 Find R_L in the network in Fig. P5.95 in order to achieve maximum power transfer.

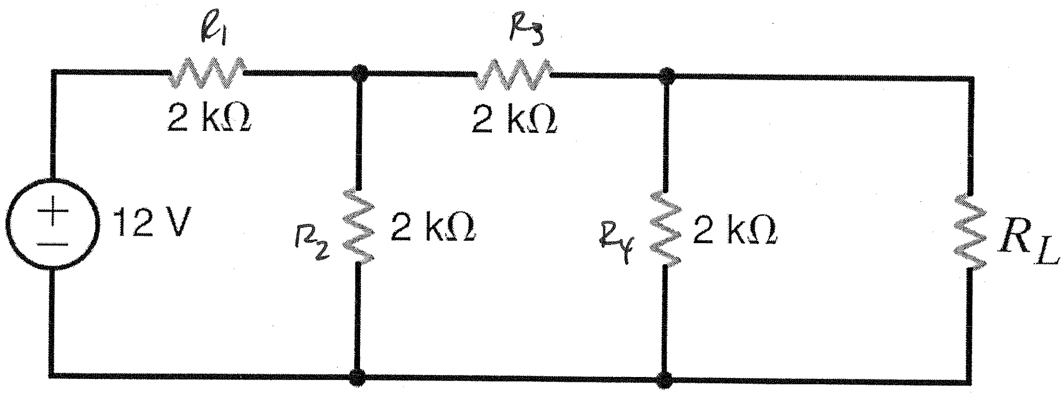
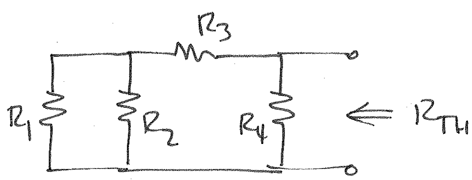


Figure P5.95

SOLUTION: Find R_{TH} !



$$R_{TH} = R_4 // \{R_3 + (R_1 // R_2)\} = 2000 // \{2000 + 1000\}$$

$$R_{TH} = 1.2\,k\Omega$$

for maximum power transfer, $\boxed{R_L = 1.2\,k\Omega}$

5.96 In the network in Fig. P5.96, find R_L, for maximum power transfer and the maximum power transferred to this load. **PSV**

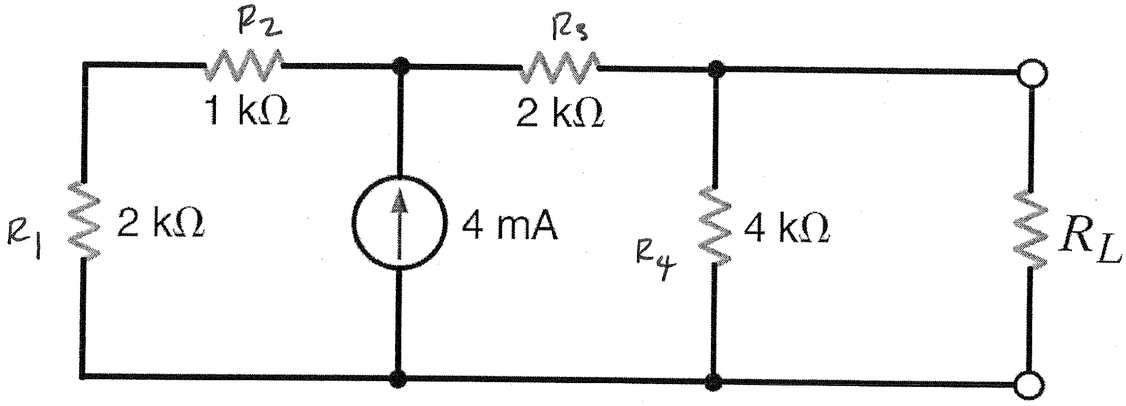
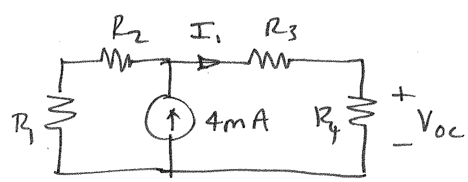


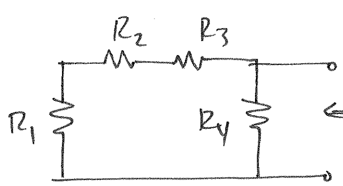
Figure P5.96

SOLUTION: Find Thevenin eq.!

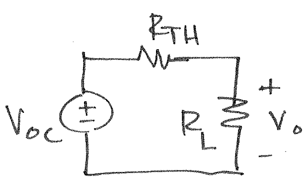


$$I_1 = \frac{4\times10^{-3}\,(R_1+R_2)}{R_1+R_2+R_3+R_4} = 1.33\text{ mA}$$

$$V_{oc} = I_1 R_4 = 5.33\text{ V}$$



$$\leftarrow R_{TH} = R_4 \,//\, (R_1+R_2+R_3) = 2.22\text{ k}\Omega$$



for maximum power transfer, $R_L = R_{TH}$
and $V_o = V_{oc}/2$

$$P_L = V_o^2/R_L = \frac{V_{oc}^2}{4R_{TH}}$$

$$\boxed{P_L = 3.2\text{ mW}}$$

5.97 Find R_L for maximum power transfer and the maximum power that can be transferred to the load in Fig. P5.97.

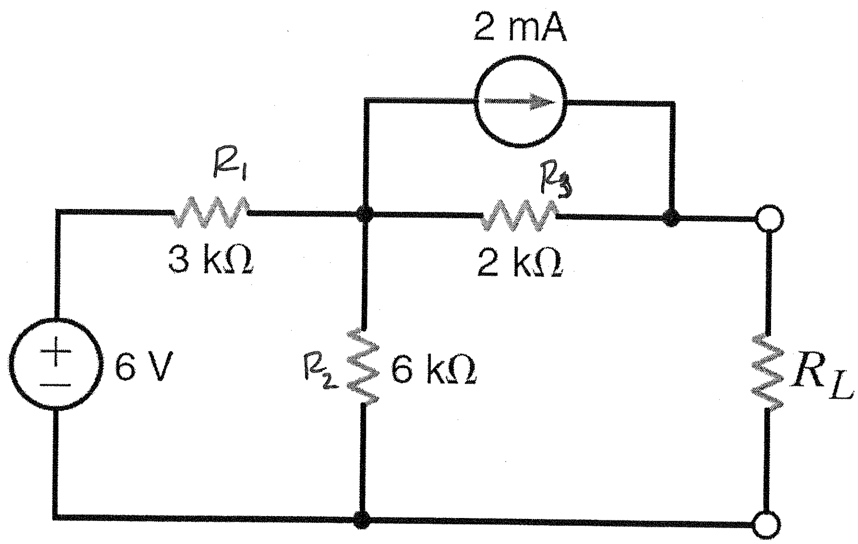
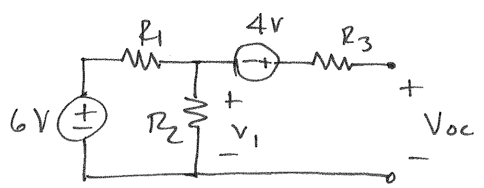


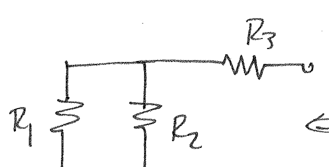
Figure P5.97

SOLUTION: Find Thevenin eq!

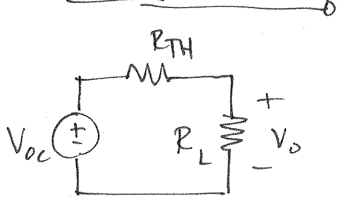


$$V_1 = 6R_2/(R_2+R_1) = 4V$$

$$V_{oc} = 4 + V_1 = 8V$$



$\Leftarrow R_{TH} = (R_1 /\!/ R_2) + R_3 = 4k\Omega$



for maximum power transfer, $R_L = R_{TH}$ and $V_o = \frac{V_{oc}}{2}$.

$$P_L = \frac{V_o^2}{R_L} = \frac{V_{oc}^2}{4R_{TH}} \qquad \boxed{P_L = 4mW}$$

5.98 Choose R_L in Fig. P5.98 for maximum power transfer.

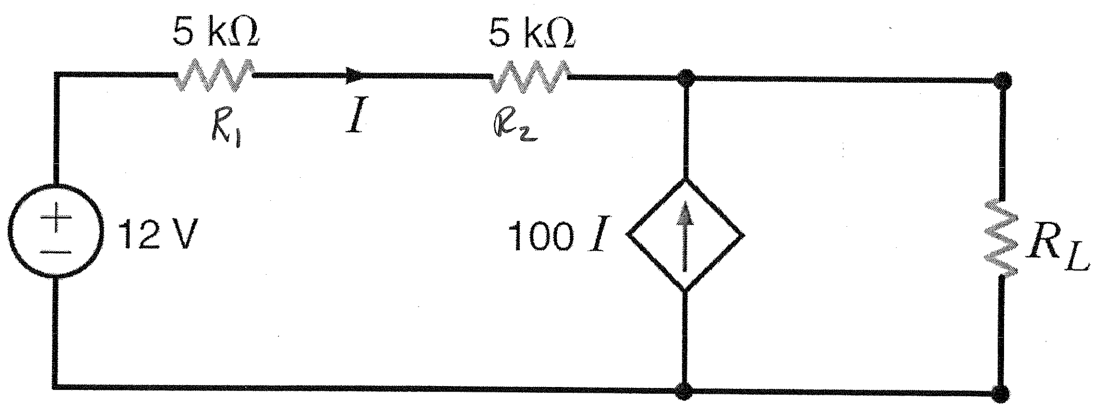
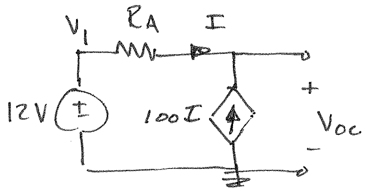


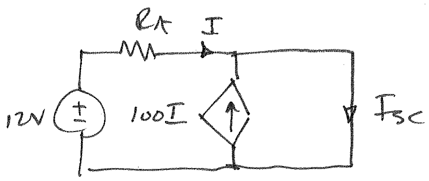
Figure P5.98

SOLUTION: find R_{TH}



$R_A = R_1 + R_2 = 10k\Omega$ $\quad V_1 = 12\,V$

$$\frac{V_1 - V_{oc}}{R_A} = I = -100I \Rightarrow I = 0 \;\&\; V_{oc} = 12V$$



$$I_{sc} = I + 100I = 101\,I$$

$$I = 12/R_A = 1.2\,mA$$

$$I_{sc} = 121.2\,mA$$

$$R_{TH} = V_{oc}/I_{sc} = 99.0\,\Omega$$

for maximum power transfer, $\boxed{R_L = 99.0\,\Omega}$

5.99 Find the value of R_L in the network in Fig. P5.99 for maximum power transfer. **PSV**

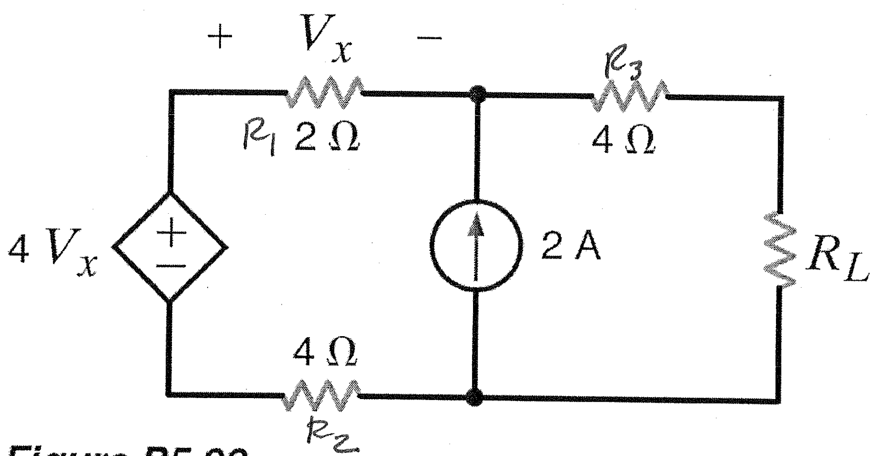
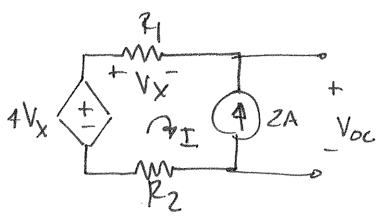


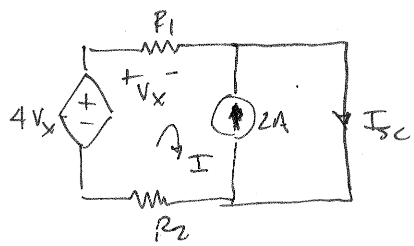
Figure P5.99

SOLUTION: find R_{TH}



$4V_x = 2I + V_{oc} + 4I \qquad I = -2 \qquad V_x = 2I$

$V_{oc} = -4V$

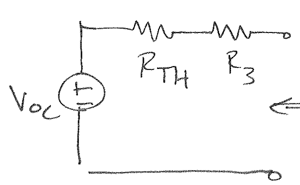


$4V_x = 2I + 4I \qquad V_x = 2I \Rightarrow I = 0$

So, $V_x = 0$

$I_{SC} = 2 + I = 2A$

$R_{TH} = V_{oc} / I_{SC} = -2\Omega$



$\Leftarrow R_{EQ} = R_{TH} + R_3 = 2\Omega$

for maximum power transfer $R_L = R_{EQ}$

$\boxed{R_L = 2\Omega}$

5.100 Calculate the maximum power that can be transferred to R_L in the circuit in Fig. P5.100.

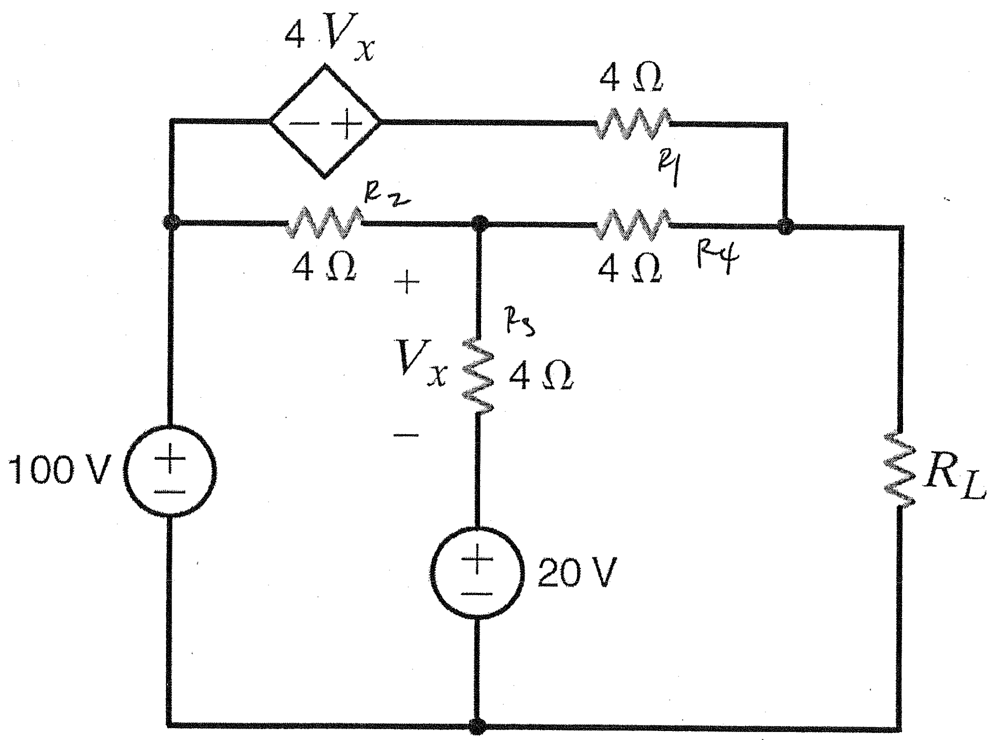
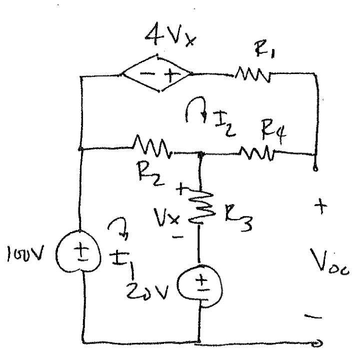
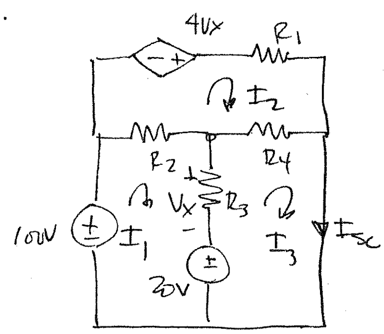


Figure P5.100

SOLUTION: Find R_{Th}.





$100 = I_1 (R_2+R_3) - R_2 I_1 - R_3 I_3 + 20$

$4V_x = -R_2 I_1 + I_2 (R_1+R_2+R_4) - R_4 I_3$

$20 = -R_3 I_1 - I_2 R_4 + I_3 (R_3+R_4)$

$I_{sc} = I_3 \qquad V_x = R_3 (I_1 - I_3)$

$I_{sc} = 55A$

$$\left.\begin{aligned} 100 &= I_1 (R_3+R_2) - R_2 I_2 + 20 \\ 4V_x &= I_2 (R_1+R_2+R_4) - I_1 R_2 \\ 20 &= -I_1 R_3 - I_2 R_4 + V_{oc} \\ V_x &= R_3 I_1 \end{aligned}\right\} V_{oc} = 660V$$

$R_{Th} = V_{oc}/I_{sc} = 12\ \Omega$

R_L for maximum power transfer is

$\boxed{R_L = 12\Omega}$

5.101 A cell phone antenna picks up a call. If the antenna and cell phone are modeled as shown in Fig. P5.101,

(a) Find R_{cell} for maximum output power.

(b) Determine the value of P_{out}.

(c) Determine the corresponding value of P_{ant}.

(d) Find v_o/v_{ant}.

(e) Determine the amount of power lost in R_{ant}.

(f) Calculate the efficiency $\eta = P_{out}/P_{ant}$.

(g) Determine the value of R_{cell} such that the efficiency is 90%.

(h) Given the change in (g), what is the new value of P_{ant}?

(i) Given the change in (g), what is the new value of P_{out}?

(j) Comment on the results obtained in (i) and (b).

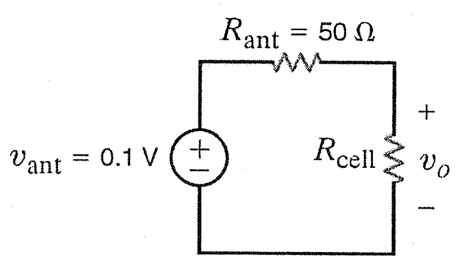


Figure P5.101

SOLUTION:

a) $R_{cell} = R_{ant} = 50\Omega$

b) $P_{out} = v_o^2/R_{cell}$ $v_o = v_{ant} R_{cell}/(R_{cell}+R_{ant}) = 50mV$, $P_{out} = 50\mu W$

c) $P_{ant} = 2P_{out} = 100\mu W$ (at maximum power transfer)

d) $v_o/v_{ant} = R_{cell}/(R_{cell}+R_{ant}) = 0.5$

e) $P_{RANT} = P_{out} = 50\mu W$

f) $\eta = P_{out}/P_{ant} = 1/2 = 50\%$

g) $\eta = \frac{P_{out}}{P_{ant}} = \frac{v_o^2/R_{cell}}{v_{ant}^2/(R_{cell}+R_{ant})} = \frac{v_{ant}^2\left(\frac{R_{cell}}{R_{cell}+R_{ant}}\right)^2/R_{cell}}{v_{ant}^2/(R_{cell}+R_{ant})}$

$\eta = \frac{R_{cell}}{R_{cell}+R_{ant}} = 0.9 \Rightarrow R_{cell} = 450\Omega$

h) $P_{ant} = v_{ant}^2/(R_{cell}+R_{ant}) = 20\mu W$

i) $P_{out} = 0.9\,P_{ant} = 18\mu W$

j) As η increases a larger PERCENTAGE of P_{ant} is transfered to P_{out}. However, P_{ant} drops faster than η rises. As a result, P_{out} decreases as η moves away from 50%.

5.102 Some young engineers at the local electrical utility are debating ways to lower operating costs. They know that if they can reduce losses, they can lower operating costs. The question is whether they should design for maximum power transfer or maximum efficiency, where efficiency is defined as the ratio of customer power to power generated. Use the model in Fig. P5.102 to analyze this issue and justify your conclusions. Assume that both the generated voltage and the customer load are constant.

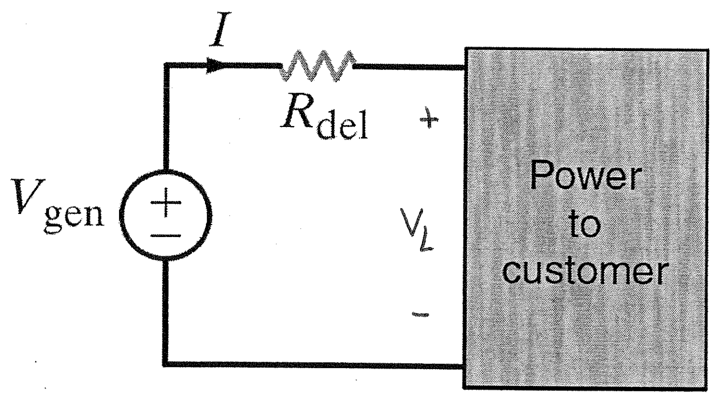


Figure P5.102

SOLUTION:

Specifications: V_{gen} fixed & I fixed.

Issue: optimize η to maximize profits.

$$P_{LOAD} = V_L I = (V_{gen} - IR_{del})I \qquad P_{GEN} = V_{gen} I$$

$$\eta = \frac{P_{LOAD}}{P_{GEN}} = \frac{(V_{gen} - IR_{del})I}{V_{gen} I} = 1 - \frac{IR_{del}}{V_{gen}} = 1 - kR_{del} \qquad k = \frac{I}{V_{gen}}$$

As $R_{del} \to 0$, $\eta \to 100\%$ No power lost in delivery! All generated power can be sold!

5.103 Find R_L for maximum power transfer and the maximum power that can be transferred in the network in Fig. P5.103.

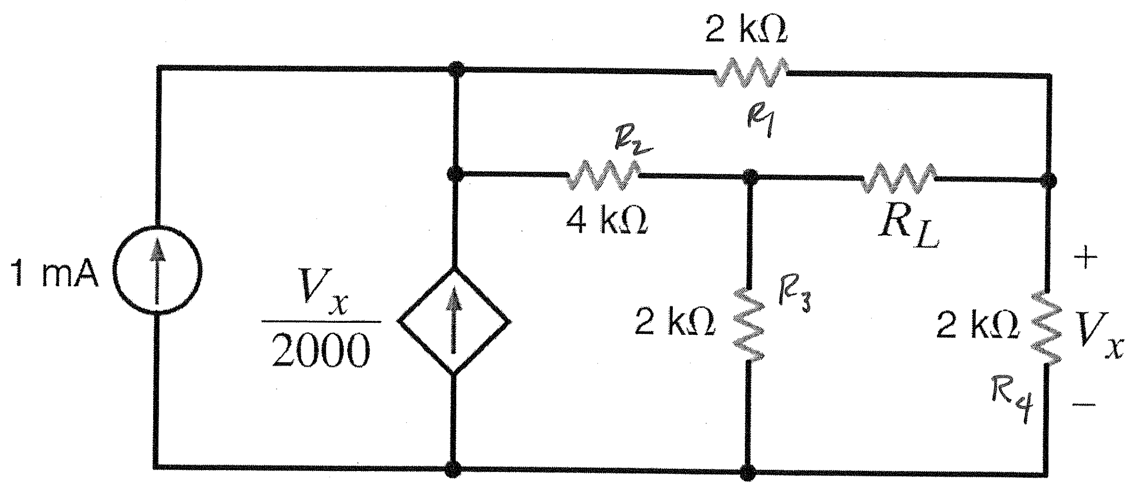
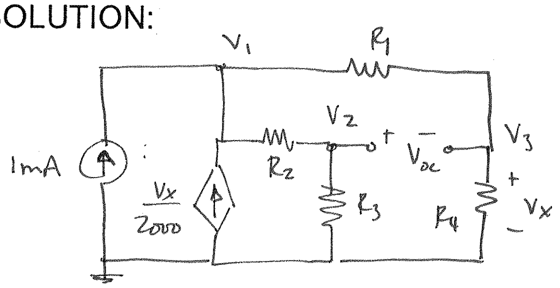


Figure P5.103

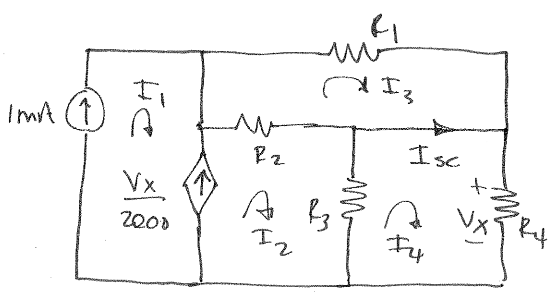
SOLUTION:



$$10^{-3} + \frac{V_x}{2000} = \frac{V_1}{R_2+R_3} + \frac{V_1}{R_1+R_4}$$

$$V_x = \frac{V_1 R_4}{R_1+R_4} = \frac{V_1}{2} \Rightarrow V_1 = 6V$$

$$V_2 = \frac{V_1 R_3}{R_2+R_3} = 2V \qquad V_3 = 3V \qquad V_{oc} = V_2 - V_3 = -1V$$



$$I_1 = 1mA \qquad I_2 - I_1 = \frac{V_x}{2000} \qquad V_x = R_4 I_4$$

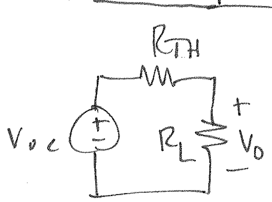
$$I_3(R_1+R_2) - R_2 I_2 = 0$$

$$I_4(R_3+R_4) - R_3 I_2 = 0$$

$$I_4 = 1mA \qquad I_3 = \frac{4}{3} mA$$

$$I_{sc} = I_4 - I_3 = 10^{-3} - 1.33\times10^{-3}$$

$$I_{sc} = -0.333mA \Rightarrow \boxed{R_{TH} = 3 k\Omega}$$

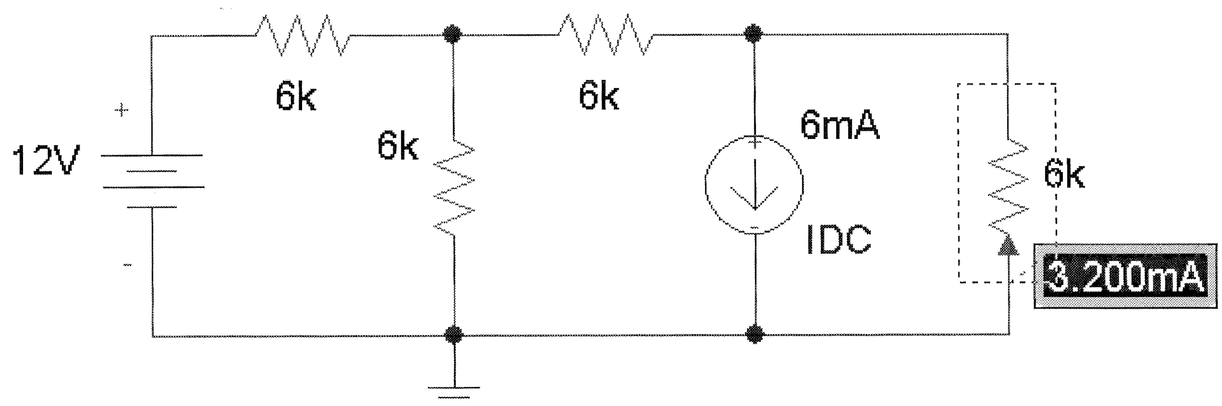


$$P_o = \frac{V_{oc}^2}{4R_{TH}} \Rightarrow \boxed{P_o = 83.3 \mu W}$$

5.104 Solve Problem 5.5 using PSPICE.

SOLUTION:

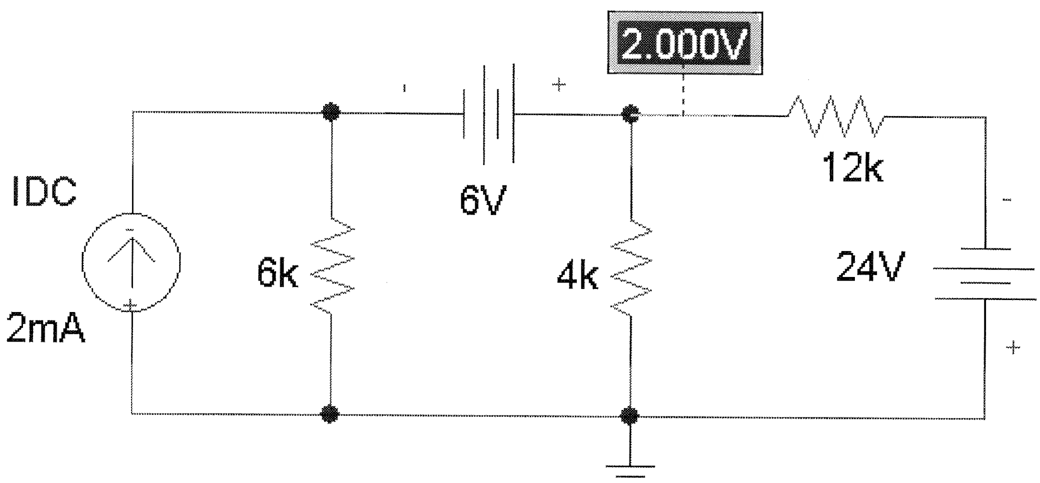
Problem 5.104



5.105 Solve Problem 5.78 using PSPICE.

SOLUTION:

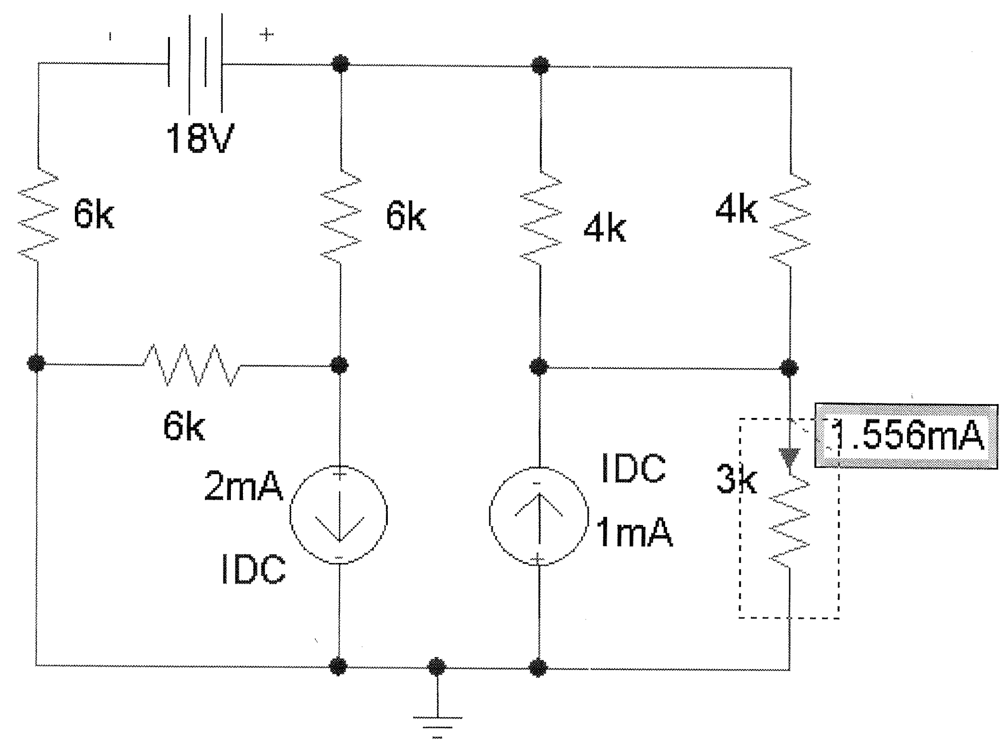
Problem 5.105



5.106 Solve Problem 5.21 using PSPICE.

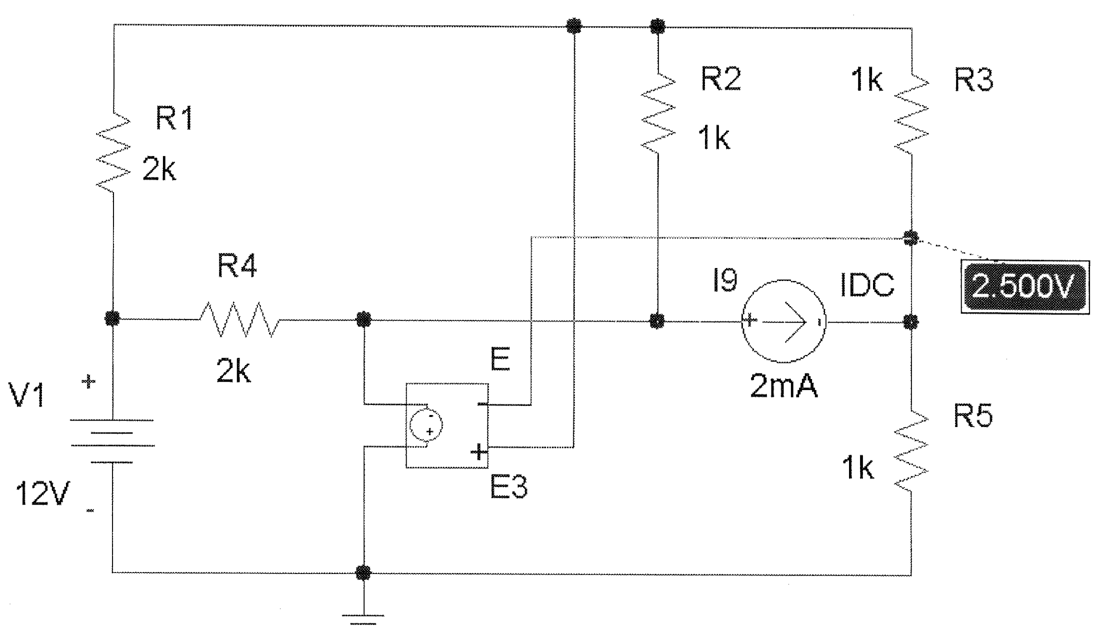
SOLUTION:

Problem 5.106



5.107 Solve Problem 5.71 using PSPICE.

SOLUTION:



5FE-1 Determine the maximum power that can be delivered to the load R_L in the network in Fig. 5PFE-1. CS

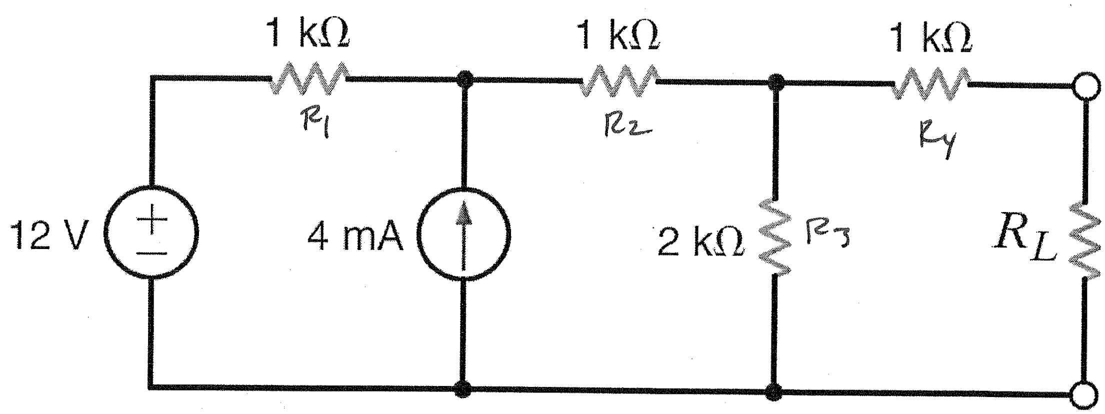
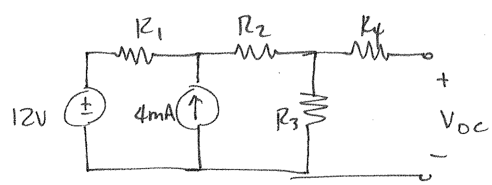


Fig. 4PFE-1

SOLUTION: Find Thevenin Eq.!



$R_{TH} = [(R_1 + R_2) // R_3] + R_4$

$R_{TH} = 2k\Omega$

Find V_{oc} by superposition:

V_{ocA} due to 12-V source

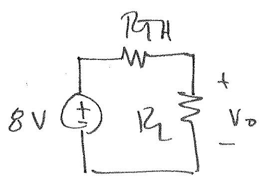
$V_{ocA} = 12\left[\frac{R_3}{R_1 + R_2 + R_3}\right]$

$V_{ocA} = 6V$

V_{ocB} due to 4-mA source

$V_{ocB} = 4\times10^{-3}\left[\frac{R_1}{R_1 + R_2 + R_3}\right] R_3$

$V_{ocB} = 2V \Rightarrow V_{oc} = 8V$



$P_{o_{max}} = \frac{V_{oc}^2}{4R_{TH}}$

$\boxed{P_{o_{max}} = 8mW}$

5FE-2 Find the value of the load R_L in the network in Fig. 5PFE-2 that will achieve maximum power transfer, and determine the value of the maximum power.

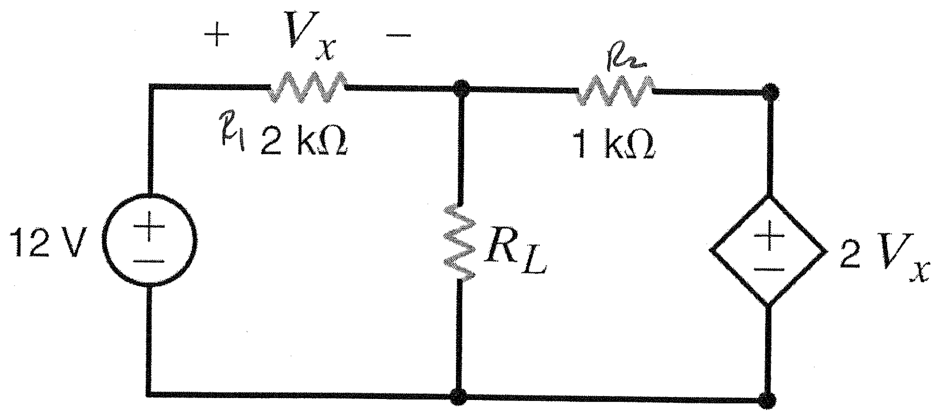
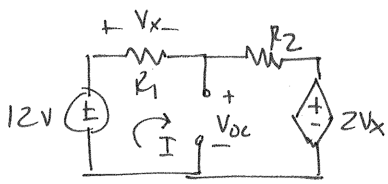


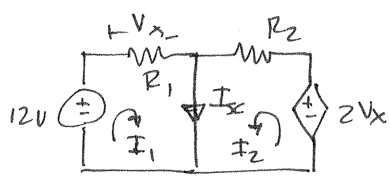
Fig. 5PFE-2

SOLUTION: Need Thevenin Eq.



$12 = IR_1 + IR_2 + 2V_x \qquad V_x = IR_1 \Rightarrow I = \frac{12}{7} mA$

$12 = IR_1 + V_{oc} \Rightarrow V_{oc} = 8.57\,V$



$I_1 = 12/R_1 = 6\,mA \qquad I_2 = 2V_x/R_2$

$V_x = 12V \Rightarrow I_2 = 24\,mA$

$I_{sc} = I_1 + I_2 = 30\,mA$

$R_{TH} = 286\,\Omega$

$P_{o_{max}} = \frac{V_{oc}^2}{4R_{TH}} \qquad \boxed{P_{o_{max}} = 64.2\,mW}$

5FE-3 Find the value of R_L in the network in Fig. 5PFE-3 for maximum power transfer to this load.

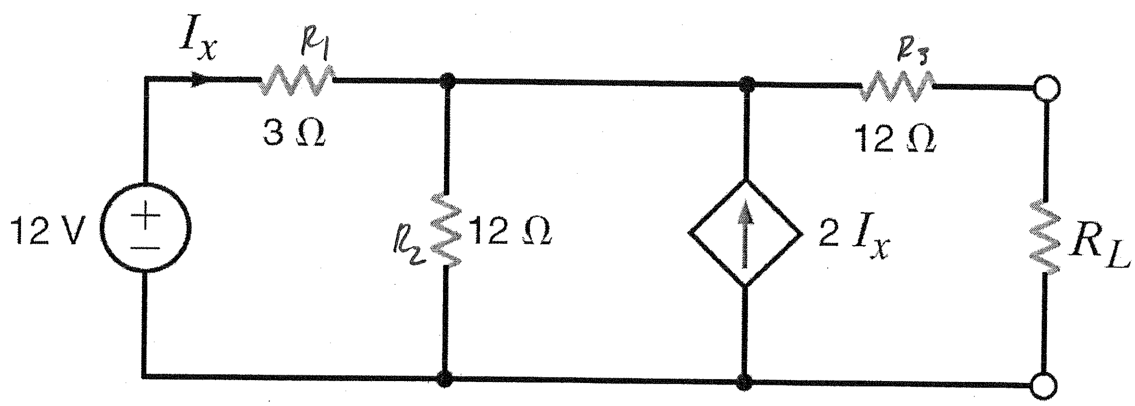
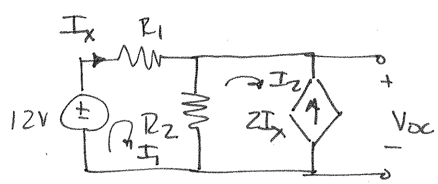


Fig. 5PFE-3

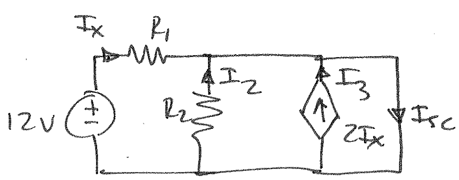
SOLUTION: Need Thevenin Eq.



$12 = (R_1+R_2)I_1 - R_2 I_2 \qquad I_1 = I_x \quad I_2 = -2I_x$

$I_1 = 12/39$ A

$V_{oc} = 12 - R_1 I_1 = 11.08\text{V}$

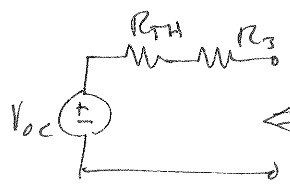


$I_x = 12/R_1 = 4A$

$I_2 = 0 \qquad I_3 = 2I_x = 8A$

$I_{sc} = I_x + I_2 + I_3 = 12A$

$R_{TH} = V_{oc}/I_{sc} = 0.92\Omega$



$\Leftarrow R_{eq} = R_{TH} + R_3 = 12.92\Omega$

for maximum power transfer,

$\boxed{R_L = 12.92\Omega}$

Chapter Six:

Capacitance and Inductance

6.1 A 6-μF capacitor was charged to 12 V. Find the charge accumulated in the capacitor.

SOLUTION:

$Q = CV = (6\times10^{-6})(12)$ $\boxed{Q = 72\,\mu C}$

6.2 A capacitor has an accumulated charge of 600 μC with 5 V across it. What is the value of capacitance?

SOLUTION:

$$C = Q/V = \frac{600 \times 10^{-6}}{5} \qquad \boxed{C = 120\mu F}$$

6.3 An uncharged 100-μF capacitor is charged by a constant current of 1 mA. Find the voltage across the capacitor after 4 s. **CS**

SOLUTION:

$$v = \frac{1}{C}\int_0^T i\,dt \qquad C = 10^{-4}\ \text{F} \qquad i = 10^{-3}\ \text{A} \qquad T = 4\ \text{s}$$

$$v = \frac{iT}{C} \qquad \boxed{v = 40\ \text{V}}$$

6.4 A 10-μF capacitor is charged by a constant current source, and its voltage is increased to 2 V in 5 s. Find the value of the constant current source.

SOLUTION:

$$v = \frac{1}{C}\int_0^5 i\,dt = 2 = \frac{i}{C}\,t\,\Big|_0^5 = \frac{5i}{C} \qquad \boxed{i = 4\,\mu A}$$

6.5 A 50-μF capacitor initially charged to −12 V is charged by a constant current of 2.5 μA. Find the voltage across the capacitor after 3 min.

SOLUTION:

$v = v_0 + \frac{1}{C}\int_0^T i\,dt$ $\quad v_0 = -12\text{V}$ $\quad T = 180\text{s}$ $\quad i = 2.5\mu\text{A}$

$\boxed{v = -3\text{V}}$

6.6 The energy that is stored in a 25-μF capacitor is $w(t) = 12\sin^2 377t$ J. Find the current in the capacitor.
CS

SOLUTION:

$$w(t) = \frac{1}{2} C v^2(t) \qquad v(t) = \sqrt{\frac{2w(t)}{C}}$$

$$i(t) = C\,dv/dt = \frac{d\sqrt{2Cw(t)}}{dt} \qquad i(t) = \sqrt{24C}\;\omega\cos\omega t$$

$$\boxed{i(t) = 9.23\cos\omega t \text{ A} \qquad \omega = 377 \text{ r/s}}$$

6.7 The voltage across a 150-μF capacitor is given by the expression $v(t) = 60 \sin 377t$ V. Find (a) the current in the capacitor and (b) the expression for the energy stored in the element.

SOLUTION:

a) $i = C\,dv/dt = 60 C \omega \cos \omega t$ $\boxed{i(t) = 3.39 \cos \omega t \text{ A}}$

b) $w(t) = \frac{1}{2} C v^2$ $\boxed{w(t) = 270 \sin^2 \omega t \text{ mJ}}$

6.8 The voltage across a 12-μF capacitor is shown in Fig. P6.8. Compute the waveform for the current in the capacitor.

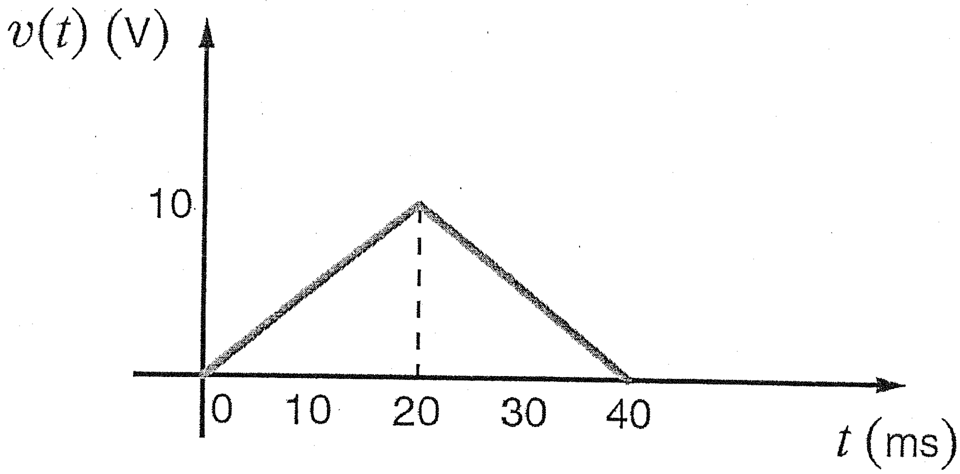


Figure P6.8

SOLUTION:

$t_1 = 20\text{ms}$ $t_2 = 40\text{ms}$ $i = c\, dv/dt$

$t < 0$ $v(t) = 0\text{ V}$ $i = 0$

$0 \le t < t_1$ $v(t) = 500t\text{ V}$ $i = 6\text{mA}$

$t_1 \le t < t_2$ $v(t) = 20 - 500t\text{ V}$ $i = -6\text{mA}$

$t > t_2$ $v(t) = 0$ $i = 0$

$$i(t) = \begin{cases} 0 & t < 0 \\ 6\text{mA} & 0 \le t < t_1 \\ -6\text{mA} & t_1 \le t < t_2 \\ 0 & t > t_2 \end{cases}$$

6.9 The current in a 100-μF capacitor is shown in Fig. P6.9. Determine the waveform for the voltage across the capacitor if it is initially uncharged. CS

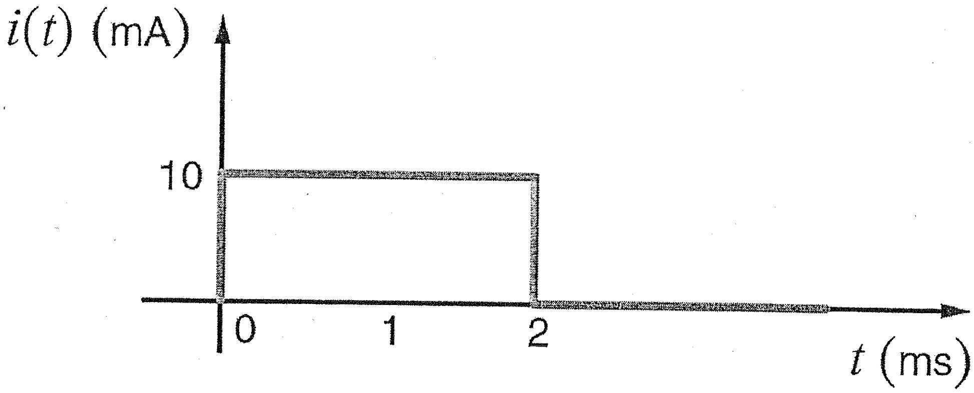


Figure P6.9

SOLUTION:

$t_1 = 2ms$ $\quad v = \frac{1}{C}\int i\,dt$

$t < 0$ $\quad i = 0$ $\quad v = 0$

$0 \le t < t_1$ $\quad i = 10mA$ $\quad v = 100t$ V

$t \ge t_1$ $\quad i = 0$ $\quad v = 0.2V$

$$v(t) = \begin{cases} 0 \text{ V} & t < 0 \\ 100t \text{ V} & 0 \le t < t_1 \\ 0.2 \text{ V} & t \ge t_1 \end{cases}$$

6.10 The voltage across a 50-μF capacitor is shown in Fig. P6.10. Compute the waveform for the current in the capacitor.

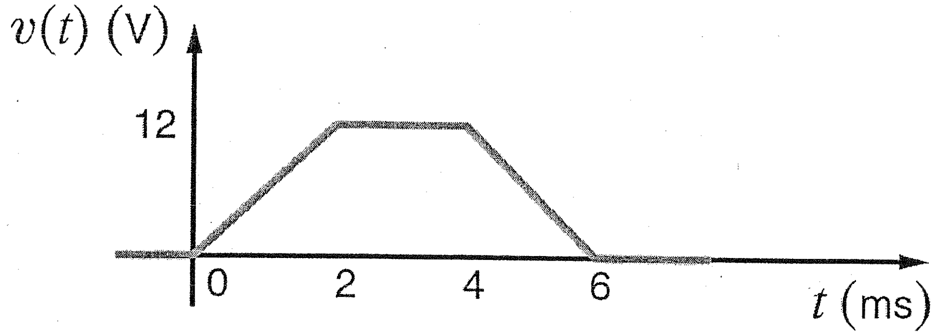


Figure P6.10

SOLUTION:

$t_1 = 2ms$ $\quad t_2 = 4ms$ $\quad t_3 = 6ms$ $\quad i = C\, dv/dt$

$t < 0$ $\quad v = 0$ $\quad i = 0$

$0 \le t < t_1$ $\quad v = 6000t$ $\quad i = 300mA$

$t_1 \le t < t_2$ $\quad v = 12$ $\quad i = 0$

$t_2 \le t < t_3$ $\quad v = 36 - 6000t$ $\quad i = -300mA$

$t \ge t_3$ $\quad v = 0$ $\quad i = 0$

$$i(t) = \begin{cases} 0 & t < 0 \\ 300mA & 0 \le t < t_1 \\ 0 & t_1 \le t < t_2 \\ -300mA & t_2 \le t < t_3 \\ 0 & t \ge t_3 \end{cases}$$

6.11 The voltage across a 25-μF capacitor is shown in Fig. P6.11. Determine the current waveform. **PSV**

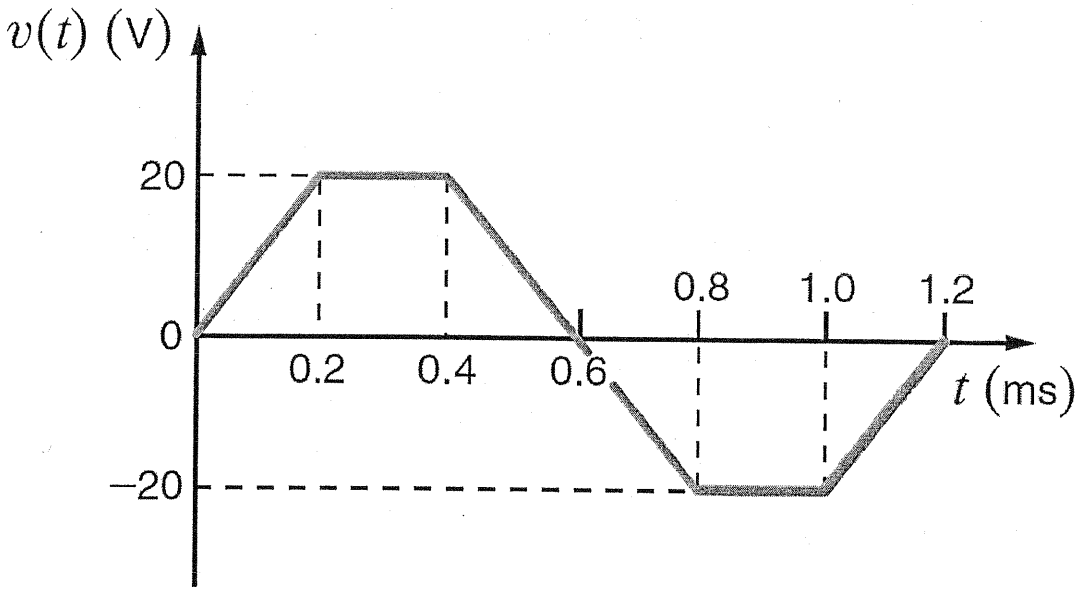


Figure P6.11

SOLUTION:

$t_1 = 0.2\text{ms}$ $t_2 = 0.4\text{ms}$ $t_3 = 0.8\text{mA}$ $t_4 = 1\text{mA}$ $t_5 = 1.2\text{ms}$ $i = C\,dv/dt$

$t < 0$ $v = 0$ $i = 0$

$0 \le t < t_1$ $v = 10^5 t$ $i = 2.5\text{A}$

$t_1 \le t < t_2$ $v = 20$ $i = 0$

$t_2 \le t < t_3$ $v = 60 - 10^5 t$ $i = -2.5\text{A}$

$t_3 \le t < t_4$ $v = 0$ $i = 0$

$t_4 \le t < t_5$ $v = -120 + 10^5 t$ $i = 2.5\text{A}$

$$i(t) = \begin{cases} 0 & t < 0 \\ 2.5 & 0 \le t < t_1 \\ 0 & t_1 \le t < t_2 \\ -2.5 & t_2 \le t < t_3 \\ 0 & t_3 \le t < t_4 \\ 2.5 & t_4 \le t < t_5 \\ 0 & t \ge t_5 \end{cases}$$

6.12 The voltage across a 2-F capacitor is given by the waveform in Fig. P6.12. Find the waveform for the current in the capacitor.

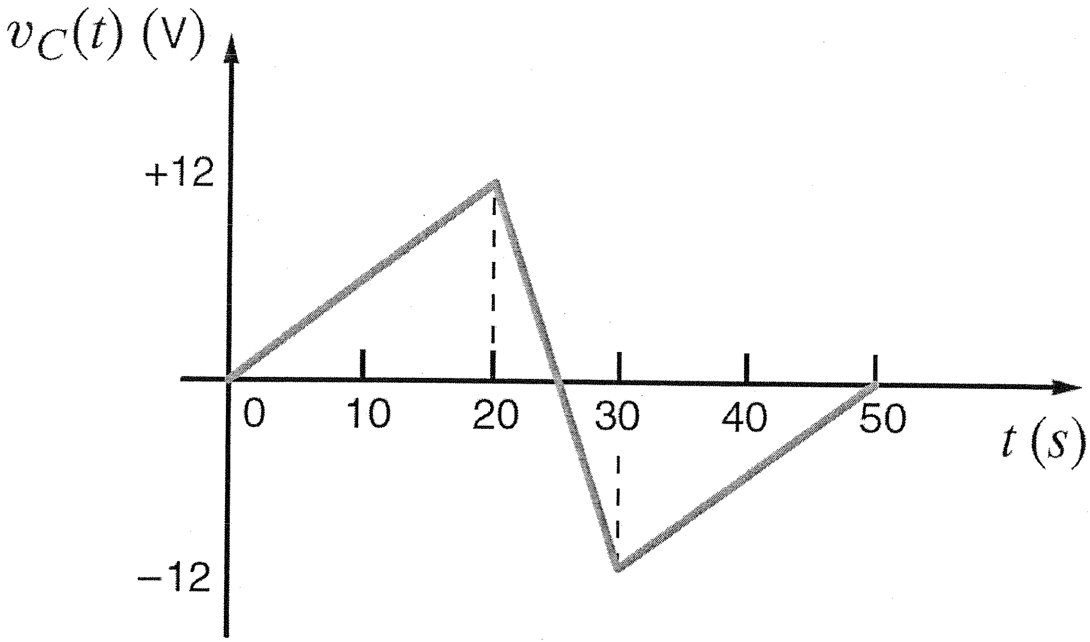


Figure P6.12

SOLUTION:

$t_1 = 20s \quad t_2 = 30s \quad t_3 = 50s \quad i = C\, dv/dt$

$t < 0$	$v = 0$	$i = 0$
$0 < t < t_1$	$v = 0.6t$ V	$i = 1.2A$
$t_1 < t < t_2$	$v = 60 - 2.4t$	$i = -4.8A$
$t_2 < t < t_3$	$v = -30 + 0.6t$	$i = 1.2A$
$t > t_3$	$v = 0$	$i = 0$

$$i(t) = \begin{cases} 0 & t < 0 \\ 1.2\text{ A} & 0 < t < t_1 \\ -4.8\text{ A} & t_1 < t < t_2 \\ 1.2\text{ A} & t_2 < t < t_3 \\ 0 & t > t_3 \end{cases}$$

6.13 The voltage across a 2-μF capacitor is given by the waveform in Fig. P6.13. Compute the current waveform. CS

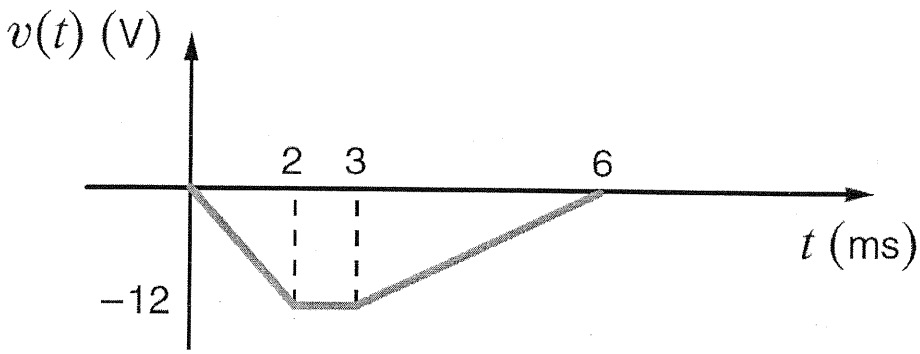


Figure P6.13

SOLUTION:

$t_1 = 2ms$ $\quad t_2 = 3ms$ $\quad t_3 = 6ms$ $\quad i = C\,dv/dt$

$t<0$	$v=0$	$i=0$
$0<t<t_1$	$v=-6000t$	$i=-12mA$
$t_1<t<t_2$	$v=-12$	$i=0$
$t_2<t<t_3$	$v=-24+4000t$	$i=8mA$
$t>t_3$	$v=0$	$i=0$

$$i(t) = \begin{cases} 0 & t<0 \\ -12\ mA & 0<t<t_1 \\ 0 & t_1<t<t_2 \\ 8mA & t_2<t<t_3 \\ 0 & t>t_3 \end{cases}$$

6.14 Draw the waveform for the current in a 24-μF capacitor when the capacitor voltage is as described in Fig. P6.14.

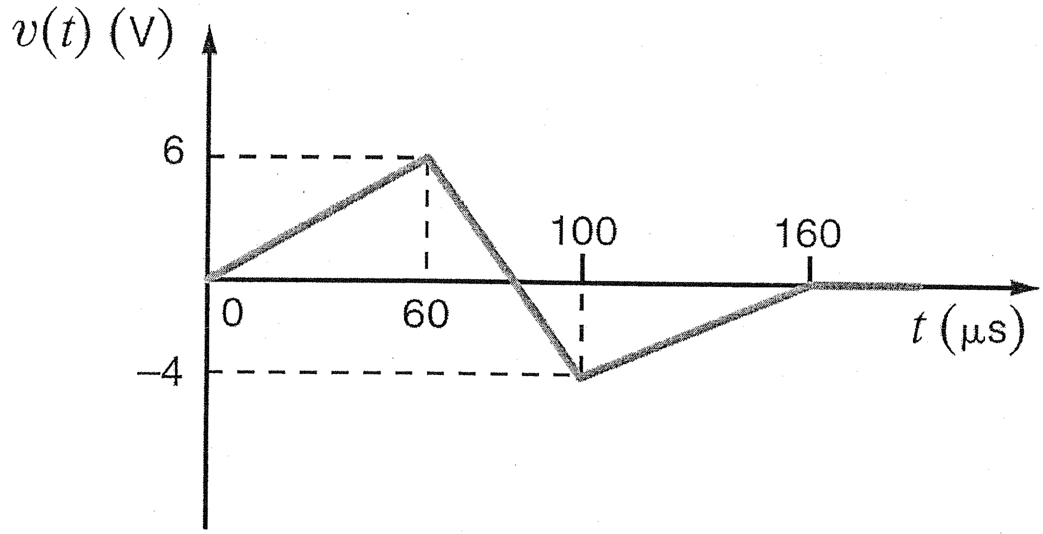


Figure P6.14

SOLUTION:

$t_1 = 60\mu s$ $\quad t_2 = 100\mu s$ $\quad t_3 = 160\mu s$ $\quad i = C\,dv/dt$

$t<0$	$v=0$	$i=0$
$0\le t<t_1$	$v=10^5 t$	$i=2.4A$
$t_1\le t<t_2$	$v=21-2.5\times10^5 t$	$i=-6A$
$t_2\le t<t_3$	$v=-\frac{32}{3}+\frac{10^6}{15}t$	$i=1.6A$
$t\ge t_3$	$v=0$	$i=0$

$$i(t)=\begin{cases} 0 & t<0 \\ 2.4A & 0\le t<t_1 \\ -2.4A & t_1\le t<t_2 \\ 1.6A & t_2\le t<t_3 \\ 0 & t\ge t_3 \end{cases}$$

6.15 Draw the waveform for the current in a 3-μF capacitor when the voltage across the capacitor is given in Fig. P6.15. CS

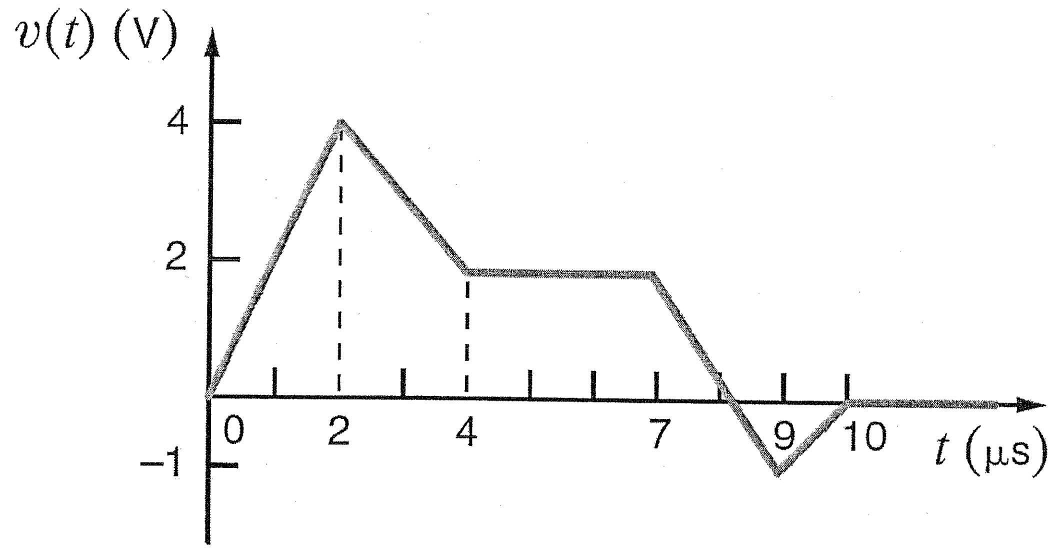


Figure P6.15

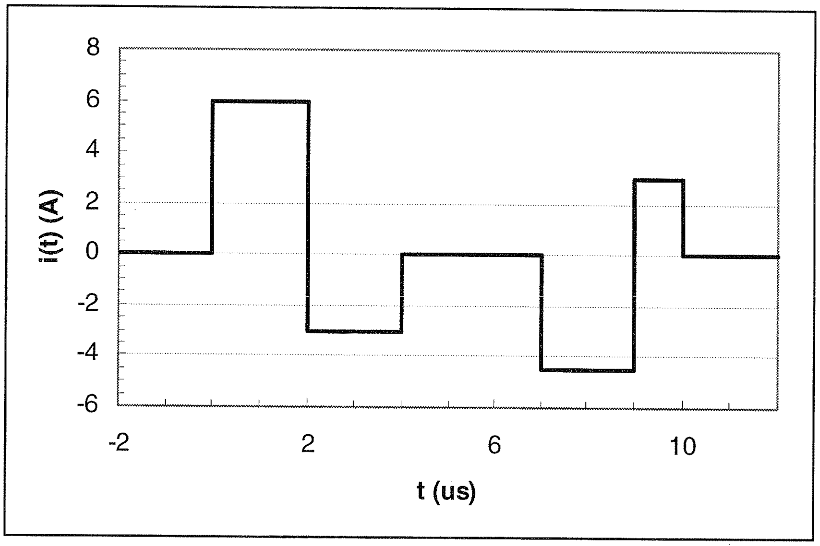
SOLUTION: $t_1 = 2\mu s \quad t_2 = 4\mu s \quad t_3 = 7\mu s \quad t_4 = 9\mu \quad t_5 = 10\mu s$

$i = c\, dv/dt$

$t < 0$	$dv/dt = 0$
$0 \le t < t_1$	$\Delta v/\Delta t = (4-0)/t_1 = 2\times10^6$
$t_1 \le t < t_2$	$\Delta v/\Delta t = (2-4)/(t_2 - t_1) = -10^6$
$t_2 \le t < t_3$	$\Delta v/\Delta t = 0$
$t_3 \le t < t_4$	$\Delta v/\Delta t = (-1-2)/(t_4 - t_3) = -1.5\times10^6$
$t_4 \le t < t_5$	$\Delta v/\Delta t = 1/(t_5 - t_4) = 10^6$
$t \ge t_5$	$\Delta v/\Delta t = 0$

$$i(t) = \begin{cases} 0 & t < 0 \\ 6A & 0 \le t < t_1 \\ -3A & t_1 \le t < t_2 \\ 0 & t_2 \le t < t_3 \\ -4.5A & t_3 \le t < t_4 \\ 3A & t_4 \le t < t_5 \\ 0 & t \ge t_5 \end{cases}$$

6.15



6.16 The voltage across a 10-μF capacitor is given by the waveform in Fig. P6.16. Plot the waveform for the capacitor current.

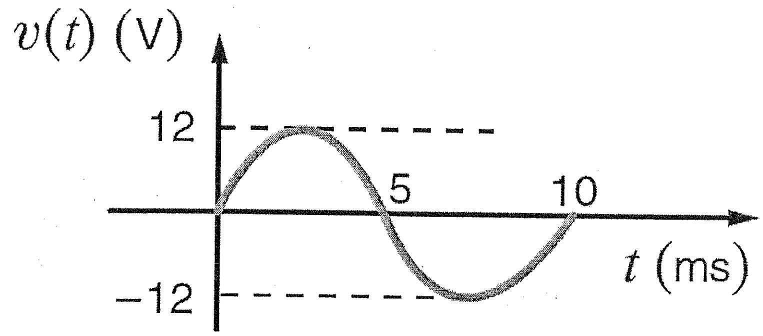


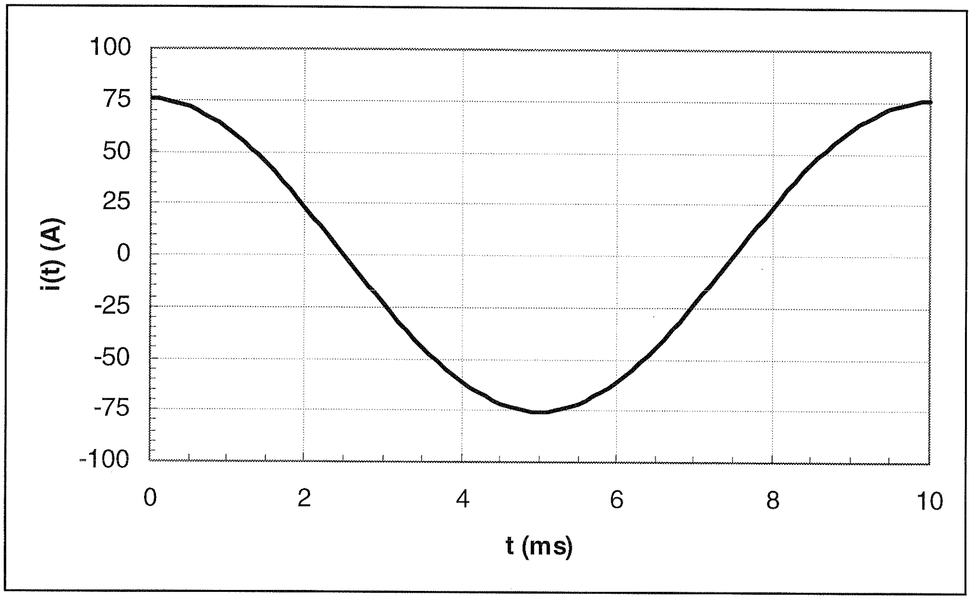
Figure P6.16

SOLUTION:

$i = C\,dv/dt$ $\quad v = 12 \sin \omega t$ $\quad \omega = \frac{2\pi}{T}$ $\quad T = 10\text{ms} \Rightarrow \omega = 200\pi$ r/s

$\boxed{i = 75.4 \cos \omega t \text{ mA} \quad \omega = 200\pi \text{ r/s}}$

6.16



6.17 The waveform for the current in a 50-μF capacitor is shown in Fig. P6.17. Determine the waveform for the capacitor voltage. **PSV**

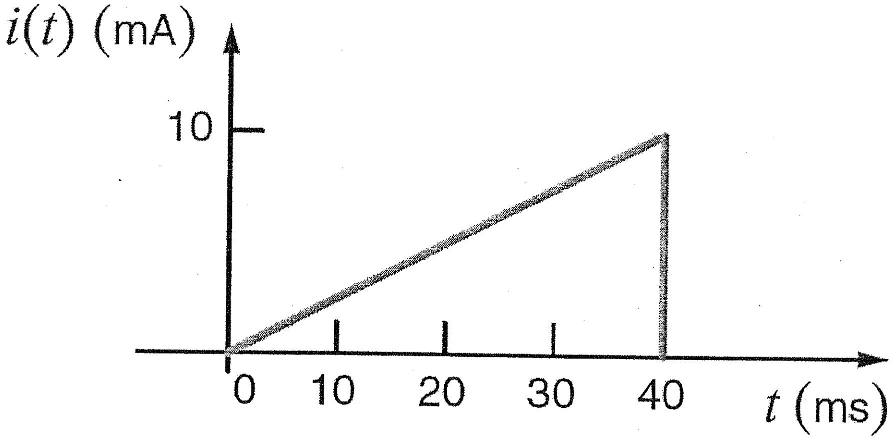


Figure P6.17

SOLUTION: $t_1 = 40\,ms$ $\quad v = \frac{1}{C}\int i\,dt + v_0$

$t < 0 \quad i(t) = 0 \quad v = 0$

$0 \le t < t_1 \quad i(t) = 0.25\,t \quad v = 2500\,t^2$

$t \ge t_1 \quad i(t) = 0 \quad v = 2500\,(40 \times 10^{-3})^2 + 0 = 4V$

$$v(t) = \begin{cases} 0 & t < 0 \\ 2500\,t^2\ V & 0 \le t < t_1 \\ 4V & t \ge t_1 \end{cases}$$

6.18 The waveform for the current in a 100-μF initially uncharged capacitor is shown in Fig. P6.18. Determine the waveform for the capacitor's voltage.

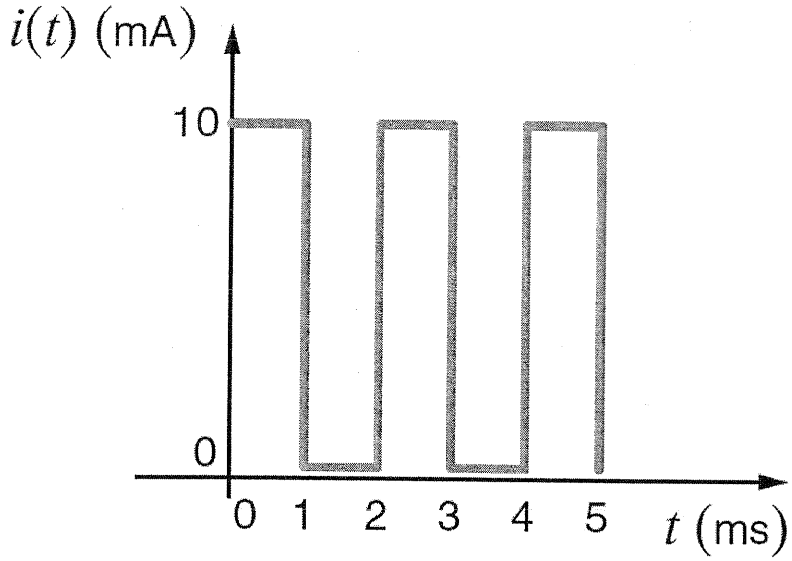


Figure P6.18

SOLUTION:

$$v_c(t) = \frac{1}{C}\int i_c(t)\,dt = 10^4 \int i_c(t)\,dt$$

for $i(t) = 10\,mA$, $\quad v_c(t) = 100t + v_0 \quad$ (v_0 = initial voltage)

$$v_c(t) = \begin{cases} 0 \text{ V} & t < 0 \\ 100t \text{ V} & 0 \le t \le 1ms \\ 0.1 \text{ V} & 1ms \le t \le 2ms \\ -0.1 + 100t \text{ V} & 2ms \le t \le 3ms \\ 0.2 \text{ V} & 3ms \le t \le 4ms \\ -0.2 + 100t \text{ V} & 4ms \le t \le 5ms \\ 0.3\text{V} & t > 5m \end{cases}$$

6.19 The waveform for the current in a 50-μF initially uncharged capacitor is shown in Fig. P6.19. Determine the waveform for the capacitor's voltage.

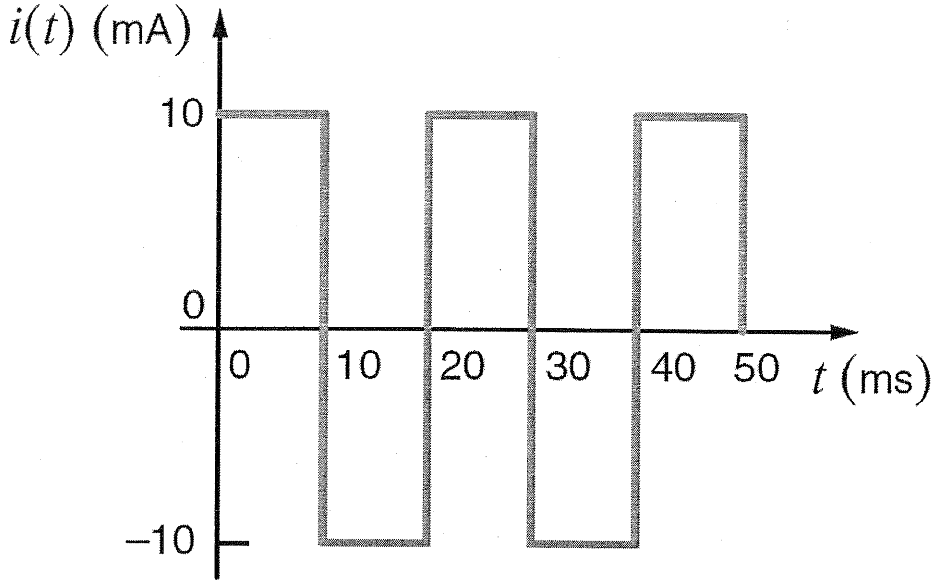


Figure P6.19

SOLUTION:

$$v_c(t) = \frac{1}{C}\int i_c(t)\,dt$$

for $i = 10\text{mA}$ $v_c(t) = 200t + V_o$ V_o = initial voltage
for $i = -10\text{mA}$ $v_c(t) = -200t + V_o$

$$v_c(t) = \begin{cases} 0 \text{ V} & t < 0 \\ 200t \text{ V} & 0 \le t \le 10\text{ms} \\ 4 - 200t \text{ V} & 10\text{ms} \le t \le 20\text{ms} \\ -4 + 200t \text{ V} & 20\text{ms} \le t \le 30\text{ms} \\ 8 - 200t \text{ V} & 30\text{ms} \le t \le 40\text{ms} \\ -8 + 200t \text{ V} & 40\text{ms} \le t \le 50\text{ms} \\ 0 \text{ V} & t > 50\text{ms} \end{cases}$$

6.20 If $v_C(t = 2\text{ s}) = 10$ V in the circuit in Fig. P6.20, find the energy stored in the capacitor and the power supplied by the source at $t = 6$ s.

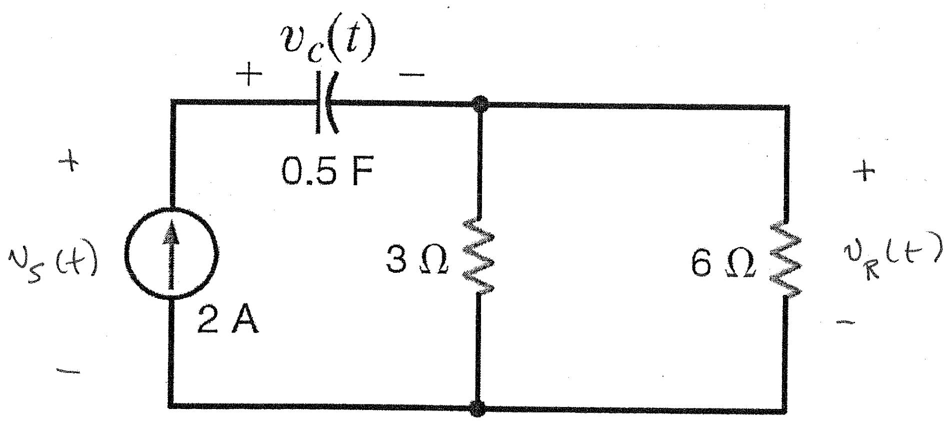


Figure P6.20

SOLUTION:

$$v_c(t_2) = \frac{1}{C}\int_{t_1}^{t_2} i_c(t)\,dt + V_o \qquad t_1 = 2s \qquad t_2 = 6s \qquad V_o = 10V$$

$$v_c(t_2) = 2(2)\,t\Big|_2^6 + 10 \qquad \boxed{v_c(t_2) = 26V}$$

$$w_c(t_2) = \frac{1}{2}Cv_c(t_2)^2 \Rightarrow \boxed{w_c(t_2) = 169J}$$

$$v_R(t_2) = i(t_2)\left[3//6\right] = 2(2) = 4V$$

$$v_s(t_2) = v_c(t_2) + v_R(t_2) = 30V$$

$$P_s(t_2) = v_s(t_2)\,i_s(t_2) = 30(2) \Rightarrow \boxed{P_s(t_2) = 60W}$$

6.21 The current in an inductor changes from 0 to 200 mA in 4 ms and induces a voltage of 100 mV. What is the value of the inductor?

SOLUTION:

$v = L\, di/dt \Rightarrow L \frac{\Delta I}{\Delta t}$ $\Delta I = 200\text{mA}$ $\Delta t = 4\text{ms}$ $v = 0.1\text{V}$

$L = v\left(\frac{\Delta t}{\Delta I}\right)$ $\boxed{L = 2\text{mH}}$

6.22 The current in a 100-mH inductor is $i(t) = 2 \sin 377t$ A. Find (a) the voltage across the inductor and (b) the expression for the energy stored in the element. CS

SOLUTION:

a) $v = L\, di/dt = 0.1\,(2)(377) \cos 377t$

$$\boxed{v(t) = 75.4 \cos 377t \quad V}$$

b) $w(t) = \frac{1}{2} L\, i(t)^2$ $\boxed{w(t) = 0.2 \sin^2 377t \quad J}$

6.23 If the current $i(t) = 1.5t$ A flows through a 2-H inductor, find the energy stored at $t = 2s$.

SOLUTION:

$$w(t) = \frac{1}{2} L\, i(t)^2$$

$$w(2) = \left(\frac{1}{2}\right)(2)\left[1.5\,(2)\right]^2 \qquad \boxed{w(2) = 9\,J}$$

6.24 The current in a 25-mH inductor is given by the expressions

$$i(t) = 0 \qquad t < 0$$

$$i(t) = 10(1 - e^{-t})\text{mA} \qquad t > 0$$

Find (a) the voltage across the inductor and (b) the expression for the energy stored in it.

SOLUTION:

a) $v = L\, di/dt$

$$v = \begin{cases} 0 \text{ V} & t<0 \\ 250e^{-t}\ \mu\text{V} & t>0 \end{cases}$$

b) $w(t) = \frac{1}{2} L\, i(t)^2$

$$w(t) = \begin{cases} 0 \text{ J} & t<0 \\ 1.25(1-e^{-t})^2\ \mu\text{J} & t>0 \end{cases}$$

6.25 Given the data in the previous problem, find the voltage across the inductor and the energy stored in it after 1 s. CS

SOLUTION:

$v(t) = 250 e^{-t} \mu V \quad t > 0$

$w(t) = 1.25 (1 - e^{-t})^2 \mu J \quad t > 0$

$v(1) = 92.0 \mu V$

$w(1) = 0.50 \mu J$

6.26 The current in a 10-mH inductor is shown in Fig. P6.26. Determine the waveform for the voltage across the inductor.

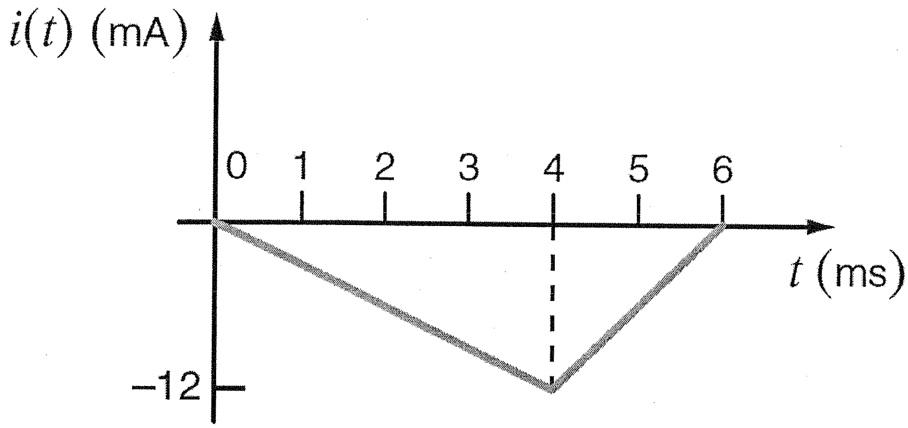


Figure P6.26

SOLUTION:

$v = L\,di/dt$

for $0 \le t \le 4ms$, $\quad di/dt = -\dfrac{12 \times 10^{-3}}{4 \times 10^{-3}} = -3\ A/s$

for $4ms \le t \le 6ms$, $\quad di/dt = +12 \times 10^{-3} / 2 \times 10^{-3} = +6\ A/s$

$$v = \begin{cases} 0\ V & t < 0 \\ -30\ mV & 0 < t \le 4ms \\ +60\ mV & 4ms < t \le 6ms \\ 0\ V & t > 6ms \end{cases}$$

6.27 The current in a 50-mH inductor is given in Fig. P6.27. Sketch the inductor voltage. CS

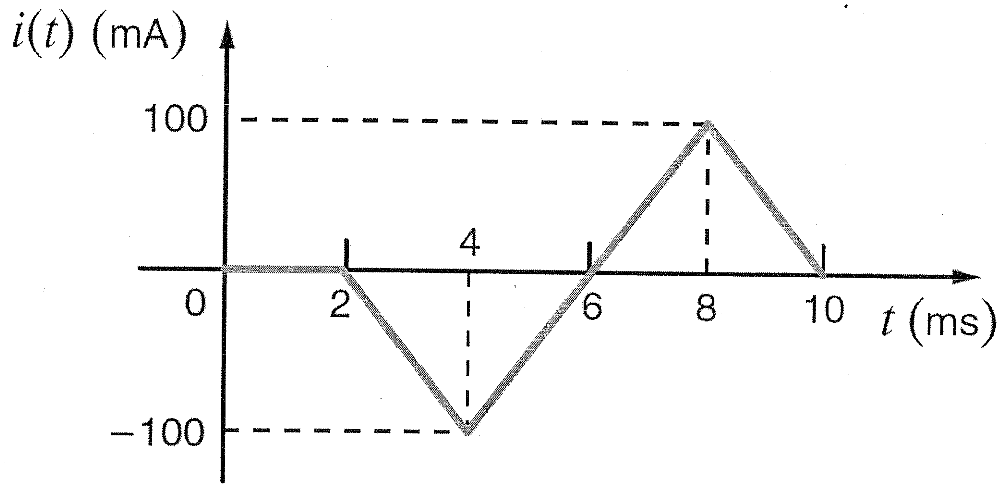
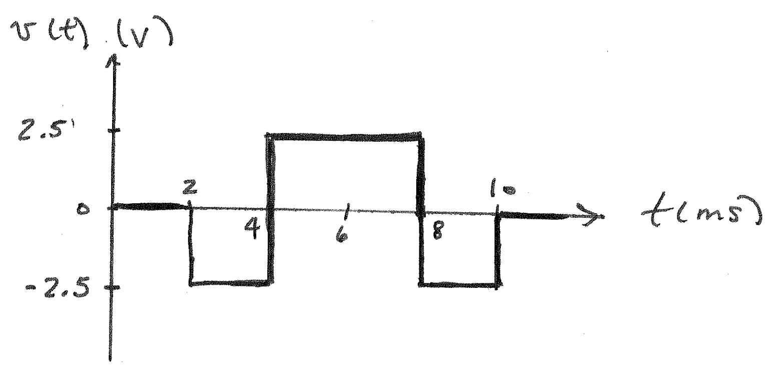


Figure P6.27

SOLUTION:

$v = L\,di/dt$ $\quad di/dt = \begin{cases} -50\ A/s & t \text{ between } 2 \,\&\, 4ms \text{ and } 8 \text{ to } 10ms \\ +50\ A/s & t \text{ between } 4ms \text{ to } 8\ ms. \end{cases}$



6.28 The current in a 50-mH inductor is shown in Fig. P6.28. Find the voltage across the inductor. **PSV**

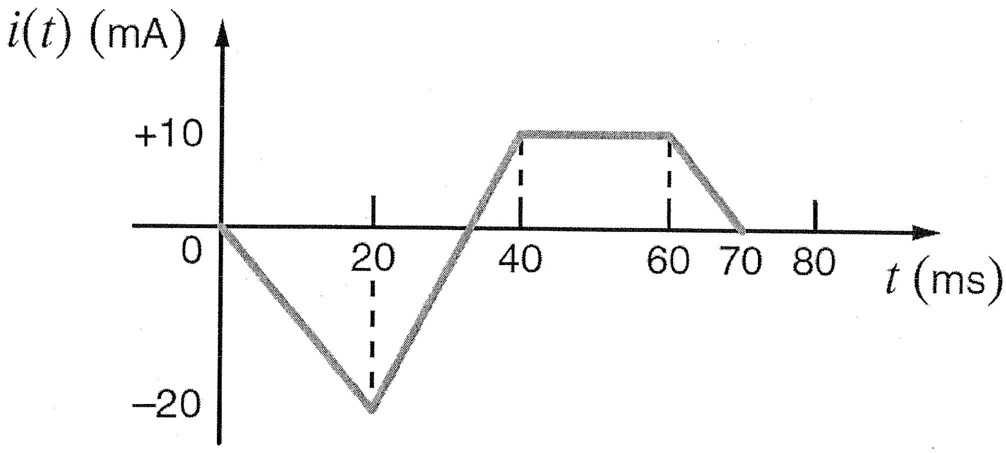


Figure P6.28

SOLUTION:

$v = L\,di/dt$

$$di/dt = \begin{cases} -1\ A/s & t \text{ between } 0 \text{ to } 20ms \\ 1.5\ A/s & t \text{ betwen } 20ms \text{ to } 40ms \\ -1\ A/s & t \text{ betwen } 60ms \text{ to } 70ms \\ 0 & \text{otherwise} \end{cases}$$

$$v(t) = \begin{cases} 0\ V & t < 0 \\ -50\ mV & 0 < t \le 20ms \\ +75\ mV & 20ms \le t \le 40ms \\ 0\ V & 40ms < t \le 60ms \\ -50\ mV & 60ms < t \le 70ms \\ 0V & t > 70ms \end{cases}$$

6.29 Draw the waveform for the voltage across a 24-mH inductor when the inductor current is given by the waveform shown in Fig. P6.29.

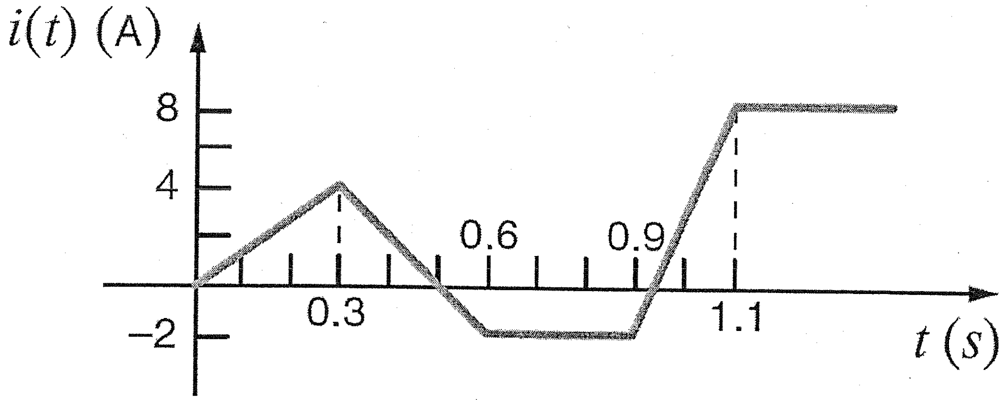


Figure P6.29

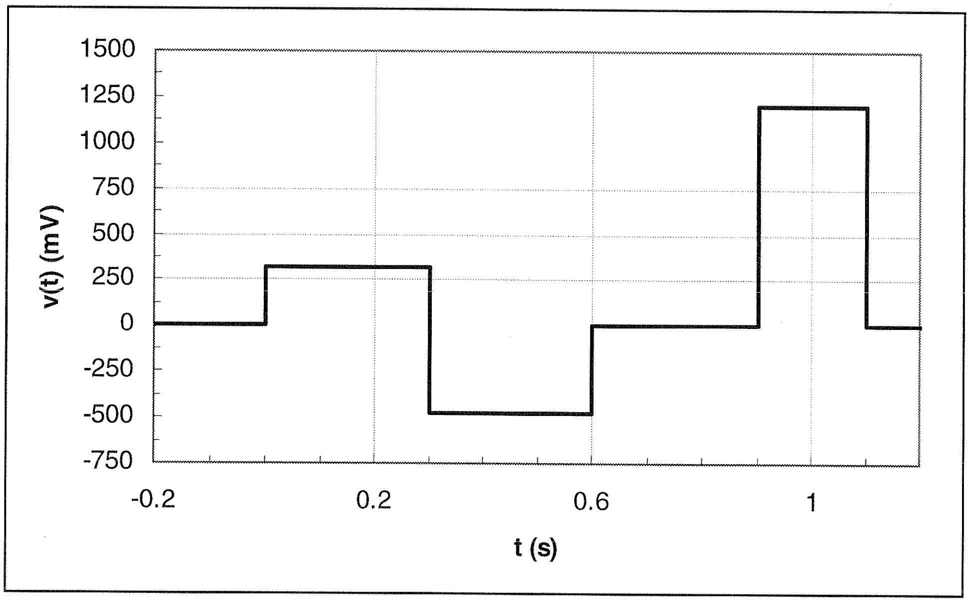
SOLUTION:

$v = L\,di/dt$

$$di/dt = \begin{cases} 40/3 \text{ A/s} & 0 \le t \le 0.3\text{ s} \\ -20 \text{ A/s} & 0.3 < t \le 0.6\text{ s} \\ 50 \text{ A/s} & 0.9 < t \le 1.1\text{ s} \\ 0 & \text{otherwise} \end{cases}$$

$$v(t) = \begin{cases} 0 & t \le 0 \\ 320\text{ mV} & 0 < t \le 0.3\text{ s} \\ -480\text{ mV} & 0.3 < t \le 0.6\text{ s} \\ 0 & 0.6 < t \le 0.9\text{ s} \\ 1200\text{ mV} & 0.9 < t \le 1.1\text{ s} \\ 0 & t > 1.1\text{ s} \end{cases}$$

6.29



6.30 The current in a 24-mH inductor is given by the waveform in Fig. P6.30. Find the waveform for the voltage across the inductor.

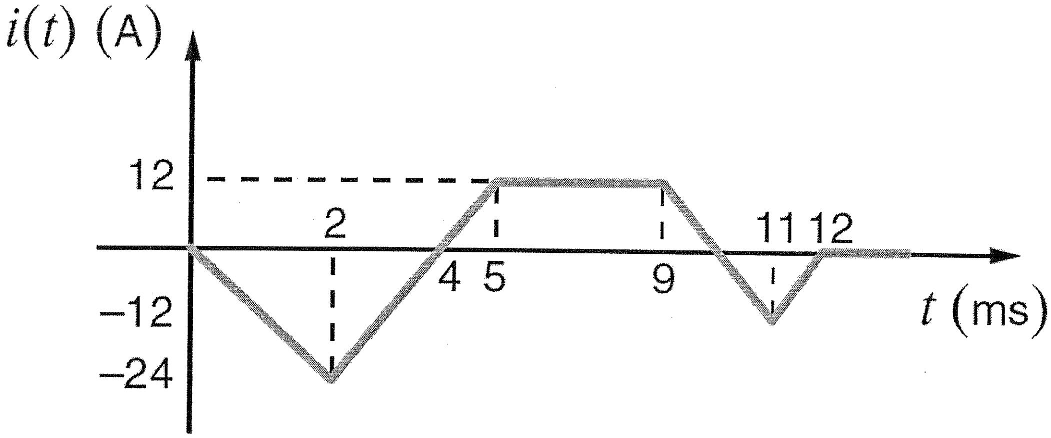


Figure P6.30

SOLUTION:

$v = L\, di/dt$

$$\frac{di}{dt} = \begin{cases} -12 \times 10^3 \text{ A/s} & 0 \le t \le 2ms \\ +12 \times 10^3 \text{ A/s} & 2ms < t \le 5ms \\ -12 \times 10^3 \text{ A/s} & 9ms \le t \le 11ms \\ +12 \times 10^3 \text{ A/s} & 11ms < t \le 12ms \end{cases}$$

$$v(t) = \begin{cases} 0 \text{ V} & t \le 0 \\ -288 \text{ V} & 0 < t \le 2ms \\ +288 \text{ V} & 2ms < t \le 5ms \\ 0 \text{ V} & 5ms < t \le 9ms \\ -288 \text{ V} & 9ms < t \le 11ms \\ +288 \text{ V} & 11ms < t \le 12ms \\ 0 \text{ V} & t > 12ms \end{cases}$$

6.31 The current in a 4-mH inductor is given by the waveform in Fig. P6.31. Plot the voltage across the inductor.

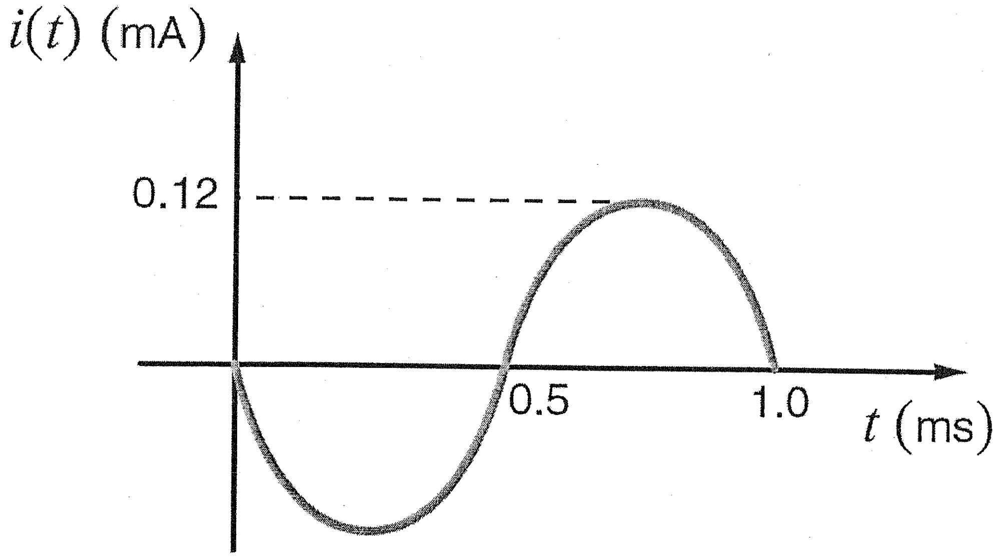
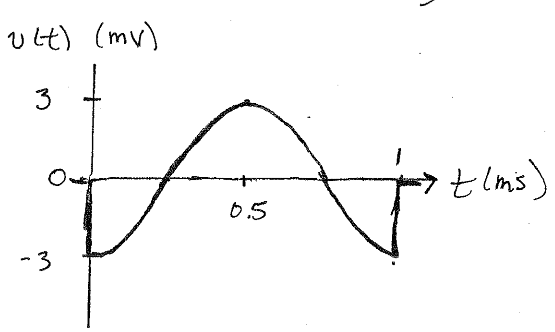


Figure P6.31

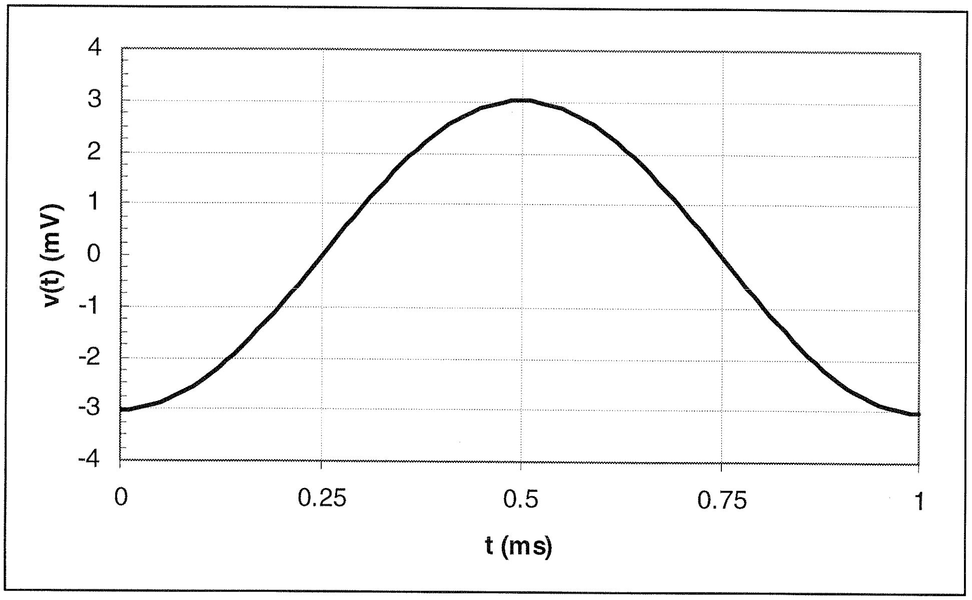
SOLUTION:

$i(t) = -120 \sin(\omega t)\ \mu A \qquad \omega = \frac{2\pi}{T} \qquad T = 1ms \Rightarrow \omega = 2000\pi\ r/s$

$i(t) = -120 \sin(2000\pi t)\ \mu A \qquad v(t) = L\frac{di}{dt} = -3.02 \cos(2000\pi t)\ mV$



6.31



6.32 The voltage across a 2-H inductor is given by the waveform shown in Fig. P6.32. Find the waveform for the current in the inductor.

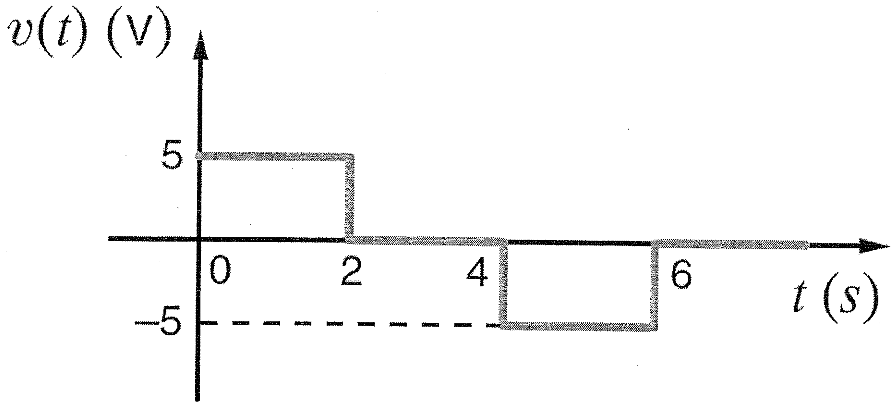


Figure P6.32

SOLUTION:

$i = \frac{1}{L}\int v(t)\,dt$

Since $v(t)$ is a constant across each time span, we can write

$$i = \frac{V}{L}t + I_0$$

V = constant voltage
I_0 = initial current.

$$i(t) = \begin{cases} 0 & t<0 \\ 2.5t \text{ A} & 0 \le t \le 2s \\ 5 \text{ A} & 2 \le t \le 4s \\ 15 - 2.5t \text{ A} & 4 < t \le 6s \\ 0 \text{ A} & t \ge 6s \end{cases}$$

6.33 The waveform for the voltage across a 20-mH inductor is shown in Fig. P6.33. Compute the waveform for the inductor current. $v(t) = 0, t < 0$. CS

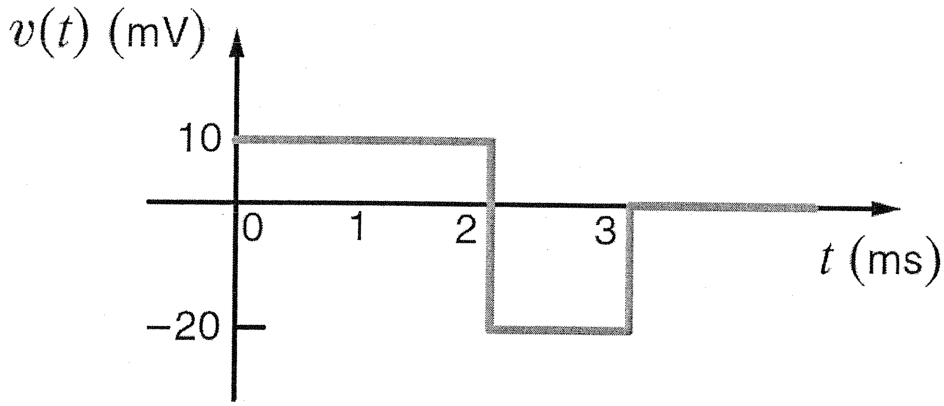


Figure P6.33

SOLUTION:

$$i = \frac{1}{L}\int v(t)\,dt$$

Since $v(t)$ is constant across three spans,

$$i(t) = \frac{V}{L}t + I_0$$

V = constant voltage
I_0 = initial current

$$i(t) = \begin{cases} 0 \text{ A} & t < 0 \\ t/2 \text{ A} & 0 \leq t \leq 2\text{ms} \\ 3\times10^{-3} - t \text{ A} & 2\text{ms} \leq t \leq 3\text{ms} \\ 0 \text{ A} & t \geq 3\text{ms} \end{cases}$$

6.34 The voltage across a 4-H inductor is given by the waveform shown in Fig. P6.34. Find the waveform for the current in the inductor. $v(t) = 0, t < 0$. **PSV**

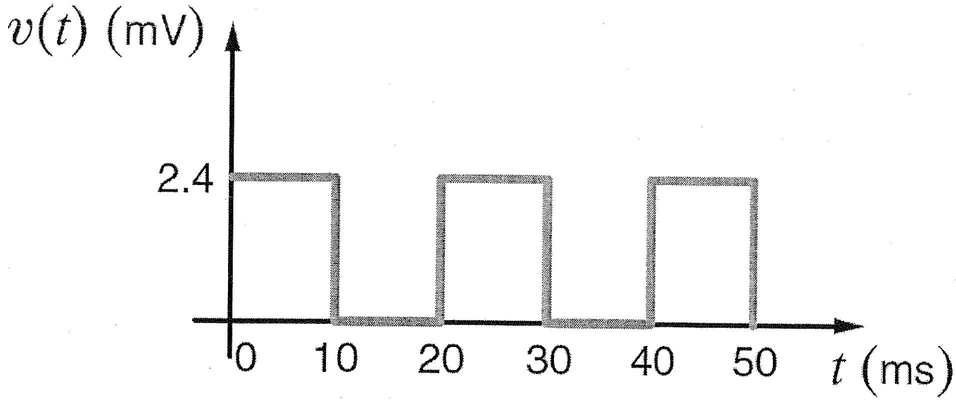


Figure P6.34

SOLUTION:

$i(t) = \frac{1}{L}\int v(t)\,dt$

$v(t)$ is constant across time spans, so

$i(t) = \frac{V}{L}t + I_0$ V = constant voltage, I_0 = initial current

$$i(t) = \begin{cases} 0 & t < 0 \\ 600t \ \mu A & 0 \le t \le 10ms \\ 6 \mu A & 10ms \le t \le 20ms \\ -6 + 600t \ \mu A & 20ms \le t \le 30ms \\ 12 \mu A & 30ms \le t \le 40ms \\ -12 + 600t \ \mu A & 40ms \le t \le 50ms \\ 18 \mu A & t \ge 50ms \end{cases}$$

6.35 The voltage across a 24-mH inductor is shown in Fig. P6.35. Determine the waveform for the inductor current. $v(t) = 0, t < 0$.

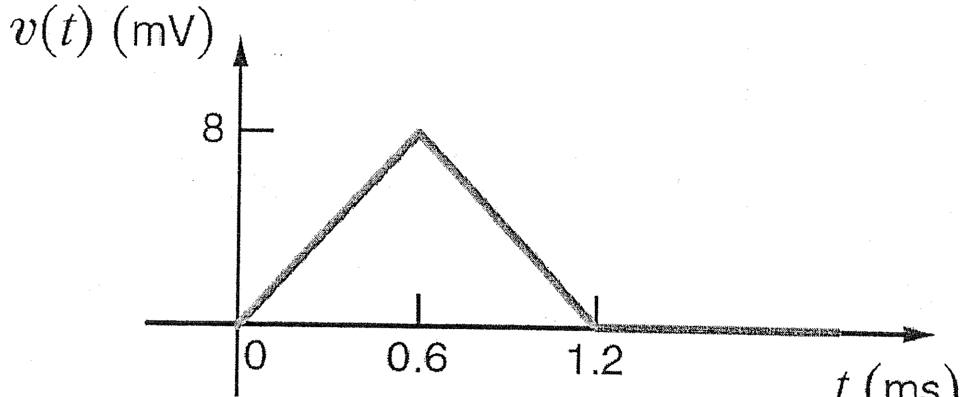


Figure P6.35

SOLUTION:

For $0 \le t \le 0.6ms$ $\quad v(t) = (40/3)t \ \ V \quad i(t) = \frac{1}{L}\int v\,dt = \frac{20}{3L}t^2 = 278t^2 A$

For $0.6m \le t \le 1.2ms$ $\quad v(t) = 16 \times 10^{-3} - \left(\frac{40}{3}\right)t$

$$i(t) = \frac{1}{L}\int v\,dt = \frac{1}{L}\left[16\times10^{-3}t - \frac{20}{3}t^2 + K\right]$$

$$i(t_1) = 100\mu A = \left[16\times10^{-3}t_1 - \frac{20}{3}t_1^{\,2} + K\right]/L \qquad t_1 = 0.6\,ms$$

$K = -200\mu A$

$$i(t) = \begin{cases} 0 \ \ A & t < 0 \\ 278t^2 \ \ A & 0 \le t \le 0.6ms \\ 0.667t - 278t^2 - 200\times10^{-6} A & 0.6ms \le t \le 1.2ms \\ 200\mu A & t \ge 1.2ms \end{cases}$$

6.36 Find the possible capacitance range of the following capacitors.

(a) 0.068 μF with a tolerance of 10%.

(b) 120 pF with a tolerance of 20%.

(c) 39 μF with a tolerance of 20%.

SOLUTION:

a) $C = 6.8\,nF \pm 10\%$ $\qquad 6.12\,nF \le C \le 7.48\,nF$

b) $C = 120\,pF \pm 20\%$ $\qquad 96\,pF \le C \le 144\,pF$

c) $C = 39\,\mu F \pm 20\%$ $\qquad 31.2\,\mu F \le C \le 46.8\,\mu F$

6.37 The capacitor in Fig. P6.37a is 51 nF with a tolerance of 10%. Given the voltage waveform in Fig. P6.37b, graph the current $i(t)$ for the minimum and maximum capacitor values.

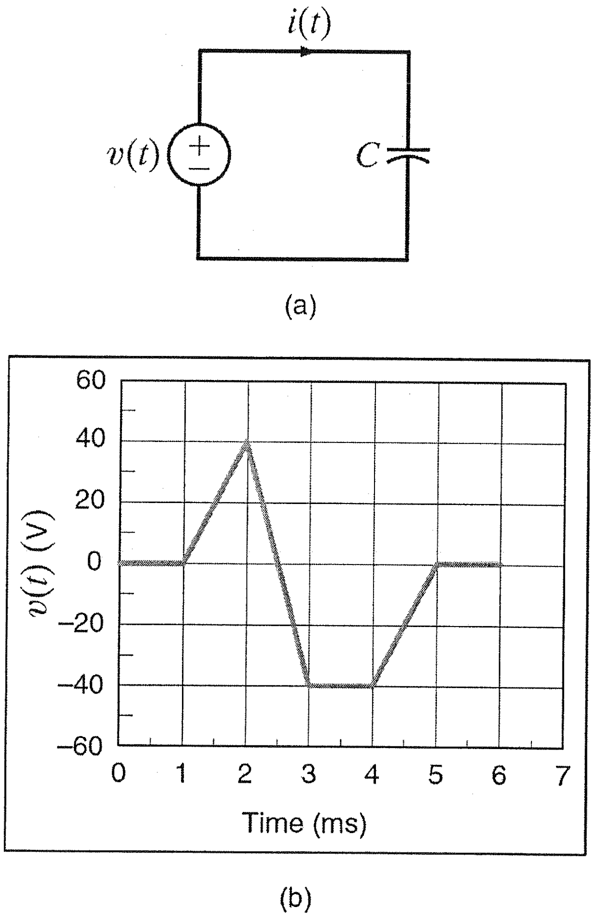
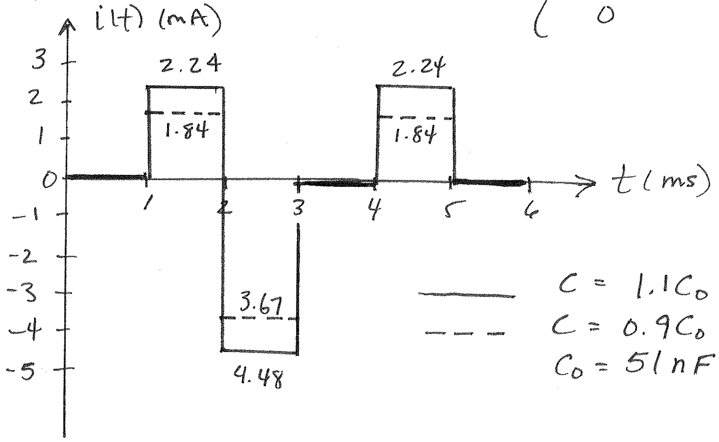


Figure P6.37

SOLUTION:

$i(t) = C\, dv/dt$

$$dv/dt = \begin{cases} 4\times10^4 \text{ V/s} & t = 1 \text{ to } 2 \text{ ms and } 4\text{–}5 \text{ ms} \\ -8\times10^4 \text{ V/s} & t = 2 \text{ to } 3 \text{ ms} \\ 0 & \text{otherwise} \end{cases}$$



———— $C = 1.1C_0$

– – – $C = 0.9C_0$

$C_0 = 51\text{ nF}$

6.38 Find the possible inductance range of the following inductors. CS

(a) 10 mH with a tolerance of 10%.

(b) 2.0 nH with a tolerance of 5%.

(c) 68 μH with a tolerance of 10%.

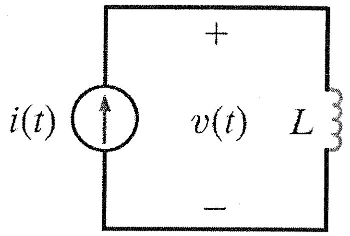
SOLUTION:

a) $L = 10\,mH \pm 10\%$ $\qquad 9\,mH \le L \le 11\,mH$

b) $L = 2\,nH \pm 5\%$ $\qquad 1.9\,nH \le L \le 2.1\,nH$

c) $L = 68\,\mu H \pm 10\%$ $\qquad 61.2\,\mu H \le L \le 74.8\,\mu H$

6.39 The inductor in Fig. P6.39a is 330 μH with a tolerance of 5%. Given the current waveform in Fig. P6.39b, graph the voltage $v(t)$ for the minimum and maximum inductor values.



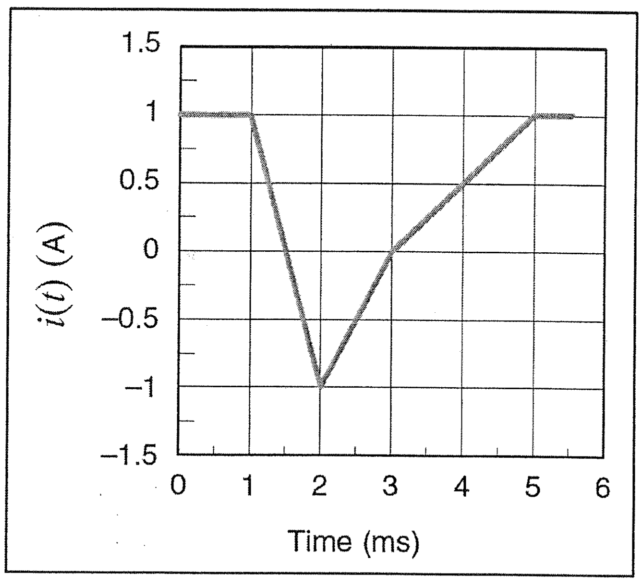
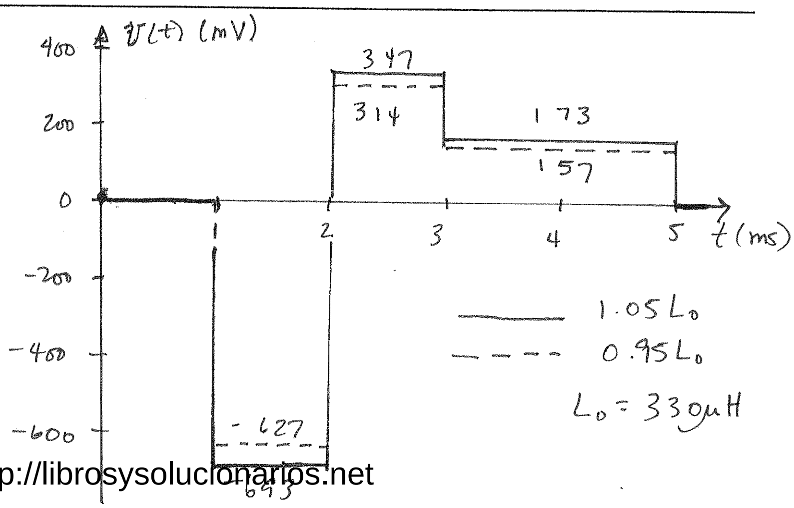


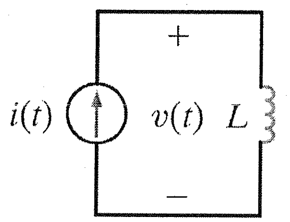
Figure P6.39

SOLUTION: $v = L\, di/dt$

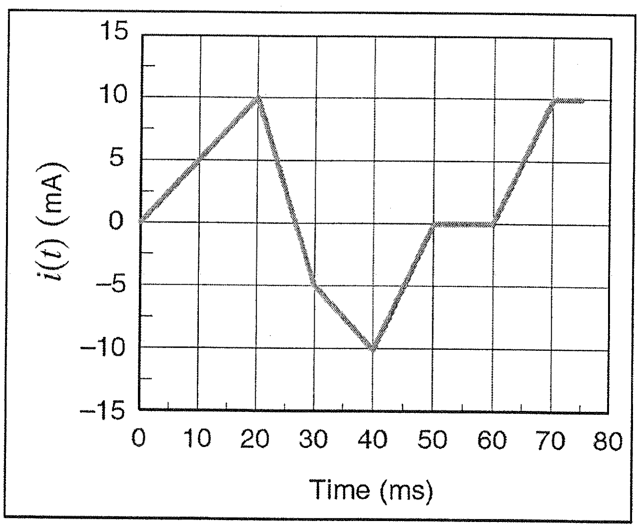
$$\frac{di}{dt} = \begin{cases} -2000 \text{ A/s} & 1 \le t \le 2 \text{ ms} \\ 1000 \text{ A/s} & 2 \le t \le 3 \text{ ms} \\ 500 \text{ A/s} & 3 \le t \le 5 \text{ ms} \\ 0 & \text{otherwise} \end{cases}$$



6.40 The inductor in Fig. P6.40a is 4.7 μH with a tolerance of 20%. Given the current waveform in Fig. 6.40b, graph the voltage $v(t)$ for the minimum and maximum inductor values.



(a)

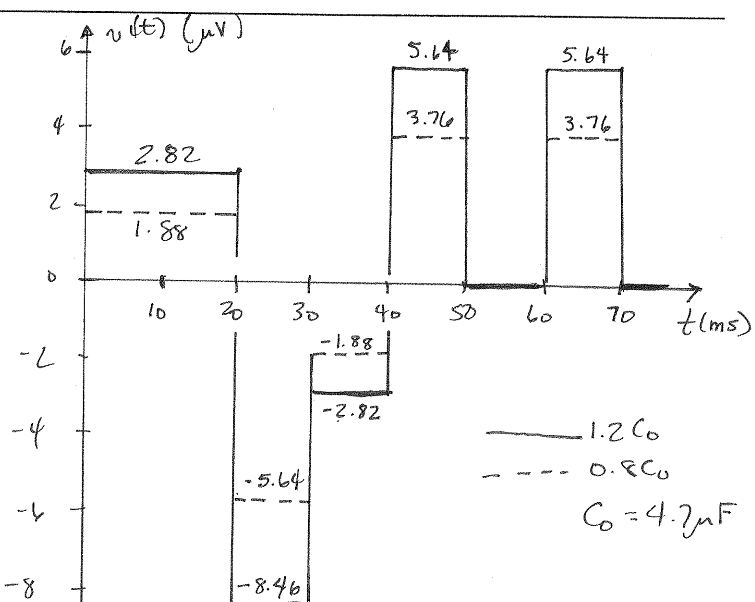


(b)

Figure P6.40

SOLUTION: $v = L\, di/dt$

$$\frac{di(t)}{dt} = \begin{cases} 1/2 \text{ A/s} & 0 \le t < 20\text{ms} \\ -1.5 \text{ ''} & 20 \le t \le 30\text{ms} \\ -0.5 \text{ ''} & 30 \le t \le 40\text{ms} \\ 1 \text{ ''} & 40 \le t \le 50\text{ms} \\ 1 \text{ ''} & 60 \le t \le 70\text{ms} \\ 0 \text{ ''} & \text{otherwise} \end{cases}$$



6.41 If the total energy stored in the circuit in Fig. P6.41 is 80 mJ, what is the value of L?

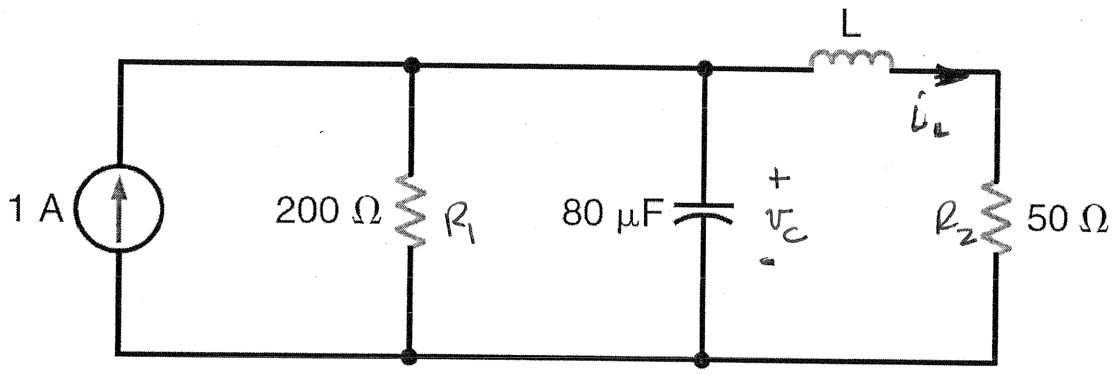


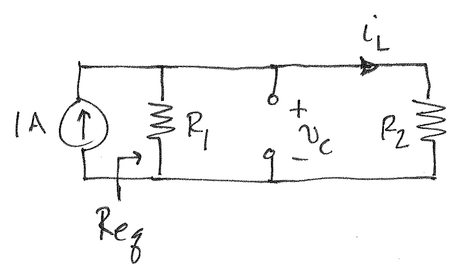
Figure P6.41

SOLUTION:

Since source (1A) is constant, v_c and i_L are also constants.

$i_c = C \frac{dv_c}{dt} = 0$ and $v_L = L \frac{di_L}{dt} = 0$

New circuit,



$R_{eq} = R_1 // R_2 = 40\Omega$

$v_c = (1A) R_{eq} = 40V$

$w_c = \frac{1}{2} C v_c^2 = 64mJ$

$w_{TOTAL} = 80mJ = w_c + w_L$

$w_L = \frac{1}{2} L i_L^2 = 16mJ$

$i_L = \frac{(1A) R_1}{R_1 + R_2} = 0.8A$

$\boxed{L = 50mH}$

6.42 Find the value of C if the energy stored in the capacitor in Fig. P6.42 equals the energy stored in the inductor.

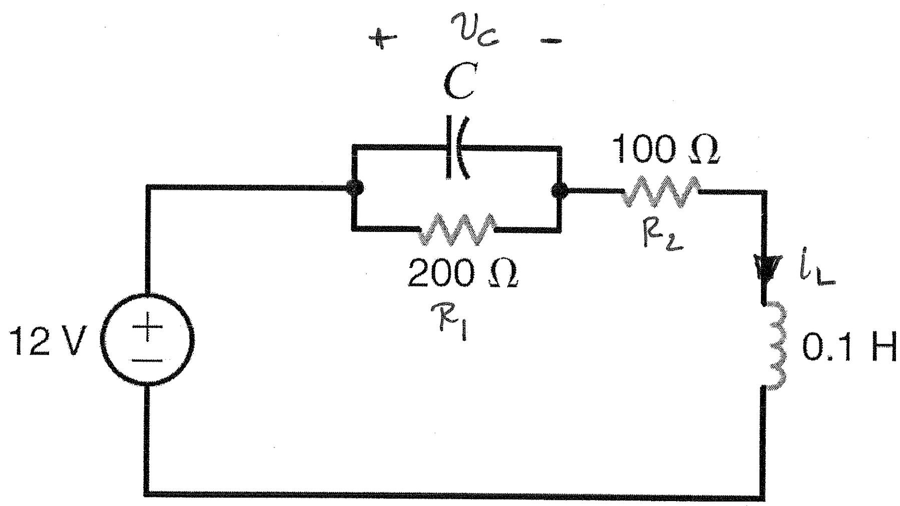


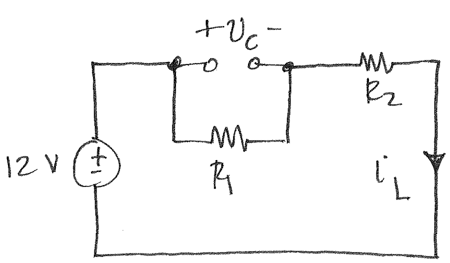
Figure P6.42

SOLUTION:

Since voltage source is constant, v_C and i_L are constant

$i_C = C\,dv_C/dt = 0$ & $v_L = L\,di_L/dt = 0$

New circuit



$$i_L = \frac{12}{R_1+R_2} = 40\,\text{mA}$$

$$v_C = \frac{12\,R_1}{R_1+R_2} = 8\,\text{V}$$

$$w_C = \frac{1}{2}C v_C^2 = w_L = \frac{1}{2}L i_L^2 \Rightarrow C = L\left(\frac{i_L}{v_C}\right)^2$$

$$\boxed{C = 2.5\,\mu\text{F}}$$

6.43 Given the network in Fig. P6.43, find the power dissipated in the 3-Ω resistor and the energy stored in the capacitor. **PSV**

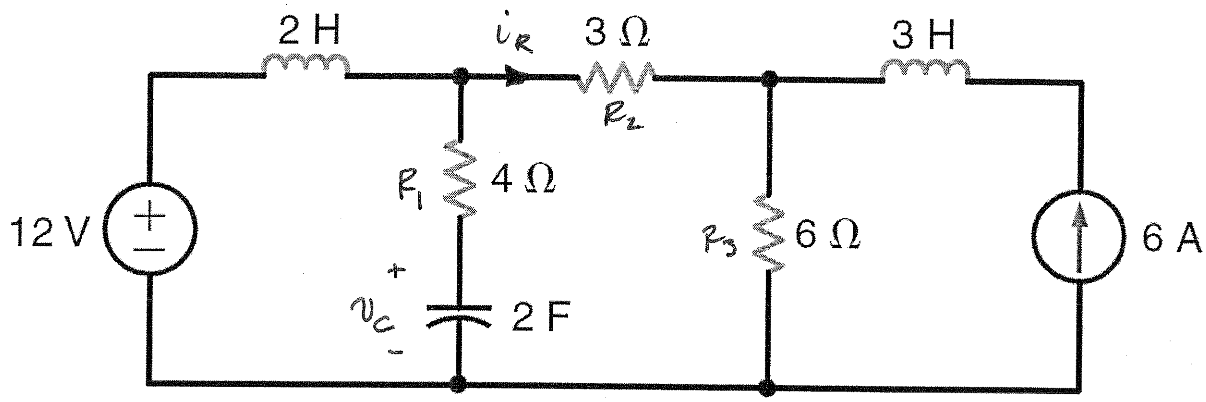
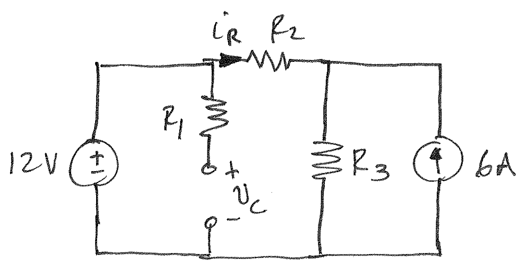


Figure P6.43

SOLUTION: Since all sources are constant, all voltages and currents are constant.

$v_L = L\, di_L/dt = 0$ & $i_c = c\, dv_c/dt = 0$

<u>New Circuit</u>



$P_{R2} = R_2 i_R^2$ $\qquad$ $w_C = \frac{1}{2} C v_c^2$

By superposition:

$$i_R = 12\left[\frac{1}{R_2+R_3}\right] - 6\left[\frac{R_3}{R_2+R_3}\right] = -\frac{8}{3}\text{ A}$$

$v_c = 12\text{ V}$

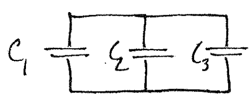
$\boxed{P_{R2} = 21.33\text{ W}}$ $\qquad$ $\boxed{w_c = 144\text{ J}}$

6.44 What values of capacitance can be obtained by interconnecting a 2-μF capacitor, a 4-μF capacitor, and an 8-μF capacitor?

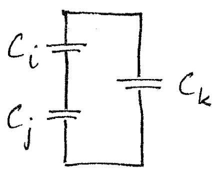
SOLUTION:

$C_1 = 2\mu F \qquad C_2 = 4\mu F \qquad C_3 = 8\mu F$

There are 4 configurations

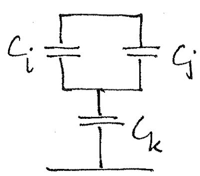


$C_{eq} = C_1 + C_2 + C_3 = 14\mu F$



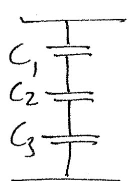
3 possibilities

$$C_{eq} = C_k + \frac{C_i C_j}{C_i + C_j} \qquad C_{eq} = \begin{cases} 9.33\mu F \\ 5.60\mu F \\ 4.67\mu F \end{cases}$$



3 possibilities

$$C_{eq} = \frac{(C_i + C_j)C_k}{C_i + C_j + C_k} \qquad C_{eq} = \begin{cases} 3.43\mu F \\ 2.86\mu F \\ 1.71\mu F \end{cases}$$



$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \qquad C_{eq} = 1.14\mu F$$

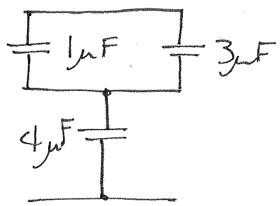
Possible values:

1.14μF	1.75μF
3.43μF	3.75μF
5.20μF	5.60μF
9.33μF	14μF

6.45 Given a 1-, 3-, and 4-μF capacitor, can they be interconnected to obtain an equivalent 2-μF capacitor? CS

SOLUTION:

Yes!

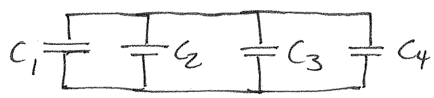


$$C_{eq} = \frac{(10^{-6} + 3\times10^{-6})\,4\times10^{-6}}{(10^{-6} + 3\times10^{-6}) + 4\times10^{-6}} = 2\mu F$$

6.46 Given four 6-μF capacitors, find the maximum value and minimum value that can be obtained by interconnecting the capacitors in series/parallel combinations.

SOLUTION:

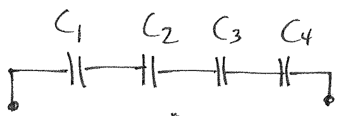
maximum C_{eq}:



all $C = 6\mu F$

$$\boxed{C_{max} = 4C = 24\mu F}$$

minimum C_{eq}:



C_{min}

$$\boxed{C_{min} = \frac{C}{4} = 1.5\mu F}$$

6.47 The two capacitors in Fig. P6.47 were charged and then connected as shown. Determine the equivalent capacitance, the initial voltage at the terminals, and the total energy stored in the network.

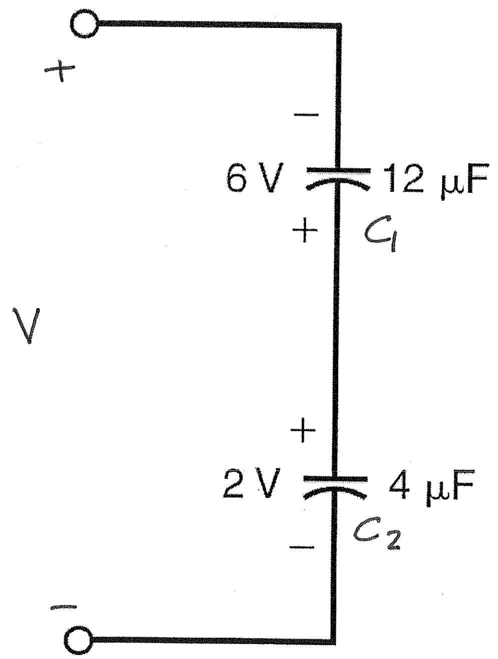


Figure P6.47

SOLUTION:

$V = -6 + 2 = -4V$ $\quad \boxed{v = -4V}$

$W = \frac{1}{2} C_1 v_1^2 + \frac{1}{2} C_2 v_2^2 \qquad v_1 = 6V \qquad v_2 = 2V$

$\boxed{W = 224 \mu J}$

$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} \qquad \boxed{C_{eq} = 3 \mu F}$

6.48 The two capacitors shown in Fig. P6.48 have been connected for some time and have reached their present values. Find V_o.

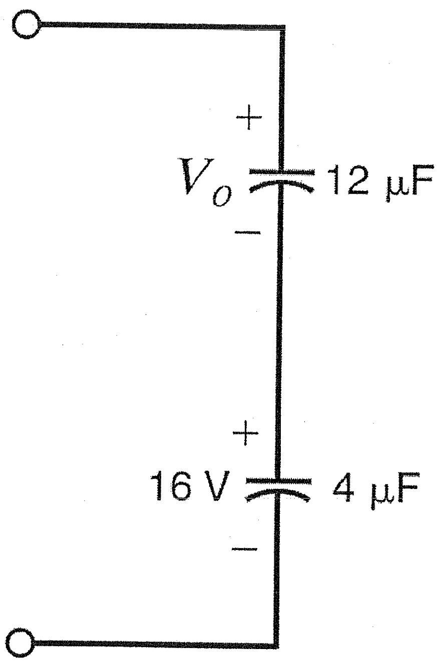


Figure P6.48

SOLUTION: Identical charge on each capacitor.

$$Q = CV = (4\times10^{-6})(16) = (12\times10^{-6})\,V_o \Rightarrow \boxed{V_o = 5.33\,V}$$

6.49 The three capacitors shown in Fig. P6.49 have been connected for some time and have reached their present values. Find V_1 and V_2. **CS**

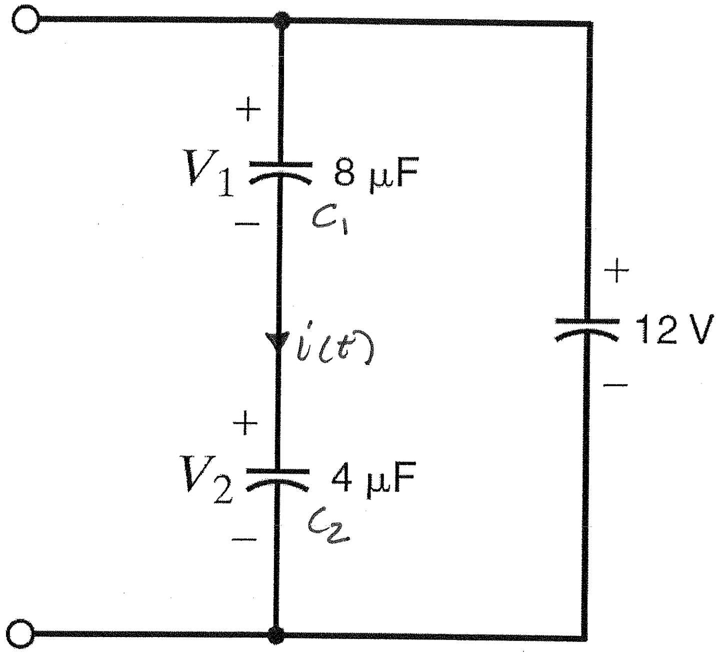


Figure P6.49

SOLUTION:

$$v_1 = \frac{1}{C_1}\int i\,dt \qquad v_2 = \frac{1}{C}\int i\,dt$$

$$\frac{v_1}{v_2} = \frac{C_2}{C_1} = \frac{1}{2} \qquad \& \quad v_1 + v_2 = 12\text{V}$$

$$\boxed{v_2 = 8\text{V}} \qquad \boxed{v_1 = 4\text{V}}$$

6.50 Select the value of C to produce the desired total capacitance of $C_T = 10\ \mu F$ in the circuit in Fig. P6.50.

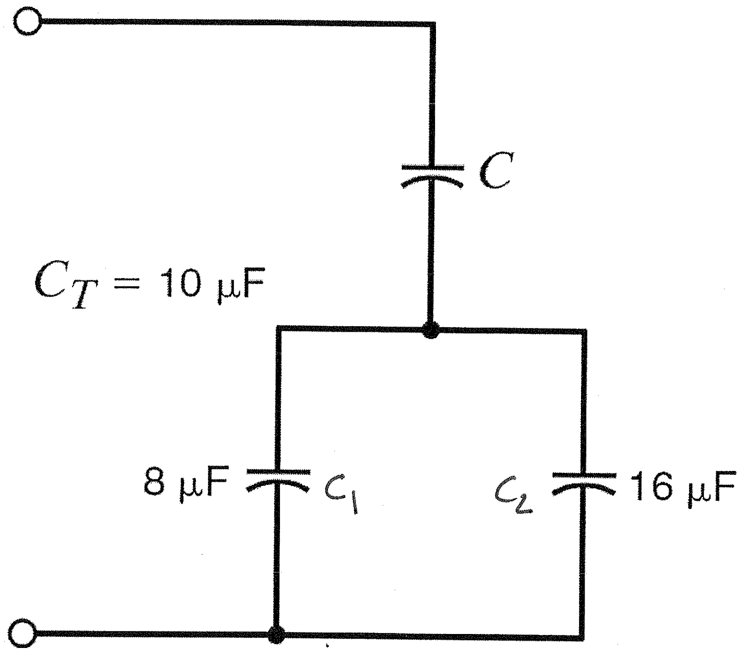


Figure P6.50

SOLUTION:

$$C_T = \frac{C(C_1+C_2)}{C+C_1+C_2} = 10\mu F$$

$$\boxed{C = 17.14\mu F}$$

6.51 Select the value of C to produce the desired total capacitance of $C_T = 1\ \mu F$ in the circuit in Fig. P6.51.

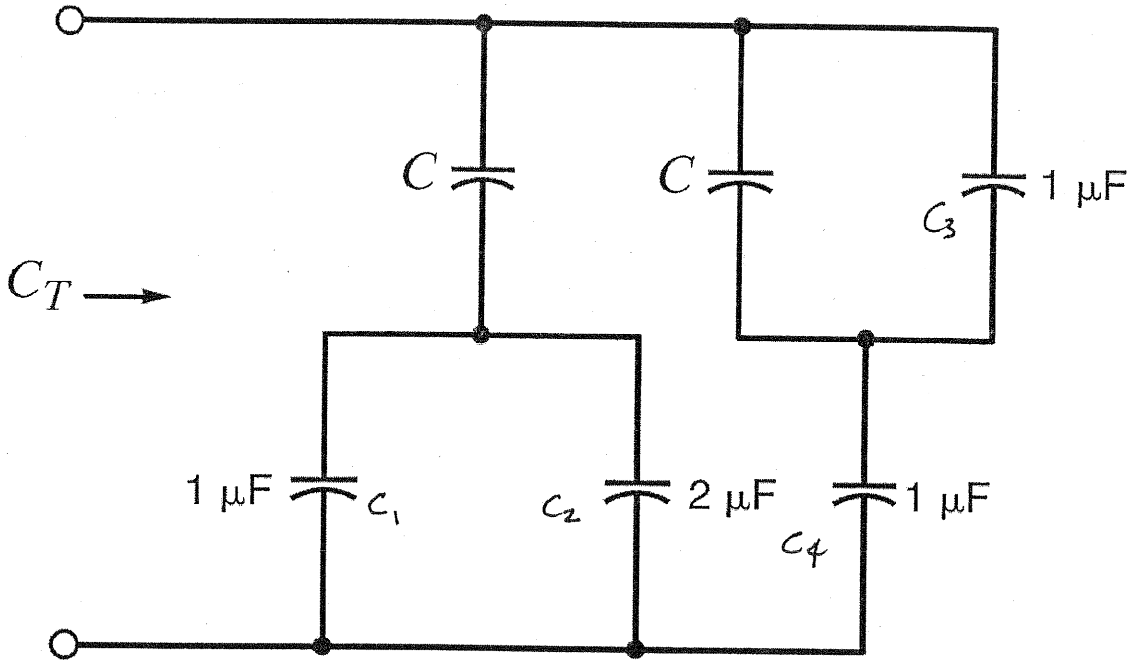


Figure P6.51

SOLUTION:

$$C_T = \frac{(C_1+C_2)C}{C_1+C_2+C} + \frac{(C_3+C)C_4}{C+C_3+C_4} = 1\mu F$$

$$\frac{3C}{3+C} + \frac{(1+C)}{2+C} = 1 \qquad \text{where } C \text{ is in } \mu F$$

$$\boxed{C = 468\ nF}$$

6.52 Find C_T in the network in Fig. P6.52 if (a) the switch is open and (b) the switch is closed.

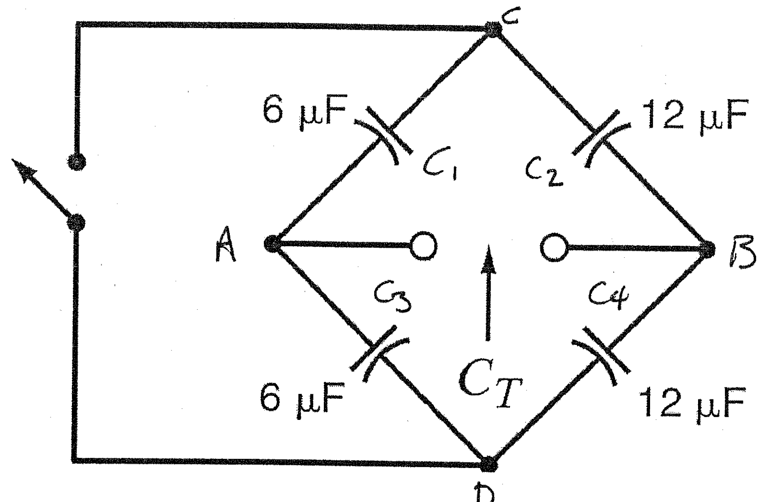
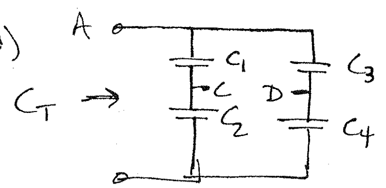


Figure P6.52

SOLUTION:

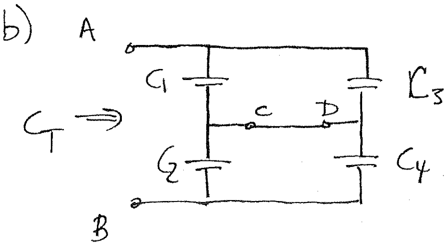
a)



$$C_T = \frac{C_1 C_2}{C_1 + C_2} + \frac{C_3 C_4}{C_3 + C_4}$$

$$C_T = 8\mu F$$

b)



$$C_T = \frac{(C_1 + C_3)(C_2 + C_4)}{C_1 + C_3 + C_2 + C_4}$$

$$\boxed{C_T = 8\mu F}$$

6.53 Find the equivalent capacitance at terminals A-B in Fig. P6.53. **PSV**

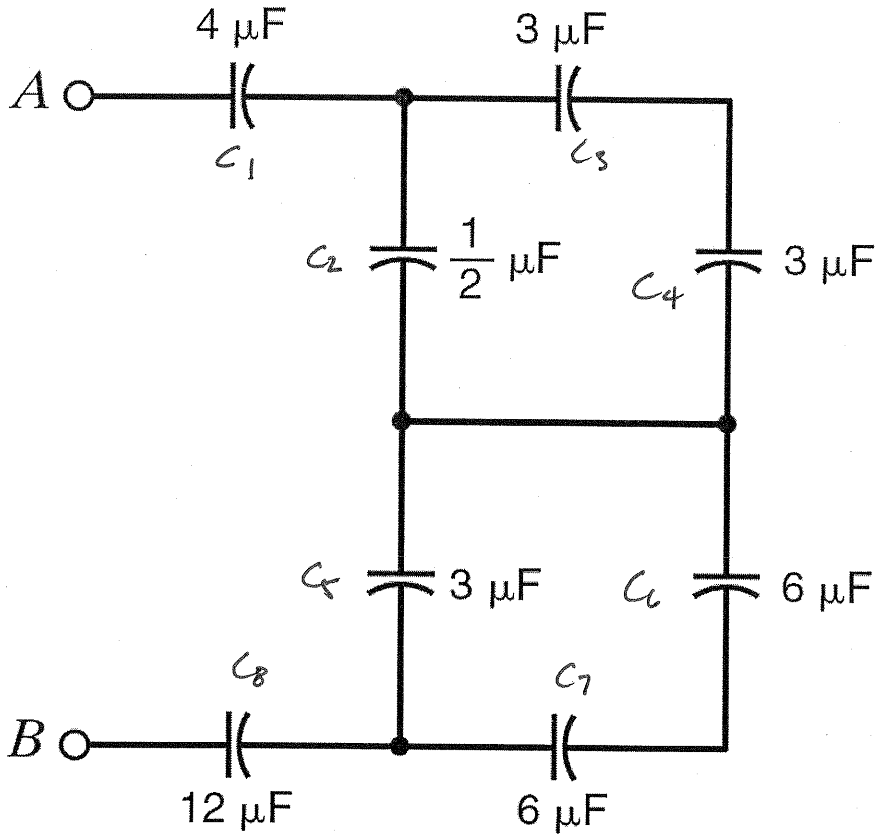


Figure P6.53

SOLUTION:

$$C_A = C_2 + \frac{C_3 C_4}{C_3 + C_4} = 2\mu F \qquad C_B = C_5 + \frac{C_6 C_7}{C_6 + C_7} = 6\mu F$$

$$\frac{1}{C_{AB}} = \frac{1}{C_1} + \frac{1}{C_A} + \frac{1}{C_B} + \frac{1}{C_8} \qquad \boxed{C_{AB} = 1\mu F}$$

6.54 Determine the total capacitance of the network in Fig. P6.54. **CS**

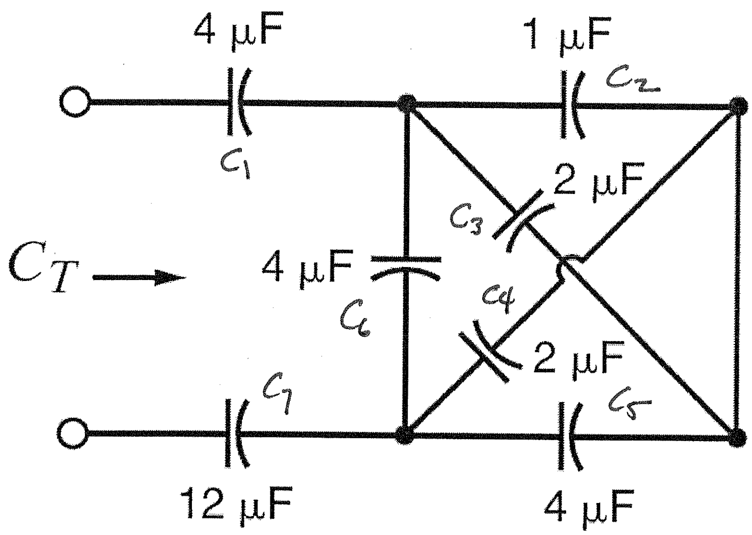
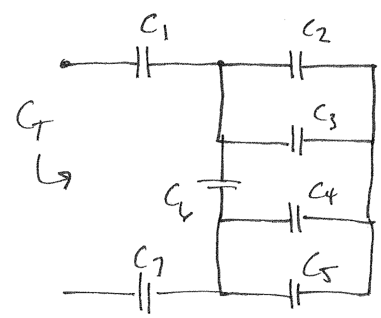


Figure P6.54

SOLUTION:

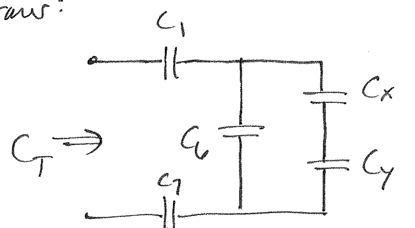
Redraw:



$C_x = C_2 + C_3 = 3\mu F$

$C_y = C_4 + C_5 = 6\mu F$

Redraw:



$C_z = C_6 + \dfrac{C_x C_y}{C_x + C_y} = 6\mu F$

$\dfrac{1}{C_T} = \dfrac{1}{C_1} + \dfrac{1}{C_z} + \dfrac{1}{C_7}$

$\boxed{C_T = 2\mu F}$

6.55 Find the total capacitance C_T shown in the network in Fig. P6.55.

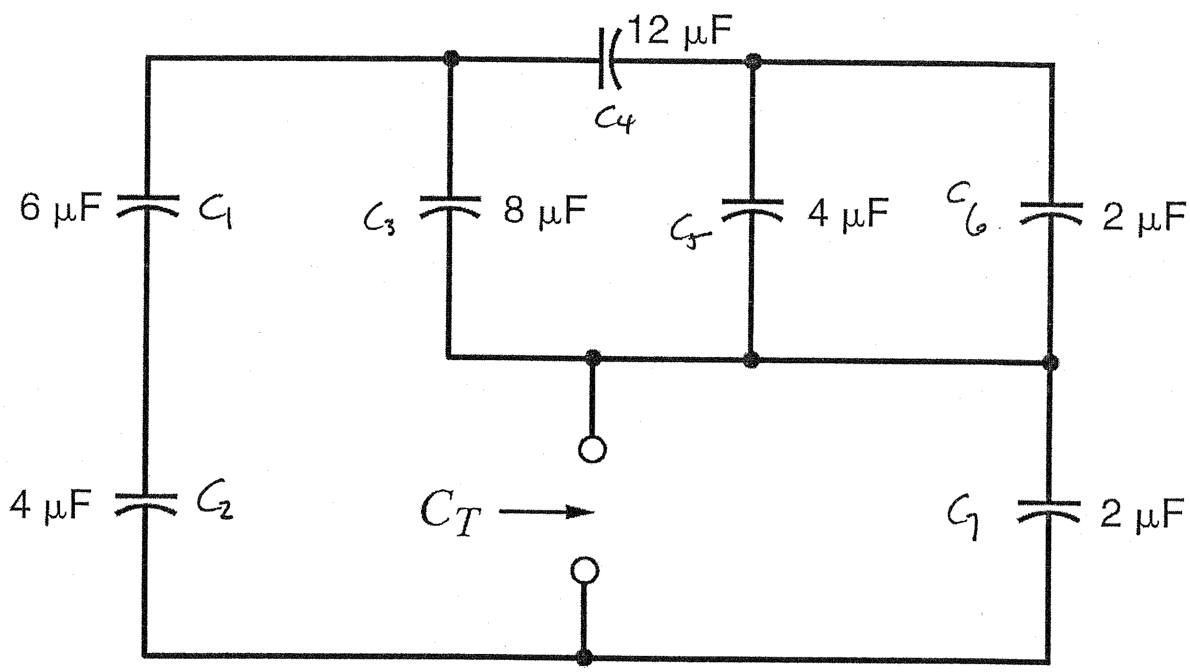
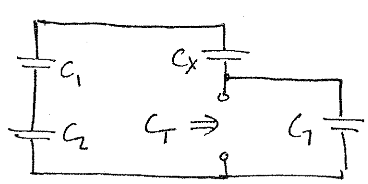


Figure P6.55

SOLUTION:



$$C_x = C_3 + \left[\frac{C_4(C_5+C_6)}{C_4+C_5+C_6}\right] = 12\mu F$$

$$\frac{1}{C_y} = \frac{1}{C_x} + \frac{1}{C_1} + \frac{1}{C_2} \Rightarrow C_y = 2\mu F$$

$$C_T = C_7 + C_y = 4\ \mu F \qquad \boxed{C_T = 4\mu F}$$

6.56 Find the total capacitance C_T of the network in Fig. P6.56.

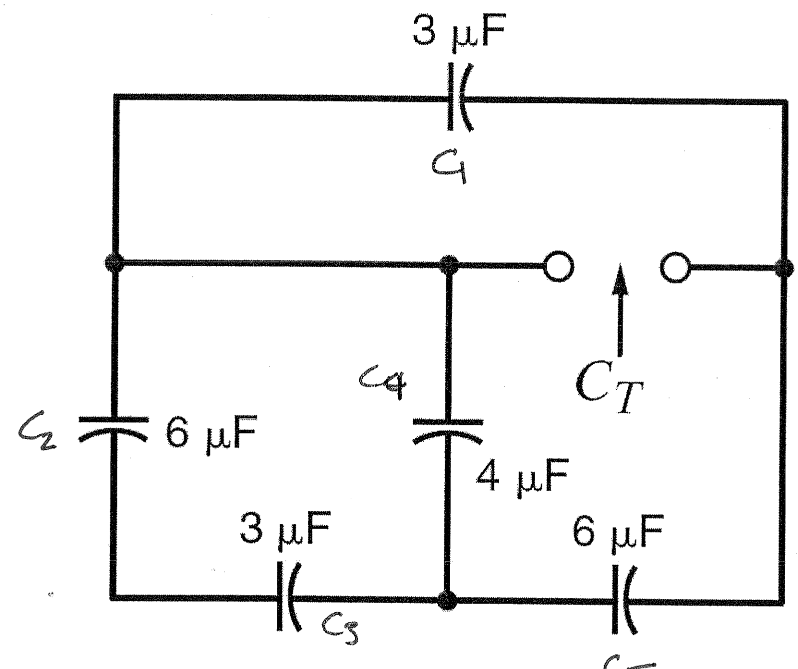
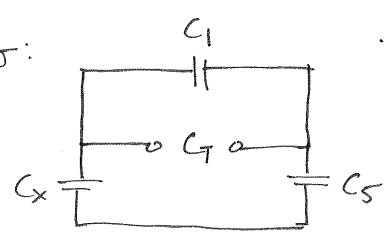


Figure P6.56

SOLUTION:

Redraw:



$$C_x = C_4 + \frac{C_2 C_3}{C_2 + C_3} = 6\mu F$$

$$C_T = C_1 + \frac{C_x C_5}{C_x + C_5}$$

$$\boxed{C_T = 6\mu F}$$

6.57 Find the total capacitance C_T of the network in Fig. P6.57.

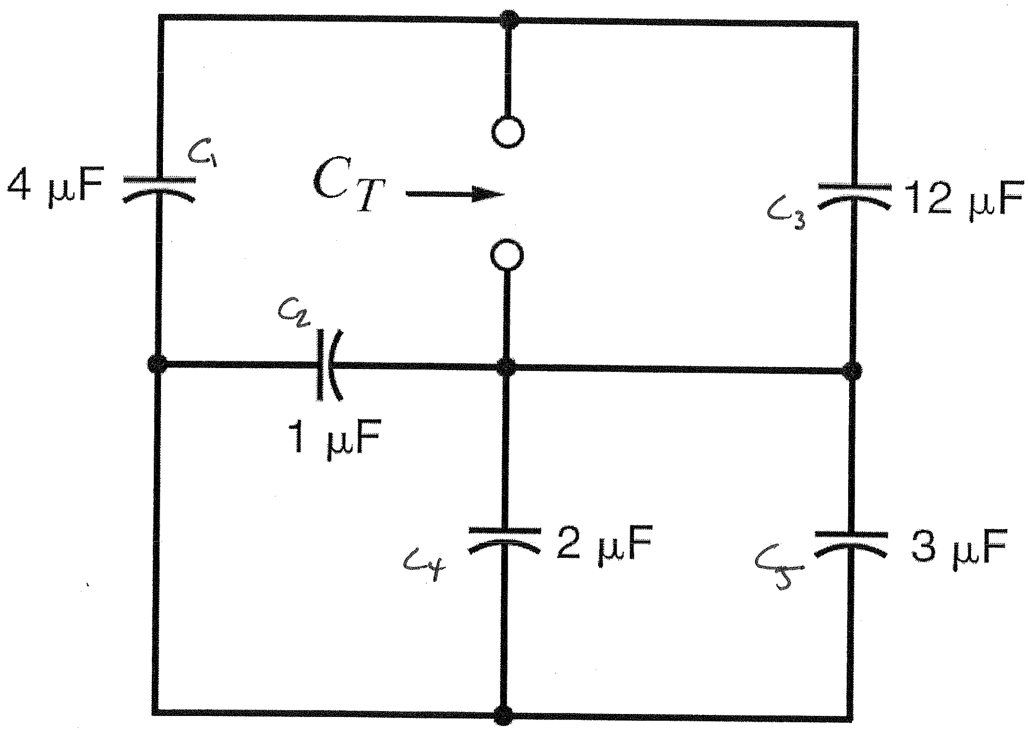
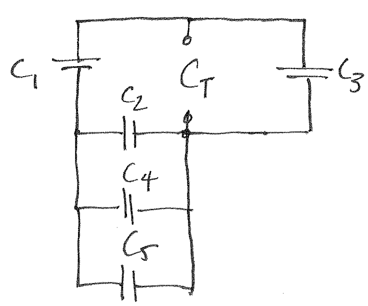


Figure P6.57

SOLUTION:

Redraw:



$$C_x = C_2 + C_4 + C_5 = 6\mu F$$

$$C_T = C_3 + \frac{C_1 C_x}{C_1 + C_x}$$

$$\boxed{C_T = 14.4\mu F}$$

6.58 In the network in Fig. P6.58, find the capacitance C_T if (a) the switch is open and (b) the switch is closed.

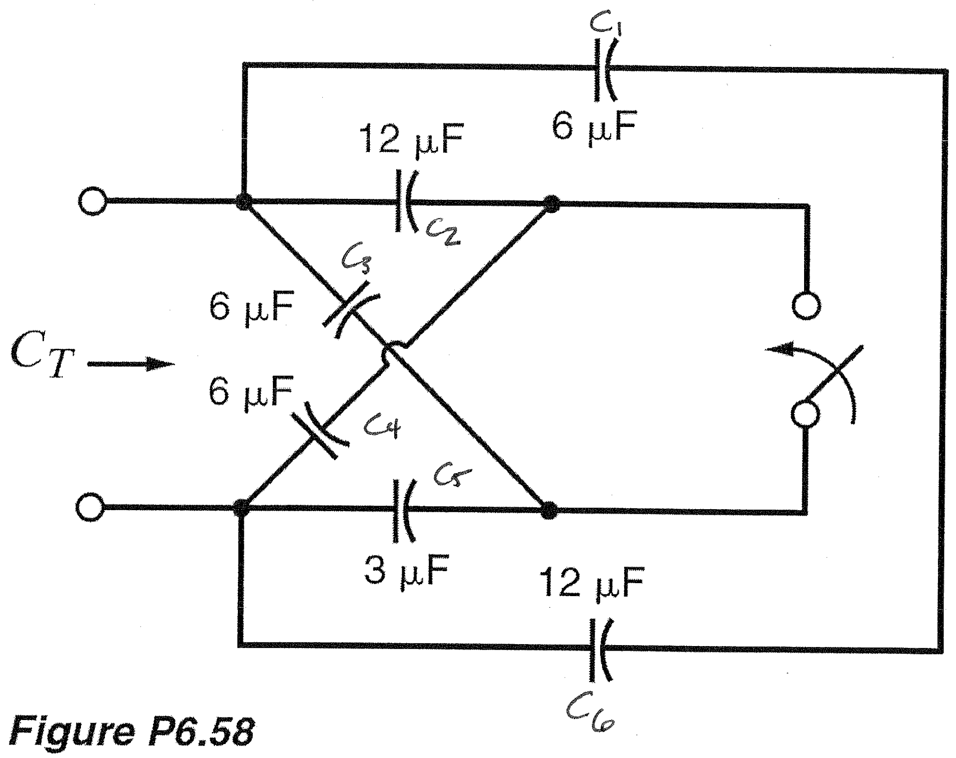
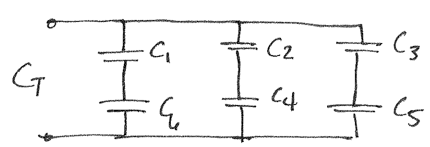


Figure P6.58

SOLUTION:

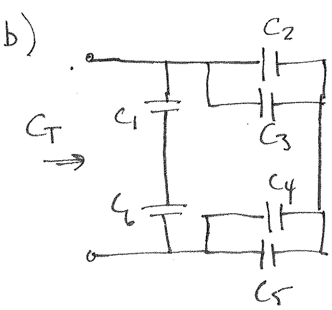
a)



$$C_T = \frac{C_1 C_6}{C_1 + C_6} + \frac{C_2 C_4}{C_2 + C_4} + \frac{C_3 C_5}{C_3 + C_5}$$

$$\boxed{C_T = 10\mu F}$$

b)



$$C_x = \frac{(C_2 + C_3)(C_4 + C_5)}{C_2 + C_3 + C_4 + C_5} = 6\mu F$$

$$C_T = C_x + \frac{C_1 C_6}{C_1 + C_6}$$

$$\boxed{C_T = 10\mu F}$$

6.59 Compute the equivalent capacitance of the network in Fig. P6.59 if all the capacitors are 6 μF.

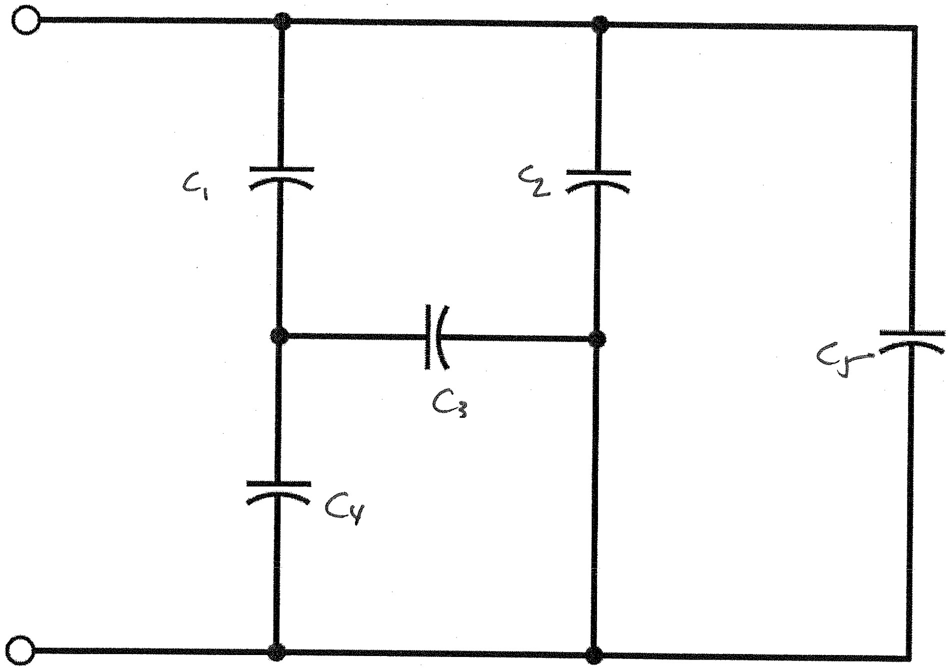
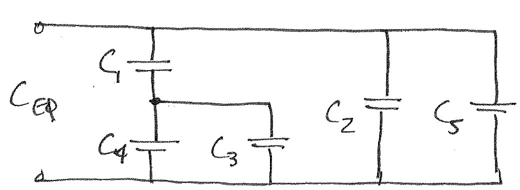


Figure P6.59

SOLUTION:

Redraw



$$C_X = \frac{(C_3+C_4)C_1}{C_3+C_4+C_1} = 4\mu F$$

$$C_T = C_X + C_2 + C_5 \qquad \boxed{C_T = 16\mu F}$$

6.60 If all the capacitors in Fig. P6.60 are 6 μF, find C_{eq}. CS

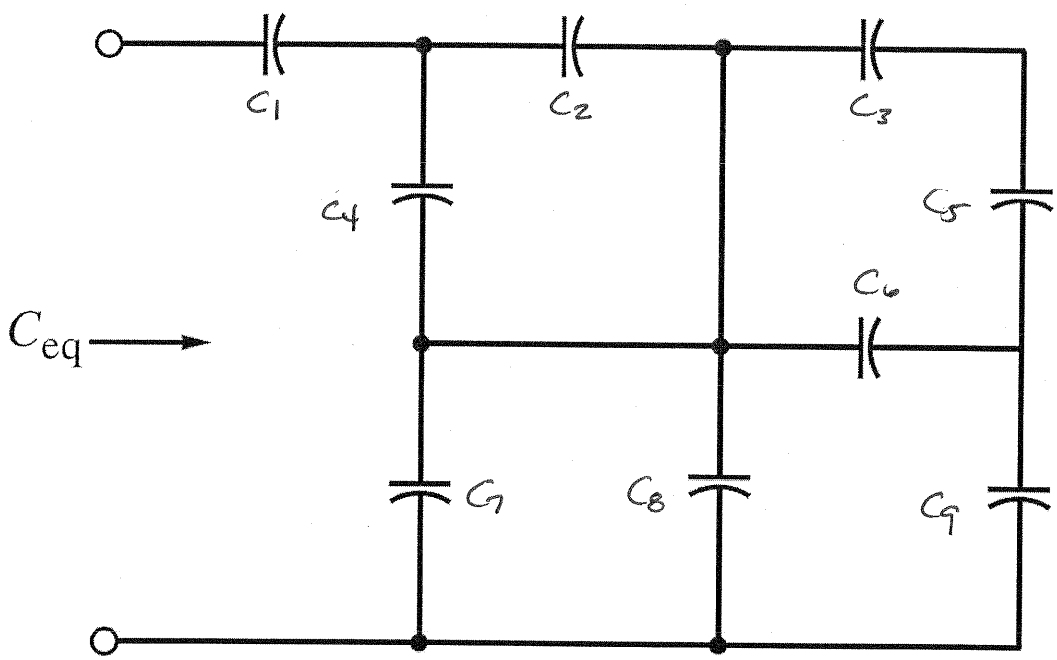
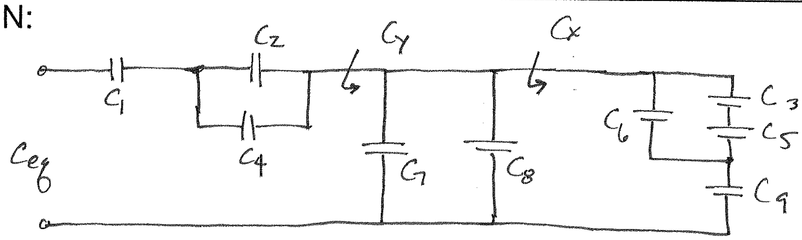


Figure P6.60

SOLUTION:

Redraw:



$$C_X = \frac{\left(\frac{C_3C_5}{C_3+C_5}+C_6\right)C_9}{\frac{C_3C_5}{C_3+C_5}+C_6+C_9} = 3.6\mu F$$

$$C_Y = C_7 + C_8 + C_X = 15.6\mu F$$

$$C_Z = C_2 + C_4 = 12\mu F$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_Z} + \frac{1}{C_Y}$$

$$\boxed{C_{eq} = 3.18\mu F}$$

6.61 Given the capacitors in Fig. P6.61, are $C_1 = 2.0\ \mu F$ with a tolerance of 2% and $C_2 = 2.0\ \mu F$ with a tolerance of 20%, find the following.

(a) The nominal value of C_{eq}.

(b) The minimum and maximum possible values of C_{eq}.

(c) The percent errors of the minimum and maximum values.

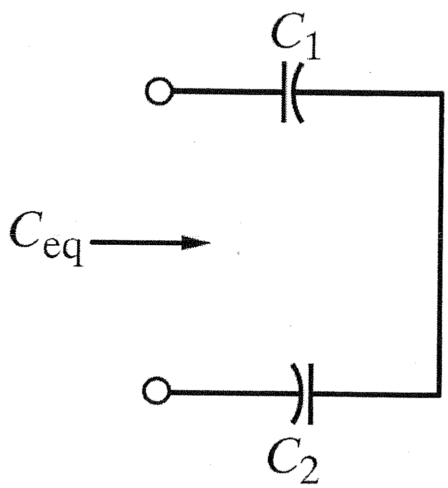


Figure P6.61

SOLUTION:

a) $C_{eq} = C_1C_2/(C_1+C_2)$ $\boxed{C_{eq} = 1\mu F}$

b) $C_{max} = C_{1max}C_{2max}/(C_{1max} + C_{2max})$

$C_{1max} = 2.04\mu F$
$C_{2max} = 2.4\mu F$

$\boxed{C_{max} = 1.103\mu F}$

$C_{1min} = 1.96\mu F$ $C_{2min} = 1.6\mu F$ $\boxed{C_{min} = 0.881\mu F}$

c) $+$ percent error $= \dfrac{C_{max} - C_{eq}}{C_{eq}} = 10.3\%$

$\boxed{\text{errors} = \begin{cases} +10.3\% \\ -11.9\% \end{cases}}$

$-$ percent error $= \dfrac{C_{min} - C_{eq}}{C_{eq}} = -11.9\%$

6.62 The capacitor values for the network in Fig. P6.62 are $C_1 = 0.1\ \mu\text{F}$ with a tolerance of 10%, $C_2 = 0.33\ \mu\text{F}$ with a tolerance of 20%, and $C_3 = 1\ \mu\text{F}$ with a tolerance of 10%. Find the following.

(a) The nominal value of C_{eq}.

(b) The minimum and maximum possible values of C_{eq}.

(c) The percent errors of the minimum and maximum values.

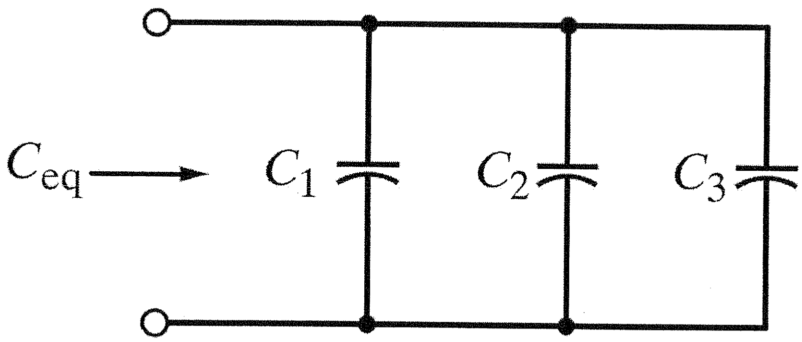


Figure P6.62

SOLUTION:

a) $C_{eq} = C_1 + C_2 + C_3 = 1.43\mu F$ $\boxed{C_{eq} = 1.43\mu F}$

b) $C_{max} = C_{1max} + C_{2max} + C_{3max}$ $C_{1max} = 0.11\mu F$ $C_{2max} = 0.396\mu F$ $C_{3max} = 1.1\mu F$

$\boxed{C_{max} = 1.606\mu F}$

$C_{1min} = 0.09\mu F$ $C_{2min} = 0.264\mu F$ $C_{3min} = 0.9\mu F$ $\boxed{C_{min} = 1.254\mu F}$

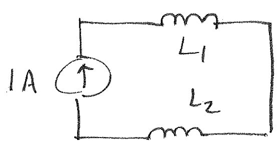
c) $+\%\ error = \dfrac{C_{max} - C_{eq}}{C_{eq}} = 12.3\%$

$\boxed{\%\ error = \pm 12.3\%}$

$-\%\ error = \dfrac{C_{min} - C_{eq}}{C_{eq}} = -12.3\%$

6.63 A 20-mH inductor and a 12-mH inductor are connected in series with a 1-A current source. Find (a) the equivalent inductance and (b) the total energy stored.

SOLUTION:



$L_1 = 20\,mH \qquad L_2 = 12\,mH$

a) $L_{eq} = L_1 + L_2 \qquad \boxed{L_{eq} = 32\,mH}$

b) $W_{TOTAL} = W_1 + W_2 = \frac{1}{2} L_1 I^2 + \frac{1}{2} L_2 I^2 \qquad I = 1A$

$\boxed{W_{TOTAL} = 16\,mJ}$

6.64 Two inductors are connected in parallel, as shown in Fig. P6.64. Find i.

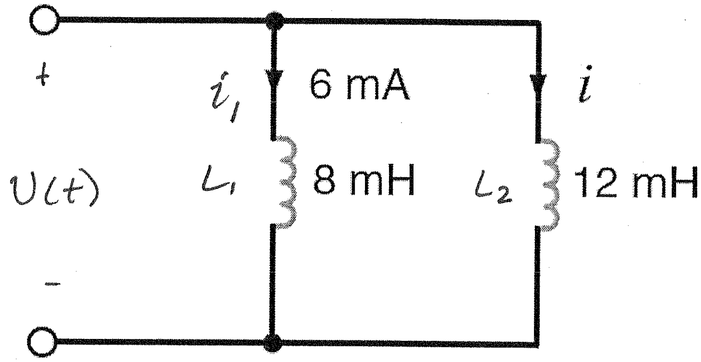


Figure P6.64

SOLUTION:

For L_1 $\quad i_1 = \frac{1}{L_1}\int v(t)\,dt$

For L_2 $\quad i = \frac{1}{L_2}\int v(t)\,dt$

$$\frac{i}{i_1} = \frac{L_1}{L_2}$$

$$i = i_1\left(L_1/L_2\right) \qquad \boxed{i = 4\text{mA}}$$

6.65 Find the value of L in the network in Fig. P6.65 so that the total inductance L_T will be 2 mH. CS

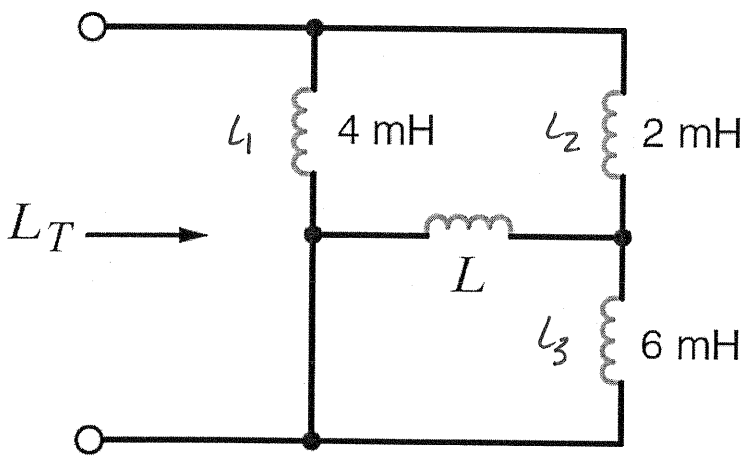
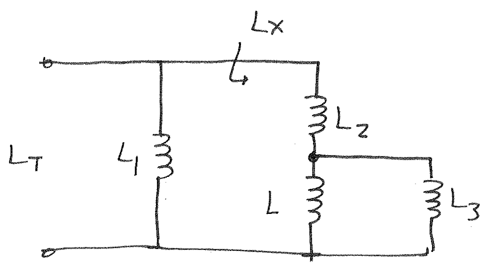


Figure P6.65

SOLUTION:

Redraw:



$$L_x = L_2 + \frac{L\,L_3}{L+L_3}$$

$$L_T = \frac{L_1 L_x}{L_1 + L_x} = 2\,mH$$

$$L_x = \frac{L_1 L_T}{L_1 - L_T} \qquad L_x = 4\,mH \qquad \frac{L\,L_3}{L+L_3} = 2\,mH$$

$$\boxed{L = 3\,mH}$$

6.66 Determine the inductance at terminals A-B in the network in Fig. P6.66. **PSV**

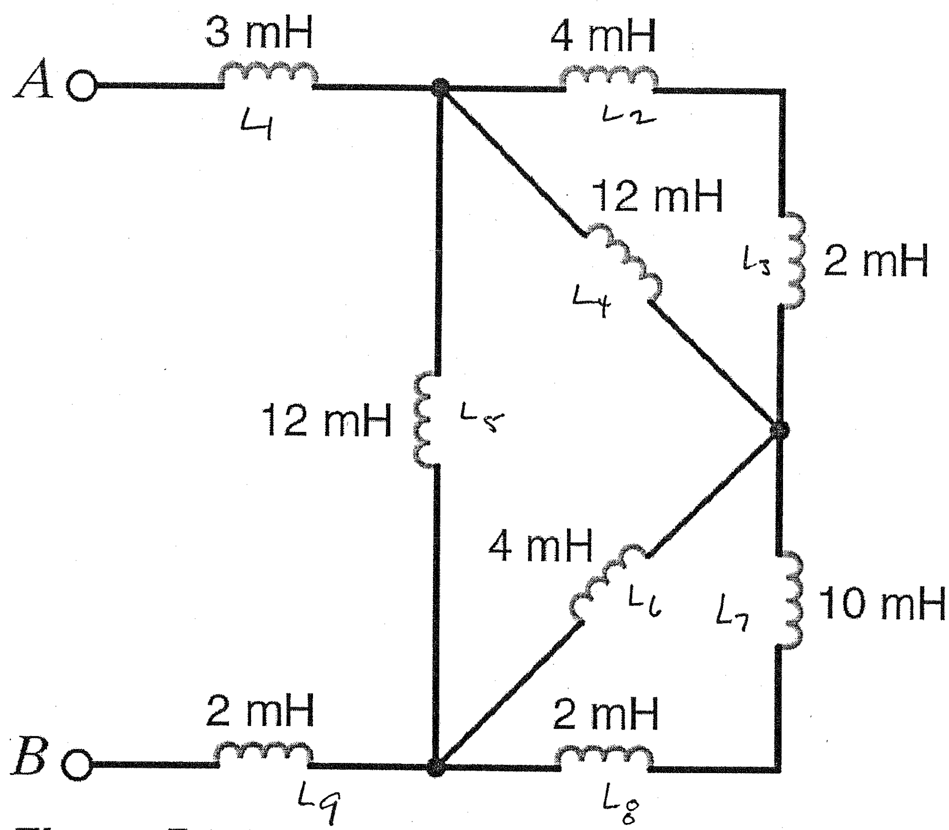
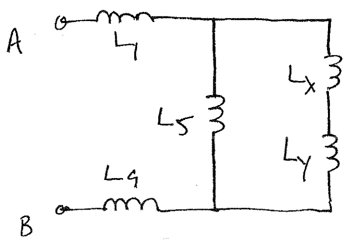


Figure P6.66

SOLUTION:

$$L_x = \frac{L_4(L_2+L_3)}{L_4+L_2+L_3} = 4mH \qquad L_y = \frac{L_6(L_7+L_8)}{L_6+L_7+L_8} = 3mH$$



$$L_z = \frac{L_5(L_x+L_y)}{L_5+L_x+L_y} = 4.42mH$$

$$L_T = L_1 + L_z + L_9 \qquad \boxed{L_T = 9.42\,mH}$$

6.67 Determine the inductance at terminals A-B in the network in Fig P6.67. cs

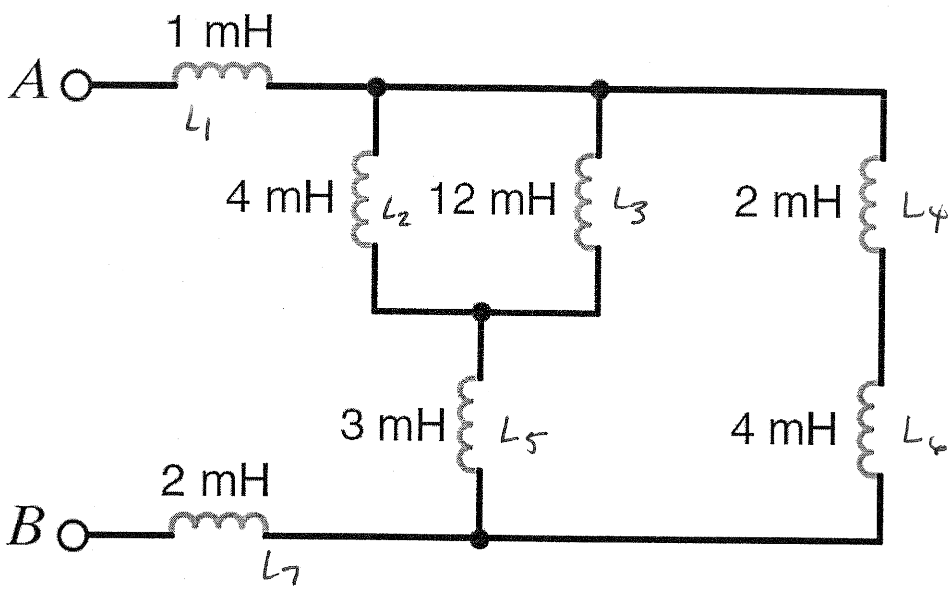
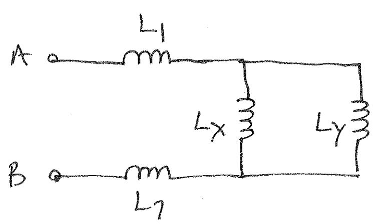


Figure P6.67

SOLUTION:



$$L_X = L_5 + \frac{L_2 L_3}{L_2 + L_3} = 6\,mH$$

$$L_Y = L_4 + L_6 = 6\,mH$$

$$L_{AB} = L_1 + L_7 + \frac{L_X L_Y}{L_X + L_Y}$$

$$\boxed{L_{AB} = 6\,mH}$$

6.68 Given the network shown in Fig. P6.68, find (a) the equivalent inductance at terminals *A*-*B* with terminals *C*-*D* short circuited, and (b) the equivalent inductance at terminals *C*-*D* with terminals *A*-*B* open circuited.

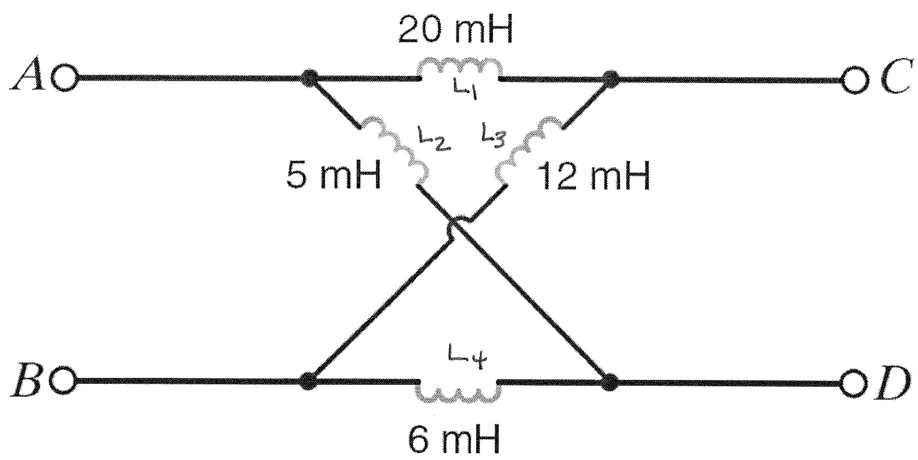
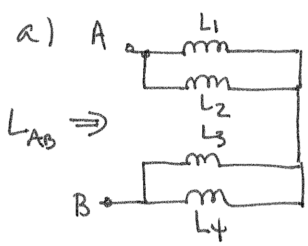


Figure P6.68

SOLUTION:

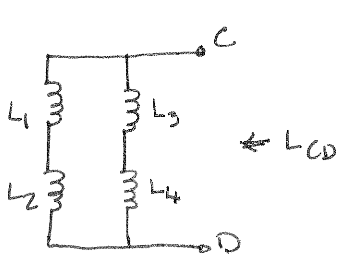
a)



$$L_{AB} = \frac{L_1 L_2}{L_1 + L_2} + \frac{L_3 L_4}{L_3 + L_4}$$

$$\boxed{L_{AB} = 8\,mH}$$

b)



$$L_{CD} = \frac{(L_1 + L_2)(L_3 + L_4)}{L_1 + L_2 + L_3 + L_4}$$

$$\boxed{L_{CD} = 10.47\,mH}$$

6.69 Find the total inductance at the terminals of the network in Fig. P6.69.

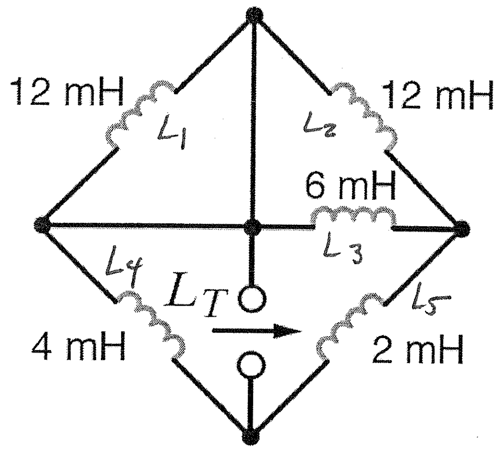
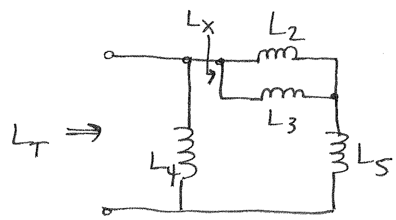


Figure P6.69

SOLUTION:

Redraw



$$L_x = \frac{L_2 L_3}{L_2 + L_3} + L_5 = 6\,mH$$

$$L_T = \frac{L_4 L_x}{L_4 + L_x}$$

$$\boxed{L_T = 2.4\,mH}$$

6.70 Compute the equivalent inductance of the network in Fig. P6.70 if all inductors are 12 mH.

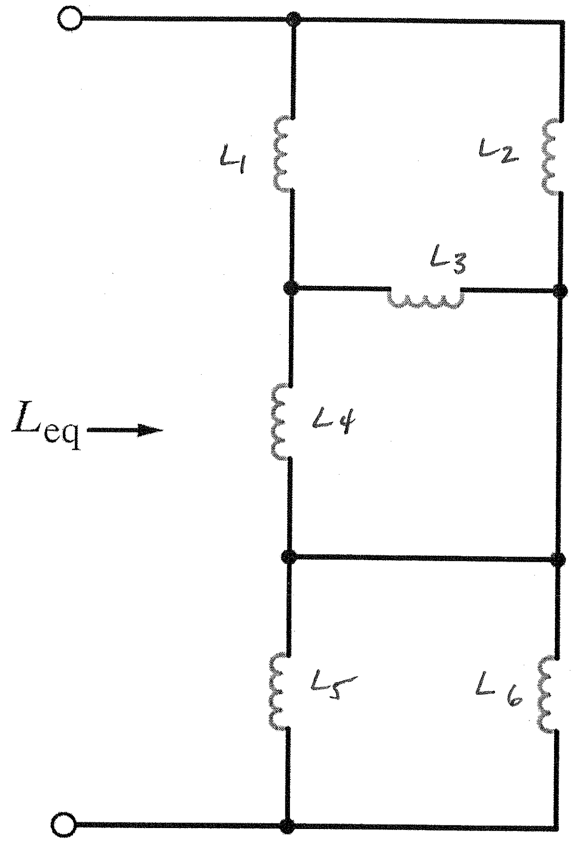
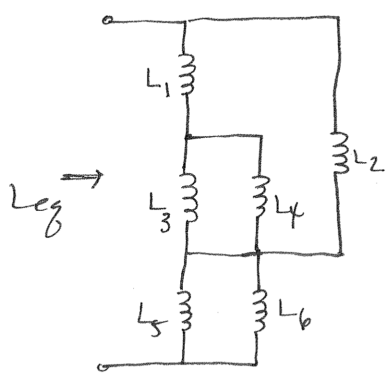


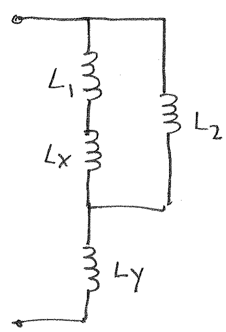
Figure P6.70

SOLUTION:

Redraw:



$\Rightarrow$



$$L_x = \frac{L_3 L_y}{L_3 + L_y} = 6\,mH$$

$$L_y = L_5 // L_6 = 6\,mH$$

$$L_z = \frac{(L_1 + L_x) L_2}{L_1 + L_x + L_2} = 7.2\,mH$$

$$L_{eq} = L_z + L_y$$

$$\boxed{L_{eq} = 13.2\,mH}$$

6.71 Find L_T in the network in Fig. P6.71 (a) with the switch open and (b) with the switch closed. All inductors are 12 mH.

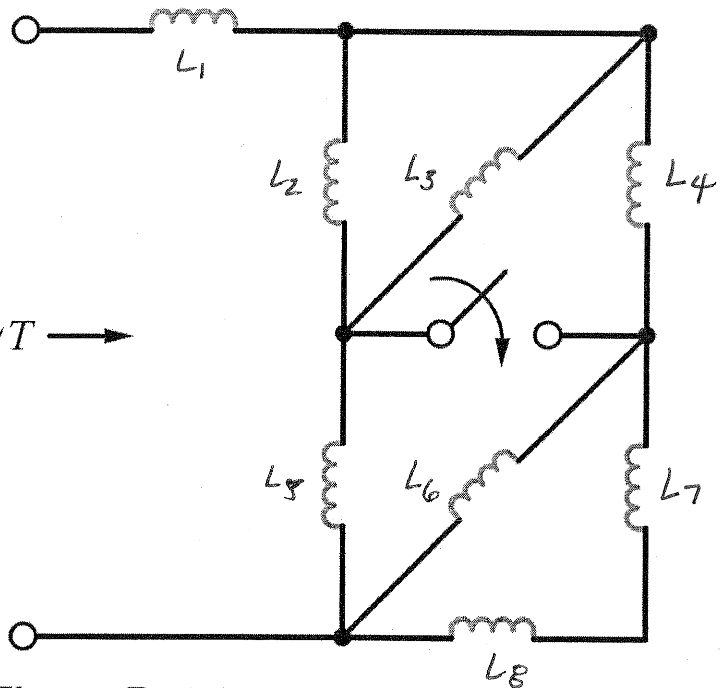
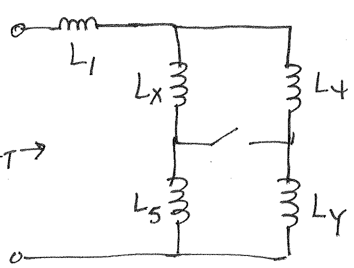


Figure P6.71

SOLUTION:



$$L_X = \frac{L_2 L_3}{L_2 + L_3} = 6\text{mH}$$

$$L_Y = \frac{L_6(L_7 + L_8)}{L_6 + L_7 + L_8} = 8\text{mH}$$

a) $L_T = L_1 + \frac{(L_X + L_5)(L_4 + L_Y)}{L_X + L_5 + L_4 + L_Y}$ $\boxed{L_T = 21.47\text{ mH}}$

b) $L_T = L_1 + \frac{(L_X L_4)}{L_X + L_4} + \frac{L_5 L_Y}{L_5 + L_Y}$ $\boxed{L_T = 20.8\text{mH}}$

6.72 For the network in Fig. P6.72, $v_S(t) = 120 \cos 377t$ V. Find $v_o(t)$.

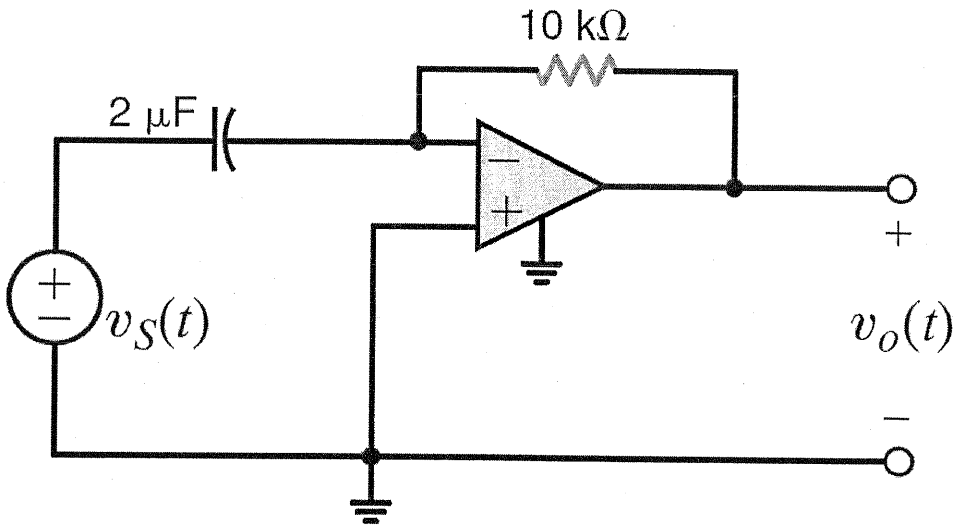


Figure P6.72

SOLUTION:

Differentiator: $v_o = -RC \dfrac{dv_S(t)}{dt}$

$$\boxed{v_o(t) = 904.8 \sin(377t) \text{ V}}$$

6.73 For the network in Fig. P6.73, $v_S(t) = 115 \sin 377t$ V. Find $v_o(t)$.

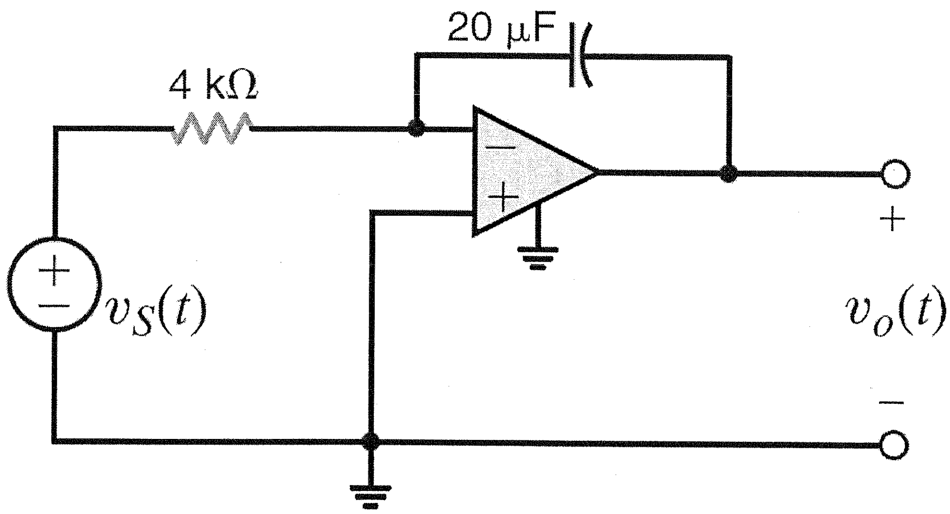


Figure P6.73

SOLUTION:

Integrator: $v_o(t) = \frac{-1}{RC} \int v_S(t)\, dt$

$$\boxed{v_o(t) = 3.81 \cos(377t) \text{ V}}$$

6.74 For the network in Fig. P6.74, $v_{S_1}(t) = 80 \cos 377t$ V and $v_{S_2}(t) = 40 \cos 377t$ V. Find $v_o(t)$.

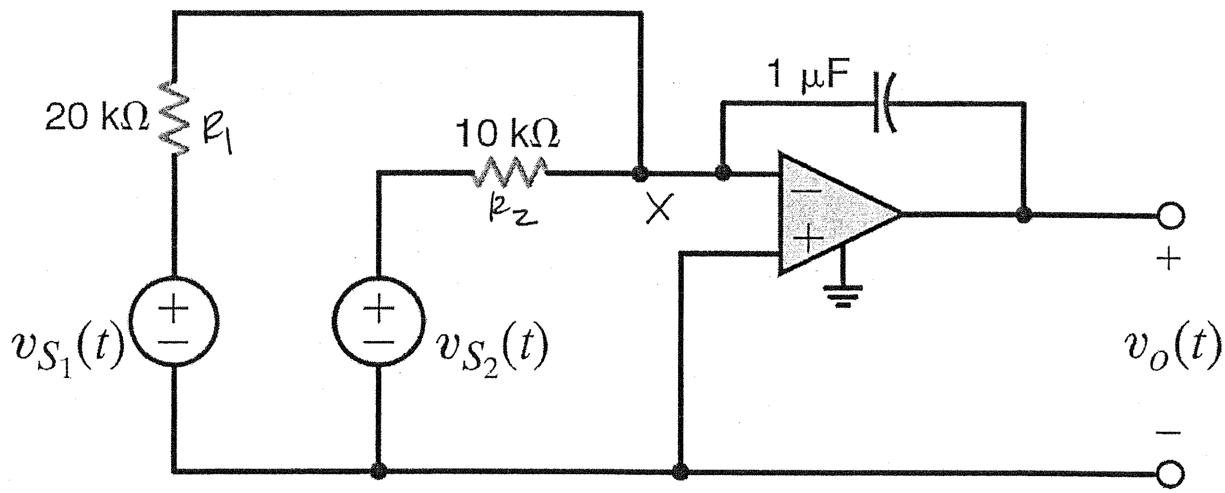


Figure P6.74

SOLUTION: Nodal analysis at node X!

$$\frac{v_{S1}}{R_1} + \frac{v_{S2}}{R_2} = -C\frac{dv_o}{dt} \qquad v_o(t) = -\frac{1}{C}\int \left(\frac{v_{S1}}{R_1} + \frac{v_{S2}}{R_2}\right) dt$$

$$v_o(t) = -\frac{8000}{\omega}\sin(\omega t)$$

$$\boxed{v_o(t) = -21.2\sin(377t)\ \text{V}}$$

6.75 For the network in Fig. P6.75, choose C such that

$$v_o = -10 \int v_S \, dt$$

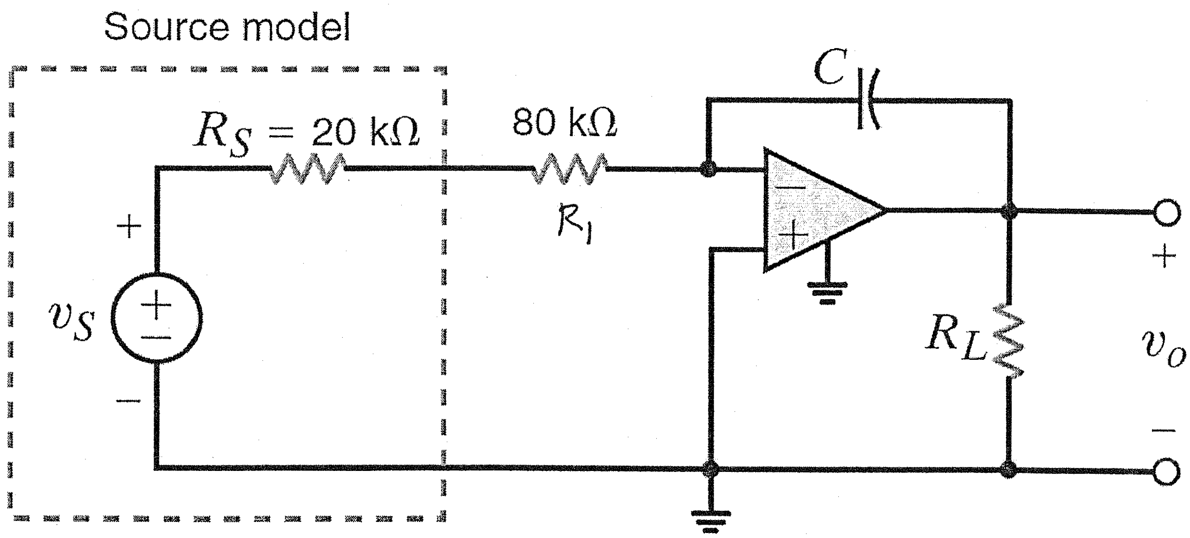


Figure P6.75

SOLUTION:

$$v_o = \frac{-1}{R_{eq} C} \int v_s \, dt \qquad R_{eq} = R_S + R_1 = 100\,\text{k}\Omega$$

Need $R_{eq} C = 0.1$ $\qquad \boxed{C = 1.0\,\mu F}$

6.76 An integrator is required that has the following performance

$$v_o(t) = 10^6 \int v_s \, dt$$

where the capacitor values must be greater than 10 nF and the resistor values must be greater than 10 kΩ.

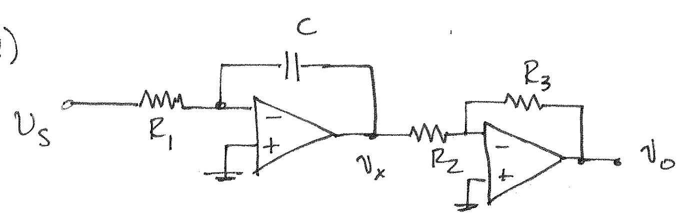
(a) Design the integrator.

(b) If ±10-V supplies are used, what are the maximum and minimum values of v_o?

(c) Suppose $V_S = 1$ V. What is the rate of change of v_o?

SOLUTION:

a)



$v_x = -\frac{1}{R_1 C}\int v_s \, dt$ $\quad v_o = -\frac{R_3}{R_2} v_x$ $\quad v_o = \frac{R_3}{C R_1 R_2}\int v_s \, dt$

Arbitrarily select: $C = 20\,nF$, $R_1 = 20\,k\Omega$ $R_2 = 20\,k\Omega$ $\Rightarrow \boxed{R_3 = 8\,M\Omega}$

b) V_o cannot exceed the supplies and so is limited to ±10V

$\boxed{V_{o\,max} = 10V \qquad V_{o\,min} = -10V}$

c) $V_s = 1V$ $\quad V_o(t) = 10^6 \int dt = 10^6 t$

$\boxed{\frac{dV_o}{dt} = 10^6 \; V/s}$

6.77 The circuit shown in Fig. P6.77 is known as a "Deboo" integrator.

(a) Express the output voltage in terms of the input voltage and circuit parameters.

(b) How is the Deboo integrator's performance different from that of a standard integrator?

(c) What kind of application would justify the use of this device?

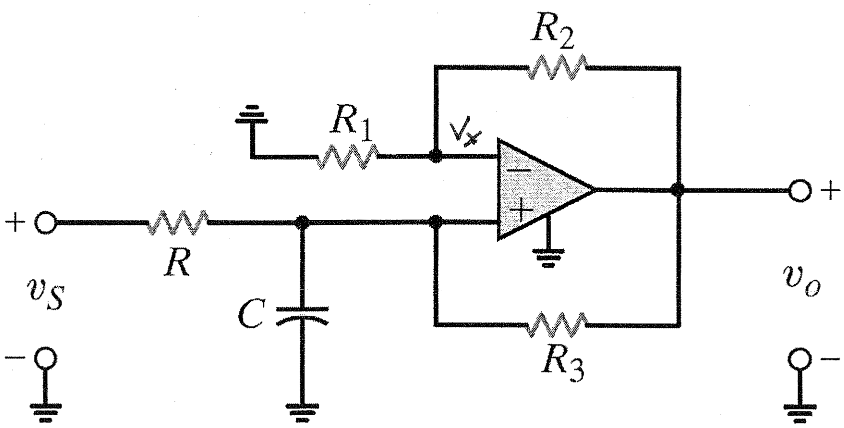


Figure P6.77

SOLUTION:

a) $v_x = v_o\left(\frac{R_1}{R_1+R_2}\right) = \alpha v_o$

$$\frac{v_s - v_x}{R} = C\frac{dv_x}{dt} + \frac{v_x - v_o}{R_3} \Rightarrow v_s = \frac{R_1 RC}{R_1+R_2}\frac{dv_o}{dt} + \left[\frac{R_1(R+R_3)}{R_3(R_1+R_2)} - \frac{R}{R_3}\right] v_o$$

$$v_o = \left(\frac{R_1+R_2}{R_1}\right)\frac{1}{RC}\int\left\{v_s(t) + \left[\frac{R}{R_3} - \frac{R_1(R+R_3)}{R_3(R_1+R_2)}\right]v_o\right\}dt$$

b) Consider the case $\frac{R_1}{R_1+R_2} = \frac{R}{R+R_3} \Rightarrow v_o = \frac{R+R_3}{R^2C}\int v_s\,dt$

Major difference is that the integration is positive

c) Where positive integration is needed!

6.78 A driverless automobile is under development. One critical issue is braking, particularly at red lights. It is decided that the braking effort should depend on distance to the light (if you're close, you better stop now) and speed (if you're going fast, you'll need more brakes). The resulting design equation is

$$\text{braking effort} = K_1\left[\frac{dx(t)}{dt}\right] + K_2 x(t)$$

where x, the distance from the vehicle to the intersection, is measured by a sensor whose output is proportional to x, $v_{\text{sense}} = \alpha x$. Use superposition to show that the circuit in Fig. P6.78 can produce the braking effort signal.

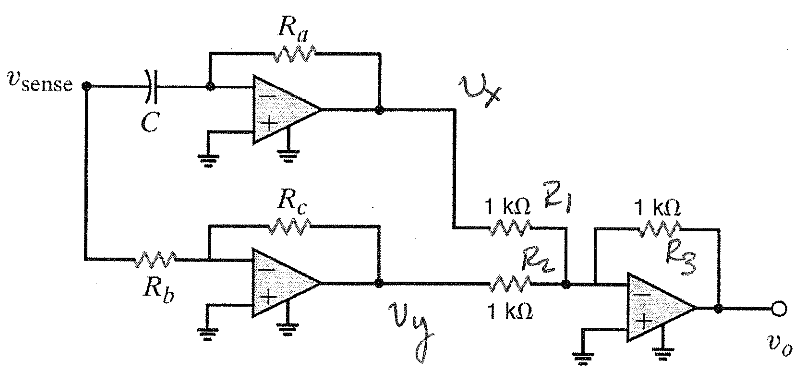


Figure P6.78

SOLUTION:

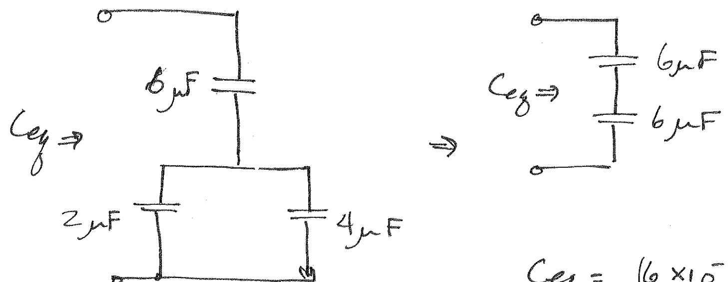
$$v_x = -R_a C \frac{d v_{sense}}{dt} \qquad v_y = -\frac{R_c}{R_b} v_{sense} \qquad v_o = -\frac{R_3}{R_1} v_x - \frac{R_3}{R_2} v_y$$

$$\boxed{v_o = \frac{\alpha R_3 R_a C}{R_1} \frac{dx}{dt} + \frac{R_3}{R_2}\frac{R_c}{R_b}\alpha x \qquad K_1 = \frac{\alpha R_3 R_a C}{R_1} \qquad K_2 = \frac{R_3 R_c}{R_2 R_b}}$$

6FE-1 Given three capacitors with values 2 μF, 4 μF, and 6 μF, can the capacitors be interconnected so that the combination is an equivalent 3 μF? CS

SOLUTION:

Yes!



$$C_{eq} = \frac{(6\times10^{-6})(6\times10^{-6})}{6\times10^{-6}+6\times10^{-6}} = 3\mu F$$

6FE-2 The current pulse shown in Fig. 5PFE-2 is applied to a 1-μF capacitor. Determine the charge on the capacitor and the energy stored.

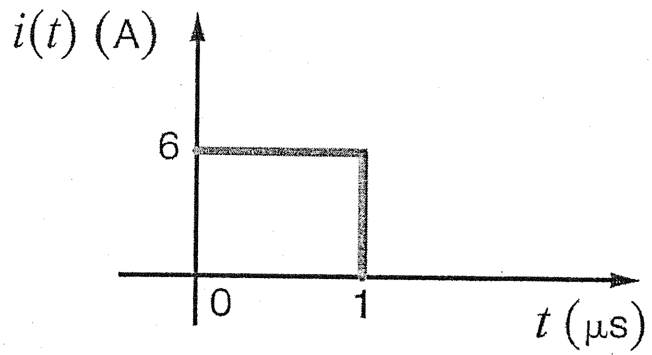


Figure 5PFE-2

SOLUTION:

$$q(t) = \int i \, dt$$

$$\boxed{q(t) = \begin{cases} 0 \quad C & t<0 \\ 6t \quad C & 0<t\le 1\mu s \\ 6 \quad \mu C & t>1\mu s \end{cases}}$$

$$w(t) = \frac{1}{2} C v^2(t) \qquad v(t) = q(t)/C = \begin{cases} 0 \quad V & t\le 0 \\ 6\times10^6 t \quad V & 0<t\le 1\mu s \\ 6 \quad V & t>1\mu s \end{cases}$$

$$\boxed{w(t) = \begin{cases} 0 \quad J & t\le 0 \\ 18\times10^6 t^2 \quad J & 0<t\le 1\mu s \\ 18 \quad \mu J & t>1\mu s \end{cases}}$$

6FE-3 The two capacitors shown in Fig. 5PFE-3 have been connected for some time and have reached their present values. Determine the energy stored in the unknown capacitor C_x. CS

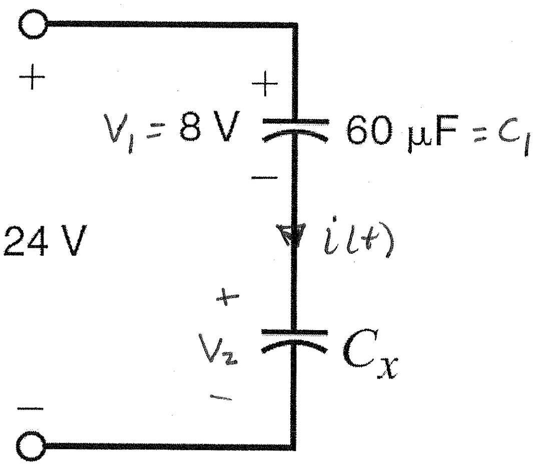


Figure 5PFE-3

SOLUTION:

$v_1 + v_2 = 24$

$v_2 = 16\text{V}$

$v_1 = \frac{1}{C_1}\int i(t)\,dt$

$v_2 = \frac{1}{C_x}\int i(t)\,dt$

$\frac{v_1}{v_2} = \frac{C_x}{C_1} = \frac{1}{2}$

$\boxed{C_x = 30\mu\text{F}}$

Chapter Seven:

First- and Second-Order Transient Circuits

7.1 Use the differential equation approach to find $v_o(t)$ for $t > 0$ in the circuit in Fig. P7.1 and plot the response including the time interval just prior to switch action.

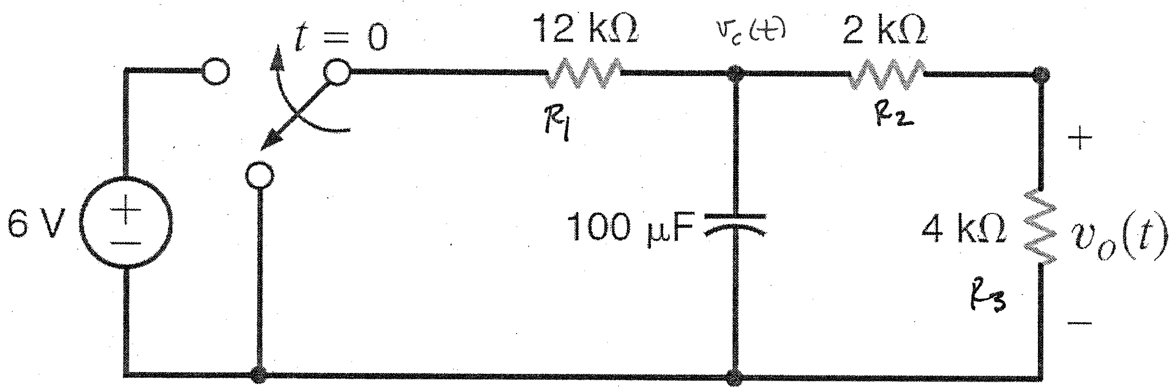


Figure P7.1

SOLUTION:

$v_c(0^-) = 0V$ for $t>0$: $\frac{6-v_c}{R_1} + \frac{v_o - v_c}{R_2} - C\frac{dv_c}{dt} = 0$ $\quad v_o = \frac{v_c R_3}{R_2+R_3} = \alpha v_c$

Multiply by $\alpha \Rightarrow \frac{6\alpha}{R_1} + \frac{v_o}{R_2}\left[\alpha - 1\right] - \frac{v_o}{R_1} - \frac{C\,dv_o}{dt} = 0$

$\frac{dv_o}{dt} + v_o\left[\frac{1}{R_1C} + \frac{1-\alpha}{R_2C}\right] - \frac{6\alpha}{R_1C} = 0$ $\quad$ let $\frac{1}{R_1C} + \frac{1-\alpha}{R_2C} = B$

assume $v_o(t) = K_1 + K_2 e^{-t/\tau}$

$-\frac{K_2}{\tau}e^{-t/\tau} + K_1B + K_2Be^{-t/\tau} - \frac{6\alpha}{R_1C} = 0$

$\tau = 1/B = C\left[\frac{R_1(R_2+R_3)}{R_1+R_2+R_3}\right]$

$K_1 = \frac{6\alpha}{R_1BC} = \frac{6R_3}{R_1+R_2+R_3}$

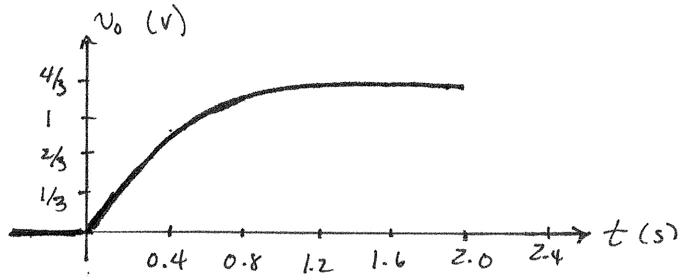
$\tau = 0.4s$ $\quad K_1 = 1.33V$

$v_o(0) = v_c(0)\alpha = 0 = K_1 + K_2 \Rightarrow K_2 = -1.33V$

$$\boxed{v_o(t) = 1.33 - 1.33e^{-2.5t}\ V}$$

$\underline{t=0^+}$: $v_C(0^+)=0$ $v_o(0^+)=\dfrac{6R_3}{R_1+R_2+R_3}=1.33V$

$\underline{t=0^-}$ $v_o(t)=0$



7.2 Use the differential equation approach to find $v_C(t)$ for $t > 0$ in the circuit in Fig. P7.2 and plot the response including the time interval just prior to closing the switch.

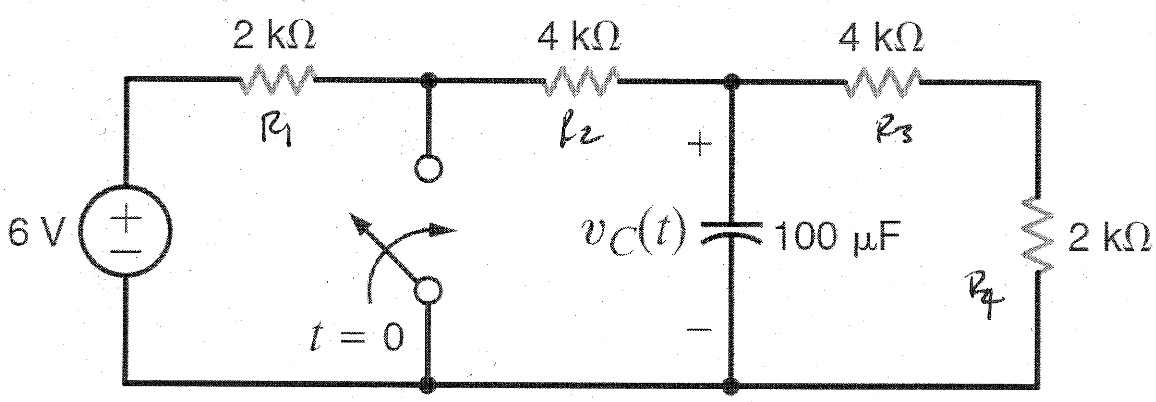


Figure P7.2

SOLUTION: $v_C(0^+) = v_C(0^-) = \frac{6\,(R_3+R_4)}{R_1+R_2+R_3+R_4} = 3V$

for $t>0$: $\frac{v_C}{R_2} + \frac{v_C}{R_3+R_4} + C\frac{dv_C}{dt} = 0$ let $v_C(t) = K_1 + K_2 e^{-t/\tau}$

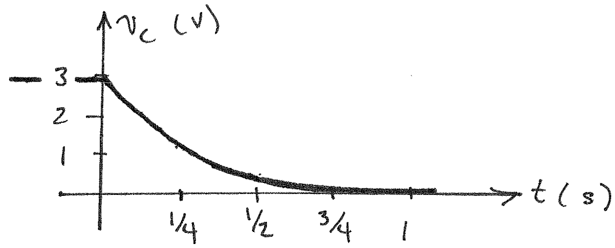
$$K_1\left(\frac{1}{R_2}+\frac{1}{R_3+R_4}\right) + K_2\left(\frac{1}{R_2}+\frac{1}{R_3+R_4}\right)e^{-t/\tau} - \frac{K_2 C}{\tau}e^{+t/\tau} = 0$$

yields $K_1 = 0$ $\tau = C\left\{\frac{R_2(R_3+R_4)}{R_2+R_3+R_4}\right\} = 0.24s$

$v_C(0^+) = 3 = K_1 + K_2 \Rightarrow K_2 = 3V$

$$\boxed{v_C(t) = 3e^{-t/0.24}\ V}$$

For $t > 0^-$: $v_C(0^-) = 3V$



7.3 Use the differential equation approach to find $v_C(t)$ for $t > 0$ in the circuit in Fig. P7.3. CS

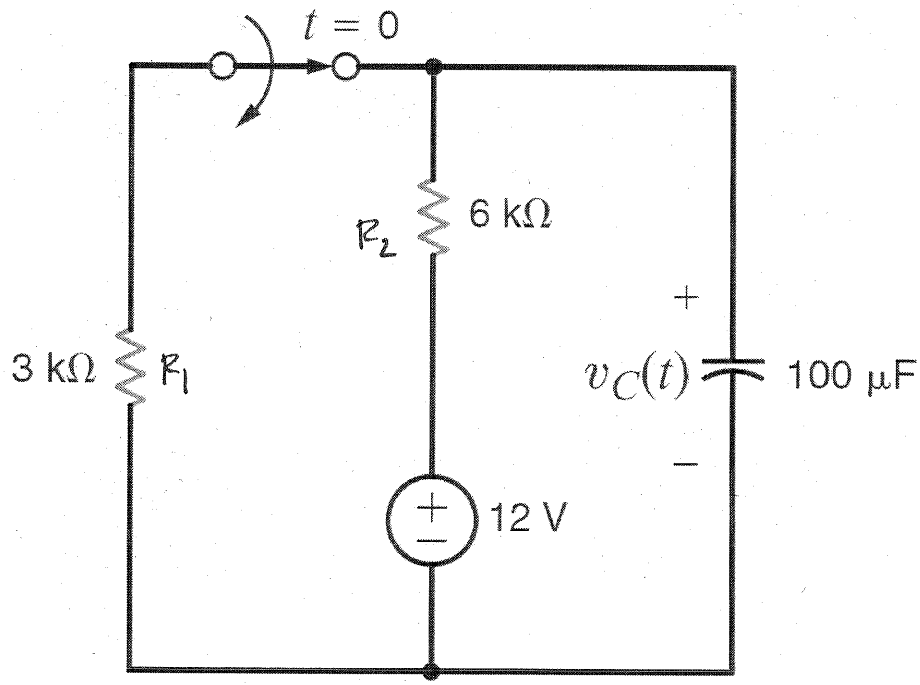


Figure P7.3

SOLUTION: $v_c(0^+) = v_c(0^-) = \frac{12\,(R_1)}{R_1+R_2} = 4V$ $\qquad v_c(t) = K_1 + K_2 e^{-t/\tau}$

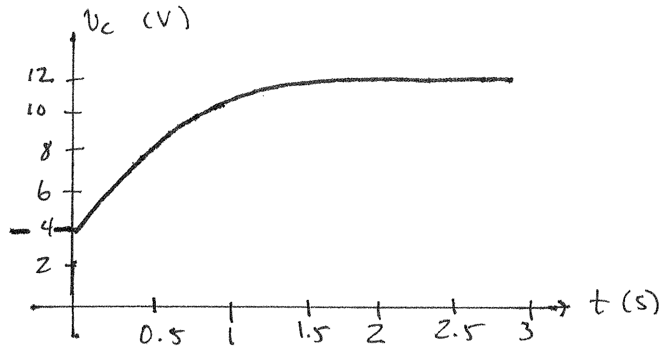
for $t>0$,

$$\frac{v_c - 12}{R_2} + C\frac{dv_c}{dt} = 0 \Rightarrow \frac{dv_c}{dt} + \frac{v_c}{R_2C} - \frac{12}{R_2C} = 0 = -\frac{K_2}{\tau}e^{-t/\tau} + \frac{K_1}{R_2C} + \frac{K_2}{R_2C}e^{-t/\tau} - \frac{12}{R_2C} = 0$$

yields : $\tau = R_2C$ $\quad K_1 = 12$ $\quad v_c(0^+) = 4 = K_1 + K_2 \Rightarrow K_2 = -8V$

$\quad = 0.6\,s$

$$\boxed{v_c(t) = 12 - 8e^{-\frac{t}{0.6}}\ V}$$


v_c (V)
12
10
8
6
4
2
0.5
1
1.5
2
2.5
3
t (s)

7.4 Use the differential equation approach to find $v_C(t)$ for $t > 0$ in the circuit in Fig. P7.4.

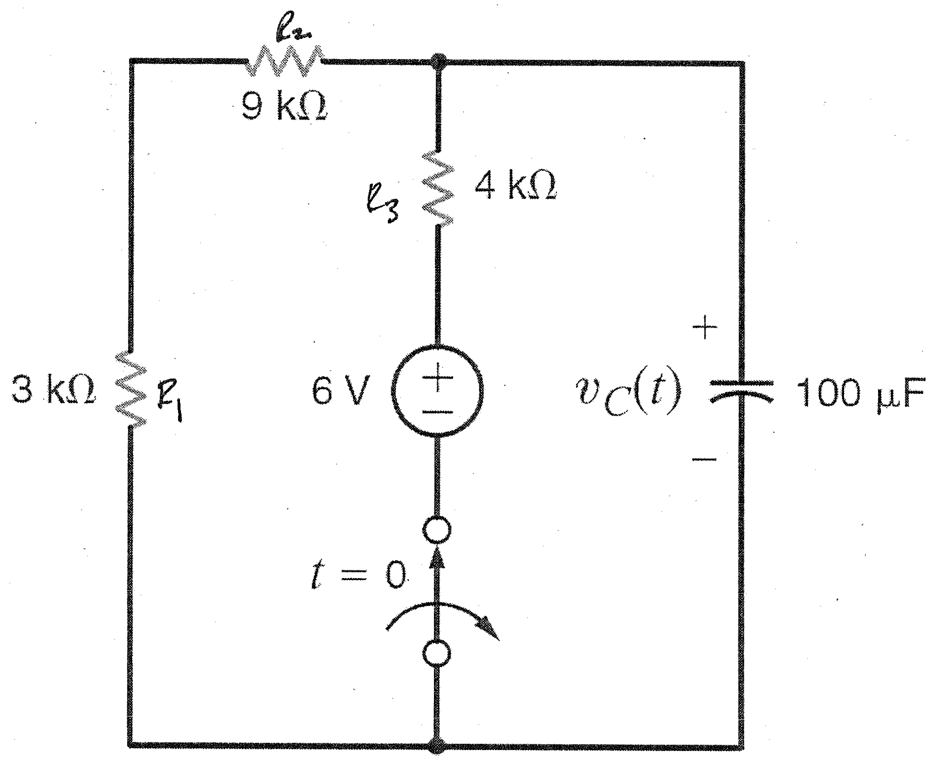


Figure P7.4

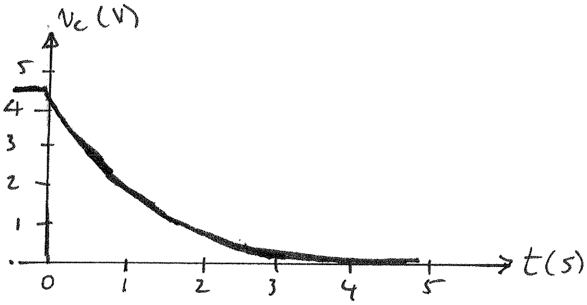
SOLUTION: $v_c(0^+) = v_c(0^-) = \frac{6\,(R_1+R_2)}{R_1+R_2+R_3} = 4.5\,V$ $\qquad v_c(t) = K_1 + K_2 e^{-t/\tau}$

for $t > 0$: $\frac{C\,dv_c(t)}{dt} + \frac{v_c(t)}{R_1+R_2} = 0 \Rightarrow \frac{dv_c}{dt} + \frac{v_c}{C(R_1+R_2)} = 0 = \frac{dv_c}{dt} + \frac{v_c}{\tau}$

$\tau = C\,(R_1+R_2) = 1.2s$ $\qquad -\frac{K_2}{\tau} + \frac{K_1}{\tau} + \frac{K_2}{\tau} = 0 \Rightarrow K_1 = 0$

$v_c(0^+) = 4.5 = K_1 + K_2 \Rightarrow K_2 = 4.5$

$$\boxed{v_c(t) = 4.5\,e^{-t/1.2}\ V}$$



v_c (V)
5
4
3
2
1
0
1
2
3
4
5
t(s)

7.5 Use the differential equation approach to find $v_C(t)$ for $t > 0$ in the circuit in Fig. P7.5 and plot the response including the time interval just prior to opening the switch. CS

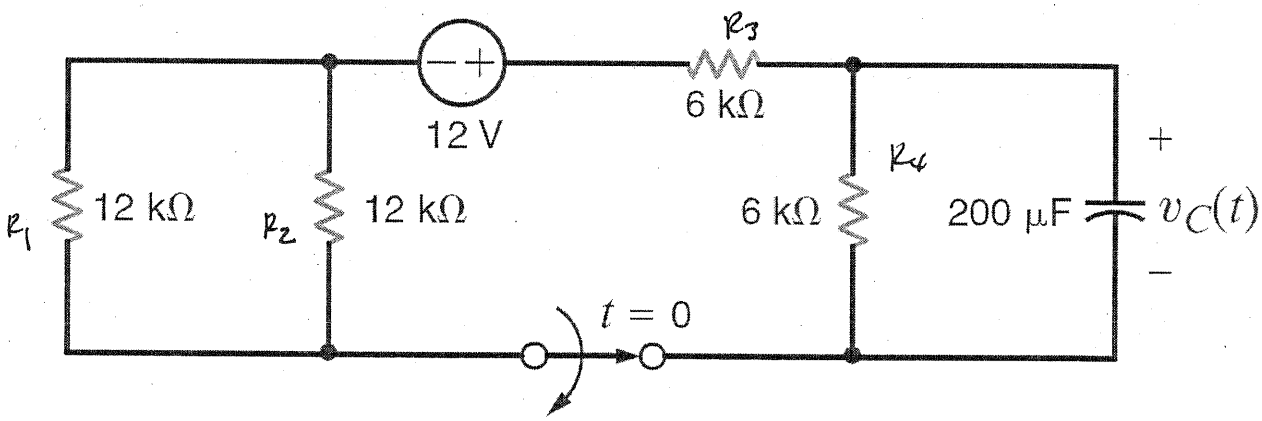


Figure P7.5

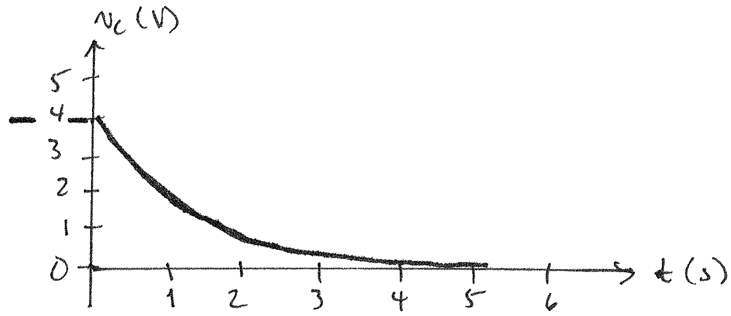
SOLUTION: $v_c(0^-) = v_c(0^+) = \dfrac{12\,R_4}{R_3 + R_4 + R_A}$ $\quad R_A = R_1 /\!/ R_2 = 6\,k\Omega \quad$ $v_c(0^+) = 4V$

for $t > 0$; $\quad \dfrac{v_c}{R_4} + C\dfrac{dv_c}{dt} = 0 \Rightarrow \dfrac{dv_c}{dt} + \dfrac{v_c}{R_4 C} = 0$

$v_c(t) = K_1 + K_2 e^{-t/\tau} \quad \Rightarrow \quad \dfrac{dv_c}{dt} + \dfrac{v_c}{\tau} = 0$

$\tau = R_4 C \qquad K_1 = 0 \qquad v_c(0^+) = 4 = K_1 + K_2 \Rightarrow K_2 = 4V$

$$\boxed{v_c(t) = 4e^{-t/1.2}\ V}$$



v_c (V)
5
4
3
2
1
0
1
2
3
4
5
6
t (s)

7.6 Use the differential equation approach to find $v_o(t)$ for $t > 0$ in the circuit in Fig. P7.6 and plot the response including the time interval just prior to opening the switch. **CS**

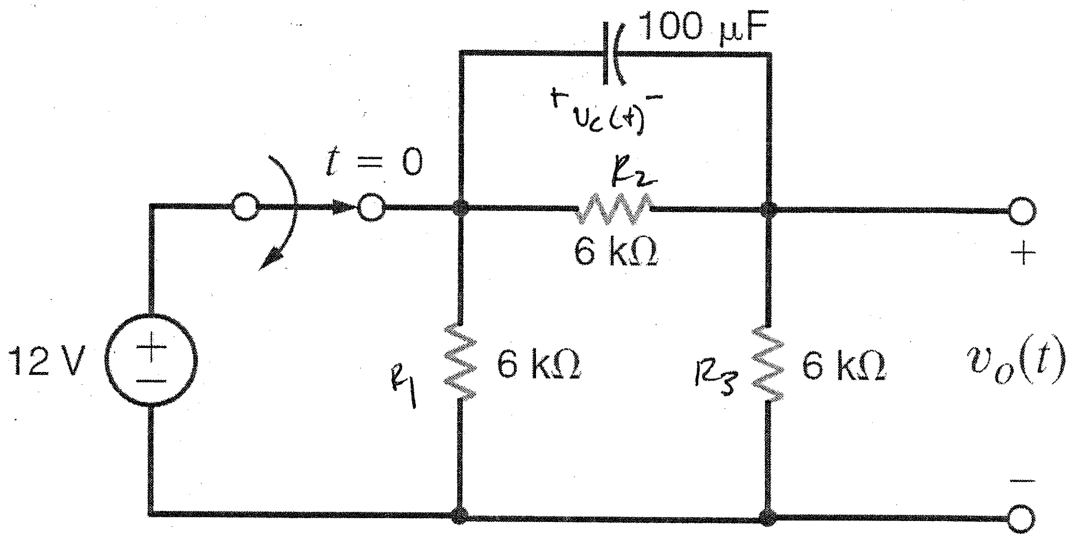


Figure P7.6

SOLUTION: $v_c(0^+) = v_c(0^-) = \dfrac{12\, R_2}{R_2+R_3} = 6\text{V}$ $\qquad v_o(t) = K_1 + K_2 e^{-t/\tau}$

for $t>0$: $\quad v_o = \dfrac{v_c\,(-R_3)}{R_1+R_3} = -\alpha v_c \quad$ & $\quad \dfrac{v_c}{R_2} + C\dfrac{dv_c}{dt} = \dfrac{v_o}{R_3}$.

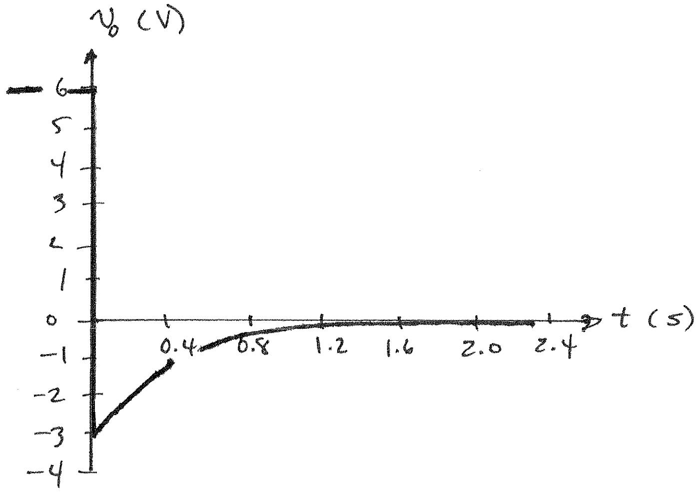
Eliminate v_c: $\quad \dfrac{dv_o}{dt} + v_o\left[\dfrac{1}{R_2C} + \dfrac{\alpha}{R_3C}\right] = 0 = \dfrac{dv_o}{dt} + \dfrac{v_o}{\tau}$

$\tau = C\left[\dfrac{R_2\,(R_1+R_3)}{R_2+R_1+R_3}\right] = 0.4s \qquad K_1 = 0$

$v_o(0^+) = v_c(0^+)\,\alpha = -3V = K_1 + K_2 \Rightarrow K_2 = -3V$

$$\boxed{v_o(t) = -3\,e^{-2.5t}\ \text{V}}$$

$\underline{t = 0^-}$: $v_c(0^-) = 6V$ $v_o(0^-) = 12 - v_c = 6V$



7.7 Use the differential equation approach to find $i_o(t)$ for $t > 0$ in the circuit in Fig. P7.7 and plot the response including the time interval just prior to closing the switch.

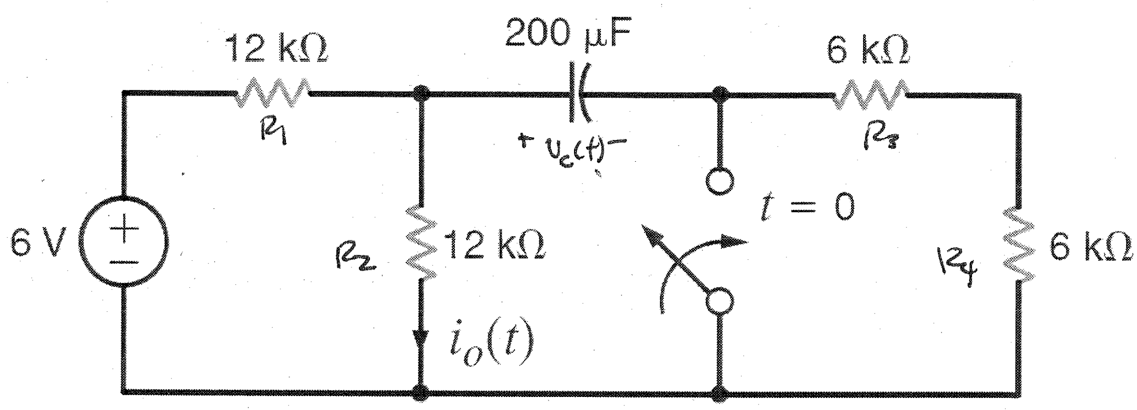


Figure P7.7

SOLUTION:
$$v_c(0^-) = v_c(0^+) = \frac{6R_2}{R_1+R_2} = 3\,V \qquad i_o(t) = \frac{v_c(t)}{R_2} \text{ for } t>0.$$

for $t>0$:
$$\frac{6-v_c}{R_1} = \frac{v_c}{R_2} + C\frac{dv_c}{dt} \Rightarrow \frac{dv_c}{dt} + v_c\left[\frac{1}{R_1C}+\frac{1}{R_2C}\right] - \frac{6}{R_1C} = 0$$

Convert to i_o:
$$\frac{di_o}{dt} + i_o\left[\frac{1}{R_1C}+\frac{1}{R_2C}\right] - \frac{6}{R_1R_2C} = 0$$

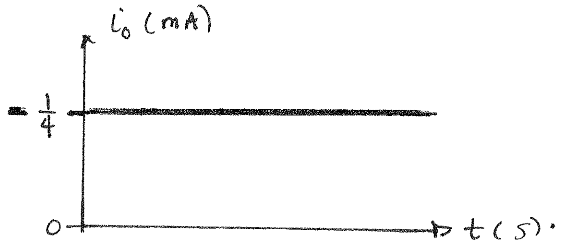
$$i_o(t) = K_1 + K_2e^{-t/\tau} \Rightarrow -\frac{K_2}{\tau}e^{-t/\tau} + (K_1+K_2e^{-t/\tau})\left[\frac{1}{R_1C}+\frac{1}{R_2C}\right] - \frac{6}{R_1R_2C} = 0$$

yields
$$\tau = C\frac{R_1R_2}{R_1+R_2} = 1.25 \qquad K_1 = \frac{6}{R_1+R_2} = 0.25\,mA$$

$$i_o(0^+) = K_1+K_2 = v_c(0^+)/R_2 = 0.25\,mA \Rightarrow K_2 = 0$$

$$\boxed{i_o(t) = 0.25\,mA}$$

$\underline{t=0^-}$: $v_c(0^-) = 3V$ $\quad i_c(0^-) = 0$ $\quad i_o(0^-) = \dfrac{6}{R_1+R_2} = 0.25mA$



7.8 Use the differential equation approach to find $v_o(t)$ for $t > 0$ in the circuit in Fig. P7.8 and plot the response including the time interval just prior to closing the switch. **CS**

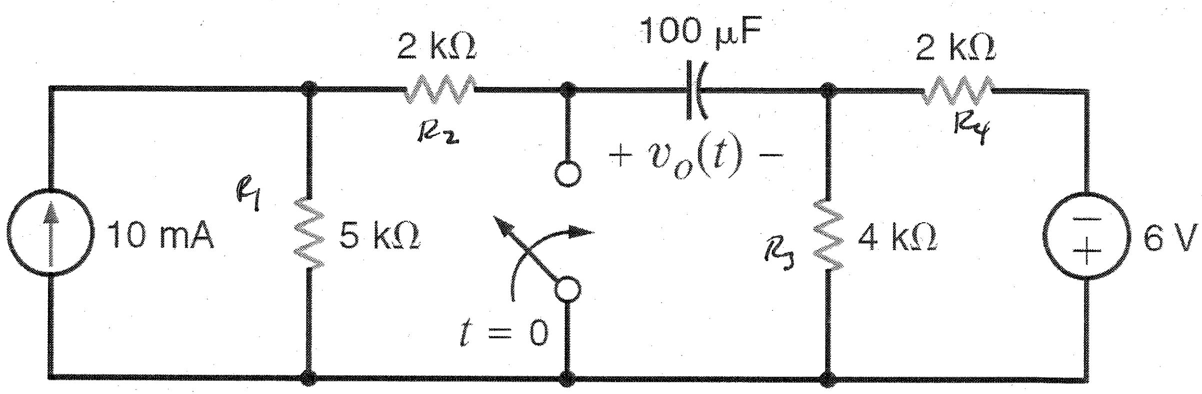
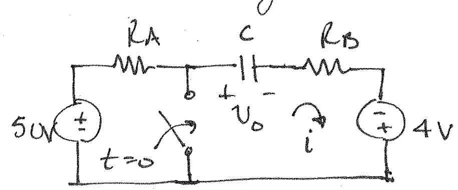


Figure P7.8

SOLUTION: Using source transformation:



$R_A = R_1 + R_2 = 7\,k\Omega \qquad R_B = R_3 // R_4 = 1.33\,k\Omega$

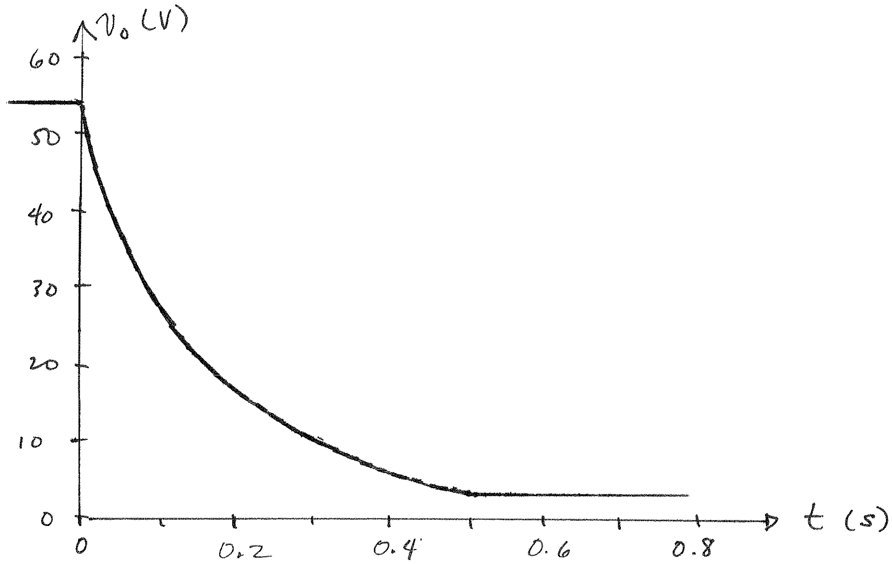
$v_o(0^+) = v_o(0^-) = 54\,V$

for $t>0$: $4 = v_o + i R_B$ & $i = C\,dv_o/dt \Rightarrow \dfrac{dv_o}{dt} + \dfrac{v_o}{R_B C} - \dfrac{4}{R_B C} = 0$

$v_o = K_1 + K_2 e^{-t/\tau} \Rightarrow \tau = R_B C$ & $K_1 = 4 \qquad \tau = 0.133\,s$

$v_o(0^+) = 54 = K_1 + K_2 \Rightarrow K_2 = 50\,V$

$$\boxed{v_o = 4 + 50 e^{-7.5t}\ V}$$


v_o (V)
60
50
40
30
20
10
0
0
0.2
0.4
0.6
0.8
t (s)

7.9 Use the differential equation approach to find $v_C(t)$ for $t > 0$ in the circuit in Fig. P7.9 and plot the response including the time interval just prior to opening the switch.

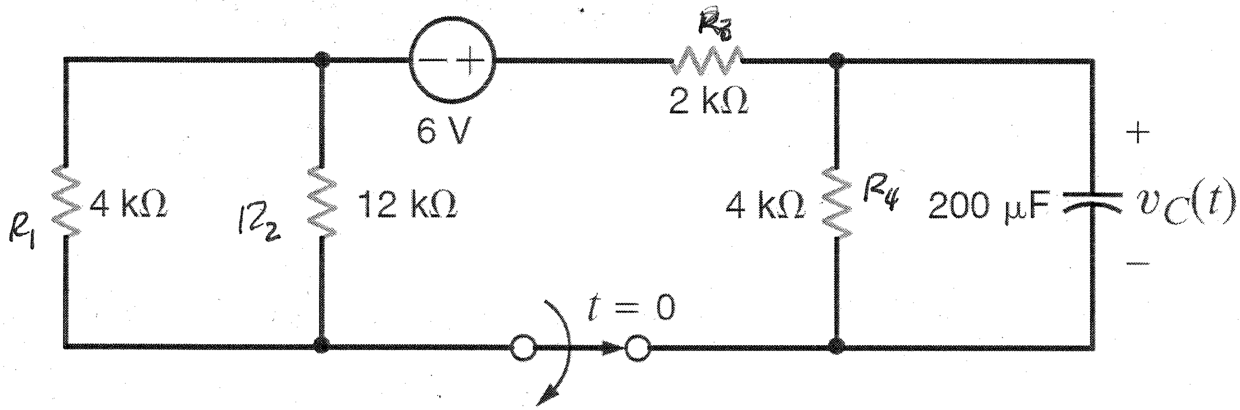
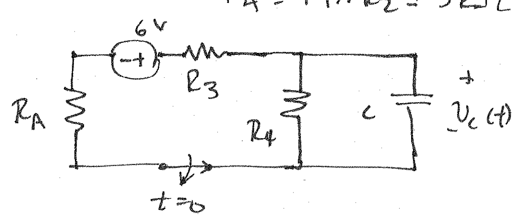


Figure P7.9

SOLUTION: $R_A = R_1 // R_2 = 3k\Omega$



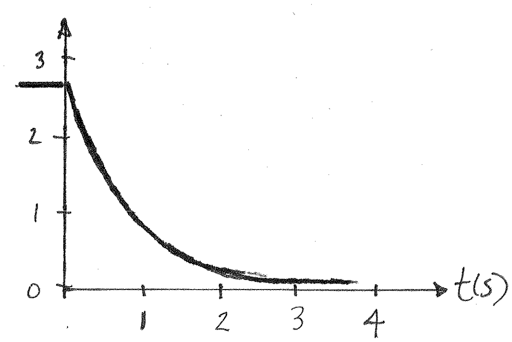
$$v_c(0^+) = v_c(0^-) = \frac{6R_4}{R_A+R_3+R_4} = \frac{8}{3}\,V$$

for $t>0$, $\frac{v_c}{R_4} + C\frac{dv_c}{dt} = 0 \Rightarrow \frac{dv_c}{dt} + \frac{v_c}{R_4C} = 0$

$v_c = K_1 + K_2e^{-t/\tau} \Rightarrow \tau = R_4C = 0.8s \qquad K_1 = 0$

$v_c(0^+) = \frac{8}{3} = K_1 + K_2 \Rightarrow K_2 = 2.67V$

$$\boxed{v_c(t) = 2.67e^{-1.25t}\ V}$$



7.10 Use the differential equation approach to find $i_o(t)$ for $t > 0$ in the circuit in Fig. P7.10 and plot the response including the time interval just prior to opening the switch. CS

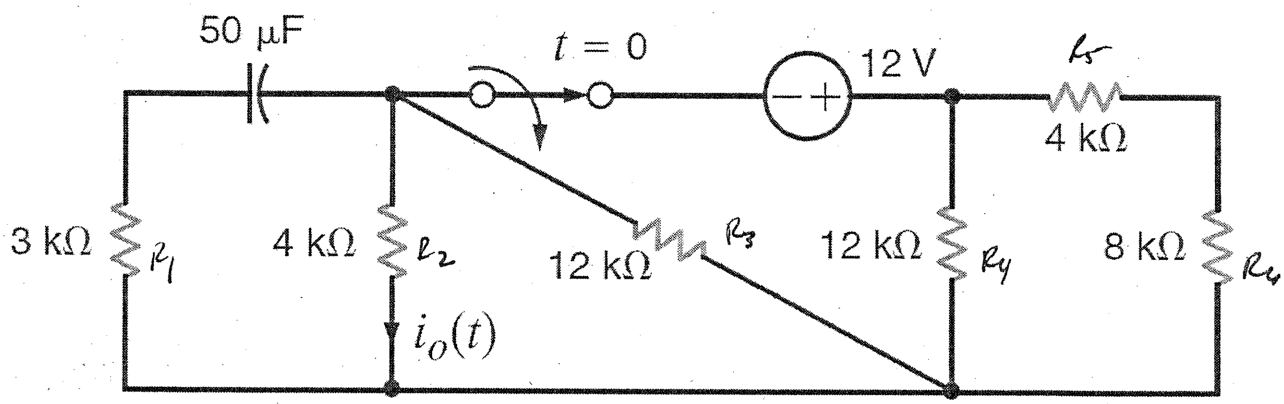
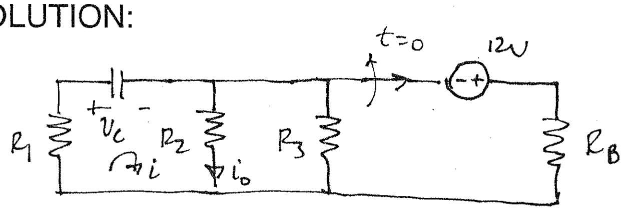


Figure P7.10

SOLUTION:



$R_B = R_4 // (R_5 + R_6) = 6\,k\Omega$

$R_A = R_2 // R_3 = 3\,k\Omega$

$v_c(0+) = v_c(0-) = \dfrac{12\,R_A}{R_A + R_B} = 4V$ $\qquad i_o(0-) = \dfrac{-v_c(0-)}{R_2} = -1mA$

For $t>0$, $\quad v_c + iR_A + iR_1 = 0 \quad \& \quad i = C\dfrac{dv_c}{dt} \quad \& \quad i_o = \dfrac{R_3}{R_2+R_3}\, i = \alpha i$

yields, $\quad \dfrac{dv_c}{dt} + \dfrac{v_c}{C(R_1+R_A)} = 0$

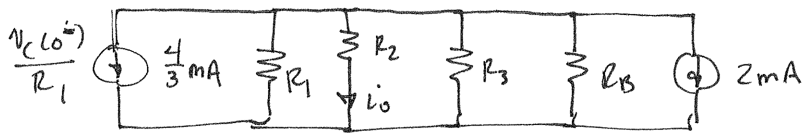
or, $\quad \dfrac{di_o}{dt} + \dfrac{i_o}{C(R_1+R_A)} = 0 \qquad$ where $i_o = K_1 + K_2 e^{-t/\tau}$

yields, $\quad \tau = C[R_1 + R_A] = 0.3s \qquad K_1 = 0$

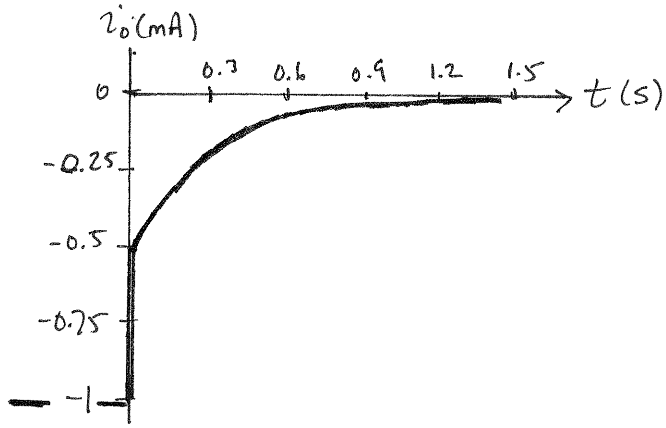
$i_o(0+) = \dfrac{-v_c(0+)}{R_1 + R_A}\;\dfrac{R_3}{R_3+R_2} = -0.5mA = K_1 + K_2 \Rightarrow K_2 = -0.5mA$

$\boxed{i_o(t) = -0.5\, e^{-t/0.3}\ mA}$

$\underline{t = 0^-}$:



$$i_o(0^-) = -\frac{(2 + 4/3)\times 10^{-3}\,(1/R_2)}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_B}} = -1\,\text{mA}$$



7.11 Use the differential equation approach to find $i_o(t)$ for $t > 0$ in the circuit in Fig. P7.11 and plot the response including the time interval just prior to opening the switch.

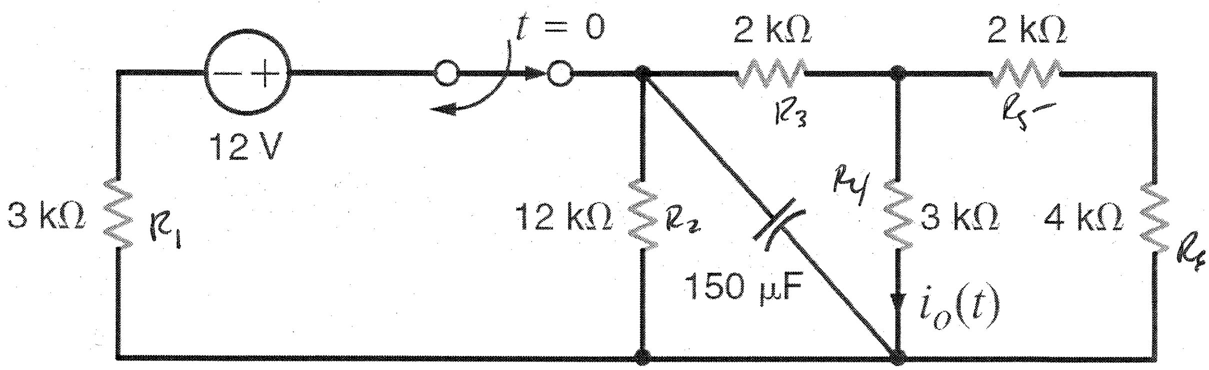
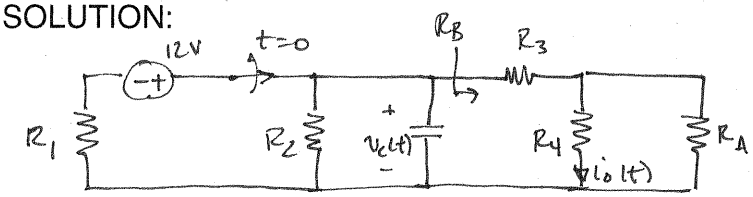


Figure P7.11

SOLUTION:



$R_A = R_5 + R_6 = 6k\Omega$

$R_B = R_3 + (R_4 // R_A) = 4k\Omega$

$R_c = R_2 // R_B = 3k\Omega$

$v_c(0^+) = v_c(0^-) = \dfrac{12\,R_c}{R_1 + R_c} = 6V$

$i_o(t) = \dfrac{v_c(t)}{R_B} \dfrac{R_A}{R_A + R_4} = \dfrac{v_c(t)}{6000}$ $\qquad i_o(0^-) = i_o(0^+) = 1mA$

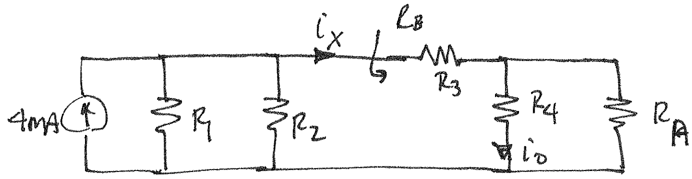
for $t>0$, $\dfrac{v_c}{R_c} + C\dfrac{dv_c}{dt} = 0 \Rightarrow \dfrac{di_o}{dt} + \dfrac{i_o}{CR_c} = 0$

$i_o = K_1 + K_2 e^{-t/\tau} \rightarrow \tau = CR_c = 0.45s \qquad K_1 = 0$

$K_1 + K_2 = i_o(0^+) = 1mA \rightarrow K_2 = 1mA$

$$\boxed{i_o(t) = e^{-t/0.45}\ mA}$$

$\underline{t = 0^-}$:

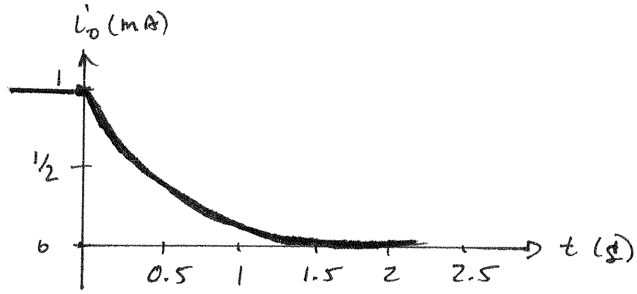


$R_B = 4\,k\Omega$

$R_A = 6\,k\Omega$

$$i_x = \frac{4\times10^{-3}\,(1/R_B)}{\frac{1}{R_B}+\frac{1}{R_1}+\frac{1}{R_2}} = 1.5\,mA$$

$$i_o = \frac{i_x R_A}{R_A + R_4} = 1\,mA$$



7.12 Use the differential equation approach to find $v_o(t)$ for $t > 0$ in the circuit in Fig. P7.12 and plot the response including the time interval just prior to opening the switch.

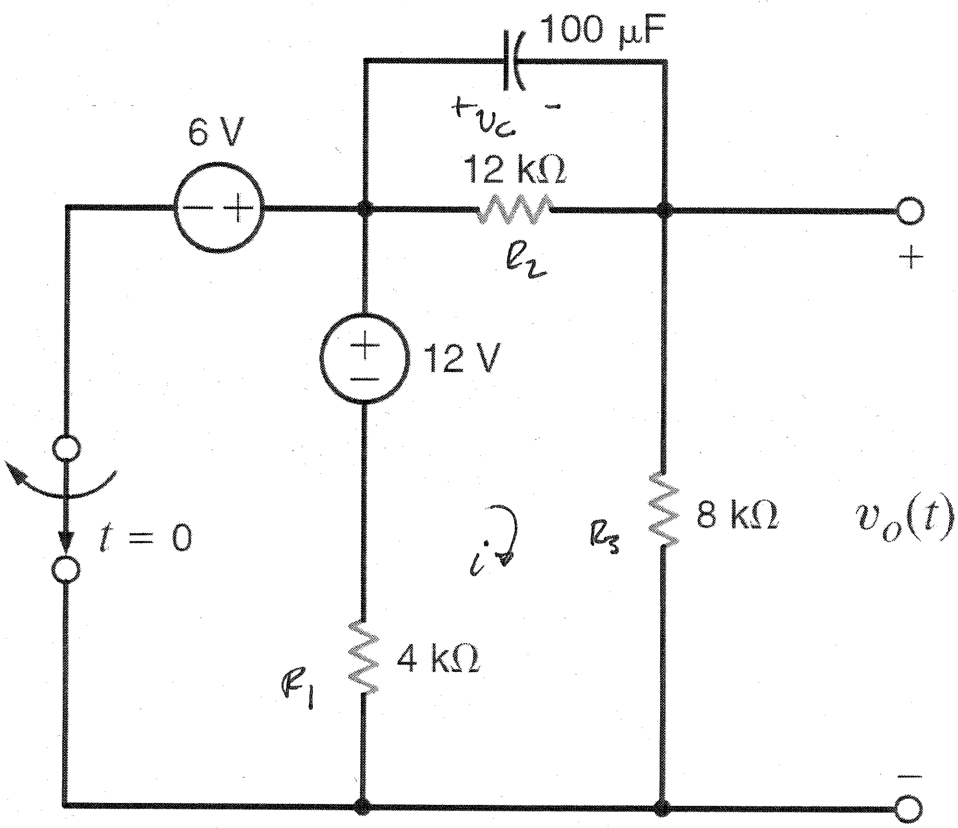


Figure P7.12

SOLUTION: $v_C(0^+) = v_C(0^-) = \frac{6R_2}{R_2+R_3} = 3.6\text{V}$ $\quad v_o(0^-) = \frac{6R_3}{R_2+R_3} = 2.4\text{V}$

for $v_o(0^+)$: $\left[12 - v_C(0^+)\right]\frac{R_3}{R_1+R_3} = v_o(0^+) = 5.6\text{V}$

for $t>0$: $\quad 12 = v_C(t) + i(R_1+R_3) \quad\quad v_o = iR_3$

and $\quad C\frac{dv_C}{dt} + \frac{v_C}{R_2} = \frac{v_o}{R_3}$

yields $\quad \frac{dv_o}{dt} + v_o\left[\frac{1}{R_2C} + \frac{\alpha}{R_3C}\right] - \frac{12\alpha}{R_2C} = 0 \quad\quad \alpha = \frac{R_3}{R_1+R_3}$

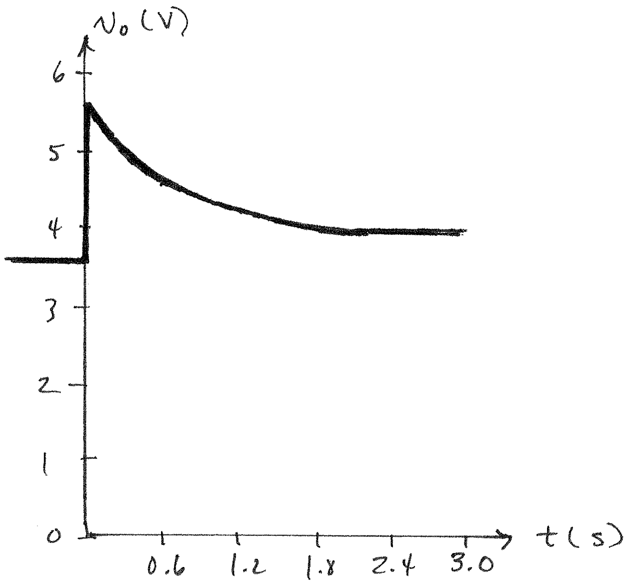
$$\tau = C\left[R_2 \,//\, (R_1 + R_3) \right] = 0.6\,s$$

$$K_1 = \frac{12\,R_3}{R_1 + R_2 + R_3} = 4V$$

$$v_o(0+) = K_1 + K_2 = 5.6V \Rightarrow K_2 = 1.6V$$

$$\boxed{v_o(t) = 4 + 1.6e^{-t/0.6}\ V}$$

$\underline{t = 0^-}$: $v_o(0^-) = 6 - v_c(0^-) = 3.6V$



7.13 Use the differential equation approach to find $v_o(t)$ for $t > 0$ in the circuit in Fig. P7.13 and plot the response including the time interval just prior to opening the switch.

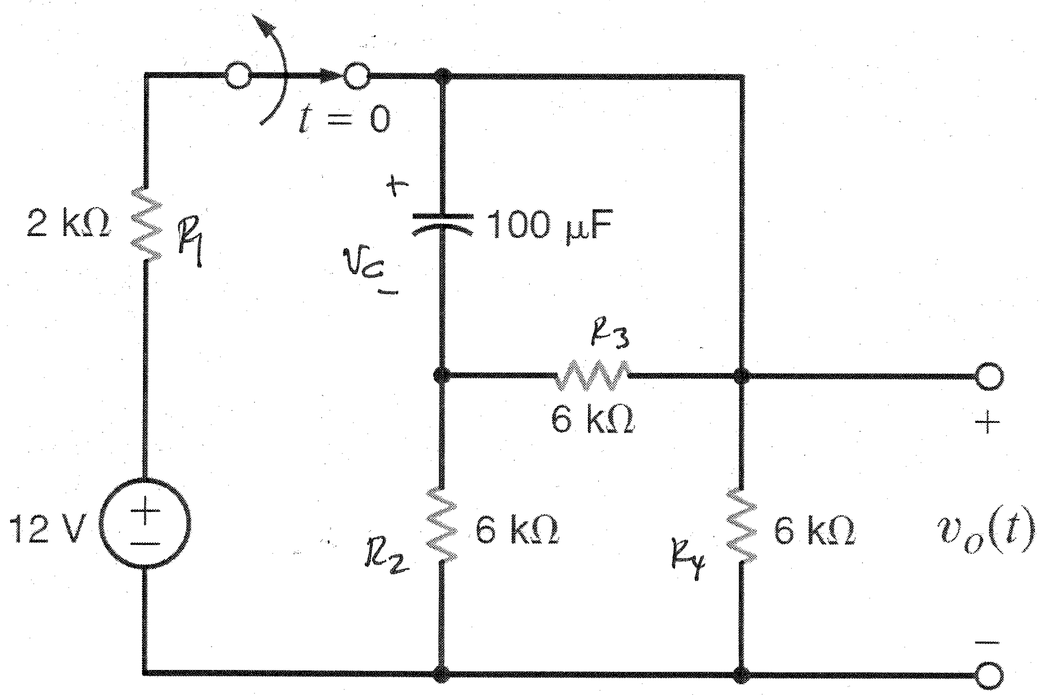
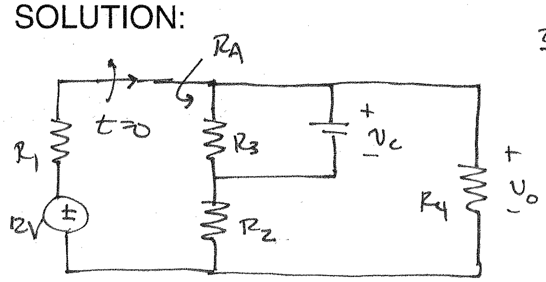


Figure P7.13

SOLUTION:



$\underline{t<0}$

$$R_A = R_4 \,//\, (R_3+R_2) = 4k\Omega$$

$$v_o(0^-) = \frac{12 R_A}{R_A + R_1} = 8V$$

$$v_c(0^-) = v_c(0^+) = \frac{v_o(0) R_3}{R_2 + R_3} = 4V$$

$\underline{t=0^+}$ $\quad v_o(0^+) = \dfrac{v_c(0^+) R_4}{R_2 + R_4} = 2 = K_1 + K_2$

$\underline{t>0}$ $\quad C\dfrac{dv_c}{dt} + \dfrac{v_c}{R_3} + \dfrac{v_o}{R_4} = 0$ and $\quad v_o = \dfrac{v_c R_4}{R_2 + R_4} = \alpha v_c$

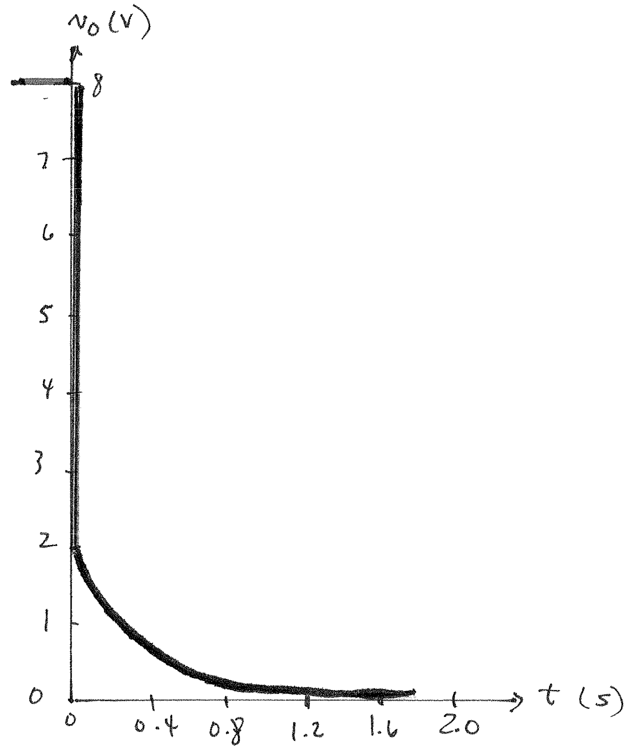
yields $\quad \dfrac{dv_o}{dt} + v_o\left[\dfrac{1}{CR_3} + \dfrac{\alpha}{CR_4}\right] = 0$

$\tau = C\left[R_3 // (R_2 + R_4)\right] = 0.4s$ $\qquad K_1 = 0$

$v_o(0+) = 2 = K_1 + K_2 \quad \Rightarrow \quad K_2 = 2$

$$\boxed{v_o(t) = 2e^{-2.5t} \text{ V}}$$

$\underline{t = 0^-}$: $\quad v_o(0^-) = 8V$



7.14 Use the differential equation approach to find $v_o(t)$ for $t > 0$ in the circuit in Fig. P7.14 and plot the response including the time interval just prior to closing the switch. PSV

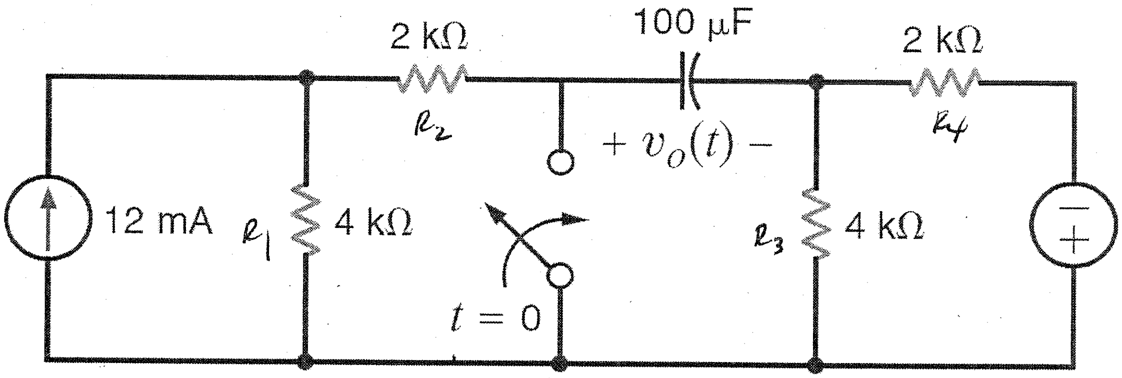
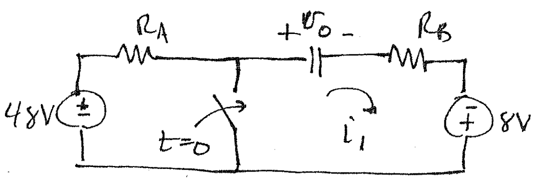


Figure P7.14

SOLUTION: By source transformation:



$R_A = R_1 + R_2 = 6k\Omega$

$R_B = R_3 // R_4 = \frac{4}{3} k\Omega$

$\underline{t<0}$ $\quad v_o(0^-) = v_o(0^+) = 56V \quad$ Note: $v_o(t) = v_c(t)$

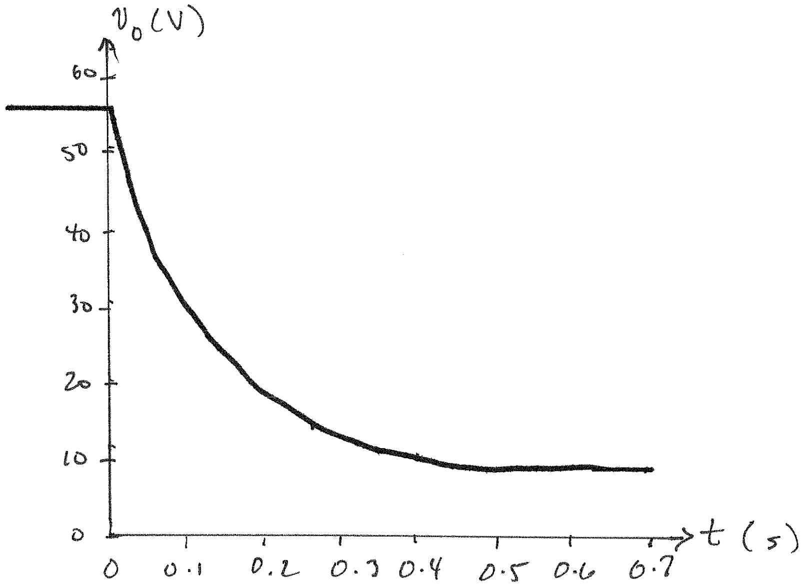
$\underline{t=0^+}$ $\quad v_o(0^+) = 56V$

$\underline{t>0}$ $\quad 8 = v_o + i_1 R_B \quad \& \quad i_1 = C\frac{dv_c}{dt} \Rightarrow \frac{dv_o}{dt} + \frac{v_o}{R_B C} - \frac{8}{R_B C} = 0$

$v_o = K_1 + K_2 e^{-t/\tau} \rightarrow \tau = R_B C = 0.133s \quad K_1 = 8V$

$K_2 = v_o(0^+) - K_1 = 48V$

$$\boxed{v_o(t) = 8 + 48e^{-7.5t} \text{ V}}$$


v_0 (V)
60
50
40
30
20
10
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
t (s)

7.15 Use the differential equation approach to find $i(t)$ for $t > 0$ in the network in Fig. P7.15.

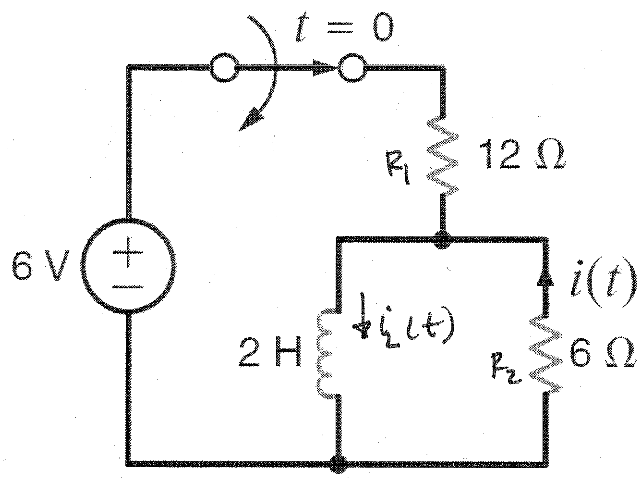


Figure P7.15

SOLUTION: $\underline{t=0^-}$: $i_L(0^-) = 6/R_1 = 0.5\text{ A} = i_L(0^+)$

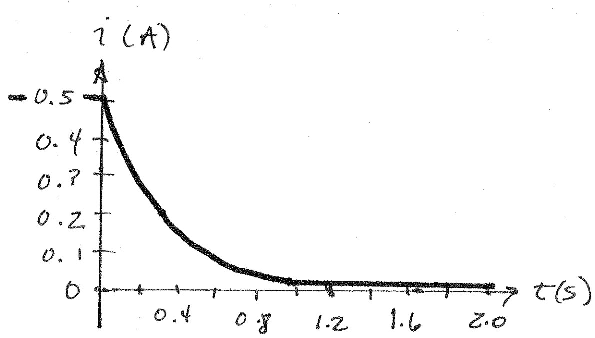
$\underline{t=0^+}$: $i(0^+) = i_L(0^+) = 0.5\text{ A} = K_1 + K_2$

$\underline{t>0}$ $\quad L\dfrac{di}{dt} + R_2 i = 0 \Rightarrow \dfrac{di}{dt} + \left(\dfrac{R_2}{L}\right) i = 0$

$i = K_1 + K_2 e^{-t/\tau} \quad \Rightarrow \quad \tau = \dfrac{L}{R_2} = \dfrac{1}{3}\text{ s} \qquad K_1 = 0$

$K_2 = i(0^+) - K_1 = 0.5\text{A}$

$$\boxed{i(t) = 0.5e^{-3t}\text{ A}}$$



7.16 Use the differential equation approach to find $i(t)$ for $t > 0$ in the circuit in Fig. P7.16 and plot the response including the time interval just prior to switch movement.

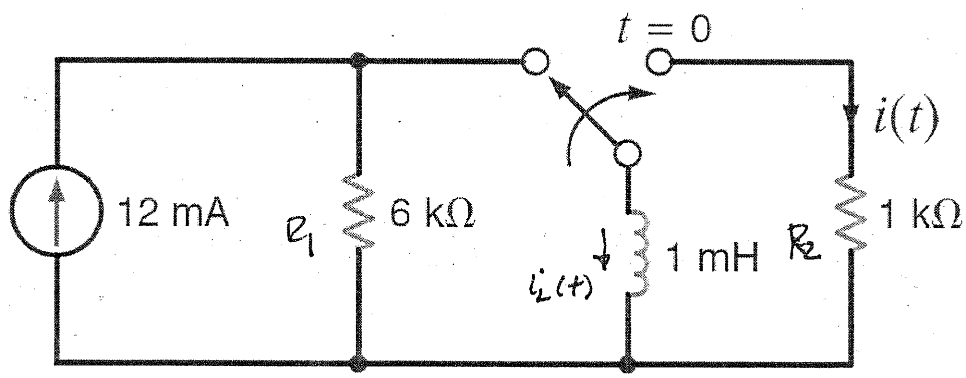


Figure P7.16

SOLUTION:

$\underline{t=0^-}$ $\quad i_L(0^-) = 12mA = i_L(0^+)$

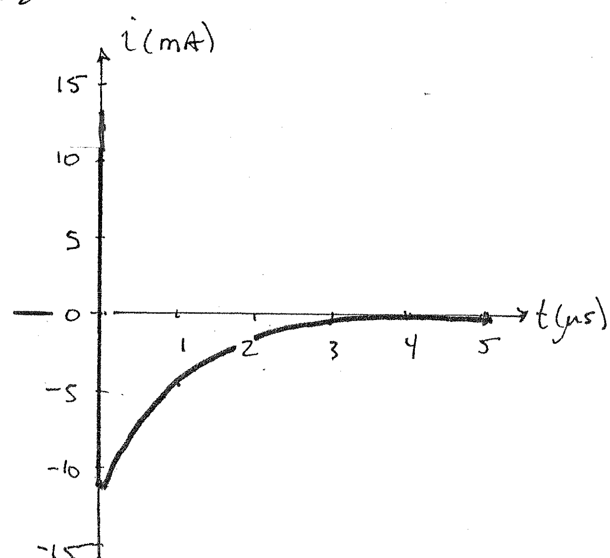
$\underline{t=0^+}$ $\quad i = -i_L = -12mA$

$\underline{t>0}$ $\quad L\frac{di_L}{dt} = R_2 i$ & $i = -i_L \Rightarrow \frac{di}{dt} + \left(\frac{R_2}{L}\right) i = 0$

$i = K_1 + K_2 e^{-t/\tau} \Rightarrow \tau = \frac{L}{R_2} = 1\mu s \qquad K_1 = 0$

$K_2 = i(0^+) - K_1 = -12mA$

$$\boxed{i(t) = -12e^{-10^6 t} \text{ mA}}$$



7.17 In the circuit in Fig. 7.17, find $i_o(t)$ for $t > 0$ using the differential equation approach.

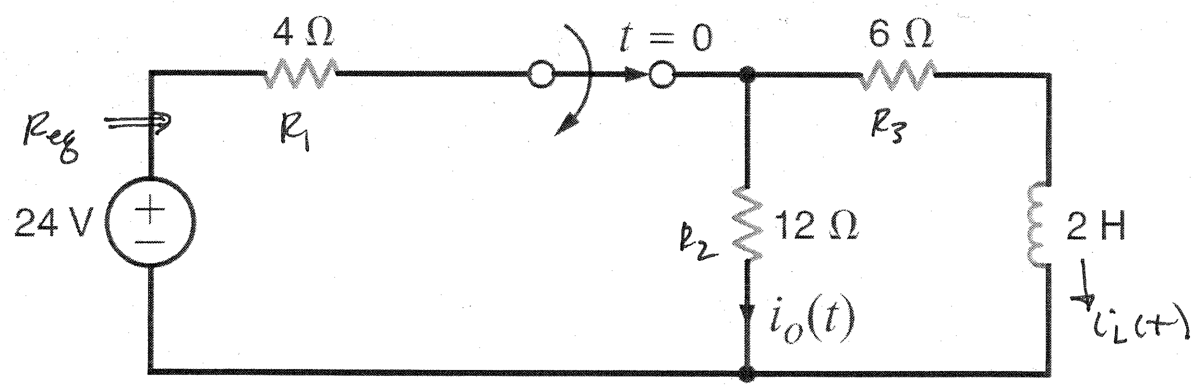


Figure P7.17

SOLUTION:

$\underline{t=0^-}$ $\quad i_o + i_L = \frac{24}{R_{EQ}} = 3 \qquad R_{EQ} = R_1 + [R_2 /\!/ R_3] = 8\Omega$

$\frac{i_o}{i_L} = \frac{R_3}{R_2} = \frac{1}{2} \quad \Rightarrow \quad i_L(0^-) = 2A = i_L(0^+)$

$\underline{t=0^+}$ $\quad i_L(0^+) = 2A = -i_o(0^+)$

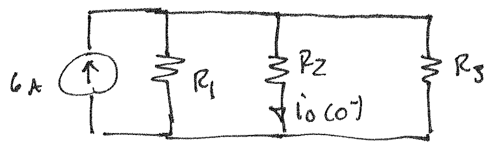
$\underline{t>0}$ $\quad L\frac{di_L}{dt} = i_o(R_2+R_3) \quad + \; i_o = -i_L \Rightarrow \frac{di_o}{dt} + i_o\frac{(R_2+R_3)}{L} = 0$

$i_o = K_1 + K_2 e^{-t/\tau} \Rightarrow \tau = \frac{L}{R_2+R_3} = \frac{1}{9}s \qquad K_1 = 0$

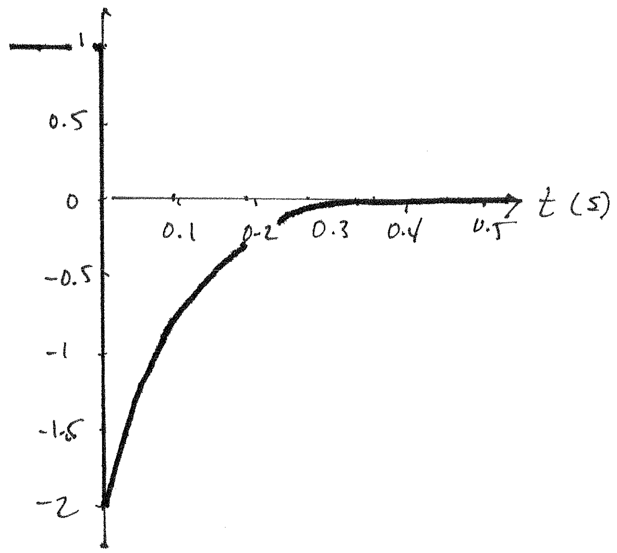
$K_2 = i_o(0^+) - K_1 = -2A$

$$\boxed{i_o(t) = -2e^{-9t} \; A}$$

$\underline{t = 0^-}$: $i_L(0^-) = 2A$



$$i_0(0^-) = \frac{6\left(\frac{1}{R_2}\right)}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} = 1A$$



7.18 Use the differential equation approach to find $i(t)$ for $t > 0$ in the circuit in Fig. P7.18 and plot the response including the time interval just prior to opening the switch.

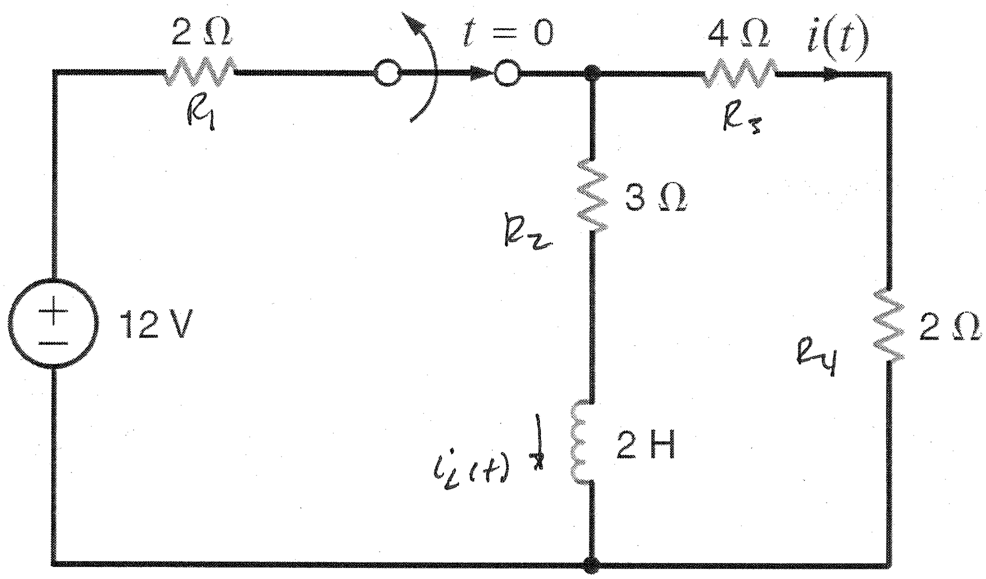
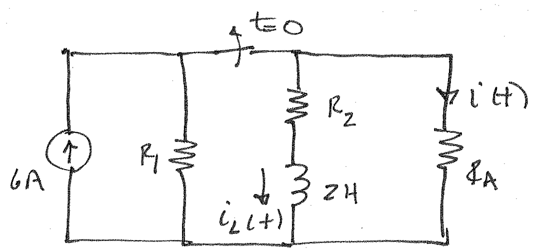


Figure P7.18

SOLUTION: By source transformation:



$R_A = R_3 + R_4 = 6\Omega$

$\underline{t=0^-}$: $i_L = \dfrac{6\,(1/R_2)}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_A}} = 2A$

$i(0^-) = \dfrac{6\,(1/R_A)}{\frac{1}{R_A}+\frac{1}{R_1}+\frac{1}{R_2}} = 1A$

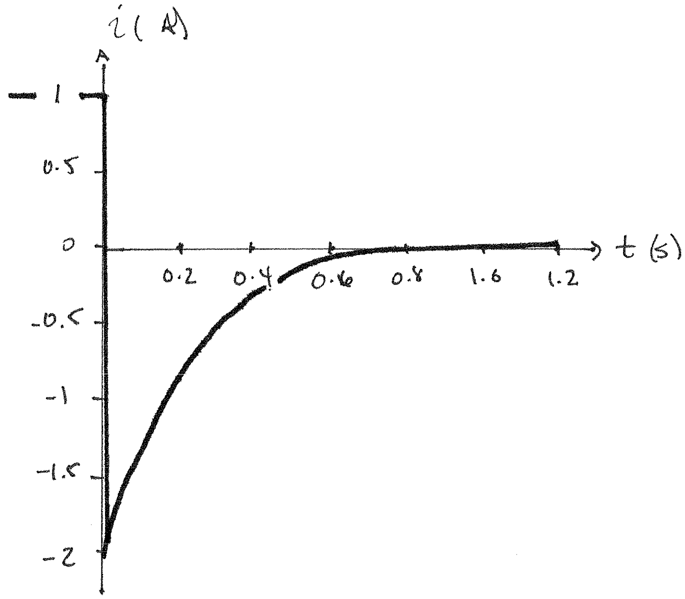
$\underline{t=0^+}$ $\quad i_L(0^+) = i_L(0^-) = 2A$

$i(0^+) = -i_L(0^+) = -2A$

$\underline{t>0}$: $L(di_L/dt) = i\,(R_A+R_2)$ & $i = -i_L \Rightarrow \dfrac{di}{dt} + \left(\dfrac{R_A+R_2}{L}\right) i = 0$

$i = K_1 + K_2\, e^{-t/\tau}$

yields $\tau = \dfrac{L}{R_A+R_2} = \dfrac{2}{9}\,s \qquad K_1 = 0 \qquad K_2 = -2 \qquad \boxed{i = -2e^{-4.5t}\ A}$


i (A)
1
0.5
0
-0.5
-1
-1.5
-2
0.2
0.4
0.6
0.8
1.6
1.2
t (s)

7.19 In the network in Fig. 7.19, find $i_o(t)$ for $t > 0$ using the differential equation approach.

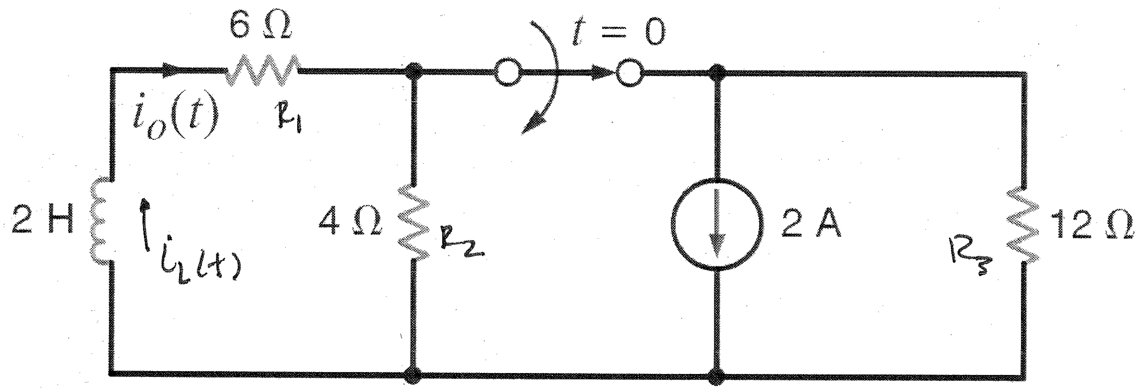


Figure P7.19

SOLUTION: $\underline{t=0^-}$: $i_L(0^-) = \dfrac{2\,(1/R_1)}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}} = \frac{2}{3}\text{ A}$

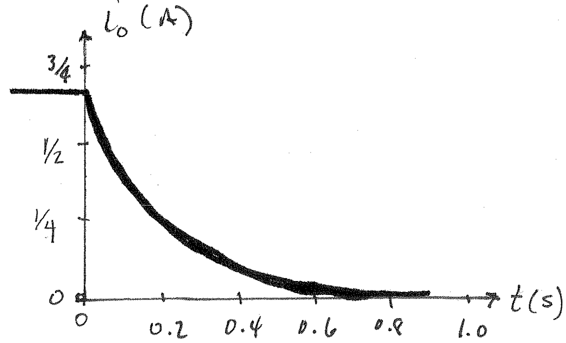
$\underline{t=0^+}$ $i_o = i_L = \frac{2}{3}\text{ A}$

$\underline{t>0}$ $L\dfrac{di_L}{dt} + i_o(R_1+R_2) = 0 \;\&\; i_L = i_o \Rightarrow \dfrac{di_o}{dt} + \dfrac{(R_1+R_2)}{L}i_o = 0$

$i_o = K_1 + K_2 e^{-t/\tau} \Rightarrow \tau = \dfrac{L}{R_1+R_2} = \frac{1}{5}\text{ s} \quad K_1 = 0$

$K_2 = i_o(0^+) - K_1 = \frac{2}{3}\text{ A}$

$$\boxed{i_o(t) = 0.67e^{-5t}\text{ A}}$$



7.20 Use the differential equation approach to find $i(t)$ for $t > 0$ in the circuit in Fig. P7.20 and plot the response including the time interval just prior to switch movement. **PSV**

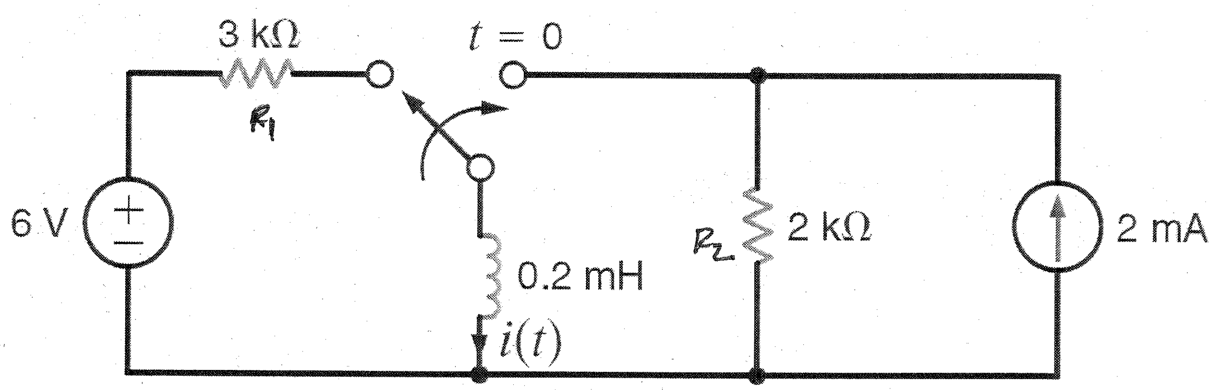
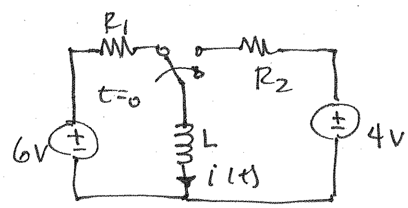


Figure P7.20

SOLUTION:



$\underline{t=0^-}$: $i(0^-) = \frac{6}{R_1} = 2mA = i(0^+)$

$\underline{t=0^+}$ $i(0^+) = 2mA$

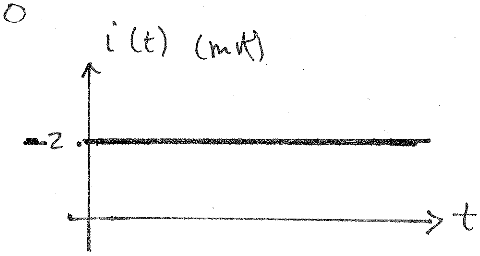
$\underline{t>0}$ $4 = R_2 i + L\, di/dt$

$$di/dt + \frac{R_2}{L} i - \frac{4}{L} = 0 \qquad i = K_1 + K_2 e^{-t/\tau}$$

$$\tau = L/R_2 = 0.1\mu s \qquad K_1 = 4/R_2 = 2mA$$

$$K_2 = i(0^+) - K_1 = 0$$

$$\boxed{i(t) = 2mA}$$



7.21 Use the differential equation approach to find $i_L(t)$ for $t > 0$ in the circuit in Fig. P7.21 and plot the response including the time interval just prior to opening the switch.

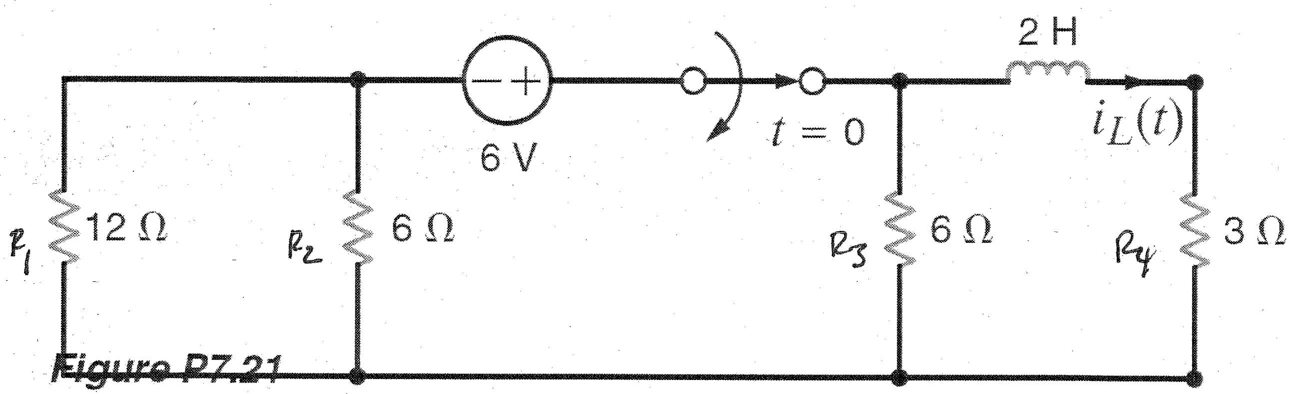
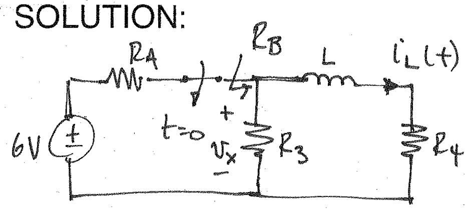


Figure P7.21

SOLUTION:



$R_A = R_1 // R_2 = 4\Omega$

$R_B = R_3 // R_4 = 2\Omega$

$\underline{t = 0^-}$: $i_L(0^-) = v_x(0^-)/R_4$ $\quad v_x = \dfrac{6 R_B}{R_B + R_A} = 2V$ $\quad i_L(0^-) = \dfrac{2}{3} A$

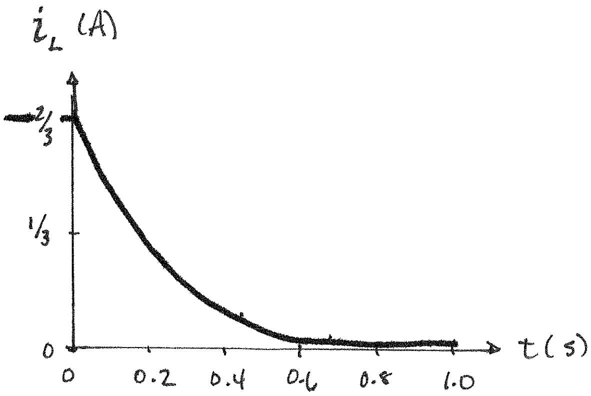
$\underline{t = 0^+}$ $\quad i_L(0^+) = i_L(0^-) = 2/3 A$

$\underline{t > 0}$: $\quad L \dfrac{di_L}{dt} + i_L (R_3 + R_4) = 0 \Rightarrow \dfrac{di_L}{dt} + \left(\dfrac{R_3 + R_4}{L}\right) i = 0$

$i = K_1 + K_2 e^{-t/\tau} \Rightarrow \tau = \dfrac{L}{R_3 + R_4} = \dfrac{2}{9} s \qquad K_1 = 0$

$K_2 = i_L(0^+) - K_1 = \dfrac{2}{3} A$

$$\boxed{i_L(t) = 0.67 e^{-4.5t} A}$$



iL (A)
2/3
1/3
0
t (s)
0
0.2
0.4
0.6
0.8
1.0

7.22 Use the differential equation approach to find $i_o(t)$ for $t > 0$ in the circuit in Fig. P7.22 and plot the response including the time interval just prior to opening the switch.

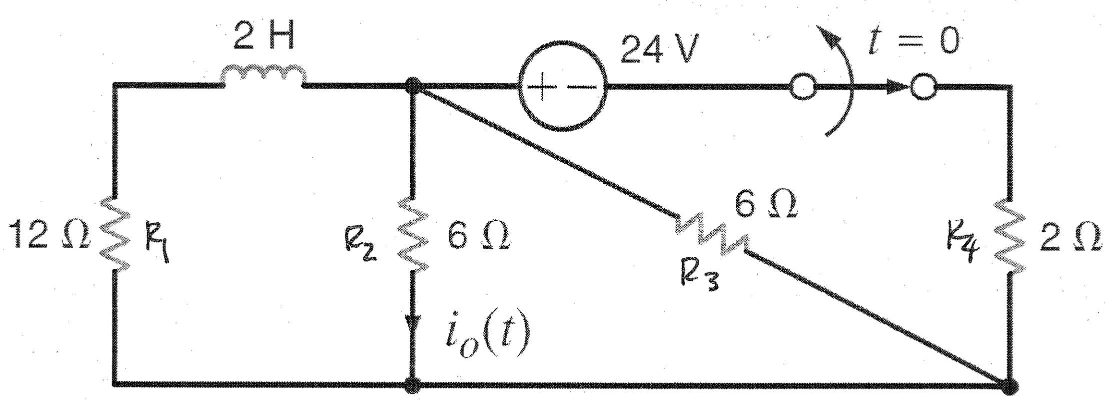
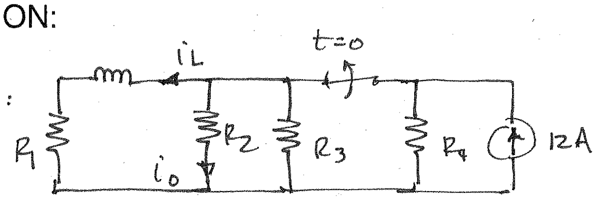


Figure P7.22

SOLUTION:

$\underline{t=0^-}$:



$$i_L(0^-) = \frac{12\,(1/R_1)}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}} = \frac{12}{11}\,A$$

$$i_o(0^-) = \frac{12A\,(1/R_2)}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}} = \frac{24}{11}\,A$$

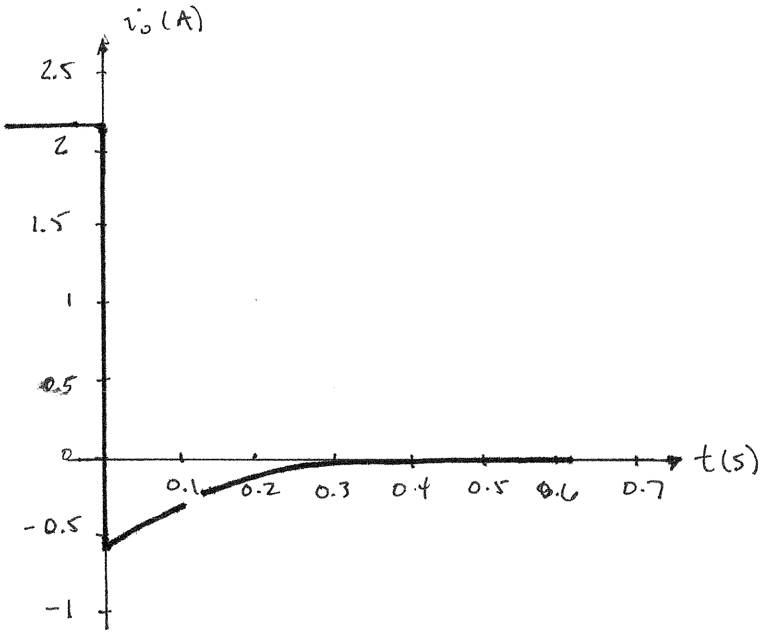
$\underline{t=0^+}$ $\quad i_L(0^+) = \frac{12}{11}A \qquad i_o(0^+) = \frac{-i_L(0^+)R_3}{R_2+R_3} = -\frac{6}{11}A$

$\underline{t>0}$: $\quad L\frac{di_L}{dt} + i_L(R_1+R_B) = 0 \qquad R_B = R_2 // R_3 \qquad i_o = \frac{-i_L R_3}{R_2+R_3}$

$$\frac{di_o}{dt} + \left(\frac{R_1+R_B}{L}\right) i_o = 0 \qquad \& \quad i_o(t) = K_1 + K_2 e^{-t/\tau}$$

$$\tau = \frac{L}{R_1+R_B} = \frac{2}{15}\,s \qquad K_1 = 0 \qquad K_2 = i_o(0^+) - K_1 = -\frac{6}{11}\,A$$

$$\boxed{i_o(t) = -0.545e^{-7.5t}\ A}$$



i_o (A)
2.5
2
1.5
1
0.5
0
-0.5
-1
0.1
0.2
0.3
0.4
0.5
0.6
0.7
t(s)

7.23 Using the differential equation approach, find $i_o(t)$ for $t > 0$ in the circuit in Fig. P7.23 and plot the response including the time interval just prior to opening the switch.

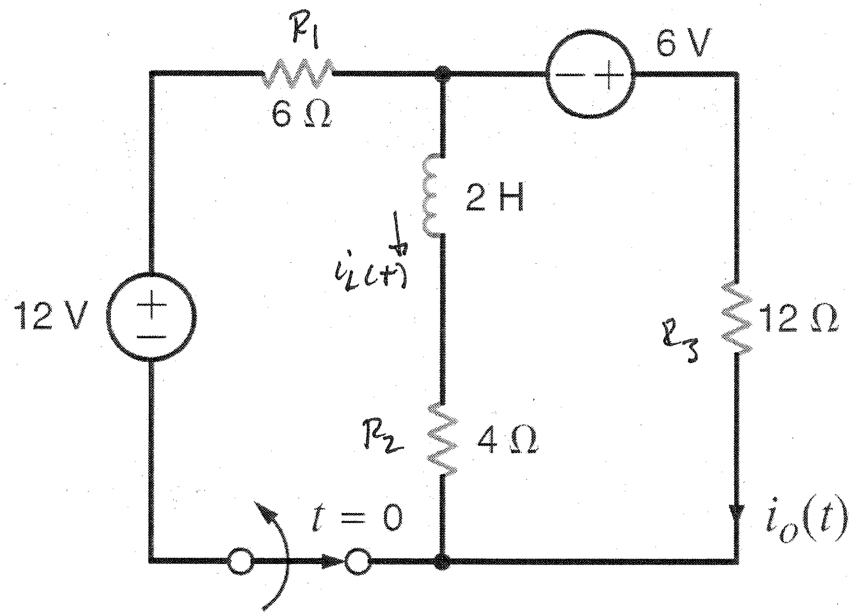
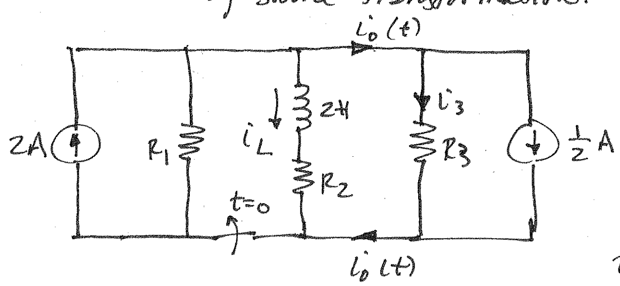


Figure P7.23

SOLUTION: By source transformation:



$\underline{t=0^-}$ $\quad i_L(0^-) = \dfrac{(2-0.5)\ 1/R_2}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}} = \dfrac{3}{4}$ A

$i_o(0^-) = i_3(0^-) + 0.5$

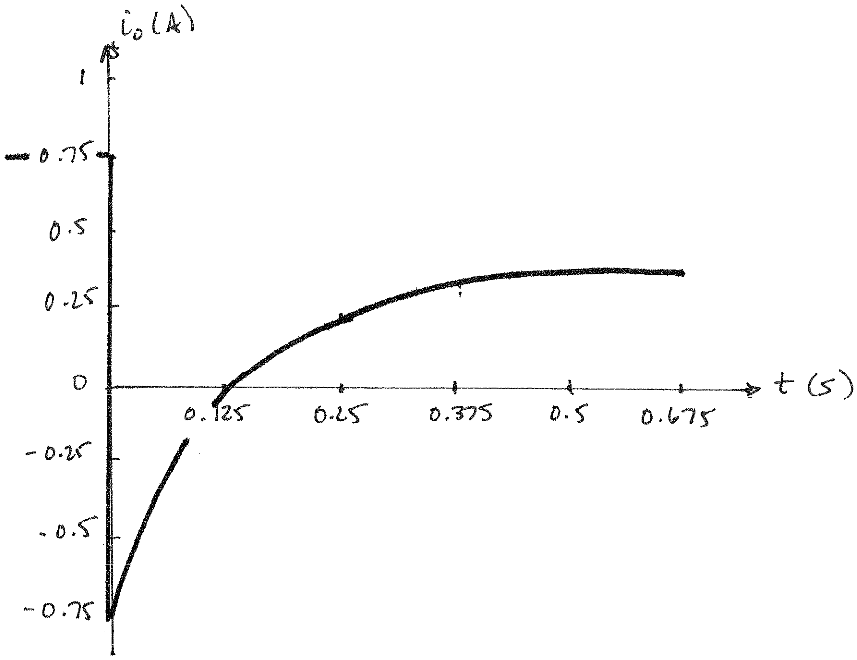
$i_3(0^-) = \dfrac{1.5\ (1/R_3)}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}} = \dfrac{1}{4}$ A $\qquad i_o(0^-) = \dfrac{3}{4}$ A

$\underline{t=0^+}$: $\quad i_o(0^+) = -i_L(0^+) = -\dfrac{3}{4}$ A

$\underline{t>0}$: $\quad 6 = R_3 i_o + R_2 i_o + L\dfrac{di_o}{dt} \Rightarrow \dfrac{di_o}{dt} + \dfrac{i_o(R_2+R_3)}{L} - \dfrac{6}{L} = 0$

$i_o = K_1 + K_2 e^{-t/\tau} \Rightarrow \tau = \dfrac{L}{R_2+R_3} = \dfrac{1}{8}$ s $\qquad K_1 = \dfrac{6}{R_2+R_3} = \dfrac{3}{8}$ A

$K_2 = i_o(0^+) - K_1 = -9/8$ A $\qquad \boxed{i_o(t) = 0.375 - 1.125e^{-8t}\ \text{A}}$



i_o (A)
1
0.75
0.5
0.25
0
-0.25
-0.5
-0.75
0.125
0.25
0.375
0.5
0.675
t (s)

7.24 Use the differential equation approach to find $v_o(t)$ for $t > 0$ in the circuit in Fig. P7.24 and plot the response including the time interval just prior to opening the switch.

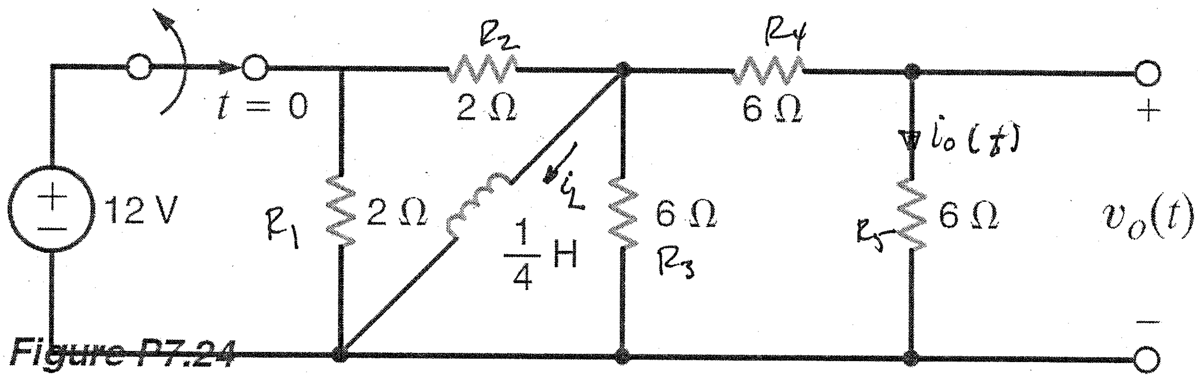


Figure P7.24

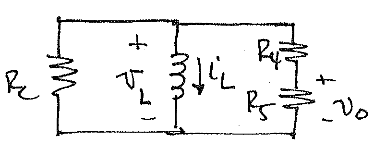
SOLUTION:

$t=0^-$: $i_L(0^-) = {}^{12}/_{R_2} = 6\text{A}$ $\qquad v_o(0^-) = 0\text{V}$

$t=0^+$: $i_L(0^+) = 6\text{ A}$ $\qquad v_o(0^+) = i_o R_5$ $\qquad i_o = \dfrac{-i_L R_C}{R_A + R_C}$ $\qquad \left\{ \begin{array}{l} R_A = R_4 + R_5 = 12\Omega \\ R_C = R_3 // [R_1 + R_2] \\ R_C = 2.4\Omega \end{array} \right.$

$v_o(0^+) = -6\text{ V}$

$t>0$



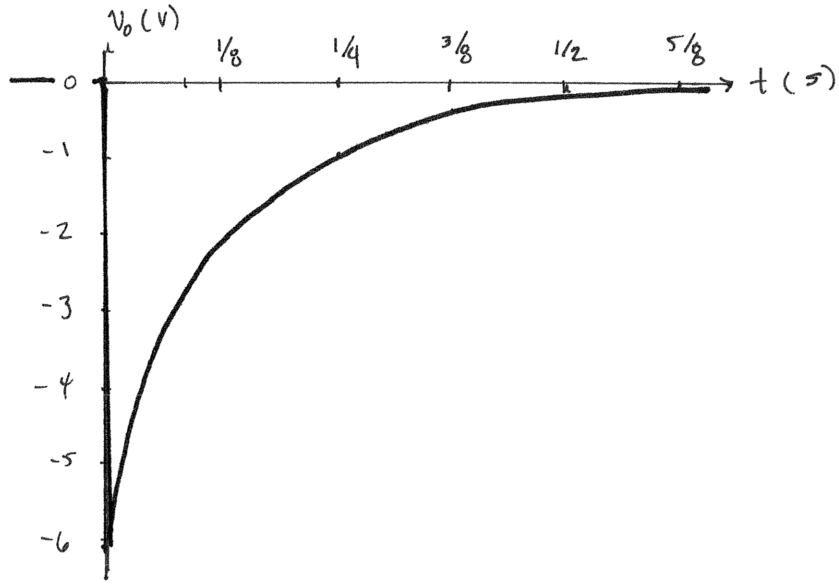
$$i_L + \frac{v_L}{R_C} + \frac{v_L}{R_A} = 0 \quad \& \quad v_L = L\frac{di_L}{dt}$$

$$\frac{di_L}{dt} + \frac{R_A R_C}{(R_A + R_C)L} i_L = 0$$

But $v_o = -\left(\dfrac{i_L R_C}{R_C + R_A}\right) R_5 \rightarrow \dfrac{dv_o}{dt} + \dfrac{R_A R_C}{(R_A + R_C)L} v_o = 0$

$v_o = K_1 + K_2 e^{-t/\tau} \rightarrow \tau = \dfrac{L(R_A + R_C)}{R_A R_C} = \dfrac{1}{8}\text{ s}$ $\qquad K_1 = 0$

$K_2 = v_o(0^+) - K_1 = -6\text{V}$ $\qquad \boxed{v_o = -6e^{-8t}\text{ V}}$



v_o (V)
1/8
1/4
3/8
1/2
5/8
t (s)
0
-1
-2
-3
-4
-5
-6

7.25 Use the differential equation approach to find $i(t)$ for $t > 0$ in the circuit in Fig. P7.25 and plot the response including the time interval just prior to opening the switch.

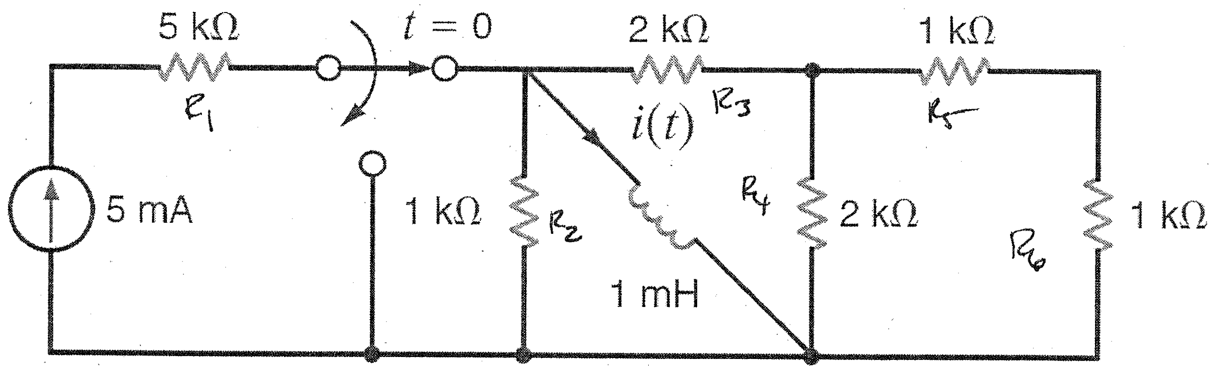


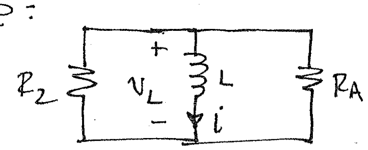
Figure P7.25

SOLUTION:

$t=0^-$: $i(0^-) = 5\,mA$

$t=0^+$: $i(0^+) = i(0^-) = 5\,mA$

$t>0$:



$$i + \frac{v_L}{R_2} + \frac{v_L}{R_A} = 0 \quad \& \quad v_L = L\frac{di}{dt}$$

$$\frac{di}{dt} + \frac{R_A R_2}{(R_A+R_2)L}\, i = 0$$

$$R_A = R_3 + \{R_4 // [R_5 + R_6]\}$$

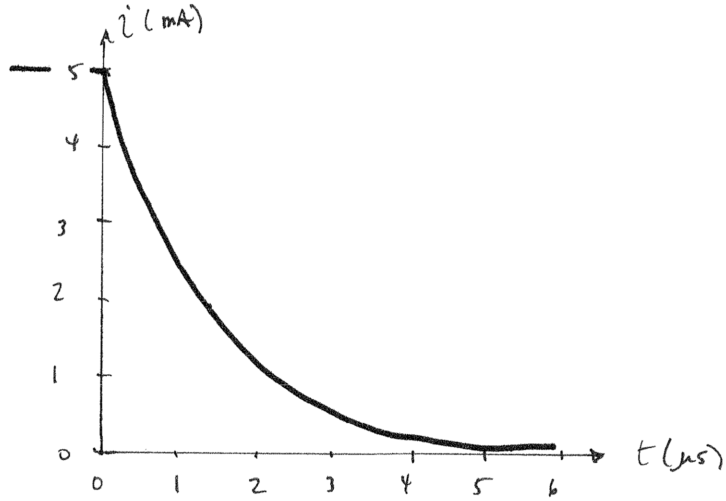
$$R_A = 3\,k\Omega$$

$$i = K_1 + K_2 e^{-t/\tau}$$

$$\tau = \frac{L(R_A+R_2)}{R_A R_2} = \frac{4}{3}\,\mu s$$

$K_1 = 0 \qquad K_2 = i(0^+) - K_1 = 5\,mA$

$$\boxed{i(t) = 5e^{-7.5\times10^5 t}\ mA}$$


i (mA)
5
4
3
2
1
0
0
1
2
3
4
5
6
t (μs)

7.26 Find $v_C(t)$ for $t > 0$ in the network in Fig. P7.26 using the step-by-step method. CS

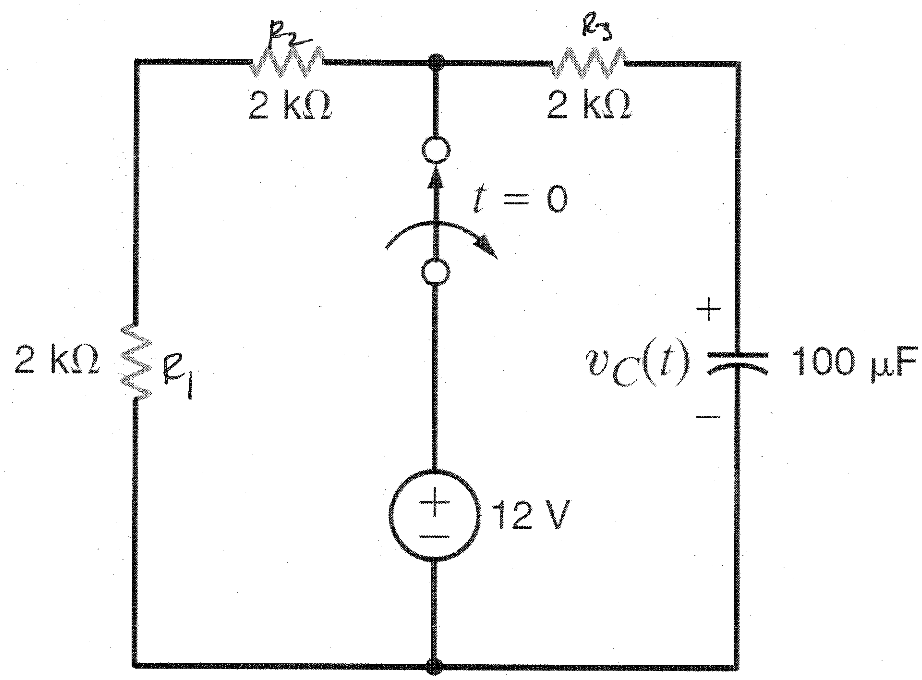
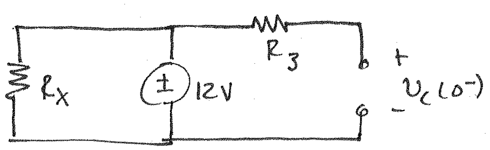


Figure P7.26

SOLUTION:

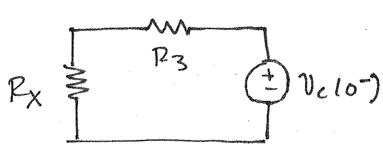
$\underline{t = 0^-}$



$R_X = R_1 + R_2 = 4k\Omega$

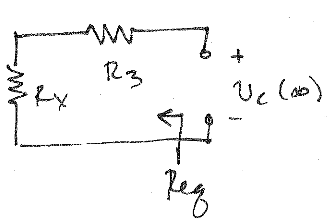
$v_c(0^-) = 12V$

$\underline{t = 0^+}$



$v_c(0^+) = v_c(0^-) = 12 = K_1 + K_2$

$\underline{t \to \infty}$



$v_c(\infty) = 0 = K_1$

$\tau = R_{eq} C = (R_X + R_3) C = 0.6 s$

$v_c(t) = K_1 + K_2 e^{-t/\tau}$

$$\boxed{v_c(t) = 12e^{-1.67t} \text{ V}}$$

7.27 Use the step-by-step method to find $i_o(t)$ for $t > 0$ in the circuit in Fig. P7.27.

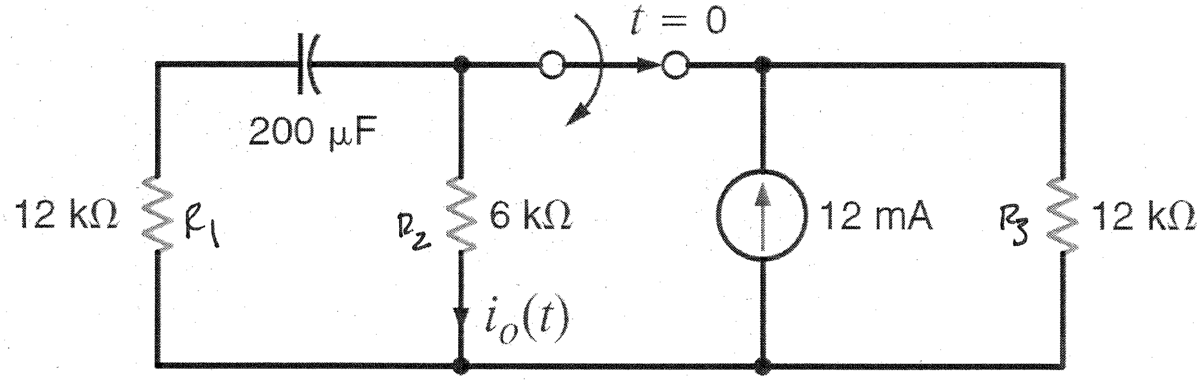
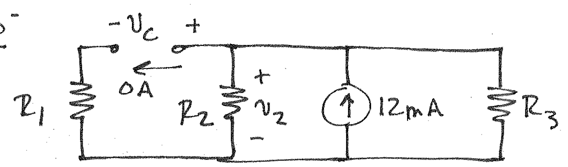


Figure P7.27

SOLUTION: $i_o(t) = K_1 + K_2 e^{-t/\tau}$

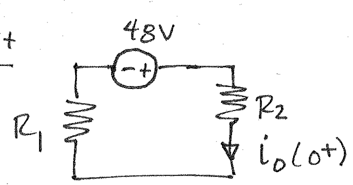
$\underline{t=0^-}$



$v_2 = v_c = 12\times10^{-3}\left[\frac{R_3 R_2}{R_3+R_2}\right]$

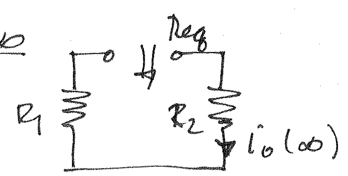
$v_c(0^-) = 48V$

$\underline{t=0^+}$



$i_o(0^+) = \frac{48}{R_1+R_2} = 2.67\ mA = K_1+K_2$

$\underline{t=\infty}$



$i_o(\infty) = 0 = K_1$

$\tau = C R_{eq} \quad R_{eq} = R_1+R_2 = 18k\Omega$

$\tau = 3.6\ s$

$$\boxed{i_o(t) = 2.67\, e^{-t/3.6}\ mA}$$

7.28 Use the step-by-step technique to find $i_o(t)$ for $t > 0$ in the network in Fig. P7.28. CS

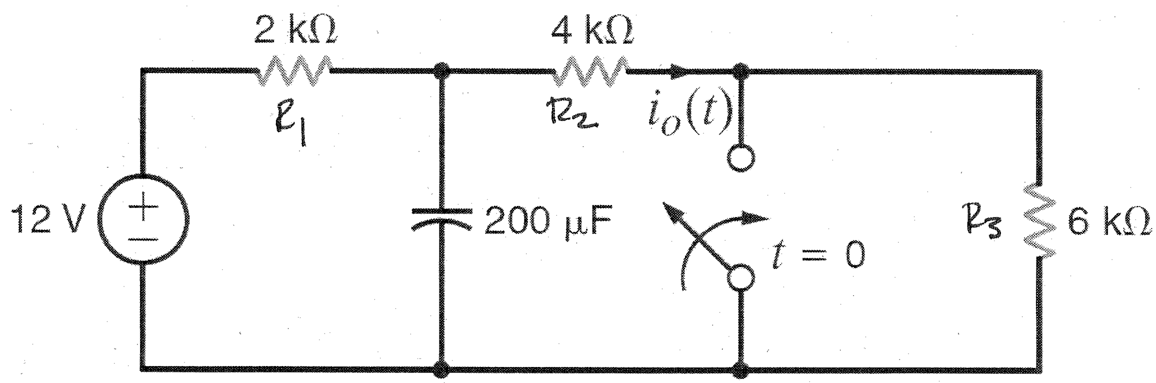
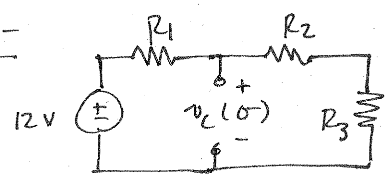


Figure P7.28

SOLUTION: $i_o(t) = K_1 + K_2 e^{-t/\tau}$

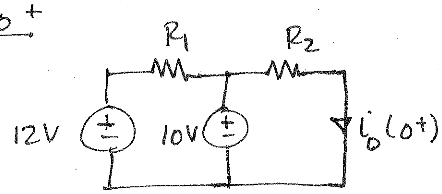
$t = 0^-$



By voltage division: $v_c(0^-) = \dfrac{12(R_2+R_3)}{R_1+R_2+R_3}$

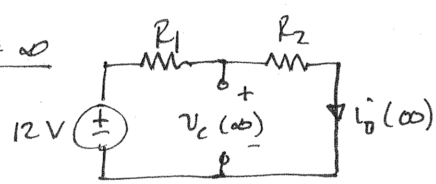
$v_c(0^-) = 10\text{V}$

$t = 0^+$



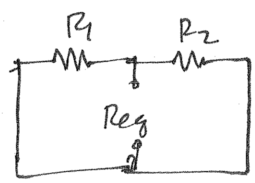
$i_o(0^+) = 10/R_2 = 2.5\text{mA} = K_1 + K_2$

$t = \infty$



$i_o(\infty) = \dfrac{12}{R_1+R_2} = 2\text{mA} = K_1$

$\tau = ?$



$\tau = CR_{eq}$ $\quad R_{eq} = R_1 // R_2 = \frac{4}{3}\text{k}\Omega$

$\tau = 0.267\text{ s}$

$$\boxed{i_o(t) = 2 + 0.5e^{-3.75t} \text{ mA}}$$

7.29 Use the step-by-step method to find $v_o(t)$ for $t > 0$ in the network in Fig. P7.29.

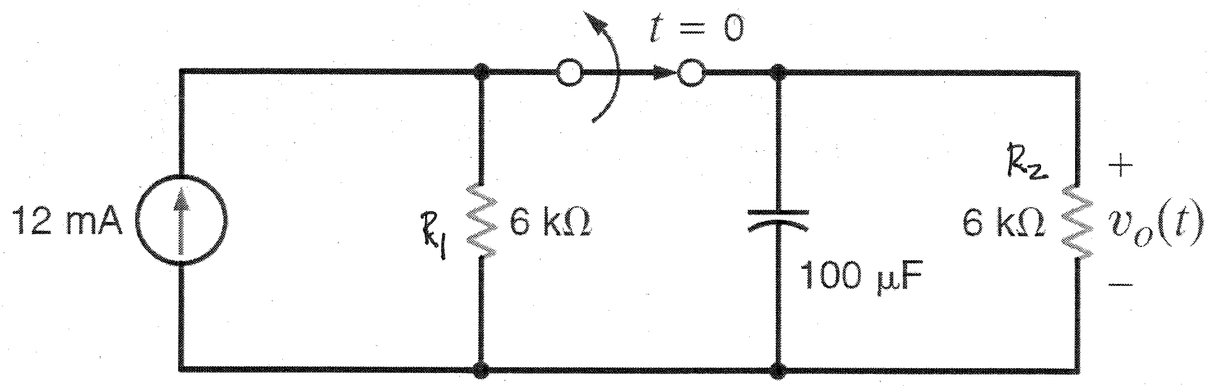
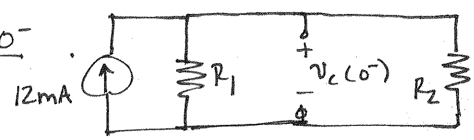


Figure P7.29

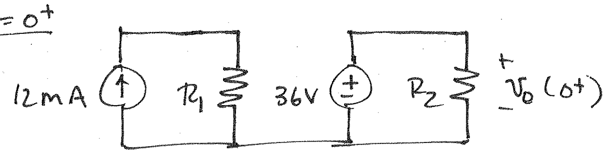
SOLUTION: $v_o(t) = K_1 + K_2 e^{-t/\tau}$

$\underline{t = 0^-}$



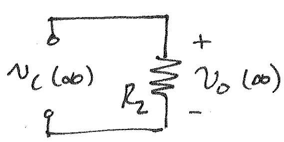
$v_c(0^-) = 12\times10^{-3}\dfrac{(R_1R_2)}{R_1+R_2} = 36V$

$\underline{t = 0^+}$



$v_o(0^+) = 36 = K_1 + K_2$

$\underline{t = \infty}$



$v_o(\infty) = 0 = K_1$

$\underline{\tau = ?}$ $\quad \tau = R_{eq}C \qquad R_{eq} = R_2 = 6k\Omega \qquad \tau = 0.6s$

$$\boxed{v_o(t) = 36e^{-t/0.6} \text{ V}}$$

7.30 Use the step-by-step method to find $i_o(t)$ for $t > 0$ in the circuit in Fig. P7.30.

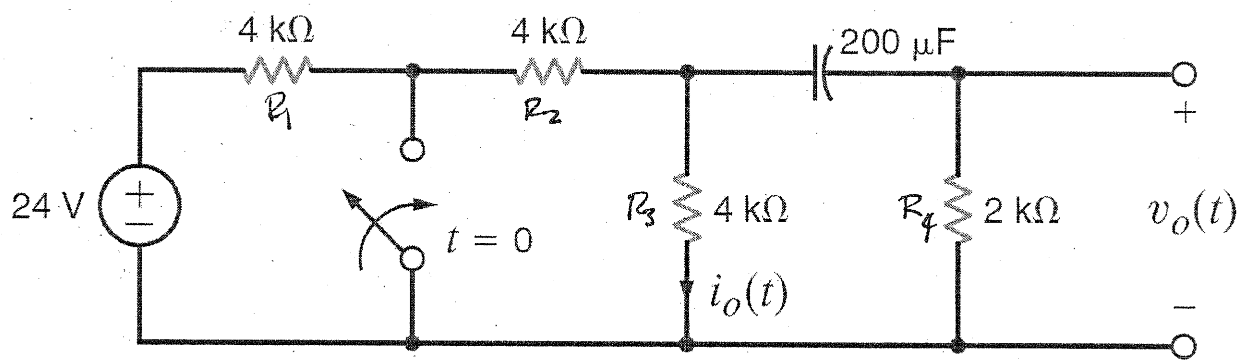
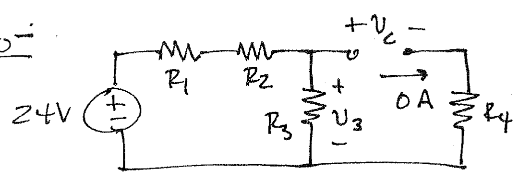


Figure P7.30

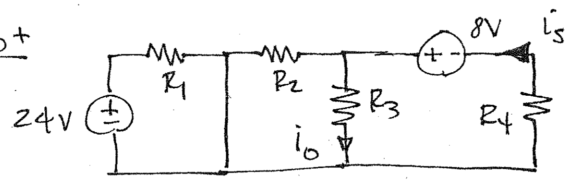
SOLUTION: $i_o(t) = K_1 + K_2 e^{-t/\tau}$

$t = 0^-$



$$v_c = v_3 = \frac{24 R_3}{R_1 + R_2 + R_3} = 8\text{V}$$

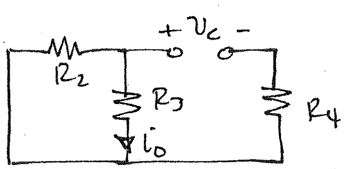
$t = 0^+$



$$i_s = \frac{8}{R_4 + R_X} \qquad R_X = \frac{R_2 R_3}{R_2 + R_3} = 2k\Omega$$

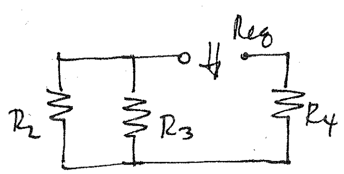
$$i_o = \frac{i_s R_2}{R_2 + R_3} = 1mA = K_1 + K_2$$

$t = \infty$



$$i_o(\infty) = 0 = K_1$$

$\tau = ?$



$$\tau = R_{eq} C \qquad R_{eq} = \frac{R_2 R_3}{R_2 + R_3} + R_4 = 4k\Omega$$

$$\tau = 0.8s$$

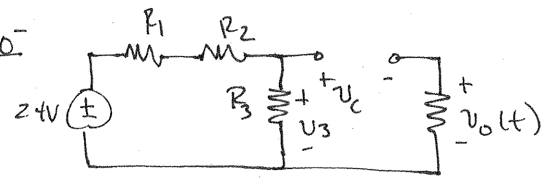
$$\boxed{i_o(t) = e^{-1.25t} \text{ mA}}$$

7.31 Find $v_o(t)$ for $t > 0$ in the network in Fig. P7.30 using the step-by-step technique.

SOLUTION:

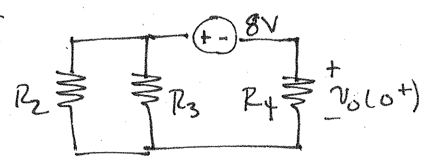
$v_o(t) = K_1 + K_2 e^{-t/\tau}$ $R_1 = R_2 = R_3 = 4k\Omega$ $R_4 = 2k\Omega$

$\underline{t = 0^-}$



$v_c = v_3 = \dfrac{24 R_3}{R_1 + R_2 + R_3} = 8V$

$\underline{t = 0^+}$



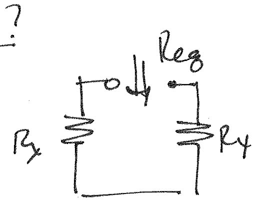
$R_x = R_2 // R_3 = 2k\Omega$

$v_o(0^+) = -\dfrac{8 R_4}{R_4 + R_x} = -4V = K_1 + K_2$

$\underline{t = \infty}$

$v_o(\infty) = 0 = K_1$

$\underline{\tau = ?}$



$\tau = R_{eq} C$ $R_{eq} = R_x + R_4 = 4k\Omega$

$\tau = 0.8s$

$$\boxed{v_o(t) = -4e^{-1.25t} \text{ V}}$$

7.32 Use the step-by-step technique to find $i_o(t)$ for $t > 0$ in the network in Fig. P7.32.

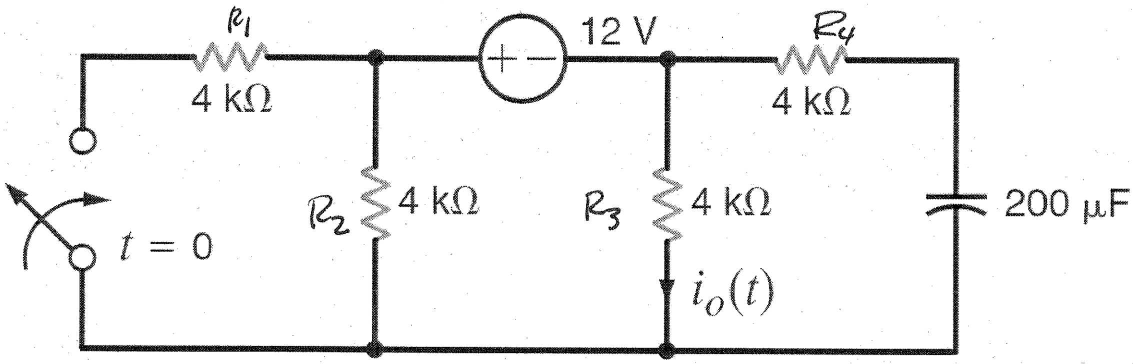
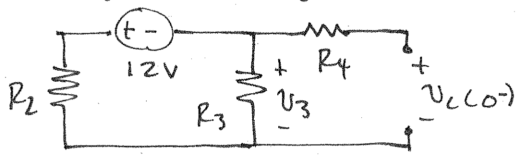


Figure P7.32

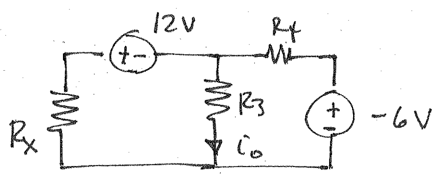
SOLUTION: $i_o(t) = K_1 + K_2 e^{-t/\tau}$

$t = 0^-$

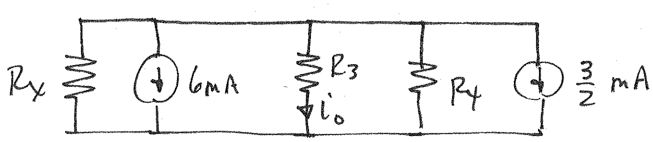


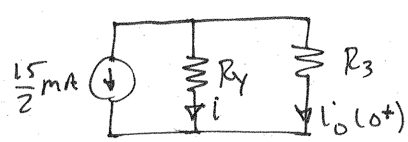
$v_c(0^-) = v_3 = -\frac{12 R_3}{R_2 + R_3} = -6V$

$t = 0^+$



$R_x = R_1 // R_2 = 2k\Omega$

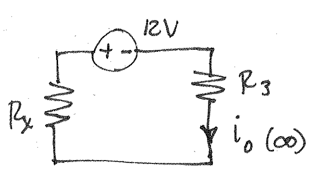




$R_y = R_x // R_4 = 4/3\ k\Omega$

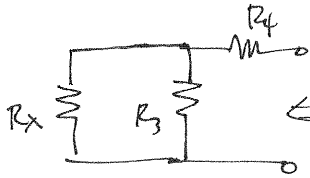
$i_o(0^+) = \frac{-15}{2000} \frac{R_y}{R_y + R_3} = -1.875\ mA = K_1 + K_2$

$t = \infty$



$i_o(\infty) = \frac{-12}{R_x + R_3} = -2mA = K_1$

$\tau = ?$



$\Leftarrow R_{eq} = R_4 + \frac{R_x R_3}{R_x + R_3} = 5.33\,k\Omega$

$\tau = R_{eq} C = 1.07\,s$

$$\boxed{i_o(t) = -2 + 0.125\, e^{-0.9375t} \quad mA}$$

7.33 Find $v_o(t)$ for $t > 0$ in the network in Fig. P7.33 using the step-by-step method. CS

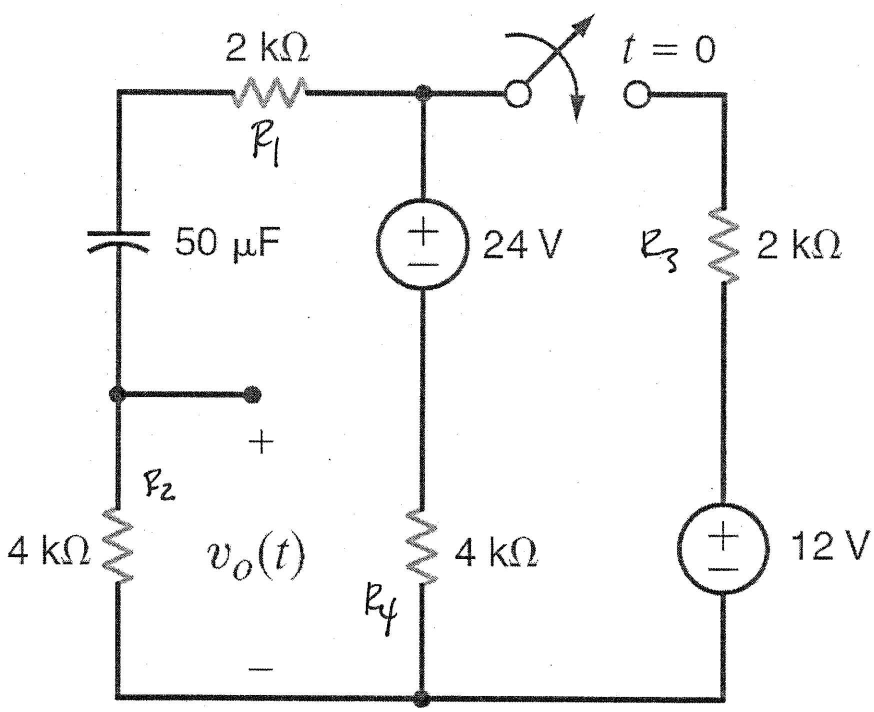
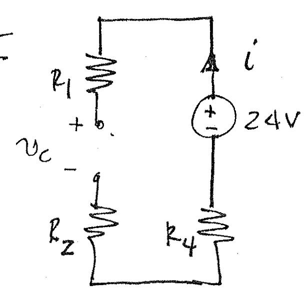


Figure P7.33

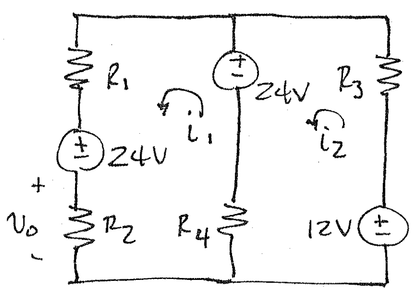
SOLUTION: $v_o(t) = K_1 + K_2 e^{-t/\tau}$

$t=0^-$



$i = 0$ so $v_c(0^-) = 24V$

$\underline{t=0^+}$



$$24 = i_1(R_1+R_2+R_4) - i_2R_4 + 24$$

or, $\quad i_1(R_1+R_2+R_4) = i_2R_4$

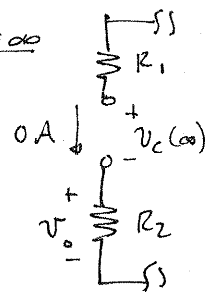
$$12 = i_2(R_3+R_4) + 24 - i_1R_4$$

or $\quad i_1R_4 - i_2(R_3+R_4) = 12$

$i_1 = -\frac{12}{11}\ mA \qquad v_o(0^+) = i_1R_2$

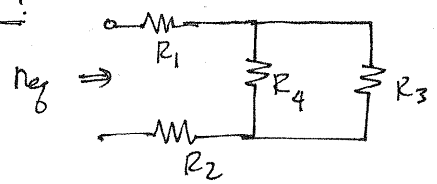
$v_o(0+) = \frac{48}{11}\ V = k_1 + k_2$

$\underline{t=\infty}$



$v_o(\infty) = 0 = k_1$

$\underline{\tau = ?}$



$R_{eq} = R_1+R_2+\frac{R_3R_4}{R_3+R_4} = 7.33k\Omega$

$\tau = 367\ ms$

$$\boxed{v_o(t) = -4.36\,e^{-2.73t}\ V}$$

7.34 Find $v_o(t)$ for $t > 0$ in the circuit in Fig. P7.34 using the step-by-step method. **PSV**

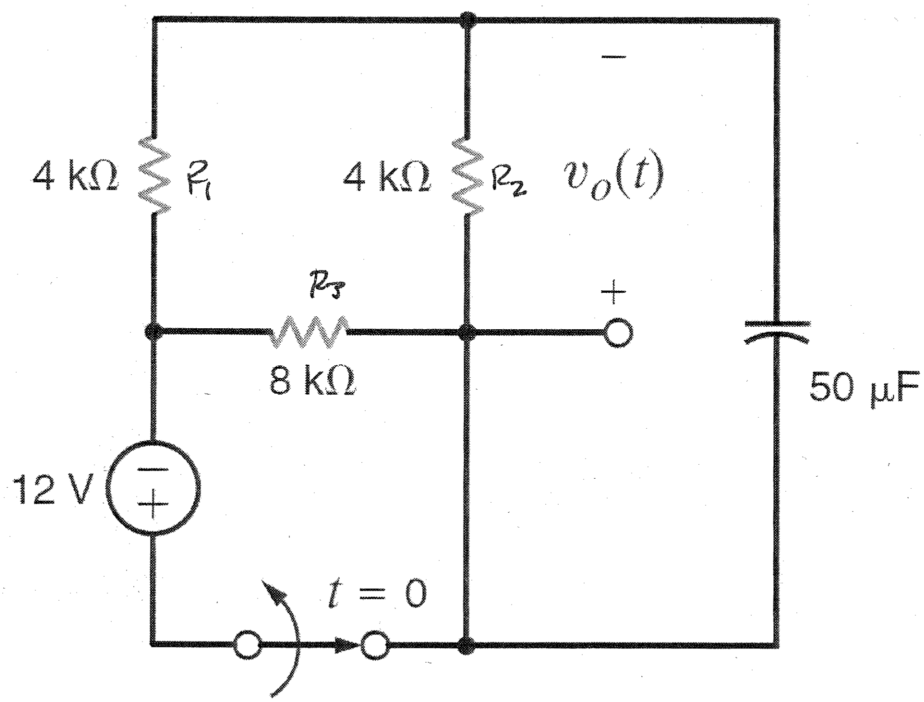
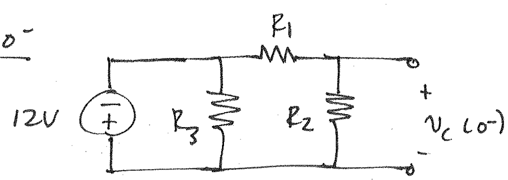


Figure P7.34

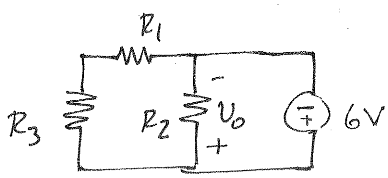
SOLUTION: $v_o(t) = K_1 + K_2 e^{-t/\tau}$

$t = 0^-$



$v_c(0^-) = -12\left(\frac{R_2}{R_1+R_2}\right) = -6\,V$

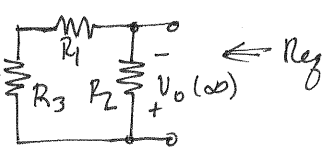
$t = 0^+$



$v_o(0+) = 6\,V = K_1 + K_2$

$t = \infty$

$v_o(\infty) = 0 = K_1$ $\quad R_{eq} = R_2\ (R_1+R_3)/(R_1+R_2+R_3) = 3\,k\Omega$



$\tau = R_{eq}C = 150\,ms$

$$\boxed{v_o(t) = 6e^{-6.67t}\ V}$$

7.35 Use the step-by-step method to find $i_o(t)$ for $t > 0$ in the network in Fig. P7.35. CS

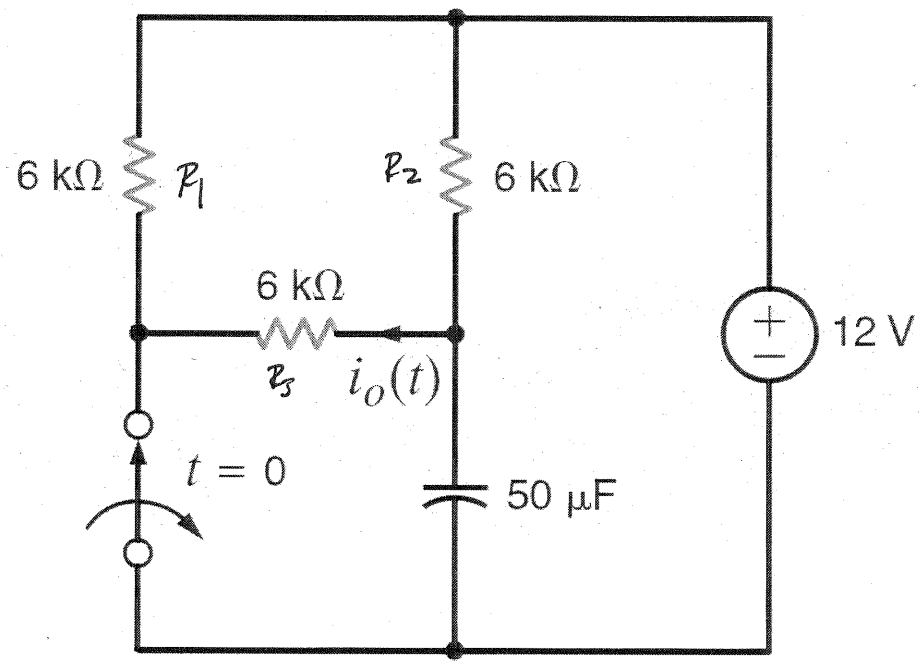
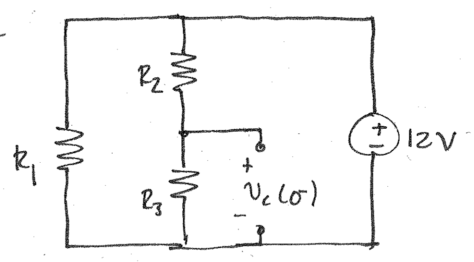


Figure P7.35

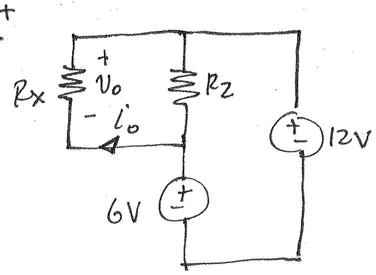
SOLUTION: $i_o(t) = K_1 + K_2 e^{-t/\tau}$

$t = 0^-$



$$v_c(0^-) = \frac{12 R_3}{R_3 + R_2} = 6V$$

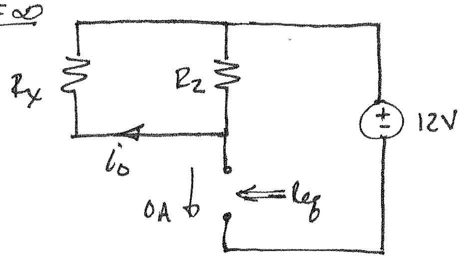
$t = 0^+$



$R_x = R_1 + R_3 = 12\,k\Omega$ $\quad v_o = 12 - 6 = 6V$

$$i_o = -\frac{v_o}{R_x} = -0.5\,mA = K_1 + K_2$$

$\underline{t=\infty}$



$i_o = 0 = K_1$

$\underline{\tau = ?}$ $R_{eq} = \dfrac{R_x R_2}{R_x + R_2} = 4k\Omega$ $\tau = R_{eq} C = 0.2s$

$$\boxed{i_o(t) = -0.5 e^{-5t} \text{ mA}}$$

7.36 Use the step-by-step technique to find $v_o(t)$ for $t > 0$ in the circuit in Fig. P7.36.

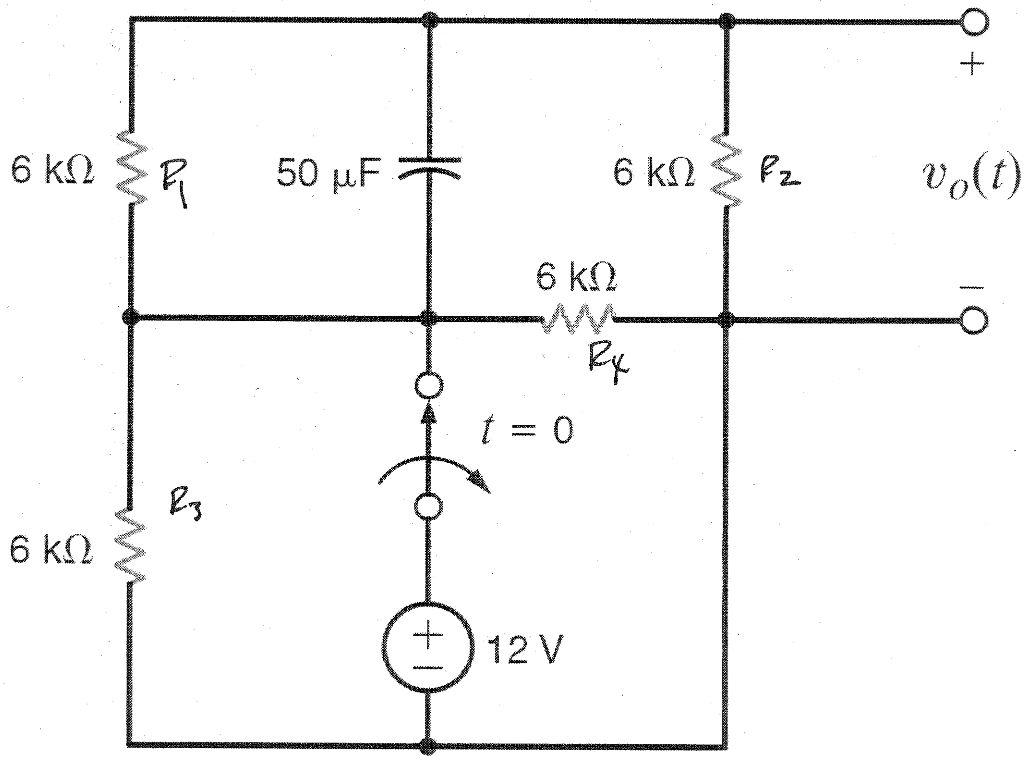
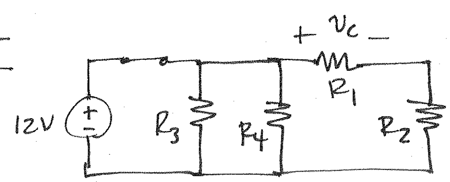


Figure P7.36

SOLUTION: $v_o(t) = K_1 + K_2 e^{-t/\tau}$

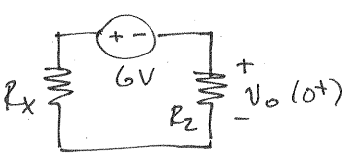
$t = 0^-$



$v_c(0^-) = \dfrac{12 R_1}{R_1 + R_2} = 6V$

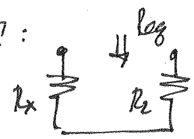
$R_x = R_3 // R_4 = 3k\Omega$

$t = 0^+$



$t = \infty$: $v_o(\infty) = 0 = K_1$

$\tau = ?$:



$R_{eq} = R_x + R_2 = 9k\Omega$

$\tau = R_{eq} C = 0.45s$

$v_o(0^+) = \dfrac{-6 R_2}{R_x + R_2} = -4V = K_1 + K_2$

$$\boxed{v_o(t) = -4e^{-2.22t} \; V}$$

7.37 Find $i_o(t)$ for $t > 0$ in the network in Fig. P7.37 using the step-by-step method. CS

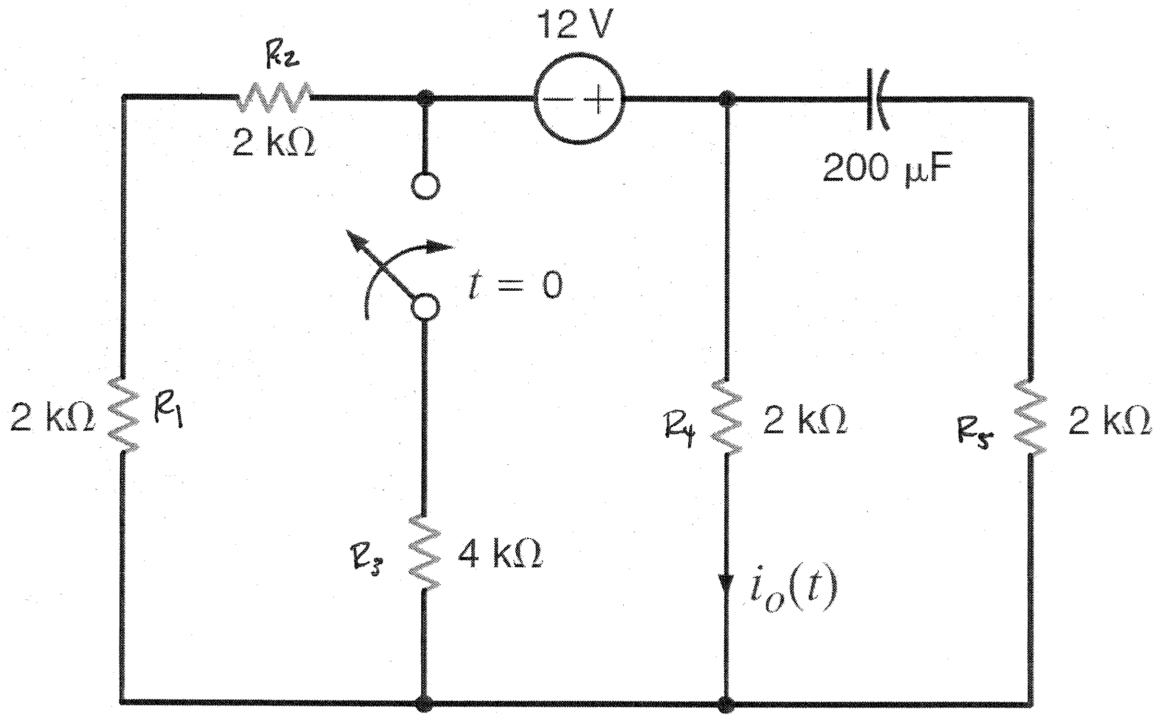
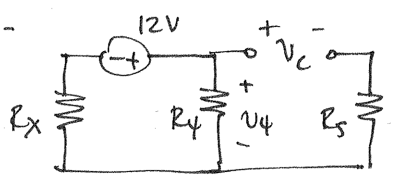


Figure P7.37

SOLUTION: $i_o(t) = K_1 + K_2 e^{-t/\tau}$

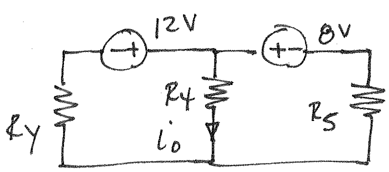
$t = 0^-$



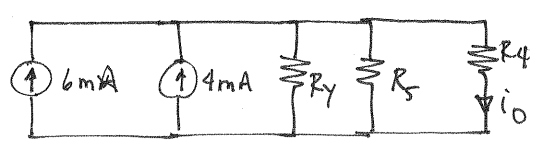
$v_C(0^-) = v_4 = \frac{12 R_4}{R_4 + R_X}$ $\quad R_X = R_1 + R_2 = 4k\Omega$

$v_C(0^-) = 8V$

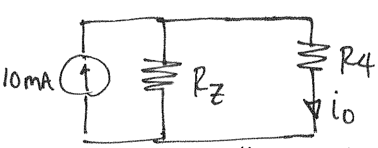
$t = 0^+$



⇒



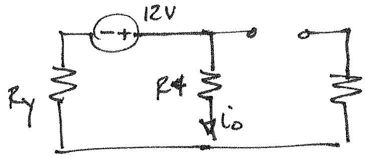
$R_Y = R_X // R_3 = 2k\Omega$



$R_Z = R_Y // R_5 = 1k\Omega$

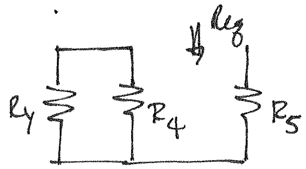
$i_o(0^+) = \frac{10^{-2} R_Z}{R_Z + R_4} = 3.33\ mA = K_1 + K_2$

$\underline{t = \infty}$



$$i_o(\infty) = \frac{+12}{R_Y + R_4} = +3\,mA = K_1$$

$\underline{\tau = ?}$



$$R_{eq} = R_5 + (R_4 /\!/ R_Y) = 3\,k\Omega$$

$$\tau = R_{eq} C = 0.6\,s$$

$$\boxed{i_o(t) = 3 + 0.33\, e^{-1.67t} \quad mA}$$

7.38 Use the step-by-step technique to find $i_o(t)$ for $t > 0$ in the network in Fig. P7.38.

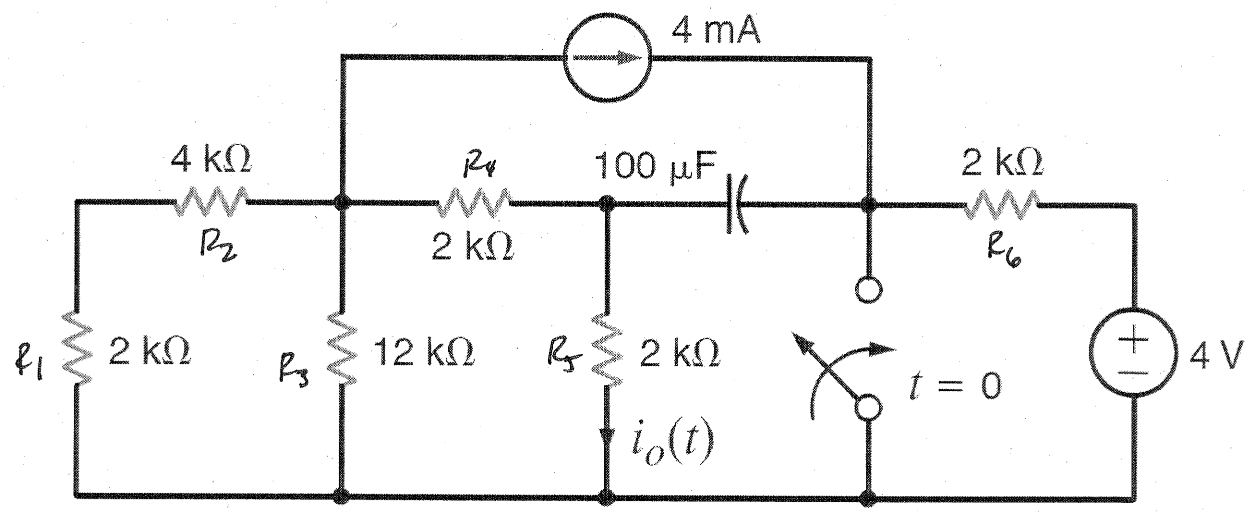
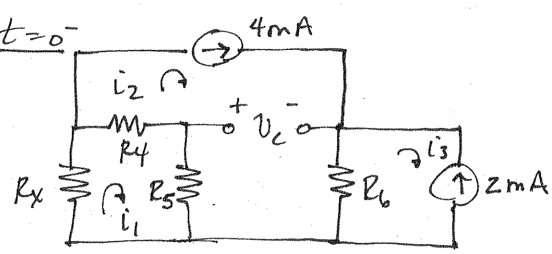


Figure P7.38

SOLUTION: $i_o(t) = K_1 + K_2 e^{-t/\tau}$

$t = 0^-$



$R_x = R_3 // (R_1 + R_2) = 4k\Omega$

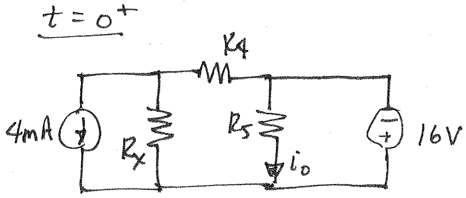
$i_2 = 4mA \qquad i_3 = -2mA$

$i_1(R_x + R_4 + R_5) - i_2(R_4 + R_5) = 0$

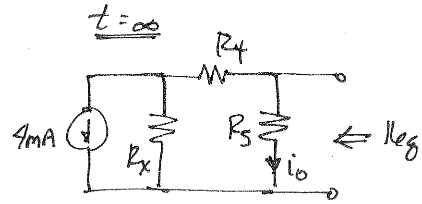
$i_1 = 2mA$

$v_C(0^-) = (i_1 - i_2)R_5 + (i_3 - i_2)R_6 = -16\text{ V}$

$t = 0^+$



$t = \infty$



$\tau = ?$

$R_{eq} = R_5 // (R_4 + R_x)$
$= 1.5k\Omega$

$\tau = R_{eq}C = 0.15S$

$i_o(0^+) = -\frac{16}{R_5} = -8mA$

$K_1 + K_2 = -8mA$

$i_o(\infty) = \frac{-4\times10^{-3} R_x}{R_x + R_4 + R_5}$

$i_o(\infty) = -2mA = K_1$

$\boxed{i_o(t) = -2 - 6e^{-6.67t}\ mA}$

7.39 Use the step-by-step method to find $i_o(t)$ for $t > 0$ in the circuit in Fig. P7.39.

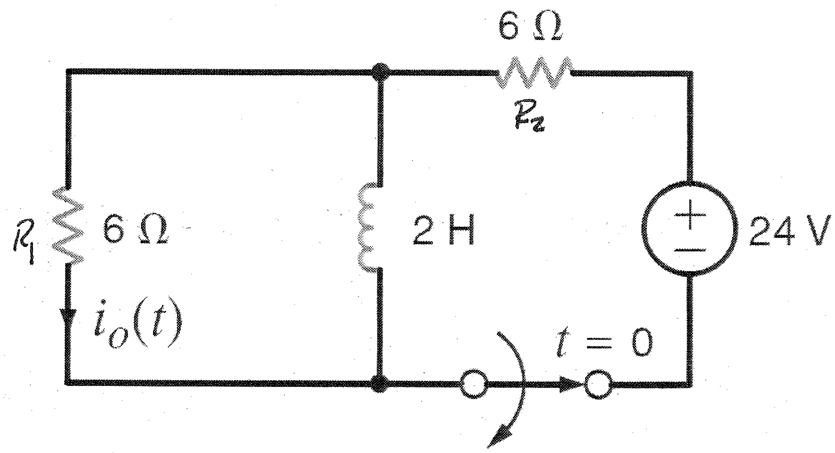
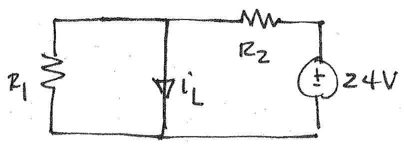


Figure P7.39

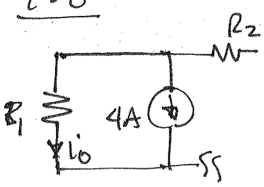
SOLUTION: $i_o(t) = K_1 + K_2 e^{-t/\tau}$

$\underline{t=0^-}$



$i_L(0^-) = 24/R_2 = 4A$

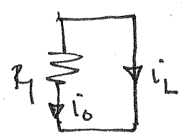
$\underline{t=0^+}$



$i_o(0^+) = -4A$

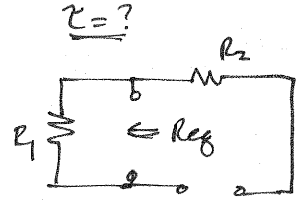
$K_1 + K_2 = -4A$

$\underline{t=\infty}$



$i_o(\infty) = 0 = K_1$

$\underline{\tau = ?}$



$R_{eq} = R_1 = 6\Omega$

$\tau = L/R_{eq} = \frac{1}{3}$ s

$$\boxed{i_o(t) = -4e^{-3t} \text{ A}}$$

7.40 Find $i_o(t)$ for $t > 0$ in the network in Fig. P7.40 using the step-by-step method.

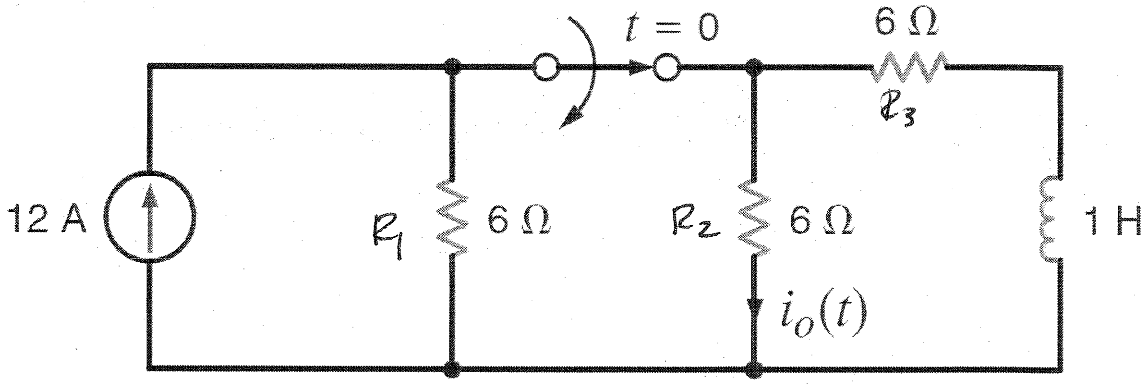
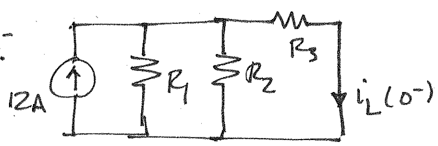


Figure P7.40

SOLUTION: $i_o(t) = K_1 + K_2 e^{-t/\tau}$

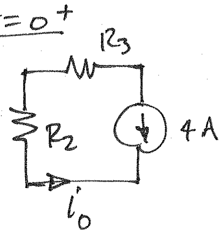
$\underline{t=0^-}$



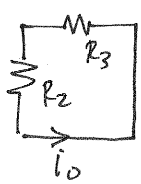
$R_x = R_1 // R_2 = 3\Omega$

$i_L(0^-) = \dfrac{12 R_x}{R_x + R_3} = 4A$

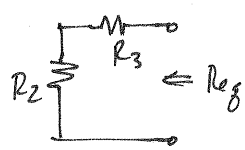
$\underline{t=0^+}$



$\underline{t=\infty}$



$\underline{\tau = ?}$



$R_{eq} = R_2 + R_3 = 12\Omega$

$\tau = L/R_{eq} = \frac{1}{12}\ s$

$i_o = -4A = K_1 + K_2$

$i_o(\infty) = 0 = K_1$

$$\boxed{i_o(t) = -4e^{-12t}\ A}$$

7.41 Find $i_o(t)$ for $t > 0$ in the network in Fig. P7.41 using the step-by-step method. CS

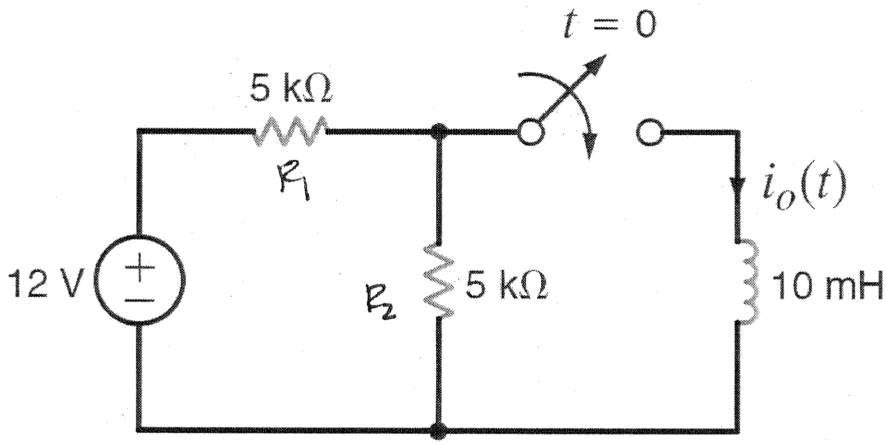
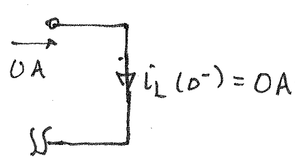


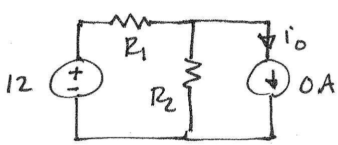
Figure P7.41

SOLUTION: $i_o(t) = K_1 + K_2 e^{-t/\tau}$

$t=0^-$

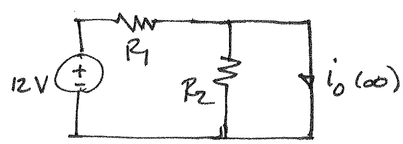


$t=0^+$



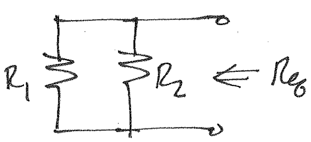
$i_o(0^+) = 0 = K_1 + K_2$

$t=\infty$



$i_o(\infty) = \frac{12}{R_1} = 2.4\text{mA} = K_1$

$\tau = ?$



$R_{eq} = R_1 // R_2 = 2.5\text{k}\Omega$

$\tau = \frac{L}{R_{eq}} = 4\mu s$

$$\boxed{i_o(t) = 2.4 - 2.4e^{-2.5\times10^5 t} \text{ mA}}$$

7.42 Find $v_o(t)$ for $t > 0$ in the network in Fig. P7.42 using the step-by-step method.

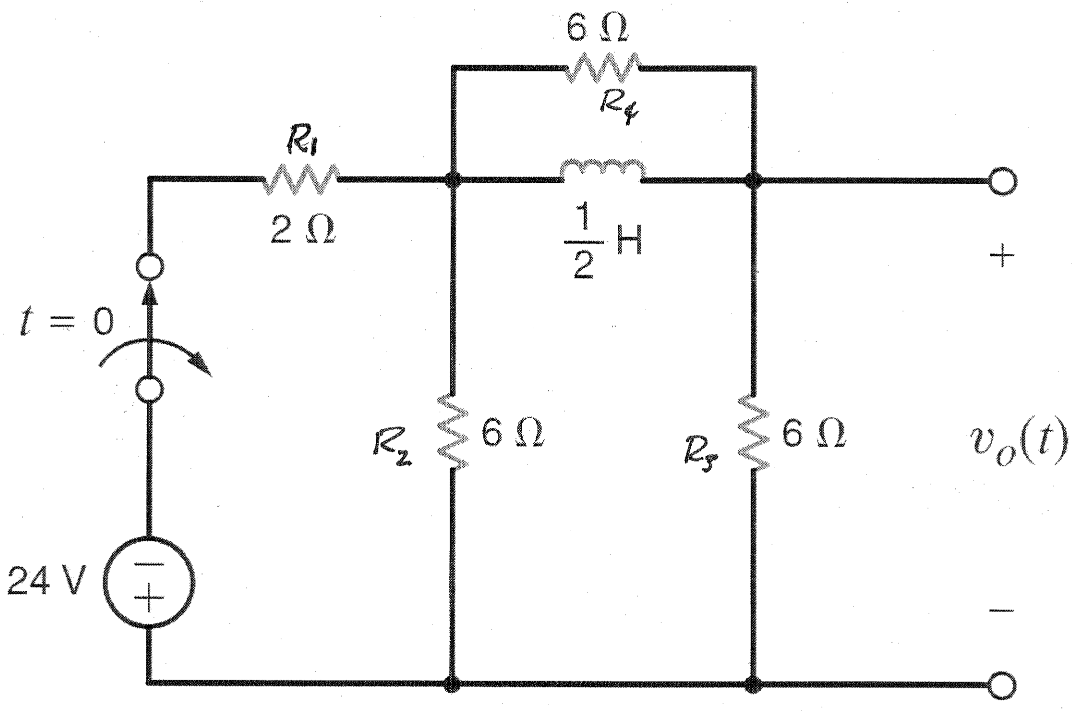
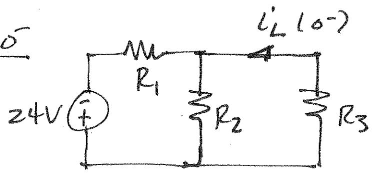


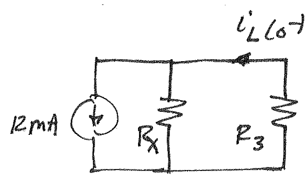
Figure P7.42

SOLUTION: $v_o(t) = K_1 + K_2 e^{-t/\tau}$

$t = 0^-$



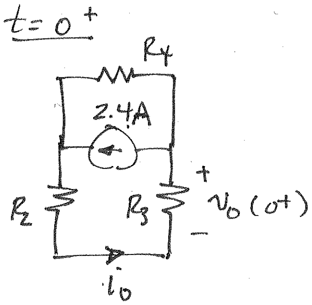
⇒



$R_x = R_1 // R_2 = 1.5k\Omega$

$$i_L(0^-) = \frac{12 \times 10^{-3} R_x}{R_x + R_3} = 2.4A$$

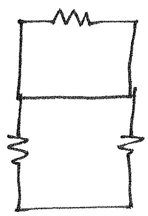
$t = 0^+$



$$i_o = \frac{2.4 R_4}{R_4 + R_3 + R_2} = 0.8\,mA$$

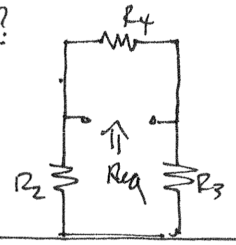
$$v_o(0^+) = -R_3\, i_o(0^+) = -4.8V = K_1 + K_2$$

$\underline{t=\infty}$



$v_o(\infty) = 0 = k_1$

$\underline{\tau = ?}$



$R_{eq} = R_4 \,//\, (R_2 + R_3) = 4\,\Omega$

$\tau = L/R_{eq} = \frac{1}{8}\ s$

$$\boxed{v_o(t) = -4.8e^{-8t}\ \text{V}}$$

7.43 Use the step-by-step method to find $v_o(t)$ for $t > 0$ in the network in Fig. P7.43. **PSV**

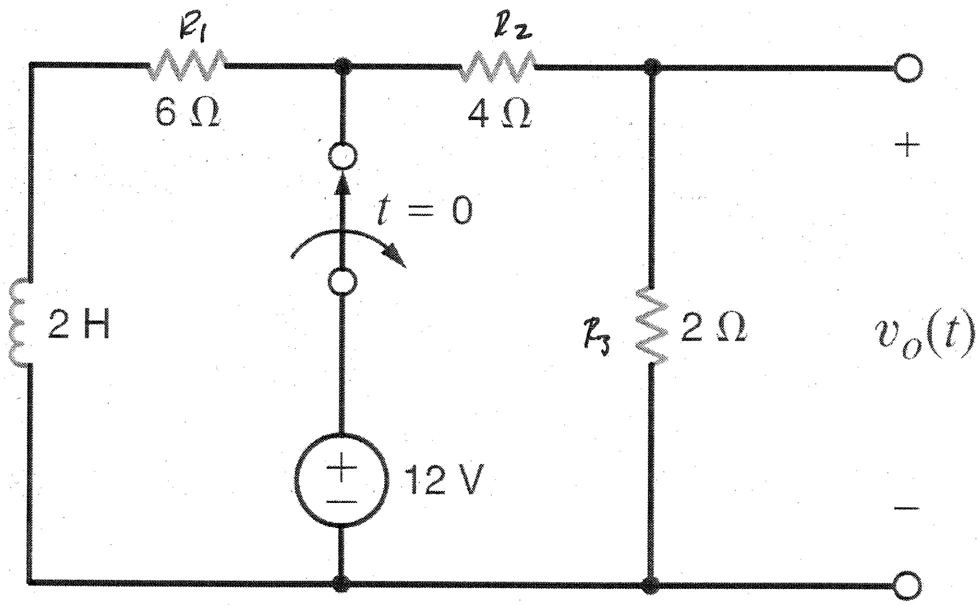
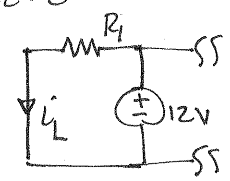


Figure P7.43

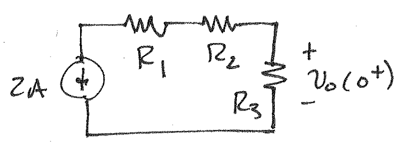
SOLUTION: $v_0(t) = K_1 + K_2 e^{-t/\tau}$

$t = 0^-$



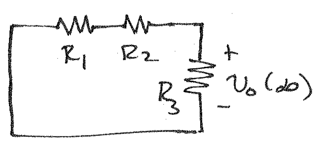
$i_L(0^-) = \frac{12}{R_1} = 2A$

$t = 0^+$



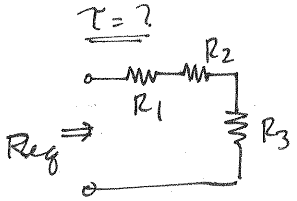
$v_0(0^+) = -2R_3 = -4V$
$= K_1 + K_2$

$t = \infty$



$v_0(\infty) = 0 = K_1$

$\tau = ?$



$R_{eq} = R_1 + R_2 + R_3 = 12\Omega$

$\tau = L/R_{eq} = \frac{1}{6}\,s$

$$\boxed{v_0 = -4e^{-6t}\ V}$$

7.44 Find $i_o(t)$ for $t > 0$ in the network in Fig. P7.44 using the step-by-step method.

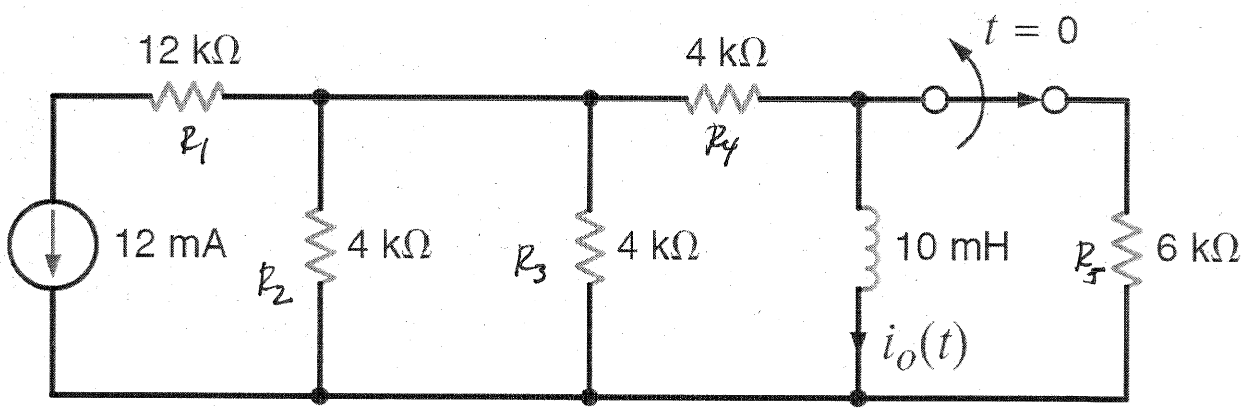
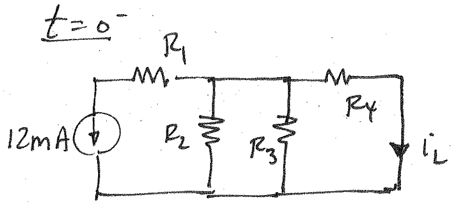


Figure P7.44

SOLUTION: $i_o(t) = K_1 + K_2 e^{-t/\tau}$

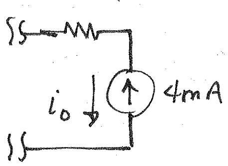
$\underline{t=0^-}$



$R_2 = R_3 = R_4 = 4k\Omega$

$i_L(0^-) = -\dfrac{12\times10^{-3}}{3} = -4mA$

$\underline{t=0^+}$



$i_o(0^+) = -4mA$
$= K_1 + K_2$

$\underline{t=\infty}$ Same as $t=0^-$!

$i_o(\infty) = -4mA = K_1$

So, $K_2 = 0$

$\boxed{i_o = -4mA}$

7.45 Use the step-by-step technique to find $v_o(t)$ for $t > 0$ in the circuit in Fig. P7.45.

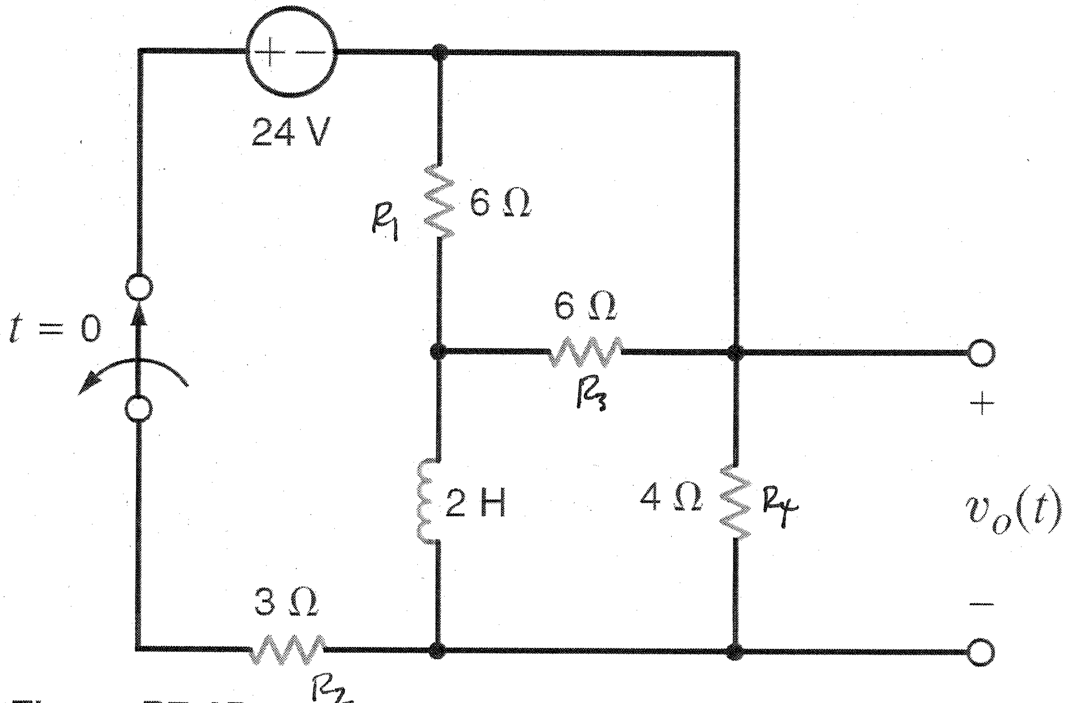
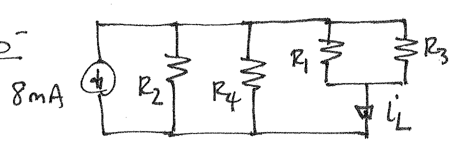


Figure P7.45

SOLUTION: $v_o(t) = K_1 + K_2 e^{-t/\tau}$

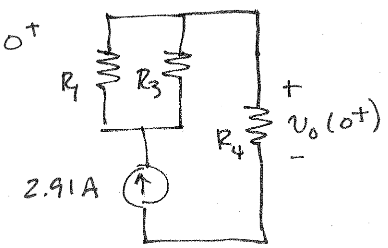
$t = 0^-$



$R_x = R_2 // R_4 = \frac{12}{7}\,\Omega$ $\quad R_y = R_1 // R_3 = 3\,\Omega$

$$i_L = \frac{-8\times10^{-3} R_x}{R_x + R_y} = -2.91\ A$$

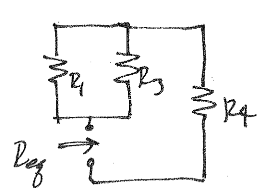
$t = 0^+$



$v_o(0^+) = 2.91\,R_4 = 11.64V$
$= K_1 + K_2$

$t = \infty$ $\quad v_o = 0 = K_1$

$\tau = ?$



$R_{eq} = R_4 + (R_1 // R_3) = 7\,\Omega$

$\tau = L/R_{eq} = \frac{2}{7}\ s$

$$\boxed{v_o(t) = 11.64\,e^{-3.5t}\ V}$$

7.46 Use the step-by-step method to find $v_o(t)$ for $t > 0$ in the network in Fig. P7.46.

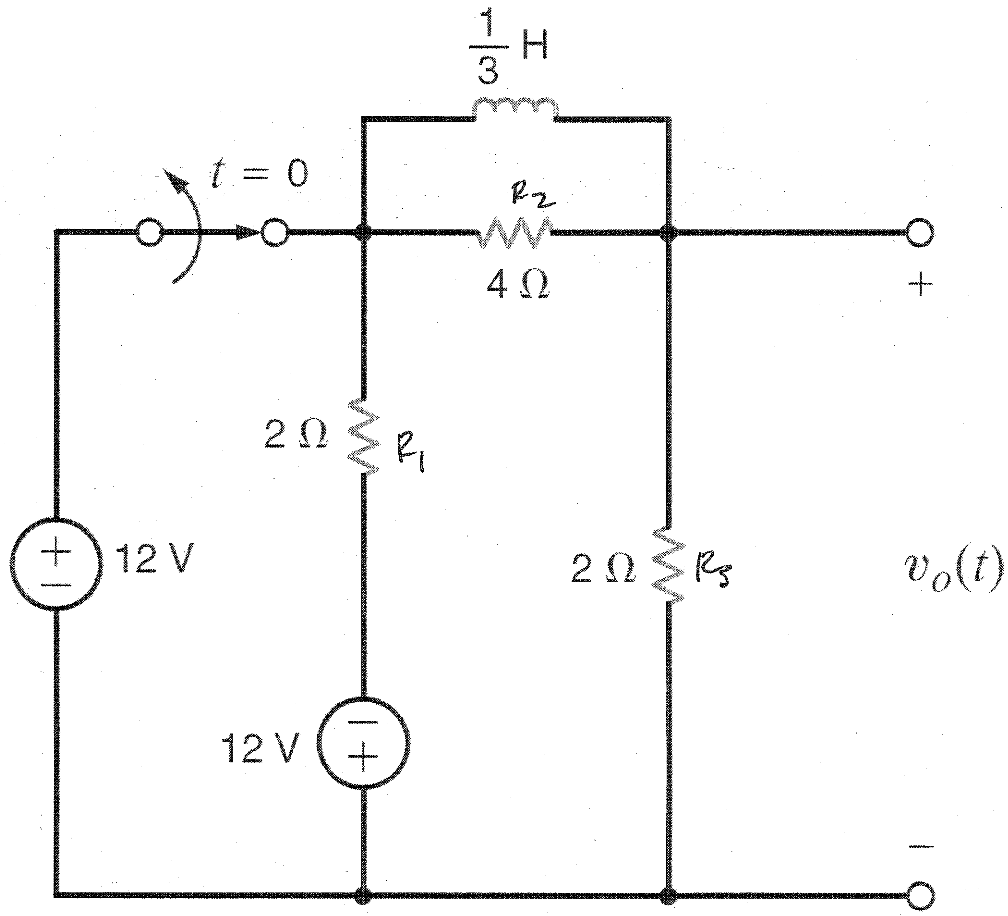
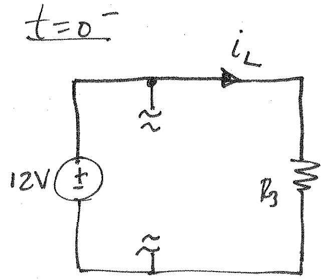


Figure P7.46

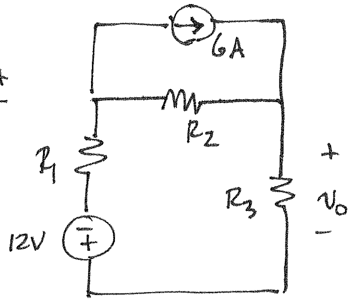
SOLUTION: $v_o(t) = K_1 + K_2 e^{-t/\tau}$

$\underline{t=0^-}$

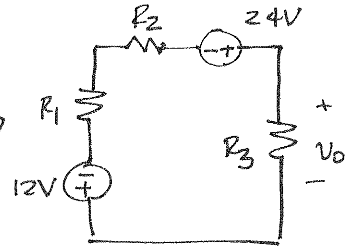


$i_L(0^-) = \frac{12}{R_3} = 6\,A$

$\underline{t = 0^+}$



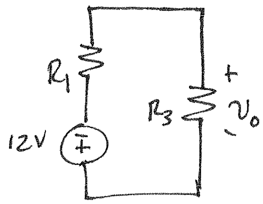
$\Rightarrow$



$$v_0 = \frac{(24-12)\,R_3}{R_1+R_2+R_3}$$

$$v_0(0^+) = 3\,V = K_1 + K_2$$

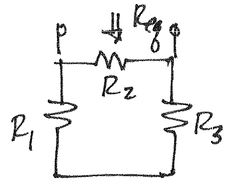
$\underline{t = \infty}$



$$v_0(\infty) = \frac{-12\,R_3}{R_1+R_3}$$

$$v_0(\infty) = -6V = K_1$$

$\underline{\tau = ?}$



$$R_{eq} = R_2 \,//\, (R_1 + R_3)$$

$$R_{eq} = 2\,\Omega$$

$$\tau = \frac{L}{R_{eq}} = \frac{1}{6}\,s$$

$$\boxed{v_0(t) = -6 + 9\,e^{-6t}\ V}$$

7.47 Find $v_o(t)$ for $t > 0$ in the circuit in Fig. P7.47 using the step-by-step method.

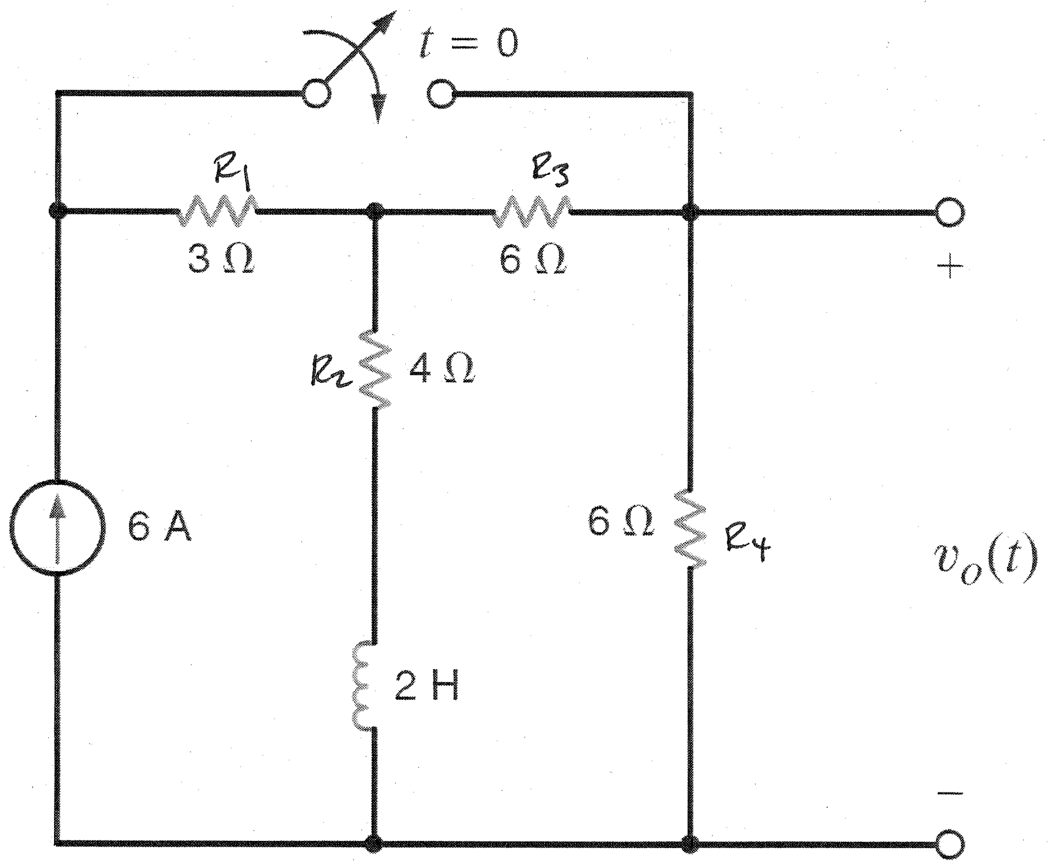
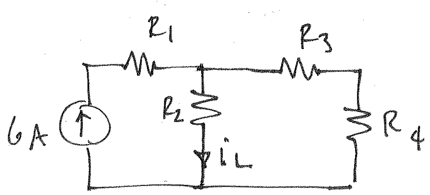


Figure P7.47

SOLUTION: $v_0(t) = K_1 + K_2 e^{-t/\tau}$

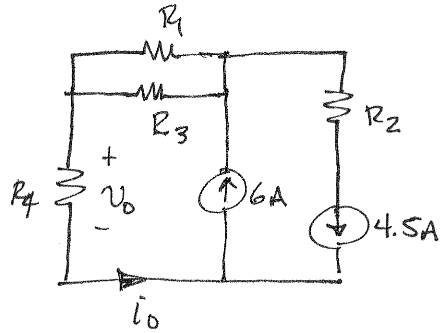
$t = 0^-$



$R_x = R_3 + R_4 = 12\Omega$

$i_L = \frac{6\,R_x}{R_x + R_2} = 4.5A$

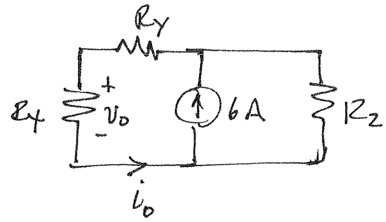
$\underline{t=0^+}$



$i_o = 6 - 4.5 = 1.5\,A$

$v_o = i_o R_4 = 9V = K_1 + K_2$

$\underline{t=\infty}$

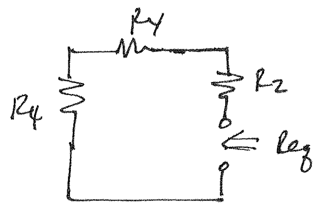


$R_y = R_1 // R_3 = 2\Omega$

$i_o = \dfrac{6\,R_2}{R_2 + R_y + R_4} \qquad i_o = 2A$

$v_o(\infty) = i_o R_4 = 12V = K_1$

$\underline{\tau = ?}$



$R_{eq} = R_2 + R_y + R_4 = 12\Omega$

$\tau = L/R_{eq} = 1/6\ s$

$$\boxed{v_o(t) = 12 - 3e^{-6t}\ V}$$

7.48 Find $v_o(t)$ for $t > 0$ in the network in Fig. P7.48 using the step-by-step technique.

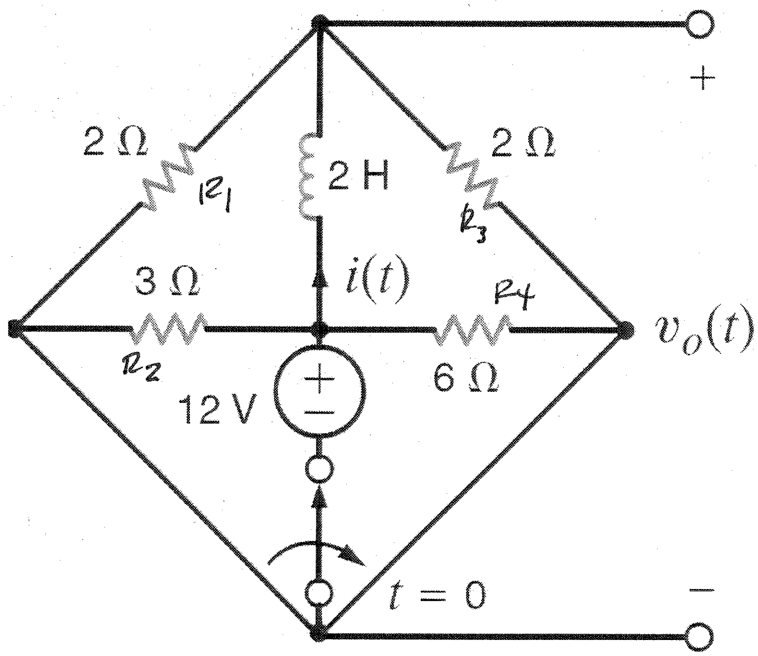
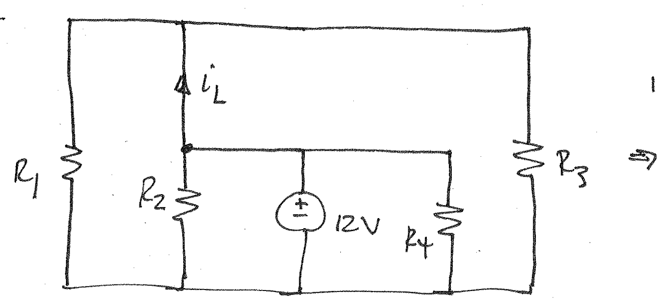
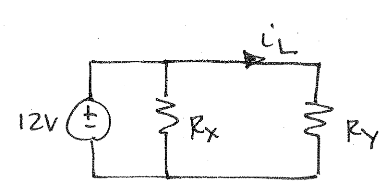


Figure P7.48

SOLUTION: $v_o(t) = K_1 + K_2 e^{-t/\tau}$

$\underline{t=0^-}$



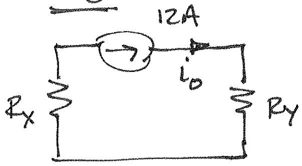


$R_x = R_2 /\!/ R_4 = 2\Omega$

$R_y = R_1 /\!/ R_3 = 1\Omega$

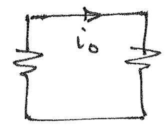
$i_L = \frac{12}{R_y} = 12\text{A}$

$\underline{t=0^+}$



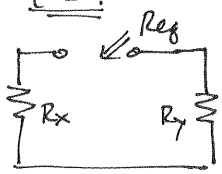
$i_o = 12A = K_1 + K_2$

$\underline{t=\infty}$



$i_o = 0 = K_1$

$\underline{\tau = ?}$



$R_{eq} = R_x + R_y$

$R_{eq} = 3\,\Omega$

$\tau = \dfrac{L}{R_{eq}} = \dfrac{2}{3}\ s$

$$\boxed{i_o = 12e^{-1.5t}\ A}$$

7.49 Use the step-by-step method to find $v_o(t)$ for $t > 0$ in the circuit in Fig. P7.49.

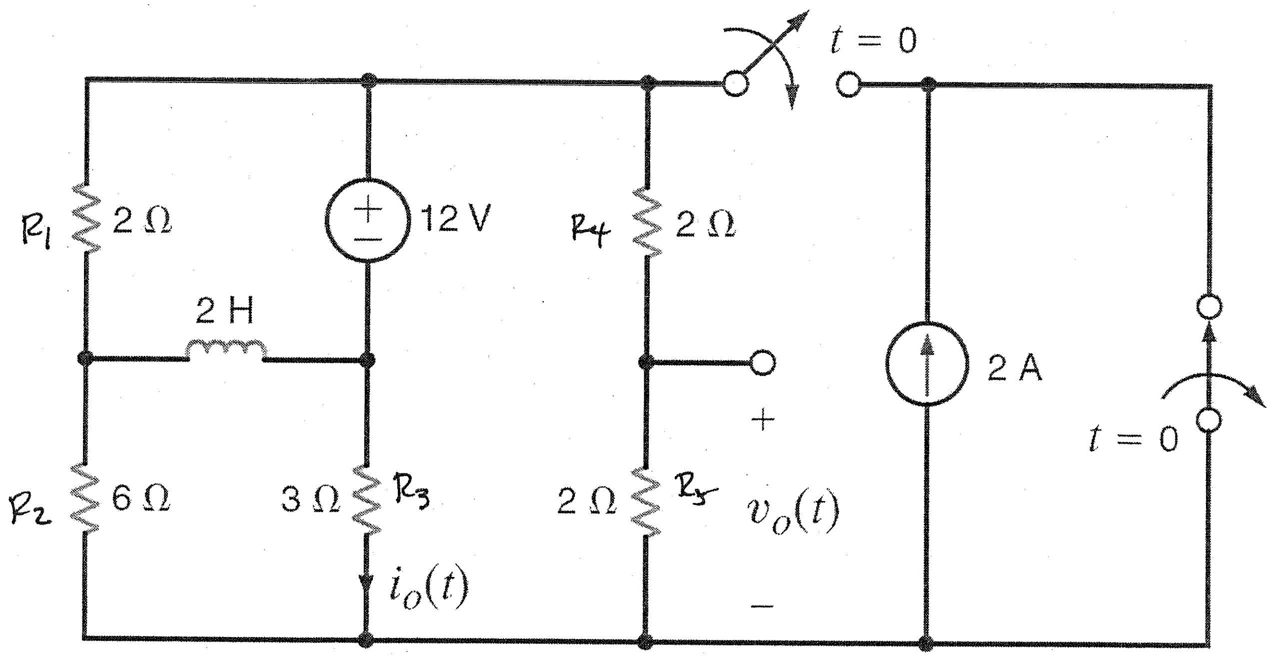
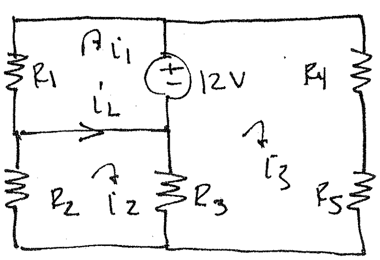


Figure P7.49

SOLUTION: $v_o(t) = K_1 + K_2 e^{-t/\tau}$

$t = 0^-$



mesh analysis:

$i_1 R_1 + 12 = 0 \Rightarrow i_1 = -6\,A$

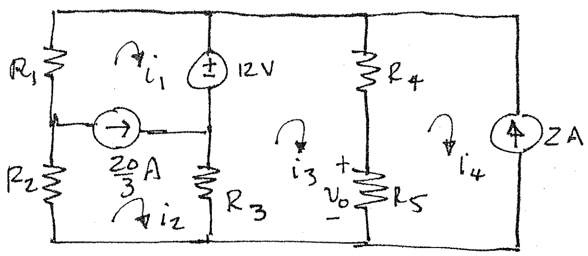
$i_2(R_2+R_3) - i_3 R_3 = 0 \Rightarrow i_3 = 3 i_2$

$12 = i_3(R_3+R_4+R_5) - i_2 R_3 \Rightarrow 7 i_3 - 3 i_2 = 12$

yields $i_3 = 2A$ & $i_2 = \frac{2}{3}\,A$

$i_L(0^-) = i_2 - i_1 = \frac{20}{3}\,A$

$\underline{t = 0^+}$



$$i_2 - i_1 = 20/3$$

$$i_4 = -2$$

$$12 = i_3(R_3 + R_4 + R_5) - i_2 R_3 - i_4(R_4 + R_5)$$

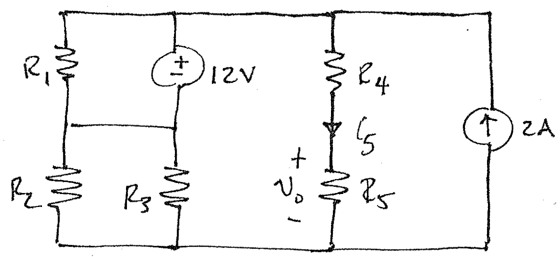
$$0 = i_1 R_1 + i_2 R_2 + i_3(R_4 + R_5) - i_4(R_4 + R_5)$$

$$i_3 = 0.706\text{A}$$

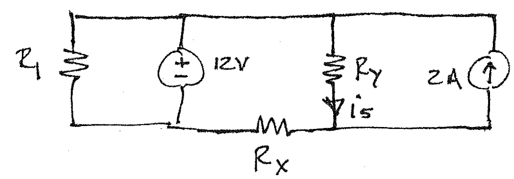
$$v_o = R_5(i_3 - i_4) = 5.41\text{V}$$

$$5.41 = K_1 + K_2$$

$\underline{t = \infty}$



$\Rightarrow$



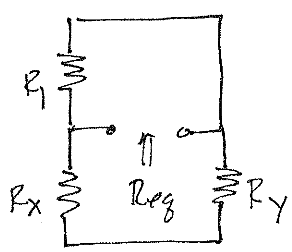
$$R_X = R_2 \parallel R_3 = 2\Omega \qquad R_Y = R_4 + R_5 = 4\Omega$$

$$v_0(\infty) = i_5 R_5 = 2 i_5$$

Find i_5 by superposition: $i_5 = \frac{12}{R_X + R_Y} + \frac{2R_X}{R_X + R_Y} = \frac{8}{3}\text{A}$

$$v_0 = \frac{16}{3} = 5.33\text{V} = K_1$$

$\underline{\tau = ?}$



$$R_{eq} = R_1 \parallel (R_X + R_Y) = 1.5\Omega$$

$$\tau = L/R_{eq} = \frac{4}{3}\text{ s}$$

$$\boxed{v_0 = 5.33 + 0.08e^{-0.75t}\text{ V}}$$

7.50 Find $i(t)$ for $t > 0$ in the circuit of Fig. P7.50 using the step-by-step method.

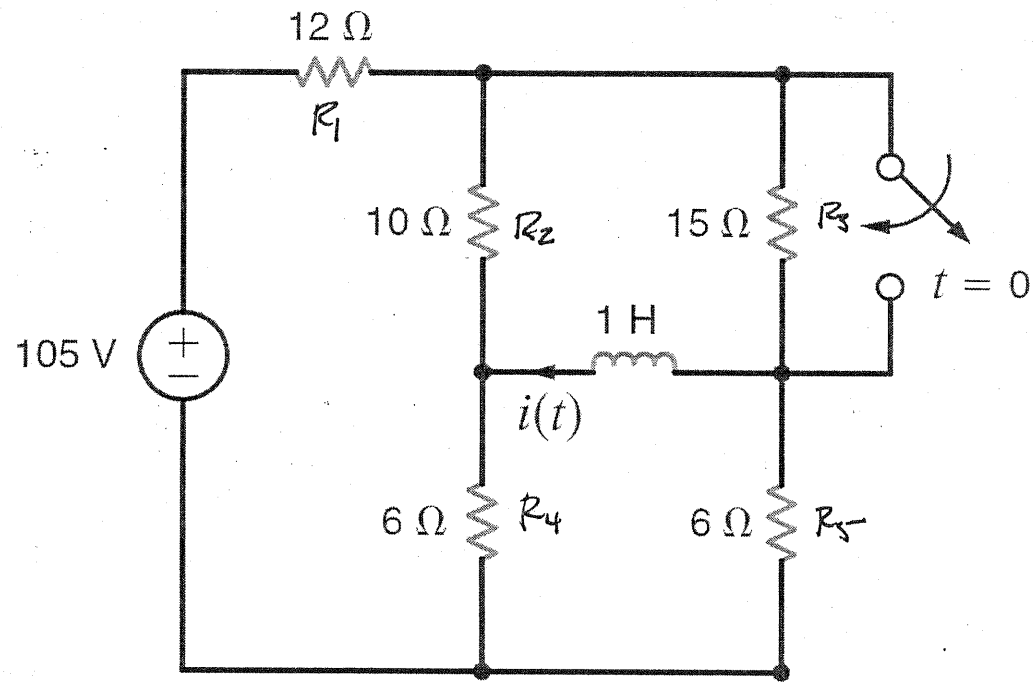
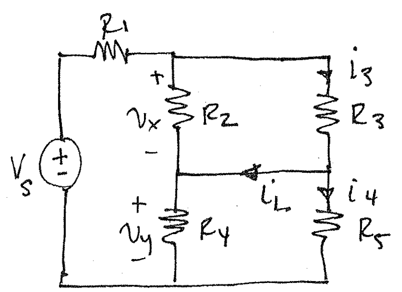


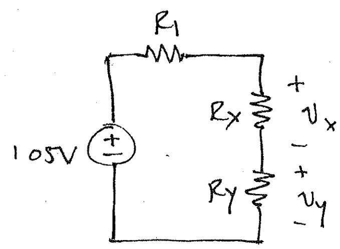
Figure P7.50

SOLUTION: $i(t) = K_1 + K_2 e^{-t/\tau}$

$t = 0^-$



$\Rightarrow$



$R_x = R_2 // R_3 = 6\,\Omega$

$R_y = R_4 // R_5 = 3\,\Omega$

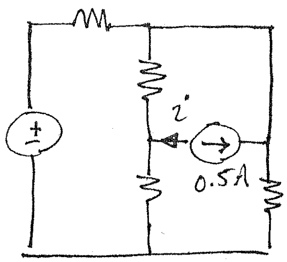
$v_x = \dfrac{105\,R_x}{R_x + R_y + R_1} = 30\,V$

$v_y = \dfrac{105\,R_y}{R_1 + R_x + R_y} = 15\,V$

$i_3 = \dfrac{v_x}{R_3} = 2\,A$ $\qquad i_4 = \dfrac{v_y}{R_5} = 2.5\,A$

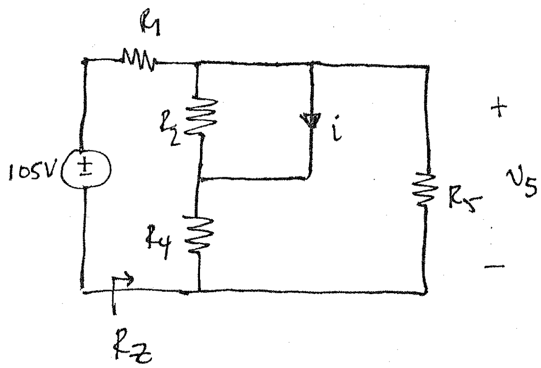
$i_L = i_3 - i_4 = -0.5\,A$

$\underline{t = 0^+}$



$$i = -0.5A = K_1 + K_2$$

$\underline{t = \infty}$

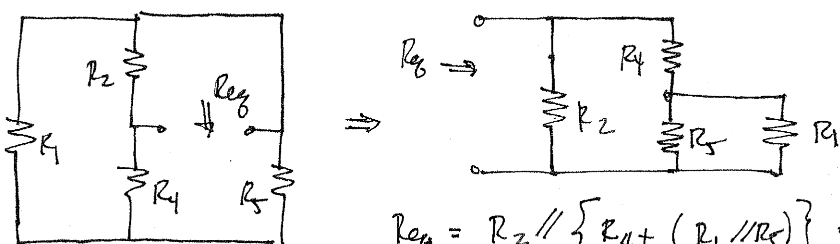


$$R_z = R_4 /\!/ R_5 = 3\,\Omega$$

$$v_5 = \frac{105 R_z}{R_z + R_1} = 21\,V$$

$$i(\infty) = \frac{v_5}{R_4} = 3.5A = K_1$$

$\underline{\tau = ?}$



$$R_{eq} = R_2 /\!/ \{R_4 + (R_1 /\!/ R_5)\} = 5\,\Omega$$

$$\tau = \frac{L}{R_{eq}} = \frac{1}{5}\,s$$

$$\boxed{i(t) = 3.5 - 4e^{-5t}\ A}$$

7.51 Find $v_C(t)$ for $t > 0$ in the circuit of Fig. P7.51 using the step-by-step method.

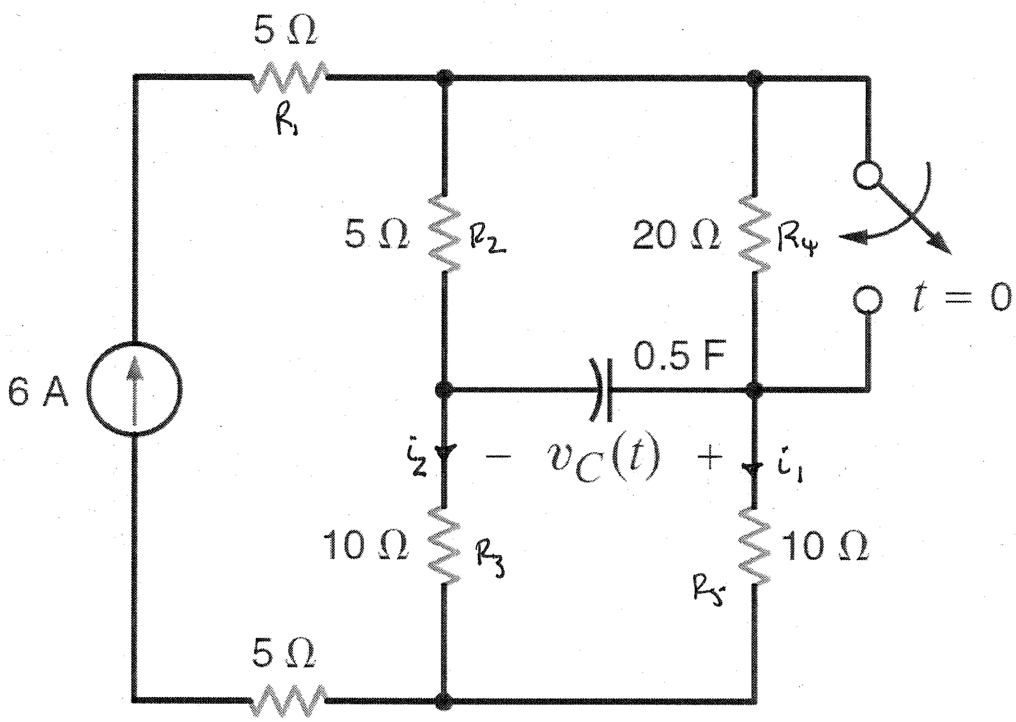


Figure P7.51

SOLUTION: $v_c(t) = K_1 + K_2 e^{-t/\tau}$

$\underline{t=0^-}$: $R_A = R_4 + R_5 = 30\Omega$ $\quad R_B = R_2 + R_3 = 15\Omega$

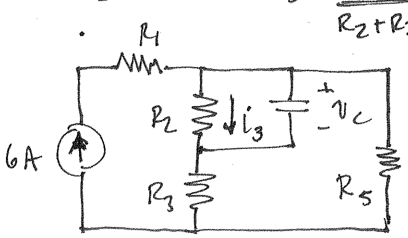
$i_1 = \dfrac{6R_B}{R_B + R_A} = 2A$ $\quad i_2 = \dfrac{6R_A}{R_A + R_B} = 4A$ $\quad v_c(0^-) = i_1 R_5 - i_2 R_3$

$v_c(0^-) = -20V$

$\underline{t=0^+}$: $v_c(0^+) = v_c(0^-) = -20 = K_1 + K_2$

$\underline{t \to \infty}$: $i_3 = \dfrac{6R_5}{R_2 + R_3 + R_5} = 2.4A$ $\quad v_c(\infty) = R_2 i_3 = 12V = K_1$

$\tau = C\left[R_2 \,/\!/\, (R_3 + R_5)\right] = 2s$



$$\boxed{v_c(t) = 12 - 32e^{-t/2} \; V}$$

7.52 Find $i(t)$ for $t > 0$ in the circuit of Fig. P7.52 using the step-by-step method.

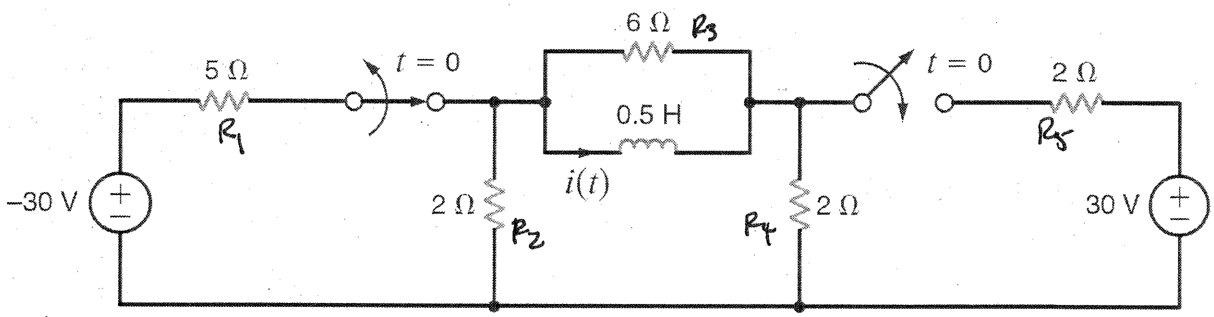
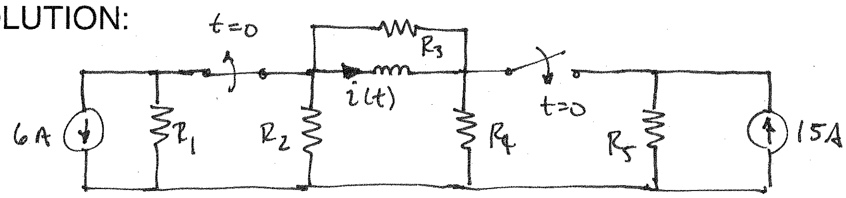


Figure P7.52

SOLUTION:

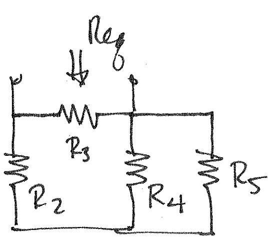


$\underline{t=0^-}$: $i(0^-) = \dfrac{-6\,(1/R_4)}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_4}} = -2.5\text{ A}$

$\underline{t=0^+}$: $i(0^+) = i(0^-) = -2.5\text{ A} = K_1 + K_2$

$\underline{t\to\infty}$: $i(\infty) = -\dfrac{15\,(1/R_2)}{\frac{1}{R_2}+\frac{1}{R_4}+1/R_5} = -5\text{A} = K_1$

$K_2 = 2.5\text{ A}$



$\tau = \dfrac{L}{R_{eq}}$

$t = \frac{1}{4}\text{ s}$

$R_{eq} = R_3 \,//\, [R_2 + R_A]$

$R_{eq} = 2\Omega$

$R_A = R_4 // R_5$

$R_A = 1\Omega$

$$\boxed{i(t) = -5 + 2.5e^{-4t}\text{ A}}$$

7.53 Find $i_o(t)$ for $t > 0$ in the circuit in Fig. P7.53 using the step-by-step method.

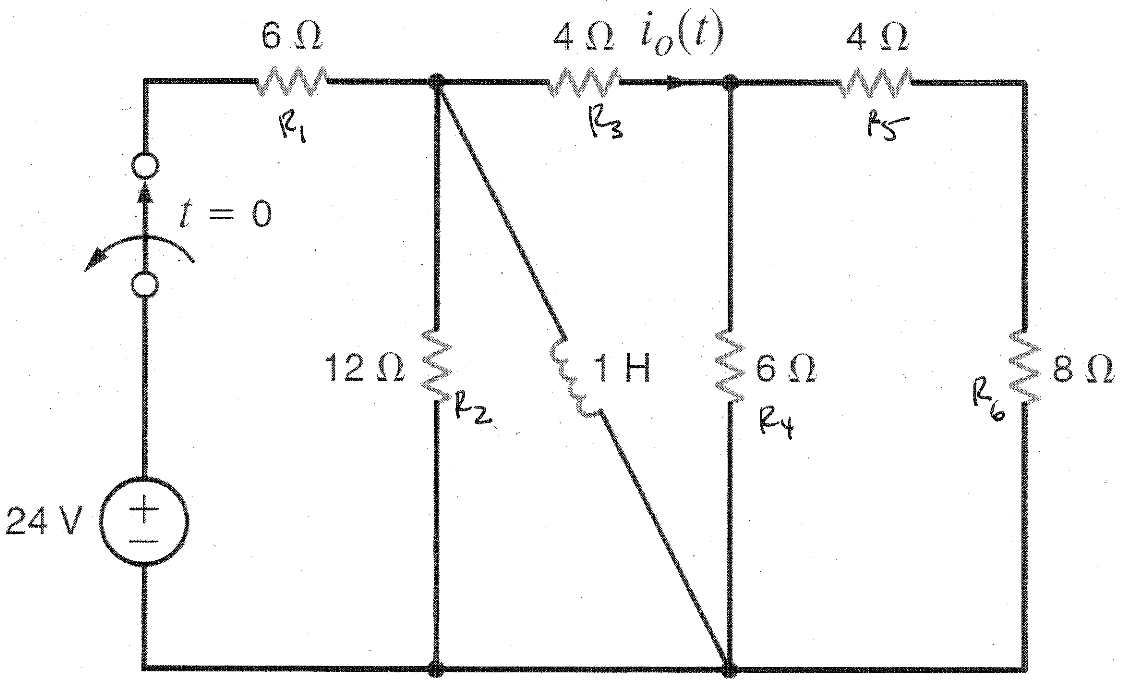
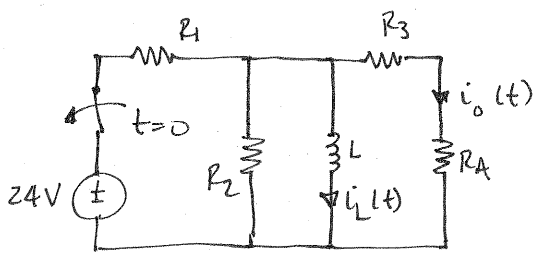


Figure P7.53

SOLUTION:



$R_A = R_4 \,//\, (R_5 + R_6) = 4\,\Omega$

$\underline{t = 0^-}$: $i_L(0^-) = \dfrac{24}{R_1} = 4\,A$

$\underline{t = 0^+}$: $i_o = -\dfrac{i_L(0^-)\, R_2}{R_2 + R_3 + R_A} = -2.4A = K_1 + K_2$

$\underline{t \to \infty}$: $i_o = 0 = K_1$

$\tau = L/R_{eq}$ $\quad R_{eq} = R_2 \,//\, (R_3 + R_A) = 4.8\,\Omega$ $\quad \tau = 0.208s$

$$\boxed{i_o(t) = -2.4e^{-4.8t}\ A}$$

7.54 Find $v_o(t)$ for $t > 0$ in the network in Fig. P7.54 using the step-by-step method.

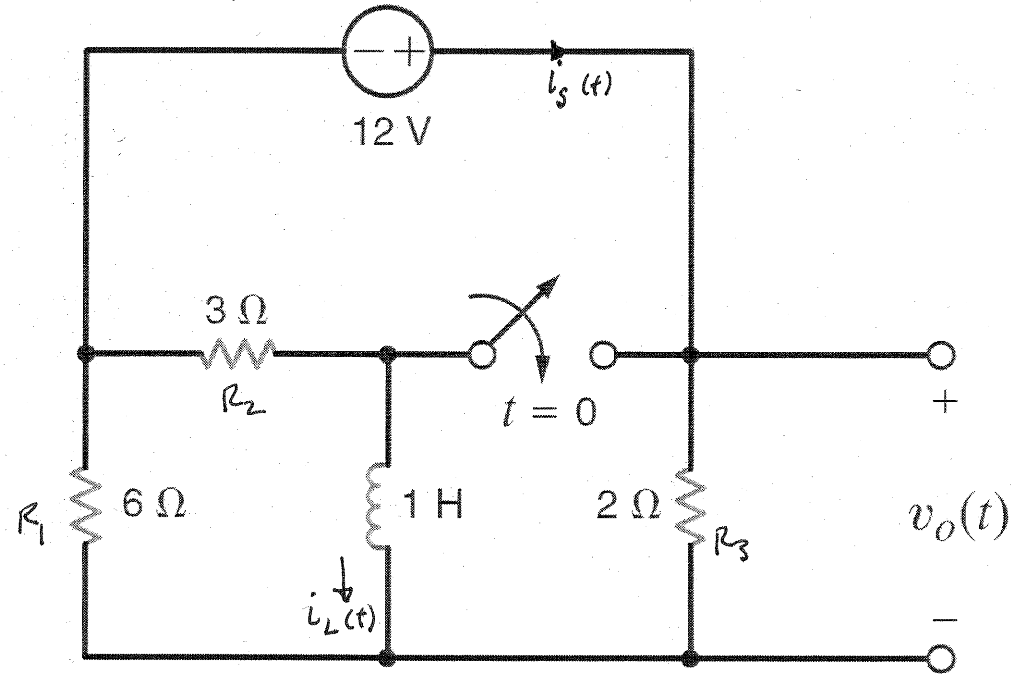
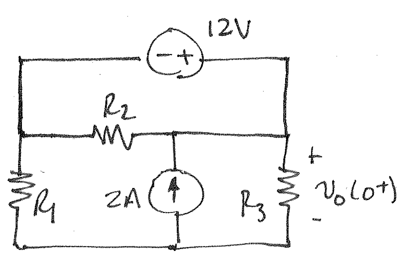


Figure P7.54

SOLUTION:

$t=0^-$: $\quad i_s = \dfrac{12}{R_3 + (R_1 // R_2)} = 3\text{A} \qquad i_L = \dfrac{-i_s R_1}{R_1 + R_2} = -2\text{A}$

$t=0^+$: By superposition: $\quad v_o(0^+) = \dfrac{12 R_3}{R_1 + R_3} + \dfrac{2 R_1 R_3}{R_1 + R_3} = 6\text{V} = K_1 + K_2$



$t \to \infty$: $\quad v_o(\infty) = 0 = K_1$

$R_{eq} = R_1 // R_3 = 1.5\,\Omega \qquad \tau = \dfrac{L}{R} = \dfrac{2}{3}\,s$

$$\boxed{v_o(t) = 6e^{-1.5t}\ \text{V}}$$

7.55 Find $i_o(t)$ for $t > 0$ in the network in Fig. P7.55 using the step-by-step method. **PSV**

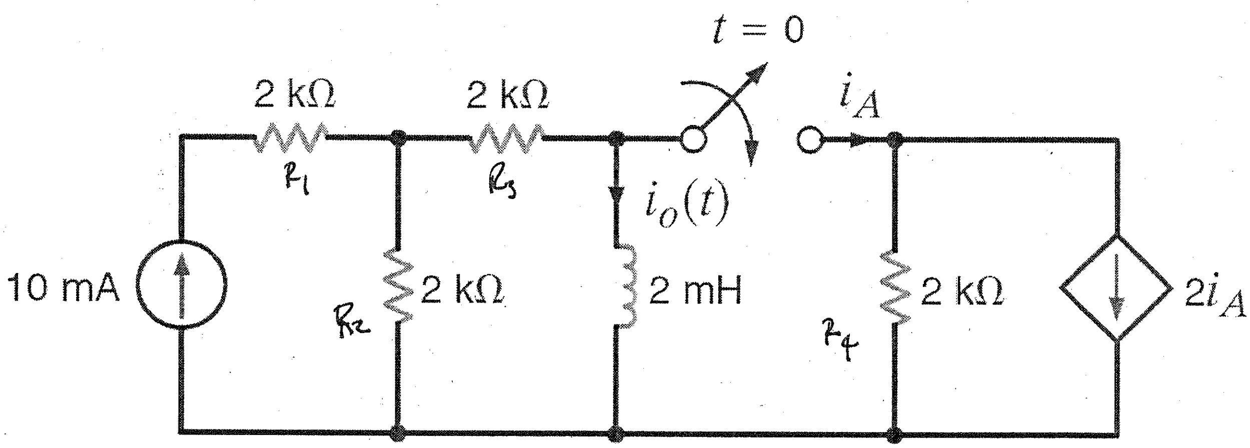
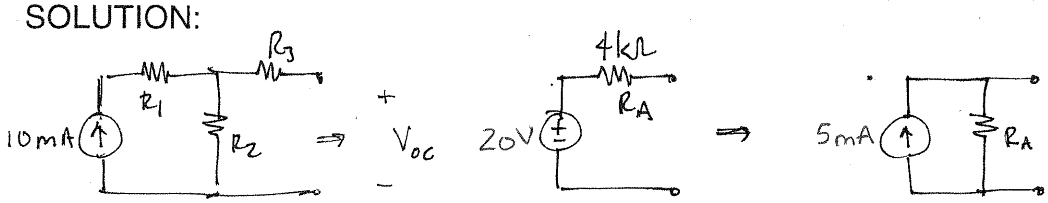
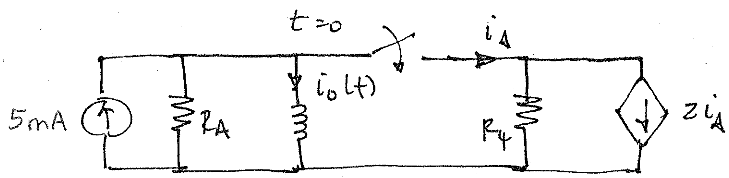


Figure P7.55

SOLUTION:

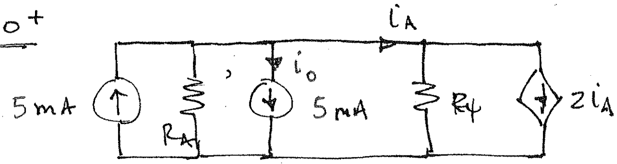




$\underline{t=0^-}$ $\quad i_A = 0$

$i_o = 5\times10^{-3} = 5\ mA$

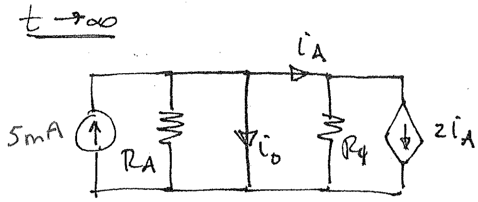
$\underline{t=0^+}$



$i_o(0^+) = i_o(0^-) = 5\ mA = K_1 + K_2$

$\underline{t\to\infty}$

$i_A = 0 \qquad i_o = 0.5mA = K_1 \Rightarrow K_2 = 0$



$$\boxed{i_o(t) = 5\ mA}$$

7.56 Find $i_L(t)$ for $t > 0$ in the circuit of Fig. P7.56 using the step-by-step method.

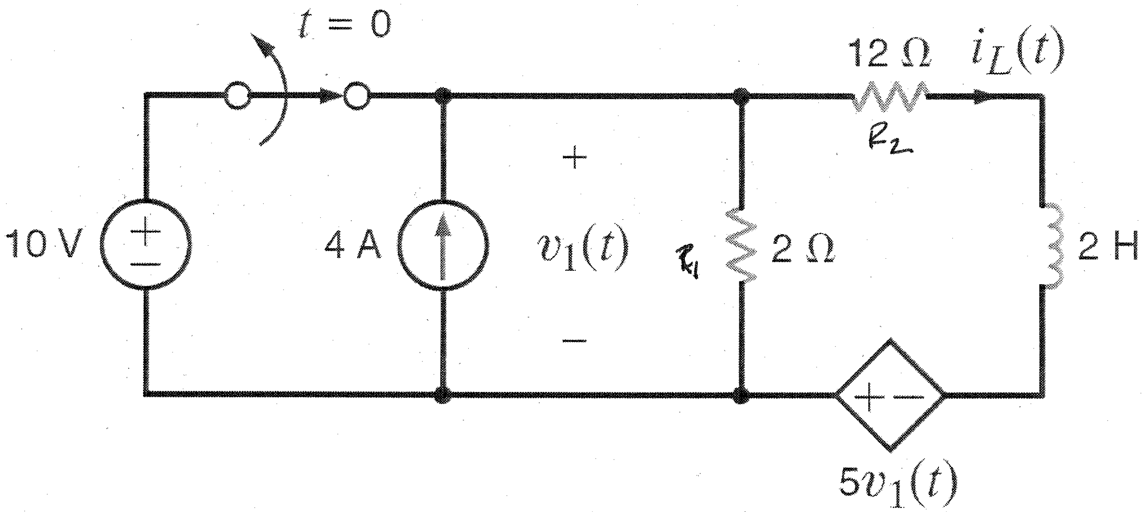


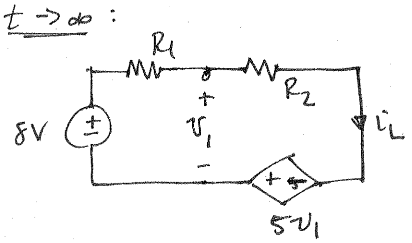
Figure P7.56

SOLUTION:

$\underline{t=0^-}$: $v_1 = 10V$ $\quad v_1 = R_2 i_L - 5v_1 \Rightarrow i_L = \frac{6v_1}{R_2} = 5A$

$\underline{t=0^+}$ $\quad i_L(0^+) = i_L(0^-) = 5A = K_1 + K_2$

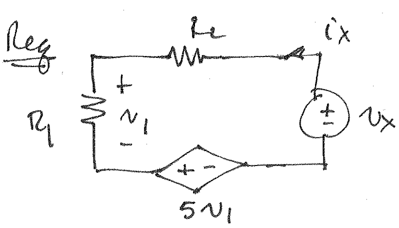
$\underline{t \to \infty}$:



$i_L = 6v_1/R_2 \Rightarrow v_1 = R_2 i_L/6$

$8 = i_L(R_1+R_2) - 5v_1 \Rightarrow i_L(\infty) = 2A = K_1$

$\underline{R_{eq}}$



$R_{eq} = v_x/i_x \qquad v_1 = i_x R_1$

$v_x = i_x(R_1+R_2) + 5v_1 = i_x(6R_1+R_2)$

$R_{eq} = 24\Omega \qquad \tau = L/R_{eq} = \frac{1}{12}\text{ s}$

$$\boxed{i_L(t) = 2 + 3e^{-12t}\text{ A}}$$

7.57 Use the step-by-step technique to find $v_o(t)$ for $t > 0$ in the network in Fig. P7.57.

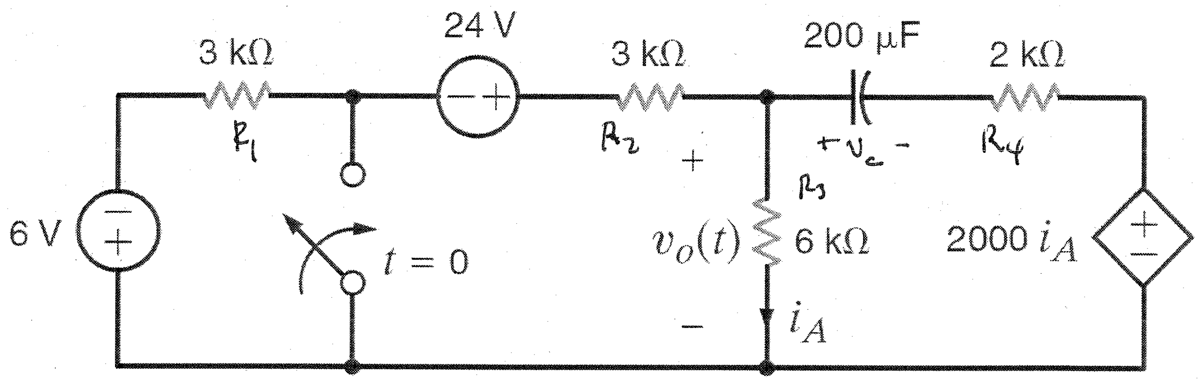


Figure P7.57

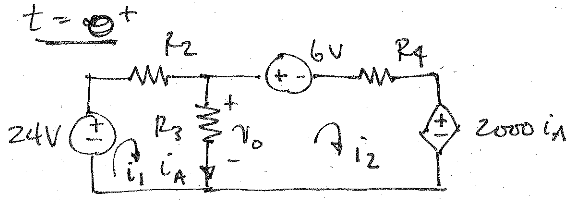
SOLUTION:

$\underline{t = 0^-}$: $\quad 6 - 24 + i_A R_1 + i_A R_2 + i_A R_3 = 0 \qquad i_A = 1.5\text{mA}$

$v_c(0^-) = v_o(0^-) - 2000\, i_A(0^-) \qquad v_o(0^-) = R_3\, i_A(0^-) = 9\text{V}$

$v_c(0^-) = 6\text{V}$

$\underline{t = 0^+}$



$24 = i_1(R_2 + R_3) - R_3 i_2$

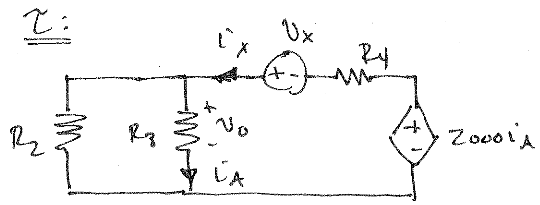
$-6 = i_2(R_3 + R_4) - R_3 i_1 + 2000\, i_A$

$i_A = i_1 - i_2$

$v_o(0^+) = i_A R_3 = 13.2\text{V} = K_1 + K_2$

$\underline{t = \infty}$: $\quad i_A = \dfrac{24}{R_2 + R_3} = 2.67\text{A} \qquad v_o = i_A R_3 = 16\text{V} = K_1$

$\underline{\tau}$:



$R_{eq} = v_x / i_x$

$v_x + 2000\, i_A = i_x(R_A + R_4)\,;\ R_A = R_2 \| R_3 = 2\text{k}\Omega$

$i_A = \dfrac{i_x R_2}{R_2 + R_3} = i_x/3 \Rightarrow R_{eq} = 3.33\text{k}\Omega$

$\tau = C R_{eq} = 0.667\text{ s}$

$$\boxed{v_o(t) = 16 - 2.8 e^{-1.5t}\ \text{V}}$$

7.58 Use the step-by-step method to find $v_o(t)$ for $t > 0$ in the network in Fig. P7.58. CS

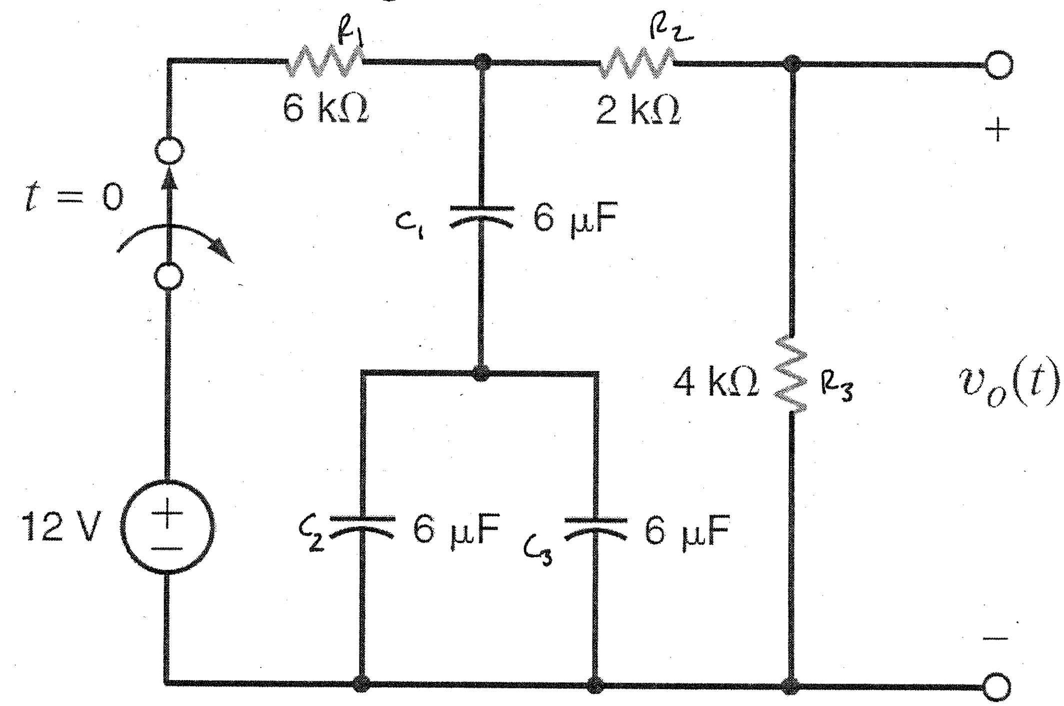
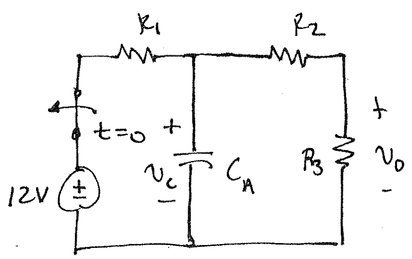


Figure P7.58

SOLUTION:



$$C_A = \frac{C_1(C_2+C_3)}{C_1+C_2+C_3} = 4\mu F$$

$\underline{t=0^-}$: $$v_c = \frac{12(R_2+R_3)}{R_1+R_2+R_3} = 6\,V$$

$\underline{t=0^+}$: $v_c = 6\,V$ $$v_o = \frac{v_c R_3}{R_2+R_3} = 4\,V = K_1+K_2$$

$\underline{t=\infty}$: $v_o = 0 = K_1$

$\underline{\tau}$: $R_{eq} = R_2 + R_3 = 6k\Omega$ $\tau = C_A R_{eq} = 24ms$

$$\boxed{v_o(t) = 4e^{-41.67t}\ V}$$

7.59 Find $i_o(t)$ for $t > 0$ in the circuit in Fig. P7.59 using the step-by-step method. CS

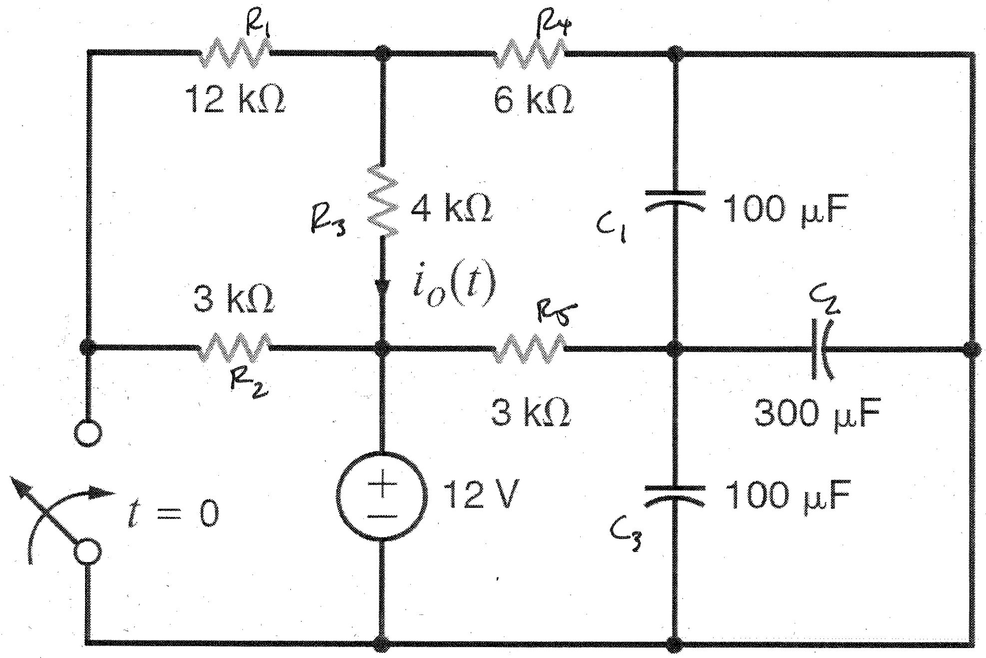
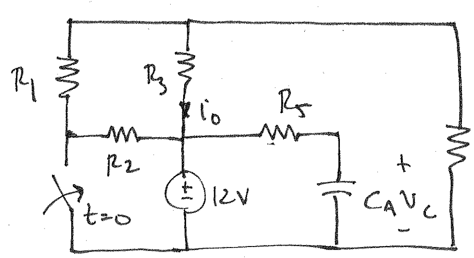


Figure P7.59

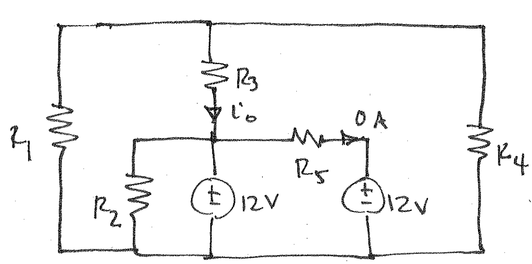
SOLUTION:



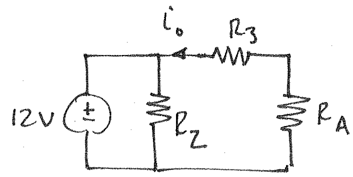
$C_A = C_1 + C_2 + C_3 = 500\mu F$

$\underline{t = 0^-}$: $v_c = 12V$

$t = 0^+$: $v_c = 12V$



$\Rightarrow$



$R_A = R_1 // R_4$
$= 4k\Omega$

$$i_o = \frac{-12}{R_3 + R_A} = -1.5mA = K_1 + K_2$$

$\underline{\underline{t = \infty}}$ Same situation as $t = 0^+$, $i_0 = -1.5mA = K_1 \Rightarrow K_2 = 0$

$i_0(t) = -1.5mA$

7.60 Find $v_o(t)$ for $t > 0$ in the network in Fig. P7.60 using the step-by-step method.

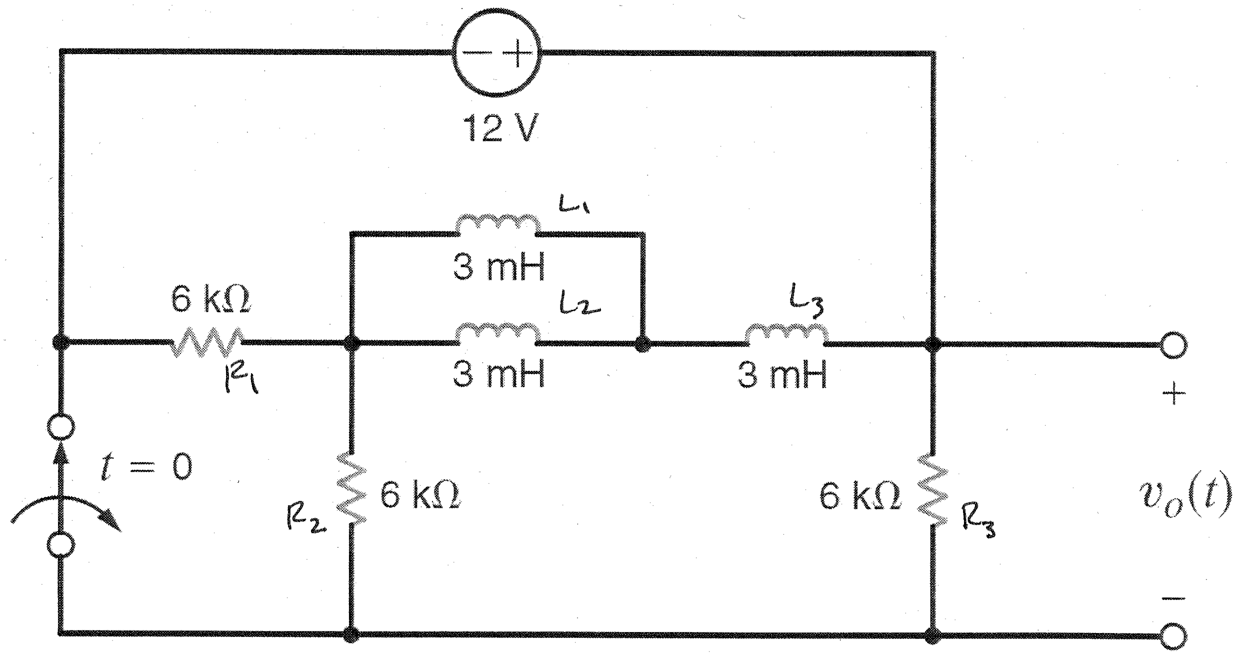
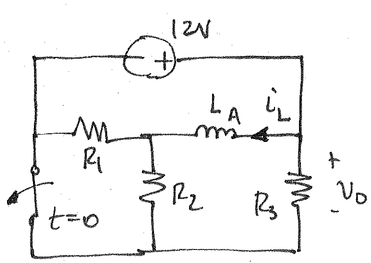


Figure P7.60

SOLUTION:

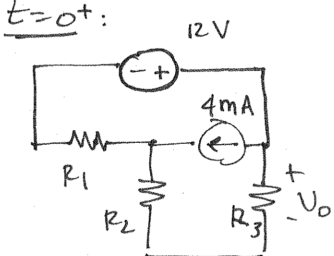


$$L_A = L_3 + \frac{L_1 L_2}{L_1 + L_2} = 4.5mH$$

$\underline{t=0^-}$

$$i_L = \frac{12}{R_1} + \frac{12}{R_2} = 4mA$$

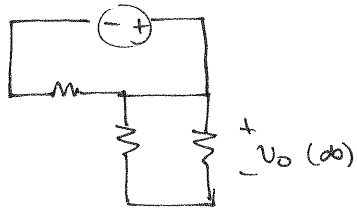
$\underline{t=0^+}$:



Superposition: $V_o = \frac{12R_3}{R_1+R_2+R_3} - \frac{4\times10^{-3} R_1}{R_1+R_2+R_3} R_3$

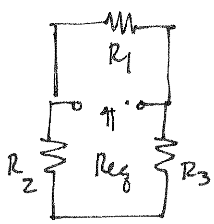
$$V_o(0+) = -4V = K_1 + K_2$$

$\underline{t = \infty}$:



$v_o(\infty) = 0\text{ V} = k_1$

$\underline{\tau = ?}$



$R_{eq} = \dfrac{R_1(R_2+R_3)}{R_1+R_2+R_3} = 4\text{k}\Omega \qquad \tau = L/R_{eq} = 1.125\ \mu s$

$$\boxed{v_o(t) = -4e^{-8.88\times10^{5}t}\ \text{V}}$$

7.61 Use the step-by-step method to find $i_o(t)$ for $t > 0$ in the network in Fig. P7.61.

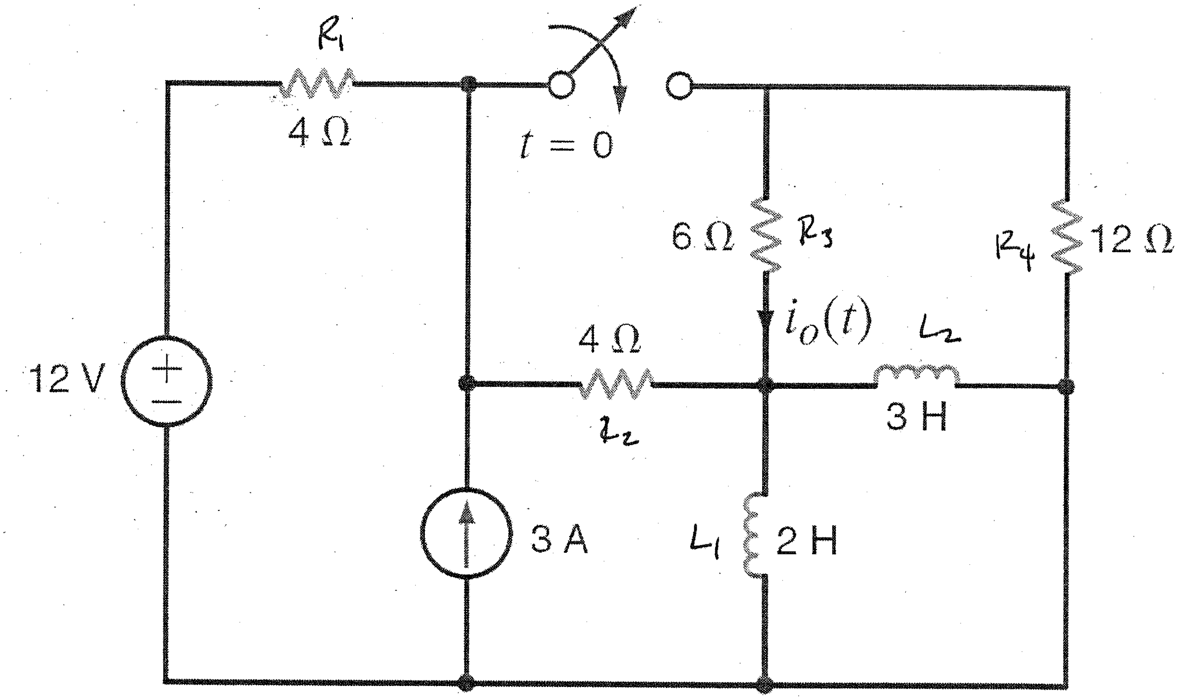
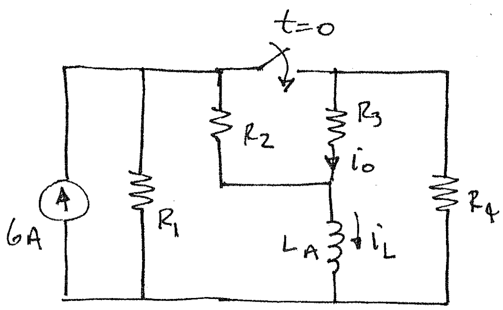


Figure P7.61

SOLUTION: Source transformation



$\underline{t=0^-}$ $\quad i_L = \dfrac{6\left(1/R_2\right)}{\frac{1}{R_1}+\frac{1}{R_2}}$

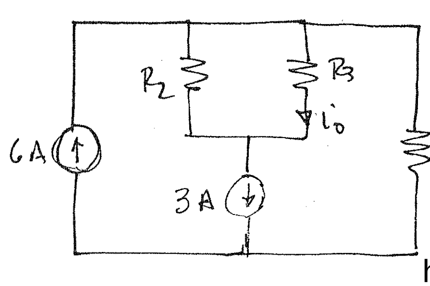
$R_A = R_2 // R_3$
$= 2.4\Omega$

$i_L = 3A$

$L_A = \dfrac{L_1L_2}{L_1+L_2} = 1.2\ H$

$\underline{t=0^+}$:

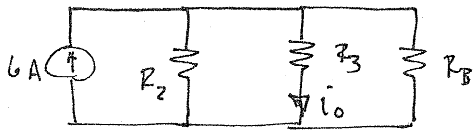
Superposition:



$i_o = \dfrac{3R_2}{R_2+R_3} = 1.2\ A = K_1 + K_2$

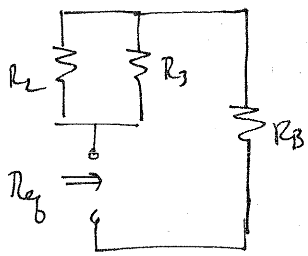
$R_B = R_1 // R_4 = 3\Omega$

$\underline{\underline{t=\infty}}$



$$i_0 = \frac{6\,(1/R_3)}{\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_B}} = \frac{4}{3}\,A = K_1$$

$\underline{\tau}$

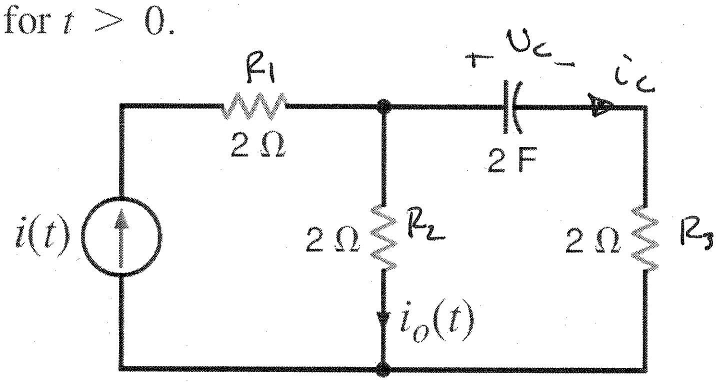


$$R_{eq} = (R_2 \,//\, R_3) + R_B = 5.4\,\Omega$$

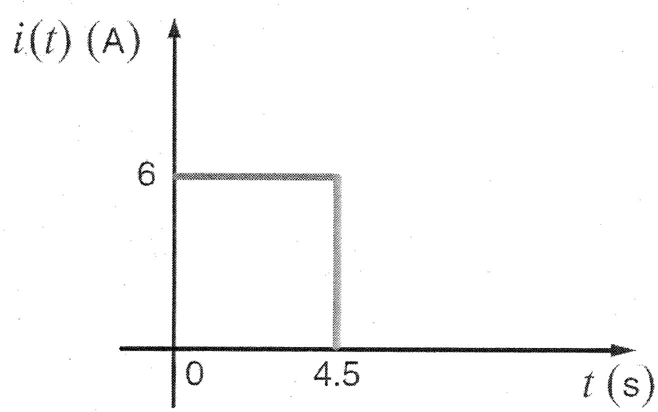
$$\tau = \frac{L_A}{R_{eq}} = 222\text{ mS}$$

$$\boxed{i_0(t) = 1.33 - 0.133\,e^{-4.5t}\ A}$$

7.62 The current source in the network in Fig. P7.62a is defined in Fig. P7.62b. The initial voltage across the capacitor must be zero. (Why?) Determine the current $i_o(t)$ for $t > 0$.



(a)



(b)

Figure P7.62

SOLUTION:

Since $i(t)$ is $\emptyset$ for $t<0$, no charge has accumulated on the capacitor and v_C must be $\emptyset$.

$v_C(t) = K_3 + K_4 e^{-t/\tau}$

$\underline{t=0^-}$: $v_C = 0$

$i_o(t) = K_1 + K_2 e^{-t/\tau}$

$\underline{t=0^+}$ $v_C = 0$, $i_o = \dfrac{i R_3}{R_2+R_3} = 3A = K_1 + K_2$

$\underline{t=\infty}$ $\quad i_c = 0\,A \qquad i_o = 6 = K_1 \qquad v_c = i_o R_2 = 12V = K_3$

$K_2 = -3A \qquad K_4 = -12V$

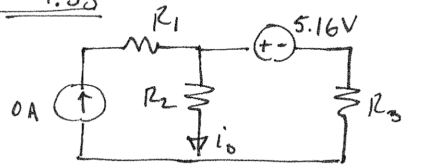
$\tau = C R_{eq} = C\,(R_2 + R_3) = 8s$

$$\left.\begin{aligned} i_o(t) &= 6 - 3e^{-t/8}\,A \\ v_c(t) &= 12 - 12e^{-t/8}\,V \end{aligned}\right\} \quad 0 \le t \le 4.5s$$

$\underline{t = 4.5s}$ $\quad v_c(4.5) = 5.16\,V \qquad i_o = K_5 + K_6 e^{-t'/\tau_2} \qquad t > 4.5s$

$t' = t - 4.5$

$\underline{t = 4.5s^+}$



$i_o = \dfrac{5.16}{R_2 + R_3} = 1.29 = K_5 + K_6$

$\underline{t \to \infty}$ $\quad i_o = 0 = K_5 \quad \Rightarrow K_6 = 1.29A$

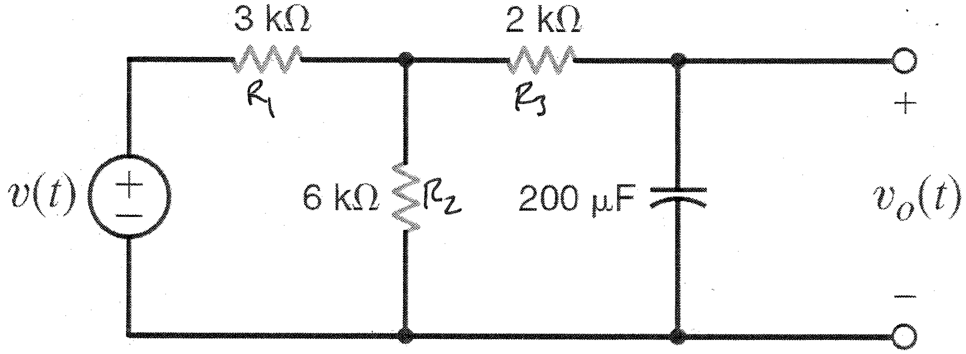
$\tau_2 = C R_{eq_2} = C\,[R_2 + R_3] = 8s$

$i_o(t) = 1.29e^{-t'/8}\,A \qquad t > 4.5s$

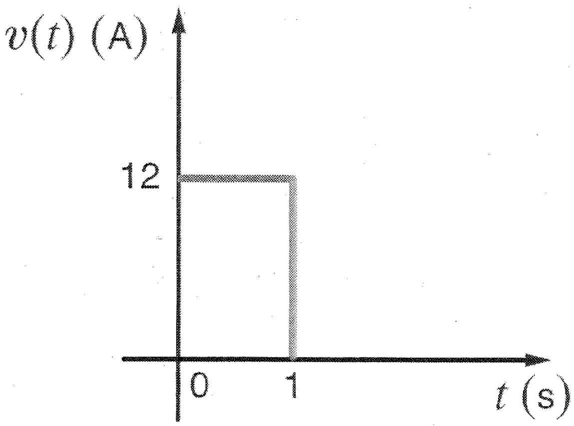
final answer

$$\boxed{i_o(t) = \begin{cases} 6 - 3e^{-t/8}\,A & 0 \le t \le 4.5s \\ 1.29e^{-(t-4.5)/8}\,A & t > 4.5s \end{cases}}$$

7.63 Determine the equation for the voltage $v_o(t)$ for $t > 0$, in Fig. P7.63a when subjected to the input pulse shown in Fig. P7.63b.



(a)



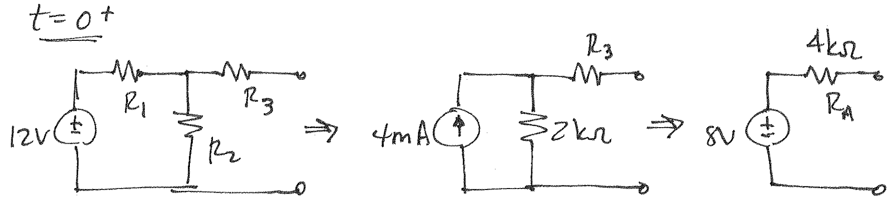
(b)

Figure P7.63

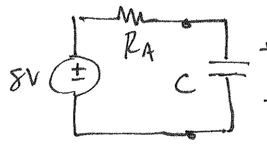
SOLUTION: $v_o = K_1 + K_2 e^{-t/\tau}$

$t = 0^-$ $v_o = 0$

$t = 0^+$



$\underline{t=0^+}$



$v_o = v_o(0^-) = 0\,V = K_1 + K_2$

$\underline{t=\infty}$: $v_o = 8V = K_1$

$\underline{\tau}$ $\tau = CR_A = 0.8s$

$v_o(t) = 8 - 8e^{-1.25t}$ V $\quad 0 < t \le 1$

for $t > 1s$, $v_o = K_3 + K_4 e^{-t'/\tau}$ $\quad t' = t - 1$

$\underline{\text{at } t = 1^-}$, $v_o = 5.71\,V$

$\underline{\text{at } t = 1^+}$, $v_o = 5.71V = K_3 + K_4$

$\underline{\text{at } t = \infty}$ $v_o = 0 = K_3 \Rightarrow K_4 = 5.71\,V$

$$\boxed{v_o = \begin{cases} 8 - 8e^{-1.25t}\ \text{V} & 0 \le t \le 1 \\ 5.71\,e^{-1.25(t-1)}\ \text{V} & t > 1 \end{cases}}$$

7.64 Find the output voltage $v_o(t)$ in the network in Fig. P7.64 if the input voltage is $v_i(t) = 5(u(t) - u(t - 0.05))$ V.

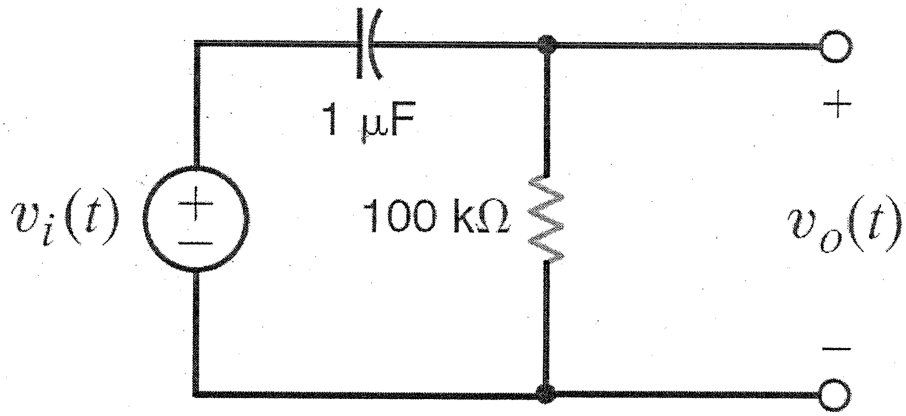
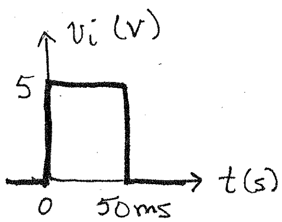


Figure P7.64

SOLUTION:



For $0 \le t \le 50ms$ $\quad v_o = K_1 + K_2 e^{-t/\tau}$

For $t > 50\,ms$ $\quad v_o = K_3 + K_4 e^{-t/\tau}$

$\underline{t = 0^-}$ $\quad v_o = 0$ & $v_c = 0\,V$

$\underline{t = 0^+}$ $\quad v_c = 0$ & $v_o = v_i = 5 = K_1 + K_2$

$\underline{t \to \infty}$ $\quad v_o = 0 = K_1 \Rightarrow K_2 = 5\,V$

$\tau = CR = 0.1\,s \Rightarrow v_o(t) = 5e^{-10t}$ $\quad 0 \le t \le 50ms$

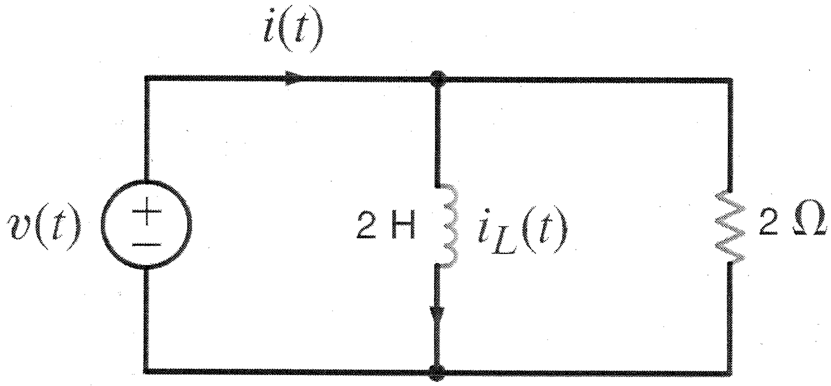
$\underline{\text{at } t = 50ms^-}$ $\quad v_o = 3.03\,V$ & $v_c = 1.97\,V$

$\underline{t = 50ms^+}$ $\quad v_c = 1.97\,V$ & $v_o = v_i - v_c = -1.97\,V = K_3 + K_4$

$\underline{t \to \infty}$ $\quad v_o = 0 = K_4 \Rightarrow v_o(t) = -1.97\,e^{-10(t-0.05)}\,V$ $\quad t > 50ms$

$$\boxed{v_o = \begin{cases} 5\,e^{-10t}\,V & 0 \le t \le 50ms \\ -1.97\,e^{-10(t-0.05)}\,V & t > 50ms \end{cases}}$$

7.65 The voltage $v(t)$ shown in Fig. P7.65a is given by the graph shown in Fig. P7.65b. If $i_L(0) = 0$, answer the following questions: (a) how much energy is stored in the inductor at $t = 3$ s?, (b) how much power is supplied by the source at $t = 4$ s?, (c) what is $i(t = 6\text{ s})$?, and (d) how much power is absorbed by the inductor at $t = 3$ s?



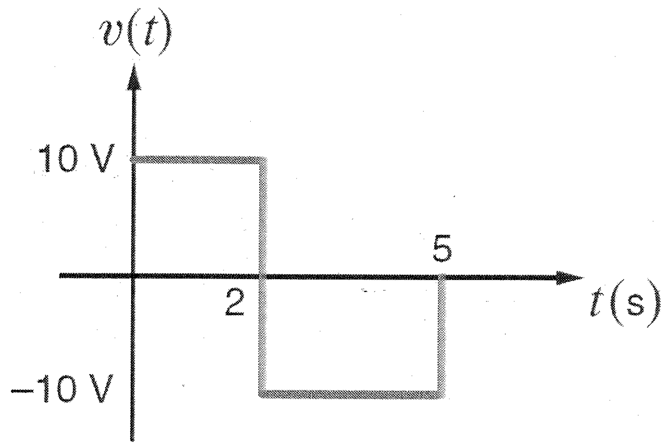


Figure P7.65

SOLUTION:

a) $w_L = \frac{1}{2} L i_L^2$ $\quad i_L(t) = \frac{1}{L}\int v_L\, dt = \frac{1}{L}\int v\, dt$

$i_L(3) = \frac{10}{2} t\Big|_0^2 - \frac{10}{2} t\Big|_2^3 = 5\text{A}$ $\quad \boxed{w_L(3) = 25\text{J}}$

b) $p_S(t) = v(t)\, i(t) = v(t)\left[i_L(t) + v(t)/R \right]$

$i_L(4) = \frac{1}{L}\int_0^4 v(t)\,dt = 5t\Big|_0^2 - 5t\Big|_2^4 = 0A$

$p_S(\cancel{t}4) = v^2(4)/R = 100/2$ $\boxed{p_S(4) = 50W}$

c) $i(6) = i_L(6) + \frac{v(6)}{R}$ $v(6) = 0$

$i_L(6) = \frac{1}{L}\int_0^6 v(\cancel{t})\,dt = 5t\Big|_0^2 - 5t\Big|_2^5 = -5\,A$

$\boxed{i(6) = -5A}$

d) $P_L = v(t)\, i_L(t)$ $i_L(3) = 5A$ $v(3) = -10V$

$\boxed{P_L(3) = -50W \text{ absorbed}}$

7.66 In the circuit in Fig. P7.66, $v_R(t) = 100e^{-400t}$ V for $t < 0$. Find $v_R(t)$ for $t > 0$.

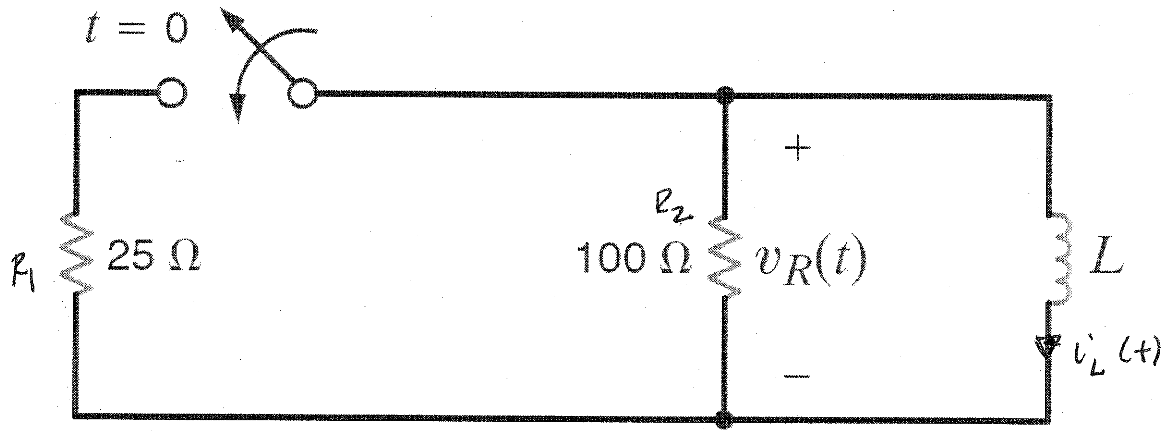


Figure P7.66

SOLUTION:

$\underline{t=0^-}$ $\quad v_R(0^-) = 100\text{V} \qquad i_L(0^-) = -\dfrac{v_R(0^-)}{R_2} = -1\text{A}$

$\tau_1 = \dfrac{L}{R_2} = \dfrac{1}{400} \Rightarrow L = \dfrac{1}{4}\text{ H}$

$\underline{t=0^+}$ $\quad i_L(0^+) = -1\text{A} \qquad v_R(0^+) = -\dfrac{i_L(0^+)\, R_2 R_1}{R_1 + R_2} = 20\text{V} = K_1 + K_2$

$\underline{t=\infty}$ $\quad v_R = 0 = K_1 \Rightarrow K_2 = 20\text{V}$

$\underline{\tau}$ $\quad \tau_2 = \dfrac{L(R_1+R_2)}{R_1 R_2} = \dfrac{1}{80}\text{ s}$

$$\boxed{v_R(t) = 20e^{-80t}\text{ V}}$$

7.67 Given that $v_{C1}(0-) = -10$ V and $v_{C2}(0-) = 20$ V in the circuit in Fig. P7.67, find $i(0+)$.

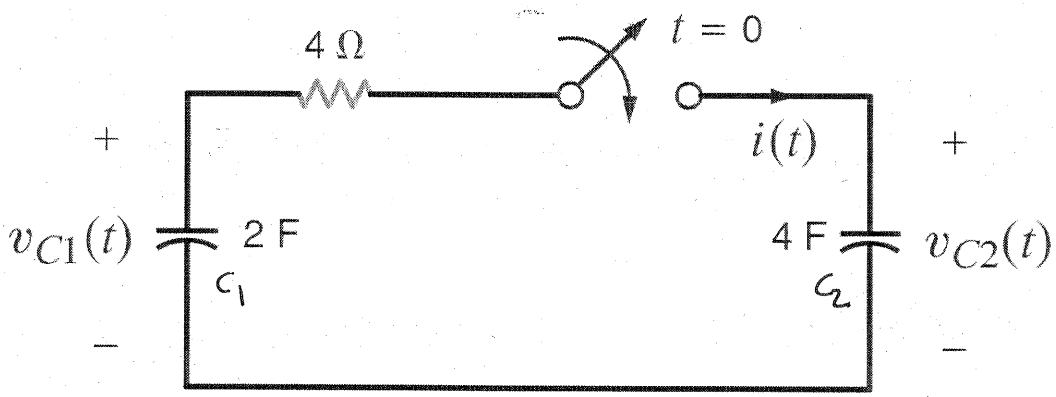
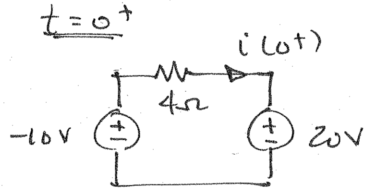


Figure P7.67

SOLUTION:

V_{C1} & V_{C2} cannot change instantaneously.

$t = 0^+$



$$i(0^+) = \frac{-10-20}{4} = -7.5A$$

$$\boxed{i(0^+) = -7.5A}$$

7.68 The switch in the circuit in Fig. P7.68 is closed at $t = 0$. If $i_1(0-) = 2$ A, determine $i_2(0+)$, $v_R(0+)$, and $i_1(t = \infty)$.

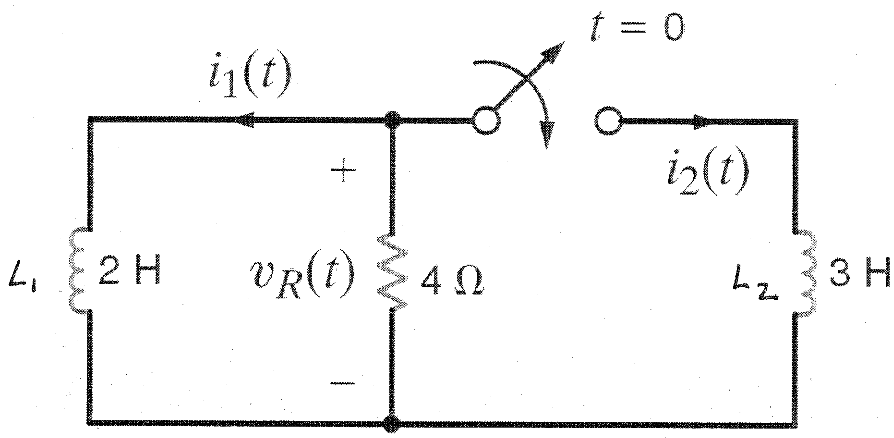


Figure P7.68

SOLUTION:

$\underline{t=0^-}$ $\quad i_1(0^-) = 2A \qquad i_2(0^-) = 0A$

$\underline{t=0^+}$ $\quad i_1(0^+) = i_1(0^-) = 2A \qquad i_2(0^+) = i_2(0^-) = 0A$

$v_R(0+) = -i_1(0+)(4) = -8V$

$$\boxed{\begin{aligned} v_R(0^+) &= -8V \\ i_2(0^+) &= 0A \\ i_1(\infty) &= 0A \end{aligned}}$$

$\underline{t=\infty}$ all $v(t)$ & $i(t) \to 0$

$i_1(\infty) = 0$

7.69 In the network in Fig. P7.69 find $i(t)$ for $t > 0$. If $v_{C1}(0-) = -10$ V, calculate $v_{C2}(0-)$.

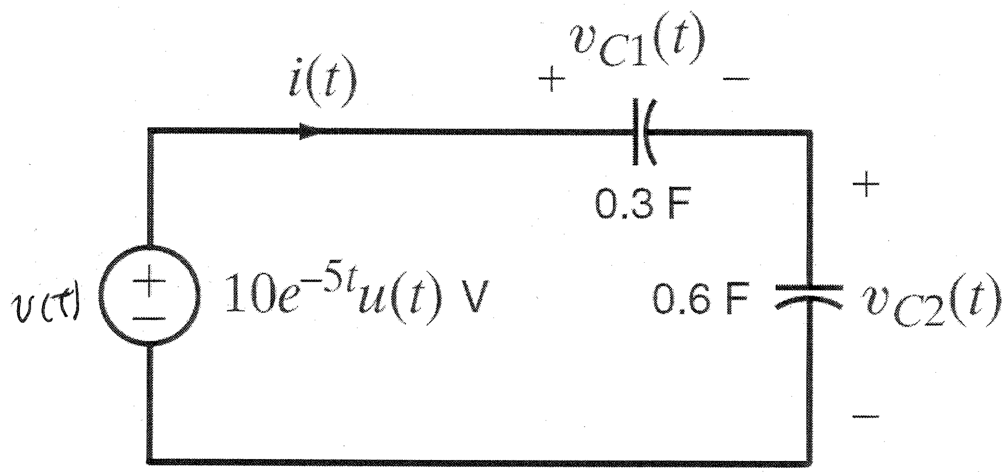
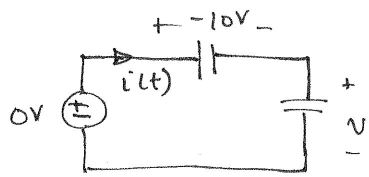


Figure P7.69

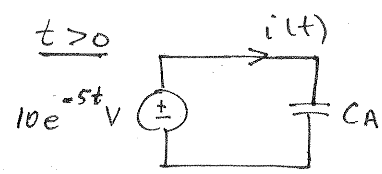
SOLUTION:

at $t = 0^-$



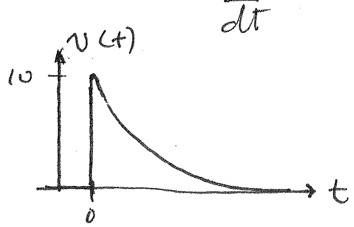
$v_{C2}(0^-) = -v_{C1}(0^-) = 10V$

$t > 0$



$C_A = \dfrac{C_1 C_2}{C_1 + C_2} = 0.2F$

$i(t) = C \dfrac{dv}{dt}$



$\dfrac{dv}{dt} = 10\,\delta(t) - 50e^{-5t} \qquad t \geq 0$

$$\boxed{\begin{array}{l} i(t) = 2\delta(t) - 10e^{-5t} \text{ A} \qquad t \geq 0 \\ v_2(0^-) = 10V \end{array}}$$

7.70 The switch in the circuit in Fig. P7.70 has been closed for a long time and is opened at $t = 0$. If $v_C(t) = 20 - 8e^{-0.05t}$ V, find R_1, R_2, and C.

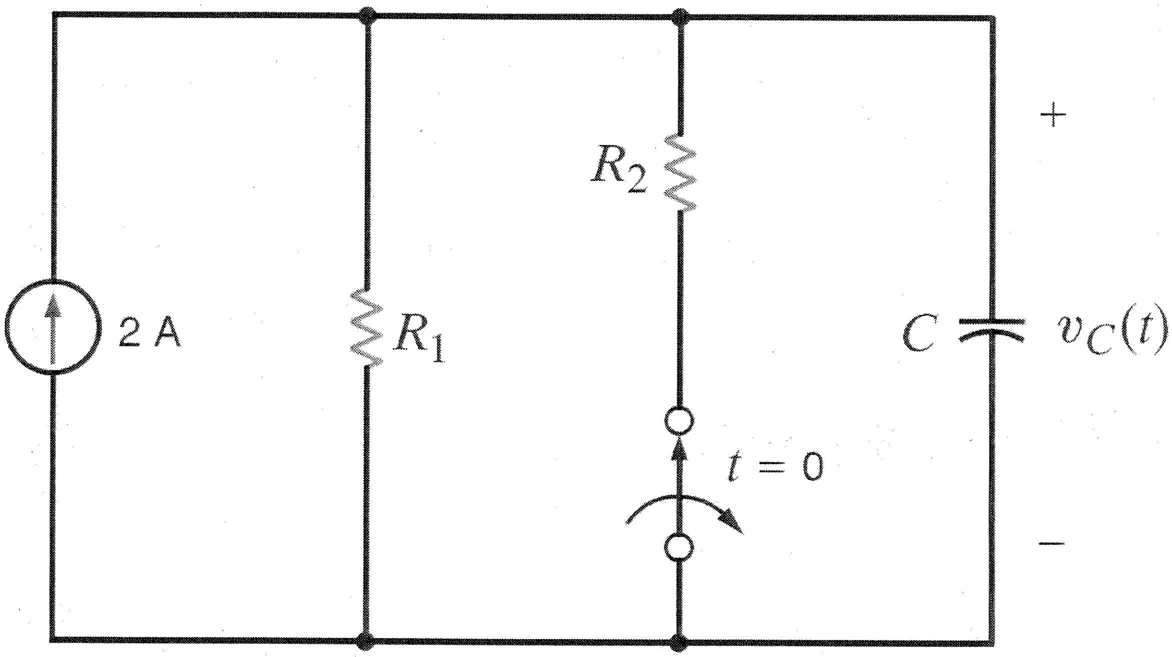


Figure P7.70

SOLUTION:

$$v_c(t) = 20 - 8e^{-t/20} \text{ V} = k_1 + k_2 e^{-t/\tau}$$

$$K_1 = v_c(\infty) = 2R_1 = 20 \qquad R_1 = 10\Omega$$

$$K_1 + K_2 = v_c(0^+) = v_c(0^-) = \frac{2(R_1 R_2)}{R_1 + R_2} = 12 \Rightarrow \qquad R_2 = 15\Omega$$

$$\tau = CR_1 = 20\text{ s} \qquad C = 2F$$

$$\boxed{\begin{array}{l} C = 2F \\ R_1 = 10\Omega \\ R_2 = 15\Omega \end{array}}$$

7.71 Given that $i(t) = 13.33e^{-t} - 8.33e^{-0.5t}$ A for $t > 0$ in the network in Fig. P7.71, find the following: (a) $v_C(0)$, (b) $v_C(t = 1\text{ s})$, and (c) the capacitance C.

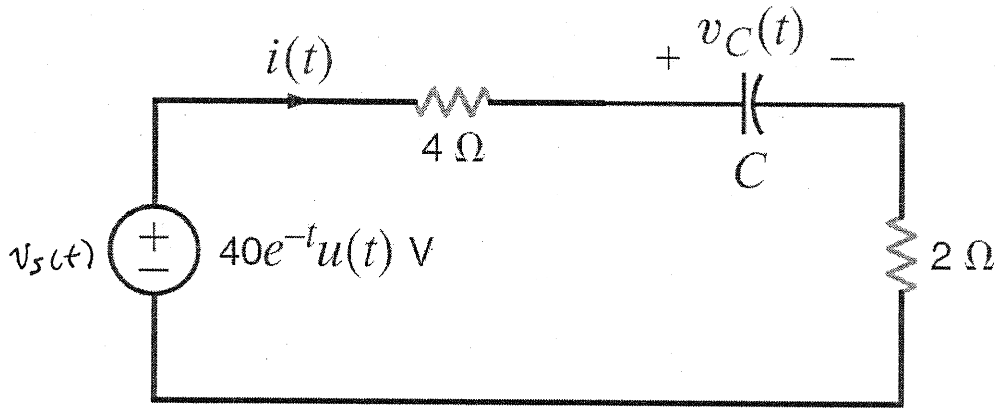


Figure P7.71

SOLUTION:

a) $v_c(0^-) = v_s(0^-) = 0\text{V} = v_c(0^+)$ $\boxed{v_c(0) = 0\text{V}}$

b) $v_c(t) = \frac{1}{C}\int i\,dt + K$

$$= \frac{1}{C}\left[16.66e^{-t/2} - 13.33e^{-t}\right] + K$$

$v_c(0) = 0 = \frac{1}{C}[3.33] + k \Rightarrow k = -3.33/C$

Need C.

c) $\tau = 2 = C[4+2] = 6C \Rightarrow \boxed{C = 1/3\text{ F}}$

Back to b)

$k = -10$ $v_c(t) = 50e^{-t/2} - 40e^{-t} - 10$

$\boxed{v_c(1) = 5.61\text{ V}}$

7.72 Given that $i(t) = 2.5 + 1.5e^{-4t}$ A for $t > 0$ in the circuit in Fig. P7.72, find R_1, R_2, and L.

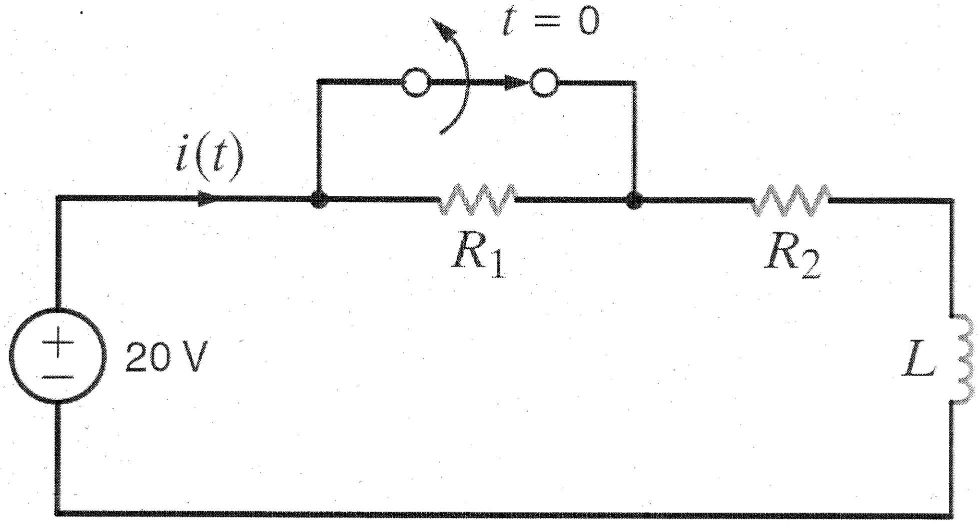


Figure P7.72

SOLUTION:

$i(t) = 2.5 + 1.5e^{-4t} = K_1 + K_2 e^{-t/\tau}$

$K_1 = 2.5 = i(\infty) = \frac{20}{R_1+R_2} \Rightarrow R_1 + R_2 = 8\,\Omega$

$K_1 + K_2 = 4 = i(0^+) = i_L(0^+) = i_L(0^-) = \frac{20}{R_2} \Rightarrow R_2 = 5\,\Omega$

$R_1 = 3\,\Omega$

$\tau = \frac{1}{4} = \frac{L}{R_1+R_2} \qquad L = 2H$

$$\boxed{\begin{array}{l} L = 2H \\ R_1 = 3\,\Omega \\ R_2 = 5\,\Omega \end{array}}$$

7.73 The differential equation that describes the current $i_o(t)$ in a network is

$$\frac{d^2 i_o(t)}{dt^2} + 6\left[\frac{di_o(t)}{dt}\right] + 4i_o(t) = 0$$

Find **(a)** the characteristic equation of the network, **(b)** the network's natural frequencies, and **(c)** the expression for $i_o(t)$.

SOLUTION:

a) $s^2 + 6s + 4 = 0$

b) $s_{1,2} = \frac{-6 \pm \sqrt{36-16}}{2} = \begin{Bmatrix} -0.764 \\ -5.24 \end{Bmatrix} = s_{1,2}$

c) $2\zeta\omega_0 = 6 \qquad \omega_0^2 = 4 \qquad \Rightarrow \zeta = 1.5$ over damped.

$$\boxed{i_o(t) = K_1 e^{-0.764t} + K_2 e^{-5.24t}.}$$

7.74 The terminal current in a network is described by the equation

$$\frac{d^2 i_o(t)}{dt^2} + 8\left[\frac{di_o(t)}{dt}\right] + 16 i_o(t) = 0$$

Find **(a)** the characteristic equation of the network, **(b)** the network's natural frequencies, and **(c)** the equation for $i_o(t)$.

SOLUTION:

a) $s^2 + 8s + 16 = 0$

b) $s_{1,2} = \frac{-8 \pm \sqrt{64-64}}{2} = -4 \text{ r/s}$

c) $2\zeta\omega_0 = 8 \quad \omega_0^2 = 16 \Rightarrow \zeta = 1 \Rightarrow$ critically damped.

$$\boxed{i_o(t) = B_1 e^{-4t} + B_2 t e^{-4t}}$$

7.75 The voltage $v_1(t)$ in a network is defined by the equation

$$\frac{d^2v_1(t)}{dt^2} + 2\left[\frac{dv_1(t)}{dt}\right] + 5v_1(t) = 0$$

Find

(a) the characteristic equation of the network.

(b) the circuit's natural frequencies.

(c) the expression for $v_1(t)$. CS

SOLUTION:

a) $s^2 + 2s + 5 = 0$

b) $s_{1,2} = \dfrac{-2 \pm \sqrt{4-20}}{2} = -1 \pm j2 \text{ r/s}$

c) underdamped!

$$v_1(t) = e^{-t}\left[A_1 \cos 2t + A_2 \sin 2t\right]$$

7.76 The output voltage of a circuit is described by the differential equation

$$\frac{d^2v_o(t)}{dt^2} + 8\left[\frac{dv_o(t)}{dt}\right] + 10v_o(t) = 0$$

Find **(a)** the characteristic equation of the circuit, **(b)** the network's natural frequencies, and **(c)** the equation for $v_o(t)$.

SOLUTION:

a) $s^2 + 8s + 10 = 0$

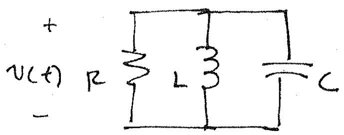
b) $s_{1,2} = \frac{-8 \pm \sqrt{64-40}}{2} = \begin{cases} -1.55 & r/s \\ -6.45 & r/s \end{cases}$

c) overdamped!

$$\boxed{v_o(t) = K_1 e^{-1.55t} + K_2 e^{-6.45t}}$$

7.77 The parameters for a parallel *RLC* circuit are $R = 1\ \Omega$, $L = 1/2$ H, and $C = 1/2$ F. Determine the type of damping exhibited by the circuit.

SOLUTION:



$$\frac{v(t)}{R} + \frac{1}{L}\int v\,dt + C\frac{dv}{dt} = 0$$

$$C\frac{d^2v}{dt^2} + \frac{1}{R}\frac{dv}{dt} + \frac{1}{L}v(t) = 0$$

$$\frac{d^2v}{dt^2} + \frac{1}{RC}\frac{dv}{dt} + \frac{1}{LC}v(t) = 0$$

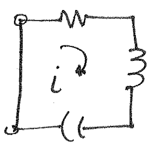
$$\omega_0^2 = \frac{1}{LC} = 4 \Rightarrow \omega_0 = 2\ r/s$$

$$2\zeta\omega_0 = 2 \Rightarrow \zeta = 1/2$$

$$\boxed{\text{underdamped}}$$

7.78 A series *RLC* circuit contains a resistor $R = 2\ \Omega$ and a capacitor $C = 1/2$ F. Select the value of the inductor so that the circuit is critically damped.

SOLUTION:



$$Ri + L\frac{di}{dt} + \frac{1}{C}\int i\,dt = 0$$

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}i(t) = 0$$

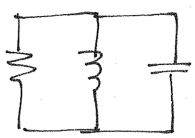
$$\frac{1}{LC} = \omega_0^2 \qquad 2\zeta\omega_0 = \frac{R}{L}$$

for $\zeta = 1$, $\quad 2\omega_0 = \frac{R}{L} \Rightarrow \frac{2}{\sqrt{LC}} = \frac{R}{L} \Rightarrow \sqrt{L} = \frac{R\sqrt{C}}{2}$

$$L = \frac{R^2C}{4}. \qquad \boxed{L = \frac{1}{2}H}$$

7.79 A parallel *RLC* circuit contains a resistor $R = 1\ \Omega$ and an inductor $L = 2$ H. Select the value of the capacitor so that the circuit is critically damped.

SOLUTION:



$$s^2 + \frac{s}{RC} + \frac{1}{LC} = 0$$

$$\omega_0^2 = \frac{1}{LC} \qquad 2\zeta\omega_0 = \frac{1}{RC} \qquad \zeta = 1$$

$$2\omega_0 = \frac{1}{RC} = \frac{2}{\sqrt{LC}} \Rightarrow \sqrt{C} = \frac{\sqrt{L}}{2R} \Rightarrow C = \frac{L}{4R^2} = \frac{1}{2}\ F$$

$$\boxed{C = \frac{1}{2}\ F}$$

7.80 For the underdamped circuit shown in Fig. P7.80, determine the voltage $v(t)$ if the initial conditions on the storage elements are $i_L(0) = 1$ A and $v_C(0) = 10$ V. CS

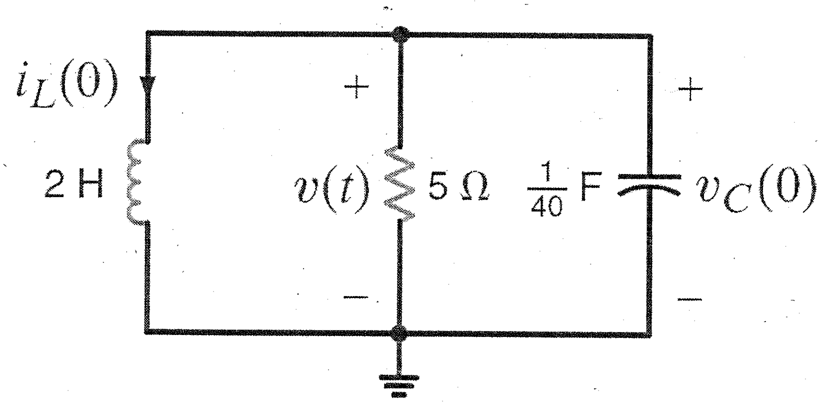


Figure P7.80

SOLUTION:

characteristic eq: $s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0 \Rightarrow s^2 + 8s + 20 = 0$

natural freq: $s_{1,2} = \frac{-8 \pm \sqrt{64-80}}{2}$ $\quad s_{1,2} = -4 \pm j2$ r/s

$v(t) = e^{-4t}\left[A_1 \cos 2t + A_2 \sin 2t\right]$

$v_c(0) = v(0) = A_1 = 10$

$i_L(0) = 1 = -\frac{v(0)}{5} - C\frac{dv}{dt}\Big|_{t=0}$ $\quad v(0) = 10$ V

$\frac{dv}{dt} = -4e^{-4t}\left[A_1 \cos 2t + A_2 \sin 2t\right] + e^{-4t}\left[-2A_1 \sin 2t + 2A_2 \cos 2t\right]$

$\frac{dv}{dt}\Big|_{t=0} = -40 + 2A_2 \quad \rightarrow \quad 1 = -2 + \frac{40}{40} - \frac{2A_2}{40} \Rightarrow A_2 = -40$ V

$$\boxed{v(t) = e^{-4t}\left[10 \cos 2t - 40 \sin 2t\right] \text{ V}}$$

7.81 In the critically damped circuit shown in Fig. P7.81, the initial conditions on the storage elements are $i_L(0) = 2$ A and $v_C(0) = 5$ V. Determine the voltage $v(t)$.

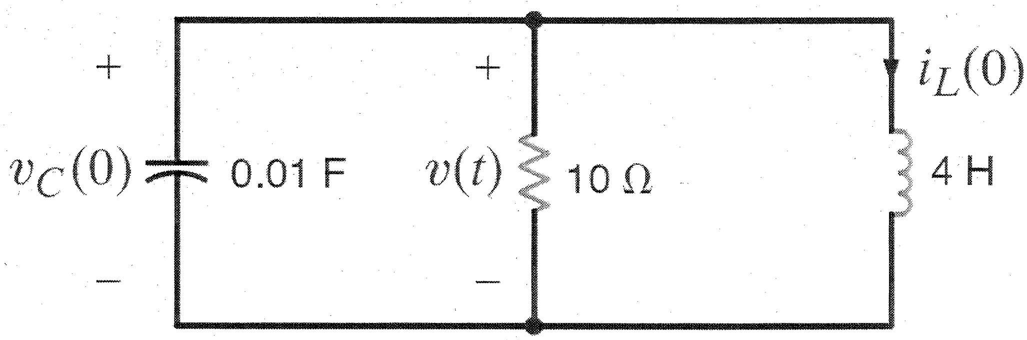


Figure P7.81

SOLUTION:

characteristic eq: $s^2 + \frac{s}{RC} + \frac{1}{LC} = 0 = s^2 + 10s + 25 = 0$

natural freq: $s_{1,2} = \frac{-10 \pm \sqrt{100-100}}{2} = -5$ critically damped!

$v(t) = B_1 e^{-5t} + B_2 t e^{-5t}$ V

$v_C(0) = v(0) = B_1 = 5$

$i_L(0) = 2 = -\frac{v(0)}{R} - C\frac{dv}{dt}\Big|_{t=0} = -\frac{1}{2} - C\frac{dv}{dt}\Big|_{t=0}$

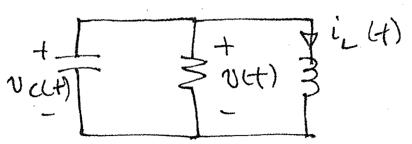
$\frac{dv}{dt}\Big|_{t=0} = \left[-5B_1 e^{-5t} + B_2 e^{-5t} - 5B_2 t e^{-5t}\right]_{t=0} = -5B_1 + B_2$

$2 = -\frac{1}{2} + \frac{25 - B_2}{100} \Rightarrow B_2 = -225$

$$\boxed{v(t) = 5e^{-5t} - 225te^{-5t} \text{ V}}$$

7.82 Given the circuit and the initial conditions from Problem 7.81, determine the current $i_L(t)$ that is flowing through the inductor.

SOLUTION:



$R = 10\Omega \quad L = 4H \quad C = 0.01F$

$v_C(0^-) = 5V \quad i_L(0^-) = 2A$

from 7.81, $\quad v(t) = 5e^{-5t} - 22.5te^{-5t}$ V

$i_L = \frac{1}{L}\int v\,dt$

from integration tables: $\int te^{-5t}dt = -\frac{te^{-5t}}{5} - \frac{1}{25}e^{-5t}$

$i_L = \frac{1}{L}\int 5e^{-5t} - 22.5te^{-5t}dt = 2e^{-5t} + 11.25te^{-5t}$ A

$$\boxed{i_L(t) = 2e^{-5t} + 11.25te^{-5t}\,A}$$

7.83 Find $v_C(t)$ for $t > 0$ in the circuit in Fig. P7.83. CS

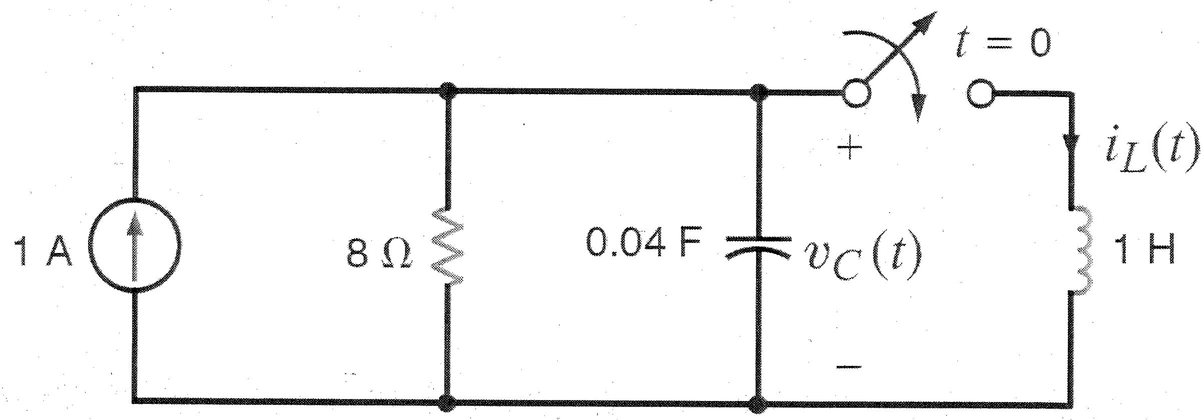


Figure P7.83

SOLUTION:

$t=0^-$: $v_c(0^-) = 8V$ $\quad i_L(0^-) = 0$

$t>0$: $\frac{d^2 v_c}{dt^2} + \frac{1}{RC}\frac{dv_c}{dt} + \frac{v_c}{LC} = 0 \Rightarrow s^2 + 3.125s + 25 = 0$

natural frequencies: $s_{1,2} = -1.5625 \pm j4.75$ r/s $= \sigma \pm j\omega$

$v_c(t) = e^{-\sigma t}\left[A_1 \cos\omega t + A_2 \sin\omega t\right]$

$v_c(0^-) = A_1 = 8$

$-i_L(0^-) = 0 = \frac{v_c}{R}\Big|_{t=0} + C\frac{dv_c}{dt}\Big|_{t=0} = \frac{A_1}{R} + C\left[A_2\omega - \sigma A_1\right]$

yields $A_2 = -2.63$ V

$$\boxed{\begin{array}{c} v_c(t) = e^{-\sigma t}\left[8\cos\omega t - 2.63\sin\omega t\right] \\ \sigma = 1.5625 \quad \omega = 4.75 \text{ r/s} \end{array}}$$

7.84 Find $v_C(t)$ for $t > 0$ in the circuit in Fig. P7.84 if $v_C(0) = 0$.

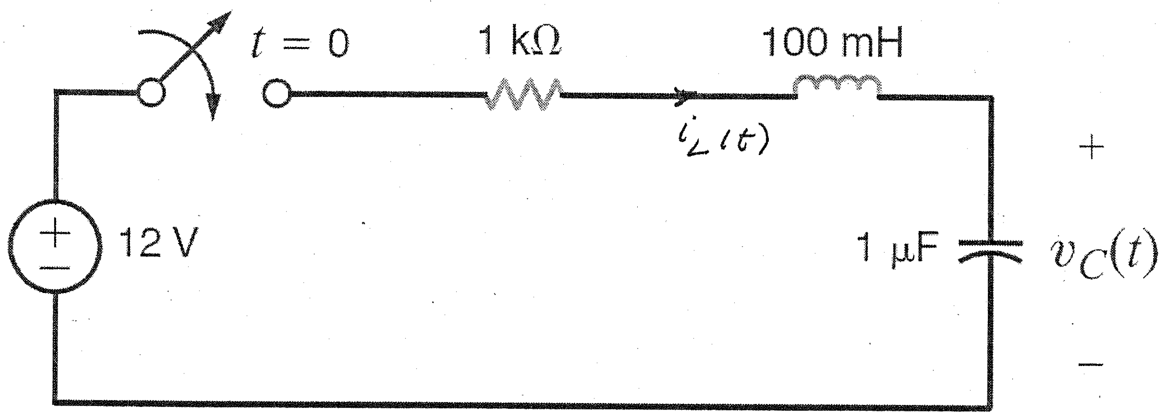


Figure P7.84

SOLUTION: $\underline{t=0^-}$: $v_c = 0 \quad i_L = 0$

$$\frac{di_L^2}{dt^2} + \frac{R}{L}\frac{di_L}{dt} + \frac{i_L}{LC} = 0 \Rightarrow s^2 + 10^4 s + 10^7 = 0$$

natural frequencies: $S_{1,2} = \begin{cases} -1127 = -\sigma_1 \\ -8873 = -\sigma_2 \end{cases}$

$$i_L(t) = K_1 e^{-1127t} + K_2 e^{-8873t} = K_1 e^{-\sigma_1 t} + K_2 e^{-\sigma_2 t}$$

$i_L(0) = 0 = K_1 + K_2$

$$12 = R i_L(0^+) + L\left.\frac{di_L}{dt}\right|_{t=0^+} + v_c(0^+) = RK_1 + RK_2 - L\sigma_1 K_1 - L\sigma_2 K_2 + v_c(0^+)$$

$v_c(0^+) = v_c(0^-) = 0V \Rightarrow K_1 = 15.5 \text{ mA} \quad K_2 = -15.5 \text{ mA}$

$$v_c(t) = \frac{1}{C}\int i_L(t)\,dt + K_3 \Rightarrow v_c(0^+) = 0 = -\frac{K_1}{C\sigma_1} - \frac{K_2}{C\sigma_2} + K_3$$

$K_3 = 12V$

$$\boxed{v_c(t) = 17.5e^{-\sigma_2 t} - 13.75e^{-\sigma_1 t} + 12 \text{ V} \quad \sigma_1 = 1127 \quad \sigma_2 = 8873}$$

7.85 Find $v_o(t)$ for $t > 0$ in the circuit in Fig. P7.85 and plot the response including the time interval just prior to closing the switch. **PSV**

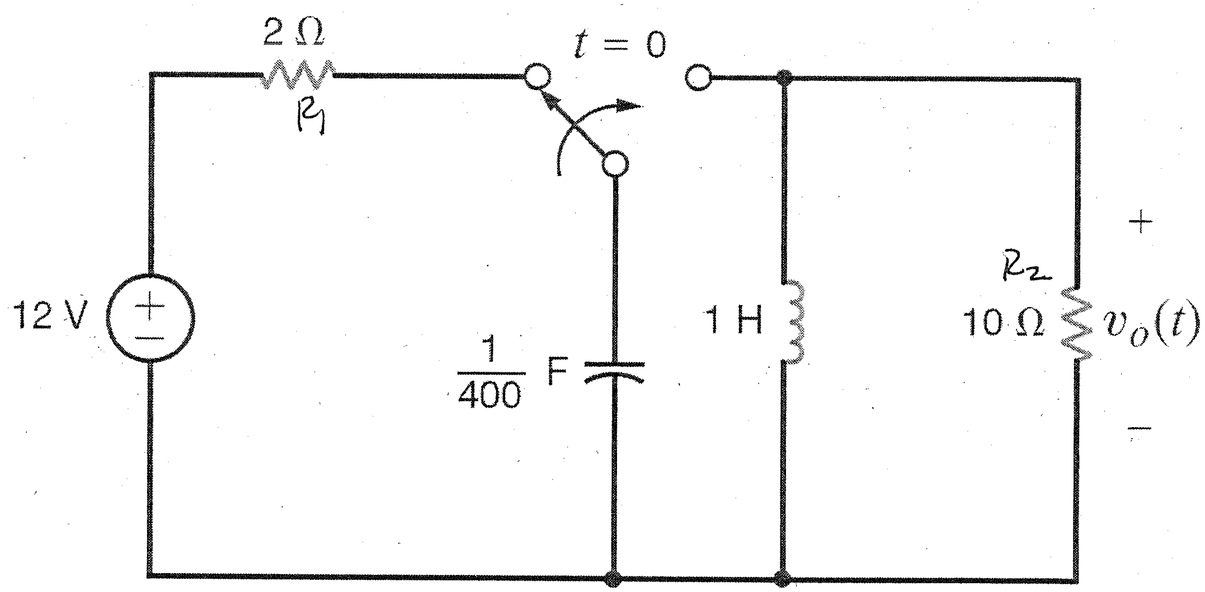


Figure P7.85

SOLUTION: $\underline{t=0^-}$ $v_C(0^-) = 12V$ $i_L(0^+) = 0 = i_L(0^-)$

$\underline{t>0}$ $\frac{d^2v_o}{dt^2} + \frac{dv_o}{dt}\left(\frac{1}{R_2C}\right) + \frac{v_o}{LC} = 0 \Rightarrow s^2 + 40s + 400 = 0$

natural frequencies: $s_{1,2} = -20$ r/s

$v_o(t) = B_1e^{-20t} + B_2te^{-20t}$

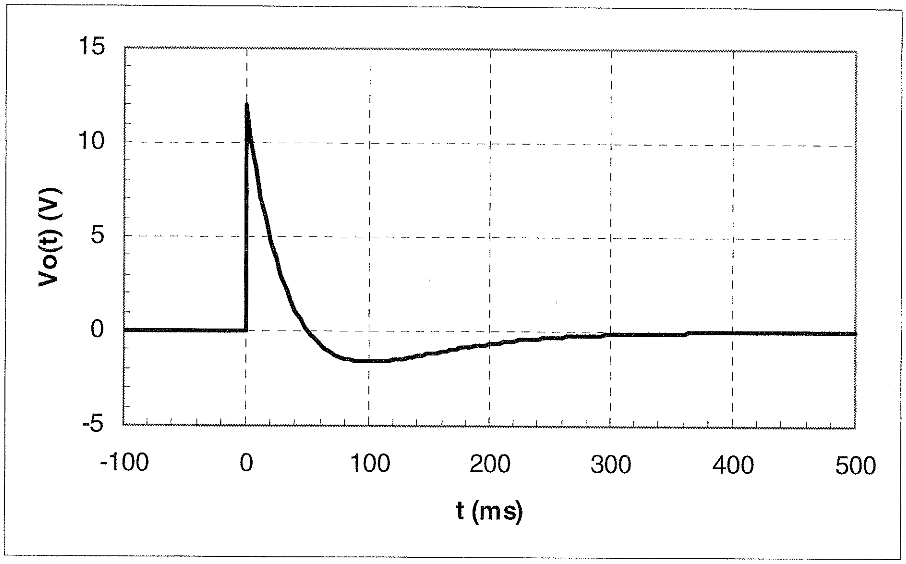
$v_o(0^+) = v_C(0^+) = 12V = B_1$

$-i_L(0^+) = 0 = \frac{v_o(0^+)}{R_2} + C\left.\frac{dv_o}{dt}\right|_{t=0^+} = \frac{12}{R_2} + C\left[-20B_1 + B_2\right]$

$B_2 = -240V$

$$\boxed{v_o(t) = 12e^{-20t} - 240te^{-20t}\ V}$$

PROBLEM 7.85



7.86 Find $v_o(t)$ for $t > 0$ in the circuit in Fig. P7.86 and plot the response including the time interval just prior to closing the switch.

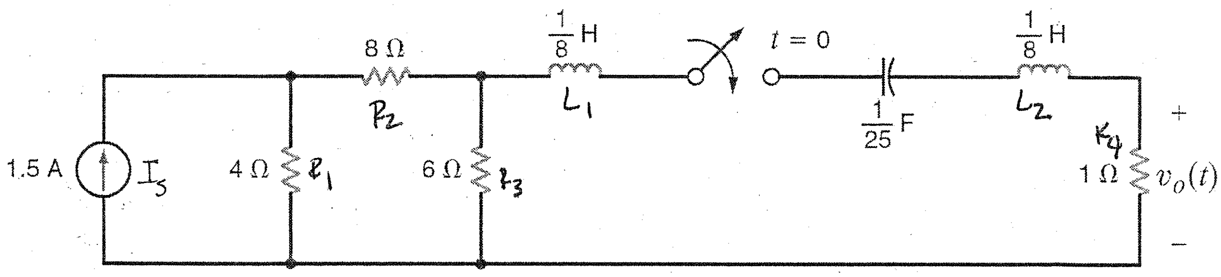
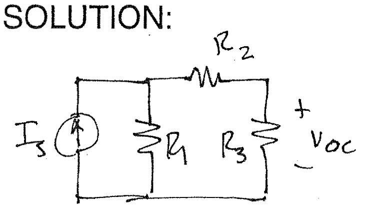
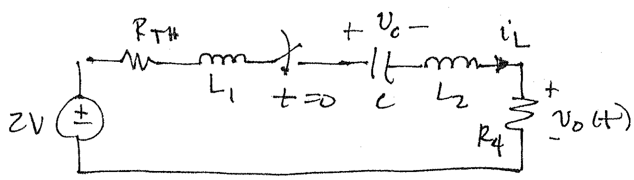


Figure P7.86

SOLUTION:



$$V_{oc} = \frac{I_s R_1}{R_1+R_2+R_3} R_3 = 2V \qquad R_{TH} = R_3 // (R_1+R_2) = 4\Omega$$



$\underline{t=0^-}$: $i_{L1} = i_{L2} = i_L = 0A$

$v_c(0^-) = 0V$

$\underline{t>0}$:

$$\frac{d^2 i_L}{dt^2} + \frac{R}{L}\frac{di_L}{dt} + \frac{1}{LC} i_L = 0 \Rightarrow s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

$R = R_{TH} + R_4$

$L = L_1 + L_2$

$$s_{1,2} = -10 \text{ r/s} \qquad i_L(t) = B_1 e^{-10t} + B_2 t e^{-10t}$$

$$i_L(0^+) = i_L(0^-) = 0 = B_1 \Rightarrow i_L(t) = B_2 t e^{-10t}$$

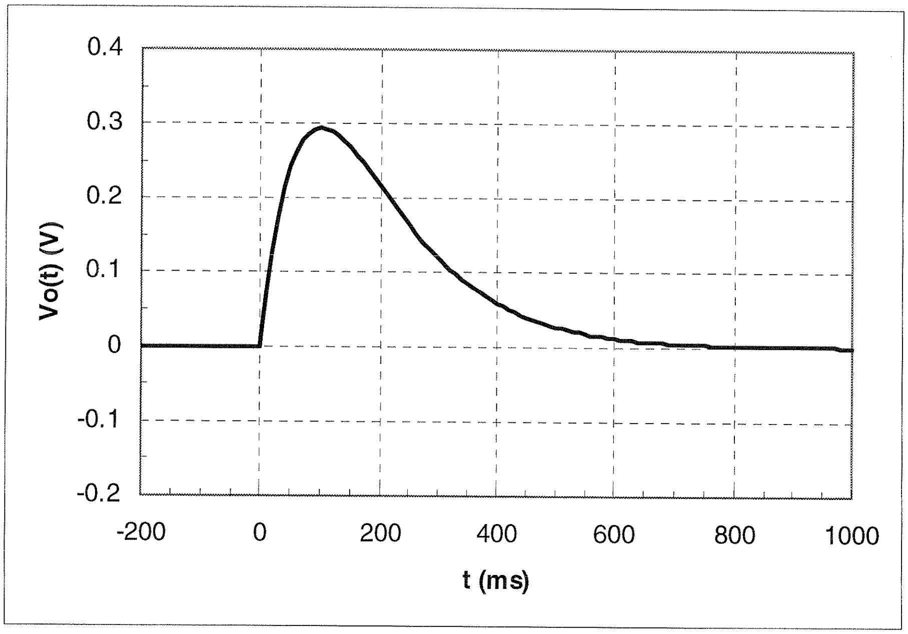
$$v_c(0^+) = 0 = 2 - R\, i_L(0^+) - L \left.\frac{di_L}{dt}\right|_{t=0} = 2 - 0 - L B_2$$

$$B_2 = \frac{2}{L} = 8$$

$$i_L(t) = 8te^{-10t} \text{ A}$$

$$v_o = R_4 i_L(t) \qquad \boxed{v_o(t) = 8te^{-10t} \text{ V}}$$

PROBLEM 7.86



7.87 In the circuit shown in Fig. P7.87, switch action occurs at $t = 0$. Determine the voltage $v_o(t)$, $t > 0$.

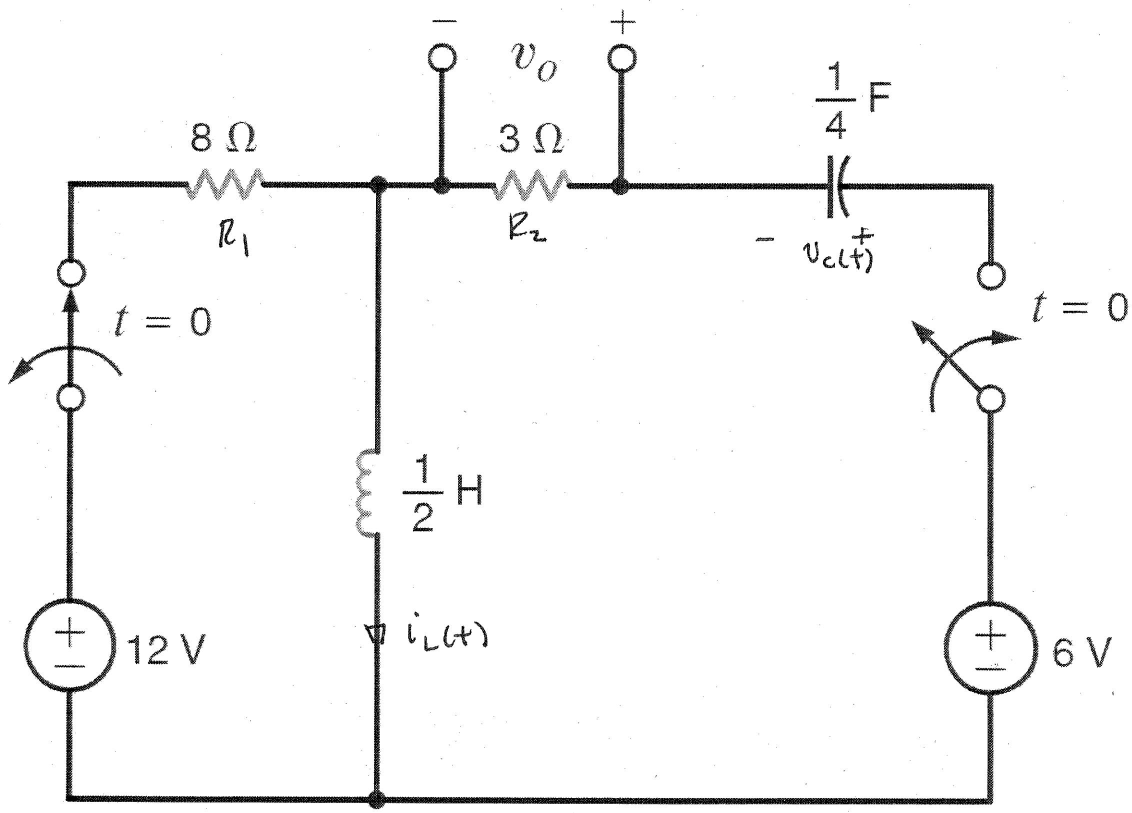


Figure P7.87

SOLUTION: $t=0^-$: $i_L(0^-) = 12/R_1 = 1.5A$ $\quad v_c(0^-) = 0V$

$t>0$: $6 = \frac{1}{C}\int i_L dt + R_2 i_L(t) + L\frac{di_L}{dt} \Rightarrow \frac{d^2 i}{dt^2} + \frac{R_2}{L}\frac{di_L}{dt} + \frac{i_L(t)}{LC} = 0$

$s^2 + 6s + 8 = 0 \Rightarrow s_{1,2} = \begin{Bmatrix} -2 = -\sigma_1 \\ -4 = -\sigma_2 \end{Bmatrix}$

$i_L(t) = K_1 e^{-\sigma_1 t} + K_2 e^{-\sigma_2 t}$ $\quad i_L(0^+) = 1.5 = K_1 + K_2$

$6 = v_c(0^+) + R_2 i_L(0^+) + L\frac{di_L}{dt}\Big|_{t=0} = 0 + R_2(1.5) - LK_1\sigma_1 - LK_2\sigma_2$

yields,

$K_1 = 4.5V$ & $K_2 = -3V$ $\quad v_o = R_2 i_L$ $\quad \boxed{v_o = 4.5e^{-2t} - 3e^{-4t}\ V}$

7.88 Find $v_o(t)$ for $t > 0$ in the circuit in Fig. P7.88 and plot the response including the time interval just prior to moving the switch.

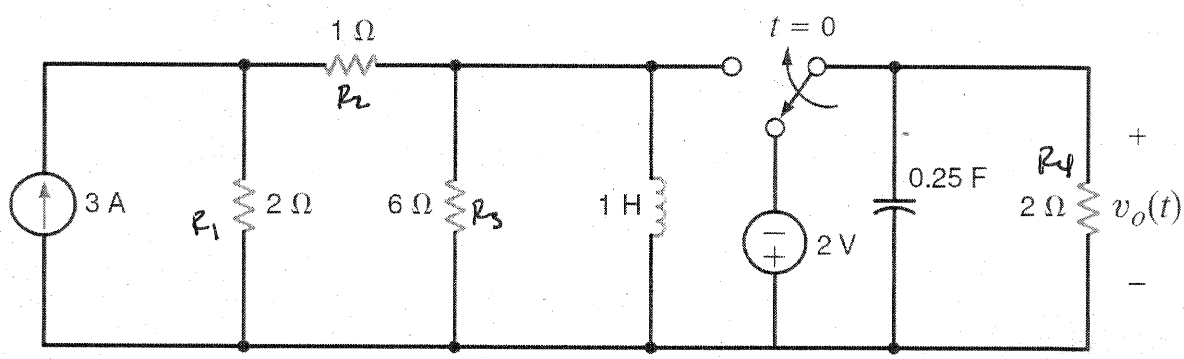
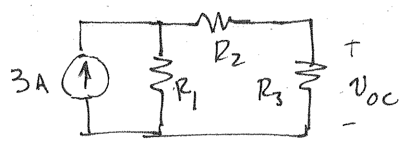


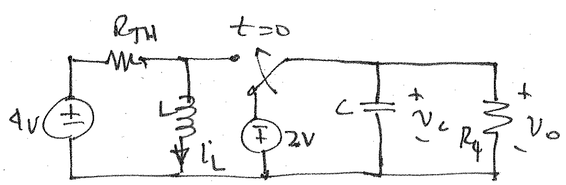
Figure P7.88

SOLUTION:



$$v_{oc} = 3\left[\frac{R_1 R_3}{R_1+R_2+R_3}\right] = 4\text{V}$$

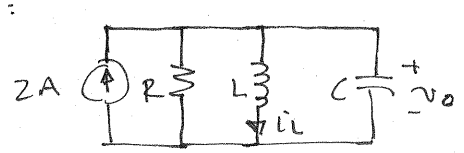
$$R_{TH} = R_3 \,//\, [R_1 + R_2] = 2\Omega$$



$\underline{t<0:}$ $i_L = 4/R_{TH} = 2\text{A}$

$v_C = -2\text{V}$

$\underline{t>0:}$



$$R = R_4 \,//\, R_{TH} = 1\Omega$$

$$\frac{d^2v_o}{dt^2} + \frac{dv_o}{dt}\left(\frac{1}{RC}\right) + \frac{v_o}{LC} = 0$$

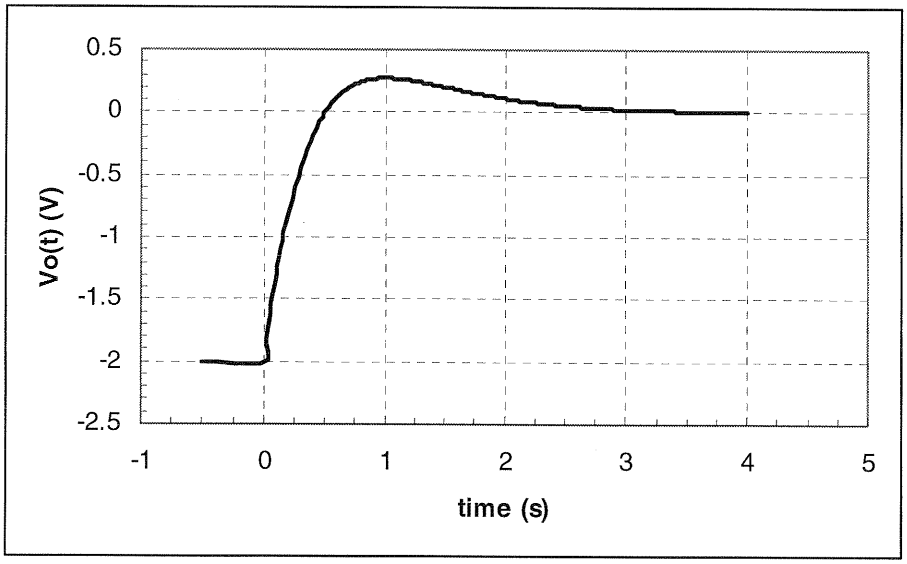
$s^2 + 4s + 4 = 0 \Rightarrow s_{1,2} = -2 \text{ r/s}$ $\quad v_o(t) = B_1 e^{-2t} + B_2 t e^{-2t}$

$v_o(0^+) = v_C(0^-) = -2\text{V} = B_1$

$$2 - i_L(0^+) = 0 = \frac{v_o(0^+)}{R} + C\left.\frac{dv_o}{dt}\right|_{t=0^+} = \frac{B_1}{R} - 2B_1C + CB_2 \Rightarrow B_2 = 4\text{V}$$

$$\boxed{v_o(t) = -2e^{-2t} + 4te^{-2t} \text{ V}}$$

PROBLEM 7.88



7.89 The switch in the circuit in Fig. P7.89 has been closed for a long time and is opened at $t = 0$. Find $i(t)$ for $t > 0$. **PSV**

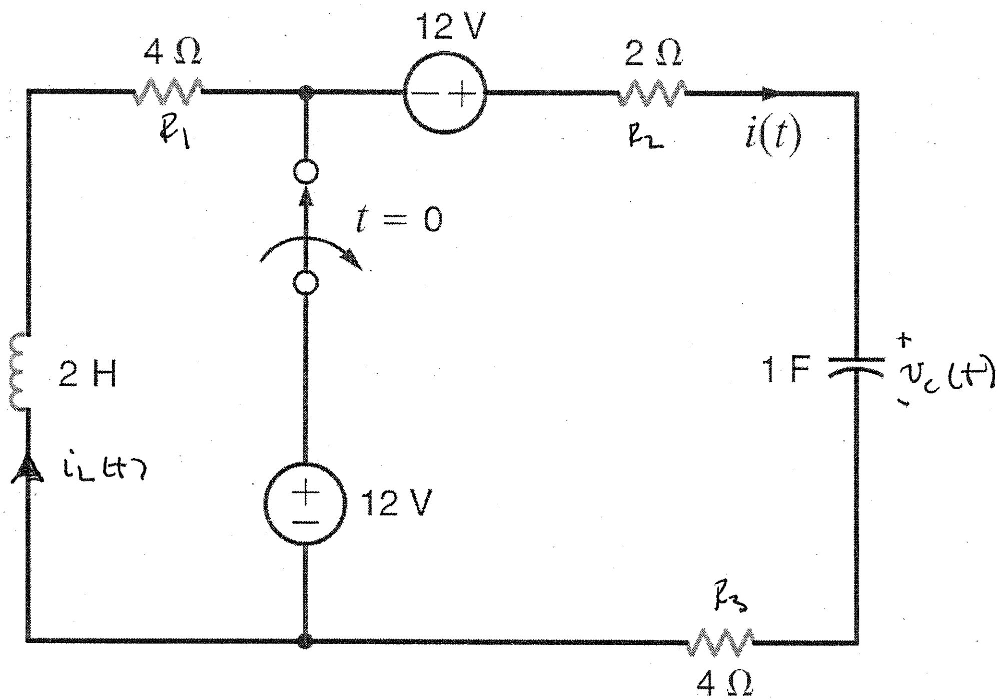


Figure P7.89

SOLUTION: $t=0^-$: $i_L(0^-) = -\frac{12}{R_1} = -3\,A$ $\quad v_c(0^-) = 24V$

$t>0$:

$$12 = (R_1+R_2+R_3)\,i(t) + \frac{1}{C}\int i(t)\,dt + L\,di(t)/dt = Ri(t) + \frac{1}{C}\int i(t)dt + L\frac{di}{dt}$$

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0 \Rightarrow s^2 + 5s + 0.5 = 0 \qquad s_{1,2} = \begin{cases} -0.10 = -\sigma_1 \\ -4.89 = -\sigma_2 \end{cases}$$

$$i(t) = K_1 e^{-\sigma_1 t} + K_2 e^{-\sigma_2 t} \qquad i(0^+) = i_L(0^+) = -3 = K_1 + K_2$$

$$v_c(0^+) = 12 - Ri(0^+) - L\left.\frac{di}{dt}\right|_{t=0^+} = 12 + 3R + LK_1\sigma_1 + LK_2\sigma_2 = 24$$

yields

$K_1 = -1.19\,A$

$K_2 = -1.81\,A$

$$\boxed{i(t) = -1.19e^{-0.10t} - 1.81e^{-4.89t}\ A}$$

7.90 The switch in the circuit in Fig. P7.90 has been closed for a long time and is opened at $t = 0$. Solve for $i(t)$ for $t > 0$.

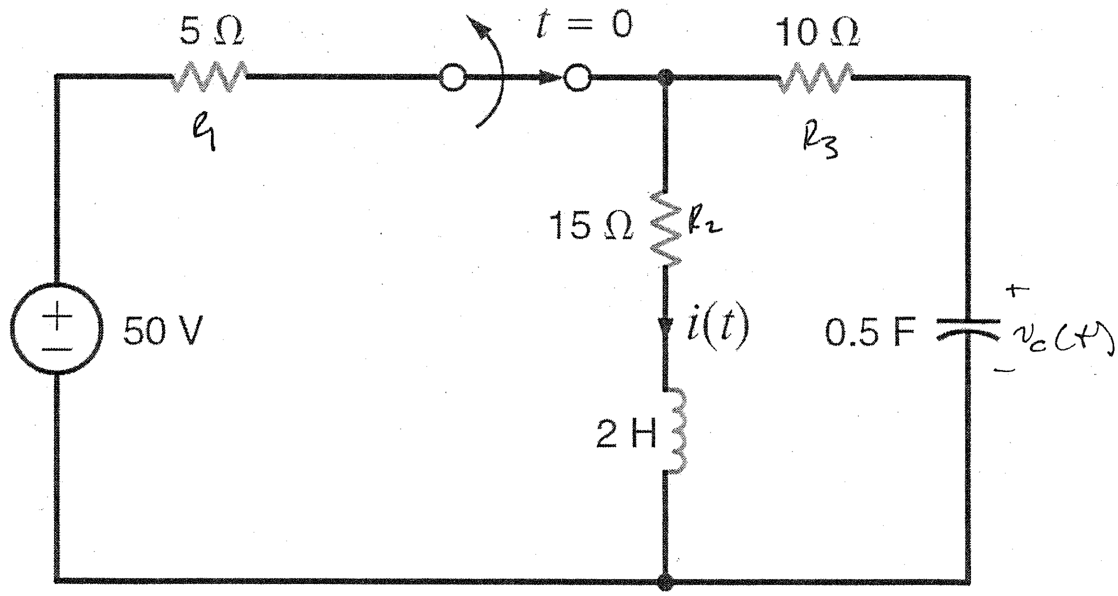


Figure P7.90

SOLUTION: $\underline{t=0^-}$: $i(0^-) = \frac{50}{R_1+R_2} = 2.5A$ $\quad v_c(0^-) = i(0^-)R_2 = 37.5V$

$\underline{t>0}$: $L\frac{di}{dt} + R\,i(t) + \frac{1}{C}\int i\,dt = 0$ $\quad R = R_2 + R_3 = 25$

$v_c(t) = -\frac{1}{C}\int i\,dt$

$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0 = s^2 + 12.5s + 1$

$s_{1,2} = \begin{Bmatrix} -0.08 = -\sigma_1 \\ -12.42 = -\sigma_2 \end{Bmatrix}$ $\quad i(t) = K_1 e^{-\sigma_1 t} + K_2 e^{-\sigma_2 t}$

$i(0^+) = K_1 + K_2 = 2.5$ $\quad v_c(0^+) = 37.5 = R\,i(0^+) + L\frac{di}{dt}\Big|_{t=0}$

yields $\quad = R(2.5) - L\sigma_1 K_1 - L\sigma_2 K_2$

$K_1 = 1.5 \quad K_2 = 1$

$$\boxed{i(t) = e^{-\sigma_2 t} + 1.5e^{\sigma_1 t}\,A \quad \sigma_1 = 0.08 \quad \sigma_2 = 12.42}$$

7.91 The switch in the circuit in Fig. P7.91 has been closed for a long time and is opened at $t = 0$. Solve for $i(t)$ for $t > 0$.

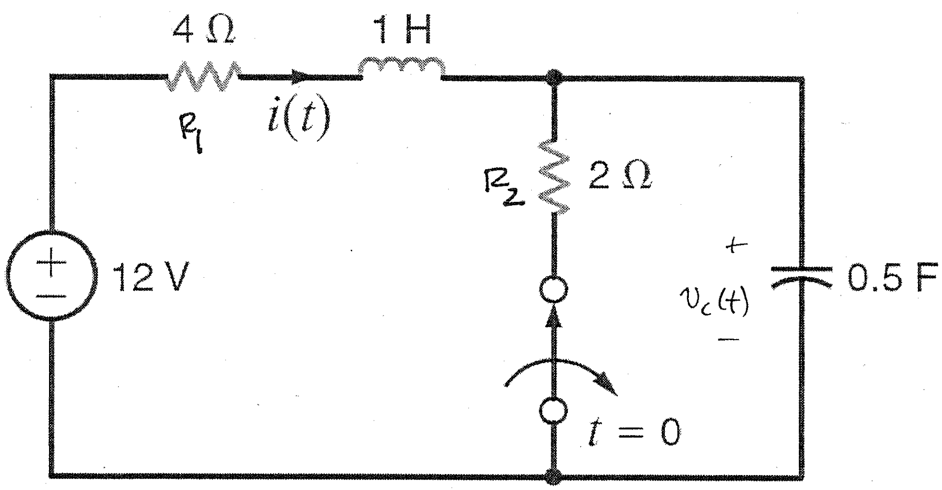


Figure P7.91

SOLUTION: $\underline{t=0^-}$: $i(0^+) = \dfrac{12}{R_1+R_2} = 2A \qquad V_c(0^+) = i(0^+)R_2 = 4V$

$\underline{t=0^+}$ $\quad 12 = L\, di/dt + R_1 i_1 + \frac{1}{C}\int i(t)dt \Rightarrow \dfrac{d^2i}{dt^2} + \dfrac{4di}{dt} + 2 = 0$

$$s^2 + 4s + 1/2 = 0 \qquad S_{1,2} = \begin{Bmatrix} -0.586 = -\sigma_1 \\ -3.41 = -\sigma_2 \end{Bmatrix}$$

$$i(t) = K_1 e^{-\sigma_1 t} + K_2 e^{-\sigma_2 t} \qquad i(0) = 2 = k_1 + k_2$$

$$v_c(0^+) = 12 - R_1 i(0^+) - L\left.\frac{di}{dt}\right|_{t=0} = 12 - 8 + \sigma_1 K_1 + \sigma_2 K_2 = 4$$

$$K_1 + K_2 = 2 \quad \& \quad \sigma_1 K_1 + \sigma_2 K_2 = 0 \Rightarrow k_1 = 2.414 \ \& \ K_2 = -0.414$$

$$\boxed{i(t) = 2.414 e^{-\sigma_1 t} - 0.414 e^{-\sigma_2 t} \text{ A} \qquad \sigma_1 = 0.586, \ \sigma_2 = 3.41}$$

7.92 The switch in the circuit in Fig. P7.92 has been closed for a long time and is opened at $t = 0$. Find $i(t)$ for $t > 0$.

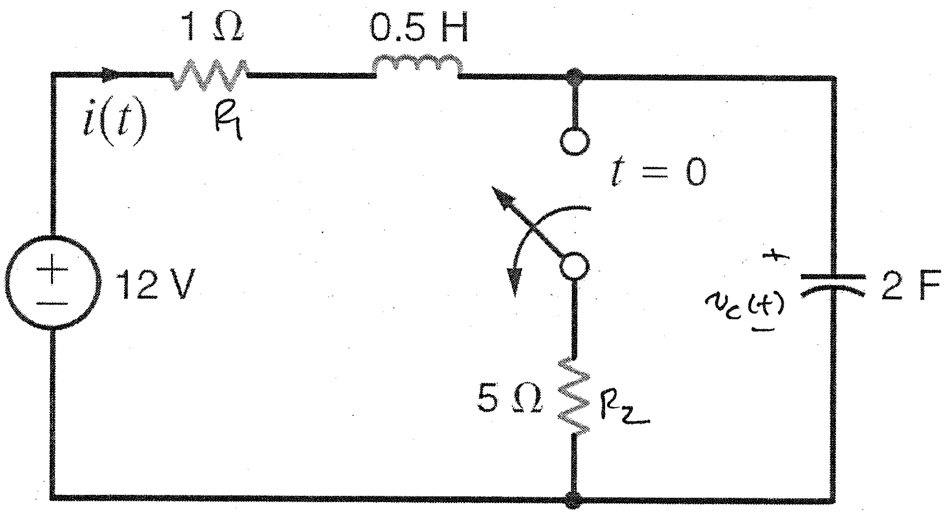


Figure P7.92

SOLUTION: $\underline{t=0^-}$: $i(0^-) = \dfrac{12}{R_1+R_2} = 2A$ $\quad v_c(0^-) = \dfrac{12\,R_2}{R_1+R_2} = 10V$

$\underline{t=0^+}$: $12 = L\,di/dt + R_1 i + \frac{1}{C}\int i\,dt \Rightarrow s^2 + \frac{R_1}{L}s + \frac{1}{LC} = 0$

$s_{1,2} = -1\ r/s \qquad i(t) = B_1 e^{-t} + B_2 t e^{-t}$

$i(0^+) = 2A = B_1 \qquad 12 = L\left.\dfrac{di}{dt}\right|_{t=0} + R_1 i(0^+) + v_c(0^+)$

$12 = -LB_1 + LB_2 + R_1B_1 + 10 \Rightarrow B_2 = 2$

$\boxed{i(t) = 2e^{-t} + 2te^{-t}}$

7.93 The switch in the circuit in Fig. P7.93 has been closed for a long time and is opened at $t = 0$. Find $i(t)$ for $t > 0$.

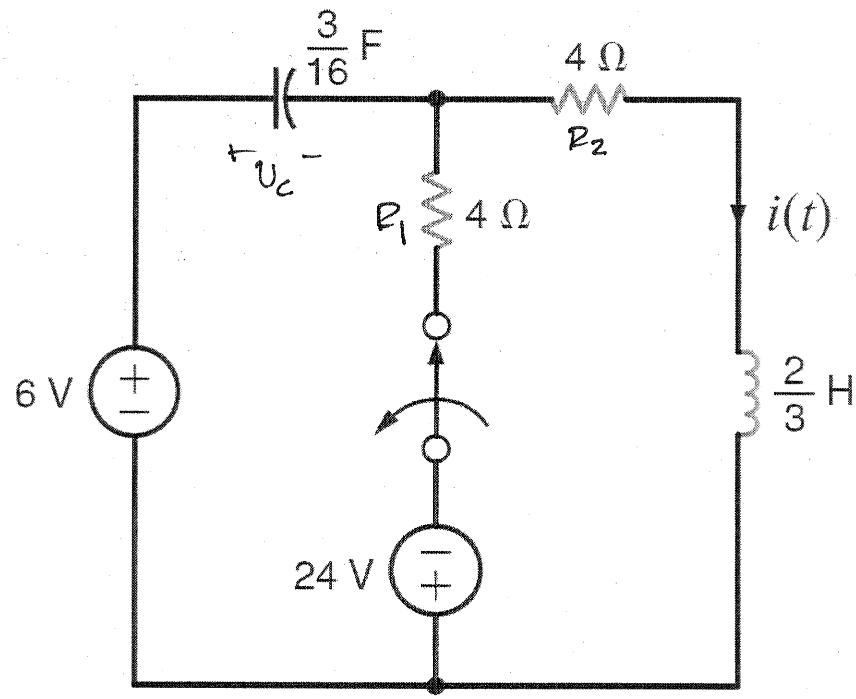


Figure P7.93

SOLUTION: $\underline{t=0^-}$: $i(0^-) = \frac{-24}{R_1+R_2} = -3A$ $\quad v_c(0^-) = 6 - i(0)R_2 = 18V$

$\underline{t=0^+}$ $\quad s^2 + \frac{R_2}{L}s + \frac{1}{LC} = 0 = s^2 + 6s + 8 \quad s_{1,2} = \begin{cases} -2 \\ -4 \end{cases}$

$i(t) = K_1 e^{-2t} + K_2 e^{-4t} \qquad i(0^+) = -3 = K_1 + K_2$

$v_c(0^+) = 6 - R_2 i(0^+) - L\frac{di}{dt}\Big|_{t=0^+} = 6 + 12 + 2L K_1 + 4L K_2 = 18$

yields, $K_1 = -6$ & $K_2 = 3 \Rightarrow \boxed{i(t) = -6e^{-2t} + 3e^{-4t}\, A}$

7.94 The switch in the circuit in Fig. P7.94 has been closed for a long time and is opened at $t = 0$. Find $i(t)$ for $t > 0$.

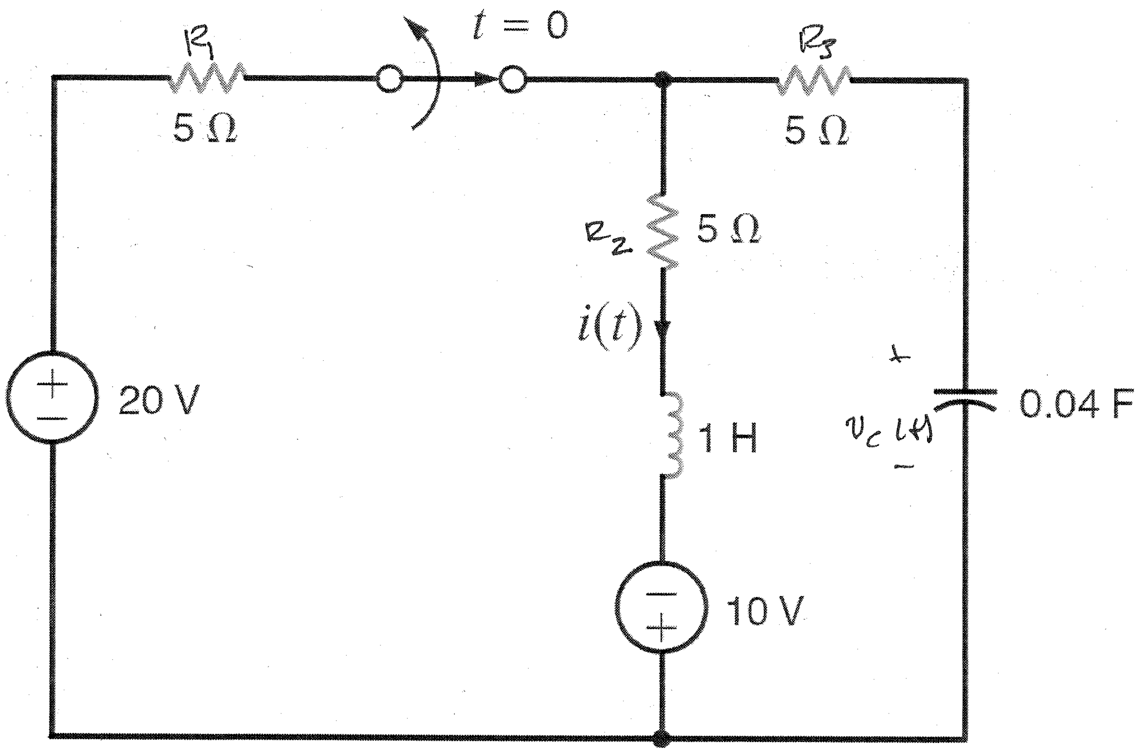


Figure P7.94

SOLUTION: $\underline{t=0^-}$: $i(0^-) = \frac{30}{R_1+R_2} = 3A$ $\quad v_c(0^-) = 20 - R_1 i(0^-) = 5V$

$\underline{t>0}$: $10 = L\frac{di}{dt} + Ri(t) + \frac{1}{C}\int i\,dt$ $\quad R = R_2+R_3 = 10\Omega$ $\quad v_c = -\frac{1}{C}\int i\,dt$

$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0 = s^2 + 10s + 25 \Rightarrow s_{1,2} = -5\ r/s$

$i(t) = B_1e^{-5t} + B_2te^{-5t}$ $\quad i(0^+) = 3 = B_1$

$-v_c(0^+) = -5 = 10 - Ri(0^+) - L\,di/dt|_{t=0^+} = 10 - 30 + L(5)B_1 - LB_2 \Rightarrow B_2 = 0$

$$\boxed{i(t) = 3e^{-5t}\ A}$$

7.95 The switch in the circuit in Fig. P7.95 has been closed for a long time and is opened at $t = 0$. Find $i(t)$ for $t > 0$.

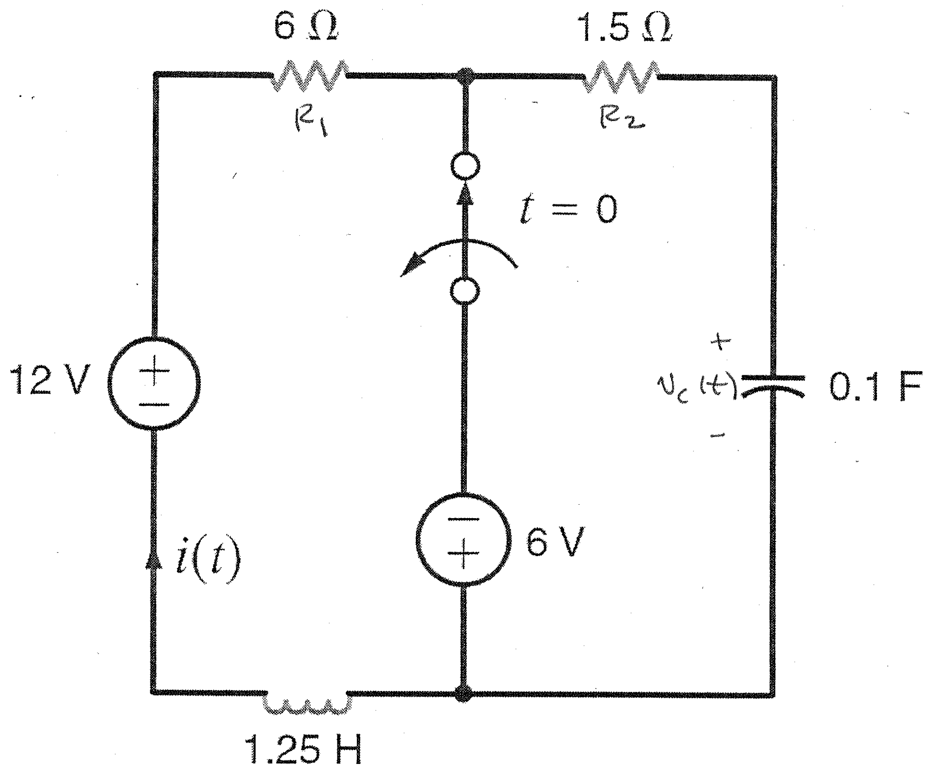


Figure P7.95

SOLUTION: $\underline{t=0^-}$: $i(0^-) = 18/R_1 = 3A$ $\quad v_c(0^-) = -6V = v_c(0^+)$

$\underline{t>0}$: $12 = Ri + \frac{1}{C}\int i\,dt + L\,di/dt$ $\quad R = R_1 + R_2 = 7.5\Omega$

$S^2 + (R/L)s + \frac{1}{LC} = s^2 + 6s + 8 \Rightarrow \sigma_1 = 2 \quad \sigma_2 = 4$

$i(t) = K_1 e^{-2t} + K_2 e^{-4t}$ & $i(0^+) = 3 = K_1 + K_2$

$\underline{\text{at } t=0^+}$: $12 = R(K_1+K_2) + \frac{1}{C}\left[-\frac{K_1}{2} - \frac{K_2}{4}\right] + K + L\left[-2K_1 - 4K_2\right] \Rightarrow K = 12$

and $v_c(0^+) = \frac{1}{C}\left[-K_1/2 - K_2/4\right] + K = -6 \Rightarrow +5K_1 + 2.5K_2 = 18$

yields $K_1 = 4.2$ & $K_2 = -1.2$ $\quad \boxed{i(t) = 4.2e^{-2t} - 1.2e^{-4t}\ A}$

7.96 Using the PSPICE *Schematics* editor, draw the circuit in Fig. P7.96, and use the PROBE utility to plot $v_C(t)$ and determine the time constants for $0 < t < 1$ ms and $1 \text{ ms} < t < \infty$. Also, find the maximum voltage on the capacitor.

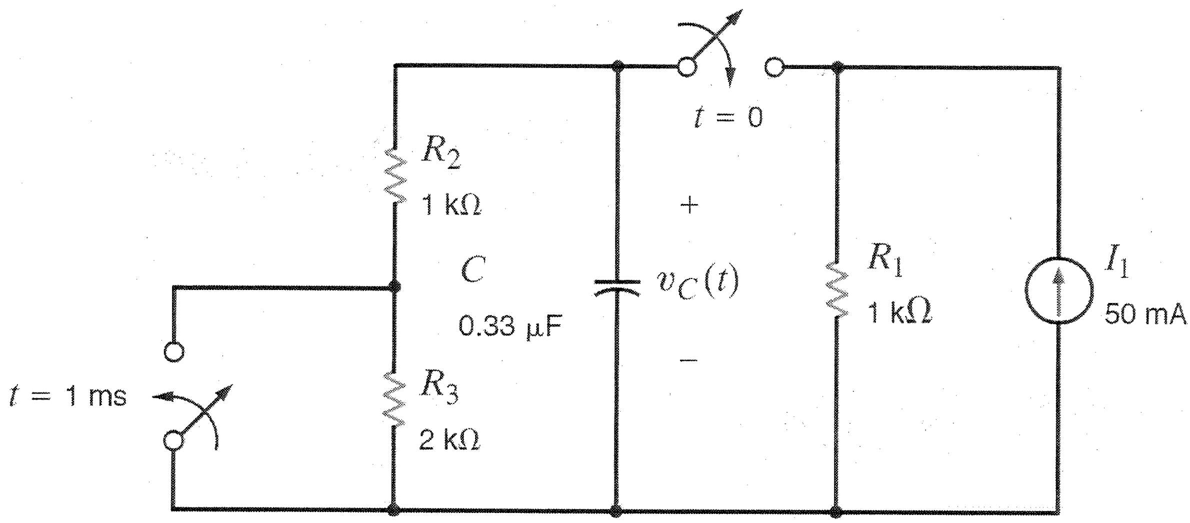
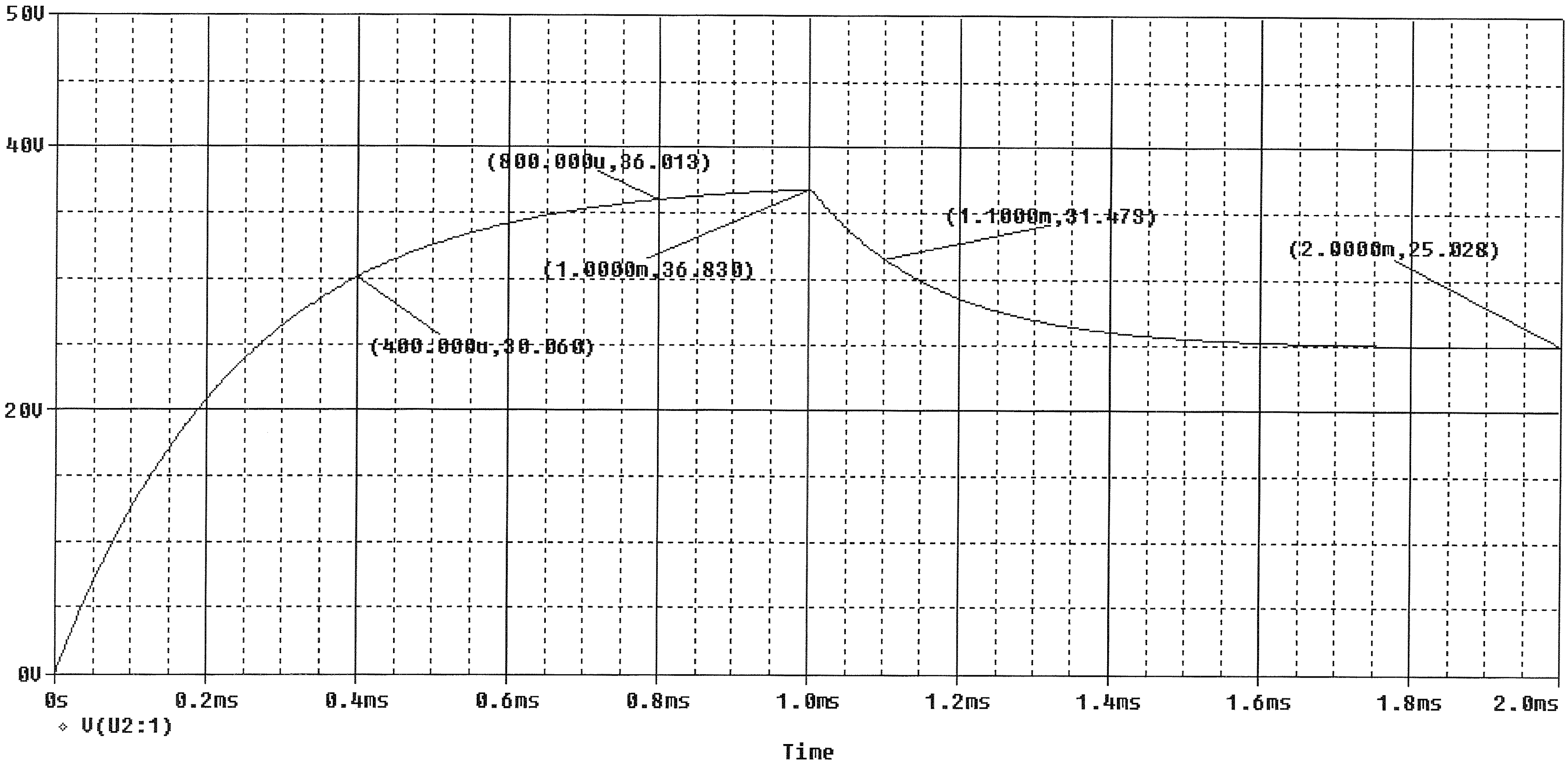


Figure P7.96

SOLUTION:

PROBLEM 7.96 PSPICE RESULTS



7.96 for $0 \le t \le 1ms$ $\quad v_c(t) = K_1 + K_2 e^{-t/\tau_1}$

$v_c(0) = 0 = K_1 + K_2 \Rightarrow K_2 = -K_1$

$v_c(t_1) = 30.6 = K_1 + K_2 e^{-t_1/\tau_1} \quad (t_1 = 0.4ms)$

$v_c(t_2) = 36.03 = K_1 + K_2 e^{-t_2/\tau_1} \quad (t_2 = 0.8ms)$

$$\frac{36.03}{30.60} = \frac{1 - e^{-t_2/\tau_1}}{1 - e^{-t_1/\tau_1}} = 1.177$$

$$1.177\, e^{-t_1/\tau_1} - e^{-t_2/\tau_1} = 0.177 = K$$

iterate!

$\tau_1 (ms)$	K
0.1	0.141
0.3	0.982
0.25	0.177 ✓

$\boxed{\tau_1 = 0.25 ms}$

for $t > 1ms$, $\quad v_c(t) = K_3 + K_4 e^{-(t-10^{-3})/\tau_2}$

$v_c(10^{-3}) = K_3 + K_4 = 36.83 \qquad v_c(\infty) = 25 = K_1 \Rightarrow K_2 = 11.83$

$v_c(t_3) = K_3 + K_4 e^{-(t_3-10^{-3})/\tau_2} = 31.48 \quad (t_3 = 1.1ms)$

yields $\boxed{\tau_3 = 0.166 ms}$

v_c max occus at $t = 1ms$ $\quad \boxed{v_{c\,max} = 36.83V}$

7.97 Using the PSPICE *Schematics* editor, draw the circuit in Fig. P7.97, and use the PROBE utility to find the maximum values of $v_L(t)$, $i_C(t)$, and $i(t)$.

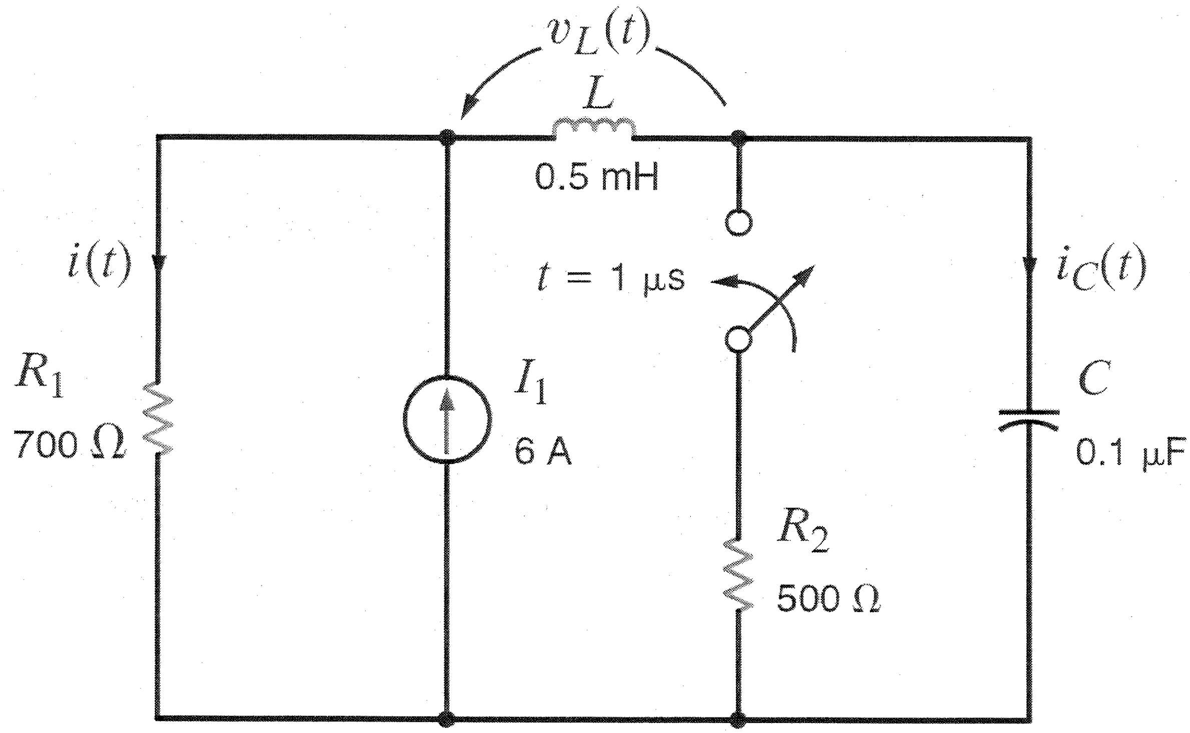


Figure P7.97

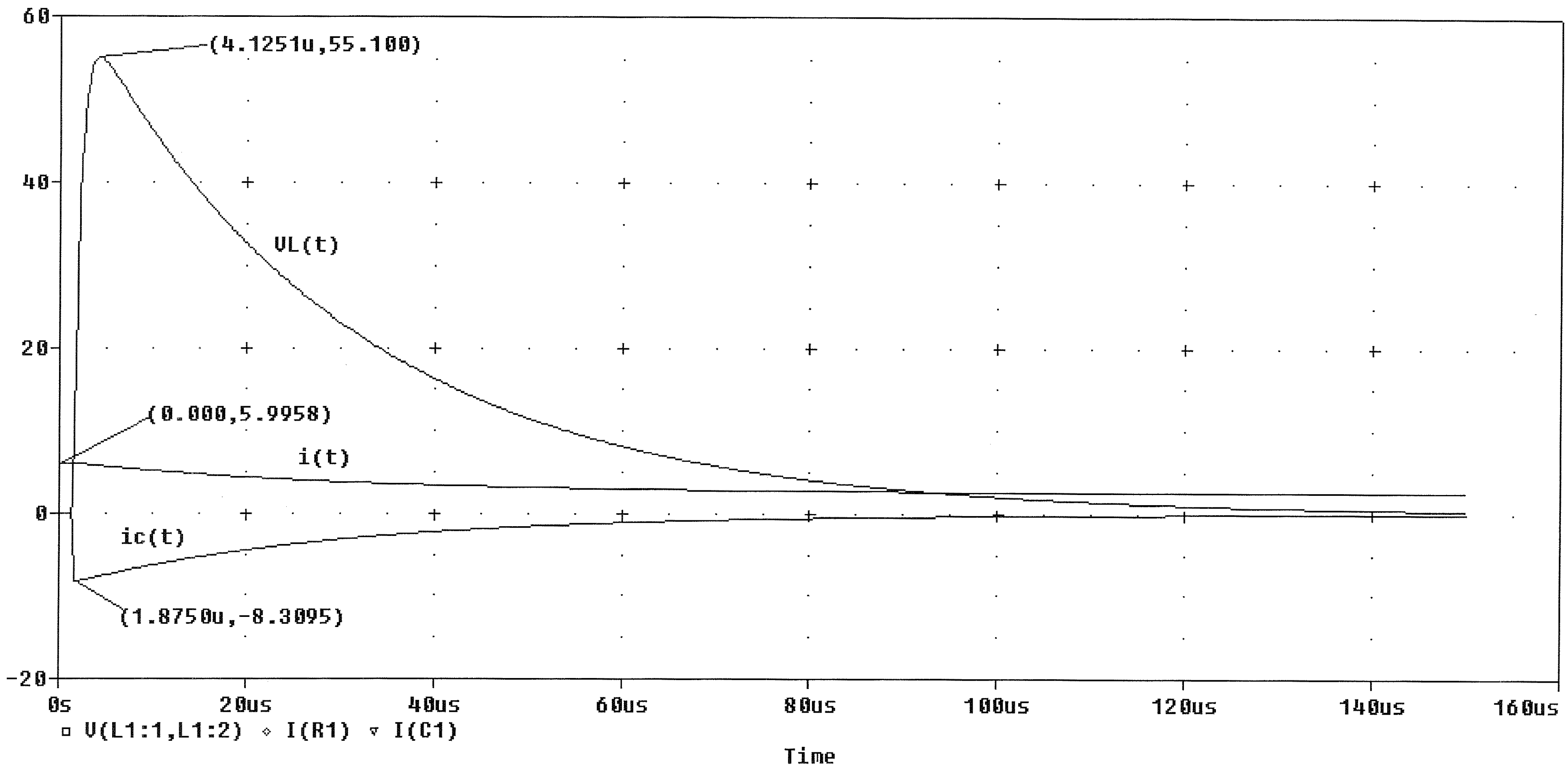
SOLUTION:

From the PSPICE simulation results,

v_L max = 55.1 V i_c max = − 8.31 A (actually the greatest deviation from ø!)

i max = 6.00 A

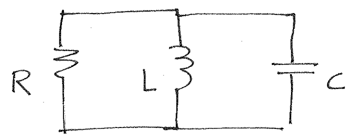
PROBLEM 7.97 PSPICE RESULTS



7.98 Design a parallel *RLC* circuit with $R \geqq 1\ \text{k}\Omega$ that has the characteristic equation

$$s^2 + 4 \times 10^7 s + 4 \times 10^{14} = 0$$

SOLUTION:



$$s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0$$

$$RC = 2.5\times10^{-8} \qquad LC = 2.5\times10^{-15}$$

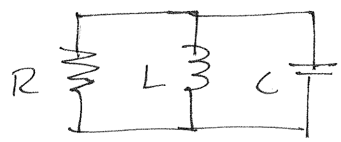
$$\frac{L}{R} = \frac{LC}{RC} = 10^{-7}$$

arbitrarily select $\boxed{L = 1\mu H \rightarrow R = 10\Omega,\ C = 2.5nF}$

7.99 Design a parallel *RLC* circuit with $R \geqq 1\ \text{k}\Omega$ that has the characteristic equation

$$s^2 + 4 \times 10^7 s + 3 \times 10^{14} = 0$$

SOLUTION:



$$s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0$$

$$RC = 0.25 \times 10^{-7} \qquad LC = 0.33 \times 10^{-14}$$

Arbitrarily choose $\boxed{L = 10\mu H \rightarrow R = 75\Omega \ \& \ C = 0.33\text{nF}}$

7.100 The curve shown in Fig. P7.100 is used to model the pressure in a vessel located in a chemical plant. We wish to design a circuit to realize this function so that we can study various parameters in the vessel, such as volume.

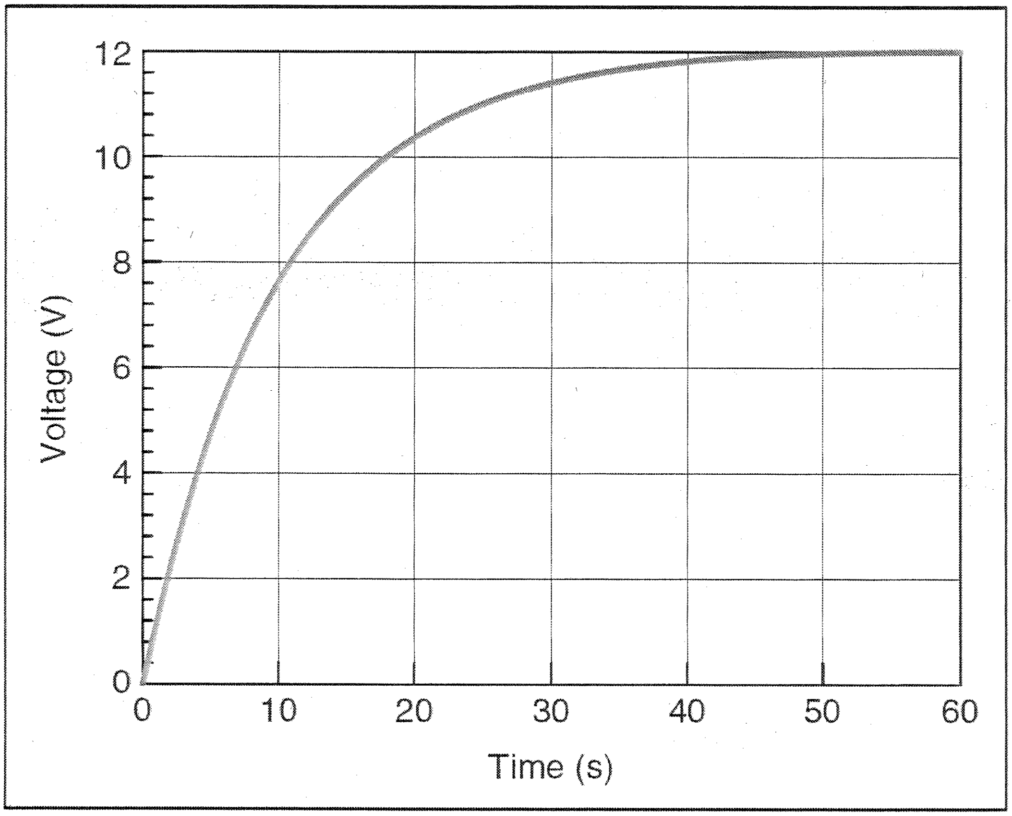
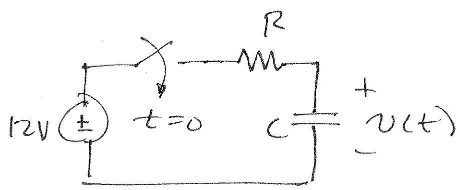


Figure P7.100

SOLUTION:

$v(t) = K_1 + K_2 e^{-t/\tau}$ $\quad v(0) = K_1 + K_2 = 0$ $\quad v(\infty) = 12 = K_1$

$t_1 = 10\,s$ $\quad v(t_1) = 7.6\,V = 12\left(1 - e^{-t_1/\tau}\right) \Rightarrow \tau = 10\,s$

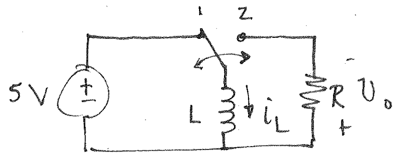


$RC = \tau = 10$

select $\boxed{C = 100\mu F \rightarrow R = 100k\Omega}$

7.101 Let us redesign the pulse generator in Example 7.14 such that a voltage with the following characteristics is created across a 10-kΩ resistor: a peak value of 250 V, a cycle time of 10,000 pulses/second, and a T_1 value of one-half the cycle time.

SOLUTION:



frequency = 10k Hz

$$\text{period} = \frac{1}{f} = 0.1\,ms$$

$$T_1 = \frac{\text{period}}{2} = 50\mu s$$

$$i_{L_{max}} = \frac{5}{L} T_1$$

$$v_{o_{max}} = i_{L_{max}} R = \frac{5}{L}(50 \times 10^{-6})(10^4) = 250$$

$$\boxed{L = 10mH}$$

Check repeatability: $\tau = \frac{L}{R} = 1\mu s$

$5\tau < T_1$? yes!!

7FE-1 In the circuit in Fig. 7PFE-1, the switch, which has been closed for a long time, opens at $t = 0$. Find the value of the capacitor voltage $v_C(t)$ at $t = 2$ s. **CS**

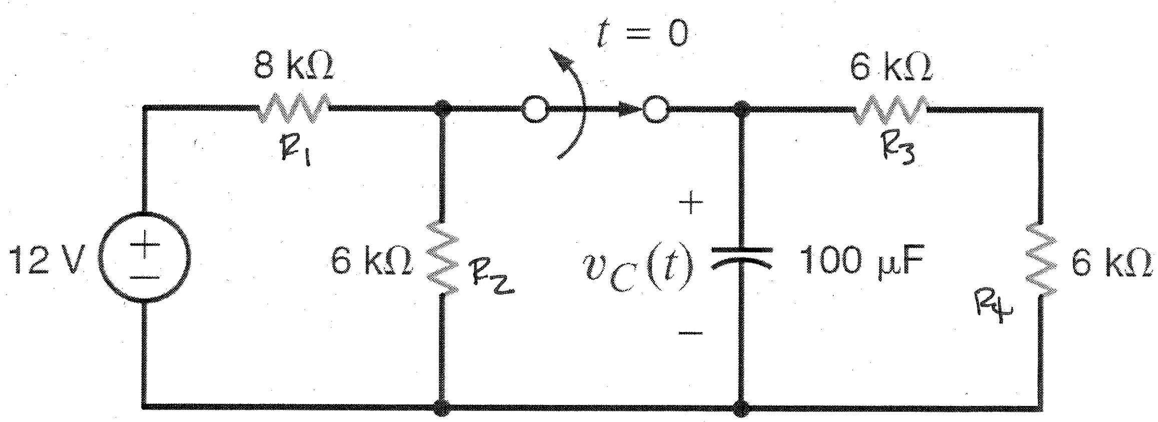
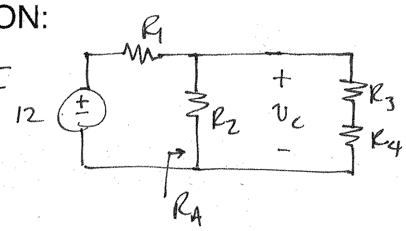


Figure 7PFE-1

SOLUTION:

$\underline{t=0^-}$



$R_A = R_2 // (R_3 + R_4) = 4k\Omega$

$v_c(0^-) = \dfrac{12 R_A}{R_A + R_1} = 4V$

$\underline{t=0^+}$: $K_1 + K_2 = v_c(0^-) = 4$

$\underline{t=\infty}$ $K_1 = 0$

$\tau = C R_{eq}$ $R_{eq} = R_3 + R_4 = 12k\Omega$ $\tau = 1.2s$

$v_c(t) = 4e^{-t/1.2}$ V $\boxed{v_c(2) = 0.756V}$

7FE-2 In the network in Fig. 7PFE-2, the switch closes at $t = 0$. Find $v_o(t)$ at $t = 1$ s.

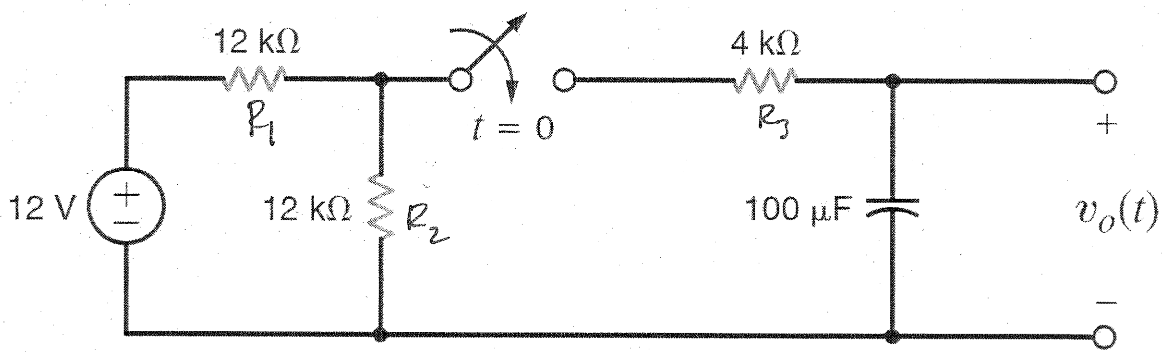


Figure 7PFE-2

SOLUTION: $v_o(t) = K_1 + K_2 e^{-t/\tau}$

$t = 0^-$: $v_c(0^-) = 0 = v_o(0^-)$ $t = 0^+$: $v_c(0^+) = 0 = v_o(0^+) = K_1 + K_2$

$t \to \infty$: $v_o(\infty) = \dfrac{12 R_2}{R_1 + R_2} = 6V = K_1$

$\tau = C R_{eq}$ $R_{eq} = R_3 + (R_1 // R_2) = 10\,k\Omega$ $\tau = 1s$

$v_o(t) = 6(1 - e^{-t})$ $\boxed{v_o(1) = 3.79V}$

7FE-3 Assume that the switch in the network in Fig. 7PFE-3 has been closed for some time. At $t = 0$ the switch opens. Determine the time required for the capacitor voltage to decay to one-half of its initially charged value. CS

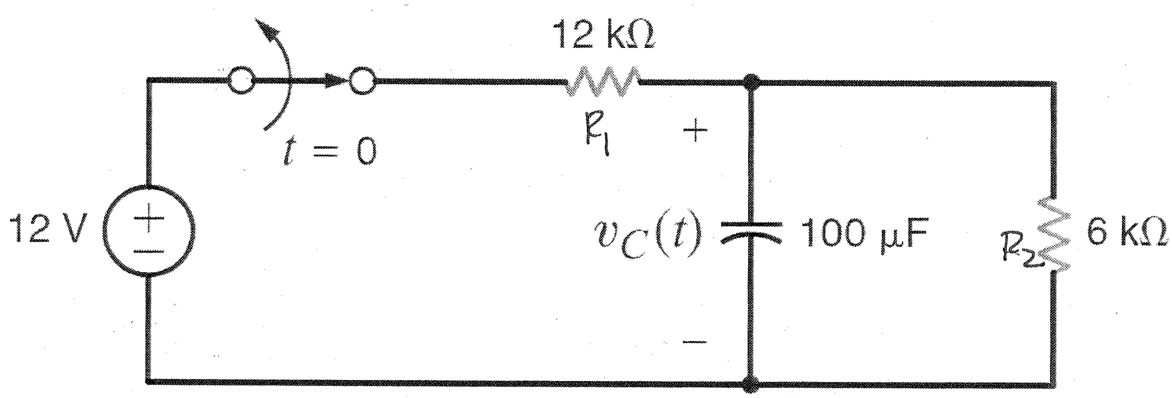


Figure 7PFE-3

SOLUTION: $v_c(t) = K_1 + K_2 e^{-t/\tau}$

$\underline{t=0^-}$: $v_c(0^-) = \dfrac{12 R_2}{R_1+R_2} = 4V = v_c(0^+)$

$\underline{t=0^+}$ $v_c(0^+) = 4 = K_1 + K_2$ $\underline{t \to \infty}$ $v_c(\infty) = 0 = K_1$

$\tau = C R_{eq} \Rightarrow R_{eq} = R_2$ $\tau = 0.6 s$

$v_c(t) = 4 e^{-t/0.6}$ V $v(t_1) = 2 = 4 e^{-t_1/0.6}$

$\boxed{t_1 = 0.416 \text{ s}}$

Chapter Eight:

AC Steady-State Analysis

8.1 Given $i(t) = 5\cos(400t - 120°)$ A, determine the period of the current and the frequency in Hertz. CS

SOLUTION:

$\omega = 400$ r/s $\quad f = \frac{\omega}{2\pi} = \frac{200}{\pi} = 63.7$ Hz $\quad T = \frac{1}{f} = 15.7$ms

$$\boxed{f = 63.7\text{Hz} \qquad T = 15.7 \text{ ms}}$$

8.2 Determine the relative phase relationship of the two waves.

$$v_1(t) = 10\cos(377t - 30°)\ \text{V}$$
$$v_2(t) = 10\cos(377t + 90°)\ \text{V}$$

SOLUTION:

$\theta_2 = 90°$ $\theta_1 = -30°$ $\theta_2 - \theta_1 = 120°$

$\boxed{v_2 \text{ leads } v_1 \text{ by } 120°}$

8.3 Given the following voltage and current

$$i(t) = 5\sin(377t - 20^\circ)\ \text{V}$$
$$v(t) = 10\cos(377t + 30^\circ)\ \text{V}$$

determine the phase relationship between $i(t)$ and $v(t)$.

SOLUTION:

$i(t) = 5\cos(377t - 20 - 90) = 5\cos(377t - 110)$

$\theta_v = 30^\circ \quad \theta_i = -110^\circ \quad \theta_v - \theta_i = 140^\circ$

$\boxed{v(t) \text{ leads } i(t) \text{ by } 140^\circ}$

8.4 Determine the phase angles by which $v_1(t)$ leads $i_1(t)$ and $v_1(t)$ leads $i_2(t)$, where

$$v_1(t) = 4\sin(377t + 25°)\text{ V}$$
$$i_1(t) = 0.05\cos(377t - 20°)\text{ A}$$
$$i_2(t) = -0.1\sin(377t + 45°)\text{ A}$$

SOLUTION:

$i_1(t) = 0.05 \sin(\omega t - 20 + 90) = 0.05 \sin(\omega t + 70°)$

$\theta_{v_1} - \theta_{i_1} = -45°$ $\boxed{v_1 \text{ ledd } i_1 \text{ by } -45°}$

$i_2(t) = 0.1 \sin(\omega t - 135°)$

$\theta_{v_1} - \theta_{v_2} = 160°$ $\boxed{v_1 \text{ lead } i_2 \text{ by } 160°}$

8.5 Calculate the current in the resistor in Fig. P8.5 if the voltage input is

(a) $v_1(t) = 10\cos(377t + 180°)$ V.

(b) $v_2(t) = 12\sin(377t + 45°)$ V.

Give the answers in both the time and frequency domains.

CS

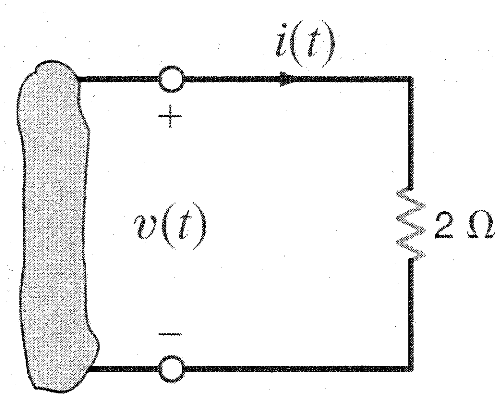


Figure P8.5

SOLUTION:

a) $i = v/R$ $\quad i(t) = 5\cos(377t + 180°)$ A $\quad \mathbf{I} = 5\angle 180°$ A

b) $i(t) = 6\sin(377t + 45°) = 6\cos(377t - 45°)$ A $\quad \mathbf{I} = 6\angle -45°$ A

8.6 Calculate the current in the inductor shown in Fig. P8.6 if the voltage input is

(a) $v_1(t) = 10\cos(377t + 45°)$ V

(b) $v_2(t) = 5\sin(377t - 90°)$ V

Give the answers in both the time and frequency domains.

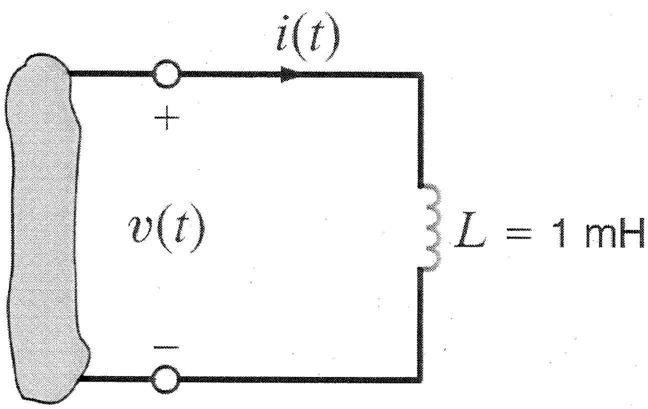
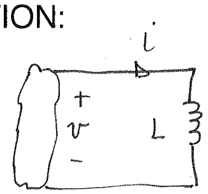


Figure P8.6

SOLUTION:

a)



$$i = \frac{1}{L}\int v\,dt = \frac{10}{377L}\sin(377t + 45°)$$

$$\boxed{\begin{array}{l} i(t) = 26.5\cos(377t - 45°)\,A \\ I = 26.5\angle{-45°}\,A \end{array}}$$

b) $$i = \frac{5}{377L}\left(-\cos(377t - 90°)\right)$$

$$\boxed{\begin{array}{l} i(t) = 13.3\cos(377t + 90)\,A \\ I = 13.3\angle 90°\,A \end{array}}$$

8.7 Calculate the current in the capacitor shown in Fig. P8.7 if the voltage input is

(a) $v_1(t) = 10\cos(377t - 30°)$ V

(b) $v_2(t) = 5\sin(377t + 60°)$ V

Give the answers in both the time and frequency domains.

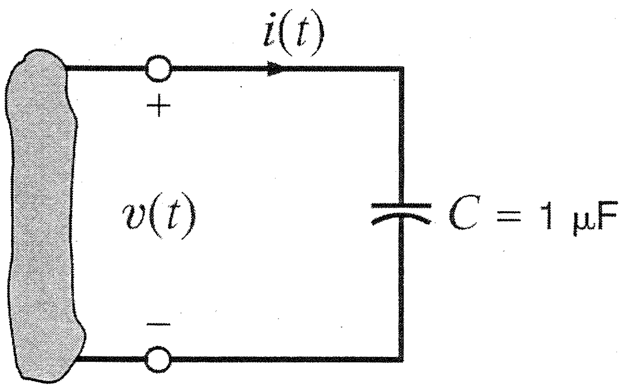


Figure P8.7

SOLUTION:

a) $i = C\frac{dv}{dt} = 10^{-6}(10)(377)\left[-\sin(377t-30°)\right]$

$i(t) = 3.77\ \sin(\omega t + 150°)$ mV

$$\boxed{\begin{array}{l} i(t) = 3.77\ \cos(377t + 60°)\ \text{mA} \\ I = 3.77\ \angle 60°\ \text{mA} \end{array}}$$

b) $i = (10^{-6})(5)(377)\ \cos(\omega t + 60°)$

$$\boxed{\begin{array}{l} i(t) = 1.89\ \cos(377t + 60°)\ \text{mA} \\ I = 1.89\ \angle 60°\ \text{mA} \end{array}}$$

8.8 Find the frequency-domain impedance, **Z**, as shown in Fig. P8.8. CS

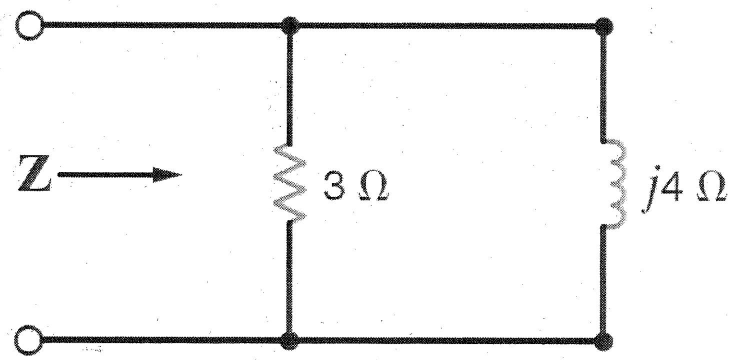


Figure P8.8

SOLUTION:

$$Z = \frac{3(j4)}{3+j4} = \frac{j12}{3+j4} = \frac{12\angle 90^\circ}{5\angle 53.1^\circ} = 2.4\angle 36.9^\circ\ \Omega$$

$$\boxed{Z = 2.4\angle 36.9^\circ\ \Omega}$$

8.9 Find the impedance, **Z**, shown in Fig. P8.9 at a frequency of 60 Hz.

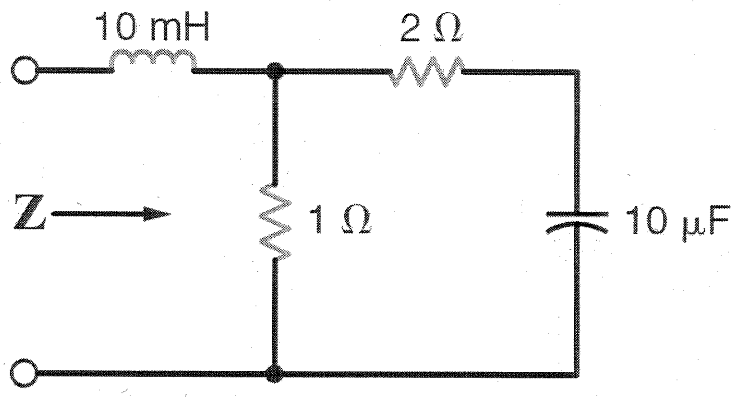
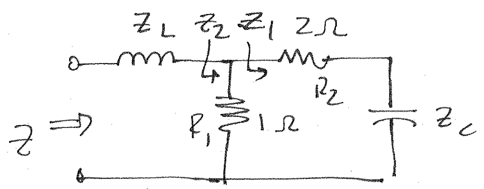


Figure P8.9

SOLUTION: $\omega = 2\pi f = 377$



$Z_L = j\omega L = j\,3.77\,\Omega$

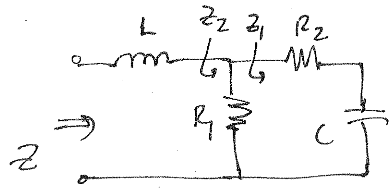
$Z_C = \dfrac{1}{j\omega C} = -j\,265\,\Omega$

$Z_1 = 2 - j265\ \Omega$ $\quad Z_2 = \dfrac{R_1 Z_1}{R_1 + Z_1}$ $\quad Z = Z_L + Z_2$

$\boxed{Z = 1.00 + j\,3.77\,\Omega}$

8.10 Find the impedance, **Z**, shown in Fig. P8.9 at a frequency of 400 Hz.

SOLUTION:



$L = 10\,mH$ $C = 10\mu F$ $R_1 = 1\Omega$ $R_2 = 2\Omega$

$Z_L = j\omega L = j25.1\Omega$

$Z_C = \frac{1}{j\omega C} = -j39.8\Omega$

$Z_1 = R_2 + Z_C$

$Z_1 = 2 - j39.8\Omega$

$Z_2 = \frac{R_1 Z_1}{R_1 + Z_1} = 1.00 - j0.02\Omega$

$Z = Z_L + Z_2$

$\boxed{Z = 25.10 \angle 87.7^\circ\ \Omega}$

8.11 In the network in Fig. P8.11, find $\mathbf{Z}(j\omega)$ at a frequency of 60 Hz. CS

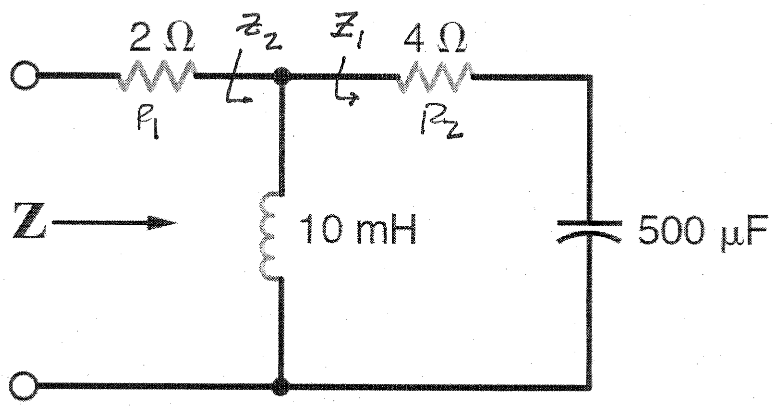


Figure P8.11

SOLUTION: $\omega = 377 \text{ r/s}$

$Z_1 = R_2 + jZ_C$ $\qquad Z_C = \frac{1}{j\omega C} = -j5.31\,\Omega$

$Z_2 = \frac{Z_L Z_1}{Z_L + Z_1}$ $\qquad Z_L = j3.77\,\Omega$

$Z = R_1 + Z_2$ $\qquad \boxed{Z = 7.11\angle 44.3°\ \Omega}$

8.12 Find the frequency-domain impedance, **Z**, shown in Fig. P8.12.

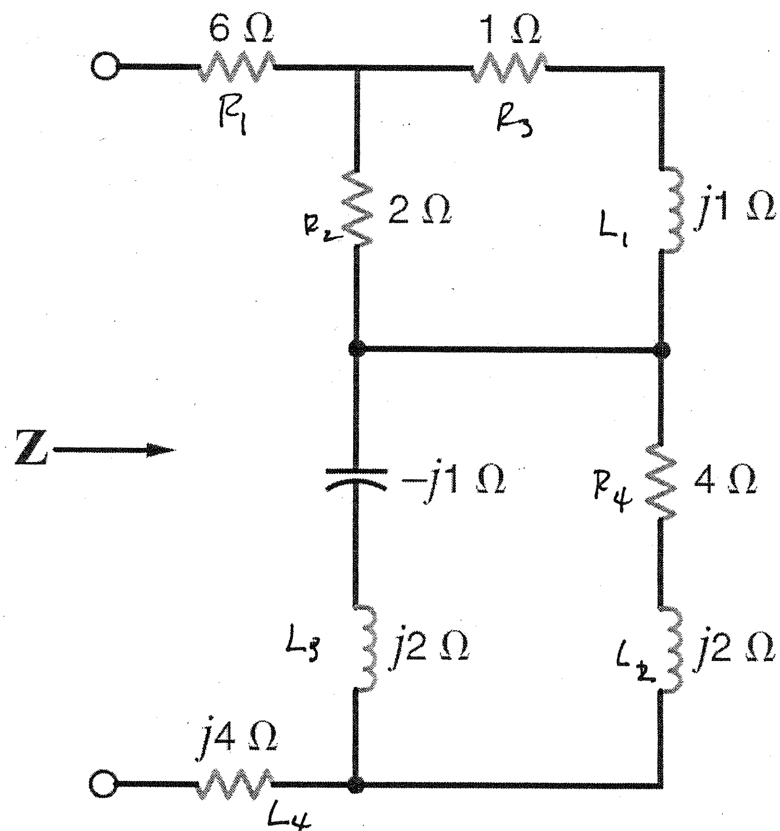
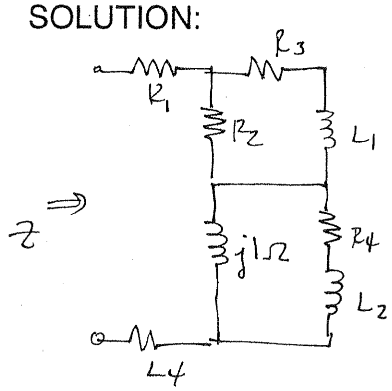


Figure P8.12

SOLUTION:



$Z_1 = R_3 + Z_{L1} = 1 + j1\ \Omega$

$Z_2 = R_4 + Z_{L2} = 4 + j2$

$Z_3 = \dfrac{Z_1 R_2}{Z_1 + R_2} = 0.8 + j0.4$

$Z_4 = \dfrac{j1\,(Z_2)}{j1 + Z_2} = 0.16 + j0.88$

$Z = R_1 + Z_3 + Z_4 + Z_{L4}$

$\boxed{Z = 8.74\ \angle 37.2^\circ\ \Omega}$

8.13 Find the frequency-domain impedance, **Z**, shown in Fig. P8.13. **PSV**

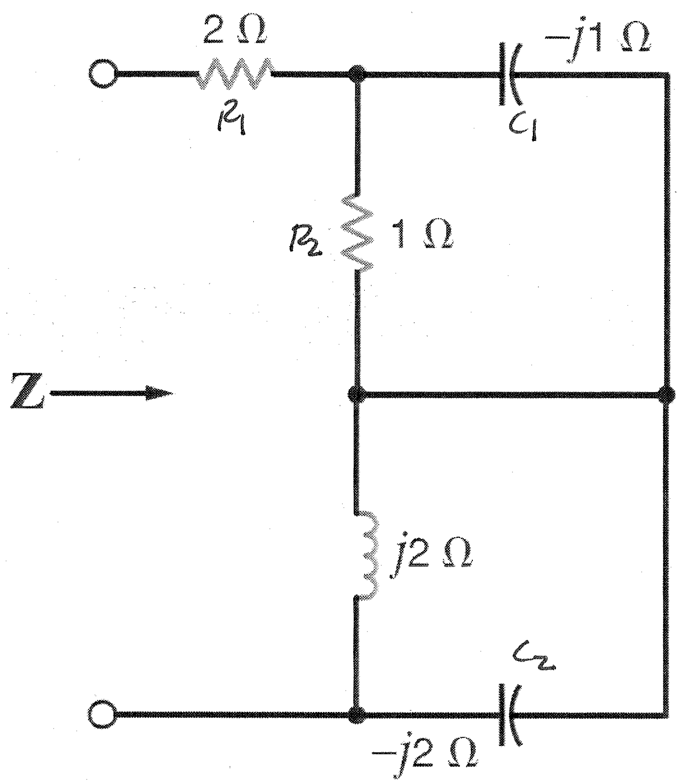


Figure P8.13

SOLUTION:

$$Z_1 = \frac{R_2 Z_{C1}}{R_2 + Z_{C1}} = 0.5 - j0.5 \qquad Z_2 = \frac{Z_L Z_{C2}}{Z_L + Z_{C2}} = \infty$$

$$Z = R_1 + Z_1 + Z_2 = \infty \qquad \boxed{Z = \infty}$$

8.14 Find **Z** in the network in Fig. P8.14. CS

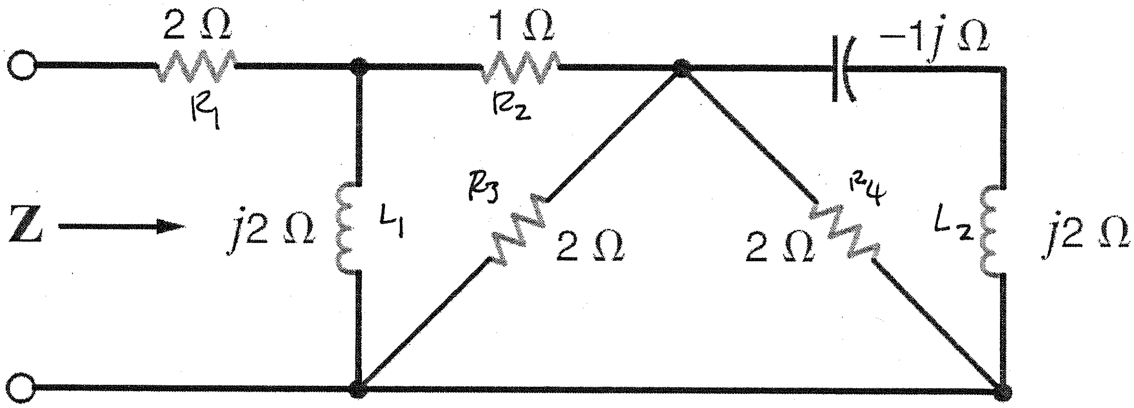


Figure P8.14

SOLUTION:

$Z_1 = Z_{L2} + Z_C = j1\,\Omega$ $\quad Z_2 = \dfrac{R_4 Z_1}{Z_1 + R_4} = 0.4 + j0.8\,\Omega$

$Z_3 = \dfrac{R_3 Z_2}{R_3 + Z_2} = 0.5 + j0.5\,\Omega$ $\quad Z_4 = R_2 + Z_3 = 1.5 + j0.5\,\Omega$

$Z_5 = \dfrac{Z_{L1} Z_4}{Z_{L1} + Z_4} = 0.706 + j\,0.824\,\Omega$ $\quad Z = R_1 + Z_5$

$Z = 2.706 + j0.824$ $\quad \boxed{Z = 2.83 \angle 16.9^\circ\,\Omega}$

8.15 Find **Z** in the network in Fig. P8.15.

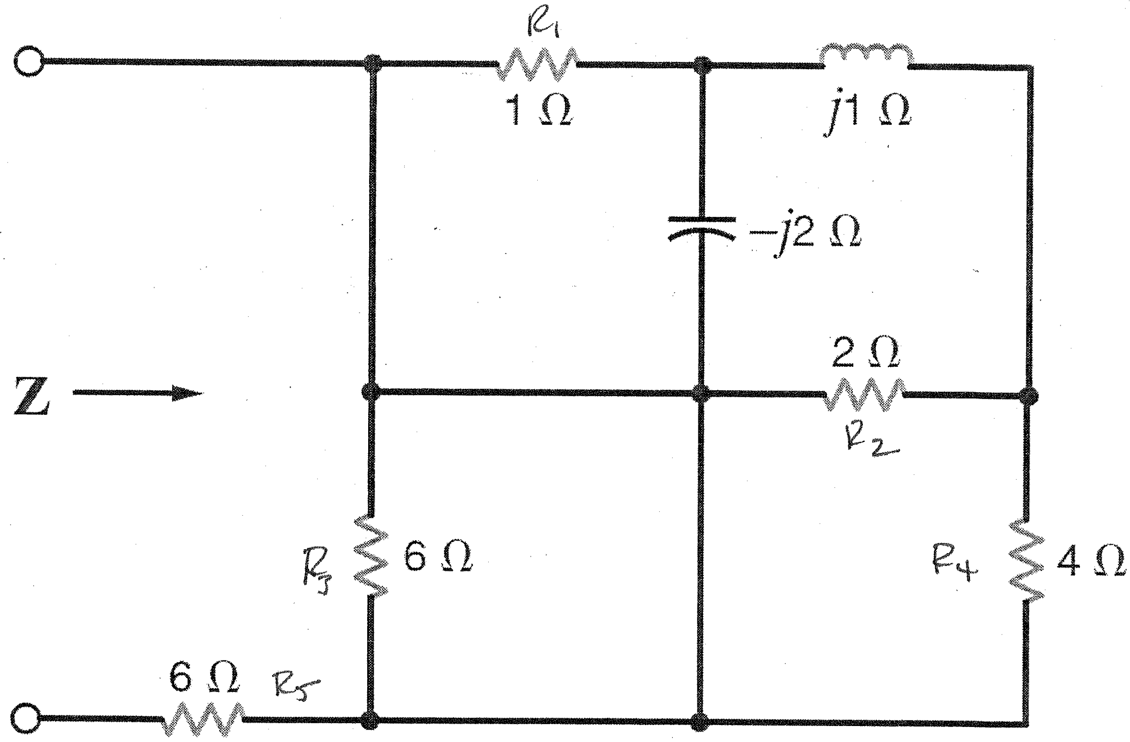
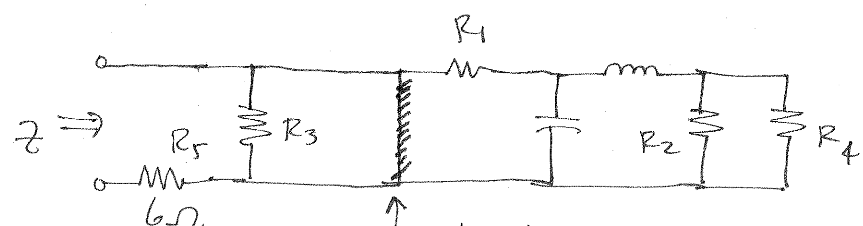


Figure P8.15

SOLUTION:

Redraw:



shorts everything but R_5!

$\boxed{Z = 6\,\Omega}$

8.16 Draw the frequency-domain circuit and calculate $i(t)$ for the circuit shown in Fig. P8.16 if $v_S(t) = 2\cos(377t)$ V.

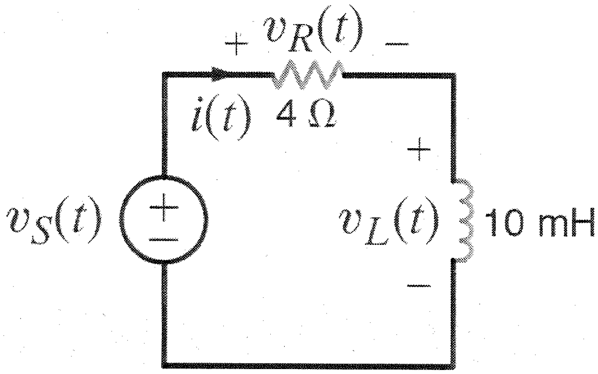
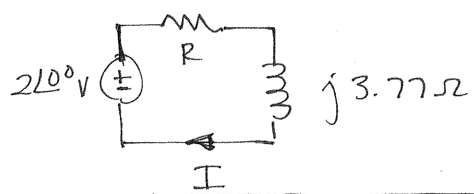


Figure P8.16

SOLUTION:



$$I = \frac{2\angle 0°}{4 + j3.77} =$$

$$\boxed{I = 0.36\angle -43.3° \text{ A}}$$

$$\boxed{i(t) = 0.36\cos(377t - 43.3°) \text{ A}}$$

8.17 Draw the frequency-domain circuit and calculate $v(t)$ for the circuit shown in Fig. P8.17 if $i_S(t) = 10\cos(377t + 30°)$ A.

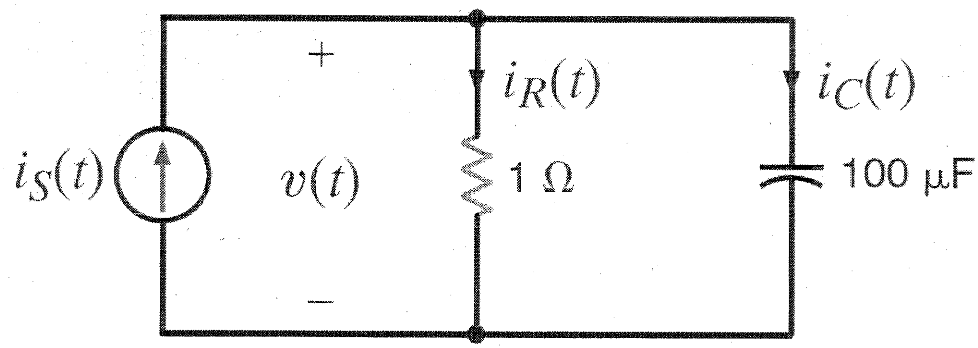
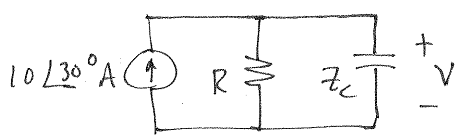


Figure P8.17

SOLUTION:



$$Z_C = \frac{1}{j\omega C} = -j26.5\,\Omega$$

$$V = \frac{10\angle 30°\,(R Z_C)}{R + Z_C}$$

$$\boxed{V = 9.99\angle 27.8°\ \text{V}}$$

$$\boxed{v(t) = 9.99\cos(377t + 27.8°)\ \text{V}}$$

8.18 Draw the frequency-domain circuit and calculate $v(t)$ for the circuit shown in Fig. P8.18 if $i_S(t) = 20\cos(377t + 120°)$ A. **CS**

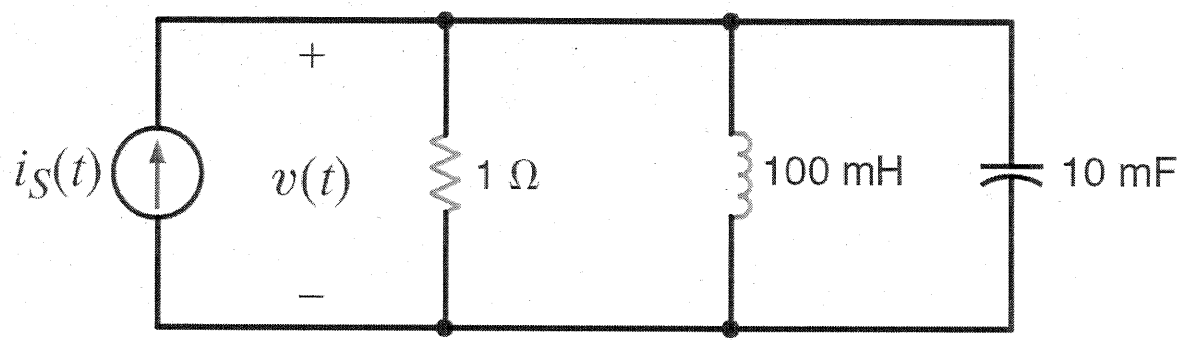
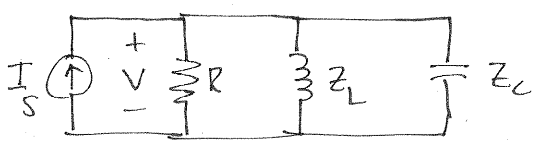


Figure P8.18

SOLUTION:



$Z_c = -j0.265\,\Omega$

$Z_L = j\,37.7\,\Omega$

$Z_{eq} = R /\!/ Z_L /\!/ Z_c = 0.258\,\angle{-74.9°}\,\Omega$

$I_s = 20\,\angle 120°$ A

$V = I_s Z_{eq}$

$\boxed{V = 5.16\,\angle 45.1°\ \text{V}}$

$\boxed{v(t) = 5.16\cos(377t + 45.1°)\ \text{V}}$

8.19 Draw the frequency-domain circuit and calculate $v(t)$ for the circuit shown in Fig. P8.19 if $i_S(t) = 2\cos(1000t + 120°)$ A.

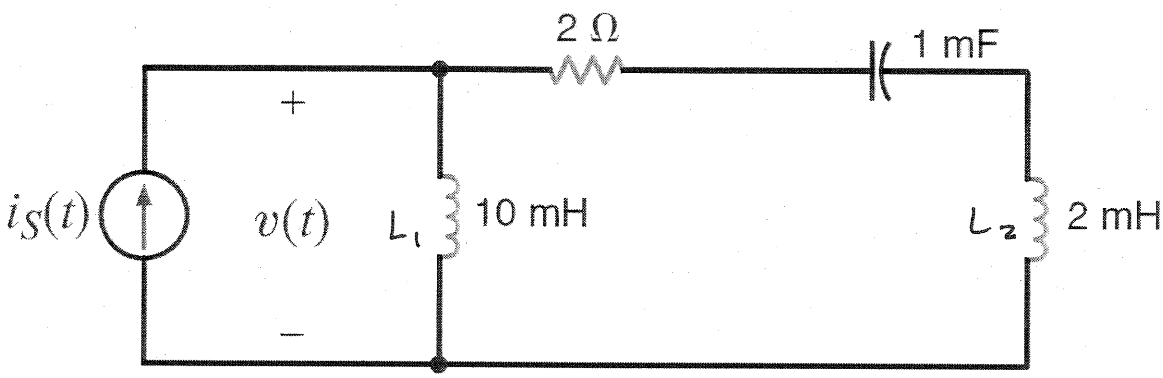
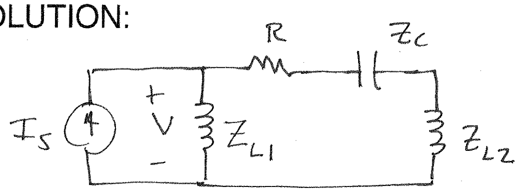


Figure P8.19

SOLUTION:



$I_S = 2\angle 120°$ A

$Z_1 = R + Z_C + Z_{L2} = 2 + j1\ \Omega$

$Z_{L1} = j10\ \Omega \quad Z_{L2} = j2\ \Omega \quad Z_C = -j1\ \Omega$

$$Z_2 = \frac{Z_{L1} Z_1}{Z_{L1} + Z_1}$$

$Z_2 = 1.6 + j1.2\ \Omega$

$V = I_S Z_2$

$$\boxed{V = 4\angle 156.9°\ \text{V}}$$

$$\boxed{v(t) = 4\cos(1000t + 156.9°)\ \text{V}}$$

8.20 Draw the frequency-domain network and calculate $v_o(t)$ in the circuit shown in Fig. P8.20 if $v_S(t)$ is $4\sin(500t + 45°)$ V and $i_S(t)$ is $1\cos(500t + 45°)$ A. Also, use a phasor diagram to determine $v_1(t)$.

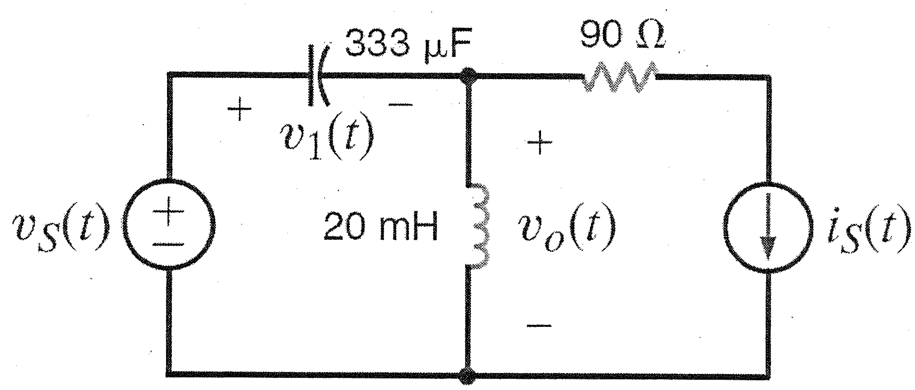
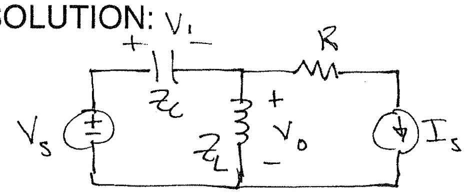


Figure P8.20

SOLUTION:



$V_S = 4\angle{-45°}$ V $\quad I_S = 1\angle{45°}$ A

$Z_C = \frac{1}{j\omega C} = -j6\,\Omega \quad Z_L = j\omega L = j10\,\Omega$

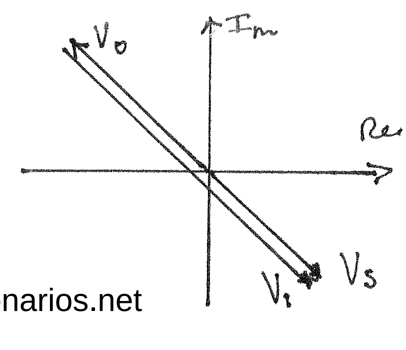
Superposition: $V_o = V_S\left[\frac{Z_L}{Z_L + Z_C}\right] - I_S\left[\frac{Z_L Z_C}{Z_L + Z_C}\right]$

$V_o = 5\angle 135°$ V

$$\boxed{v_o(t) = 5\cos(500t + 135°)\ \text{V}}$$

$V_1 = V_S - V_o = 4\angle{-45°} - 5\angle 135 = 9\angle{-45°}$

$$\boxed{v_1(t) = 9\cos(500t - 45°)\ \text{V}}$$



8.21 Draw the frequency-domain network and calculate $v_o(t)$ in the circuit shown in Fig. P8.21 if $i_1(t)$ is $200\cos(10^5 t + 60°)$ mA, $i_2(t)$ is $100\sin(10^5 t + 90°)$ mA, and $v_S(t) = 10\sin(10^5 t)$ V. Also, use a phasor diagram to determine $v_C(t)$.

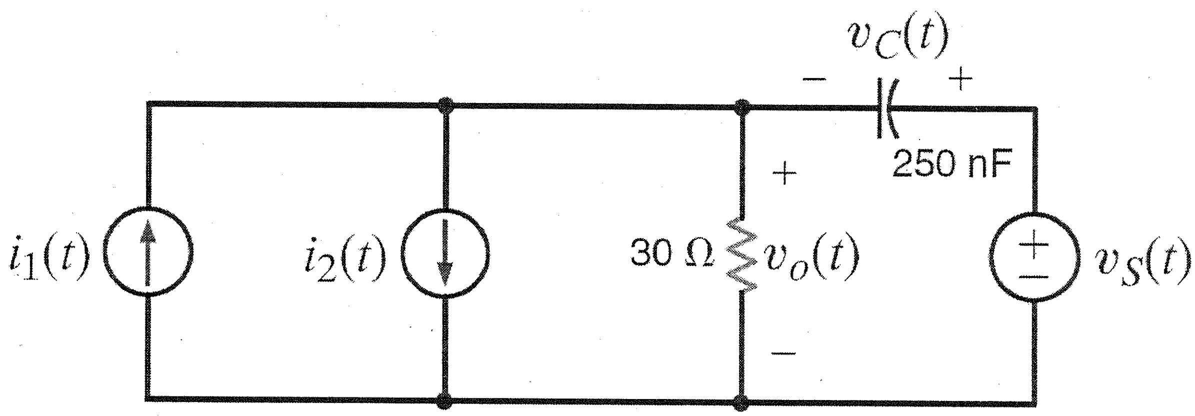
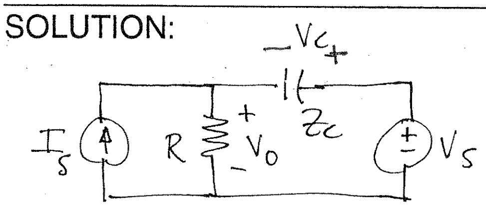


Figure P8.21

SOLUTION:



$I_1 = 200\angle 60°$ mA $\quad I_2 = 100\angle 0°$ mA

$I_S = I_1 - I_2 = 173\angle 90°$ mA

$V_S = 10\angle -90°$ V

$Z_C = \dfrac{1}{j\omega C} = -j\,40\,\Omega$

Superposition:

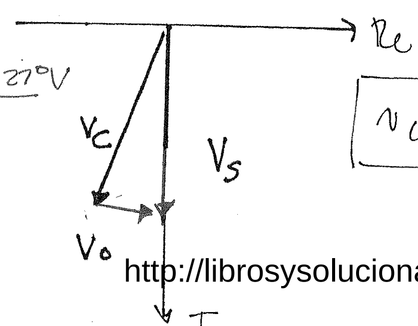
$$V_o = \frac{I_S R Z_C}{R + Z_C} + \frac{V_S R}{R + Z_C}$$

$V_o = 7.30\angle -2.16$ V

$$\boxed{v_o(t) = 7.30\cos(10^5 t - 2.16°)\ \text{V}}$$

$V_C = V_S - V_o$

$V_C = 12.2\angle -127°$ V



$$\boxed{v_C(t) = 12.2\cos(10^5 t - 127°)\ \text{V}}$$

8.22 The impedance of the network in Fig. P8.22 is found to be purely real at $f = 400$ Hz. What is the value of C?

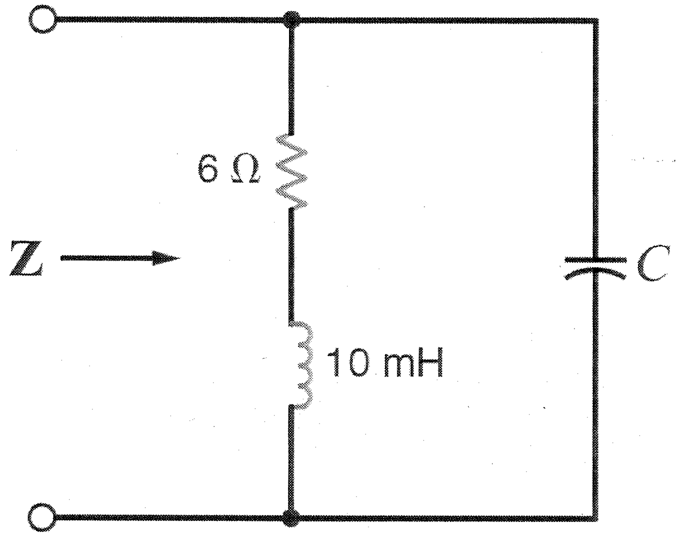


Figure P8.22

SOLUTION:

$$Z = \frac{(6 + j\omega L)(-j/\omega C)}{6 + j(\omega L - 1/\omega C)} = R_{eq}$$

Requires angles of numerator = angle of denominator

$$\frac{-\frac{6}{\omega C}}{L/C} = \frac{\omega L - 1/\omega C}{6} \Rightarrow \frac{-36}{\omega L} = \omega L - \frac{1}{\omega C}$$

$$C = \frac{L}{36 + \omega^2 L^2}$$

$$\boxed{C = 15.0\,\mu F}$$

8.23 In the circuit shown in Fig. P8.23, determine the value of the inductance such that the current is in phase with the source voltage. **PSV**

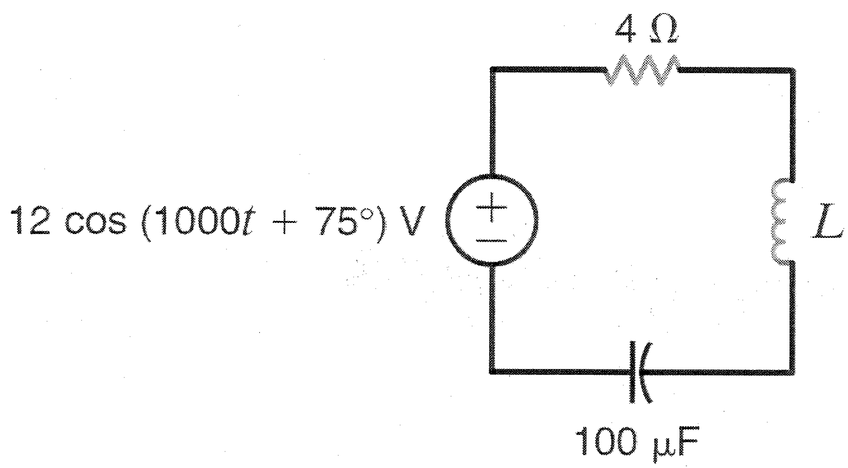
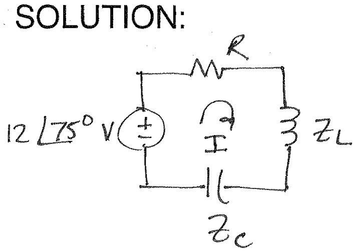


Figure P8.23

SOLUTION:



If V_s & I are in phase, then

$$\frac{V_s}{I} = R_{eq} + j0 = R + Z_L + Z_C$$

or $Z_L + Z_C = 0$

$Z_L = j1000L \qquad Z_C = -j10\,\Omega \qquad \Rightarrow Z_L = j10\,\Omega$

$$\boxed{L = 10\,mH}$$

8.24 The impedance of the box in Fig. P8.24 is $5 + j4\ \Omega$ at 1000 rad/s. What is the impedance at 1300 rad/s?

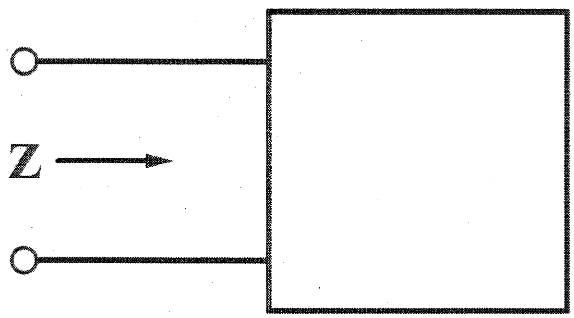


Figure P8.24

SOLUTION:

$Z = 5 + j4\ \Omega = R_{eq} + j\omega L_{eq}$ @ $\omega = 1000$ r/s $\Rightarrow L_{eq} = 4$ mH

at $\omega = 1300$ r/s $Z = R_{eq} + j(1300)(0.004)$

$$\boxed{Z = 5 + j5.2\ \Omega}$$

8.25 The admittance of the box in Fig. P8.25 is $0.1 + j0.2$ S at 500 rad/s. What is the impedance at 300 rad/s?

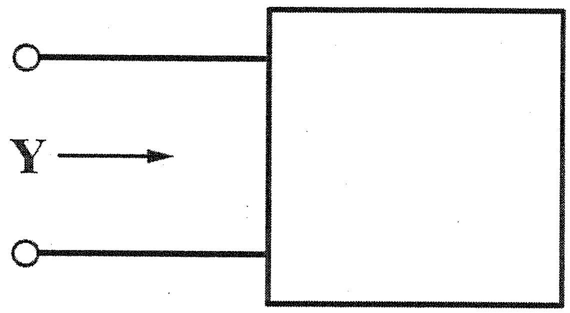


Figure P8.25

SOLUTION:

$$Y = 0.1 + j0.2 \text{ S} = \frac{1}{R_{eq}} + \frac{1}{\frac{1}{j\omega C_{eq}}} \qquad \omega C_{eq} = 0.2 \quad @ \; 500 \text{ r/s}$$

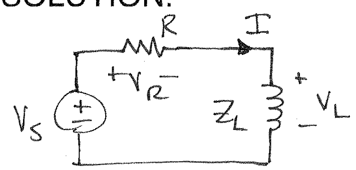
$C_{eq} = 400\mu F$

at 300 r/s $\boxed{Y = 0.1 + j\,0.12 \text{ S}}$

$$\boxed{Z = 1/Y = 4.10 - j4.92\ \Omega}$$

8.26 The voltages $v_R(t)$ and $v_L(t)$ in the circuit shown in Fig. P8.16 can be drawn as phasors in a phasor diagram. Use a phasor diagram to show that $v_R(t) + v_L(t) = v_S(t)$.

SOLUTION:

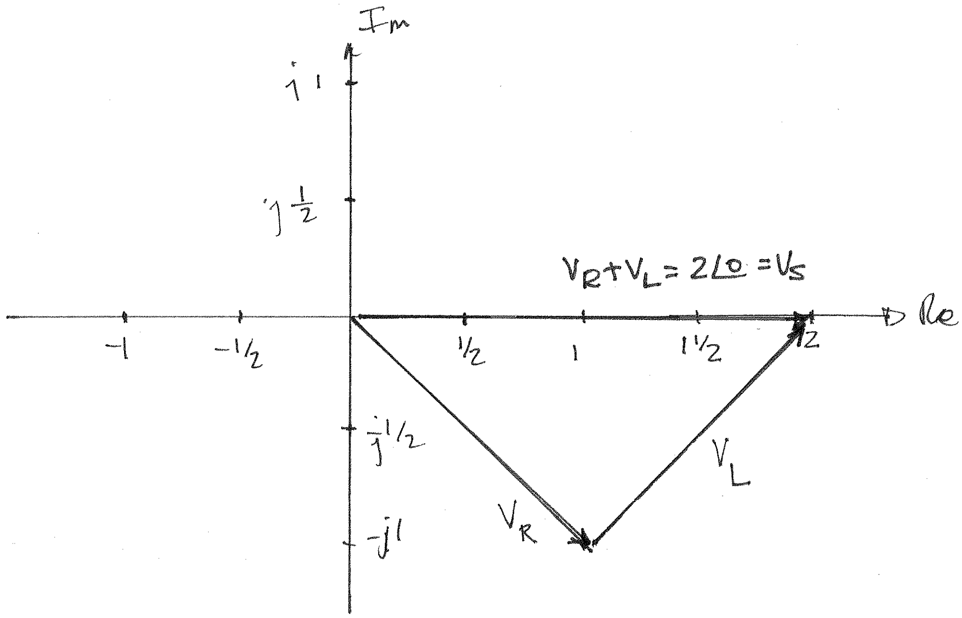


$R = 4\,\Omega \qquad L = 10\,mH \qquad \omega = 377\,r/s$

$Z_L = j\,3.77\,\Omega \qquad V_S = 2\angle 0^\circ\ V$

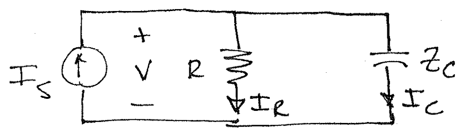
$I = \dfrac{V_S}{R + j\omega L} = 0.36\angle -43.3^\circ\ A$

$V_R = RI = 1.46\angle -43.3^\circ\ V \qquad V_L = Z_L I = 1.37\angle 46.7^\circ\ V$



8.27 The currents $i_R(t)$ and $i_C(t)$ in the circuit shown in Fig. P8.17 can be drawn as phasors in a phasor diagram. Use the diagram to show that $i_R(t) + i_C(t) = i_S(t)$.

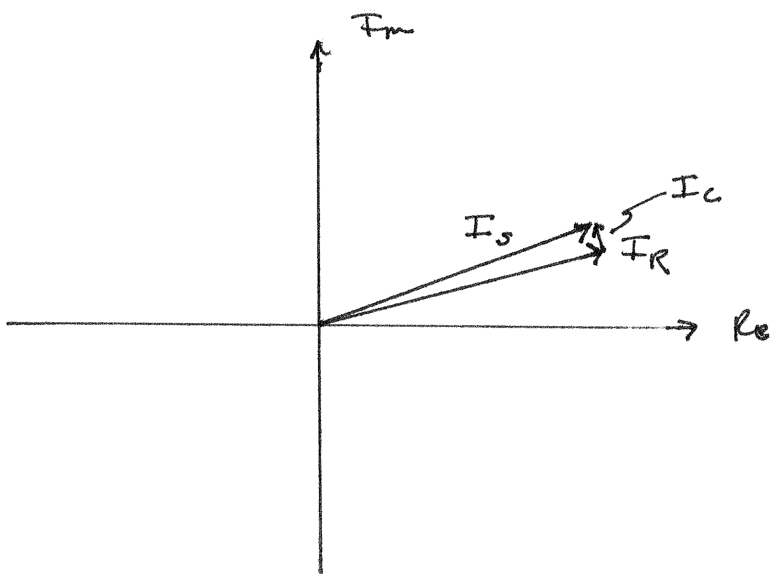
SOLUTION:



$I_S = 10\angle 30°\ A$ $R = 1\Omega$ $C = 100\mu F$

$\omega = 377$ $Z_C = -j26.5\Omega$

$$I_R = \frac{I_S Z_C}{R + Z_C} = 9.993\angle 27.8°\ A \qquad I_C = \frac{I_S R}{R + Z_C} = 0.377\angle 118°\ A$$



8.28 The currents $i_R(t)$ and $i_C(t)$ in the circuit shown in Fig. P8.28 can be drawn as phasors in a phasor diagram. Use the diagram to show that $i_R(t) + i_C(t) = i_S(t)$.
CS

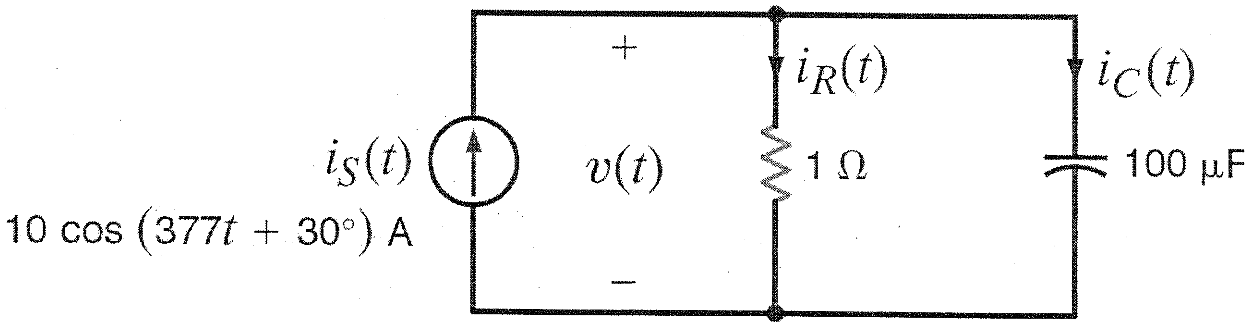
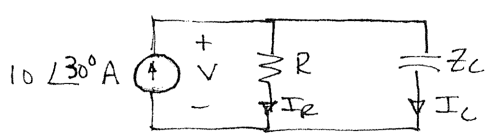


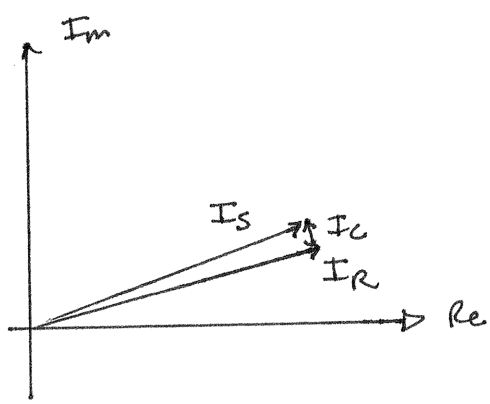
Figure P8.28

SOLUTION:



$Z_C = \frac{1}{j\omega C} = -j26.5\,\Omega$

$I_R = \frac{I_S Z_C}{Z_C + R} = 9.993\,\angle 27.8^\circ\ \text{A}$ $\qquad I_C = \frac{I_S R}{R + Z_C} = 0.377\,\angle 118^\circ\ \text{A}$



8.29 Draw the frequency-domain network and calculate $v_o(t)$ in the circuit shown in Fig. P8.29 if $i_S(t)$ is $300\sin(10^4 t - 45°)$ mA. Also, using a phasor diagram, show that $i_1(t) + i_2(t) = i_S(t)$. CS

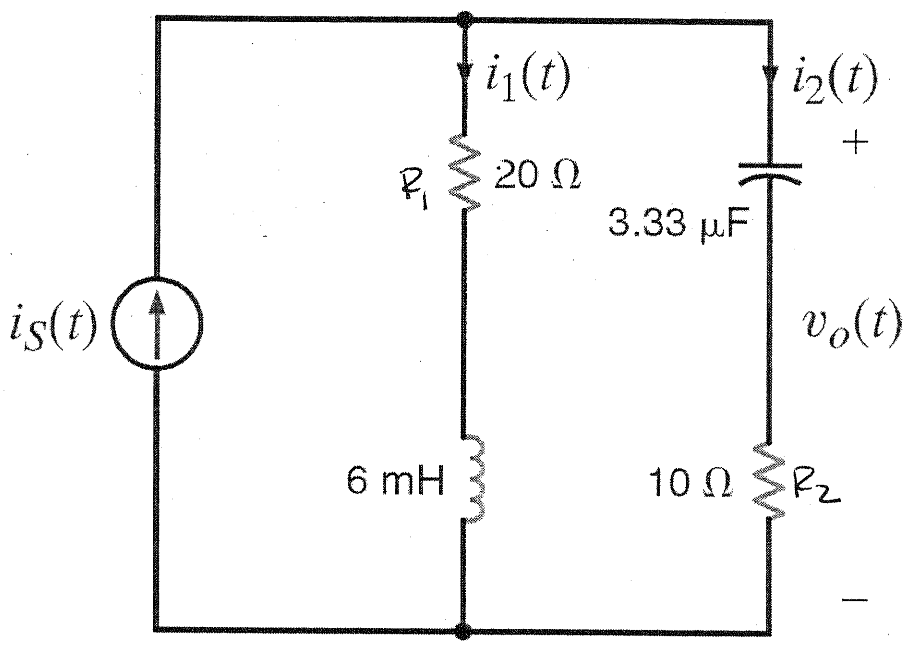
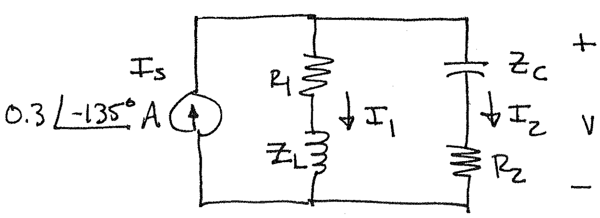


Figure P8.29

SOLUTION:



$Z_L = j60\,\Omega \qquad Z_C = -j30$

$Z_1 = R_1 + Z_L = 20 + j60\,\Omega$

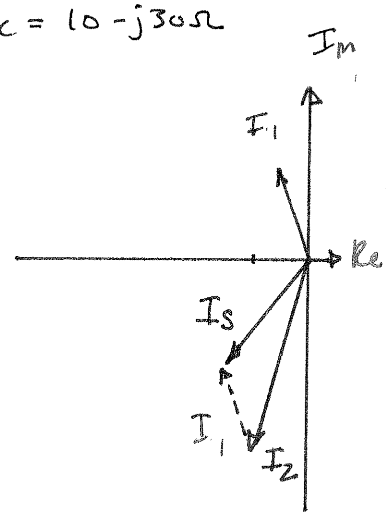
$Z_2 = R_2 + Z_C = 10 - j30\,\Omega$

$I_1 = I_S Z_2/(Z_1+Z_2) = 0.224\angle 108.4°$ A

$I_2 = I_S Z_1/(Z_1+Z_2) = 0.448\angle -108.4°$ A

$V_o = I_2 Z_2 = 14.1\angle 180$ V

$\boxed{v_o(t) = 14.1\cos(10^4 t + 180°)\text{V}}$



8.30 Find the value of C in the circuit shown in Fig. P8.30 so that $\mathbf{Z}$ is purely resistive at the frequency of 60 Hz.

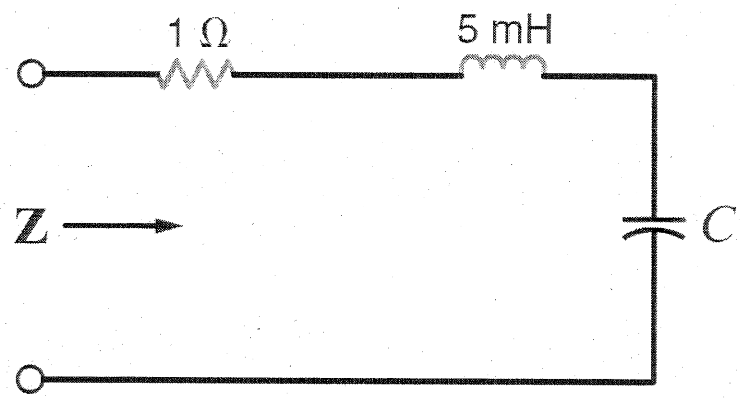


Figure P8.30

SOLUTION:

$Z = R + Z_L + Z_C = R_{eq}$ requires $\omega L = \frac{1}{\omega C}$

$C = \frac{1}{\omega^2 L}$ $\boxed{C = 1.41\,mF}$

8.31 Find the frequency at which the circuit shown in Fig. P8.31 is purely resistive.

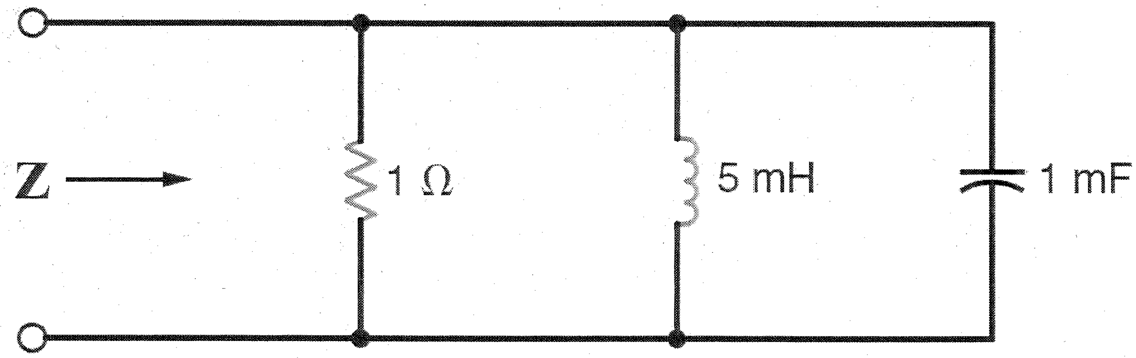


Figure P8.31

SOLUTION:

$Z = R_{eq} \qquad Y = \frac{1}{R_{eq}} = G_{eq} = \frac{1}{R} + \frac{1}{Z_L} + \frac{1}{Z_C}$

Requires $\frac{1}{\omega L} = \omega C \Rightarrow \omega = \frac{1}{\sqrt{LC}} \qquad \omega = 447.2\ r/s$

$f = \frac{\omega}{2\pi} \qquad \boxed{f = 71.2\ \text{Hz}}$

8.32 In the circuit shown in Fig. P8.32, determine the frequency at which $i(t)$ is in phase with $v_S(t)$.

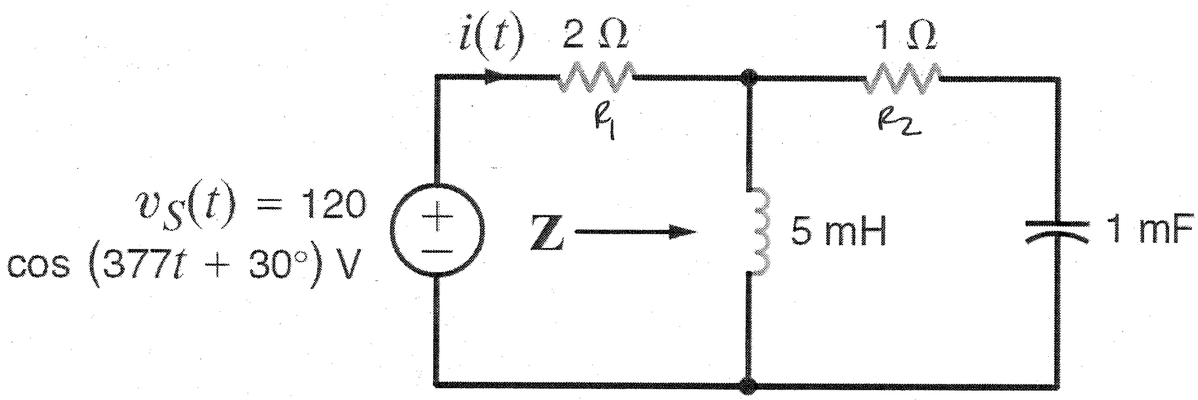


Figure P8.32

SOLUTION: for $i(t)$ and $V_S(t)$ to be in phase, Z must be real.

$Z_1 = R_2 + Z_C = 1 - \frac{j}{\omega C}$ $\qquad Z_2 = \frac{Z_L Z_1}{Z_L + Z_1}$ must be real!

$$Z_2 = \frac{L/C + j\omega L}{1 + j(\omega L - \frac{1}{\omega C})} = R_{eq} \Rightarrow \frac{\omega L}{L/C} = \frac{\omega L - 1/\omega C}{1}$$

$(\omega C)^2 = \omega^2 LC - 1$ $\qquad \omega^2 (LC - C^2) = 1$ $\qquad \omega = \frac{1}{\sqrt{LC - C^2}}$

$\boxed{\omega = 500 \text{ r/s}}$

8.33 In the circuit shown in Fig. P8.33, determine the value of the inductance such that $v(t)$ is in phase with $i_S(t)$.

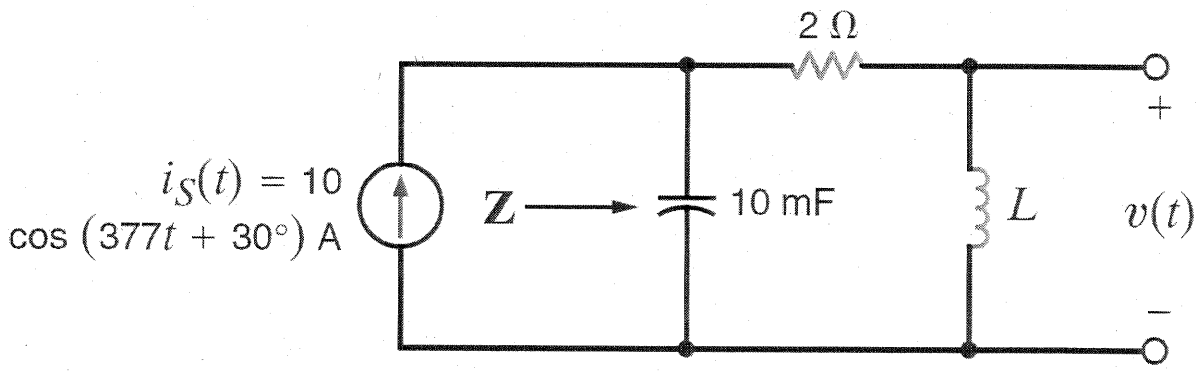
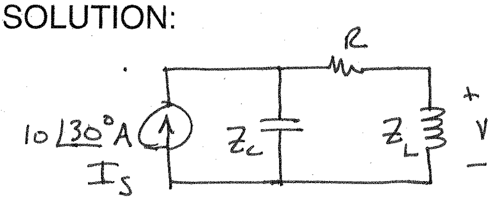


Figure P8.33

SOLUTION:



$$V = \frac{I_S Z_C Z_L}{R + Z_L + Z_C}$$

$$\frac{V}{I_S} = \frac{Z_C Z_L}{R + Z_L + Z_C} = \frac{L/C}{R + j\left(\omega L - \frac{1}{\omega C}\right)} = R_{eq} + j0$$

Requires $\omega L = \frac{1}{\omega C} \Rightarrow L = \frac{1}{\omega^2 C}$ $\boxed{L = 0.704 \text{ mH}}$

8.34 Find the current **I** shown in Fig. P8.34.

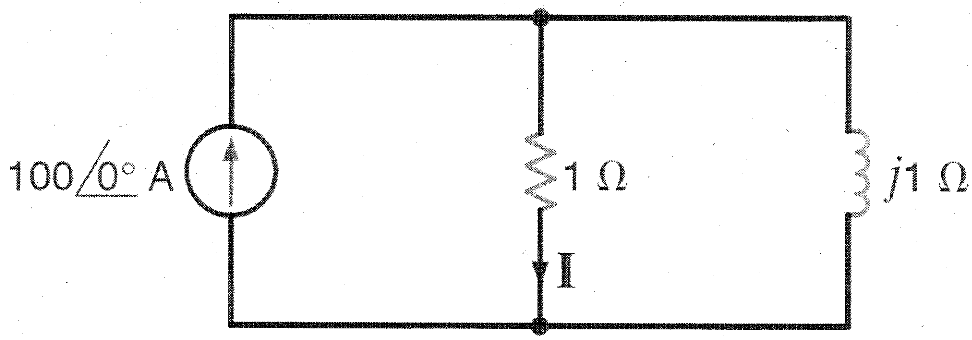


Figure P8.34

SOLUTION:

$$I = 100\angle 0^\circ \left(\frac{j1}{1+j1}\right) \qquad \boxed{I = 70.7\angle 45^\circ \text{ A}}$$

8.35 Find the voltage **V** shown in Fig. P8.35.

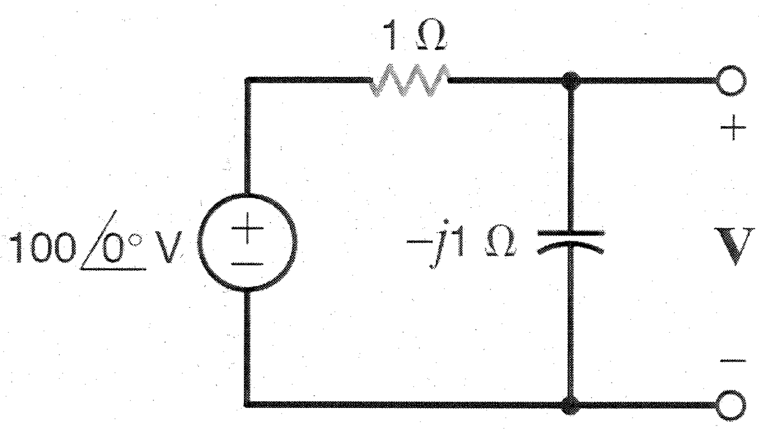


Figure P8.35

SOLUTION:

$$V = 100\angle 0^\circ \left(\frac{-j1}{1-j1}\right)$$

$$\boxed{V = 70.7\angle -45^\circ\ V}$$

8.36 Find the frequency-domain voltage $\mathbf{V}_o$, as shown in Fig. P8.36. CS

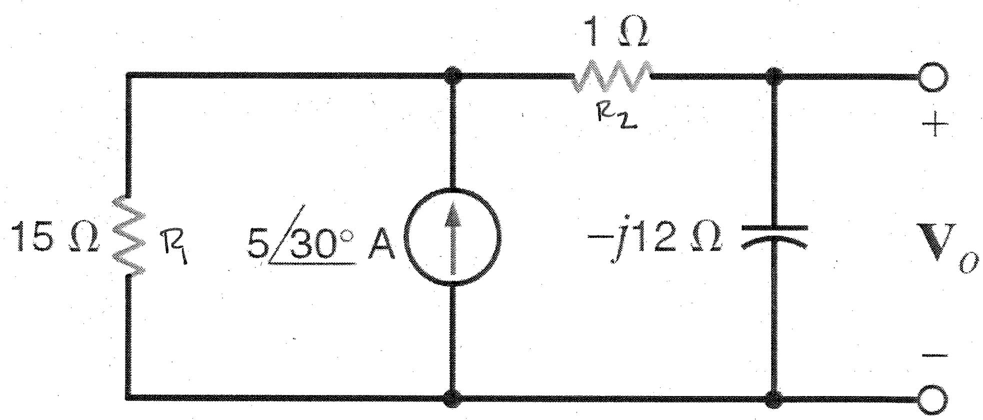
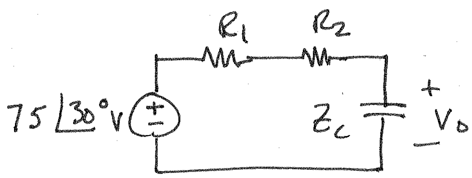


Figure P8.36

SOLUTION:

Source transformation:



$$V_o = \frac{75\angle 30\, Z_c}{Z_c + R_1 + R_2}$$

$$\boxed{V_o = 45\angle -23.1^\circ \text{ V}}$$

8.37 Find the voltage, $\mathbf{V}_o$, shown in Fig. P8.37.

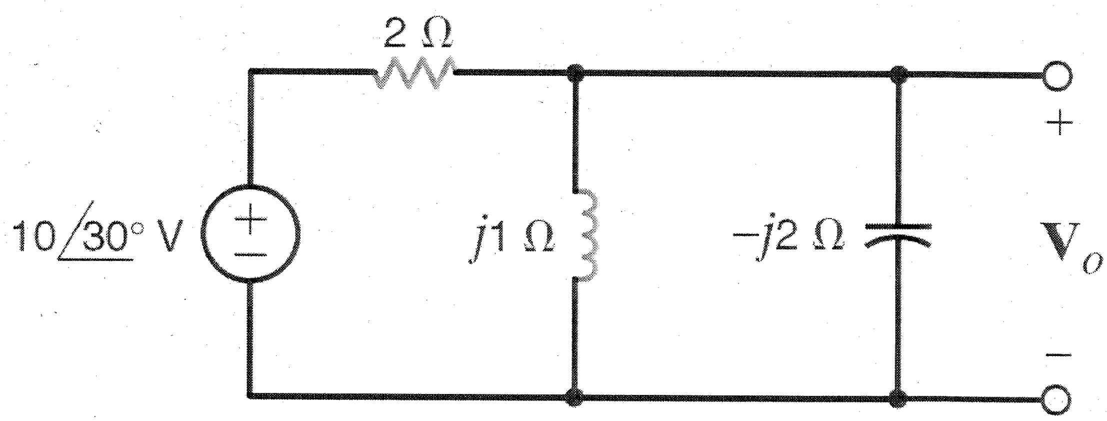
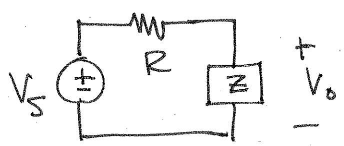


Figure P8.37

SOLUTION:



$$Z = \frac{Z_L Z_C}{Z_L + Z_C} = j2\,\Omega$$

$$V_o = \frac{V_s Z}{R + Z}$$

$$\boxed{V_o = 7.07 \angle 75^\circ \text{ V}}$$

8.38 Given the network in Fig. P8.38, determine the value of $\mathbf{V}_o$ if $\mathbf{V}_S = 24\,\underline{/0^\circ}$ V.

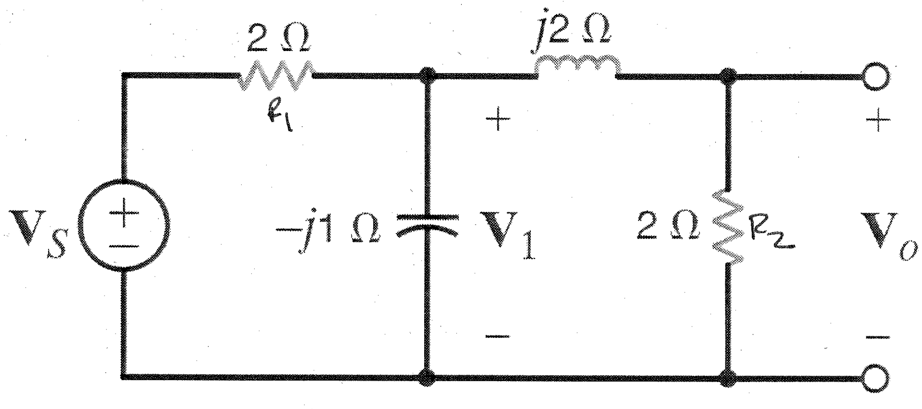


Figure P8.38

SOLUTION:

$Z_1 = R_2 + Z_L = 2 + j2\,\Omega$ $\quad Z_2 = \dfrac{Z_1 Z_C}{Z_1 + Z_C} = \dfrac{2-j2}{2+j1}$

$V_1 = V_S \dfrac{Z_2}{Z_2 + R_1} = 8 - j8$ V $\quad V_o = \dfrac{V_1 R_2}{R_2 + Z_L} = \dfrac{8-j8}{1+j1} = 8\,\underline{/-90^\circ}$ V

$\boxed{V_o = 8\,\underline{/-90^\circ}\text{ V}}$

8.39 Find $\mathbf{V}_S$ in the network in Fig. P8.39, if $\mathbf{V}_1 = 4\angle 0°$ V.
CS

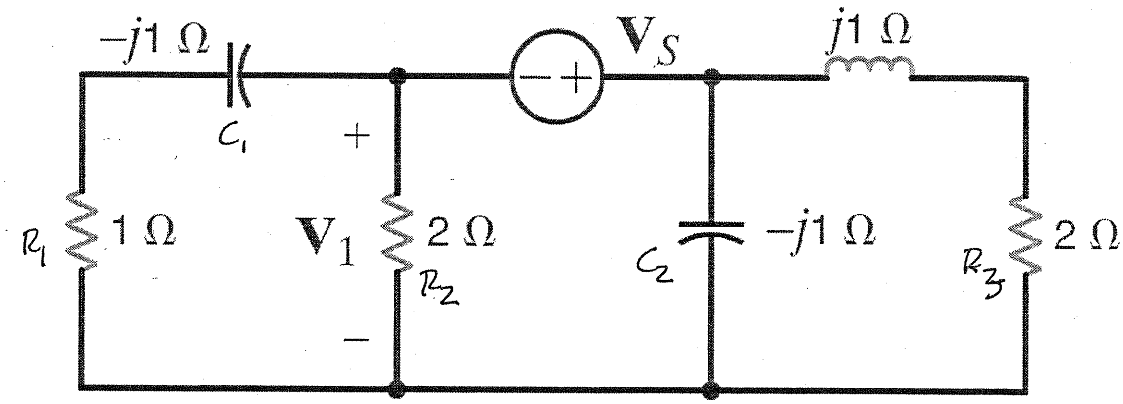
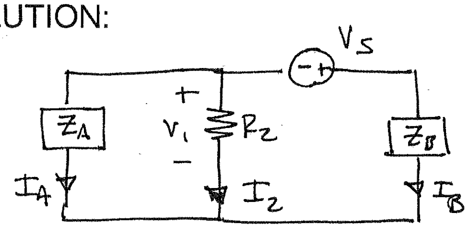


Figure P8.39

SOLUTION:



$Z_A = 1 - j1\ \Omega$

$Z_B = \dfrac{-j1(2+j1)}{2} = \dfrac{1-j2}{2}\ \Omega$

$I_2 = V_1/R_2 = 2\angle 0°\ A$ $\quad I_A = \dfrac{V_1}{Z_A} = 2 + j2\ A$ $\quad -I_B = I_A + I_2 = 4 + j2\ A$

$V_S = I_B Z_B - V_1 = -8 + j3\ V$

$$\boxed{V_S = 8.54\angle +159.4°\ \ V}$$

8.40 Find $\mathbf{V}_o$ in the network in Fig. P8.40. PSV

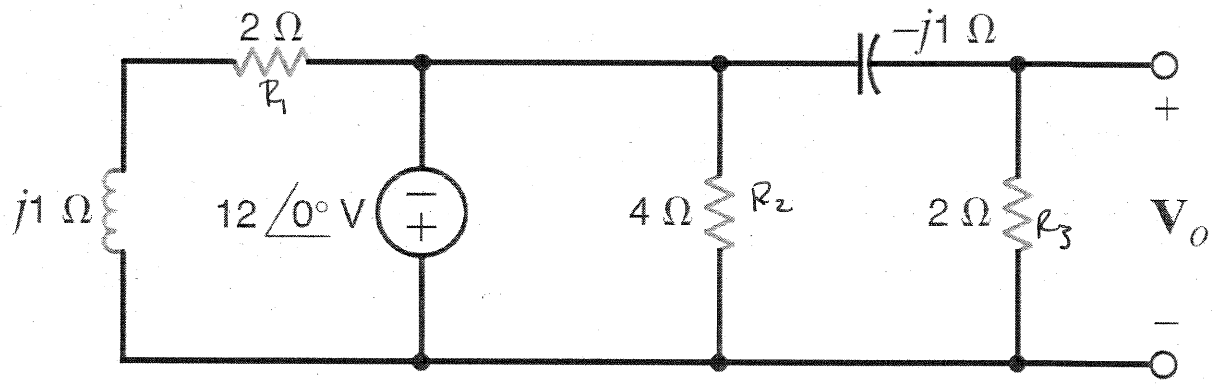
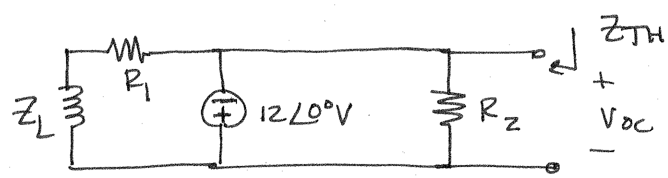


Figure P8.40

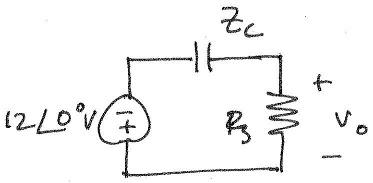
SOLUTION:

Use Thevenin's Theorem:



$V_{oc} = 12\angle 180° V$

$Z_{TH} = 0\ \Omega$



$$V_o = -12\angle 0° \left(\frac{R_3}{R_3 + Z_C}\right)$$

$$\boxed{V_o = 10.7\ \angle -153.4°\ V}$$

8.41 If $\mathbf{V}_1 = 4\underline{/0^\circ}$ V, find $\mathbf{I}_o$ in Fig. P8.41. CS

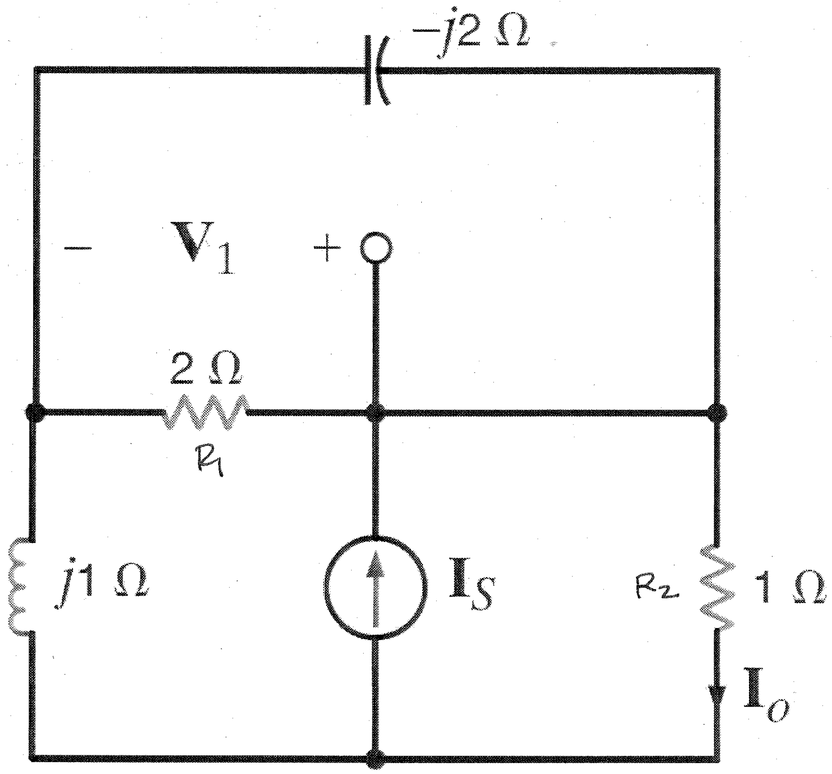
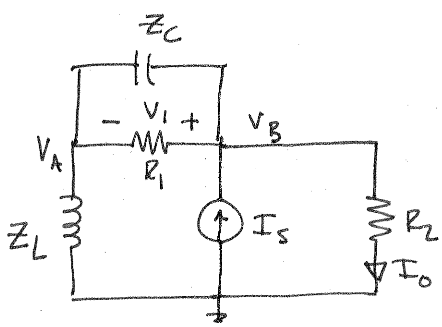


Figure P8.41

SOLUTION:



Nodal: $\dfrac{V_1}{R_1} + \dfrac{V_1}{Z_C} = \dfrac{V_A}{Z_L} \Rightarrow V_A = -2 + j2\ \text{V}$

$V_B = V_A + V_1 = V_A + 4 = 2 + j2\ \text{V}$

$I_o = \dfrac{V_B}{R_2} \Rightarrow \boxed{I_o = 2.83\,\underline{/45^\circ}\ \text{A}}$

8.42 In the network in Fig. P8.42 $\mathbf{I}_o = 4\angle 0°$ A find $\mathbf{I}_x$.

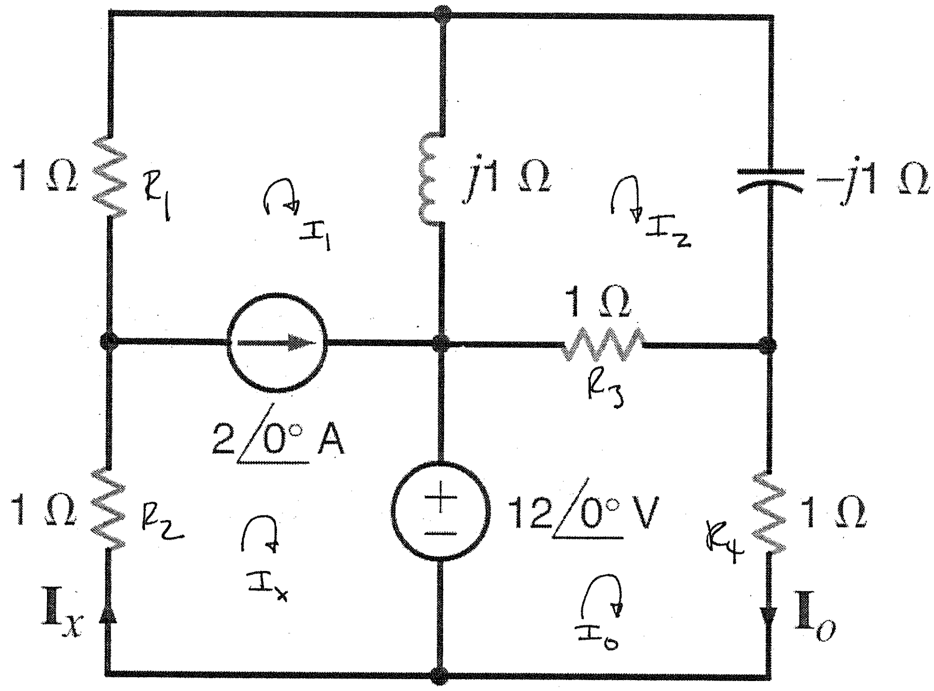


Figure P8.42

SOLUTION:

$I_x - I_1 = 2\angle 0$ $\Downarrow$ $I_1 = I_x - 2$

$12 = 2I_o - I_2$ $\Downarrow$ $I_2 = -4A$

$I_x + I_1 - j1\, I_2 + I_o = 0$ $\Downarrow$ $2I_x - 2 + j4 + 4 = 0$

$$\boxed{I_x = -1 - j2 \text{ A} = 2.24\angle -116.6° \text{ A}}$$

8.43 If $\mathbf{I}_o = 4\angle 0^\circ$ A in the circuit in Fig. P8.43, find $\mathbf{I}_x$.

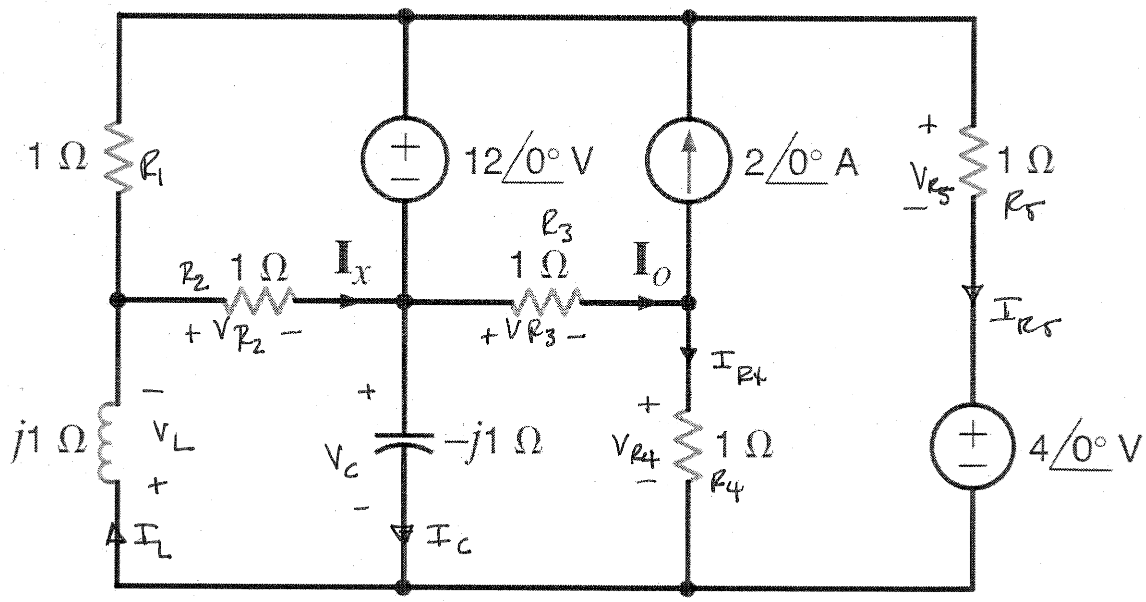


Figure P8.43

SOLUTION:

$V_{R3} = I_o R_3 = 4\angle 0^\circ$ V $\quad I_{R4} = I_o - 2\angle 0^\circ = 2\angle 0^\circ$ A $\quad V_C = V_{R3} + V_{R4} = 6\angle 0^\circ$ V

$I_C = V_C / {-j1} = j6$ A $\quad V_C + 12 = V_{R5} + 4 \Rightarrow V_{R5} = 14$ V

$I_{R5} = V_{R5}/R_5 = 14$ A $\quad I_L = I_C + I_{R4} + I_{R5} = 16 + j6$ A

$V_L = I_L(j1) = -6 + j16$ V $\quad V_L + V_{R2} + V_C = 0 \Rightarrow V_{R2} = -j16$ V

$I_x = \dfrac{V_{R2}}{R_2} = -j16$ A $\quad \boxed{I_x = -j16 \text{ A} = 16\angle -90^\circ \text{ A}}$

8.44 If $\mathbf{I}_o = 4\angle 0^\circ$ A in the network in Fig. P8.44, find $\mathbf{I}_x$.

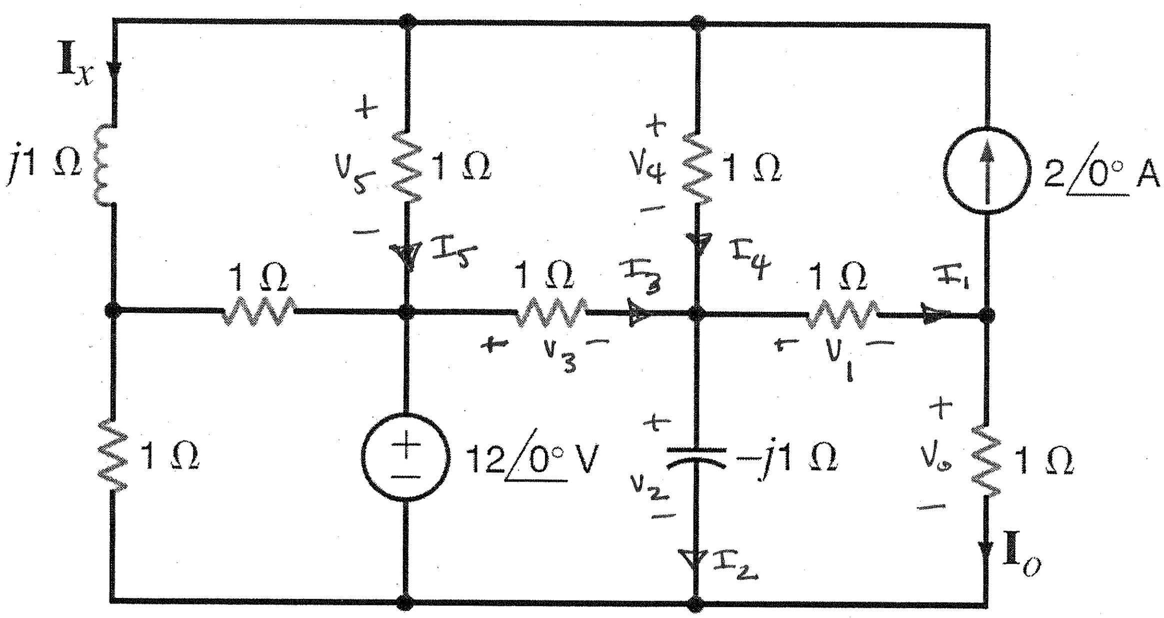


Figure P8.44

SOLUTION: $V_o = (1)I_o = 4\angle 0^\circ$ V

$I_1 = 2 + I_o = 6\angle 0^\circ$ A

$V_1 = 1 I_1 = 6\angle 0^\circ$ V

$V_2 = V_1 + V_o = 10\angle 0^\circ$ V

$I_2 = V_2 / -j1 = j10$ A

$V_3 = 12\angle 0 - V_2 = 2\angle 0$ V

$I_3 = V_3 / 1 = 2\angle 0$ A

$I_4 = I_1 + I_2 - I_3 = 4 + j10$ A

$V_4 = (1) I_4 = 4 + j10$ A

$V_5 = V_4 - V_3 = 2 + j10$ V

$I_5 = V_5 / 1 = 2 + j10$ A

$I_x = 2 - I_4 - I_5$

$$\boxed{I_x = -4 - j20 \text{ A}}$$

8.45 In the network in Fig. P8.45, $\mathbf{V}_o$ is known to be $4\,\underline{/45°}$ V. Find **Z**.

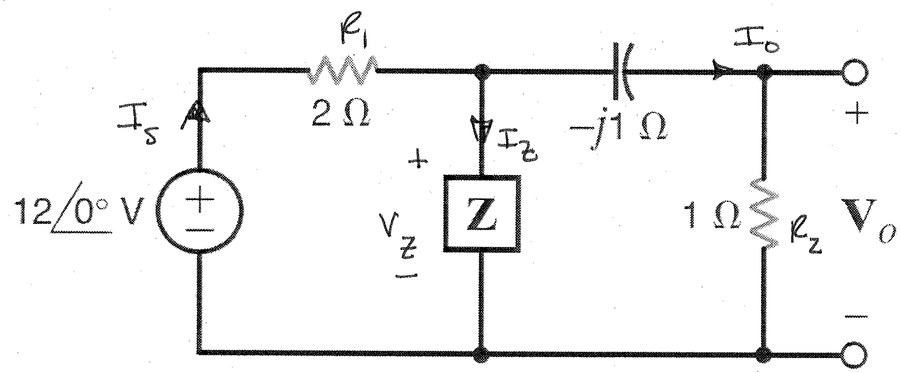


Figure P8.45

SOLUTION:

$I_o = \dfrac{V_o}{R_2} = 4\,\angle 45°\ \text{A}$ $\quad V_Z = I_o(R_2 - j1) = 4\sqrt{2}\,\angle 0°\ \text{V}$

$I_s = \dfrac{12\,\angle 0° - V_Z}{R_1} = 3.17\angle 0°\ \text{A}$ $\quad I_Z = I_s - I_o = 2.85\,\angle -83.1°\ \text{A}$

$$\boxed{Z = \frac{V_Z}{I_Z} = 1.98\,\angle 83.1°\ \Omega}$$

8.46 In the network in Fig. P8.46, $\mathbf{V}_1 = 2\angle 45^\circ$ V. Find $\mathbf{Z}$.

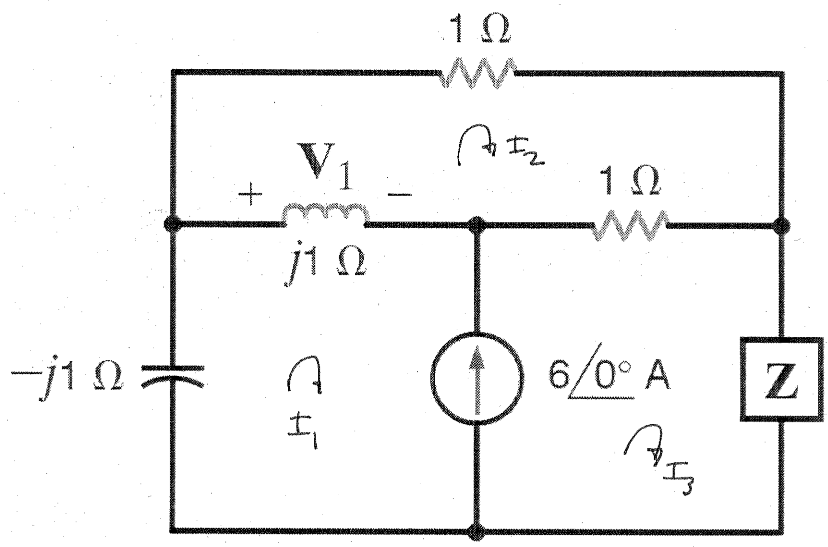


Figure P8.46

SOLUTION:

① $I_3 - I_1 = 6\angle 0^\circ \text{A}$ ② $I_3(1+Z) - I_2(1+j1) = 0$

③ $I_2(2+j1) - j1 I_1 - I_3 = 0$ ④ $\frac{V_1}{j1} = 2\angle -45^\circ = I_1 - I_2$

Solve for Z, yields

$$\boxed{\begin{array}{l} Z = -0.508 + j0.586\ \Omega \\ Z = 0.776\angle 130.9^\circ\ \Omega \end{array}}$$

8.47 Find $\mathbf{V}_o$ in the circuit in Fig. P8.47.

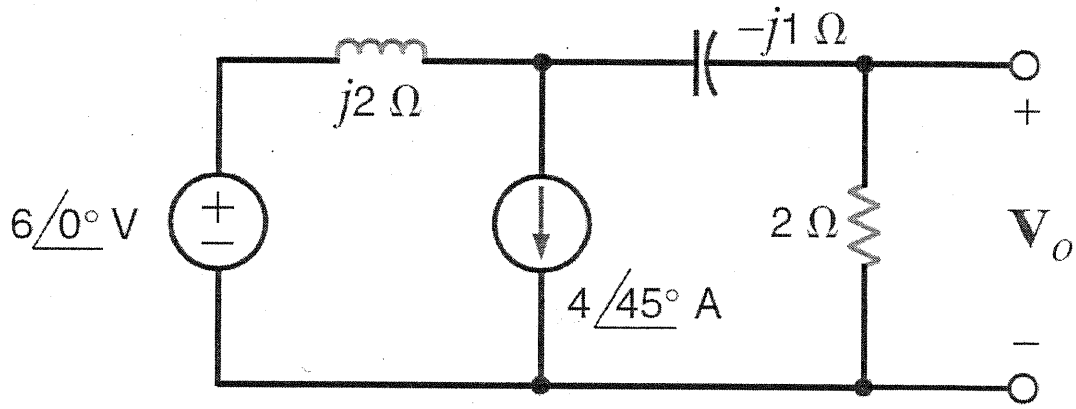
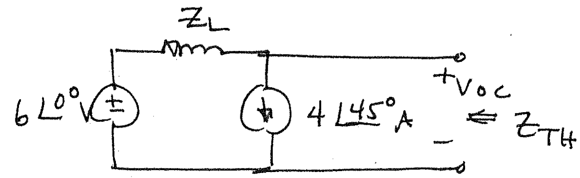


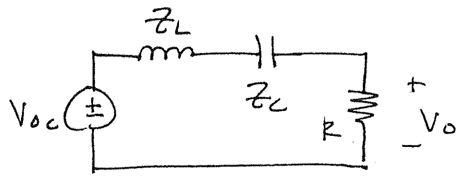
Figure P8.47

SOLUTION:

Thevenin eq.



$Z_{TH} = Z_L = j2$



Superposition

$V_{oc} = 6\angle 0° - 4\angle 45° (j2)$

$V_{oc} = 13.0\angle -25.9°$ V

$V_o = \dfrac{V_{oc}\,(2)}{2 + j2 - j1}$

$\boxed{V_o = 11.6\angle -52.5°\ \text{V}}$

8.48 Using nodal analysis, find $\mathbf{I}_o$ in the circuit in Fig. P8.48.

CS

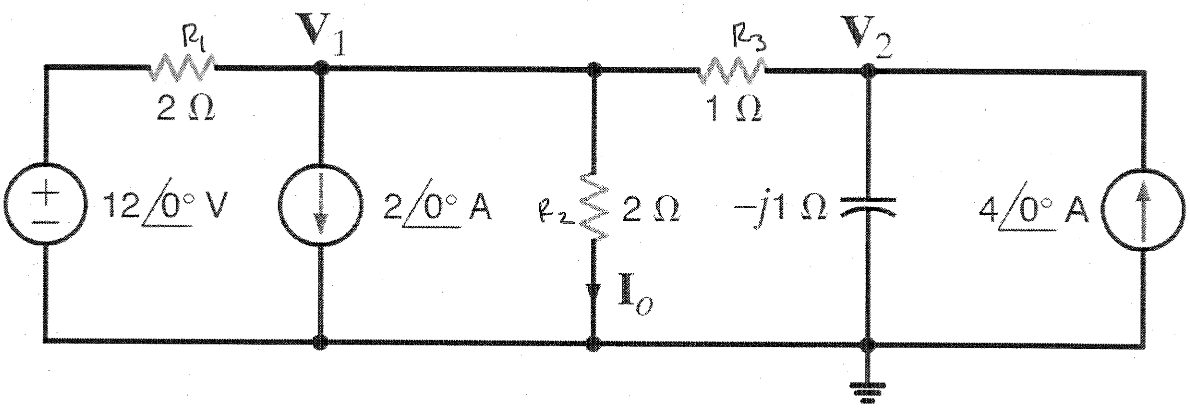


Figure P8.48

SOLUTION:

$$\frac{V_1-12}{2} + 2 + \frac{V_1}{2} + \frac{V_1 - V_2}{1} = 0 \Rightarrow 2V_1 - V_2 = 4$$

$$\frac{V_2 - V_1}{1} + \frac{V_2}{-j1} = 4 \Rightarrow -V_1 + V_2(1+j1) = 4$$

$$\begin{bmatrix} 2 & -1 \\ -1 & 1+j1 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$$

$$I_o = \frac{V_1}{2} \qquad \boxed{I_o = 2\angle{-36.9^\circ}\ A} \Leftarrow V_1 = 4\angle{-36.9^\circ}\ V$$

8.49 Determine $\mathbf{V}_o$ in the circuit in Fig. P8.49.

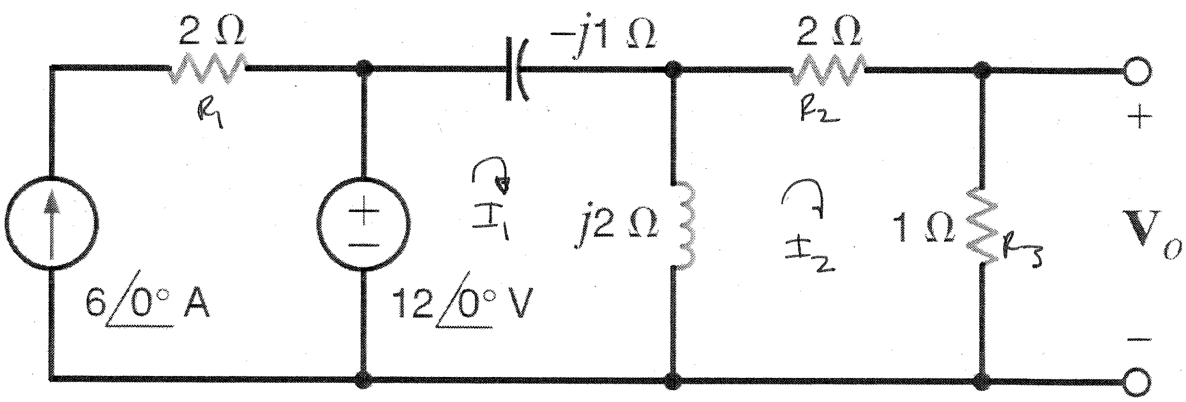


Figure P8.49

SOLUTION:

① $12 = I_1(j1) - j2 I_2$ ② $-j2 I_1 + I_2(3+j2) = 0$

$$\begin{bmatrix} j1 & -j2 \\ -j2 & 3+j2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \end{bmatrix} \Rightarrow I_2 = 6.66\angle 33.67^\circ \text{ A}$$

$V_o = R_3 I_2$ $\boxed{V_o = 6.66\angle 33.67^\circ \text{ V}}$

8.50 Using nodal analysis, find $\mathbf{I}_o$ in the circuit in Fig. P8.50.

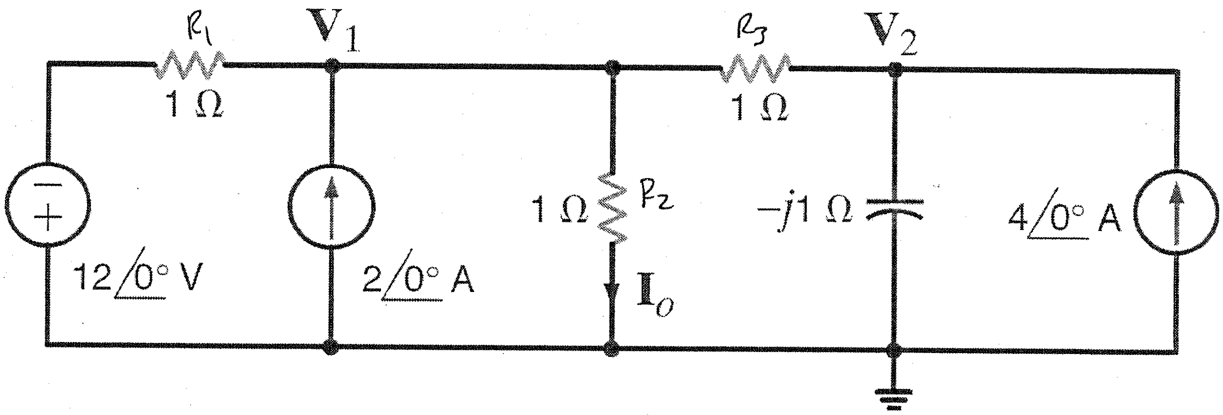


Figure P8.50

SOLUTION:

① $\frac{V_1+12}{1} + \frac{V_1}{1} + \frac{V_1 - V_2}{1} = 2\angle 0°$ $I_0 = \frac{V_1}{R_2}$

or, $3V_1 - V_2 = -10$

② $\frac{V_2 - V_1}{1} + \frac{V_2}{-j1} = 4 \Rightarrow -V_1 + V_2(1+j1) = 4$

$\begin{bmatrix} 3 & -1 \\ -1 & 1+j1 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} -10 \\ 4 \end{bmatrix} \rightarrow V_1 = 3.23 \angle -177° \text{ V}$

$\boxed{I_0 = 3.23 \angle -177° \text{ A}}$

8.51 Use nodal analysis to find $\mathbf{I}_o$ in the circuit in Fig. P8.57.

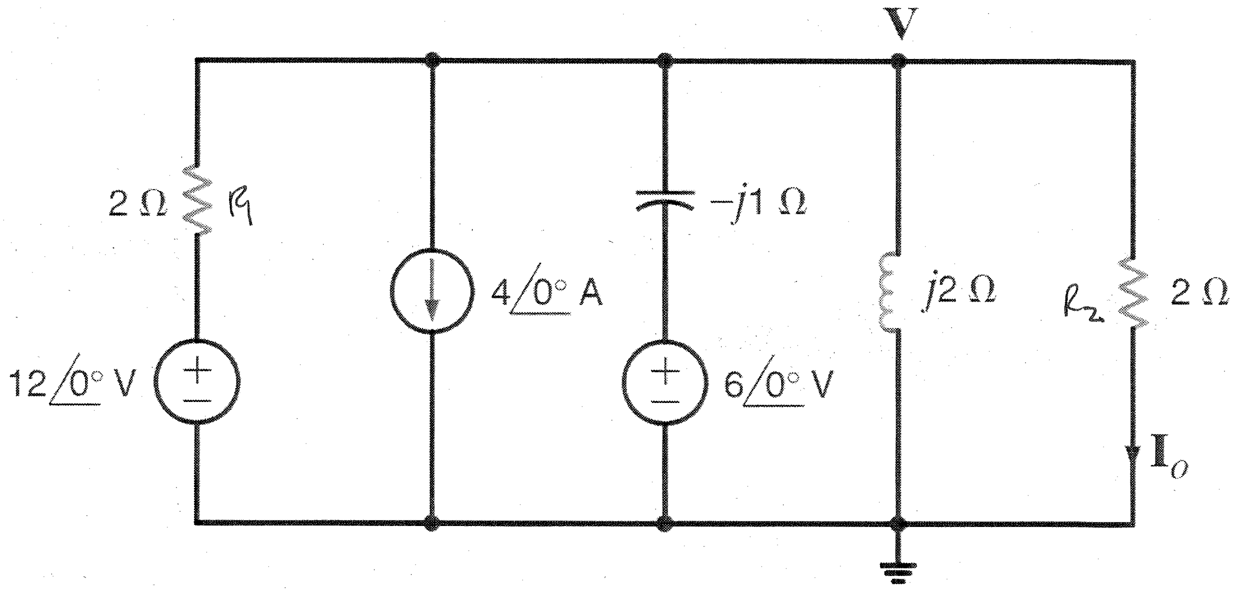


Figure P8.51

SOLUTION:

$$\frac{V-12}{2} + 4 + \frac{V-6}{-j1} + \frac{V}{j2} + \frac{V}{2} = 0 \qquad \frac{V}{R_2} = I_o$$

$$V\left(1 + j\tfrac{1}{2}\right) = 2 + j6 \qquad V = \frac{4+j12}{2+j1} = 5.66\angle 45^\circ \text{ V}$$

$$\boxed{I_o = 2.83\angle 45^\circ \text{ A}}$$

8.52 Find $\mathbf{V}_o$ in the network in Fig. P8.52. CS

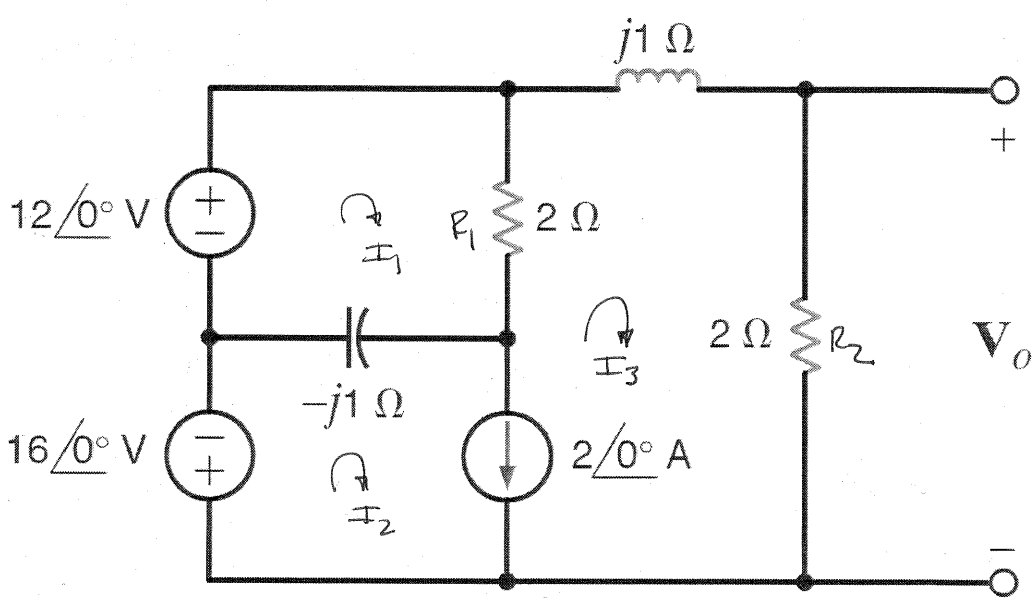


Figure P8.52

SOLUTION:

① $12 = I_1(2 - j1) + j1 I_2 - 2I_3$

② $-16 + 12 = I_3(2 + j1) = -4 \Rightarrow I_3 = \dfrac{-4}{2+j1} = 1.79\angle 153.4° A$

$V_o = R_2 I_3$ $\boxed{V_o = 3.58\angle 153.4° \ V}$

8.53 Find $\mathbf{V}_o$ in the network in Fig. P8.53 using nodal analysis. **PSV**

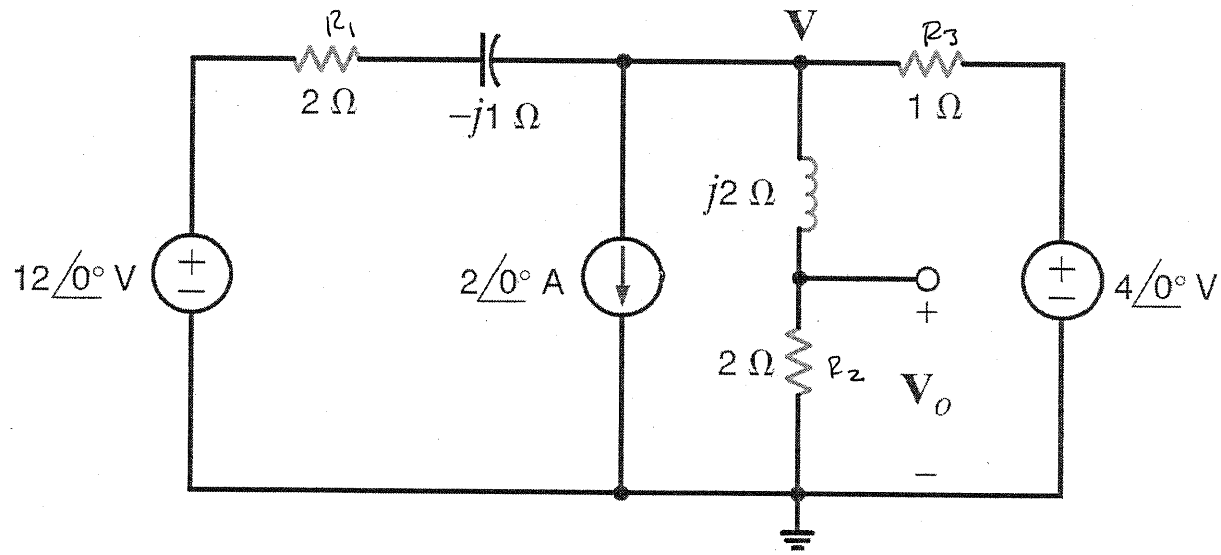


Figure P8.53

SOLUTION:

$$\frac{V-12}{2-j1} + 2 + \frac{V}{2+j2} + \frac{V-4}{1} = 0 \qquad V_o = \frac{V\,(2)}{2+j2}$$

$$\boxed{V_o = 3.09\ \angle -23.8^\circ\ \text{V}}$$

8.54 Find $\mathbf{I}_o$ in the circuit in Fig. P8.54 using nodal analysis.

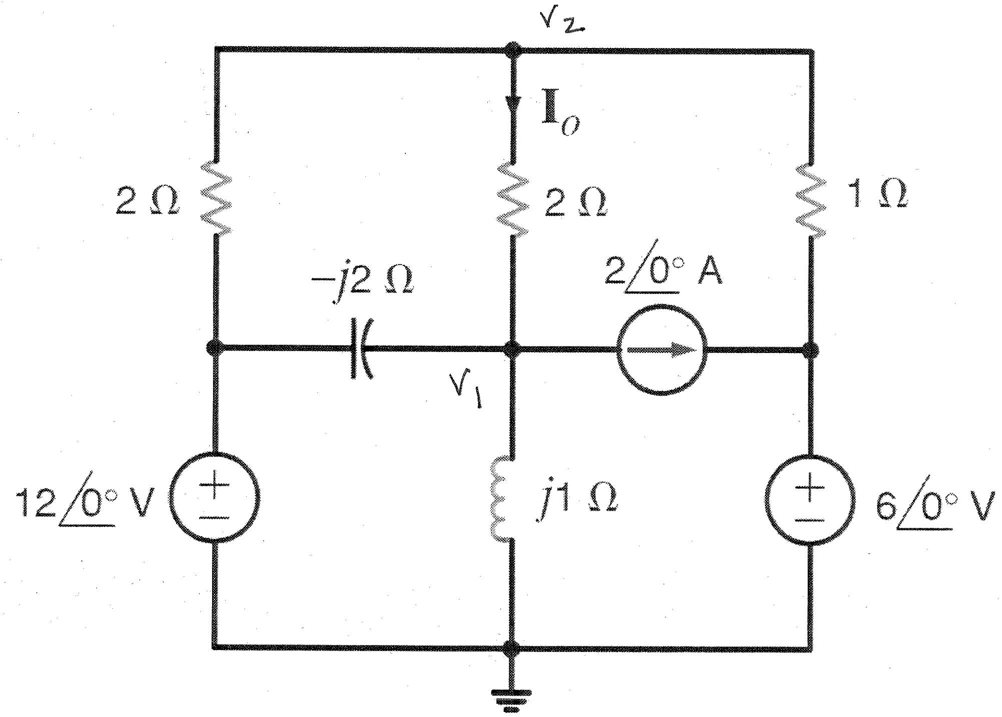


Figure P8.54

SOLUTION:

@ V_1: $\frac{V_1-12}{-j2}+\frac{V_1}{j1}+\frac{V_1-V_2}{2}=-2 \Rightarrow V_1(1-j1)-V_2=-4+j12$

@ V_2: $\frac{V_2-12}{2}+\frac{V_2-V_1}{2}+\frac{V_2-6}{1}=0 \Rightarrow -\frac{V_1}{2}+2V_2=12$

$$\begin{bmatrix} 1-j1 & -1 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}=\begin{bmatrix} -4+j12 \\ 24 \end{bmatrix} \qquad I_0=\frac{V_2-V_1}{2}$$

$V_1=-6.72+j7.04$ V
$V_2=4.32+j1.26$ V

$$\boxed{I_0=6.12\angle{-25.5^\circ}\ \text{A}}$$

8.55 Use the supernode technique to find $\mathbf{I}_o$ in the circuit in Fig. P8.55.

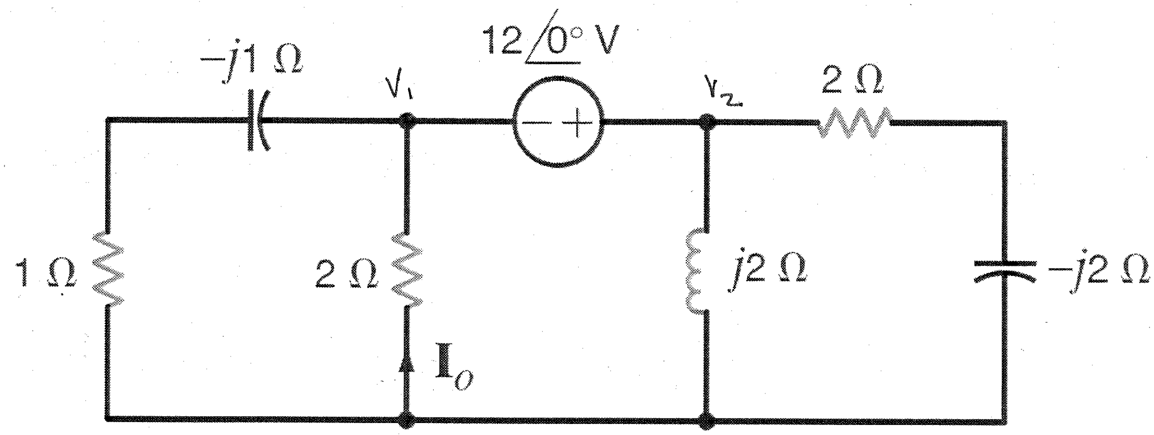


Figure P8.55

SOLUTION:

① $\frac{V_1}{1-j1} + \frac{V_1}{2} + \frac{V_2}{j2} + \frac{V_2}{2-j2} = 0 \Rightarrow \quad V_1[8+j4] + V_2[2-j2] = 0$

② $V_2 - V_1 = 12\angle 0°$

$$\begin{bmatrix} 8+j4 & 2-j2 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 12\angle 0° \end{bmatrix} \Rightarrow V_1 = 3.33\angle 123.7° \text{ V}$$

$I_o = -\frac{V_1}{2}$ $\boxed{I_o = 1.67\angle -56.3° \text{ A}}$

8.56 Find $\mathbf{I}_o$ in the network in Fig. P8.56.

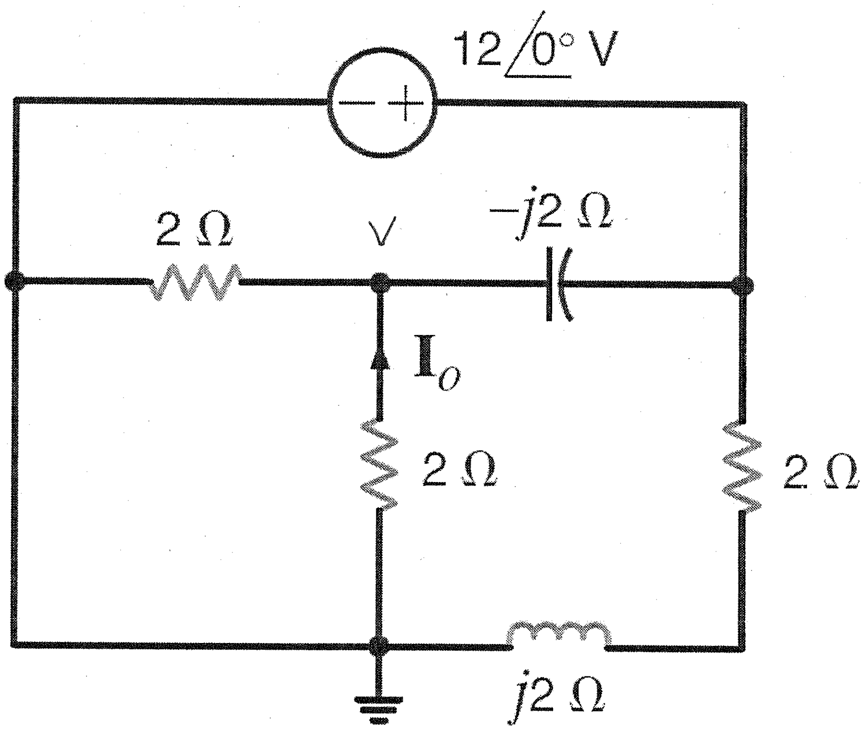


Figure P8.56

SOLUTION:

$$\frac{V}{2} + \frac{V}{2} + \frac{V - 12\angle 0}{-j2} = 0 \Rightarrow V\left(1 + j/2\right) = +j6 \Rightarrow V = 5.37\angle 63.4°\ V$$

$I_o = -V/2$ $\boxed{I_o = 2.69\angle -167°\ A}$

8.57 Find $\mathbf{V}_o$ in the network in Fig. P8.57.

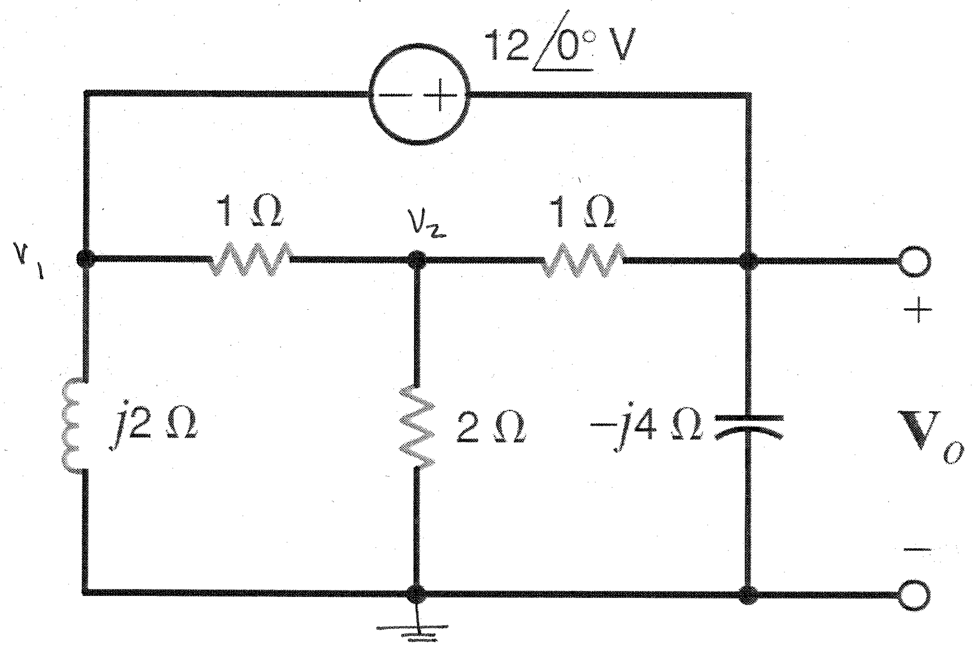


Figure P8.57

SOLUTION:

Nodal: @ V_2: $\frac{V_2 - V_1}{1} + \frac{V_2 - V_0}{1} + \frac{V_2}{2} = 0 \Rightarrow -V_1 + 2.5V_2 - V_0 = 0$

@ GND: $\frac{V_1}{j2} + \frac{V_2}{2} + \frac{V_0}{-j4} = 0 \Rightarrow 2V_1 + j2V_2 - V_0 = 0$

and, $V_0 - V_1 = 12\angle 0^\circ$ V

$$\begin{bmatrix} -1 & 2.5 & -1 \\ 2 & j2 & -1 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 12 \end{bmatrix} \Rightarrow \boxed{V_0 = 13.7\angle -36.2^\circ \text{ V}}$$

8.58 Use nodal analysis to find $\mathbf{V}_o$ in the circuit in Fig. P8.58.

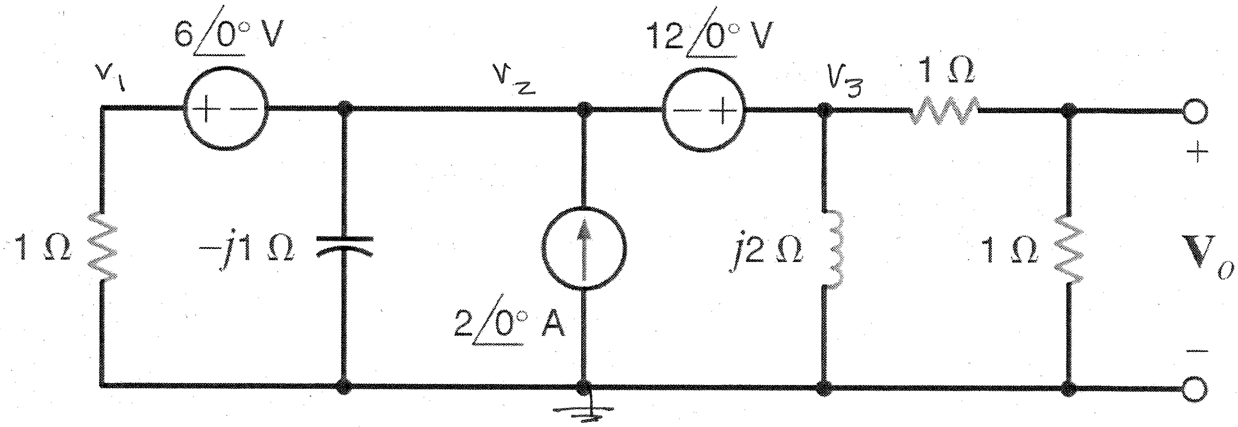


Figure P8.58

SOLUTION:

$V_1 - V_2 = 6\angle 0°$ $\quad V_3 - V_2 = 12\angle 0°$ $\quad V_3 - V_o = V_o \Rightarrow V_3 = 2V_o$

at GND NODE: $\frac{V_1}{1} + \frac{V_2}{-j1} + \frac{V_3}{j2} + \frac{V_o}{1} = 2\angle 0° \Rightarrow V_1 + jV_2 - \frac{jV_3}{2} + V_o = 2$

$$\begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & -2 \\ 1 & j1 & -j/2 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_o \end{bmatrix} = \begin{bmatrix} 6 \\ 12 \\ 0 \\ 2 \end{bmatrix} \Rightarrow \boxed{V_o = 4.56\angle 37.9° \text{ V}}$$

8.59 The low-frequency equivalent circuit for a common-emitter transistor amplifier is shown in Fig. P8.59. Compute the voltage gain $\mathbf{V}_o/\mathbf{V}_S$.

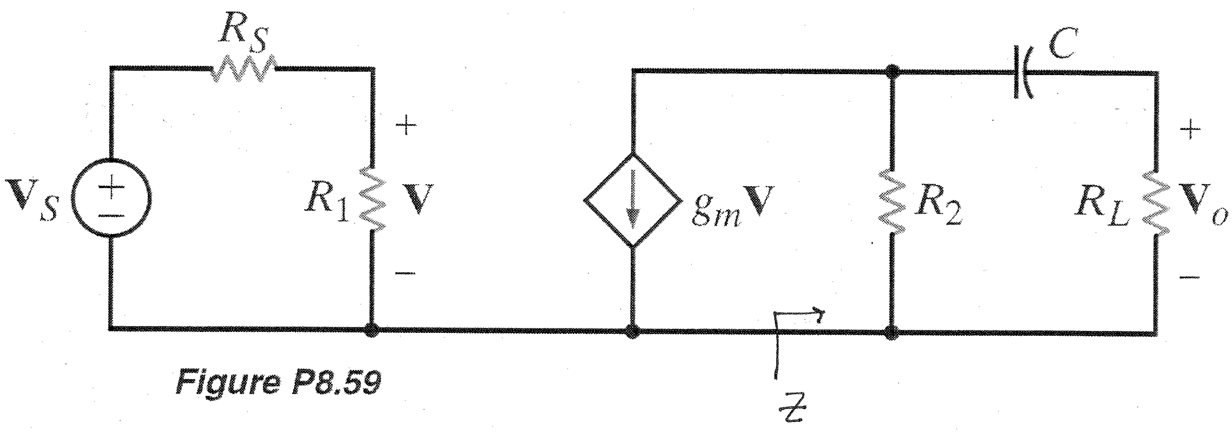


Figure P8.59

SOLUTION:

$$\frac{V}{V_S} = \frac{R_1}{R_1 + R_S} \qquad V_o = -g_m V Z$$

$$Z = \frac{R_2(R_L + Z_C)}{R_2 + R_L + Z_C} \qquad \frac{V_o}{V_s} = -g_m\left(\frac{R_1 R_2}{R_1 + R_S}\right)\left(\frac{j\omega C R_L + 1}{j\omega C(R_L + R_2) + 1}\right)$$

$$\boxed{\frac{V_o}{V_S} = -g_m R_2\left(\frac{R_1}{R_1 + R_S}\right)\left[\frac{j\omega C R_L + 1}{j\omega C(R_L + R_2) + 1}\right]}$$

8.60 Use nodal analysis to find $\mathbf{V}_o$ in the circuit in Fig. P8.60.
PSV

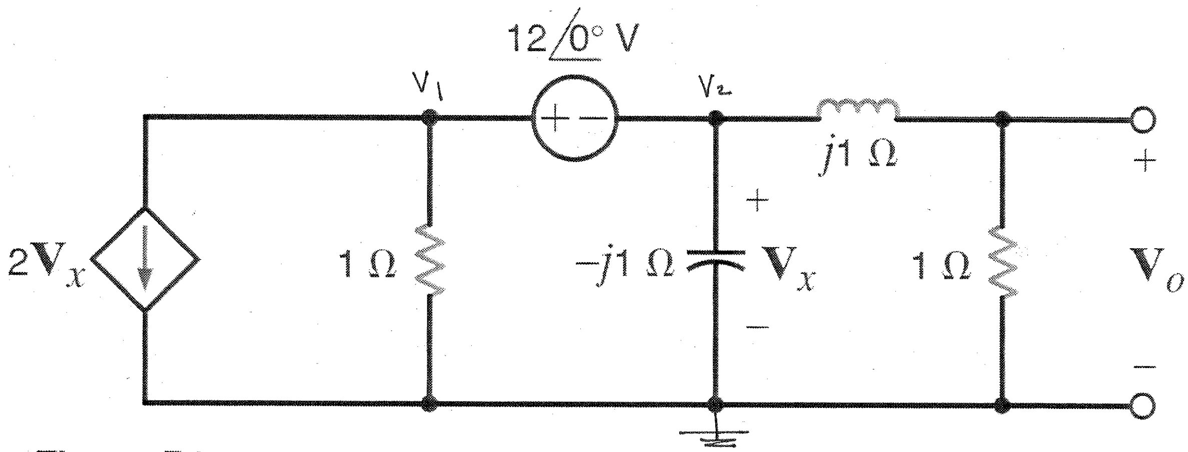


Figure P8.60

SOLUTION:

$V_1 - V_2 = 12\angle 0°\ \text{V}$ @ V_o: $\dfrac{V_2 - V_o}{j1} = \dfrac{V_o}{1} \Rightarrow V_2 - V_o(1+j1) = 0$

@ GND: $2V_x + \dfrac{V_1}{1} + \dfrac{V_2}{-j1} + \dfrac{V_o}{1} = 0$ where $V_x = V_2$

so

$$V_1 + V_2(2+j1) + V_o = 0$$

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1-j1 \\ 1 & 2+j1 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_o \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \boxed{V_o = 2.4\angle 126.9°\ \text{V}}$$

8.61 Find the voltage across the inductor in the circuit shown in Fig. P8.61 using nodal analysis.

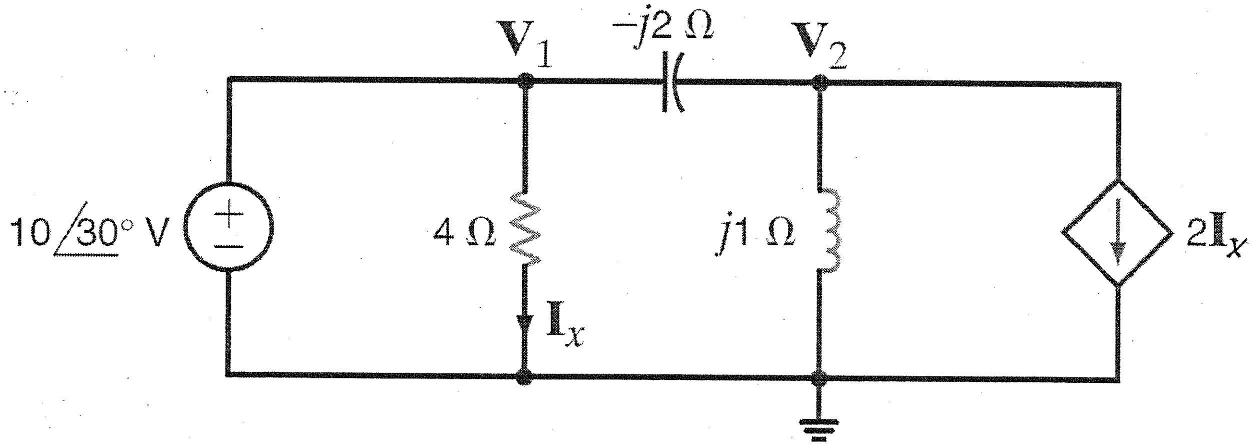


Figure P8.61

SOLUTION:

$V_1 = 10\angle 30°$ & $\dfrac{-V_1 + V_2}{-j2} + \dfrac{V_2}{j1} + 2I_x = 0$ where $I_x = \dfrac{V_1}{4}$

yields; $V_1(1+j1) + V_2 = 0$

$$\begin{bmatrix} 1 & 0 \\ 1+j1 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 10\angle 30° \\ 0 \end{bmatrix} \Rightarrow \boxed{V_2 = 14.1\angle -105° \text{ V}}$$

8.62 Use nodal analysis to find $\mathbf{I}_o$ in the circuit in Fig. P8.62.

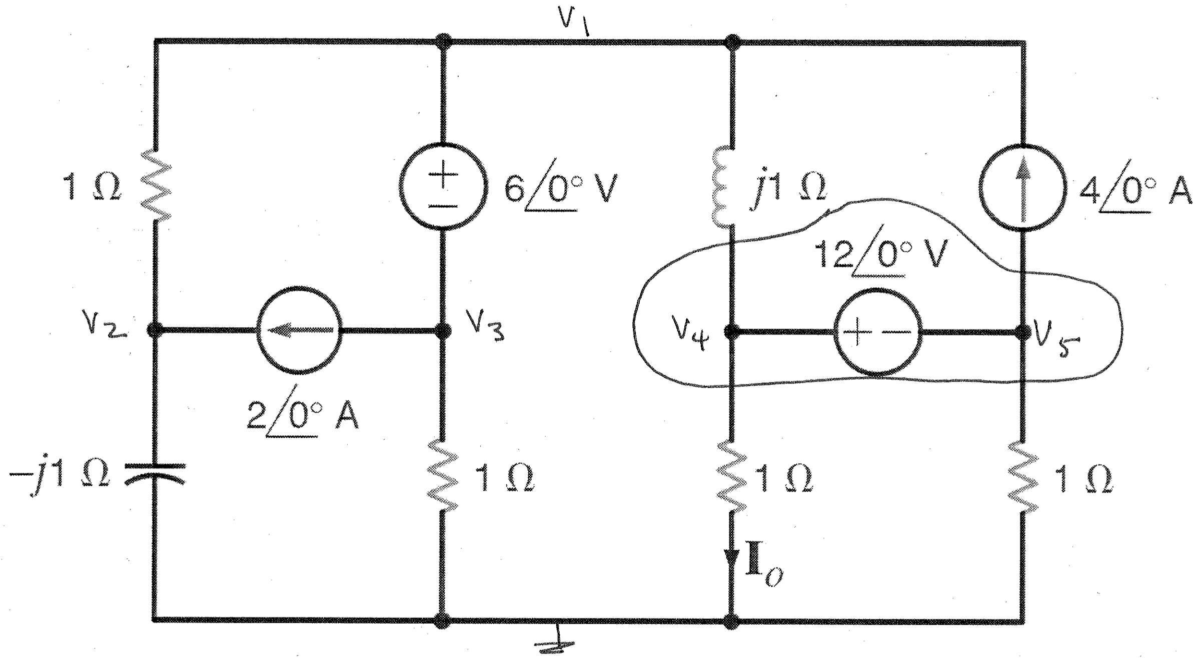


Figure P8.62

SOLUTION:

$V_1 - V_3 = 6\angle 0^\circ \qquad V_4 - V_5 = 12\angle 0^\circ \qquad I_o = V_4/1 = V_4$

@ V_2: $\quad \frac{V_2 - V_1}{1} + \frac{V_2}{-j1} = 2 \Rightarrow -V_1 + V_2(1+j1) = 2$

@ GND: $\quad \frac{V_2}{-j1} + \frac{V_3}{1} + \frac{V_4}{1} + \frac{V_5}{1} = 0 \Rightarrow jV_2 + V_3 + V_4 + V_5 = 0$

@ super node: $\quad \frac{V_4 - V_1}{j1} + \frac{V_4}{1} + \frac{V_5}{1} = -4 \Rightarrow jV_1 + V_4(1-j1) + V_5 = -4$

$$\begin{bmatrix} 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ -1 & 1+j1 & 0 & 0 & 0 \\ 0 & j1 & 1 & 1 & 1 \\ j1 & 0 & 0 & 1-j1 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{bmatrix} = \begin{bmatrix} 6 \\ 12 \\ 2 \\ 0 \\ -4 \end{bmatrix} \Rightarrow$$

$V_4 = 3.96 \angle -14.2^\circ$ V

$\boxed{I_o = 3.96 \angle -14.2 \text{ A}}$

8.63 Use mesh analysis to find $\mathbf{V}_o$ in the circuit shown in Fig. P8.63.

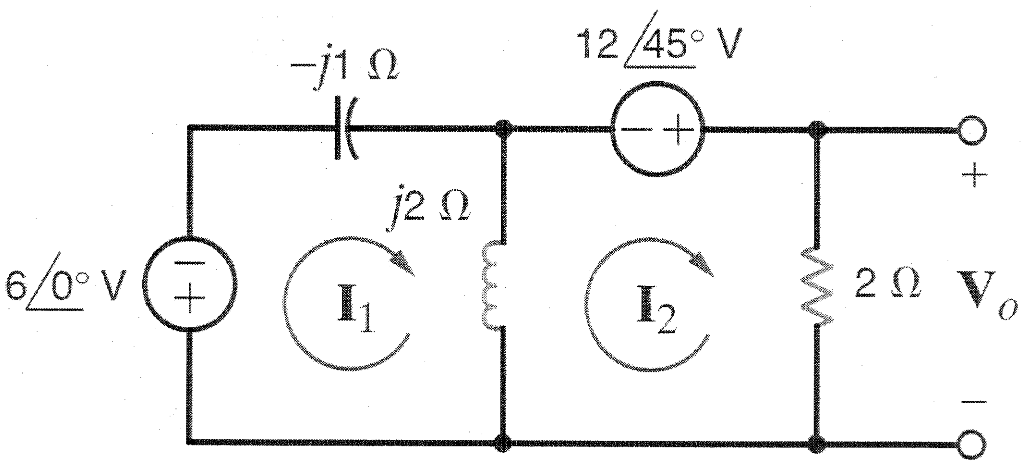


Figure P8.63

SOLUTION:

$$-6\angle 0^\circ = I_1(j1) - I_2(j2) \Rightarrow \quad jI_1 - j2I_2 = -6\angle 0^\circ$$

$$12\angle 45^\circ = I_2(2+j2) - j2I_1 \Rightarrow \quad -j2I_1 + (2+j2)I_2 = 12\angle 45^\circ$$

$$\begin{bmatrix} j1 & -j2 \\ -j2 & 2+j2 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} -6\angle 0 \\ 12\angle 45^\circ \end{bmatrix} \Rightarrow \quad I_2 = 3.25\angle 157.5^\circ \text{ A}$$

$$V_o = 2I_2 \qquad \boxed{V_o = 6.50\angle 157.5^\circ \text{ V}}$$

8.64 Use mesh analysis to find $\mathbf{V}_o$ in the circuit shown in Fig. P8.64.

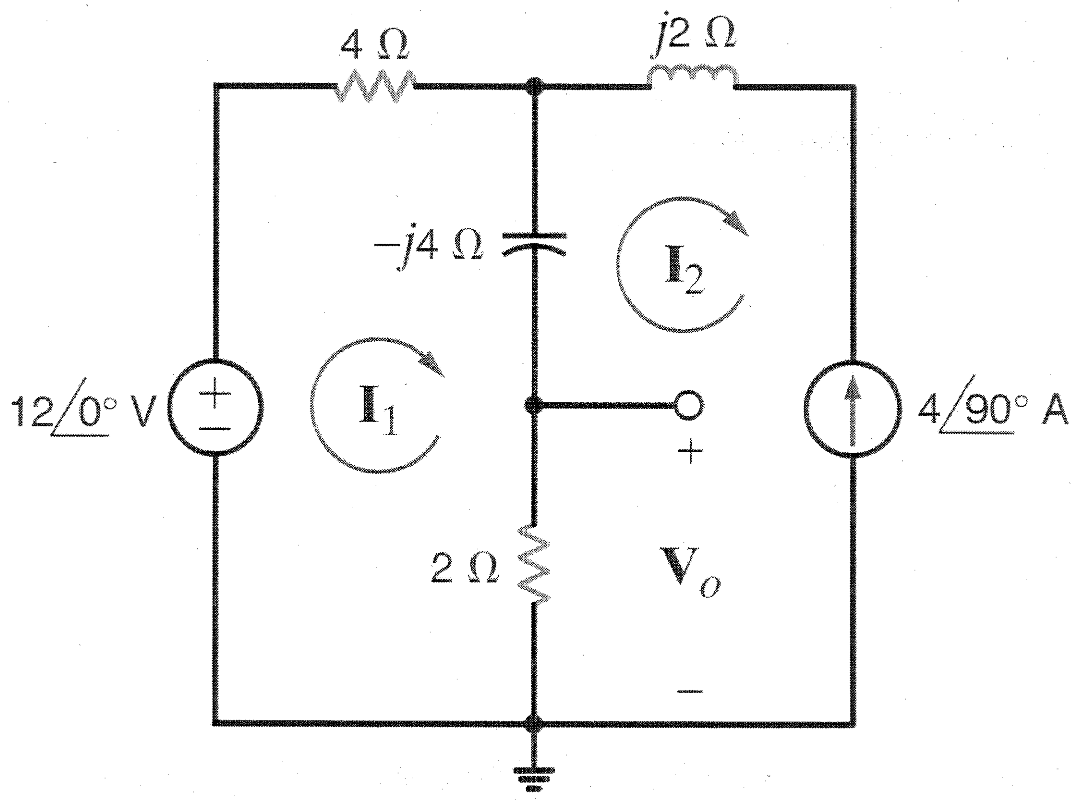


Figure P8.64

SOLUTION:

$12\angle 0 = I_1(6-j4) - I_2(2-j4) \Rightarrow I_1(6-j4) + I_2(-2+j4) = 12+j0$

$I_2 = -4\angle 90^\circ = -j4\text{ A}$ $\qquad V_o = 2(I_1 - I_2)$

$$\begin{bmatrix} 6-j4 & -2+j4 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 12 \\ -j4 \end{bmatrix} \Rightarrow \begin{cases} I_1 = 0.154 - j1.23\text{ A} \\ I_2 = -j4\text{ A} \end{cases}$$

$$\boxed{V_o = 5.55\angle 86.8^\circ\text{ V}}$$

8.65 Find $\mathbf{V}_o$ in the circuit in Fig. P8.65 using mesh analysis.

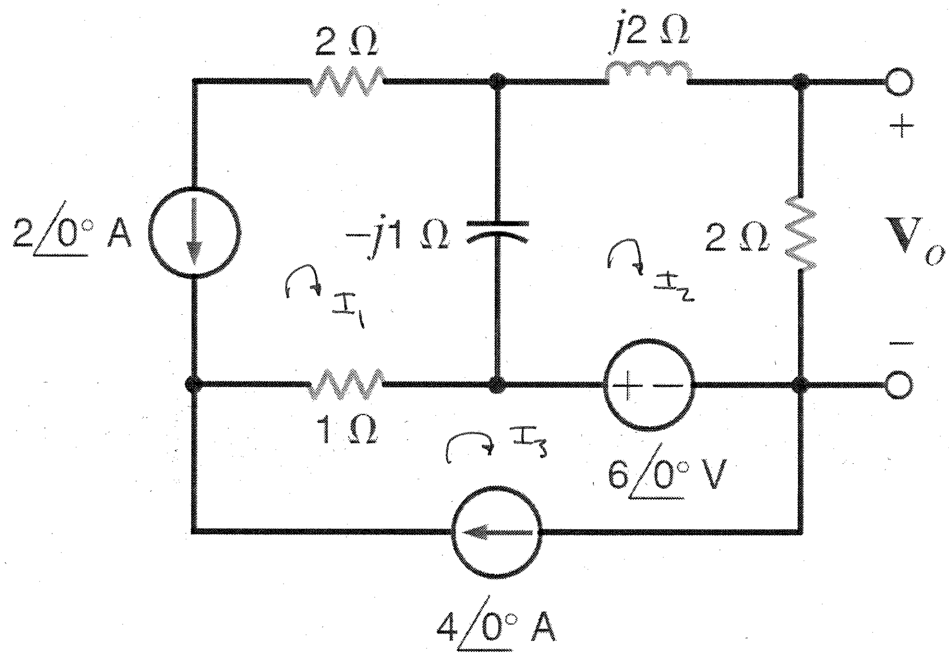


Figure P8.65

SOLUTION:

$I_1 = -2\angle 0^\circ$ A $\quad$ $I_3 = 4\angle 0^\circ$ A $\quad$ $6\angle 0 = jI_1 + I_2(2 + j1)$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ j1 & 2+j1 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} -2+j0 \\ 4+j0 \\ 6+j0 \end{bmatrix} \Rightarrow I_2 = 2.83\angle -8.13^\circ \text{ A}$$

$V_o = 2I_2$ $\quad$ $\boxed{V_o = 5.66\angle -8.13^\circ \text{ V}}$

8.66 Use mesh analysis to find $\mathbf{V}_o$ in the circuit in Fig. P8.66.

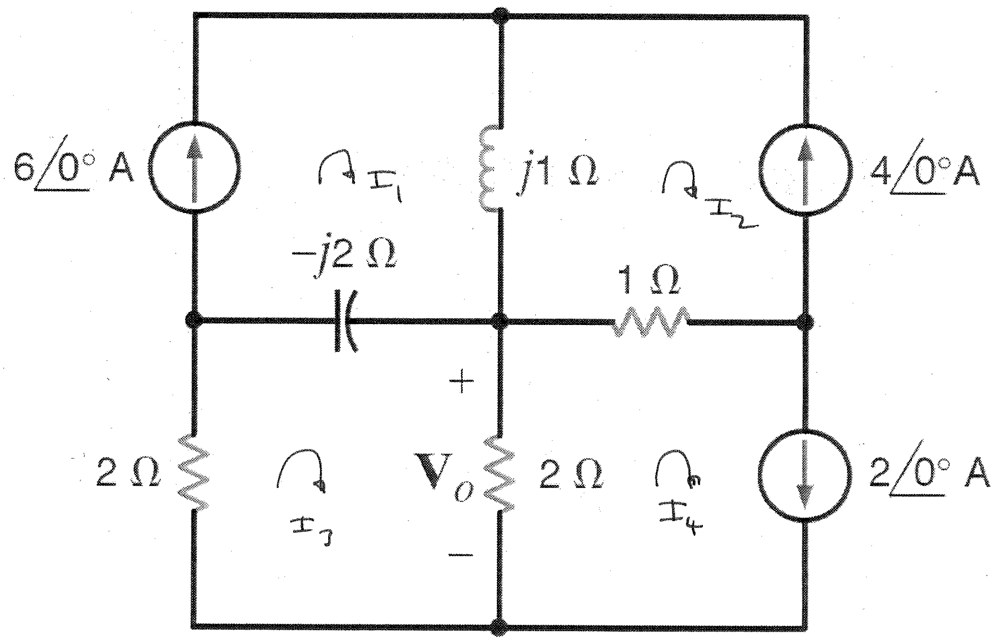


Figure P8.66

SOLUTION:

$I_1 = 6\angle 0°\text{ A} \qquad I_2 = -4\angle 0°\text{ A} \qquad I_4 = 2\angle 0°\text{ A}$

$j2I_1 + I_3(4-j2) - 2I_4 = 0 \qquad V_o = 2\,(I_3 - I_4)$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ j2 & 0 & 4-j2 & -2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} 6 \\ -4 \\ 2 \\ 0 \end{bmatrix} \Rightarrow \begin{matrix} I_3 = 2-j2\text{ A} \\ I_4 = 2\text{ A} \end{matrix}$$

$$\boxed{V_o = 4\angle -90°\text{ V}}$$

8.67 Using loop analysis and MATLAB, find $\mathbf{I}_o$ in the network in Fig. P8.67. CS

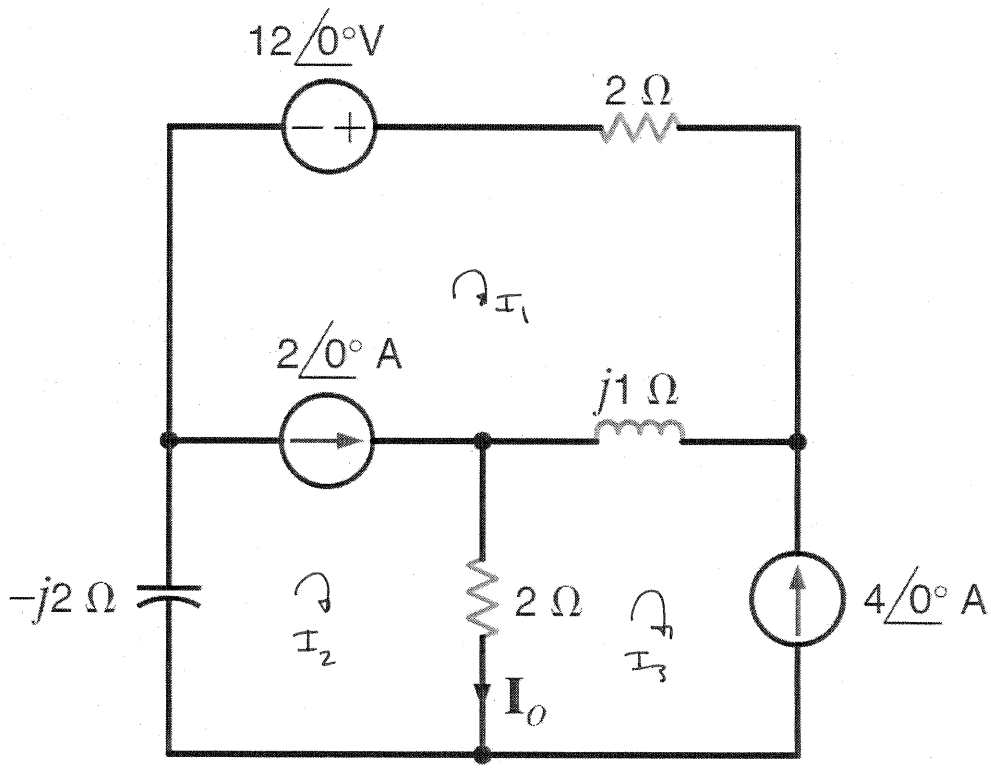


Figure P8.67

SOLUTION:

$I_2 - I_1 = 2\angle 0°\ \text{A}$ $\quad I_3 = -4\angle 0°\ \text{A}$ $\quad 12 = I_1(2+j1) + I_2(2-j2) - I_3(2+j1)$

$$\begin{bmatrix} -1 & 1 & 0 \\ 0 & 0 & 1 \\ 2+j1 & 2-j2 & -2-j1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -4 \\ 12 \end{bmatrix}$$

MATLAB
```
» Z=[-1 1 0;0 0 1;2+1i 2-2i -2-1i];
» V=[2;-4;12];
» I=inv(z)*v

I =   0   + 0i
      2.0 + 0i
     -4.0
```

$I_o = I_2 - I_3$ $\quad \boxed{I_o = 6\angle 0°\ \text{A}}$

8.68 Find $\mathbf{V}_o$ in the network in Fig. P8.68.

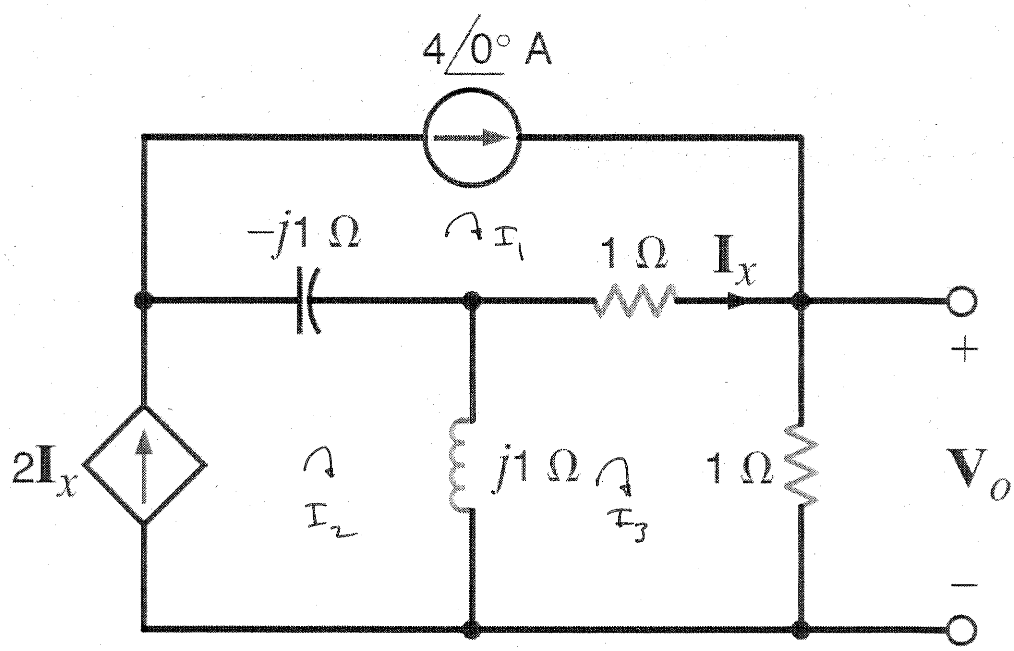


Figure P8.68

SOLUTION:

$I_1 = 4\angle 0° \text{ A} \qquad I_2 = 2I_x = 2I_3 - 2I_1 \Rightarrow +2I_1 + I_2 - 2I_3 = 0$

$-I_1 - jI_2 + I_3(2+j1) = 0$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & -2 \\ -1 & -j1 & 2+j1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 4+j0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow I_3 = 4\angle -36.9° \text{ A}$$

$V_o = (1)I_3 \qquad \boxed{V_o = 4\angle -36.9° \text{ V}}$

8.69 Find $\mathbf{V}_o$ in the network in Fig. P8.69. PSV

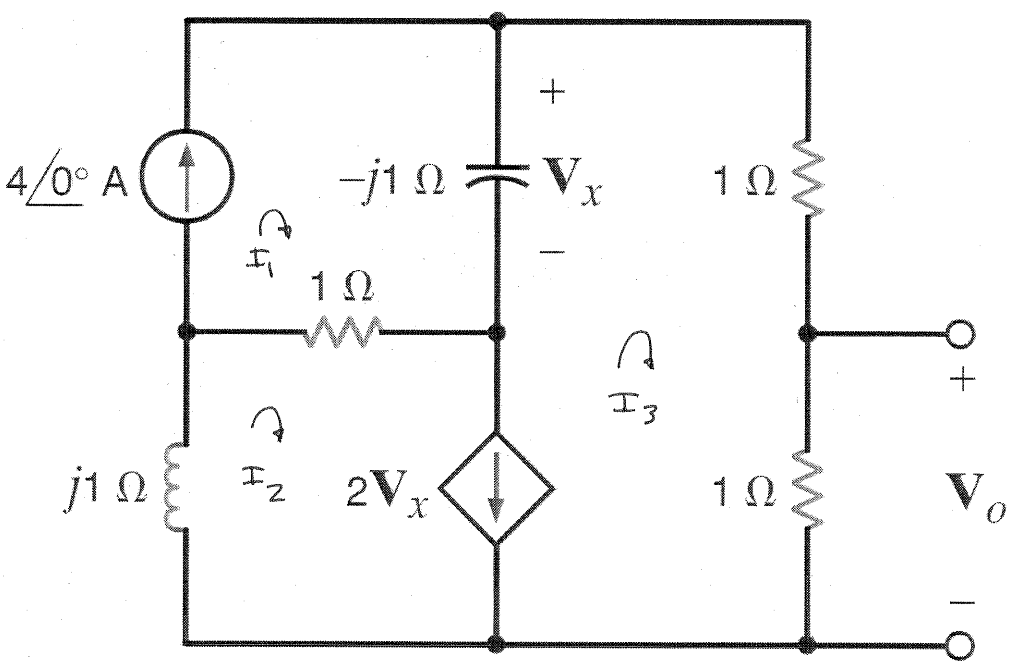


Figure P8.69

SOLUTION:

$I_1 = 4\angle 0°$ A $\qquad I_2 - I_3 = 2V_x = 2(-j1)(I_1 - I_3) \Rightarrow j2I_1 + I_2 + I_3(-1-j2) = 0$

and, $\quad jI_2 + (I_2 - I_1)1 - j1(I_3 - I_1) + 2I_3 = 0 \qquad$ & $V_o = I_3(1)$

or, $\quad I_1(-1+j1) + I_2(1+j1) + I_3(2-j1) = 0$

$$\begin{bmatrix} 1 & 0 & 0 \\ j2 & 1 & -1-j2 \\ -1+j1 & 1+j1 & 2-j1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \quad I_3 = 2.53\angle 71.6° \text{ A}$$

$$\boxed{V_o = 2.53\angle 71.6° \text{ V}}$$

8.70 Use loop analysis to find $\mathbf{I}_o$ in the network in Fig. P8.70.

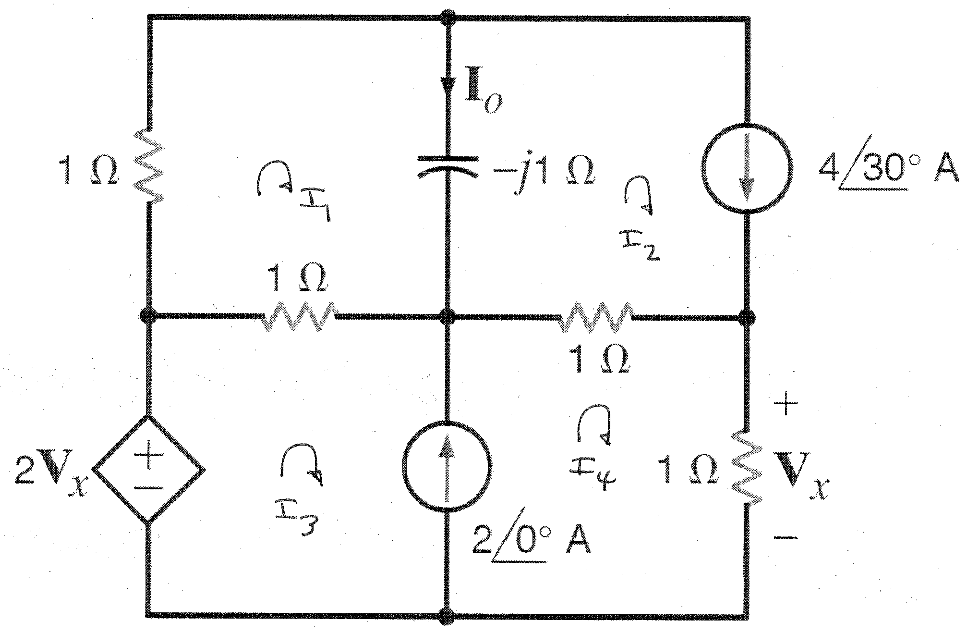


Figure P8.70

SOLUTION: $I_o = I_1 - I_2$

$I_2 = 4\angle 30^\circ$ A $\qquad I_4 - I_3 = 2\angle 0$ A $\qquad I_1(2-j1) + jI_2 - I_3 = 0$

$$\left.\begin{aligned} 2V_x &= -I_1 - I_2 + I_3 + 2I_4 \\ \text{and} \quad V_x &= (1)\, I_4 \end{aligned}\right\} \quad -I_1 - I_2 + I_3 = 0$$

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 2-j1 & j1 & -1 & 0 \\ -1 & -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} 4\angle 30 \\ 2\angle 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{aligned} I_1 &= 4\angle 30^\circ \text{ A} \\ I_2 &= 4\angle 30^\circ \text{ A} \end{aligned}$$

$$\boxed{I_o = 0 \text{ A}}$$

8.71 Use superposition to find $\mathbf{V}_o$ in the network in Fig. P8.71.

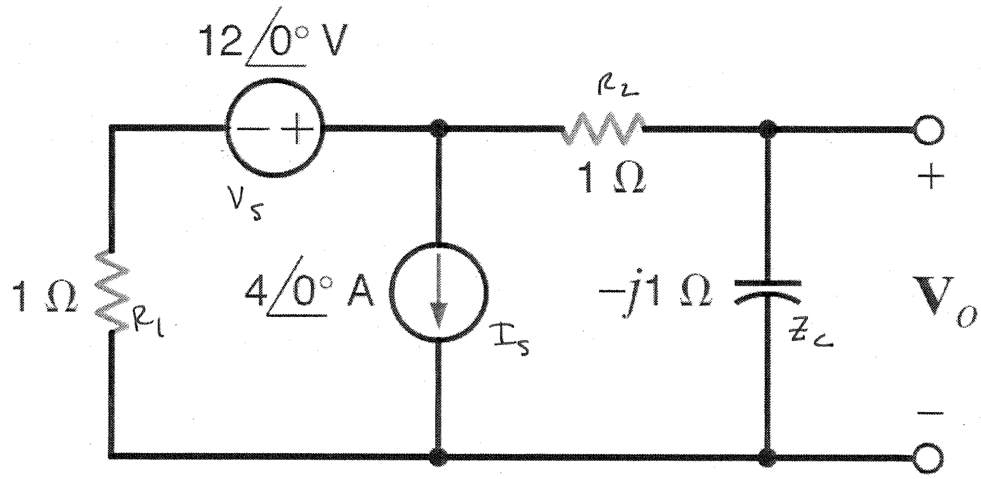
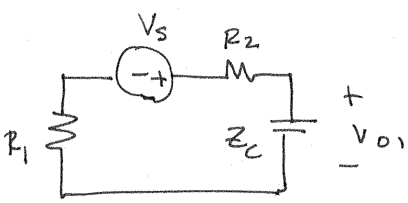
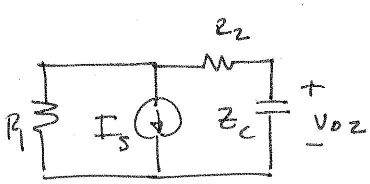


Figure P8.71

SOLUTION:



$$V_{o1} = \frac{V_s Z_C}{R_1 + R_2 + Z_C} \qquad V_{o1} = 2.4 - j4.8\ V$$



$$V_{o2} = \frac{-I_s R_1}{R_1 + R_2 + Z_C} Z_C \qquad V_{o2} = -0.8 + j1.6\ V$$

$$V_o = V_{o1} + V_{o2} \qquad \boxed{V_o = 3.58 \angle -63.4^\circ\ V}$$

8.72 Using superposition, find $\mathbf{V}_o$ in the circuit in Fig. P8.72.

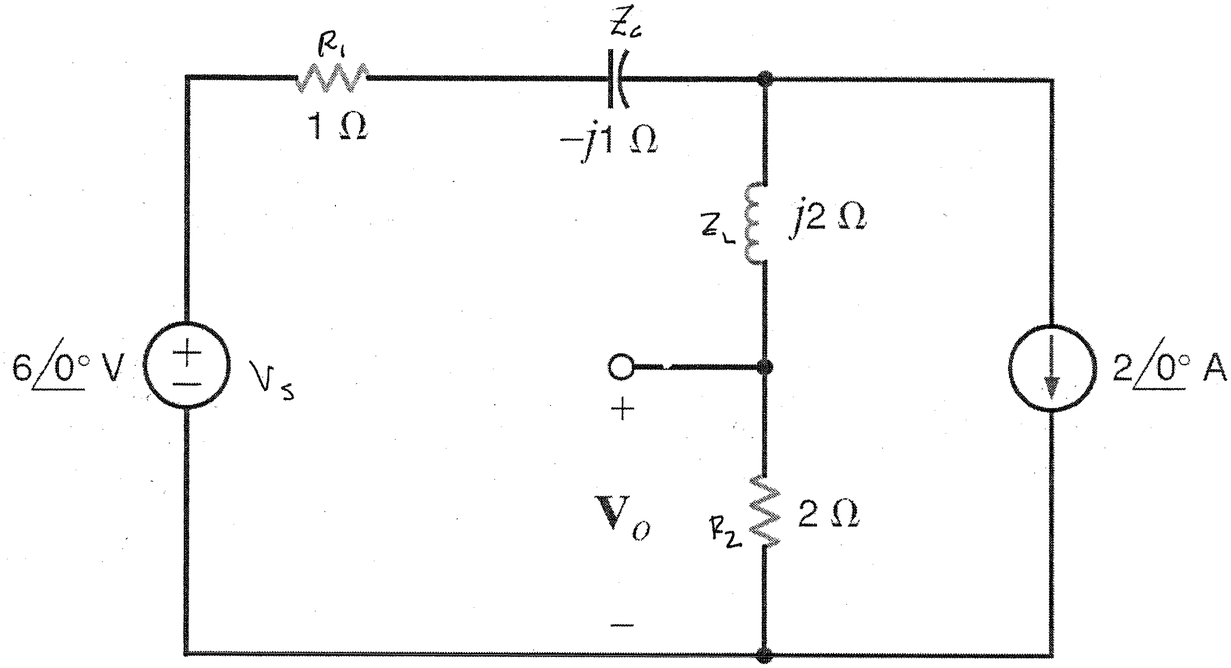
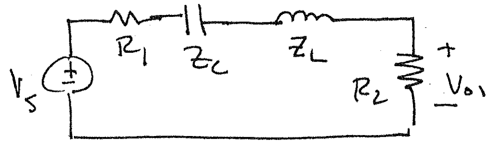


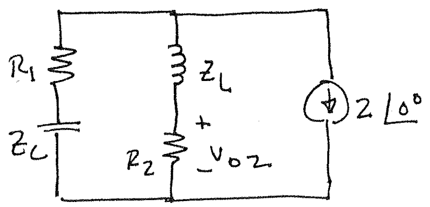
Figure P8.72

SOLUTION:



$$V_{o1} = \frac{V_s R_2}{R_1 + R_2 + Z_L + Z_C}$$

$$V_{o1} = 3.79 \angle -18.4° \text{ V}$$



$$V_{o2} = \frac{-2\,(R_1 + Z_C)}{R_1 + Z_C + R_2 + Z_L} R_2$$

$$V_{o2} = 1.79 \angle 116.6° \text{ V}$$

$$V_o = V_{o1} + V_{o2}$$

$$\boxed{V_o = 2.83 \angle 8.13 \text{ V}}$$

8.73 Find $\mathbf{V}_o$ in the network in Fig. P8.73 using superposition.
CS

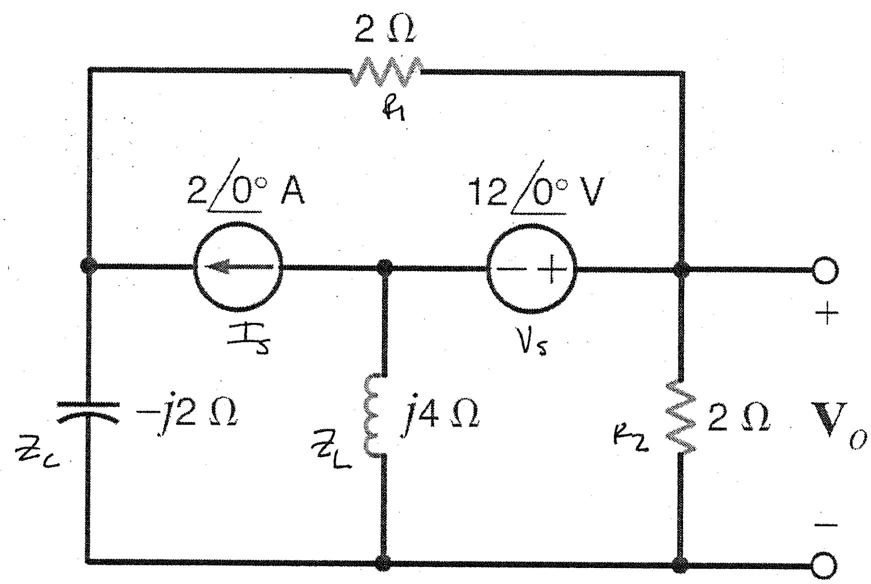
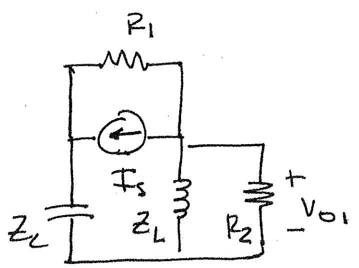


Figure P8.73

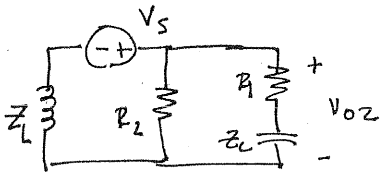
SOLUTION:



$$Z_A = \frac{Z_L R_2}{Z_L + R_2} = \frac{4}{2-j1}$$

$$Z_B = Z_C + Z_A = \frac{2-j2}{2-j1}$$

$$V_{O1} = \frac{I_s R_1}{R_1 + Z_B}\left(\frac{-Z_L R_2}{Z_L + R_2}\right) = -1.33 - j1.33 \text{ V}$$



$$Z_x = R_1 + X_c$$

$$Z_y = R_2 Z_x / (R_2 + Z_x)$$

$$V_{O2} = V_s Z_y / (Z_y + Z_L) = 0 - j4$$

$$V_o = V_{O1} + V_{O2}$$

$$\boxed{V_o = 5.50\angle -104° \text{ V}}$$

8.74 Use both superposition and MATLAB to determine $\mathbf{V}_o$ in the circuit in Fig. P8.74.

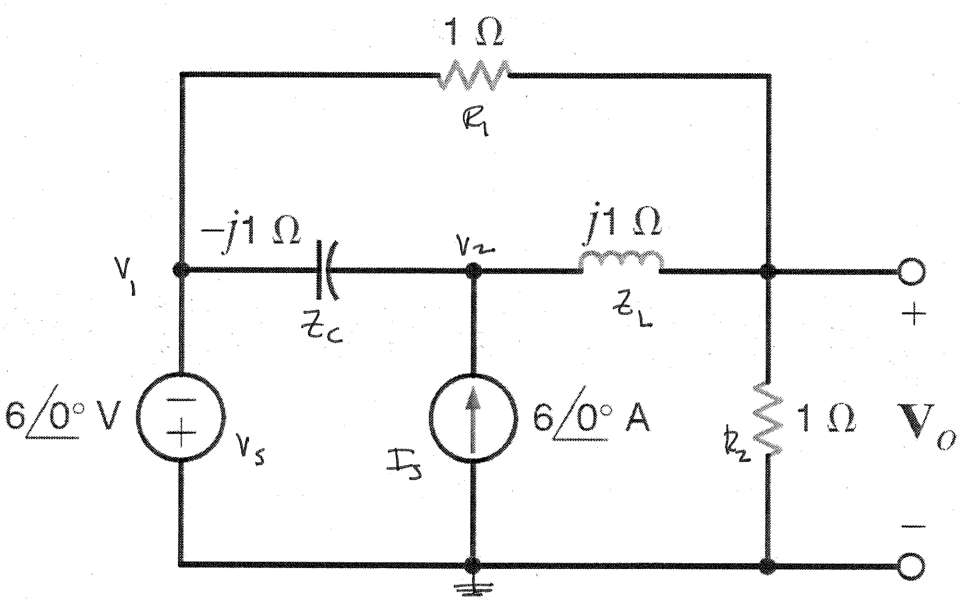
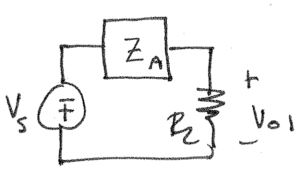


Figure P8.74

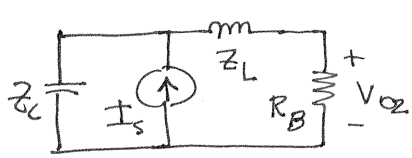
SOLUTION:

Superposition



$Z_A = R_1 \parallel (Z_L + Z_C)$
$= 0$

$V_{o1} = -V_s = -6\angle 0° V$



$R_B = R_1 \parallel R_2 = ½\,\Omega$

$V_{o2} = I_s Z_C R_B / (R_B + Z_L + Z_C) = 6\angle -90° V$

MATLAB

$V_1 = -6\angle 0° V \quad ; \quad \frac{V_2 - V_1}{Z_C} + \frac{V_2 - V_0}{Z_L} = I_s$

$$\frac{V_0 - V_1}{R_1} + \frac{V_0 - V_2}{Z_L} + \frac{V_0}{R_2} = 0$$

$$\begin{bmatrix} -j1 & 0 & j1 \\ -1 & j1 & 2-j1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_0 \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \\ -6 \end{bmatrix}$$

```
» y=[-1i 0 1i;-1 1i 2-1i;1 0 0];
»  i=[6;0;-6];
» v=inv(y)*i

v =   -6.0000
       6.0000 -12.0000i
      -6.0000 - 6.0000i
```

$\boxed{V_0 = 8.49 \angle -135° \; V}$

8.75 Find $\mathbf{V}_o$ in the network in Fig. P8.75 using superposition.
PSV

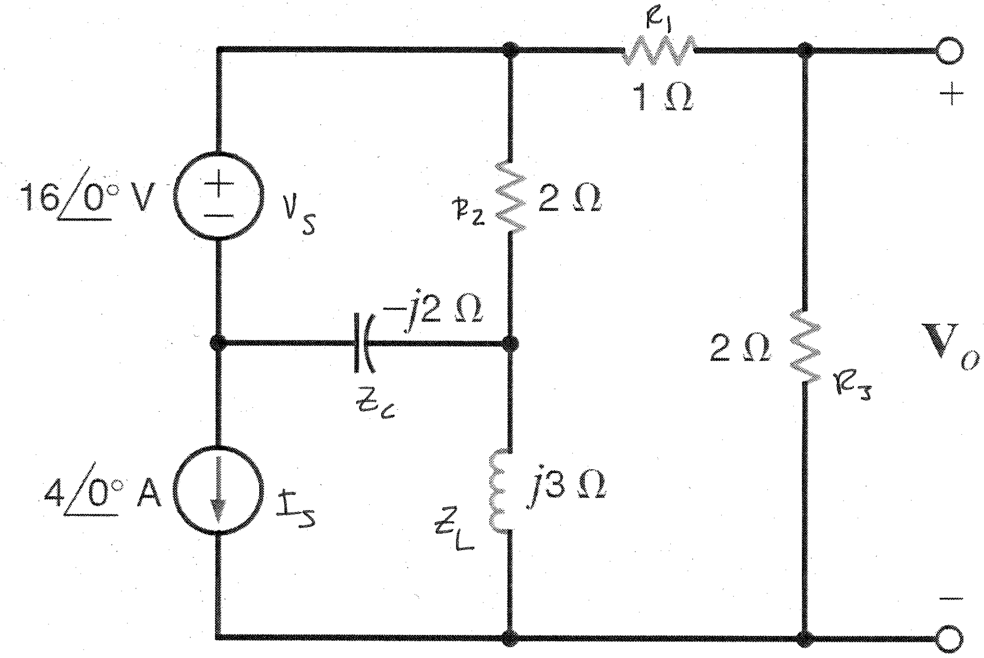
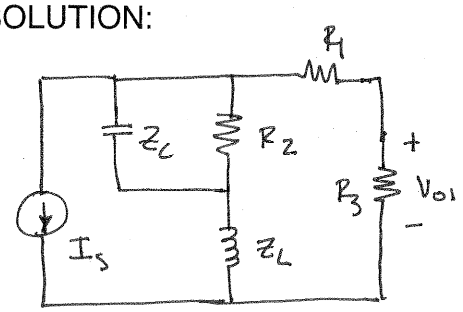


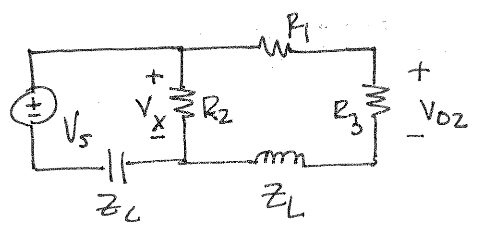
Figure P8.75

SOLUTION:



$$Z_A = R_2 \,//\, Z_C \qquad Z_B = Z_L + Z_A$$

$$V_{o1} = \frac{-I_S Z_B}{Z_B + R_1 + R_3} R_3 = -3.2 - j2.4\,V$$



$$Z_x = R_1 + R_3 + Z_L \qquad Z_Y = R_2 \,//\, Z_x$$

$$V_x = \frac{V_S Z_Y}{Z_C + Z_Y} \qquad V_{o2} = \frac{V_x R_3}{R_1 + R_3 + Z_L}$$

$$V_{o2} = 4.8 + j1.6\,V$$

$$\boxed{V_o = V_{o1} + V_{o2} = 1.79\,\angle 26.6^\circ\,V}$$

8.76 Use source exchange to determine $\mathbf{V}_o$ in the network in Fig. P8.76.

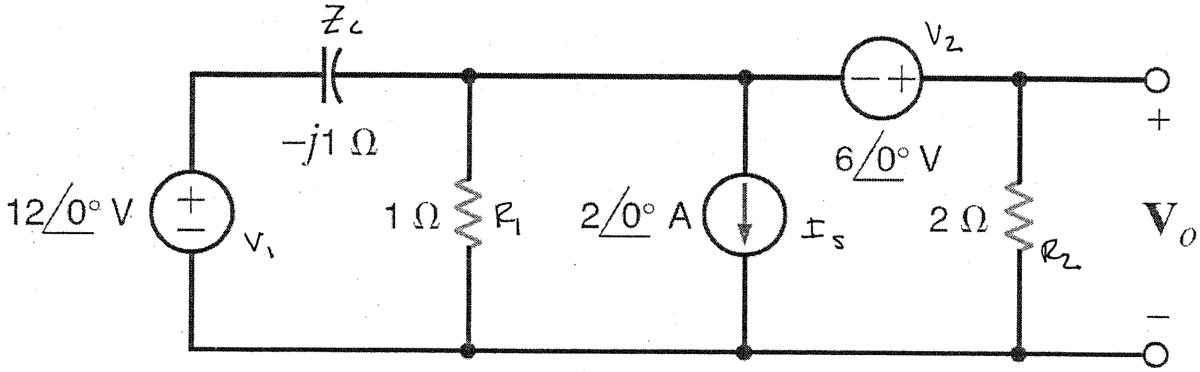
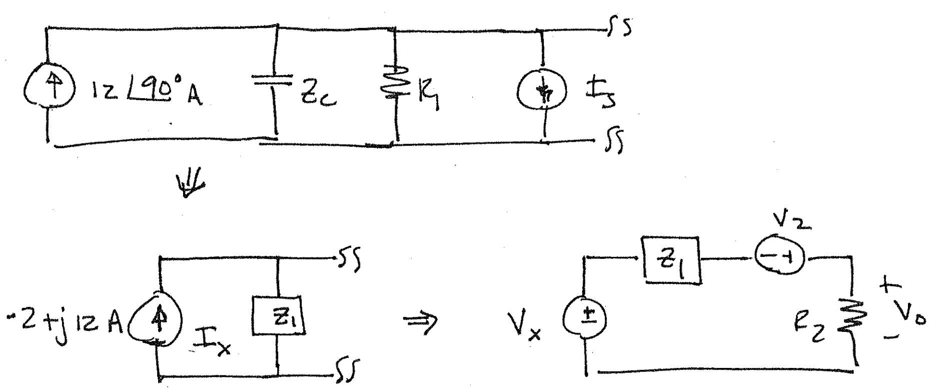


Figure P8.76

SOLUTION:



$Z_1 = R_1 // Z_c$

$V_x = I_x Z_1$

$$V_o = \frac{(V_x + V_2)\,R_2}{R_2 + Z_1}$$

$$\boxed{V_o = 10.2\,\angle 43.8^\circ \text{ V}}$$

8.77 Use source exchange to find the current $\mathbf{I}_o$ in the network in Fig. P8.77. CS

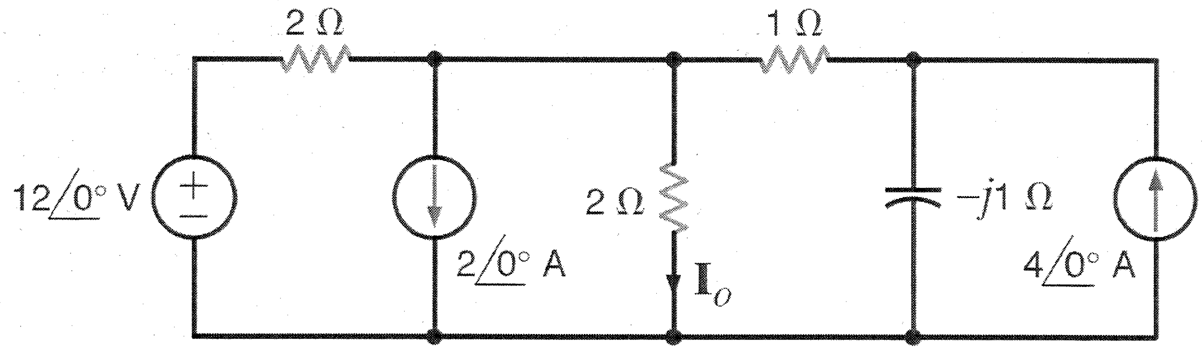
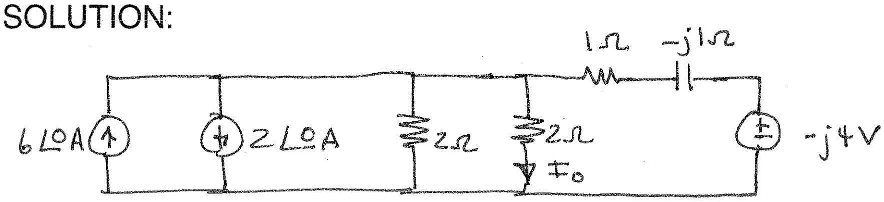
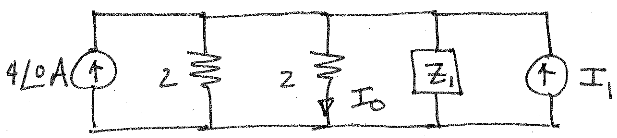


Figure P8.77

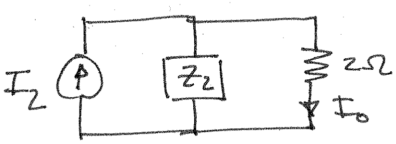
SOLUTION:





$$Z_1 = 1 - j1\ \Omega$$

$$I_1 = -j4/Z_1$$



$$I_2 = 4\angle 0 + I_1$$

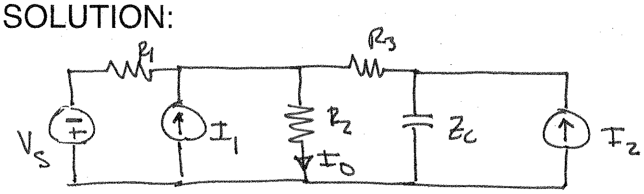
$$Z_2 = 2Z_1/(2+Z_1)$$

$$I_o = \frac{I_2 Z_2}{Z_2 + 2}$$

$$\boxed{I_o = 2\ \angle -36.9^\circ\ \text{A}}$$

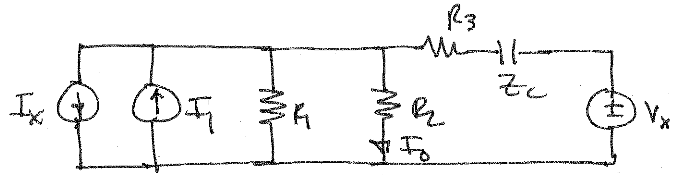
8.78 Use source transformation to determine $\mathbf{I}_o$ in the network in Fig. P8.50.

SOLUTION:



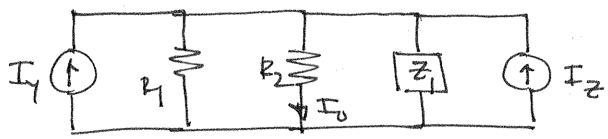
$R_1 = R_2 = R_3 = 1\Omega \quad Z_C = -j1\Omega$

$V_s = 12\angle 0° V \quad I_1 = 2\angle 0 A \quad I_2 = 4\angle 0 A$



$I_x = V_s / R_1 = 12\angle 0° A$

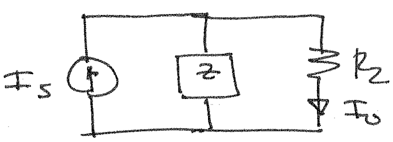
$V_x = I_2 Z_C = -j4\ V$



$I_y = I_1 - I_x = -10A$

$I_Z = \dfrac{V_x}{Z_1} = 2 - j2\ A$

$Z_1 = R_3 - Z_C = 1 - j1\ \Omega$



$I_S = I_y + I_Z = -8 - j2\ A$

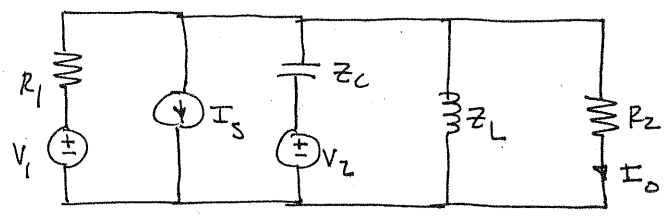
$Z = \dfrac{Z_1 R_1}{Z_1 + R_1} = \dfrac{1 - j1}{2 - j1}$

$I_o = \dfrac{I_S Z}{Z + R_2}$

$\boxed{I_o = 3.23\angle -177.3°\ A}$

8.79 Use source transformation to determine $\mathbf{I}_o$ in the network in Fig. P8.51.

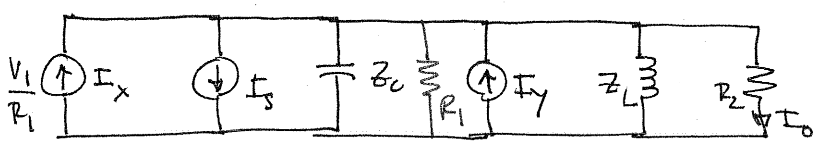
SOLUTION:



$R_1 = R_2 = 2\Omega \qquad Z_C = -j1\,\Omega$

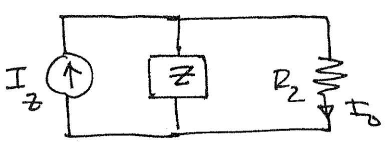
$Z_L = j2\Omega \qquad I_S = 4\angle 0°\,A$

$V_1 = 12\angle 0°\,V \qquad V_2 = 6\angle 0°\,V$



$I_X = V_1/R_1 = 6\angle 0°\,A$

$I_Y = V_2/Z_C = 6\angle 90°\,A$



$I_Z = I_X + I_Y - I_S = 2 + j6\,A$

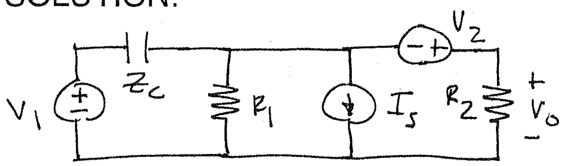
$Z = \left[\frac{1}{R_1} + \frac{1}{Z_C} + \frac{1}{Z_L}\right]^{-1} = 1 - j1\,\Omega$

$$I_o = \frac{I_Z Z}{Z + R_2}$$

$$\boxed{I_o = 2.83\angle 45°\,A}$$

8.80 Use source transformation to determine $\mathbf{V}_o$ in the network in Fig. P8.76.

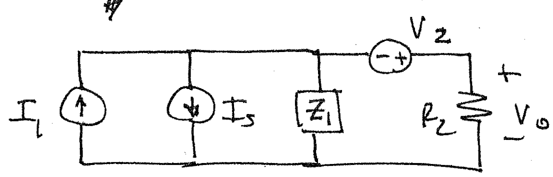
SOLUTION:



$Z_c = -j1\,\Omega \quad R_1 = 1\,\Omega \quad R_2 = 2\,\Omega$

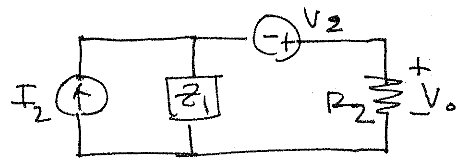
$V_1 = 12\angle 0^\circ\,V \quad I_s = 2\angle 0^\circ\,A$

$V_2 = 6\angle 0^\circ\,V$



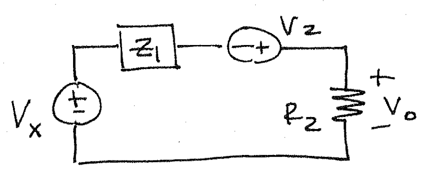
$I_1 = V_1 / Z_c = j12\,A$

$Z_1 = \dfrac{R_1 Z_c}{R_1 + Z_c} = 0.5 - j0.5\,\Omega$



$I_2 = I_1 - I_s = -2 + j12\,A$

$V_x = I_2 Z_1 = 5 + j7\,V$

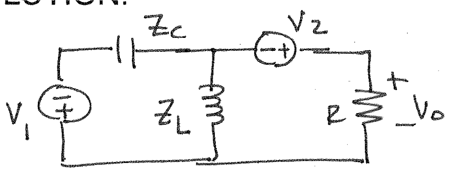


$V_o = \dfrac{(V_x + V_2)R_2}{R_2 + Z_1}$

$\boxed{V_o = 10.2\angle 43.8^\circ\,V}$

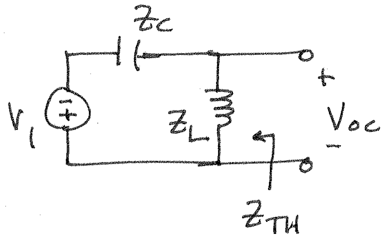
8.81 Using Thévenin's theorem, find $\mathbf{V}_o$ in the network in Fig. P8.63.

SOLUTION:



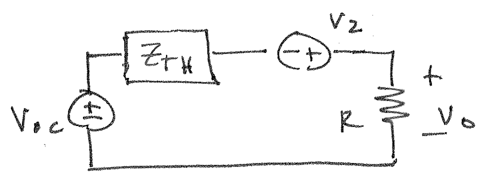
$V_1 = 6\angle 0^\circ\ \text{V} \qquad V_2 = 12\angle 45^\circ\ \text{V}$

$Z_C = -j1\,\Omega \qquad Z_L = j2\,\Omega \qquad R = 2\,\Omega$



$$V_{oc} = \frac{-V_1 Z_L}{Z_L + Z_C} = -12\angle 0^\circ\ \text{V}$$

$$Z_{TH} = \frac{Z_L Z_C}{Z_L + Z_C} = -j2\,\Omega$$

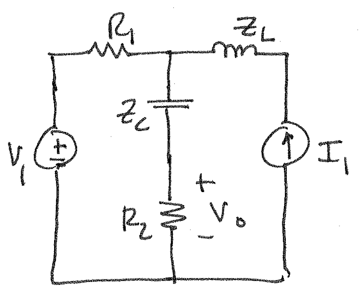


$$V_o = \frac{(V_{oc} + V_2)R}{R + Z_{TH}}$$

$$\boxed{V_o = 6.50\angle 157.5^\circ\ \text{V}}$$

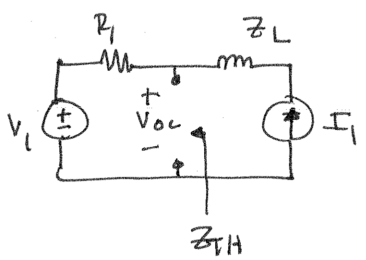
8.82 Use Thévenin's theorem to find $\mathbf{V}_o$ in the circuit in Fig. P8.64. CS

SOLUTION:



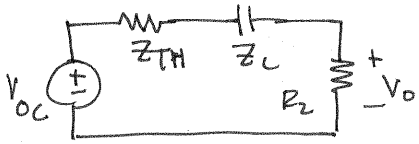
$R_1 = 4\Omega \quad R_2 = 2\Omega \quad Z_L = j2\Omega \quad Z_C = -j4\Omega$

$V_1 = 12\angle 0° V \quad I_1 = 4\angle 90° A$



$V_{oc} = V_1 + I_1 R_1 = 12 + j16 V$

$Z_{TH} = R_1 = 4\Omega$

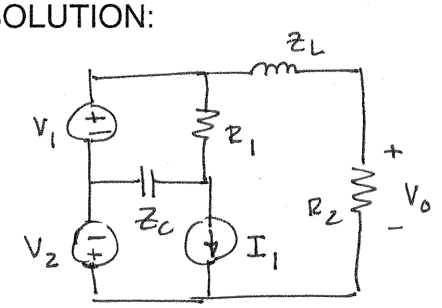


$$V_o = \frac{V_{oc} R_2}{R_2 + Z_C + Z_{TH}}$$

$$\boxed{V_o = 5.55\angle 86.8° \ V}$$

8.83 Solve Problem 8.52 using Thévenin's theorem.

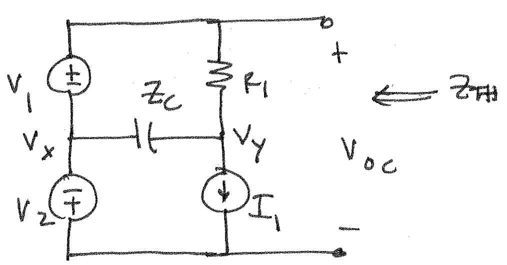
SOLUTION:



$V_1 = 12\angle 0° \text{ V}$ $\quad V_2 = 16\angle 0° \text{ V}$

$I_1 = 2\angle 0° \text{ A}$ $\quad R_1 = R_2 = 2\,\Omega$

$Z_L = j1\,\Omega$ $\quad Z_C = -j1\,\Omega$

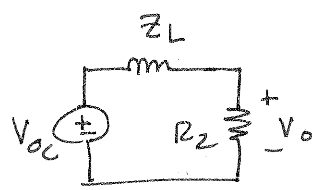


$V_{oc} - V_x = V_1 \qquad V_x = -V_2$

$$\frac{V_{oc} - V_y}{R_1} = \frac{V_y - V_x}{Z_C} + I_1$$

$V_{oc} = V_1 - V_2 = -4\angle 0° \text{ V}$

$Z_{TH} = 0\,\Omega$ $\quad$ ($V_1 - V_2$ path shorts)



$$V_o = \frac{V_{oc} R_2}{R_2 + Z_L}$$

$$\boxed{V_o = 3.58\angle 153.4° \text{ V}}$$

8.84 Apply Thévenin's theorem twice to find $\mathbf{V}_o$ in the circuit in Fig. P8.84.

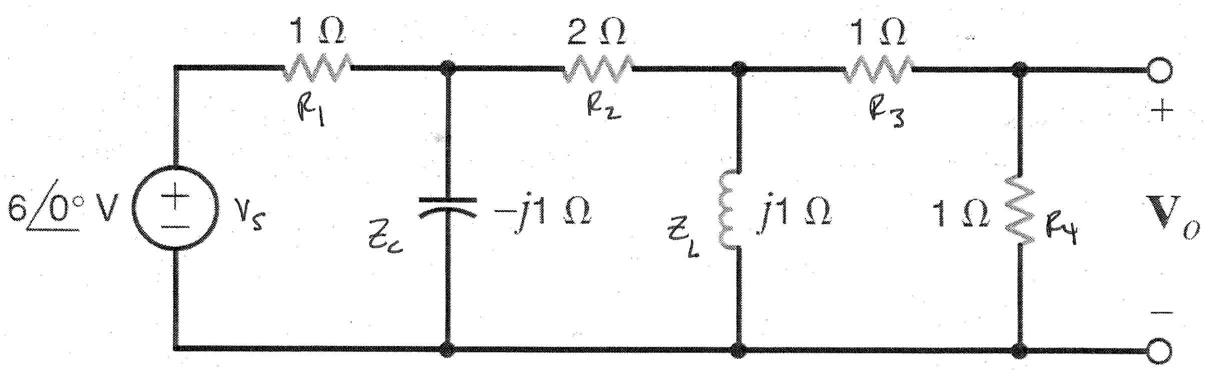
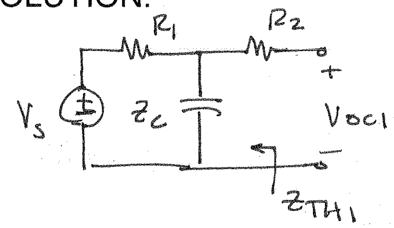


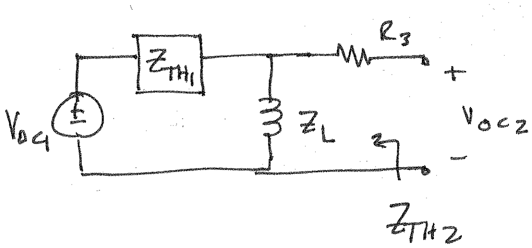
Figure P8.84

SOLUTION:



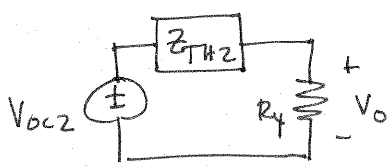
$$V_{oc1} = \frac{V_s Z_c}{Z_c + R_1} = 3 - j3 \text{ V}$$

$$Z_{TH1} = R_2 + \frac{R_1 Z_c}{R_1 + Z_c} = \frac{2 - j3}{1 - j1} \ \Omega$$



$$V_{oc2} = \frac{V_{oc1} Z_L}{Z_L + Z_{TH1}} = 1.66 \angle 33.7° \text{ V}$$

$$Z_{TH2} = R_3 + \frac{Z_L Z_{TH1}}{Z_L + Z_{TH1}} = 1.66 \angle 33.7° \ \Omega$$

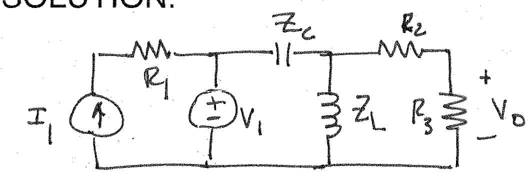


$$V_o = \frac{V_{oc2} R_4}{R_4 + Z_{TH2}}$$

$$\boxed{V_o = 0.651 \angle 12.5° \text{ V}}$$

8.85 Solve Problem 8.49 using Thévenin's theorem.

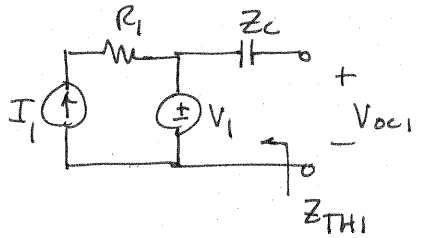
SOLUTION:



$R_1 = R_2 = 2\,\Omega \quad R_3 = 1\,\Omega$

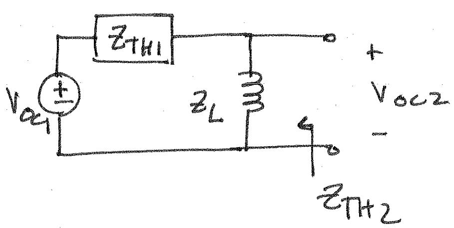
$Z_L = j1\,\Omega \quad Z_C = -j1\,\Omega$

$I_1 = 6\angle 0^\circ\,A \quad V_1 = 12\angle 0^\circ\,V$



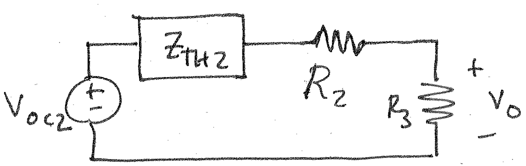
$V_{oc1} = V_1 = 12\angle 0^\circ\,V$

$Z_{TH1} = Z_C = -j1\,\Omega$



$$V_{oc2} = \frac{V_{oc1} Z_L}{Z_L + Z_C} = 24\angle 0^\circ\,V$$

$$Z_{TH2} = \frac{Z_L Z_{TH1}}{Z_L + Z_{TH1}} = -j2\,\Omega$$



$$V_o = \frac{V_{oc2} R_3}{R_2 + R_3 + Z_{TH2}}$$

$$\boxed{V_o = 6.66\angle 33.67^\circ\,V}$$

8.86 Use Thévenin's theorem to find $\mathbf{V}_o$ in the network in Fig. P8.86.

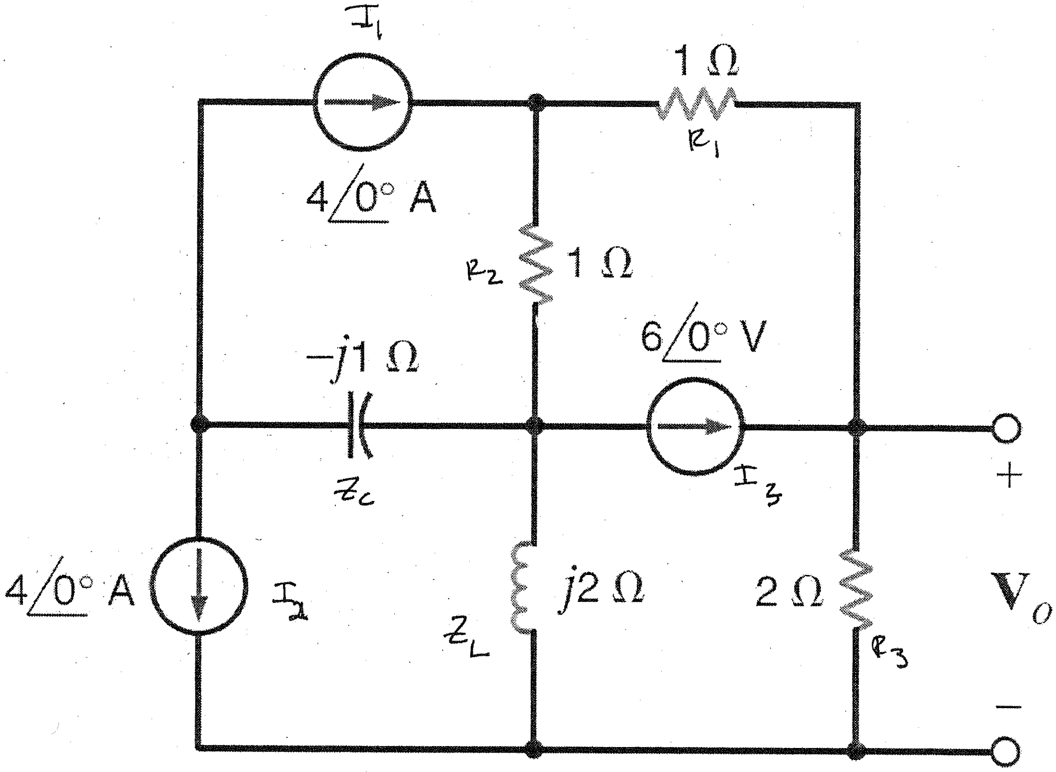
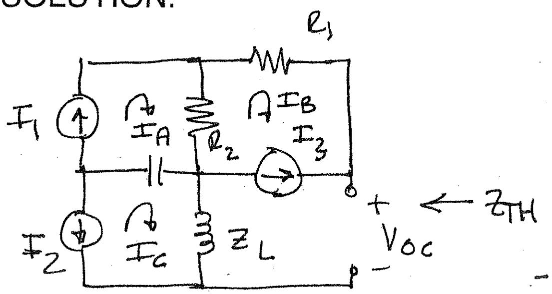


Figure P8.86

SOLUTION:



$I_A = I_1 = 4\angle 0° A$

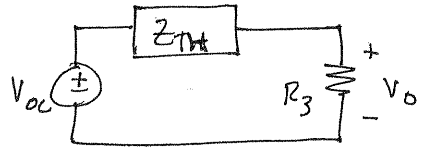
$I_B = -I_3 = -6\angle 0° A$

$I_C = -I_2 = -4\angle 0° A$

$-Z_L I_C + R_2(I_B - I_A) + R_1 I_B + V_{oc} = 0$

$Z_{TH} = R_1 + R_2 + Z_L = 2 + j2\,\Omega$

$V_{oc} = 16 - j8\ V$



$$V_o = \frac{V_{oc} R_3}{R_3 + Z_{TH}}$$

$$\boxed{V_o = 4\angle -53.1°\ V}$$

8.87 Find $\mathbf{V}_o$ in the network in Fig. P8.87 using Thévenin's theorem.

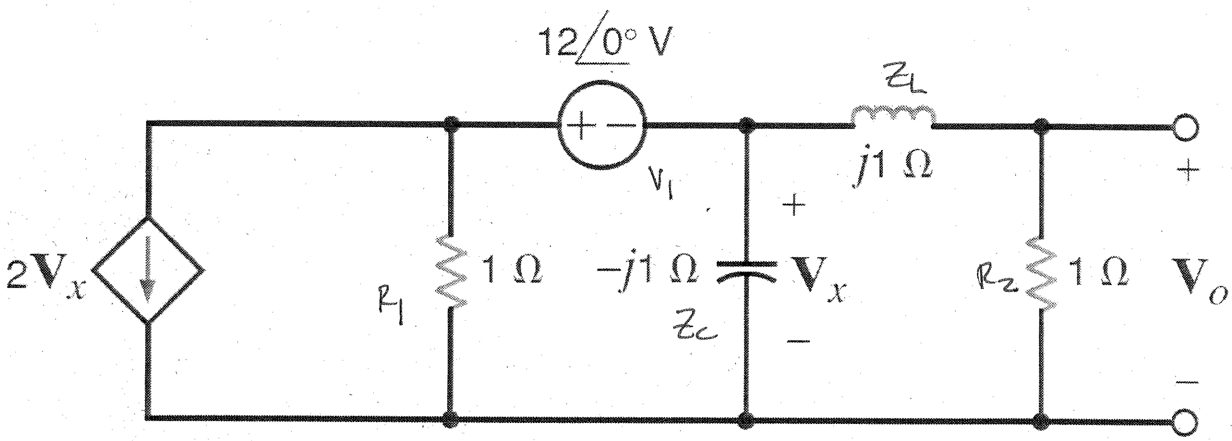
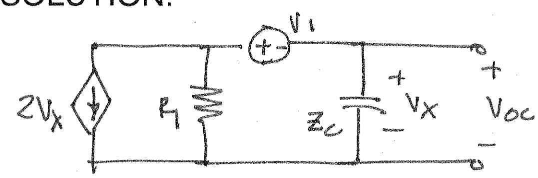


Figure P8.87

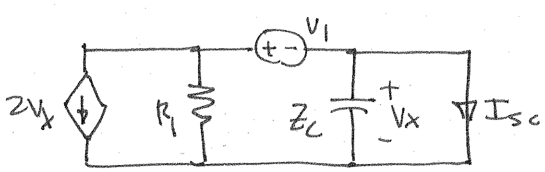
SOLUTION:



$V_x = V_{oc}$

$$2V_{oc} + \frac{V_{oc}+V_1}{R_1} + \frac{V_{oc}}{Z_C} = 0$$

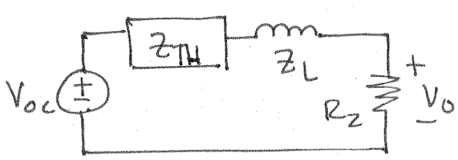
$$V_{oc} = -3.6 + j1.2\ \text{V}$$



$V_x = 0$

$$\frac{V_1}{R_1} + I_{sc} = 0 \qquad I_{sc} = -12\angle 0^\circ\ \text{A}$$

$$Z_{TH} = V_{oc}/I_{sc} = 0.3 - j0.1\ \Omega$$



$$V_o = \frac{V_{oc} R_2}{R_2 + Z_L + Z_{TH}}$$

$$\boxed{V_o = 2.4\angle 127^\circ\ \text{V}}$$

8.88 Find the Thévenin's equivalent for the network in Fig. P8.88 at the terminals *A-B*. CS

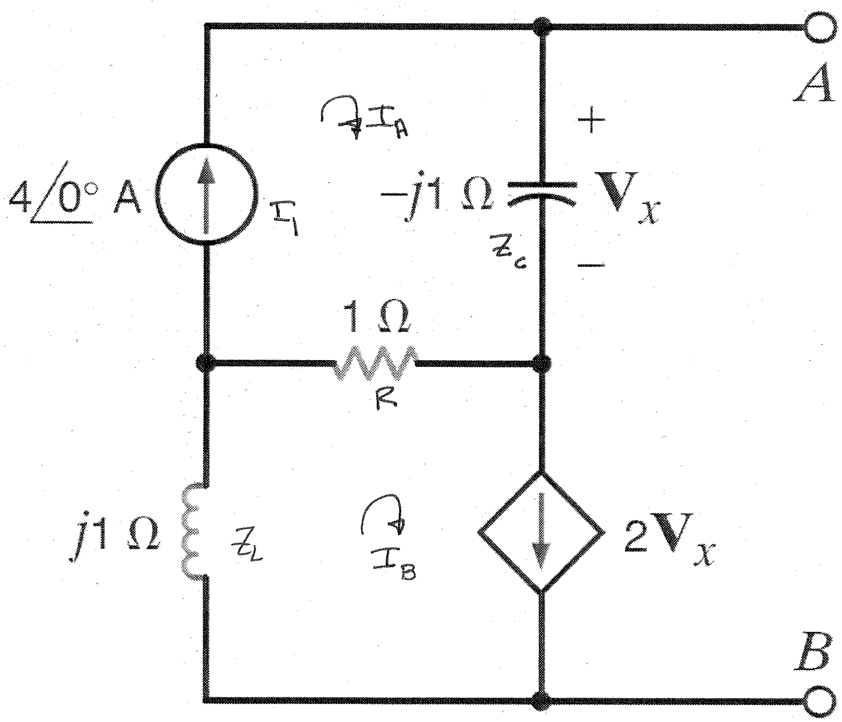


Figure P8.88

SOLUTION:

$\underline{V_{AB} = V_{oc}}$

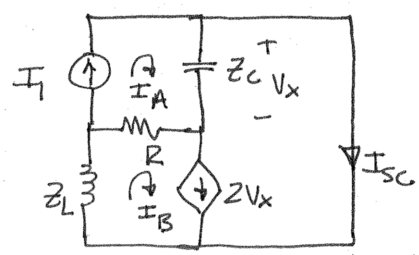
$I_A = 4\angle 0°$ A

$V_x = Z_c I_A = -j4$ V

$I_B = 2V_x = -j8$ A

$Z_L I_B + R(I_B - I_A) - Z_c I_A + V_{oc} = 0$

$V_{oc} = -4 + j4$ V



$I_A = I_1 = 4\angle 0°$A

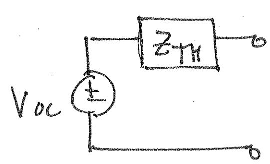
$V_x = Z_c(I_1 - I_{sc})$

$I_B - I_{sc} = 2V_x$

$Z_c(I_{sc} - I_A) + Z_L I_B + R(I_B - I_A) = 0$

$I_{sc} = 2.53\ \angle 18.4°$ A

$Z_{TH} = V_{oc}/I_{sc} = 2.24\ \angle 117°\ \Omega$



8.89 Given the network in Fig. P8.89, find the Thévenin's equivalent of the network at the terminals *A*-*B*.

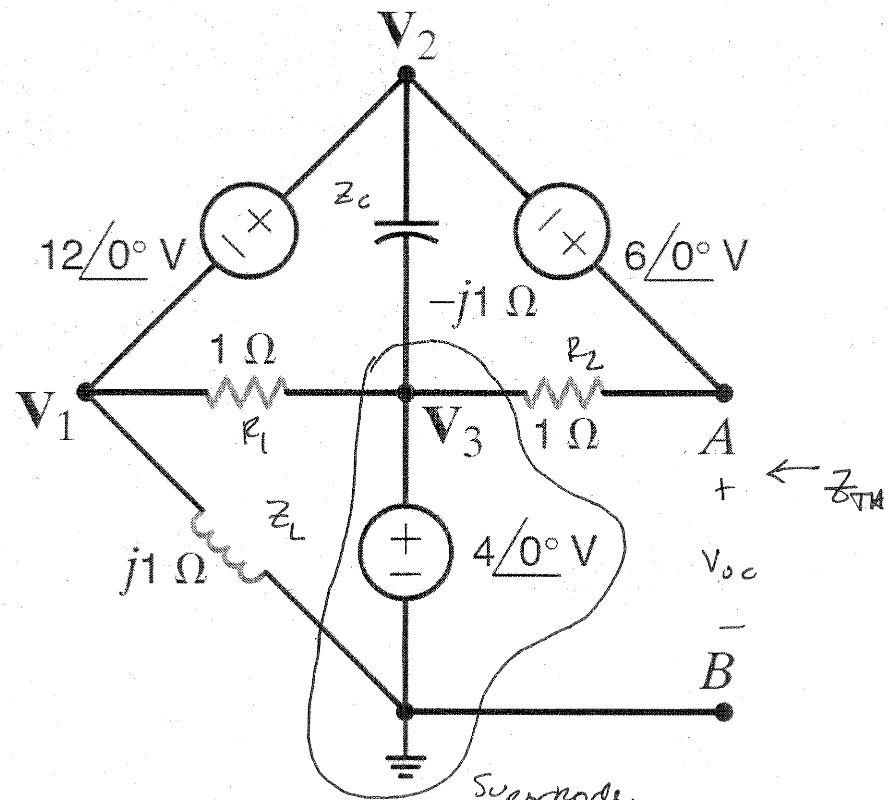


Figure P8.89

SOLUTION:

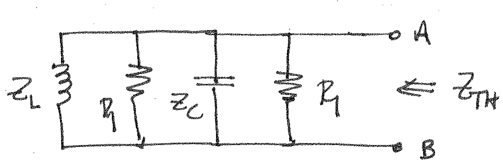
$V_2 - V_1 = 12\angle 0° V \qquad V_{oc} - V_2 = 6\angle 0° V \qquad V_3 = 4\angle 0° V$

Supernode:

$$\frac{V_3 - V_2}{Z_C} + \frac{V_3 - V_1}{R_1} + \frac{-V_1}{Z_L} + \frac{V_3 - V_{oc}}{R_2} = 0$$

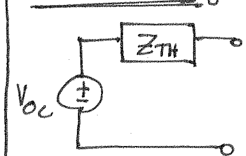
$V_{oc} = 13.6 \angle -17.1° \ V$

Z_{TH}



$$\frac{1}{Z_{TH}} = \frac{1}{Z_C} + \frac{1}{R_1} + \frac{1}{Z_C} + \frac{1}{R_1} \Rightarrow Z_{TH} = 0.5\Omega$$

Thevenin Eq



$V_{oc} = 13.6 \angle -17.1° V$

$Z_{TH} = 0.5\Omega$

8.90 Find $\mathbf{V}_x$ in the circuit in Fig. P8.90 using Norton's theorem.

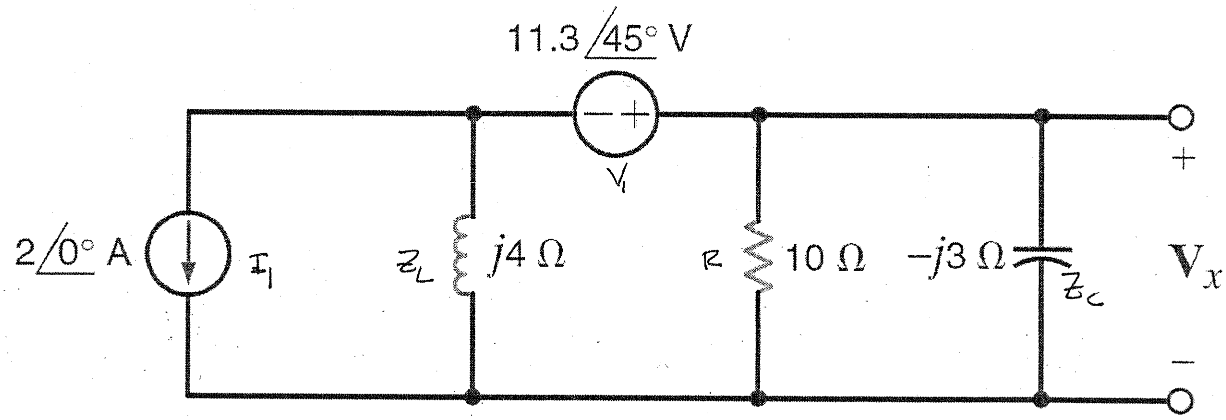
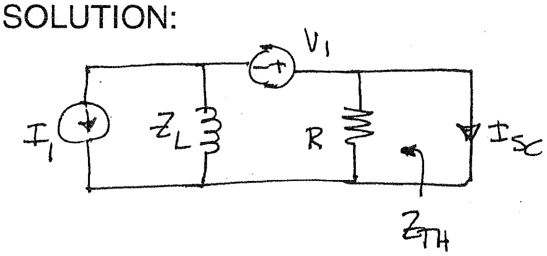


Figure P8.90

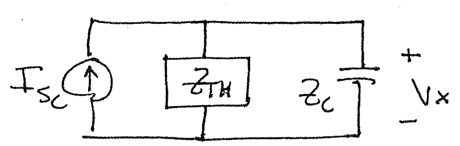
SOLUTION:



Superposition:

$$I_{SC} = -I_1 + \frac{V_1}{Z_L} = -j2\,A$$

$$Z_{TH} = \frac{R Z_L}{R + Z_L} = 5.90 - j11.8\,\Omega$$



$$V_x = I_{SC} Z \qquad Z = \frac{Z_{TH} Z_C}{Z_{TH} + Z_C}$$

$$\boxed{V_x = 15.4 \angle -129.8^\circ\,V}$$

8.91 Find $\mathbf{I}_o$ in the network in Fig. P8.91 using Norton's theorem.

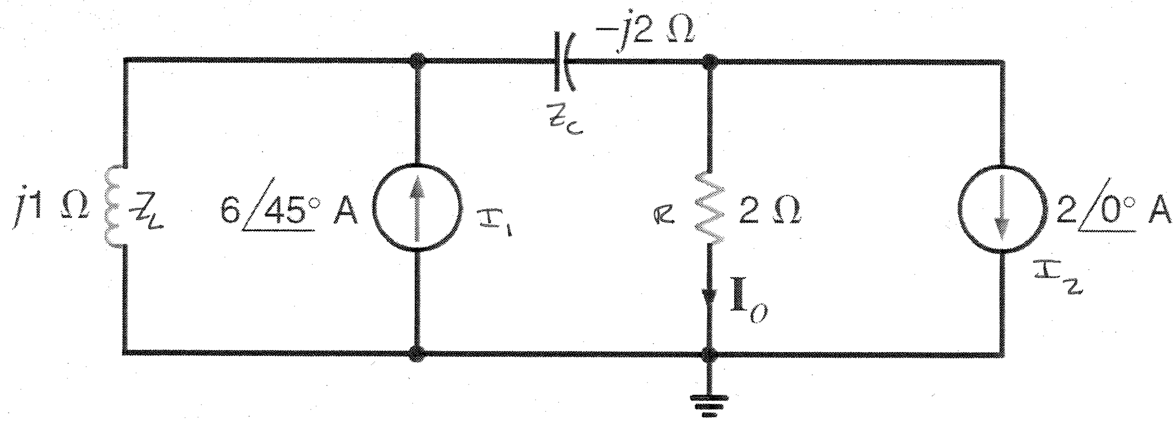
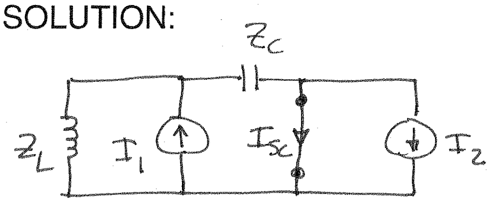


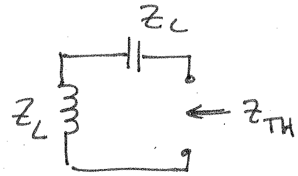
Figure P8.91

SOLUTION:

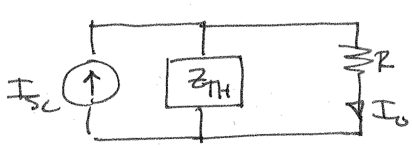


Superposition:

$$I_{SC} = -I_2 + \frac{I_1 Z_L}{Z_L + Z_C}$$



$$Z_{TH} = Z_L + Z_C = -j1\,\Omega$$



$$I_o = \frac{I_{SC}\, Z_{TH}}{R + Z_{TH}}$$

$$\boxed{I_o = 3.38\,\angle 151^\circ \text{ A}}$$

8.92 Apply both Norton's theorem and MATLAB to find $\mathbf{V}_o$ in the network in Fig. P8.92.

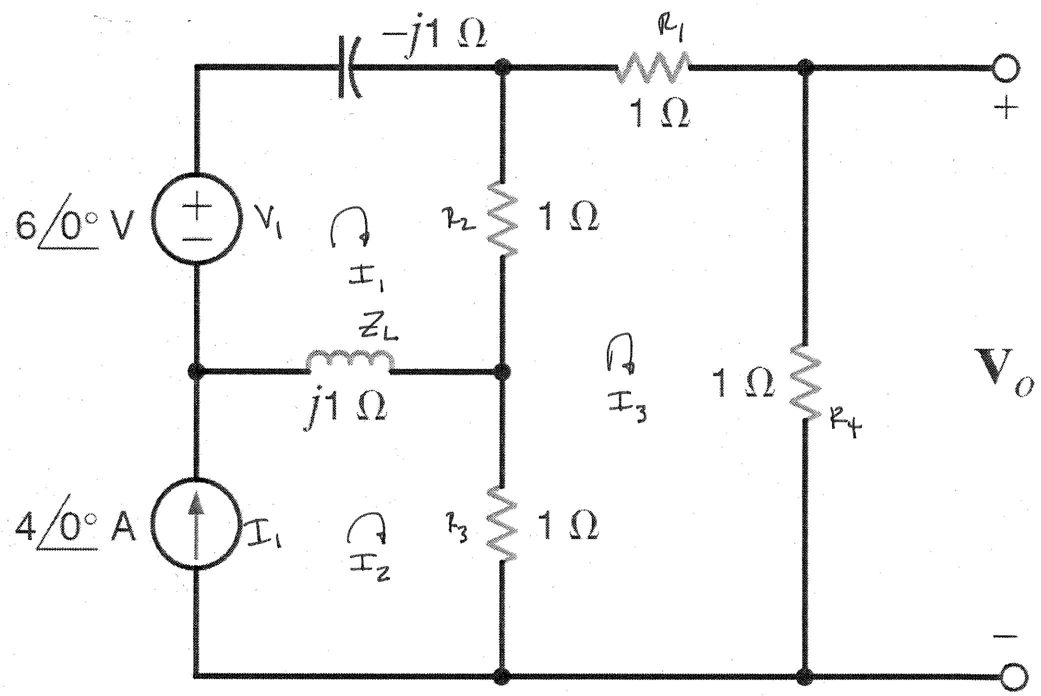


Figure P8.92

SOLUTION:

MATLAB:

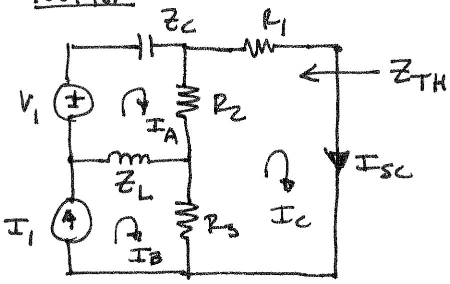
$$\left\{\begin{array}{l} 6 = I_1(1) - j1 I_2 - I_3 \\ I_2 = 4\angle 0° \\ 0 = -I_1 - I_2 + I_3(4) \end{array}\right\} \Rightarrow \begin{bmatrix} 1 & -j1 & -1 \\ 0 & 1 & 0 \\ -1 & -1 & 4 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \\ 0 \end{bmatrix}$$

$V_0 = R_4 I_3$

```
» z=[1 -1i -1;0 1 0;-1 -1 4];
» v=[6;4;0];
» i=inv(z)*v

i = 9.3333 + 5.3333i
    4.0000
    3.3333 + 1.3333i
```

$$\boxed{V_0 = 3.59\angle 21.8° \text{ V}}$$

Norton



$I_{SC} = I_C$

① $V_1 = I_A(R_2 + Z_L + Z_C) - Z_L I_B - R_2 I_C$

or, $V_1 = I_A - jI_B - I_C = 6\angle 0°$

② $I_B = I_1 = 4\angle 0°\,A$

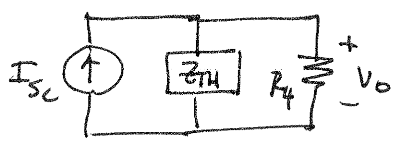
③ $-R_2 I_A - R_3 I_B + I_C(R_1 + R_2 + R_3) = 0$

or $-I_A - I_B + 3I_C = 0$

yields $I_C = I_{SC} = 5 + j2\ A$

$Z_{TH} = R_1 + Z + R_3$ $\quad Z = \dfrac{R_2(Z_C + Z_L)}{R_2 + Z_C + Z_L} = 0$

$Z_{TH} = 2\Omega$



$$V_o = I_{SC}\frac{R_4 Z_{TH}}{R_4 + Z_{TH}}$$

$$\boxed{V_o = 3.59\angle 21.8°\ V}$$

8.93 Find $\mathbf{V}_o$ using Norton's theorem for the circuit in Fig. P8.93. CS

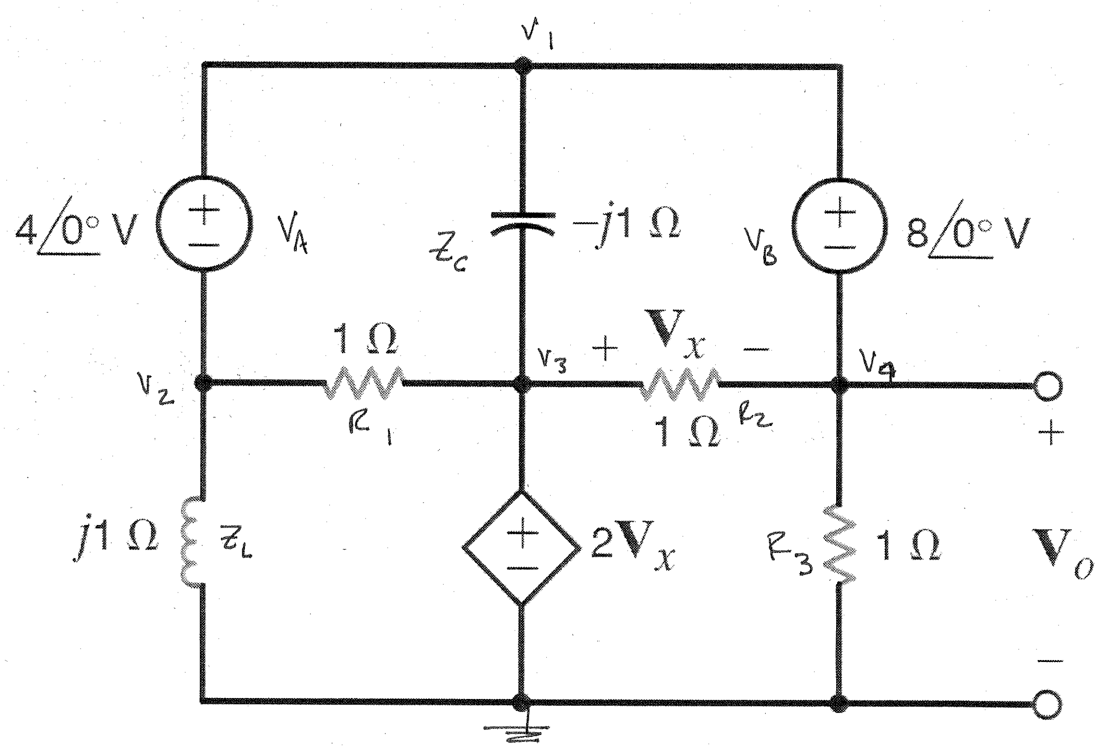
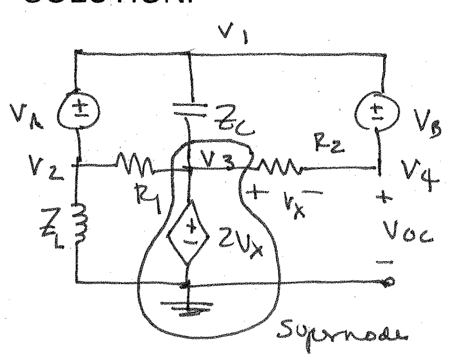


Figure P8.93

SOLUTION:



$V_{oc} = V_4$

$V_1 - V_2 = 4\angle 0°\,V \qquad V_1 - V_4 = 8\angle 0°\,V$

yield $V_1 = 8 + V_4$ & $V_2 = 4 + V_4$

$2V_x = V_3 = 2(V_3 - V_4) \Rightarrow V_3 = 2V_4$

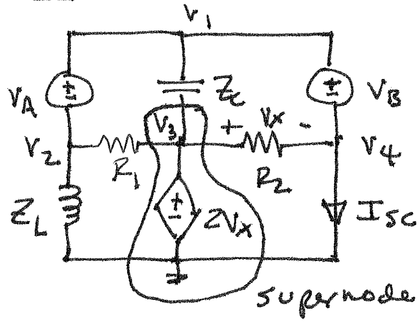
Supernode:

$$\frac{V_3 - V_2}{R_1} + \frac{V_3 - V_1}{Z_C} + \frac{V_3 - V_4}{R_2} - \frac{V_2}{Z_L} = 0$$

or, $V_3(2+j1) + V_2(-1+j1) - j1V_1 - V_4 = 0$

yields $V_{oc} = V_4 = 2\angle 0°\,V$

$\underline{I_{SC}}$



$V_4 = 0$

$V_3 - V_4 = V_x \quad \& \quad V_3 = 2V_x \Rightarrow V_3 = 0$

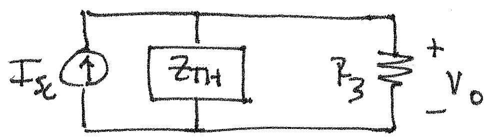
$V_1 = V_B = 8\angle 0° \text{ V}$

$V_1 - V_2 = V_A = 4\angle 0° \Rightarrow V_2 = 4\angle 0° \text{ V}$

at supernode: $\dfrac{V_3 - V_1}{Z_C} + \dfrac{V_3 - V_4}{R_2} + \dfrac{V_3 - V_2}{R_1} + \dfrac{0 - V_2}{Z_L} = I_{SC}$

yields $I_{SC} = -4 - j4 \text{ A}$

$Z_{TH} = \dfrac{V_{oc}}{I_{SC}} = 0.354\angle -135° \ \Omega$



$V_0 = \dfrac{I_{SC}\left[Z_{TH} R_3\right]}{Z_{TH} + R_3}$

$\boxed{V_0 = 4\angle 90° \text{ V}}$

8.94 Use Norton's theorem to find $\mathbf{V}_o$ in the network in Fig. P8.94. **PSV**

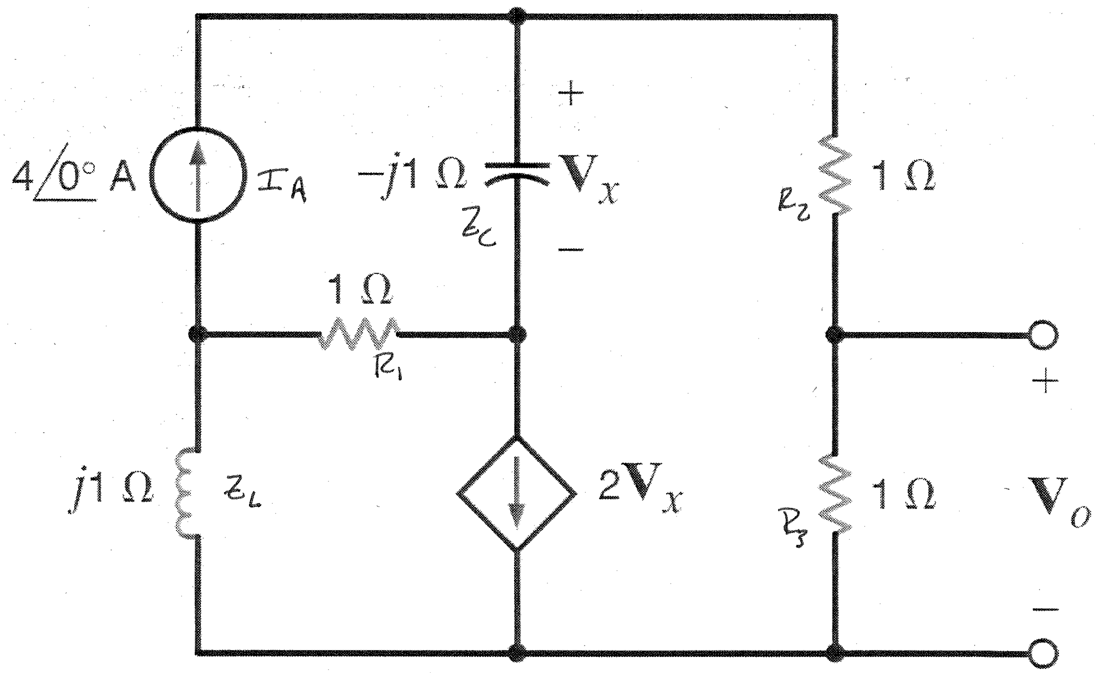
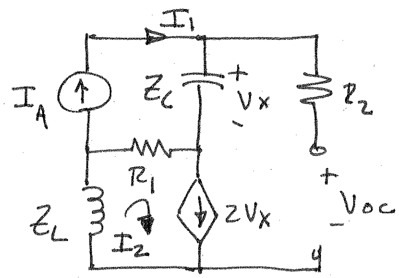


Figure P8.94

SOLUTION:

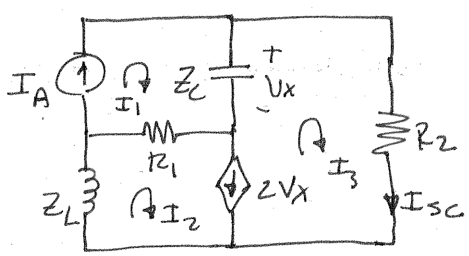


$I_1 = I_A = 4\angle 0° A$

$I_2 = 2V_x = 2Z_C I_1 = -j8\ A$

$Z_L I_2 + R_1(I_2 - I_1) - Z_C I_1 + V_{oc} = 0$

yields $V_{oc} = -4 + j4\ V$



$I_1 = 4\angle 0°$ $\quad 2V_x = I_2 - I_3 = 2Z_C(I_1 - I_3)$

or, $-I_2 + I_3(1+j2) = j8$

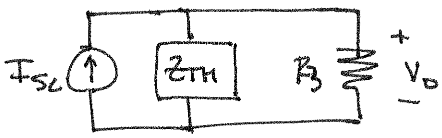
$I_2(Z_L + R_1) + I_3(Z_C + R_2) - I_1(R_1 + Z_C) = 0$

or $-I_1(1-j1) + I_2(1+j1) + I_3(1-j1) = 0$

$I_{sc} = I_3$

yields $I_{sc} = 2 + j2\ A$

$$Z_{TH} = \frac{V_{OC}}{I_{SC}} = j2\,\Omega$$



$$V_o = I_{SC}\left[\frac{R_3 Z_{TH}}{R_3 + Z_{TH}}\right]$$

$$\boxed{V_o = 2.53\angle 71.6^\circ \text{ V}}$$

8.95 Use MATLAB to find the node voltages in the network in Fig. P8.95.

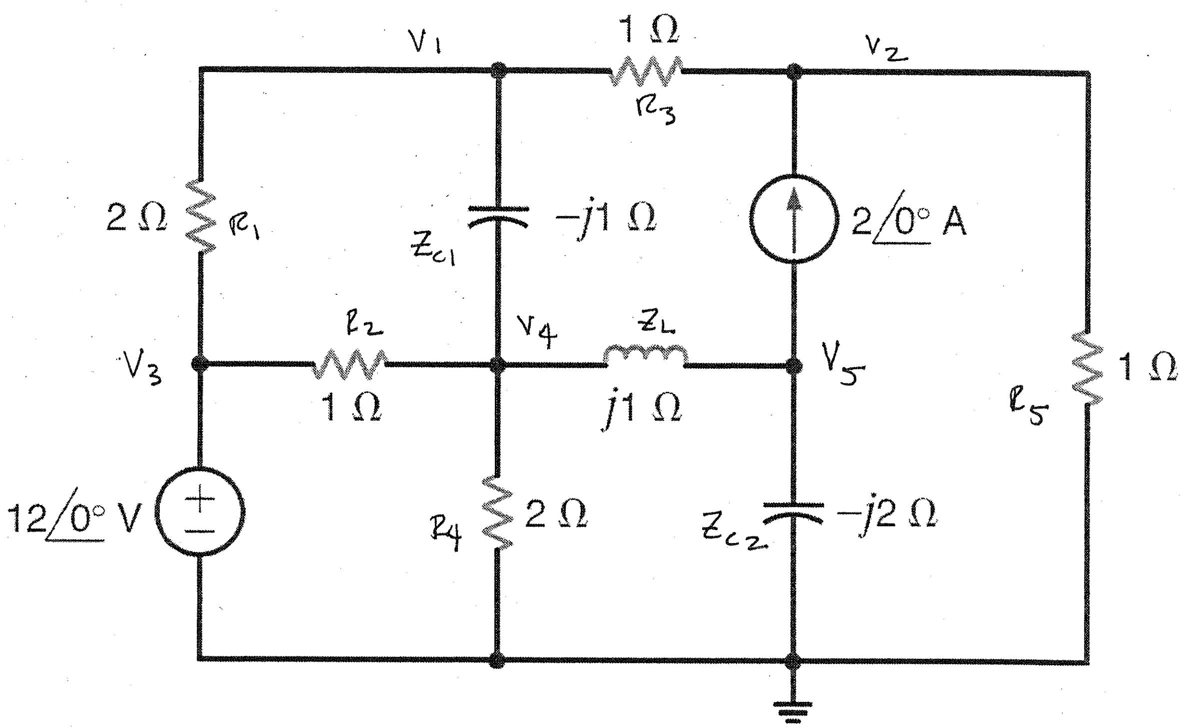


Figure P8.95

SOLUTION:

$$\frac{V_1 - V_3}{R_1} + \frac{V_1 - V_2}{R_3} + \frac{V_1 - V_4}{Z_{C1}} = 0 \Rightarrow V_1[3 + j2] - 2V_2 - V_3 - j2V_4 = 0$$

$$\frac{V_2 - V_1}{R_3} + \frac{V_2}{R_5} = 2\angle 0 \Rightarrow -V_1 + 2V_2 = 2\angle 0^\circ$$

$$\frac{V_4 - V_1}{Z_{C1}} + \frac{V_4 - V_5}{Z_L} + \frac{V_4}{R_4} + \frac{V_4 - V_3}{R_2} = 0 \Rightarrow -j2V_1 - 2V_3 + 3V_4 + j2V_5 = 0$$

$$\frac{V_5 - V_4}{Z_L} + \frac{V_5}{Z_{C2}} = -2\angle 0^\circ \rightarrow -j2V_4 + jV_5 = 4$$

$$V_3 = 12\angle 0^\circ$$

$$
\begin{bmatrix} 3+j2 & -2 & -1 & -j2 & 0 \\ -1 & 2 & 0 & 0 & 0 \\ -j2 & 0 & -2 & 3 & j2 \\ 0 & 0 & 0 & -j2 & j1 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 0 \\ 4 \\ 12 \end{bmatrix}
$$

```
» y=[3+2i -2 -1 -2i 0;-1 2 0 0 0;-2i 0 -2 3 2i;0 0 0 -2i 1i;0 0 1 0 0];

» i=[0;2;0;4;12];

» v=inv(y)*i

v =

   6.5800 - 2.0600i
   4.2900 - 1.0300i
  12.0000 + 0.0000i
   4.5200 - 1.6400i
   9.0400 - 7.2800i
```

8.96 Use MATLAB to find $\mathbf{I}_o$ in the network in Fig. P8.96.

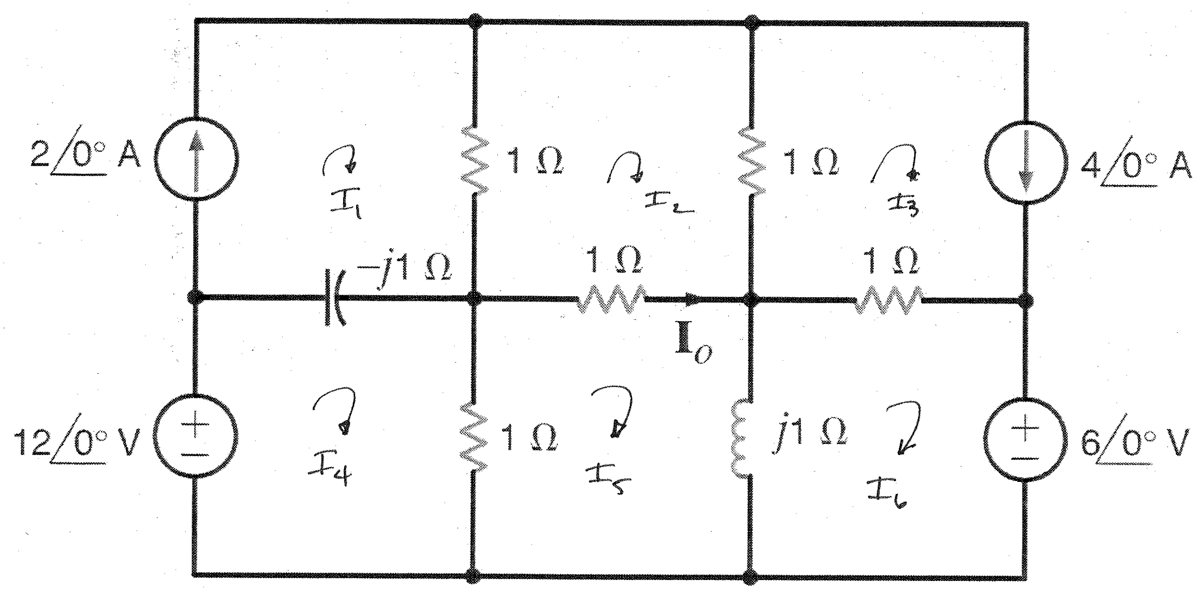


Figure P8.96

SOLUTION:

$I_1 = 2\angle 0° A$ $\quad I_3 = 4\angle 0° A$ $\quad -I_1 + 3I_2 - I_3 - I_5 = 0$

$12 = j1 I_1 + I_4(1-j1) - I_5$ $\quad -I_2 - I_4 + I_5(2+j1) - j1 I_6 = 0$

$-I_3 - j1 I_5 + I_6(1+j1) = -6$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ -1 & 3 & -1 & 0 & -1 & 0 \\ j1 & 0 & 0 & 1-j1 & -1 & 0 \\ 0 & -1 & 0 & -1 & 2+j1 & -j1 \\ 0 & 0 & -1 & 0 & -j1 & 1+j1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 0 \\ 12 \\ 0 \\ -6 \end{bmatrix}$$

$I_o = I_5 - I_2$

```
» z=[1 0 0 0 0 0;0 0 1 0 0 0;-1 3 -1 0 -1 0;
  1i 0 0 1-1i -1 0;0 -1 0 -1 2+1i -1i;0 0 -1 0 -1i 1+1i];
» v=[2;4;0;12;0;-6];
» i=inv(z)*v

i = 2.0
    3.6 + 0.8i
    4.0
    8.2 + 8.6i
    4.8 + 2.4i
    0.2 + 4.6i
```

$I_5 = 4.8 + j2.4 A$

$I_2 = 3.6 + j0.8 A$

$\boxed{I_o = 2\angle 53.1° A}$

8.97 Find $\mathbf{V}_o$ in the circuit in Fig. P8.97 using MATLAB.

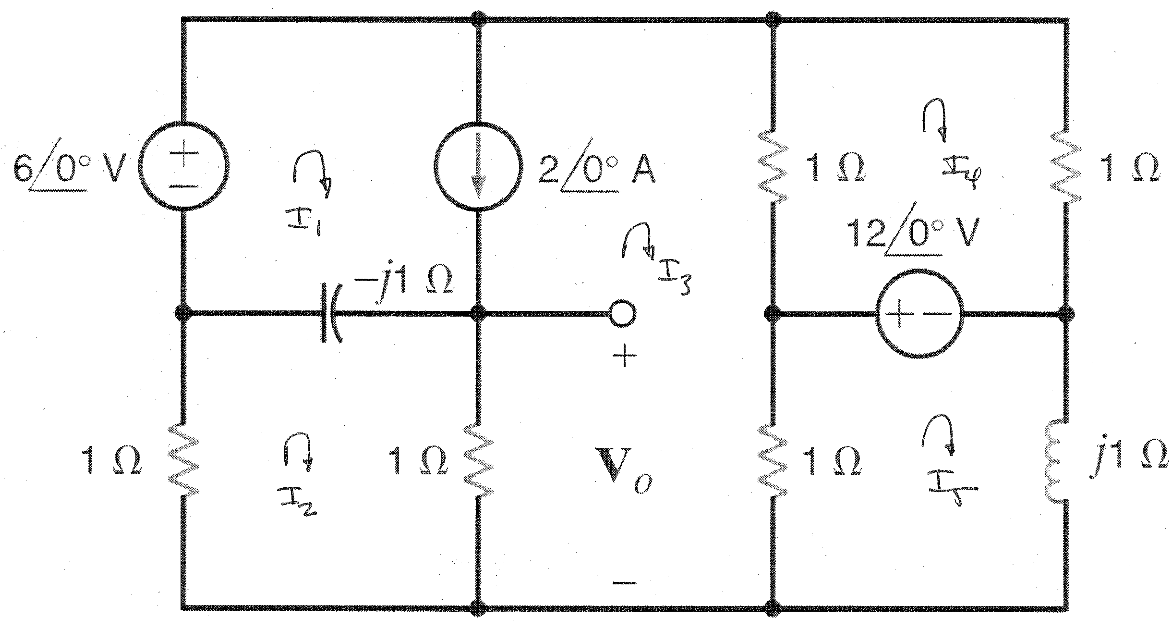


Figure P8.97

SOLUTION:

$I_1 - I_3 = 2\angle 0$ $\quad j1 I_1 + I_2(2-j1) - I_3 = 0$ $\quad 12\angle 0° = -I_3 + I_4(2)$

$-12 = -I_3 + (1+j1) I_5$ $\quad 6 = I_4 + j1\, I_5 + I_2$

$$\begin{bmatrix} 1 & 0 & -1 & 0 & 0 \\ j1 & 2-j1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 2 & 0 \\ 0 & 0 & -1 & 0 & 1+j1 \\ 0 & 1 & 0 & 1 & j1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 12 \\ -12 \\ 6 \end{bmatrix}$$

$V_0 = 1\,[I_2 - I_3]$

```
» z=[1 0 -1 0 0;1i 2-1i -1 0 0;0 0 -1 2 0;0 0 -1 0 1+1i;0 1 0 1 1i];
» v=[2;0;12;-12;6];
» i=inv(z)*v

i = 6.1509 + 3.4717i
    3.5849 + 0.4528i
    4.1509 + 3.4717i
    8.0755 + 1.7358i
   -2.1887 + 5.6604i
```

$I_2 = 3.59 + j\,0.45\text{ A}$

$I_3 = 4.15 + j\,3.42\text{ A}$

$\boxed{V_0 = 3.07\angle -101°\text{ V}}$

8.98 Determine $\mathbf{V}_o$ in the network in Fig. P8.98 using MATLAB.

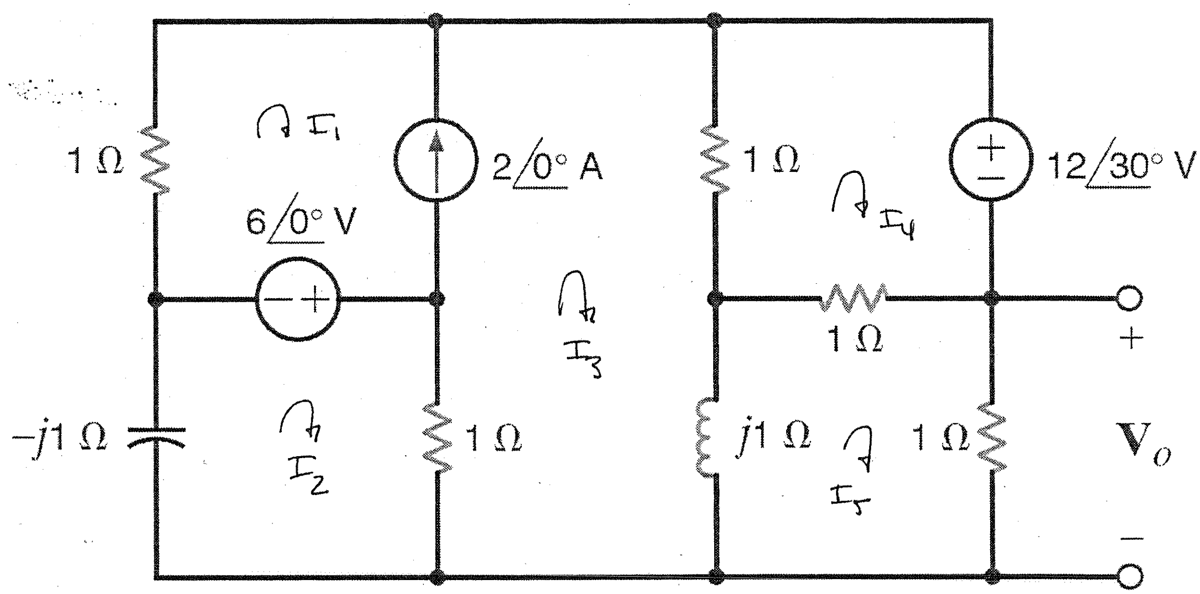


Figure P8.98

SOLUTION:

$I_3 - I_1 = 2\angle 0^\circ$ $\quad I_2(1-j1) - I_3 = 6\angle 0$ $\quad -I_3 + 2I_4 - I_5 = -12\angle 30$

$-j1I_3 - I_4 + I_5(2+j1) = 0$ $\quad I_1 - jI_2 + I_5 = -12\angle 30^\circ$

$$\begin{bmatrix} -1 & 0 & 1 & 0 & 0 \\ 0 & 1-j1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & -j1 & -1 & 2+j1 \\ 1 & -j1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \\ -12\angle 30 \\ 0 \\ -12\angle 30 \end{bmatrix}$$

$V_o = 1(I_5)$

```
» z=[-1 0 1 0 0;0 1-1i -1 0 0;0 0 -1 2 -1;0 0 -1i -1 2+1i;1 -1i 0 0 1];

» v=[2;6;-10.4-6.0i;0;-10.4-6.0i];

» i=inv(z)*v

i =

  -5.7798 - 1.9339i
   2.0771 + 0.1431i
  -3.7798 - 1.9339i
  -9.4716 - 4.9615i
  -4.7633 - 1.9890i
```

$I_5 = -4.76 - j1.99$ A

$V_o = (1)I_5$

$$\boxed{V_o = 5.16\angle -157^\circ \text{ V}}$$

8.99 Find $\mathbf{I}_o$ in the network in Fig. P8.99 using MATLAB.

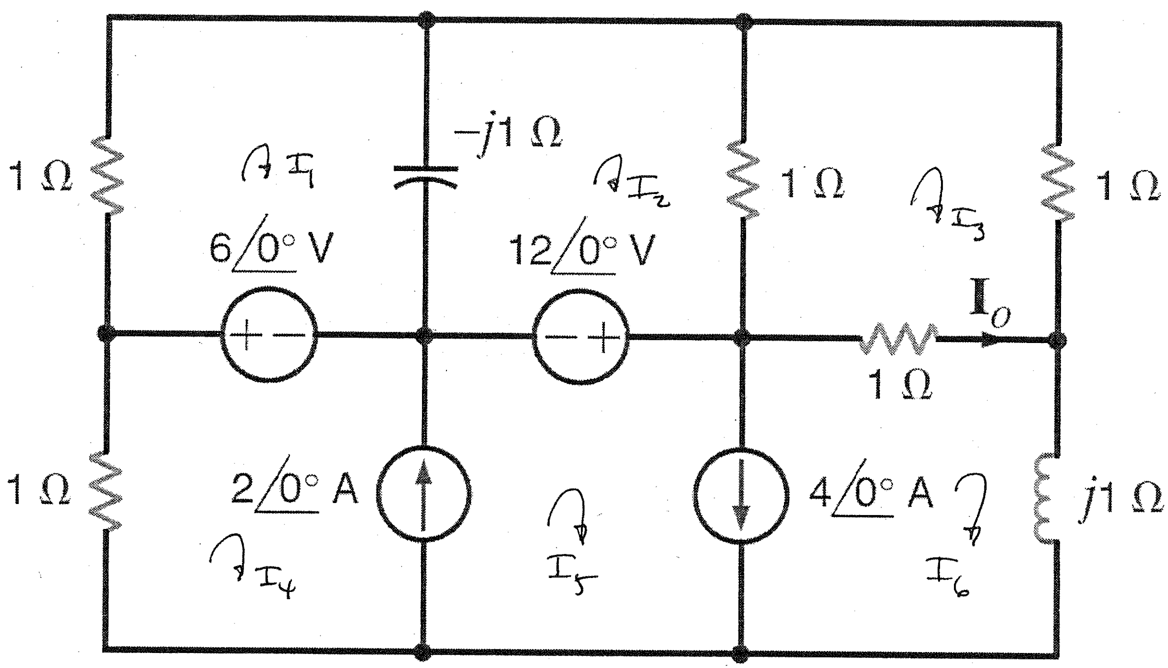


Figure P8.99

SOLUTION:

$(1-j1)I_1 + jI_2 = 6\angle 0 \qquad jI_1 + I_2(1-j1) - I_3 = -12\angle 0°$

$-I_2 + I_3(3) - I_6 = 0 \qquad I_5 - I_4 = 2\angle 0° \qquad I_5 - I_6 = 4\angle 0°$

$I_1 + I_3 + jI_6 + I_4 = 0$

$$\begin{bmatrix} 1-j1 & j1 & 0 & 0 & 0 & 0 \\ j1 & 1-j1 & -1 & 0 & 0 & 0 \\ 0 & -1 & 3 & 0 & 0 & -1 \\ 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 1 & 0 & 1 & 1 & 0 & j1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} = \begin{bmatrix} 6 \\ -12 \\ 0 \\ 2 \\ 4 \\ 0 \end{bmatrix}$$

```
» z=[1-1i 1i 0 0 0 0;1i 1-1i -1 0 0 0;
    0 -1 3 0 0 -1;0 0 0 -1 1 0;
    0 0 0 0 1 -1;1 0 1 1 0 1i];

» v=[6;-12;0;2;4;0];

» i=inv(z)*v

i =

  -2.2800 + 3.0400i
  -5.3200 - 5.2400i
  -1.6000 - 2.2000i
   2.5200 - 1.3600i
   4.5200 - 1.3600i
   0.5200 - 1.3600i
```

$I_o = I_6 - I_3$

$I_6 = 0.52 - j1.36 \qquad I_3 = -1.6 - j2.2A$

$$\boxed{I_o = 2.28\angle 21.62°\ A}$$

8.100 Use MATLAB to determine $\mathbf{I}_o$ in the network in Fig. P8.100.

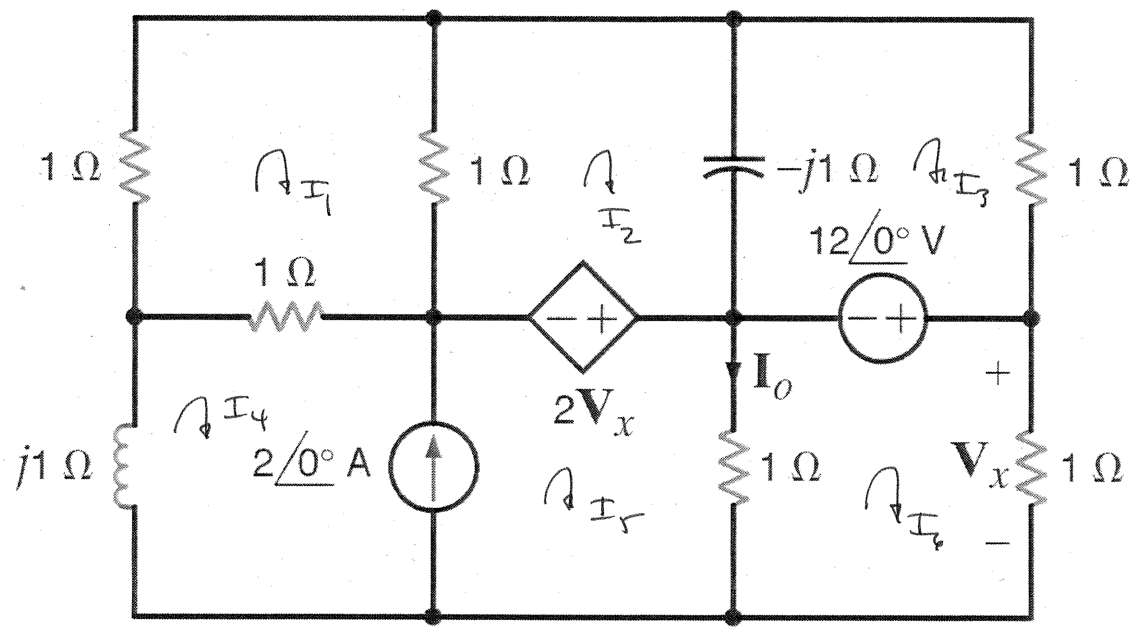


Figure P8.100

SOLUTION:

$3I_1 - I_2 - I_4 = 0 \qquad -I_1 + I_2(1-j1) + j1\, I_3 = -2V_x \qquad V_x = I_6\,[1]$

$j1 I_2 + I_3(1-j1) = -12\angle 0 \qquad I_5 - I_4 = 2\angle 0^\circ \qquad -I_5 + I_6(2) = 12\angle 0^\circ$

$I_1 + I_3 + jI_4 + I_6 = 0$

$$\begin{bmatrix} 3 & -1 & 0 & -1 & 0 & 0 \\ -1 & 1-j1 & j1 & 0 & 0 & 2 \\ 0 & j1 & 1-j1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 2 \\ 1 & 0 & 1 & j1 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -12 \\ 2 \\ 12 \\ 0 \end{bmatrix}$$

```
» z=[3 -1 0 -1 0 0;-1 1-1i 1i 0 0 2;
    0 1i 1-1i 0 0 0;0 0 0 -1 1 0;
    0 0 0 0 -1 2;1 0 1 1i 0 1];
» v=[0;0;-12;2;12;0];
» i=inv(z)*v

i =
   -5.7379 - 2.8552i
  -20.2345 + 3.1862i
  -14.5241 + 5.7103i
    3.0207 -11.7517i
    5.0207 -11.7517i
    8.5103 - 5.8759i
```

$I_o = I_5 - I_6$

$$\boxed{I_o = 6.83\angle{-121^\circ}\ \text{A}}$$

8.101 Find $\mathbf{I}_o$ in the circuit in Fig. P8.101 using MATLAB.

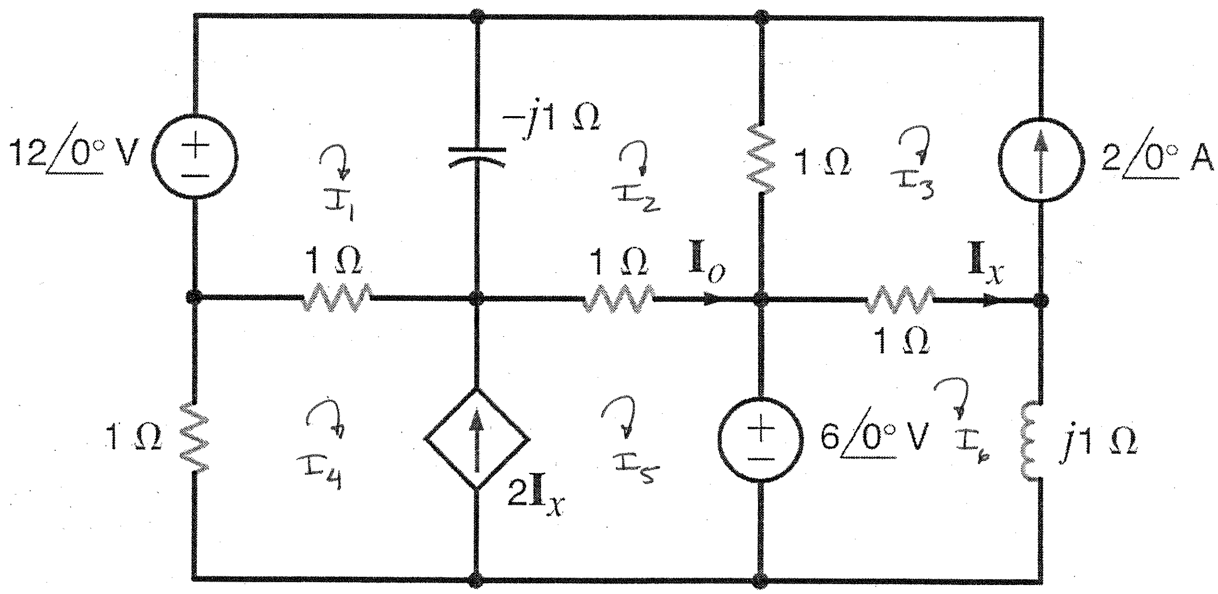


Figure P8.101

SOLUTION: $I_3 = -2\angle 0°\ A$ $\qquad I_5 - I_4 = 2I_x = 2(I_6 - I_3) \Rightarrow 2I_3 - I_4 + I_5 - 2I_6 = 0$

$12\angle 0° = I_1(1-j1) + I_2 - I_4$ $\qquad I_1(j1) + I_2(2-j1) - I_3 - I_5 = 0$

$6\angle 0° = -I_3 + I_6(1+j1)$ $\qquad -I_1 - I_2 + I_4(2) + I_5 = -6\angle 0$

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & -1 & 1 & -2 \\ 1-j1 & j1 & 0 & -1 & 0 & 0 \\ j1 & 2-j1 & -1 & 0 & -1 & 0 \\ -1 & -1 & 0 & 2 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1+j1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \\ 12+j0 \\ 0 \\ -6+j0 \\ 6+j0 \end{bmatrix}$$

$I_o = I_5 - I_2 = 2.12 - j0.471$

$$\boxed{I_o = 2.17\angle -12.5°\ A}$$

8.102 Use MATLAB to find $\mathbf{I}_o$ in the network in Fig. P8.102.

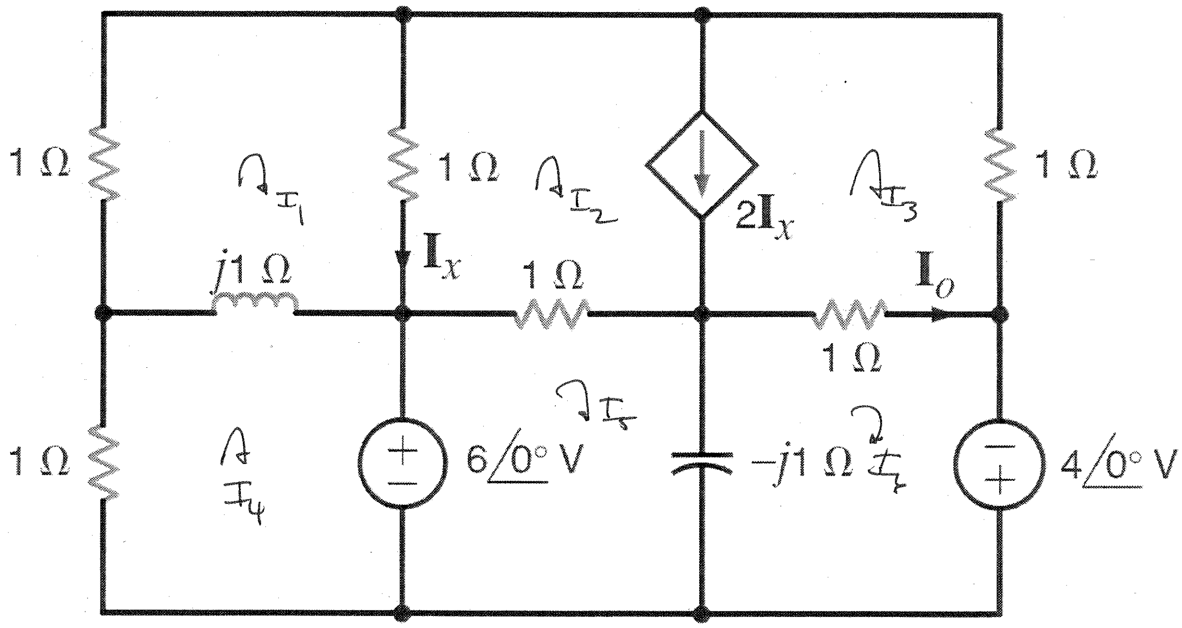


Figure P8.102

SOLUTION:

$I_1(2+j1) - I_2 - j1 I_4 = 0$ $\quad$ $I_2 - I_3 = 2I_x = 2(I_1 - I_2) \Rightarrow -2I_1 + 3I_2 - I_3 = 0$

$-j1 I_1 + I_4(1+j1) = -6\angle 0°$ $\quad$ $-I_2 + I_5(1-j1) + j1 I_6 = 6\angle 0°$

$-I_3 + jI_5 + I_6(1-j1) = 4\angle 0$ $\quad$ $I_1 + I_3 + I_4 = 4\angle 0$

$$\begin{bmatrix} 2+j1 & -1 & 0 & -j1 & 0 & 0 \\ -2 & 3 & -1 & 0 & 0 & 0 \\ -j1 & 0 & 0 & 1+j1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1-j1 & j1 \\ 0 & 0 & -1 & 0 & j1 & 1-j1 \\ 1 & 0 & 1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -6 \\ 6 \\ 4 \\ 4 \end{bmatrix}$$

```
» z=[2+1i -1 0 -1i 0 0;-2 3 -1 0 0 0;
     -1i 0 0 1+1i 0 0;0 -1 0 0 1-1i 1i;
     0 0 -1 0 1i 1-1i;1 0 1 1 0 0];
» v=[0;0;-6;6;4;4];
» i=inv(z)*v

i =
  -0.7170 - 1.5094i
   1.9623 - 1.1321i
   7.3208 - 0.3774i
  -2.6038 + 1.8868i
   9.4566 - 1.5019i
   9.8264 - 0.0075i

» io=i(6)-i(3)
io = 2.51 + 0.400i
```

$I_o = I_6 - I_3$

$$\boxed{I_o = 2.53\angle 8.40°}$$

8.103 Use both a nodal analysis and a loop analysis, each in conjunction with MATLAB, to find $\mathbf{I}_o$ in the network in Fig. P8.103.

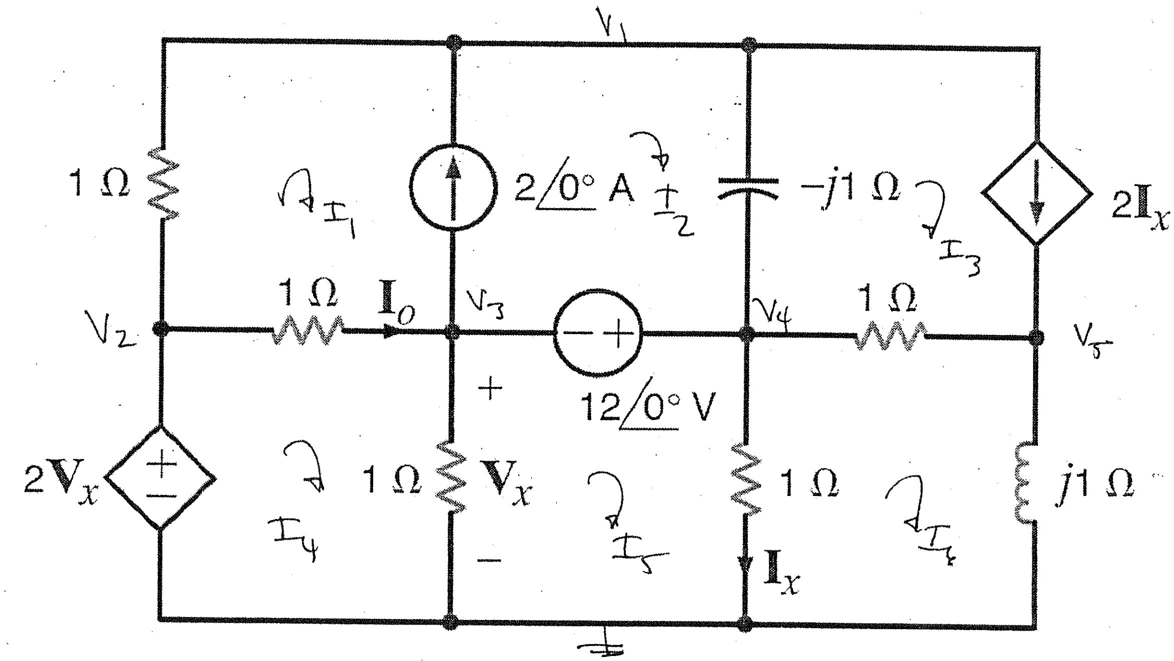


Figure P8.103

SOLUTION:

Loop: $I_2 - I_1 = 2\angle 0°$ $\quad I_3 = 2I_x = 2[I_5 - I_6] \Rightarrow I_3 - 2I_5 + 2I_6 = 0$

$-I_1 + 2I_4 - I_5 = 2V_x = 2[I_4 - I_5](1) \Rightarrow \quad -I_1 + I_5 = 0$

$-I_4 + 2I_5 - I_6 = 12\angle 0 \qquad - I_3 - I_5 + (2+j1)I_6 = 0$

$2I_1 - jI_2 + jI_3 - I_4 = -12\angle 0$

$$\begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -2 & 2 \\ -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 0 & -1 & 2+j1 \\ 2 & -j1 & j1 & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 0 \\ 12 \\ 0 \\ -12 \end{bmatrix}$$

$\boxed{I_o = I_4 - I_1 = 17.7\angle -137° \text{ A}}$

Nodal: $V_2 = 2V_x = 2V_3 \Rightarrow \quad V_2 - 2V_3 = 0 \qquad V_4 - V_3 = 12\angle 0^\circ$

$$\frac{V_1 - V_2}{1} + \frac{V_1 - V_4}{-j1} + 2I_x = 2\angle 0^\circ \qquad I_x = \frac{V_4}{1} \Rightarrow \quad V_1(1+j1) - V_2 + V_4(2-j1) = 2\angle 0^\circ$$

$$\frac{V_5 - V_4}{1} + \frac{V_5}{j1} - 2I_x = 0 \Rightarrow \quad -3V_4 + V_5(1-j1) = 0$$

$$\frac{V_3 - V_2}{1} + \frac{V_3}{1} + \frac{V_4 - V_1}{-j1} + \frac{V_4 - V_5}{1} + \frac{V_4}{1} = -2\angle 0^\circ$$

$$-jV_1 - V_2 + 2V_3 + V_4(2+j1) - V_5 = -2\angle 0^\circ$$

$$\begin{bmatrix} 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 1+j1 & -1 & 0 & 2-j1 & 0 \\ 0 & 0 & 0 & -3 & 1-j1 \\ -j1 & -1 & 2 & 2+j1 & -1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 12 \\ 2 \\ 0 \\ -2 \end{bmatrix}$$

$I_o = (V_2 - V_3)/1$

$$\boxed{I_o = 17.7\angle -137^\circ \text{ A}}$$

MATLAB for LOOP
```
» z=[-1 1 0 0 0 0;0 0 1 0 -2 2;-1 0 0 0 1 0;
     0 0 0 -1 2 -1;0 0 -1 0 -1 2+1i;2 -1i 1i -1 0 0];

» v=[2;0;0;12;0;-12];

» i=inv(z)*v;

» io=i(4)-i(1)

io = -13.0 -12.0i
```

MATLAB for NODAL
```
» y=[0 1 -2 0 0;0 0 -1 1 0;1+1i -1 0 2-1i 0;
     0 0 0 -3 1-1i;-1i -1 2 2+1i -1];

» i=[0;12;2;0;-2];

» v=inv(y)*i;

» io=v(2)-v(3)

io = -13.0 -12.0i
```

8.104 Use Thévenin's theorem, in conjunction with MATLAB, to determine $\mathbf{I}_o$ in the network in Fig. P8.104.

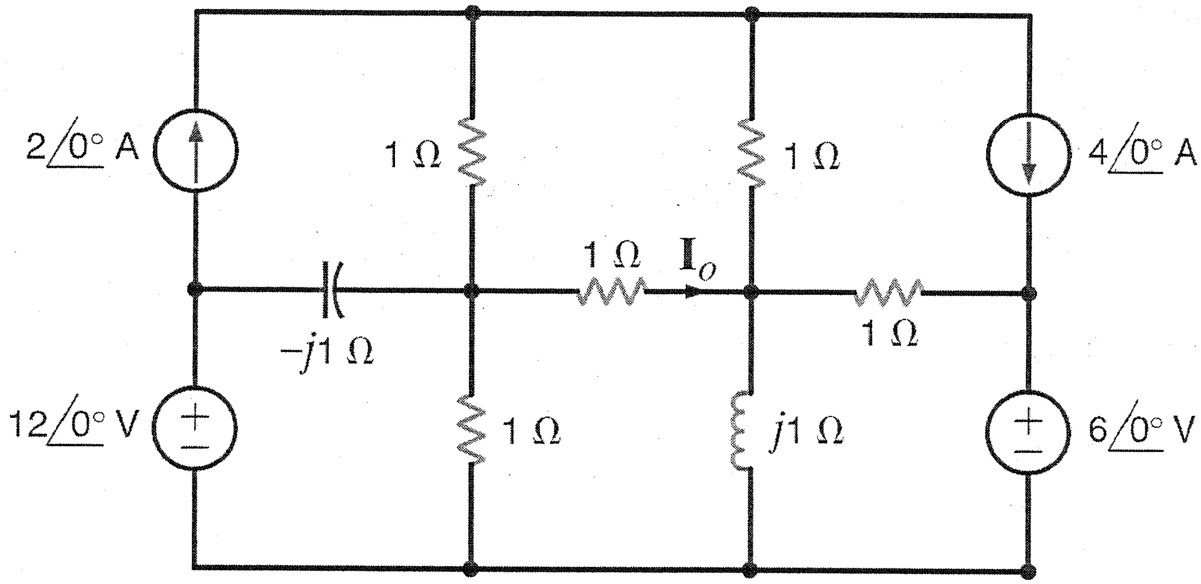
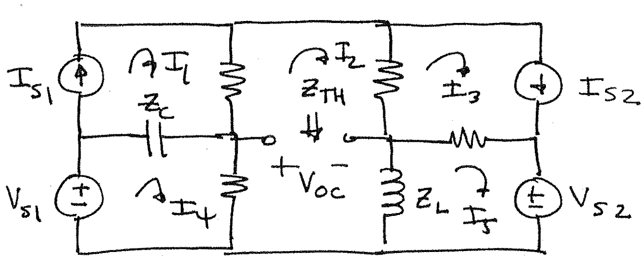


Figure P8.104

SOLUTION:



$Z_C = -j1\,\Omega \quad Z_L = j1\,\Omega$

all $R = 1\,\Omega$

$2\angle 0° = I_1 \qquad I_3 = 4\angle 0°\text{ A} \qquad j1 I_1 + I_4(1-j1) - I_2 = 12\angle 0°$

$0 = -I_1 - I_3 - I_4 - jI_5 + I_2(3+j1) \qquad -I_3 - jI_2 + I_5(1+j1) = -6\angle 0°$

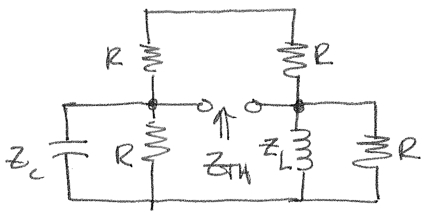
$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ j1 & -1 & 0 & 1-j1 & 0 \\ 0 & -j1 & -1 & 0 & 1+j1 \\ -1 & 3+j1 & -1 & -1 & -j1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 12 \\ -6 \\ 0 \end{bmatrix}$$

$(I_2 - I_4)1 + V_{oc} + j1(I_2 - I_5) = 0$

$V_{oc} = I_4 - I_2 + j1(I_5 - I_2)$

```
» z=[1 0 0 0 0;0 0 1 0 0;1i -1 0 1-1i 0;0 -1i -1 0 1+1i;-1 3+1i -1 -1 -1i];

» v=[2;4;12;-6;0];

» i=inv(z)*v

i =
   2.0000
   4.0000 + 1.3333i
   4.0000
   8.3333 + 7.6667i
   0.3333 + 3.6667i

» voc=i(4)-i(2)+1i*(i(5)-i(2))

voc = 2.0000 + 2.6667i
```

$V_{oc} = 3.33 \angle 53.1^\circ \text{ V}$

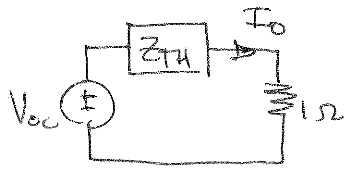


$$Z_{TH} = 2R \,//\, \left[(Z_C // R) + (Z_L // R) \right]$$

$$= 2 \,//\, \left[\tfrac{1}{2} - j\tfrac{1}{2} + \tfrac{1}{2} + j\tfrac{1}{2} \right]$$

$$= 2 // 1 = 2/3 \ \Omega$$

$$I_o = \frac{V_{oc}}{Z_{TH} + 1}$$



$$\boxed{I_o = 2 \angle 53.1^\circ \text{ A}}$$

8.105 Using the PSPICE *Schematics* editor, draw the circuit in Fig. P8.105. At what frequency are the magnitudes of $i_C(t)$ and $i_L(t)$ equal?

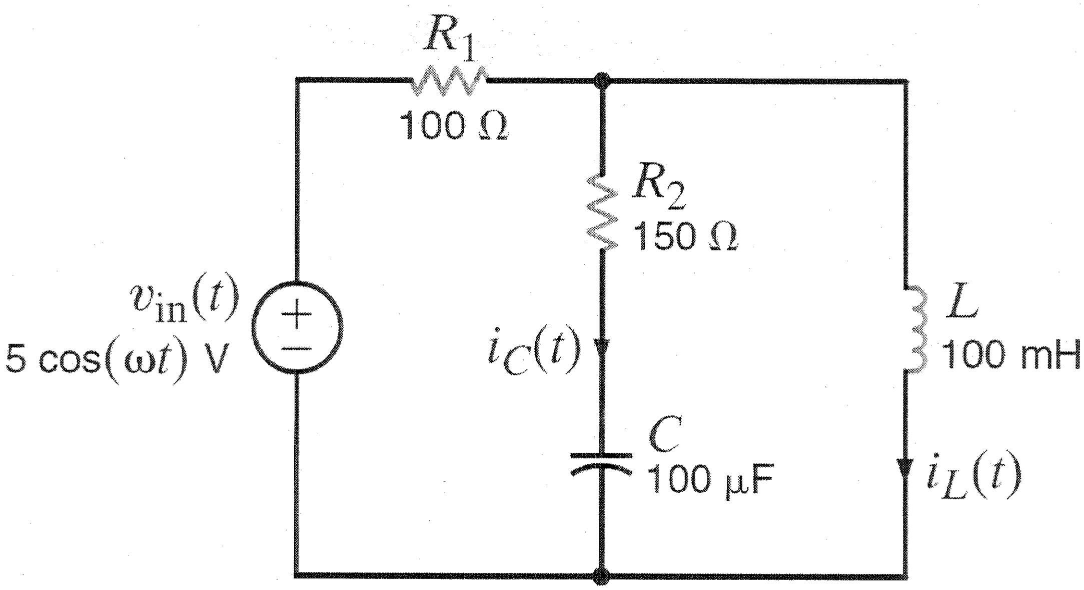
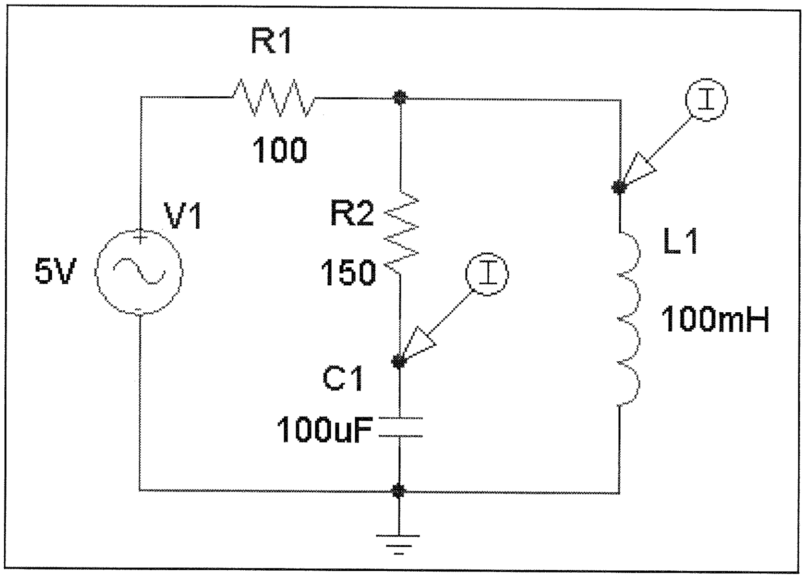
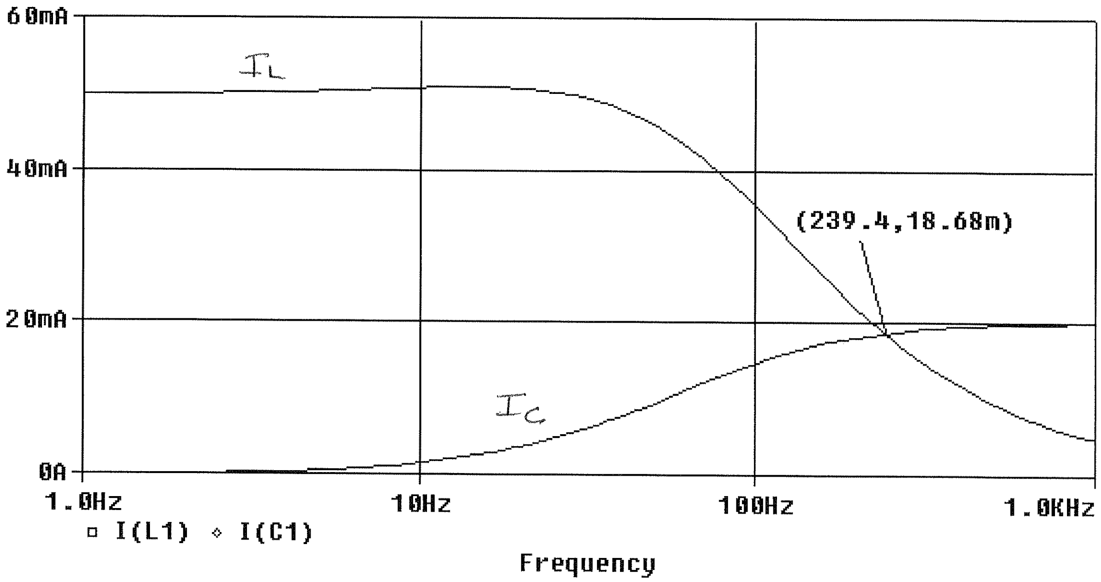


Figure P8.105

SOLUTION: $\boxed{f = 239.4\,\text{Hz}}$


R1
100
V1
5V
R2
150
C1
100uF
L1
100mH


60mA
40mA
20mA
0A
1.0Hz
10Hz
100Hz
1.0KHz
(239.4,18.68m)
I_L
I_C
□ I(L1) ◇ I(C1)
Frequency

8.106 Using the PSPICE *Schematics* editor, draw the circuit in Fig. P8.106. At what frequency are the phases of $i_1(t)$ and $v_x(t)$ equal?

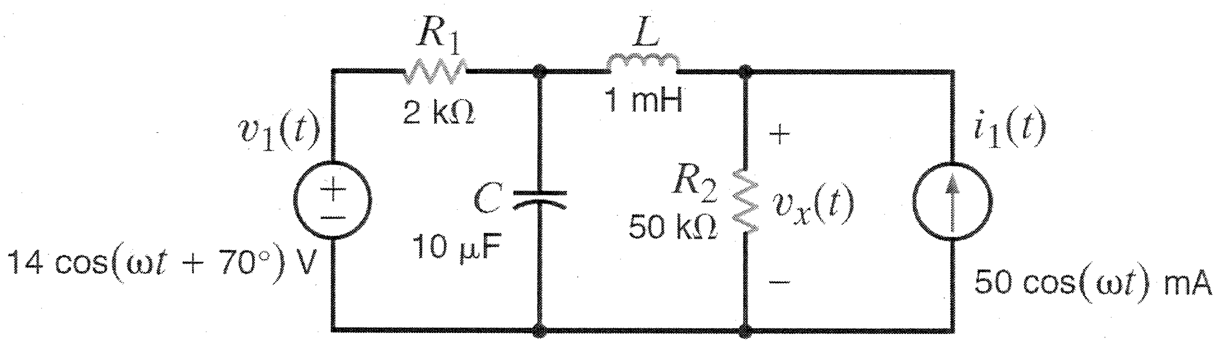
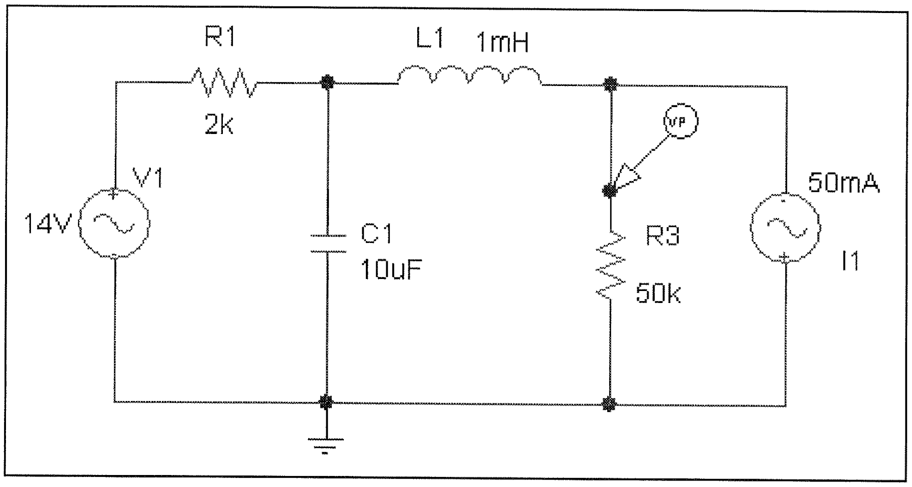
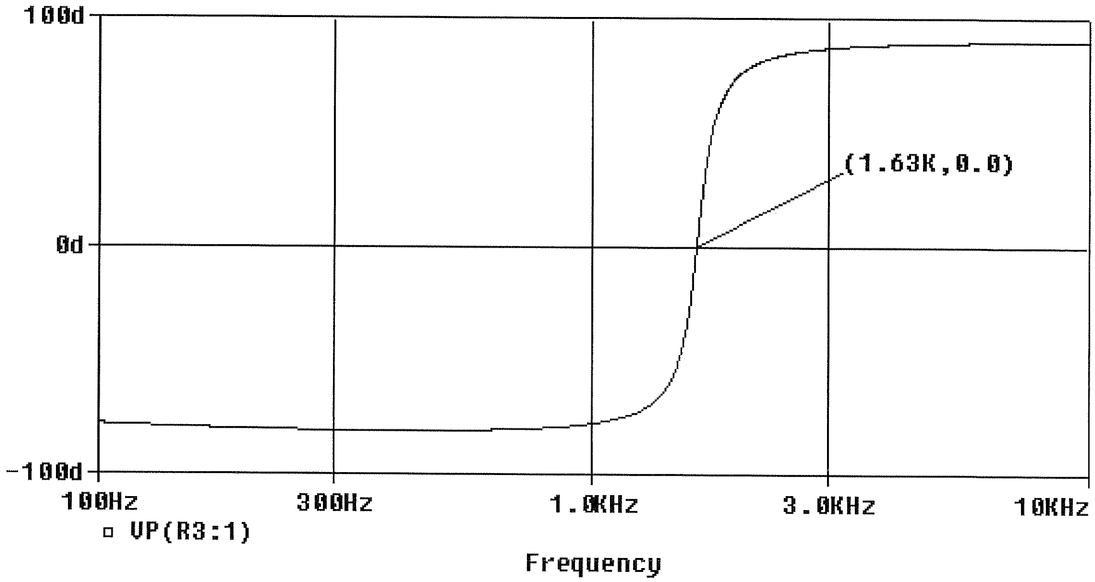


Figure P8.106

SOLUTION: Phase of $i_1(t) = 0°$ find frequency where phase of $v(t)$ is also $0°$.

$\boxed{f = 1.63\text{kHz}}$

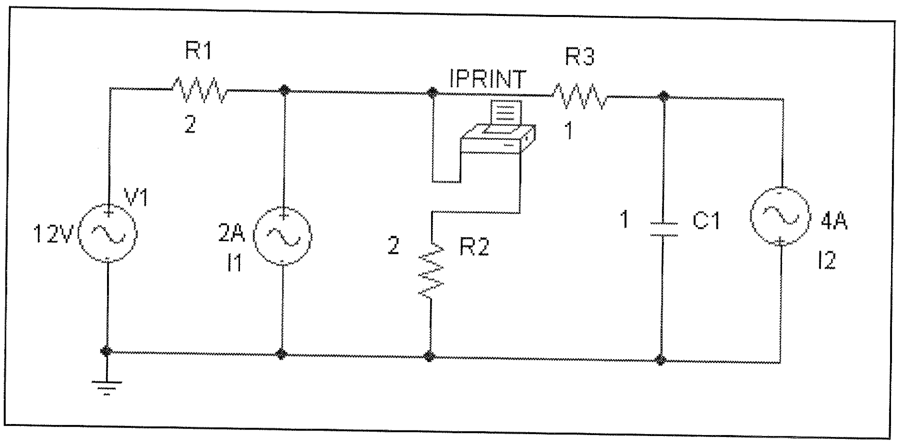

R1
2k
L1 1mH
V1
14V
C1
10uF
VP
R3
50k
50mA
I1


100d
0d
-100d
(1.63K,0.0)
100Hz
300Hz
1.0KHz
3.0KHz
10KHz
VP(R3:1)
Frequency

8.107 Solve Problem 8.48 using PSPICE.

SOLUTION:

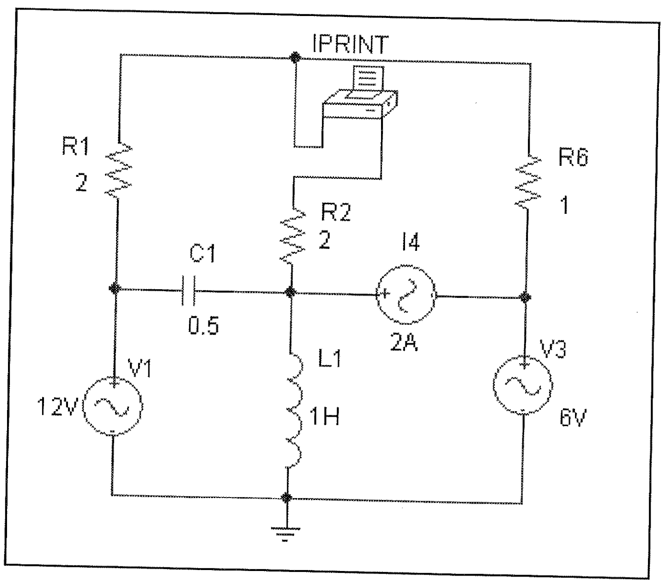
f (Hz)	Imag (A)	Iphase (°)
0.159	2.0	-36.87



8.108 Solve Problem 8.54 using PSPICE.

SOLUTION:

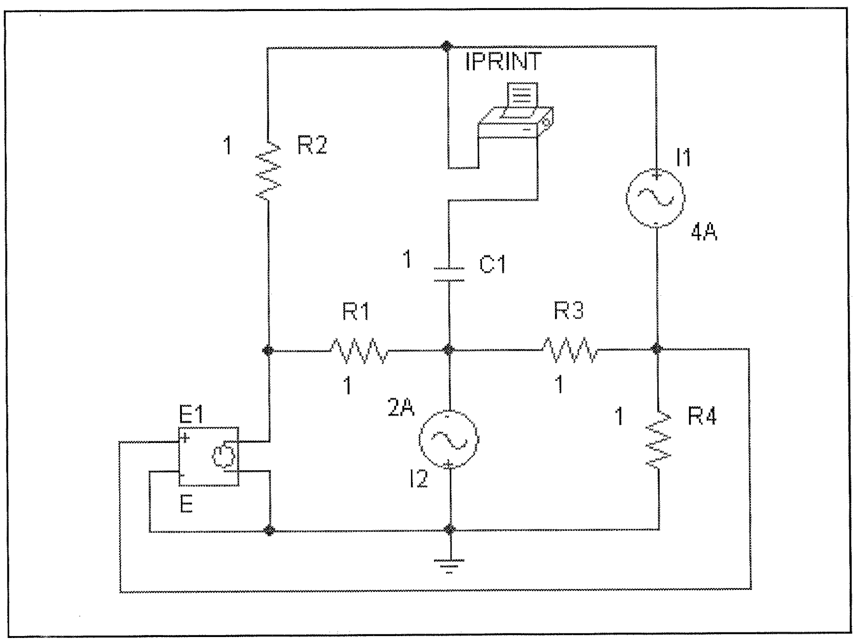
f(Hz)	Imag (A)	Iphase (°)
0.159	6.11	-25.5



8.109 Solve Problem 8.70 using PSPICE.

SOLUTION:

f (Hz)	Imag (A)	Iphase (°)
0.159	1.776E-15	1.800E+02



8.110 Physical inductors are essentially coils of "long" pieces of wire, usually copper with a very thin enamel coating for insulation. Since copper has some resistivity, inductors have some resistance. In most inductors, the coils touch each other. As we learned earlier, conductors in close proximity have capacitance between them. Since the enamel insulation is so thin, the capacitance in an inductor is larger than you would expect for coils of plastic coated wire. Thus, one practical electrical model of a specific inductor is shown in Fig. 8.110. Develop an equation for the inductor's impedance and determine the frequency at which the impedance is real.

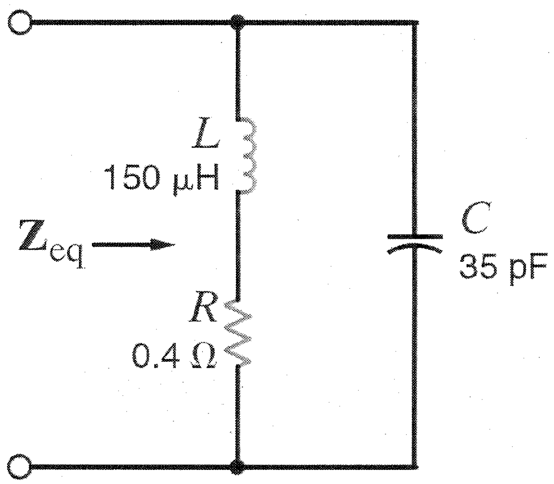


Figure P8.110

SOLUTION: $Z_{eq} = Z_C(R+Z_L) / (R+Z_L+Z_C) = (R+j\omega L) / [1-\omega^2 LC + j\omega CR]$

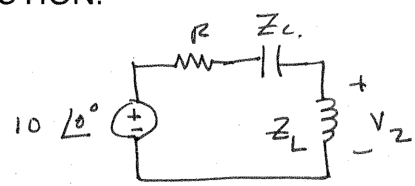
$Z_{eq} = R_{eq} + j0$ requires numerator & denominator have same angle.

$\frac{\omega L}{R} = \frac{\omega CR}{1-\omega^2 LC} \Rightarrow L - \omega^2 L^2 C = R^2 C \Rightarrow \omega^2 = \frac{1}{LC} - \left(\frac{R}{L}\right)^2$

$\omega = 13.88$ Mr/s $\quad \boxed{f = 2.21 \text{ MHz}}$

8.111 We have available a sinusoidal voltage, $v_1(t) = 10\cos\left[2\pi\left(10^3 t\right)\right]$ V. We are asked to design a circuit that will produce a second voltage that has the same magnitude as $v_1(t)$ but leads it by 60°. A good starting point for the design is an *RC* voltage divider.

SOLUTION:



$$\frac{V_2}{10\angle 0} = 1\angle 60° = \frac{Z_L}{R + Z_L + Z_C}$$

$$1\angle 60° = \frac{j\omega L}{R + j\omega L - j/\omega C} = \frac{\omega L\angle 90°}{Z_D\angle \theta_D}$$

$$90° - \theta_D = 60° \Rightarrow \theta_D = 30° = \tan^{-1}\left(\frac{\omega L - 1/\omega C}{R}\right)$$

$$\frac{\omega L - 1/\omega C}{R} = 0.577$$

Arbitrarily select $R = 100\Omega$ yields $\omega L - \frac{1}{\omega C} = 57.7\Omega$

Also, $\dfrac{(\omega L)^2}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = 1 \Rightarrow (\omega L)^2 = 13323$

$\omega = 2000\pi$ r/s $\Rightarrow L = 18.4\,mH$ & $C = 2.76\,\mu F$

$$\boxed{R = 100\,\Omega \quad L = 18.4\,mH \quad C = 2.76\,\mu F}$$

8FE-1 Find $\mathbf{V}_o$ in the network in Fig. 8PFE-1. CS

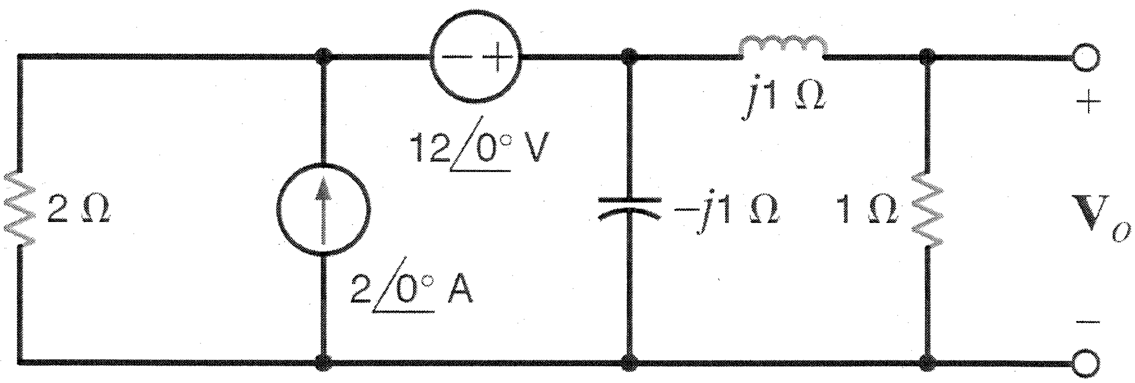
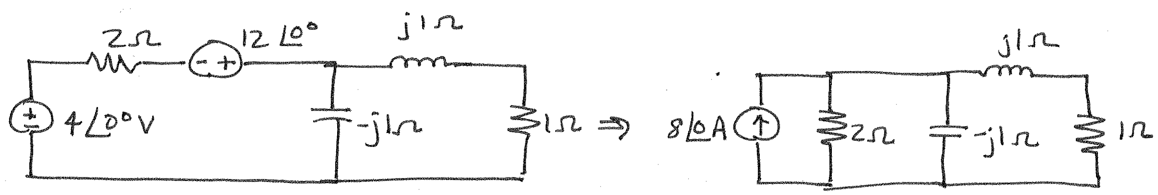


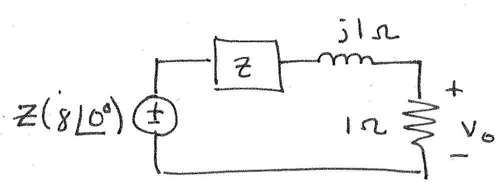
Figure 8PFE-1

SOLUTION:

Use source transformation:



$$Z = 2(-j1)/(2-j1) = -j2/(2-j1)$$



$$V_o = \frac{Z(8\angle 0^\circ)(1)}{1+j1+Z}$$

$$\boxed{V_o = 5.06\angle -71.6^\circ \text{ V}}$$

8FE-2 Find $\mathbf{V}_o$ in the circuit in Fig. 8PFE-2.

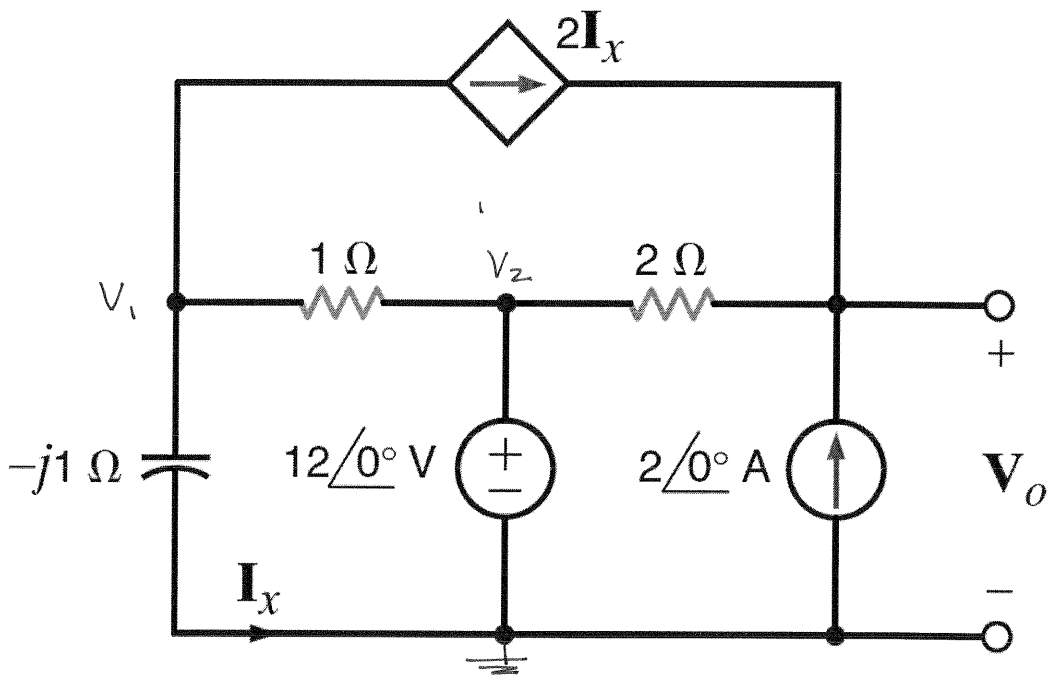


Figure 8PFE-2

SOLUTION:

$V_2 = 12\angle 0° \text{ V}$ $\quad \dfrac{V_1 - V_2}{1} + \dfrac{V_1}{-j1} + 2I_x = 0 \quad I_x = \dfrac{V_1}{-j1} \Rightarrow V_1(1+j3) - V_2 = 0$

$\dfrac{V_2 - V_o}{2} + 2I_x + 2\angle 0 = 0 \Rightarrow V_2 - V_o + j4I_1 = -4$

$$\begin{bmatrix} 1+j3 & -1 & 0 \\ j4 & 1 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_o \end{bmatrix} = \begin{bmatrix} 0 \\ -4 \\ 12 \end{bmatrix}$$

$\boxed{V_o = 30.8\angle 8.97° \text{ V}}$

8FE-3 Find $\mathbf{V}_o$ in the network in Fig. 8PFE-3. CS

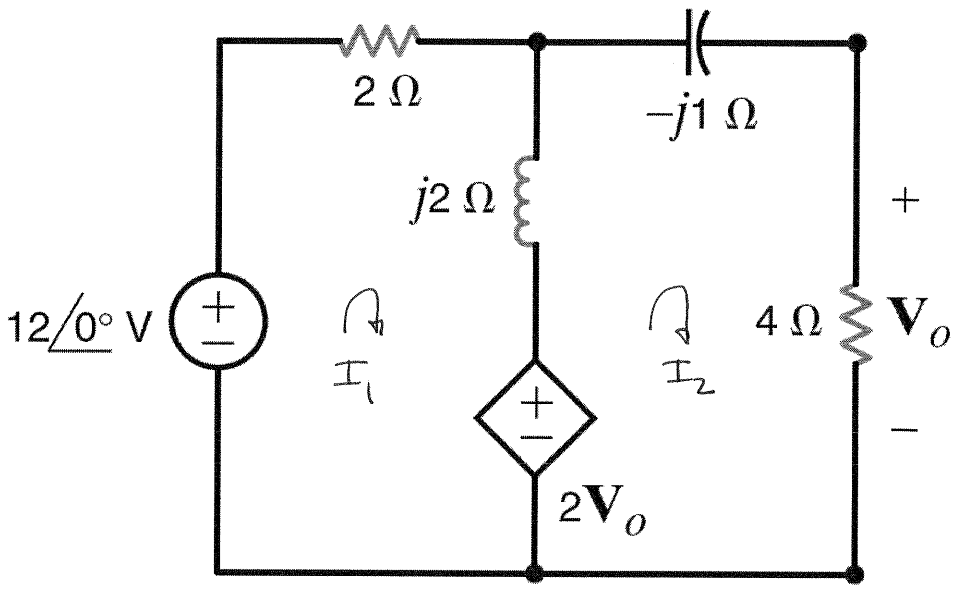


Figure 8PFE-3

SOLUTION:

$$12\angle 0 = (2+j2)I_1 - j2I_2 + 2V_o \qquad V_o = 4I_2$$

$$12\angle 0^\circ = I_1(2+j2) + I_2(8-j2)$$

$$12\angle 0 = 2I_1 + I_2(4-j1) \qquad V_o = 4I_2$$

$$\begin{bmatrix} 2+j2 & 8-j2 \\ 2 & 4-j1 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 12 \\ 12 \end{bmatrix}$$

$$I_2 = 2.06\angle -31.0^\circ \text{ A}$$

$$\boxed{V_o = 8.23\angle -31.0^\circ \text{ V}}$$

8FE-4 Determine the midband (where the coupling capacitors can be ignored) gain of the single-stage transistor amplifier shown in Fig. 8PFE-4.

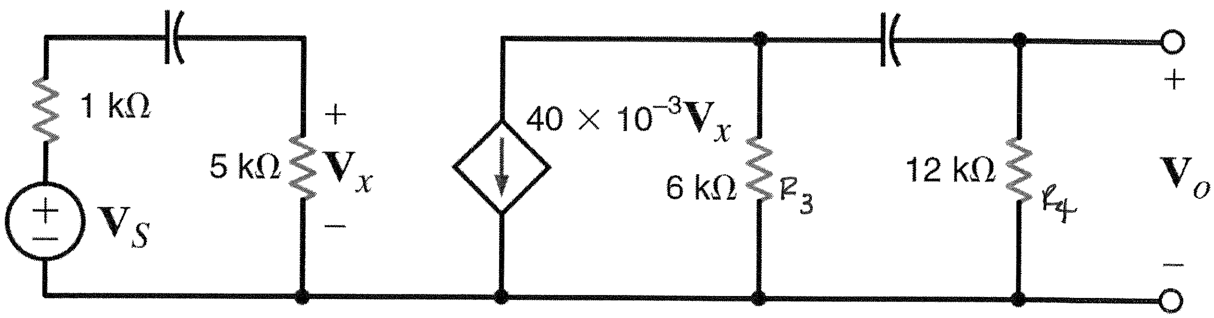


Figure 8PFE-4

SOLUTION:

At midband Z_C is small!

$$\frac{V_x}{V_S} = \frac{5000}{5000+1000} = 5/6$$

$$V_o = -40\times10^{-3}\, V_x \left\{ \frac{R_3 R_4}{R_3+R_4} \right\} \Rightarrow \frac{V_o}{V_x} = -40\times10^{-3}\,(4000) = -160$$

$$\frac{V_o}{V_S} = \frac{V_x}{V_S}\,\frac{V_o}{V_x} \qquad \boxed{\frac{V_o}{V_S} = -133}$$

Chapter Nine:

Steady-State Power Analysis

9.1 Determine the equations for the current and the instantaneous power in the network in Fig. P9.1. **CS**

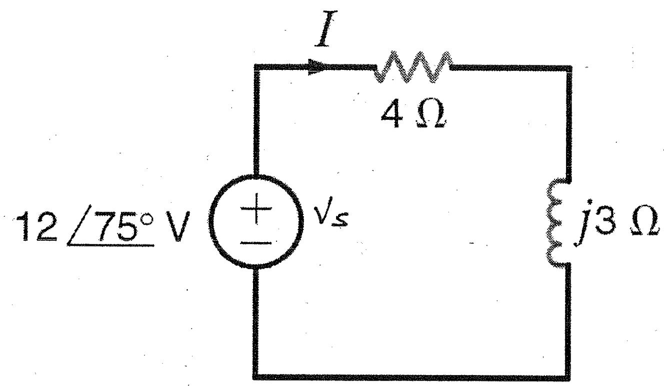


Figure P9.1

SOLUTION:

$$I = \frac{12\angle 75^\circ}{4+j3}$$

$$\boxed{\begin{array}{l} I = 2.4\angle 38.1^\circ \text{ A} \\ i(t) = 2.4\cos(\omega t + 38.1^\circ) \text{ A} \end{array}}$$

$$p(t) = i(t)\,v_s(t) \qquad v_s(t) = 12\cos(\omega t + 75^\circ) \text{ V}$$

$$p(t) = 28.8\left[\cos(\omega t + 38.1^\circ)\cos(\omega t + 75^\circ)\right]$$

$$\boxed{p(t) = 14.4\cos(2\omega t + 113.1^\circ) + 11.5 \quad \text{W}}$$

9.2 Determine the equations for the voltage and the instantaneous power in the network in Fig. P9.2.

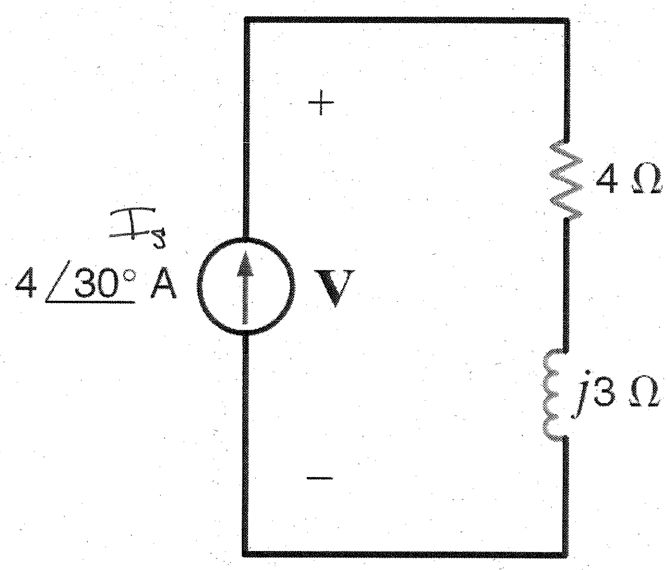


Figure P9.2

SOLUTION:

$V = 4\angle 30 \, (4+j3) = 20\angle 66.9° \text{V}$

$\boxed{\begin{array}{l} V = 20\angle 66.9° \text{V} \\ v(t) = 20 \cos(\omega t + 66.9°) \text{V} \end{array}}$

$i_s(t) = 4\cos(\omega t + 30°)$ A

$p(t) = i_s(t)\,v(t) = 80\cos(\omega t + 30°)\cos(\omega t + 66.9°)$ W

$\boxed{p(t) = 40\cos(2\omega t + 96.9°) + 32.0 \text{ W}}$

9.3 The voltage and current at the input of a circuit are given by the expressions

$$v(t) = 170 \cos(\omega t + 30°) \text{ V}$$
$$i(t) = 5 \cos(\omega t + 45°) \text{ A}$$

Determine the average power absorbed by the circuit.

SOLUTION:

$$P = \frac{V_M I_M}{2} \cos(\theta_v - \theta_i) = \frac{170(5)}{2} [\cos(30 - 45)]$$

$$\boxed{P = 411 \text{ W}}$$

9.4 The voltage and current at the input of a network are given by the expressions

$$v(t) = 6 \cos \omega t \text{ V}$$

$$i(t) = 4 \sin \omega t \text{ A}$$

Determine the average power absorbed by the network.

SOLUTION:

$$P = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i)$$

$$i(t) = 4 \cos(\omega t - 90°) \Rightarrow \theta_i = -90°$$

$$\boxed{P = 0\text{W}}$$

9.5 Find the average power absorbed by the resistor in the circuit shown in Fig. P9.5 if $v_1(t) = 10\cos(377t + 60°)$ V and $v_2(t) = 20\cos(377t + 120°)$ V.

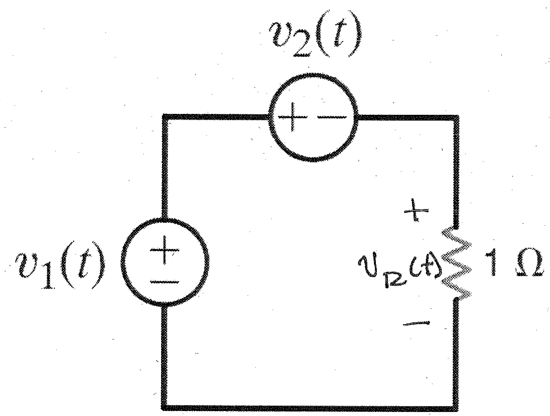


Figure P9.5

SOLUTION:

$V_1 = 10\angle 60°$ V $\qquad V_2 = 20\angle 120°$ V

$V_R = V_1 - V_2 = 17.3\angle -30°$ V

$P_R = \dfrac{V_m^2}{2R} \Rightarrow \boxed{P = 150\text{W}}$

9.6 Find the average power absorbed by the resistor in the circuit shown in Fig. P9.6. Let $i_1(t) = 4\cos(377t + 60°)$ A, $i_2(t) = 6\cos(754t + 10°)$ A, and $i_3(t) = 4\cos(377t - 30°)$ A.

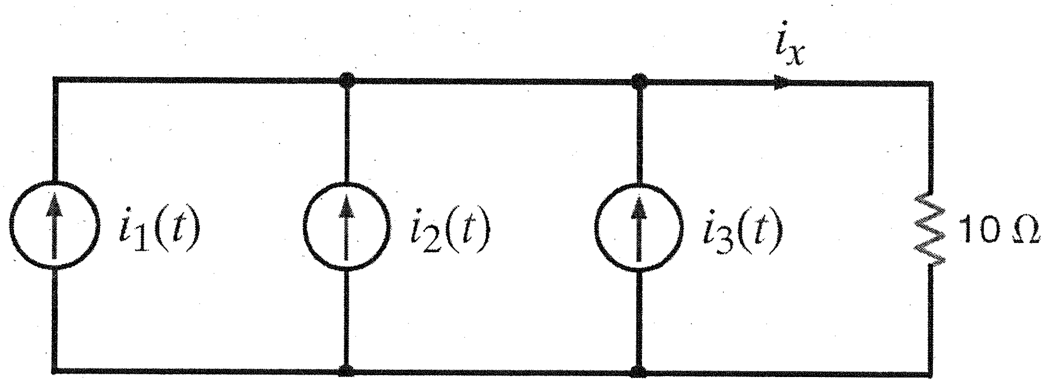


Figure P9.6

SOLUTION:

$$I_x = I_1 + I_2 + I_3 = 4\angle 60° + 6\angle 10° + 4\angle -30° = 11.64\angle 12.4°\ A$$

$$P = \frac{I_M^2}{2} R \qquad \boxed{P = 678\ W}$$

9.7 Compute the average power absorbed by each of the elements to the right of the dashed line in the circuit shown in Fig. P9.7.

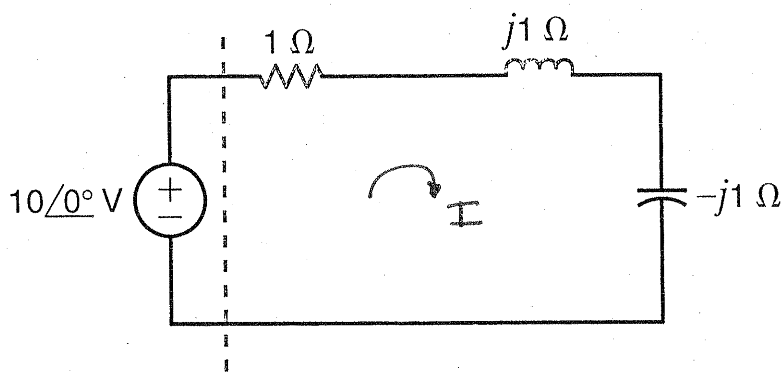


Figure P9.7

SOLUTION

The inductor & capacitor consume zero average power. As for the resistor,

$$P_R = \frac{|I|^2}{2} R$$

and

$$I = \frac{10\angle 0°}{1 + j1 - j1} = 10\angle 0° \text{ A}$$

$$P_R = \frac{(10)^2}{2}(1) = 50\text{W}$$

$$\boxed{\begin{aligned} P_R &= 50\text{ W} \\ P_L &= 0\text{W} \\ P_C &= 0\text{W} \end{aligned}}$$

9.8 Determine the average power supplied by each source in the network shown in Fig. P9.8. CS

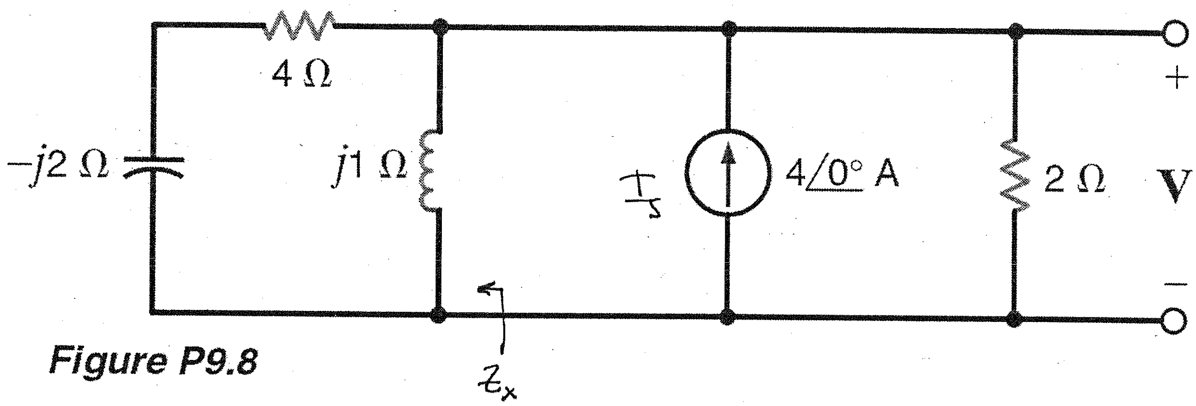
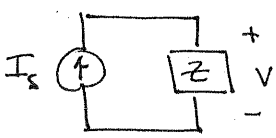


Figure P9.8

SOLUTION:



$Z = \dfrac{2Z_x}{2+Z_x}$ $\qquad Z_x = \dfrac{j1\,(4-j2)}{4-j1} = 1.09\,\angle 77.5^\circ\ \Omega$

$Z = 0.539 + j\,0.692\ \Omega = R_{eq} + j\,X_{eq}$

Only the resistive part of Z consumes average power.

$P = \dfrac{I_M^{\,2}}{2} R_{eq}$ $\qquad P = \dfrac{(4)^2}{2}(0.539)$ $\qquad \boxed{P = 4.31\,W}$

9.9 Find the average power absorbed by the network shown in Fig. P9.9.

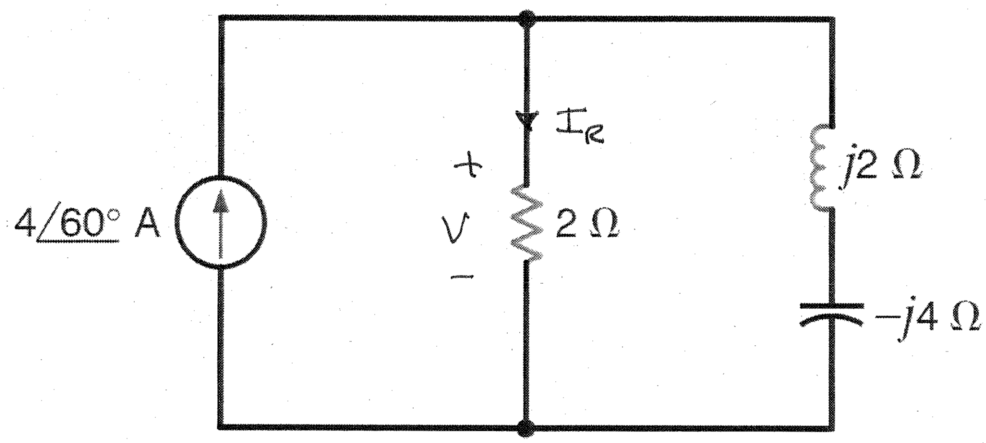


Figure P9.9

SOLUTION:

$$I_R = 4\angle 60° \left[\frac{-j2}{2-j2}\right] = 2.83\angle 15° \text{ A}$$

$$V = 2I_R = 5.66\angle 15° \text{ V}$$

$$P = \frac{V_m I_{RM}}{2} \qquad \boxed{P = 8 \text{ W}}$$

9.10 Given the network in Fig. P9.10, find the power supplied and the average power absorbed by each element. **PSV**

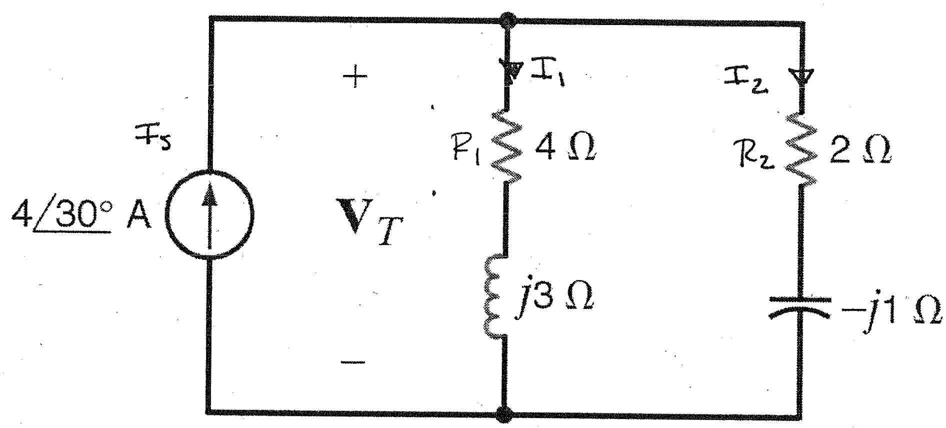


Figure P9.10

SOLUTION:

Let $Z_1 = 4+j3\Omega$ and $Z_2 = 2-j1\Omega$ & $Z_T = \dfrac{Z_1Z_2}{Z_1+Z_2} = 1.75-j0.25\Omega$

$V_T = 4\angle 30°\, Z_T = 7.07\angle 21.9°$ V

for the source: $p_S(t) = \dfrac{4(7.07)}{2}\left[\cos(2\omega t + 68.13°) + \cos 8.13°\right]$

$$\boxed{p_s(t) = 14.14\cos(2\omega t + 51.9°) + 14.0 \text{ W}}$$

$I_1 = V_T/Z_1 = 1.41\angle -15°$ A $\quad I_2 = V_T/Z_2 = 3.16\angle 48.4°$ A

$P_{R1} = \dfrac{I_{1M}^2}{2} R_1 = 4.0$ W $\quad P_{R2} = \dfrac{I_{2M}^2}{2} R_2 = 10$ W

$P_{R1} = 4.0$W	$P_L = 0$
$P_{R2} = 10$ W	$P_C = 0$

9.11 Given the network in Fig. P9.11, determine which elements are supplying power, which ones are absorbing power, and how much power is being supplied and absorbed.

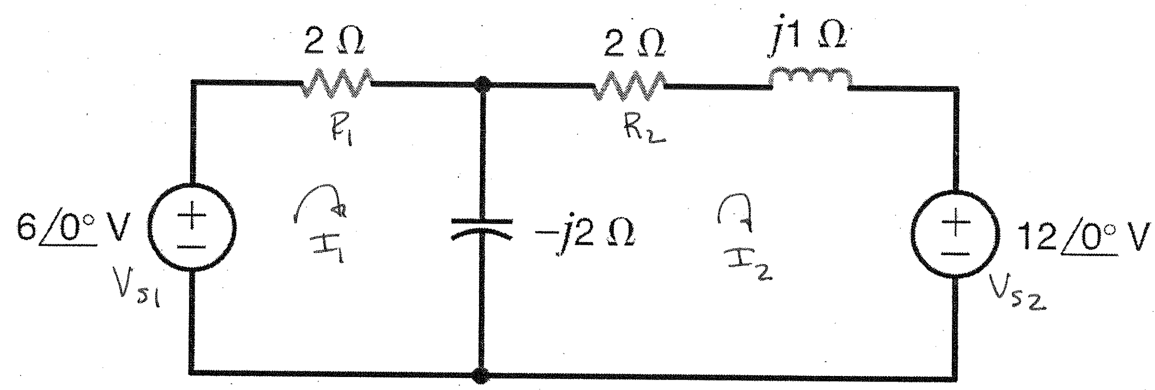


Figure P9.11

SOLUTION:

$6 = I_1(2-j2) + j2I_2$ & $-12 = j2I_1 + I_2(2-j1)$

$$\begin{bmatrix} 2-j2 & j2 \\ j2 & 2-j1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 6 \\ -12 \end{bmatrix} \Rightarrow \begin{cases} I_1 = 2.55\angle 101° \text{ A} \\ I_2 = 3.16\angle -161° \text{ A} \end{cases}$$

$P_{R_1} = \dfrac{I_{1M}^2}{2} R_1 = 6.5$ W absorbed $\qquad P_L = 0$

$P_{R2} = \dfrac{I_{2M}^2}{2} R_2 = 10$ W absorbed $\qquad P_C = 0$

$P_{S1} = \dfrac{V_{S1M} I_{1M}}{2} \cos(\theta_{V_{S1}} - \theta_{I_1}) = -1.46$ W supplied

So, V_{S1} absorbs 1.46W (V_{S1} & I_1 have active sign format)

$P_{S2} = \dfrac{V_{S2M} I_{2M}}{2} \cos(\theta_{V_{S2}} - \theta_{I_2}) = -17.96$ W

So, V_{S2} delivers 17.96W (V_{S2} & I_2 have passive sign format)

9.12 Given the network in Fig. P9.12, show that the power supplied by the sources is equal to the power absorbed by the passive elements.

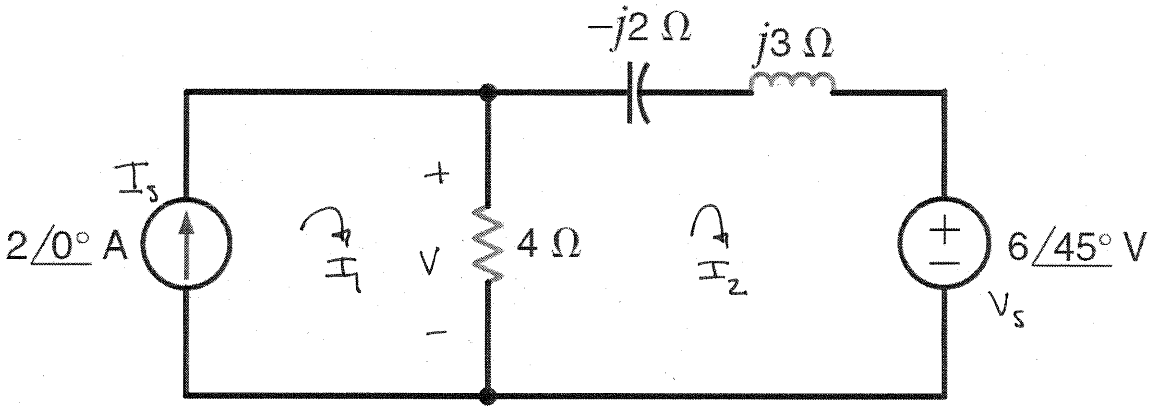


Figure P9.12

SOLUTION:

$I_1 = 2\angle 0°$ A $\quad -4I_1 + I_2(4+j1) = -6\angle 45 \quad \begin{bmatrix} 1 & 0 \\ -4 & 4+j1 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 2 \\ -6\angle 45° \end{bmatrix}$

$I_2 = 1.37\angle -62.5°$ A $\quad V = 4(I_1 - I_2) = 7.32\angle 41.8°$ V

I_s supplies $\quad P_{I_s} = \frac{I_{SM} V_M}{2} \cos(\theta_V - \theta_{I_S}) \Rightarrow P_{I_s} = 5.46$ W

V_s absorbs $\quad P_{V_s} = \frac{V_{SM} I_{2M}}{2} \cos(\theta_{V_S} - \theta_{I_2}) \Rightarrow P_{V_s} = -1.24$ W

So, V_s actually delivers 1.24 W

$P_R = \frac{V_m^2}{2R} = 6.70$ W $\qquad P_L = P_C = 0$ W

$\left.\begin{array}{l} \text{Power supplied} = 5.46 + 1.24 = 6.70\text{ W} \\ \text{Power absorbed} = P_R = 6.70\text{ W} \end{array}\right\}$ balance!

9.13 Calculate the average power absorbed by the 1-Ω resistor in the network shown in Fig. P9.13.

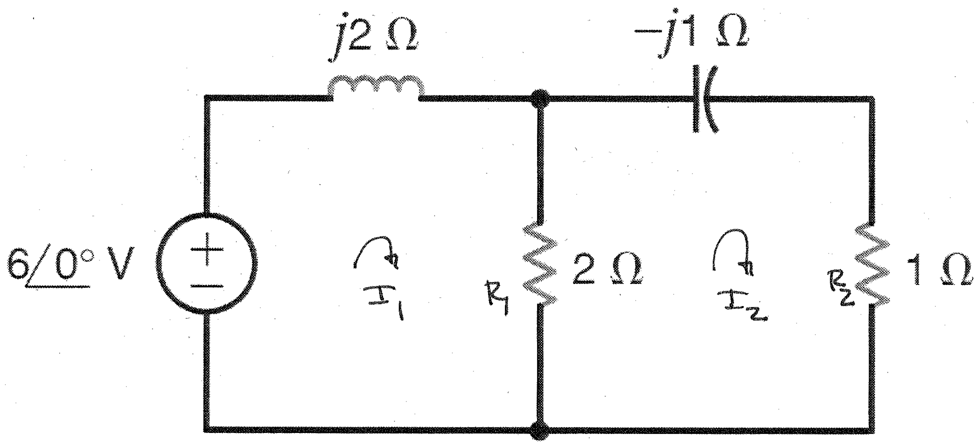


Figure P9.13

SOLUTION:

$6\angle 0 = I_1(2+j2) - 2I_2$ & $-2I_1 + I_2(3-j1) = 0$

$\begin{bmatrix} 2+j2 & -2 \\ -2 & 3-j1 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \end{bmatrix} \Rightarrow I_2 = 2.12\angle -45° \text{ A}$

$P_{1\Omega} = \dfrac{I_{2M}^2}{2}(R_2)$ $\boxed{P_{1\Omega} = 2.25 \text{W}}$

9.14 Given the network in Fig. P9.14, find the average power supplied to the circuit. CS

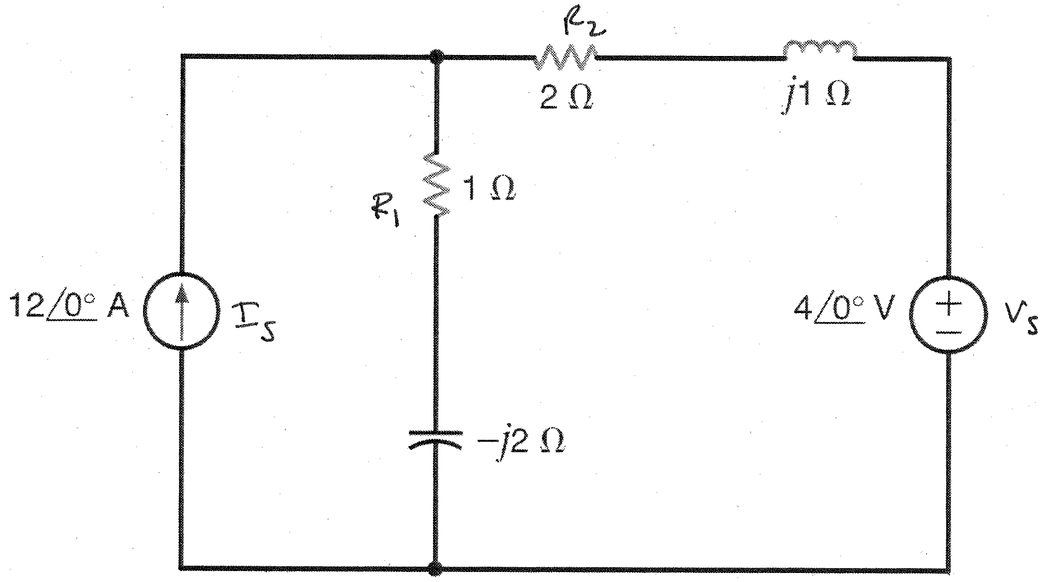
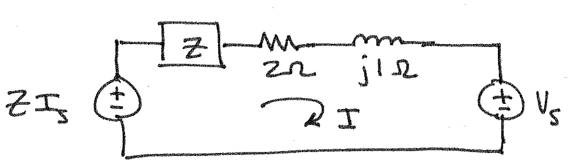


Figure P9.14

SOLUTION:

Let $Z = 1 - j2\Omega$ & use source transformation.



$ZI_S - V_S = I(Z + 2 + j1)$

$I = 8\angle{-53.1°}$ A

$P_{R_1} = \frac{I_M^2}{2} R_1 = 32W$

$P_{R_2} = \frac{I_M^2}{2} R_2 = 64W$

$P_L = P_C = 0$

$P_{\text{supplied to circuit}} = P_{R_1} + P_{R_2} = 96W$

9.15 Given $v_S(t) = 100 \cos 100t$ volts, find the average power supplied by the source and the current $i_2(t)$ in the network in Fig. P9.15.

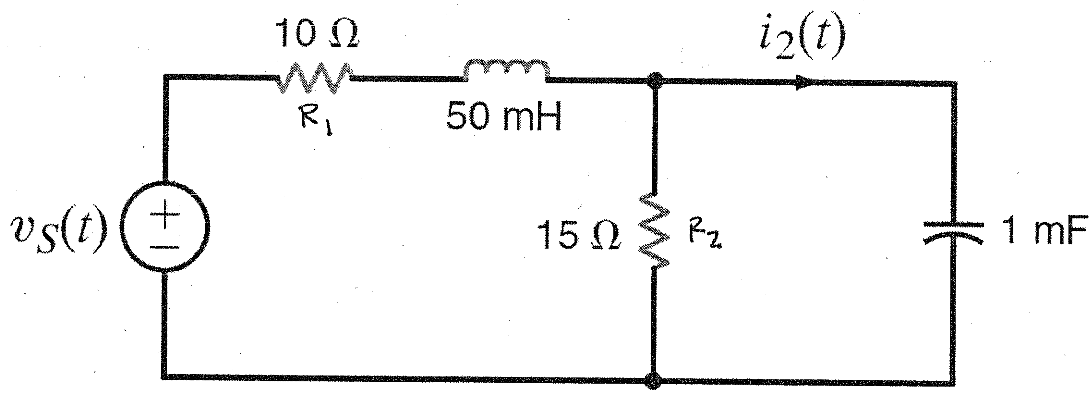
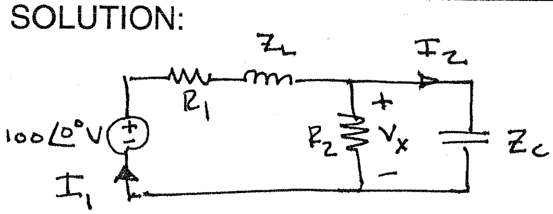


Figure P9.15

SOLUTION:



$Z_L = j5\,\Omega \qquad Z_C = -j10\,\Omega$

Let $Z_x = \dfrac{R_2 Z_C}{R_2 + Z_C} = \dfrac{-j30}{3 - j2}$

$V_x = \dfrac{100\angle 0 \; Z_x}{R_1 + Z_L + Z_x} \Rightarrow V_x = 56.4 \angle -48.8° \text{ V}$

$I_2 = V_x / Z_C = 5.64 \angle 41.2° \text{ A}$ $\quad \boxed{i_2(t) = 5.64 \cos(100t + 41.2°) \text{ A}}$

Let $Z_y = R_1 + Z_L + Z_x = 14.7 \angle -7.49° \,\Omega$

$I_1 = V_S / Z_y = 6.78 \angle 7.49° \text{ A}$

$P_{V_S} = \dfrac{V_{SM} I_{1M}}{2} \cos(0 - 7.49°)$ $\quad \boxed{P_{V_S} = 336 \text{ W}}$

9.16 If $i_g(t) = 0.5 \cos 2000t$ A, find the average power absorbed by each element in the circuit in Fig. P9.16.

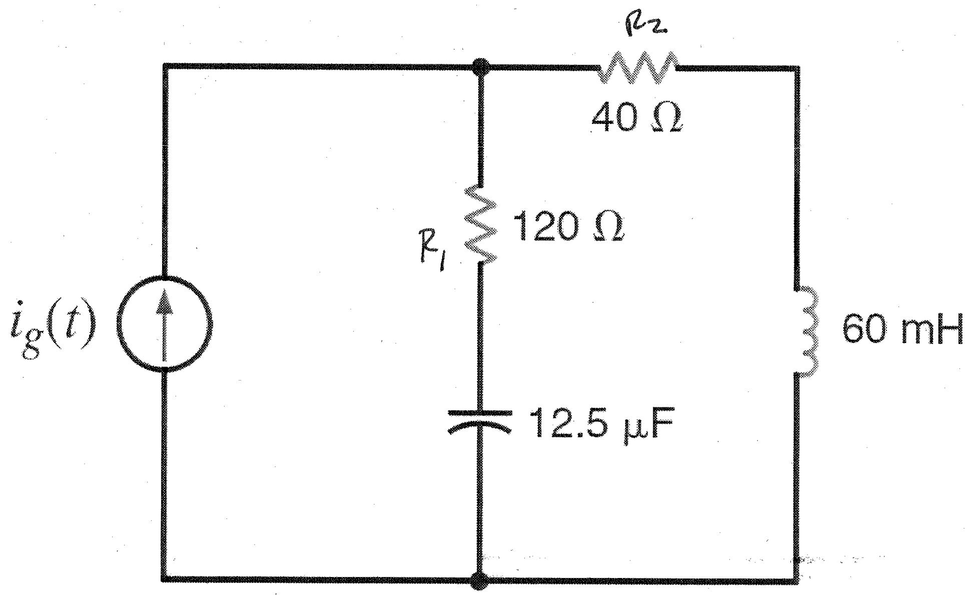
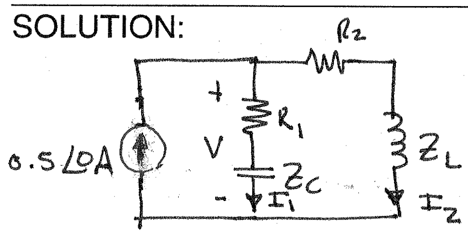


Figure P9.16

SOLUTION:



$Z_L = j120\,\Omega \qquad Z_C = -j40\,\Omega$

Let $Z_1 = R_1 + Z_C = 120 - j40\,\Omega$

$Z_2 = R_2 + Z_L = 40 + j120\,\Omega$

$Z_3 = Z_1 Z_2/(Z_1 + Z_2) = 80 + j40\,\Omega$

$V = (0.5\angle 0^\circ) Z_3$

$V = 44.7 \angle 26.6^\circ$ V

$I_1 = V/Z_1 = 0.354 \angle 45^\circ$ A $\qquad I_2 = V/Z_2 = 0.354 \angle -45^\circ$ A

$P_{R1} = \dfrac{I_{1m}^2}{2} R_1 = 7.5\text{W} \qquad P_{R2} = \dfrac{I_{2m}^2}{2} R_2 = 2.5\text{W}$

$P_{I_S} = -\dfrac{I_{sm} V_m}{2} \cos(0 - 26.6^\circ) = -10.0$ W

$P_L = P_C = 0$ W

9.17 Calculate the average power absorbed by the 1-Ω resistor in the network shown in Fig. P9.17.

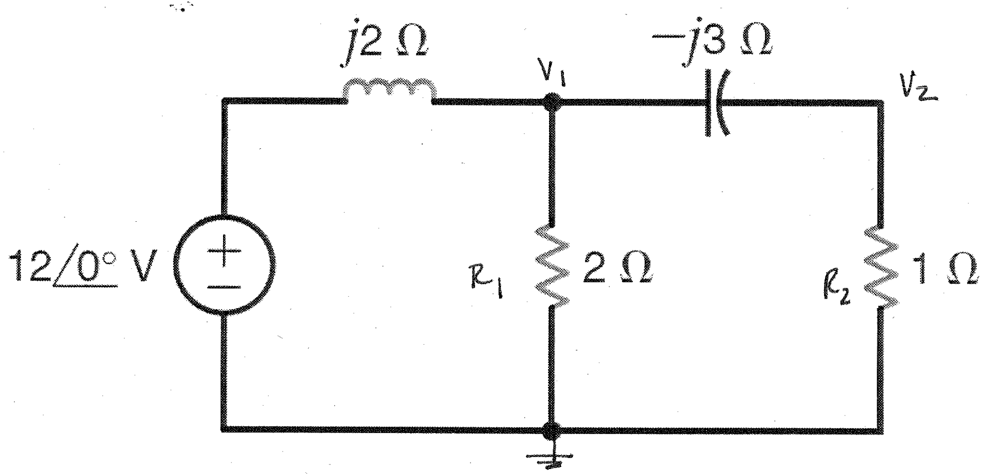


Figure P9.17

SOLUTION:

$$\frac{V_1 - 12}{j2} + \frac{V_1}{2} + \frac{V_1}{1-j3} = 0 \Rightarrow V_1 = 3 - j9 \text{ V} = 9.49\angle -71.6^\circ \text{ V}$$

$$V_2 = \frac{V_1 (1)}{1-j3} = 3\angle 0^\circ \text{ V}$$

$$P_{1\Omega} = \frac{V_{2m}^2}{2R_2}$$

$$\boxed{P_{1\Omega} = 4.5\text{W}}$$

9.18 Find the average power supplied and/or absorbed by each element in Fig. P9.18.

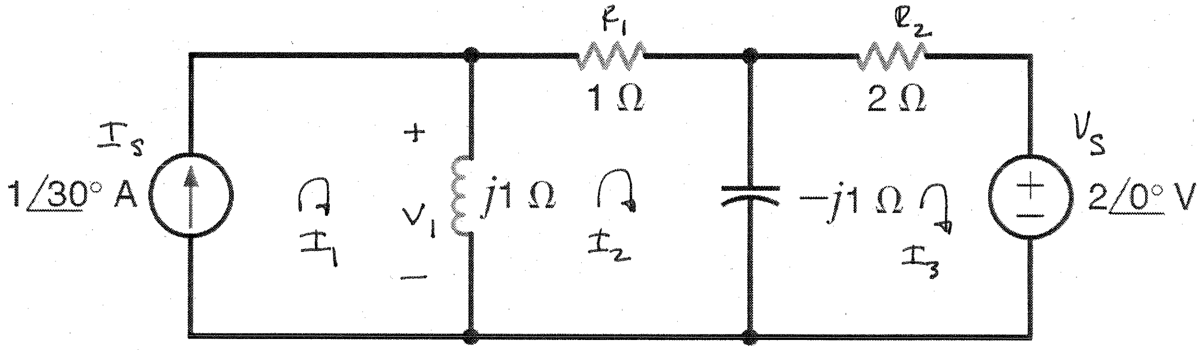


Figure P9.18

SOLUTION:

$I_1 = 1\angle 30^\circ$ A $\qquad -j1 I_1 + I_2(1) + j1 I_3 = 0 \qquad j1 I_2 + I_3(2 - j1) = -2\angle 0^\circ$

$$\begin{bmatrix} 1 & 0 & 0 \\ -j1 & 1 & j1 \\ 0 & j1 & 2-j1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 1\angle 30^\circ \\ 0 \\ -2\angle 0^\circ \end{bmatrix} \Rightarrow \begin{array}{l} I_1 = 1.0\angle 30^\circ \text{ A} \\ I_2 = 1.34\angle 110^\circ \text{ A} \\ I_3 = 0.392\angle 174^\circ \text{ A} \end{array}$$

$P_L = P_C = 0 \qquad P_{R_1} = \frac{I_{2m}^2}{2} R_1 = 0.90\text{ W} \qquad P_{R_2} = \frac{I_{3m}^2}{2} R_2 = 0.154\text{ W}$

$V_1 = j1(I_1 - I_2) \Rightarrow V_1 = 1.53\angle 60.4^\circ$ V

$P_{I_s} = \frac{V_{1m} I_{sm}}{2} \cos(60.4 - 30) = 0.660$ W supplied

$P_{V_s} = -\frac{V_{sm} I_{3m}}{2} \cos(0 - 174^\circ) = 0.394$W supplied

power supplied = power absorbed

9.19 Determine the average power absorbed by the 4-Ω in the network shown in Fig. P9.19. **PSV**

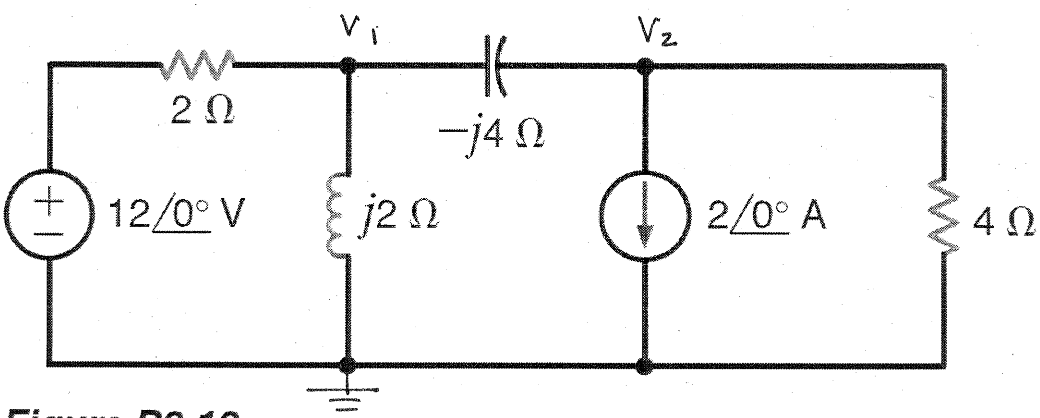


Figure P9.19

SOLUTION:

$$\frac{V_1 - 12\angle 0}{2} + \frac{V_1}{j2} + \frac{V_1 - V_2}{-j4} = 0 \Rightarrow V_1(2 - j1) - jV_2 = 24$$

$$\frac{V_2 - V_1}{-j4} + 2\angle 0^\circ + \frac{V_2}{4} = 0 \Rightarrow -jV_1 + V_2(1 + j1) = -8$$

$$\begin{bmatrix} 2-j1 & -j1 \\ -j1 & 1+j1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 24 \\ -8 \end{bmatrix} \qquad V_2 = 8.68 \angle 102.6^\circ \text{ V}$$

$$P_{4\Omega} = \frac{V_{2M}^2}{2(4)} \qquad \boxed{P_{4\Omega} = 9.42 \text{ W}}$$

9.20 Given the network in Fig. P9.20, find the total average power supplied and the average power absorbed in the 4-Ω resistor.

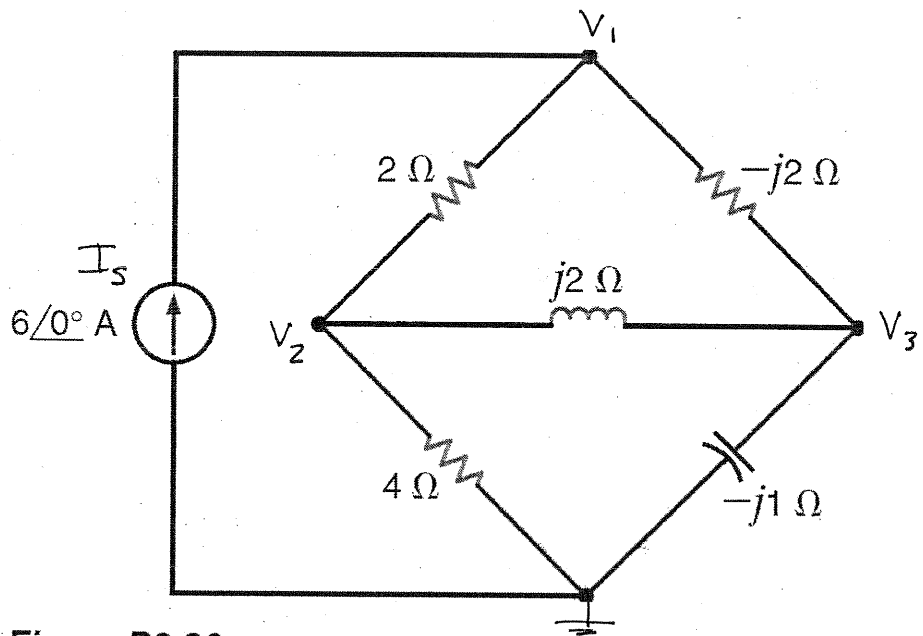


Figure P9.20

SOLUTION:

$$\frac{V_1 - V_2}{2} + \frac{V_1 - V_3}{-j2} = 6\angle 0^\circ \Rightarrow V_1(1 + j1) - V_2 - jV_3 = 12$$

$$\frac{V_2 - V_1}{2} + \frac{V_2 - V_3}{j2} + \frac{V_2}{4} = 0 \Rightarrow -2V_1 + V_2(3 - j2) + j2V_3 = 0$$

$$\frac{V_3 - V_1}{-j2} + \frac{V_3 - V_2}{j2} + \frac{V_3}{-j1} = 0 \Rightarrow -V_1 + V_2 + 2V_3 = 0$$

$$\begin{bmatrix} 1+j1 & -1 & -j1 \\ -2 & 3-j2 & j2 \\ -1 & 1 & 2 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{array}{l} V_1 = 16.7\angle -54.4\ V \\ V_2 = 8.82\angle -36.0^\circ\ V \end{array}$$

$$P_{I_s} = \frac{I_{sm}V_{1m}}{2}\cos(-51.1 - 0) \Rightarrow \boxed{P_{I_s} = 29.2\,W}$$

$$P_{4\Omega} = \frac{V_{2m}^2}{2(4)} \Rightarrow \boxed{P_{4\Omega} = 9.73\,W}$$

9.21 Determine the average power supplied to the network in Fig. P9.21.

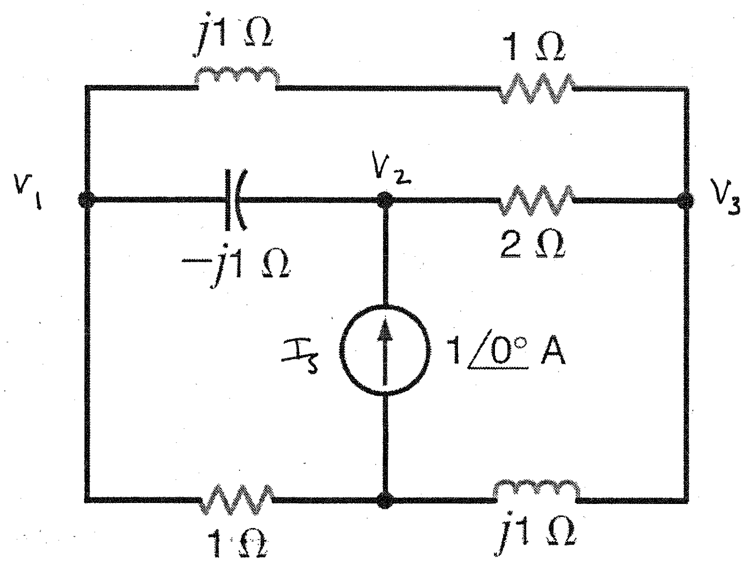


Figure P9.21

SOLUTION:

$$\frac{V_1 - V_2}{-j1} + \frac{V_1 - V_3}{1+j1} + \frac{V_1}{1} = 0 \Rightarrow V_1(3+j1) - j2V_2 + V_3(-1+j1) = 0$$

$$\frac{V_2 - V_1}{-j1} + \frac{V_2 - V_3}{2} = 1\angle 0^\circ \Rightarrow -j2V_1 + V_2(1+j2) - V_3 = 2\angle 0$$

$$\frac{V_1}{1} + \frac{V_3}{j1} = 1\angle 0 \Rightarrow V_1 - jV_3 = 1\angle 0$$

$$\begin{bmatrix} 3+j1 & -j2 & -1+j1 \\ -j2 & 1+j2 & -1 \\ 1 & 0 & -j1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} \Rightarrow V_2 = 1.01\angle -17.7^\circ \ V$$

$$P_{I_s} = \frac{I_{sm} V_{2m}}{2} \cos(-40.4 - 0) \qquad \boxed{P_{I_s} = 481 \text{ mW}}$$

9.22 Find the average power absorbed by the 2-Ω resistor in the circuit shown in Fig. P9.22.

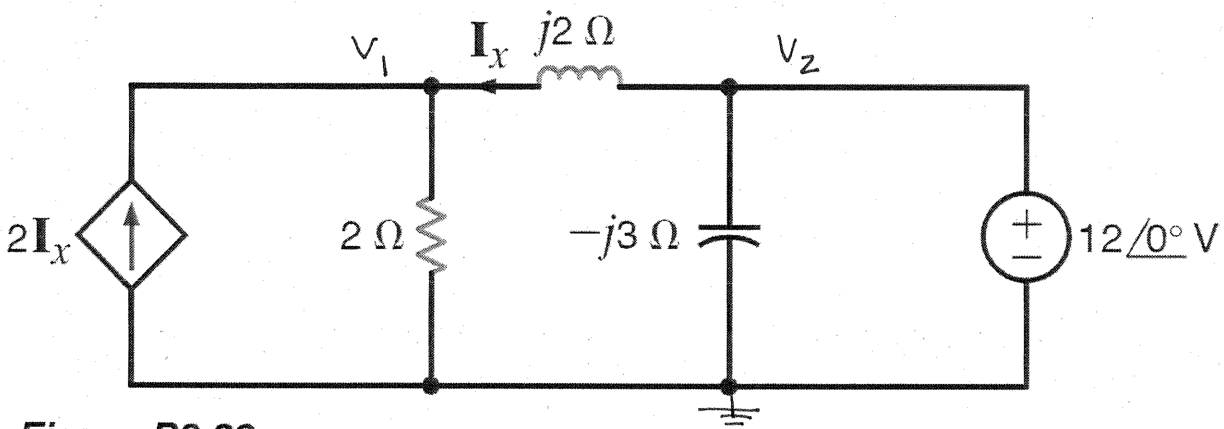


Figure P9.22

SOLUTION:

$$I_x + 2I_x = \frac{V_1}{2} \qquad I_x = \frac{V_2 - V_1}{j2} \quad \Rightarrow \quad V_1(3+j1) - 3V_2 = 0$$

$$V_2 = 12\angle 0^\circ \quad \Rightarrow \quad V_1 = 11.4\angle -18.4^\circ \text{ V}$$

$$P_{2\Omega} = \frac{V_{1m}^2}{2(2)} \qquad \boxed{P_{2\Omega} = 32.5 \text{ W}}$$

9.23 Determine the average power absorbed by a 2-Ω resistor connected at the output terminals of the network shown in Fig. P9.23.

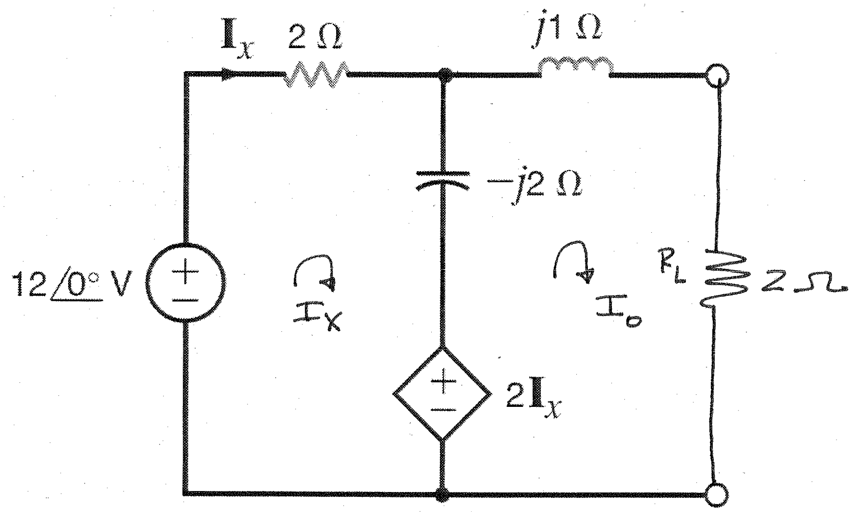


Figure P9.23

SOLUTION:

$12 = I_x(2-j2) + 2I_x + j2I_o \Rightarrow \quad 12\angle 0 = I_x(4-j2) + j2I_o$

$2I_x = j2I_x + I_o(2-j1) \Rightarrow \quad 0 = I_x(-2+j2) + I_o(2-j1)$

$$\begin{bmatrix} 4-j2 & j2 \\ -2+j2 & 2-j1 \end{bmatrix}\begin{bmatrix} I_x \\ I_o \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \end{bmatrix}$$

$I_o = 3.15\ \angle -23.2°\ A$

$P_{R_L} = \dfrac{I_{oM}^2}{2} R_L$

$\boxed{P_{R_L} = 9.92\ W}$

9.24 Determine the average power absorbed by the 2-kΩ output resistor in Fig. P9.24.

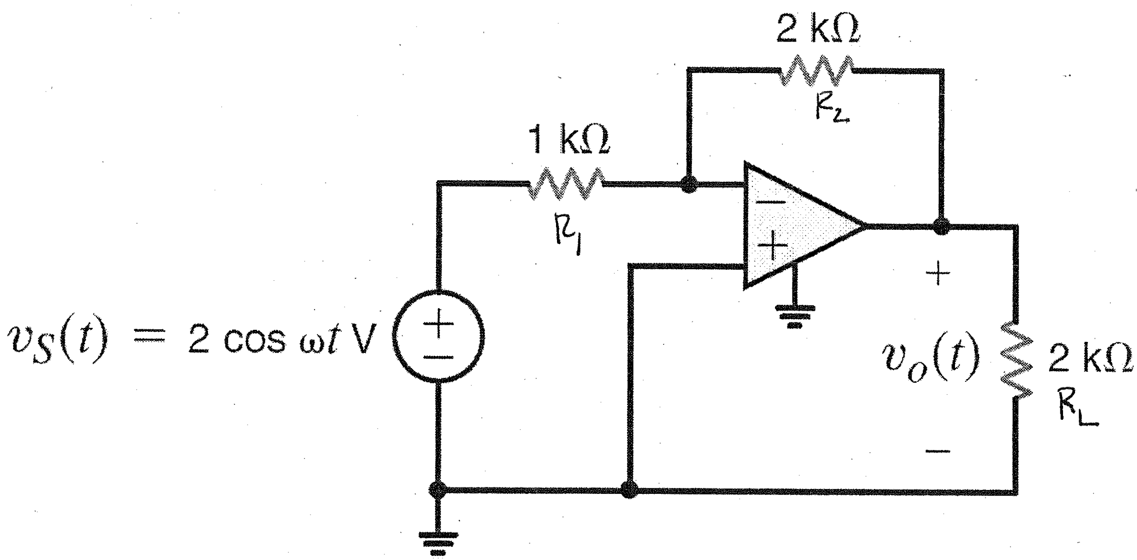


Figure P9.24

SOLUTION:

$\frac{v_o(t)}{v_S(t)} = -\frac{R_2}{R_1} = -2$ $\quad v_o(t) = -4\cos(\omega t) = 4\cos(\omega t + 180°)$ V

$V_o = 4 \angle 180°$ V $\quad P_{R_L} = \frac{V_{om}^2}{2R_L}$ $\quad \boxed{P_{R_L} = 4\,\text{mW}}$

9.25 Determine the average power absorbed by the 4-kΩ resistor in Fig. P9.25.

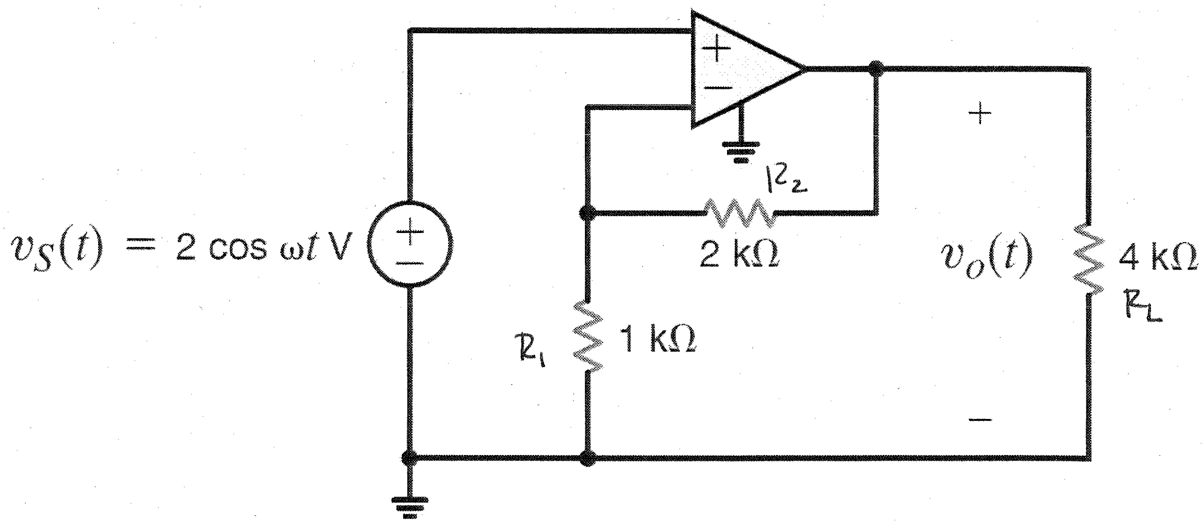


Figure P9.25

SOLUTION:

$V_S = 2\angle 0°$ V $\qquad V_o = V_S\left(1 + R_2/R_1\right) = 6\angle 0°$ V

$$P_{R_L} = \frac{V_{om}^2}{2R_L} \qquad \boxed{P_{R_L} = 4.5\,\text{mW}}$$

9.26 Determine the impedance $\mathbf{Z}_L$ for maximum average power transfer and the value of the maximum power transferred to $\mathbf{Z}_L$ for the circuit shown in Fig. P9.26.

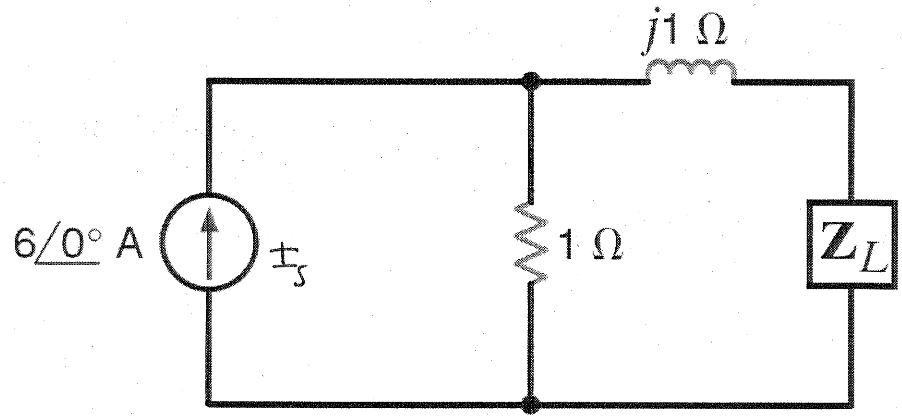
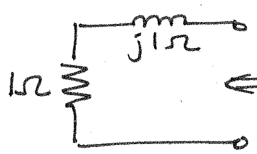


Figure P9.26

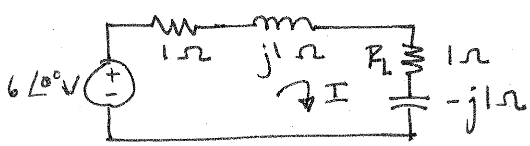
SOLUTION:



$\Leftarrow Z_{TH} = 1 + j1\,\Omega$

for max. power transfer, $Z_L = Z_{TH}^* = 1 - j1$

$$\boxed{Z_L = 1 - j1\,\Omega}$$



$$I = \frac{6\angle 0}{2} = 3\angle 0^\circ$$

$$P_{MAX} = \frac{I_M^2}{2} R_L$$

$$\boxed{P_{MAX} = 4.5\,W}$$

9.27 Determine the impedance $\mathbf{Z}_L$ for maximum average power transfer and the value of the maximum average power transferred to $\mathbf{Z}_L$ for the circuit shown in Fig. P9.27. CS

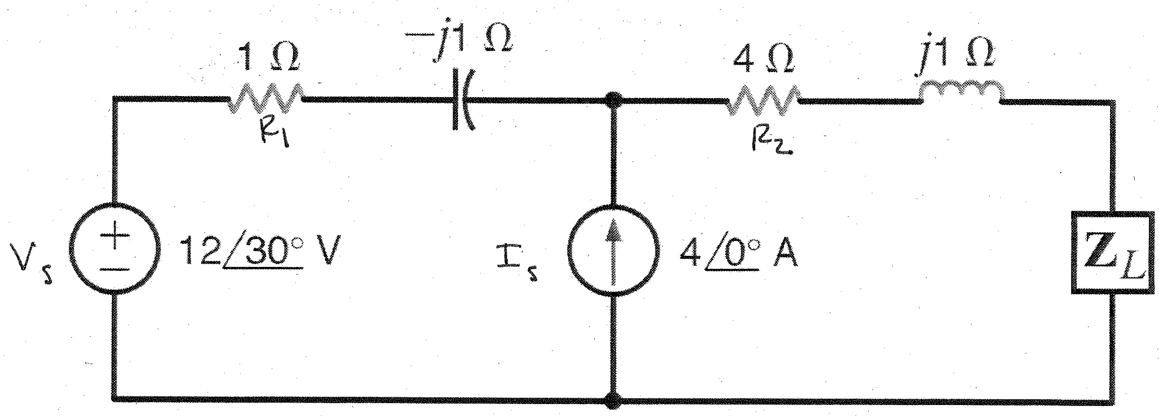
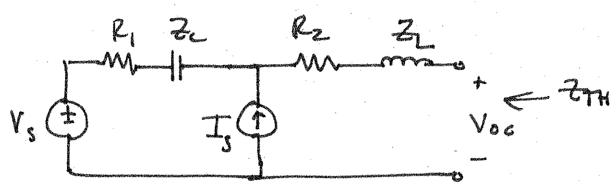


Figure P9.27

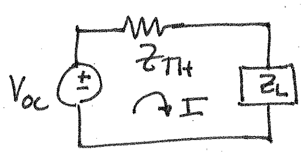
SOLUTION:

Thevenin eq



Superposition: $V_{OC} = V_S + I_S(R_1 + Z_C) \Rightarrow V_{OC} = 14.5\angle 7.91°\ V$

$Z_{TH} = R_1 + Z_C + R_2 + Z_L = 5\ \Omega$



Z_L is purely resistive, so for max. power transfer,

$R_L = |Z_{TH}| = 5\ \Omega$

$$I = \frac{V_{OC}}{Z_{TH} + Z_L} = 1.45\angle 7.91°\ A \qquad P_L = \frac{1}{2} I_m^2 R_L$$

$$\boxed{P_L = 5.26\ W}$$

9.28 Determine the impedance $\mathbf{Z}_L$ for maximum average power transfer and the value of the maximum average power absorbed by the load in the network shown in Fig. P9.28.

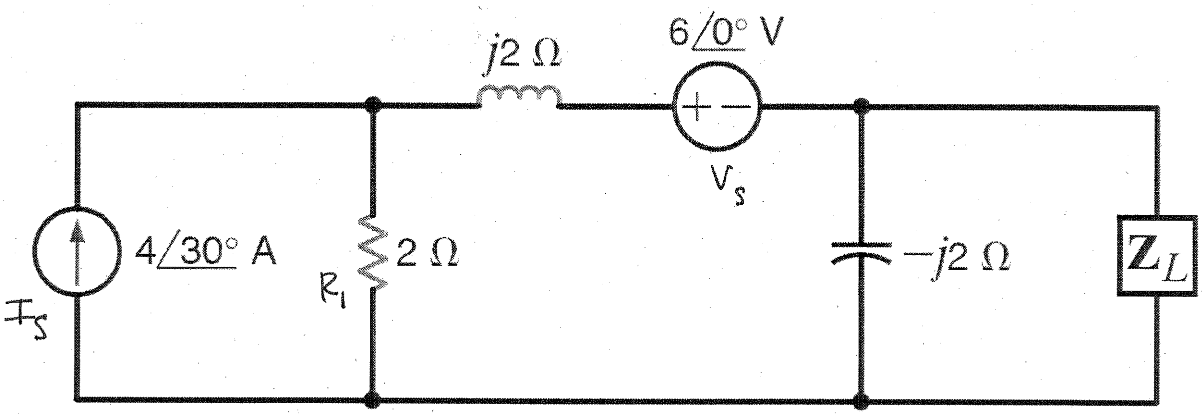
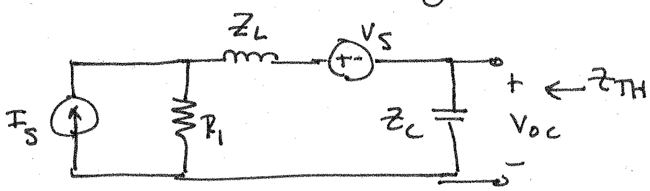


Figure P9.28

SOLUTION: Thevenin eq:

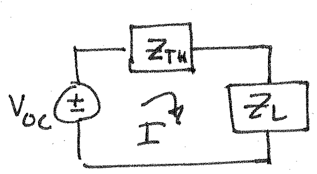


$$Z_{TH} = \frac{(R_1 + Z_L)\, Z_C}{R_1 + Z_L + Z_C}$$

$$Z_{TH} = 2 - j2\ \Omega$$

Superposition:

$$V_{OC} = \frac{I_S R_1 Z_C}{R_1 + Z_L + Z_C} - \frac{V_S Z_C}{R_1 + Z_L + Z_C} = 4.10\angle{-13.1^\circ}\ \text{V}$$



for max power transfer, $Z_L = Z_{TH}^* = 2 + j2\,\Omega$

$$\boxed{Z_L = 2 + j2\,\Omega}$$

$$P_L = \frac{1}{2} I_M^2 R_L \qquad I = \frac{V_{OC}}{Z_{TH} + Z_L} = 1.03\angle{-13.1^\circ}\ \text{A}$$

$$\boxed{P_L = 1.05\ \text{W}}$$

9.29 Determine the impedance $\mathbf{Z}_L$ for maximum average power transfer and the value of the maximum average power absorbed by the load in the network shown in Fig. P9.29.

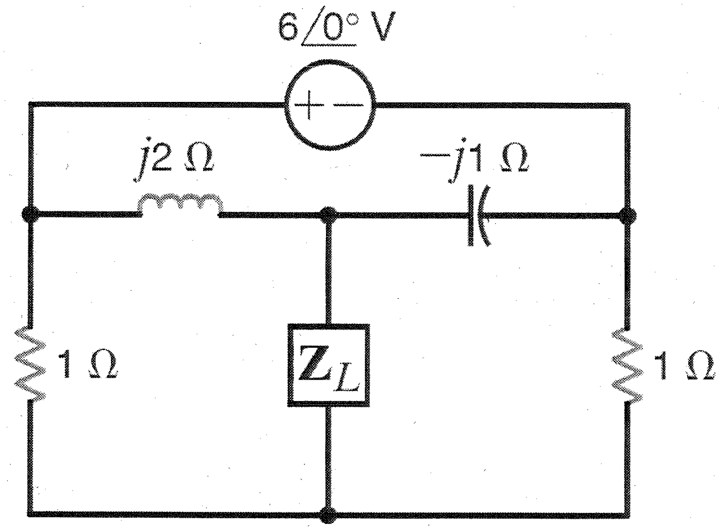
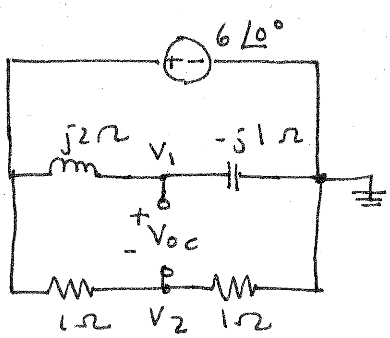


Figure P9.29

SOLUTION: Thevenin eq.



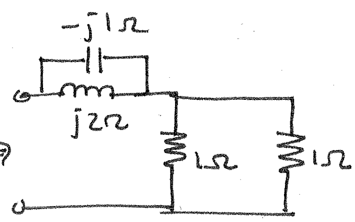
$$V_1 = 6\angle 0 \left[\frac{-j1}{j2-j1}\right] = -6\angle 0^\circ \text{ V}$$

$$V_2 = 6\angle 0\left[\frac{1}{1+1}\right] = 3\angle 0^\circ$$

$$V_{oc} = V_1 - V_2 = -9\angle 0^\circ = 9\angle 180^\circ \text{ V}$$

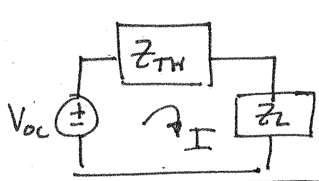
Z_{TH} redraw ⇒

Z_{TH} ⇒



$$Z_{TH} = \frac{j2(-j1)}{j2-j1} + \frac{1(1)}{1+1}$$

$$Z_{TH} = \frac{1}{2} - j2\,\Omega$$



$$P_L = \frac{I_m^2}{2} R_L \qquad I = \frac{V_{oc}}{Z_{TH}+Z_L} = 9\angle 180^\circ \text{A}$$

$$\boxed{Z_L = Z_{TH}^* = \frac{1}{2} + j2\,\Omega}$$

$$\boxed{P_L = 20.25 \text{ W}}$$

9.30 Repeat Problem 9.29 for the network in Fig. P9.30.

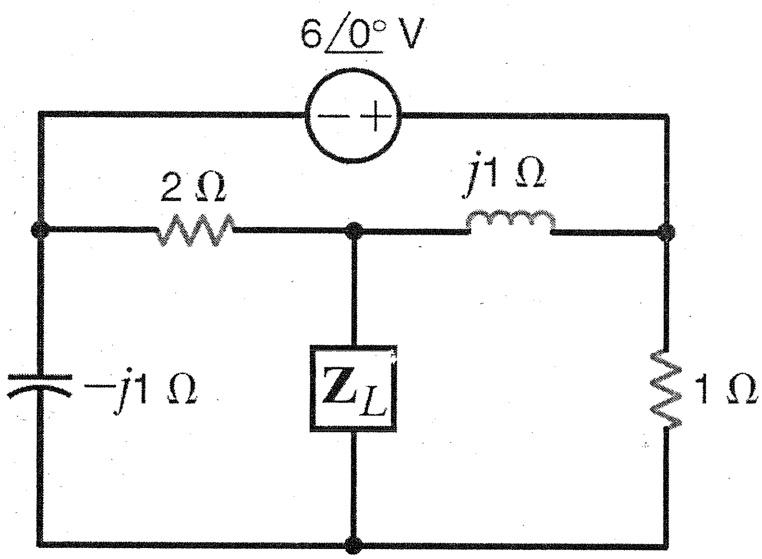
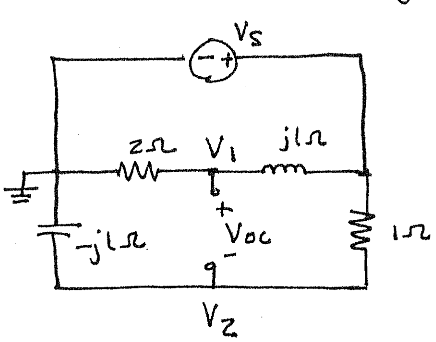


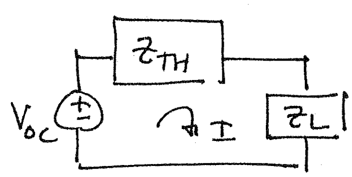
Figure P9.30

SOLUTION: Thevenin eq.



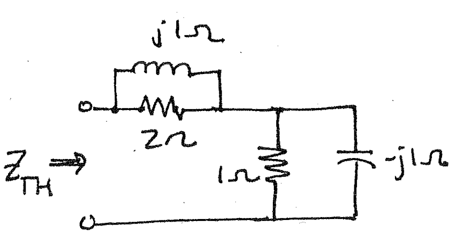
$$V_1 = V_S\left[\frac{2}{2+j1}\right] \qquad V_2 = V_S\left[\frac{-j1}{1-j1}\right]$$

$$V_{oc} = V_1 - V_2 = 1.90\,\angle 18.4^\circ\ V$$



$$I = \frac{V_{oc}}{Z_{TH}+Z_L} = 1.06\,\angle 18.4^\circ\ A$$

Redraw for Z_{TH}



$$P_L = \frac{I_M^2}{2} R_L$$

$$\boxed{P_L = 0.501\,W}$$

$$Z_{TH} = \frac{2(j1)}{2+j1} + \frac{1(-j1)}{1-j1}$$

$$Z_{TH} = 0.9 + j0.3\ \Omega$$

$$\boxed{Z_L = Z_{TH}^* = 0.9 + j\ldots\ \Omega}$$

9.31 Determine the impedance $\mathbf{Z}_L$ for maximum average power transfer and the value of the maximum average power transferred to $\mathbf{Z}_L$ for the circuit shown in Fig. P9.31. **PSV**

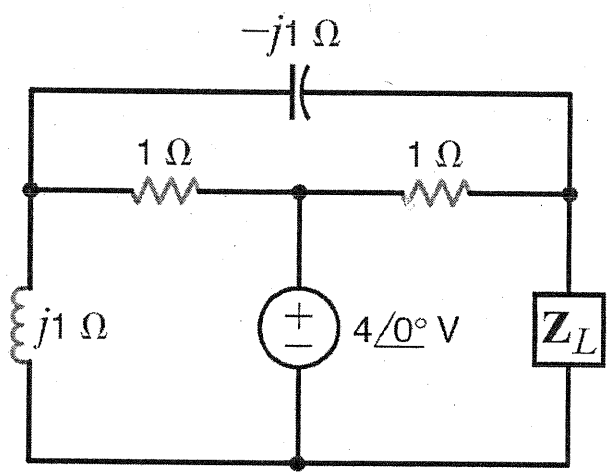
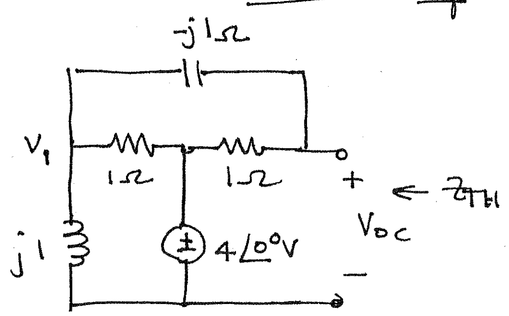


Figure P9.31

SOLUTION: Thevenin eq.



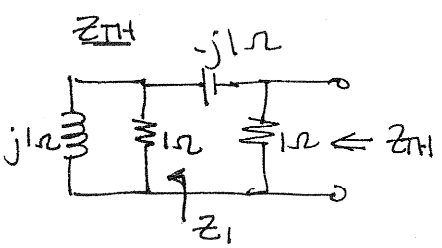
$$\frac{V_1 - V_{oc}}{-j1} + \frac{V_1}{j1} + \frac{V_1 - 4}{1} = 0$$

$$V_1 = 4 + (j1)\,V_{oc}$$

$$\frac{V_1 - V_{oc}}{-j1} = \frac{V_{oc} - 4}{1}$$

$$-j1\,V_1 + V_{oc}(1 + j1) = 4$$

yields $V_{oc} = 2.4 + j0.8\,V$



$$Z_L = Z_{TH}^{*} = 0.4 + j0.2\,\Omega$$

$$P_L = \frac{1}{2} I_m^2 R_L$$

$$z_1 = 1(j1)/(1+j) = j1/(1+j1)\,\Omega$$

$$I = V_{oc}/(Z_{TH} + Z_L) = 3 + j1\ A$$

$$Z_{TH} = 1[z_1 - j1]/\{1 + z_1 - j1\}$$

$$\boxed{P_L = 2\ W}$$

$$Z_{TH} = 0.4 - j\,0.2\,\Omega$$

9.32 In the network in Fig. P9.32, find $\mathbf{Z}_L$ for maximum average power transfer and the maximum average power transferred. CS

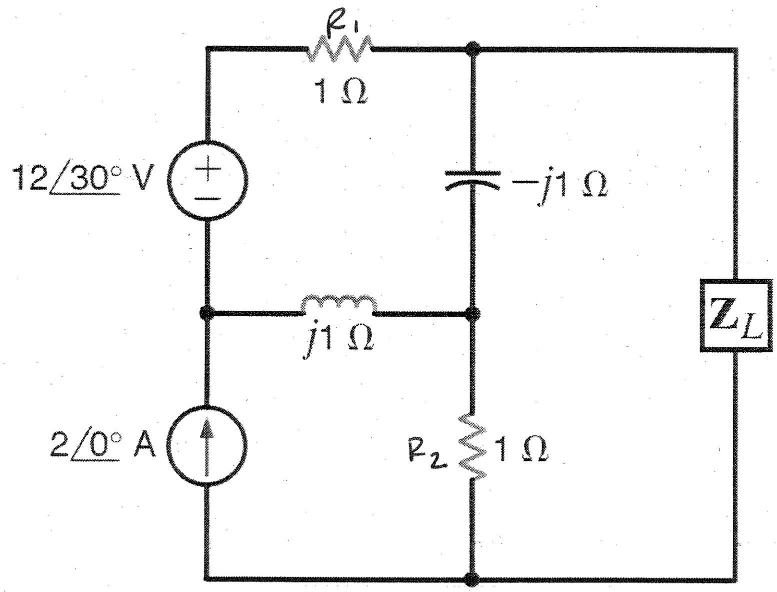
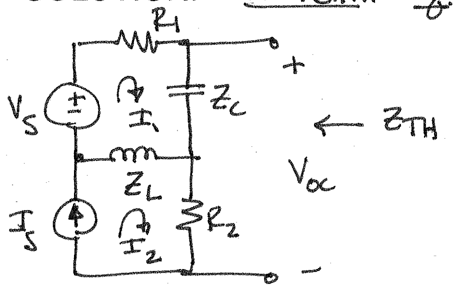


Figure P9.32

SOLUTION: Thevenin eq.



$V_S = I_1(1-j1+j1) - j1\,I_2 = 12\angle 30°$

$\Downarrow$

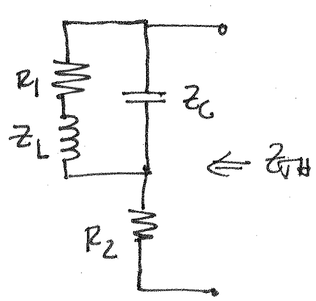
$I_1 - j1\,I_2 = 12\angle 30°$

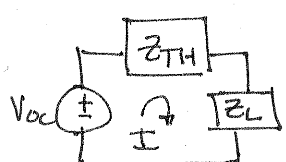
$I_2 = 2\angle 0°$

yields $\quad I_1 = 13.1\angle 37.6°\,A$

$V_{oc} = I_1(-j1) + I_2(1) = 13.1\angle -52.4°\,V$

Z_{TH}





$I = \dfrac{V_{oc}}{Z_{TH}+Z_L} = 3.28\angle -52.4°\,A$

$\boxed{Z_L = Z_{TH}^* = 2+j1\,\Omega}$

$P_L = \frac{1}{2} I_M^2 R_L$

$\boxed{P_L = 10.7\ W}$

$Z_1 = R_1 + Z_L = 1+j1\,\Omega$

$Z_{TH} = R_2 + Z_C \,//\, Z_1 = 2-j1\,\Omega$

9.33 Repeat Problem 9.32 for the network in Fig. P9.33.

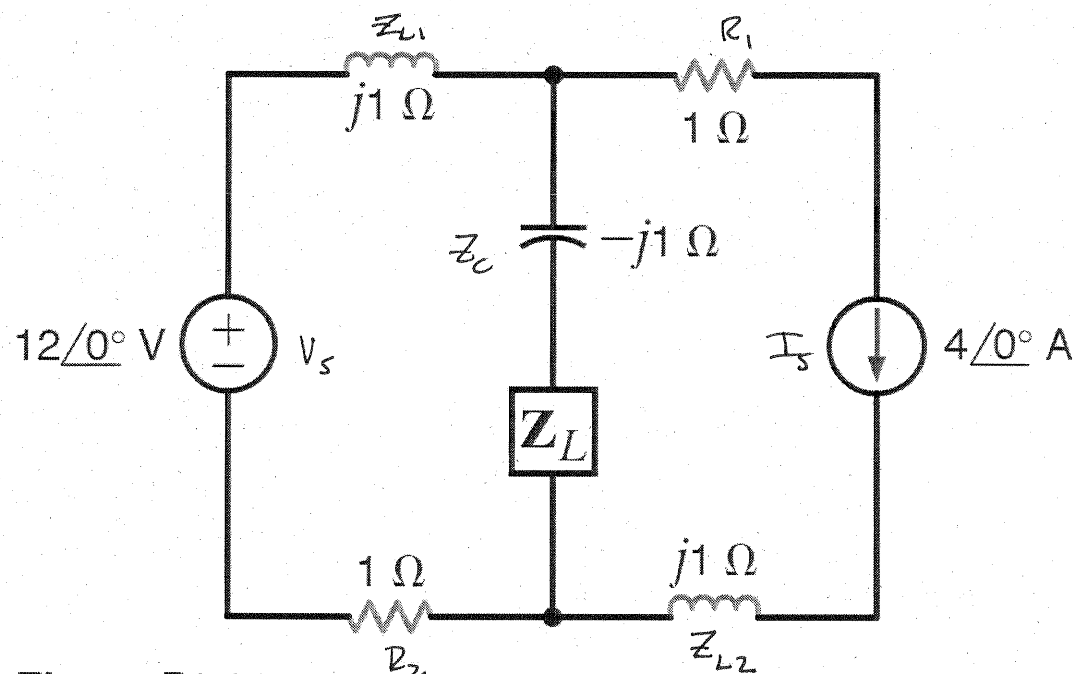
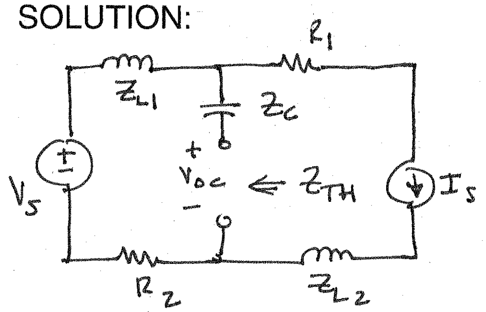


Figure P9.33

SOLUTION:



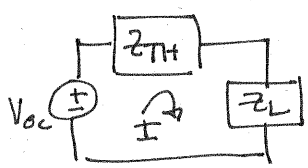
$$V_S = Z_{L1} I_S + V_{oc} + R_2 I_S$$

$$V_{oc} = 12\angle 0 - j1\,(4\angle 0) - 1\,(4\angle 0)$$

$$V_{oc} = 8 - j4 \text{ V}$$

Let $Z_1 = R_2 + Z_{L1} = 1 + j1\ \Omega$

$$Z_{TH} = Z_C + Z_1 = 1\ \Omega$$



$$I = \frac{V_{oc}}{Z_{TH} + Z_L} = \frac{8 - j4}{2} = 4 - j2 = 4.47\angle -26.6° \text{ A}$$

$$\boxed{Z_L = Z_{TH}^* = 1\ \Omega}$$

$$P_L = \frac{1}{2} I_M^2 R_L$$

$$\boxed{P_L = 10.0 \text{ W}}$$

9.34 Repeat Problem 9.32 for the network in Fig. P9.34. CS

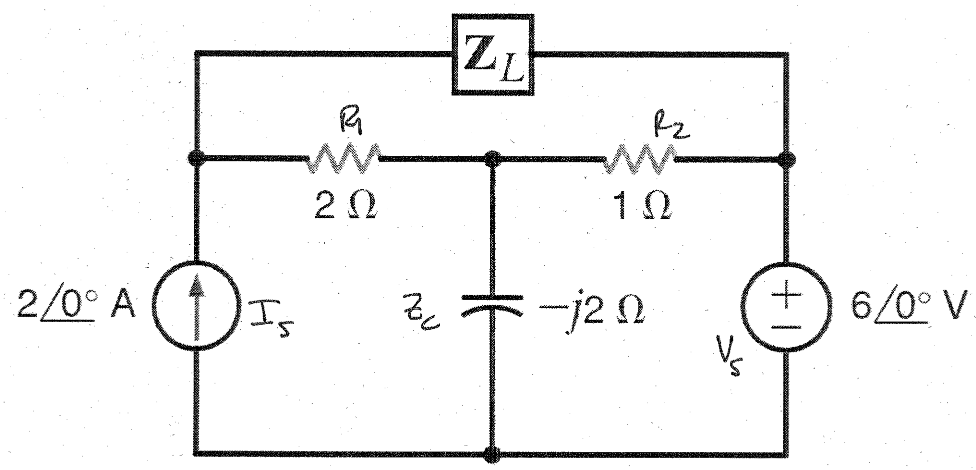
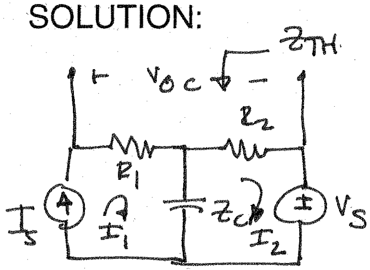


Figure P9.34

SOLUTION:

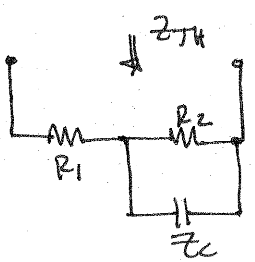


$I_1 = 2\angle 0°\text{ A}$

$j2 I_1 + I_2(1 - j2) = -6\angle 0°$

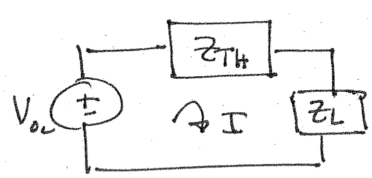
$I_2 = \dfrac{-6 - j4}{1 - j2} = 0.4 - j3.2\text{ A}$

$V_{oc} = I_1 R_1 + I_2 R_2 = 4.4 - j3.2\text{ V}$



$Z_{TH} = R_1 + \dfrac{(R_2 Z_c)}{R_2 + Z_c} = 2.8 - j0.4\,\Omega$

$Z_L = Z_{TH}^*$ $\boxed{Z_L = 2.8 + j0.4\,\Omega}$



$I = \dfrac{V_{oc}}{Z_{TH} + Z_L} = 0.972\angle -36.0°\text{ A}$

$P_L = \frac{1}{2} I_m^2 R_L$ $\boxed{P_L = 1.32\text{ W}}$

9.35 Repeat Problem 9.32 for the network in Fig. P9.35.

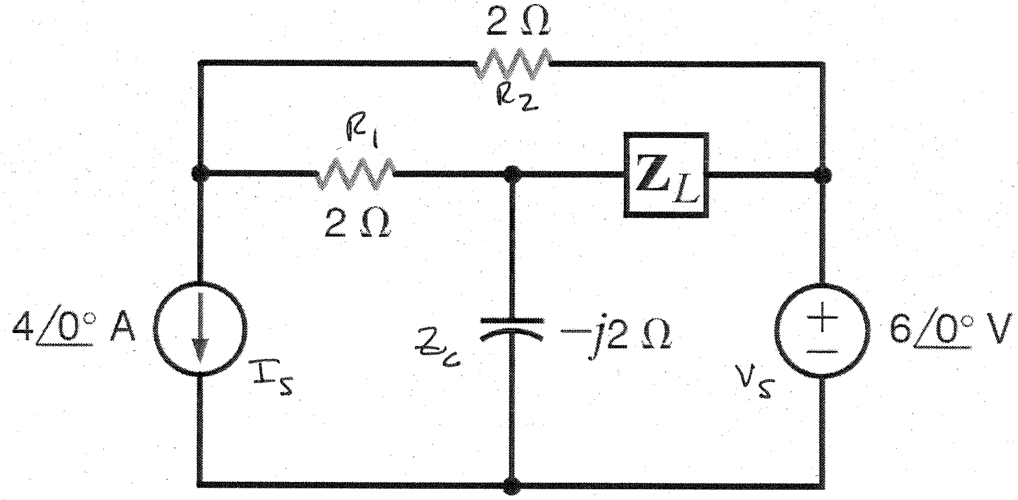
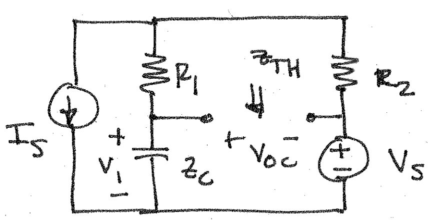


Figure P9.35

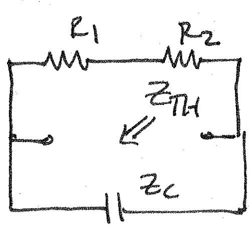
SOLUTION:



$V_{oc} = V_1 - V_s = V_1 - 6\angle 0°$

Use superposition to find V_1

$$V_1 = V_S\left\{\frac{Z_C}{R_1+R_2+Z_C}\right\} - \frac{I_S R_2 Z_C}{R_1+R_2+Z_C}$$

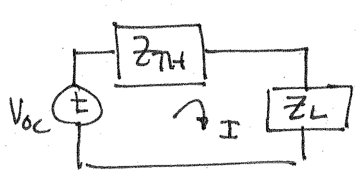


$V_1 = -0.4 + j0.8\text{ V}$

$V_{oc} = -6.4 + j0.8\text{ V}$

$Z_{TH} = Z_C(R_1+R_2) / [R_1+R_2+Z_C]$

$Z_{TH} = 0.8 - j1.6\,\Omega$



$$\boxed{Z_L = Z_{TH}^* = 0.8 + j1.6\,\Omega}$$

$$I = \frac{V_{oc}}{Z_{TH}+Z_L} = 4.03\angle 173°\text{ A}$$

$P_L = \frac{1}{2} I_M^2 R_L$ $\quad \boxed{P_L = 6.50\text{W}}$

9.36 Repeat Problem 9.32 for the network in Fig. P9.36.

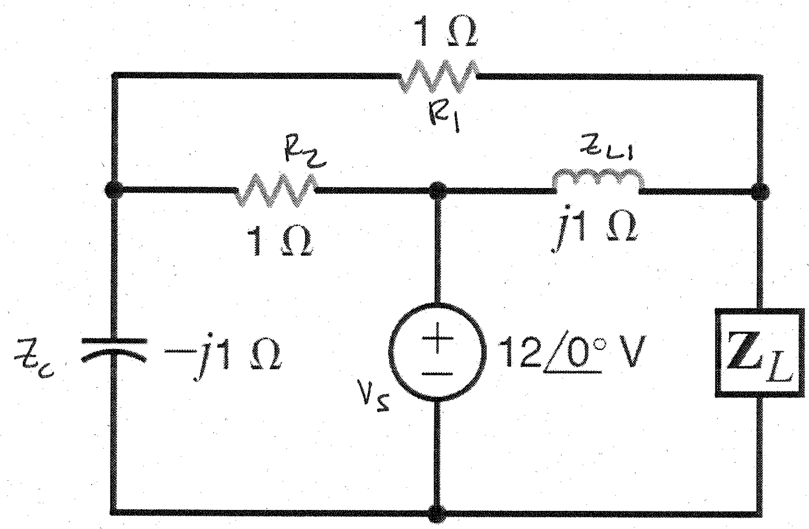
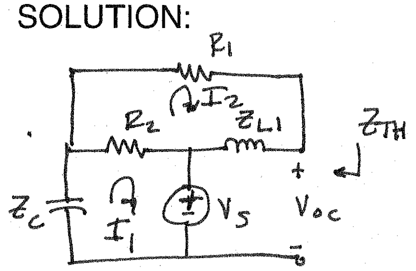


Figure P9.36

SOLUTION:

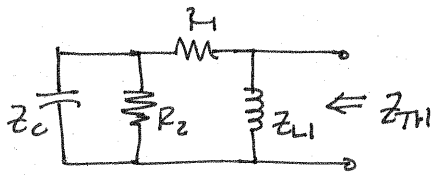


$$I_1(1-j1) - I_2 = -12\angle 0$$

$$-I_1 + I_2(2+j1) = 0$$

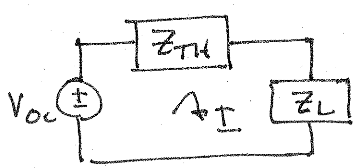
$$\begin{bmatrix} 1-j1 & -1 \\ -1 & 2+j1 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} -12 \\ 0 \end{bmatrix}$$

$$I_2 = -4.8 - j2.4\text{A}$$



$$V_{oc} = I_2(j1) + V_s = 14.4 - j4.8\text{V}$$

Let $Z_1 = \dfrac{R_2 Z_C}{R_2 + Z_C} = 0.5 - j0.5$ $\qquad Z_{TH} = \dfrac{Z_{L1}[R_1 + Z_1]}{Z_{L1} + R_1 + Z_1} = 0.6 + j0.8\,\Omega$



$$\boxed{Z_L = Z_{TH}^* = 0.6 - j0.8\,\Omega}$$

$$I = \frac{V_{oc}}{Z_{TH} + Z_L} = 12.6\angle -18.4^\circ\text{ A}$$

$$P_L = \tfrac{1}{2} I_m^2 R_L \qquad \boxed{P_L = 47.6\text{W}}$$

9.37 Determine the impedance $\mathbf{Z}_L$ for maximum average power transfer and the value of the maximum average power absorbed by the load in the network shown in Fig. P9.37. **CS**

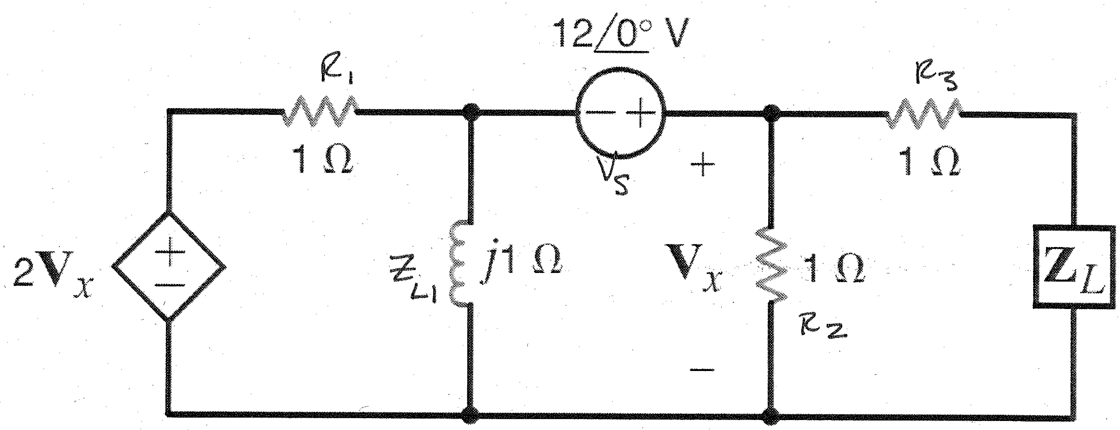
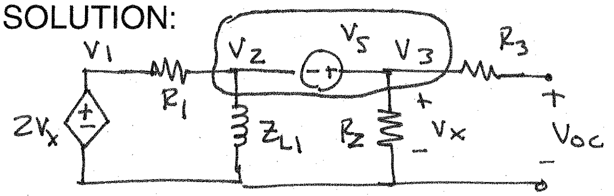


Figure P9.37

SOLUTION:



$V_1 = 2V_x \qquad V_x = V_3 = V_{oc}$

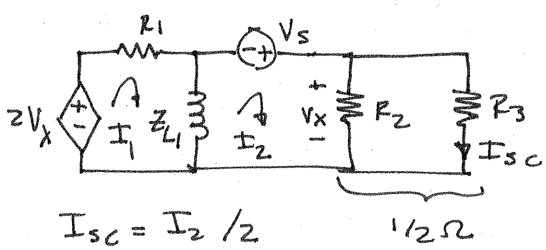
$\hookrightarrow V_1 = 2V_{oc}$

$V_3 - V_2 = V_s = 12\angle 0$

@ supernode: $\dfrac{V_2 - V_1}{R_1} + \dfrac{V_2}{Z_{L1}} + \dfrac{V_3}{R_2} = 0$

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & -1 & 1 \\ -1 & 1-j1 & 1 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 12 \\ 0 \end{bmatrix}$$

$V_3 = V_{oc} = 12 + j12\text{ V}$



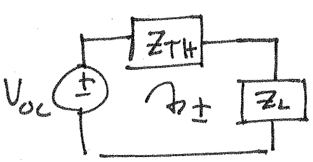
$I_{sc} = I_2/2$

$V_x = I_2/2$

$2V_x = I_1(1+j1) - jI_2 \Rightarrow I_1 = I_2$

$V_s = \dfrac{I_2}{2} + I_2(j1) - I_1(j1) = 12\angle 0°$

$I_2 = 24\angle 0°\text{ A} \qquad I_{sc} = 12\angle 0°\text{ A}$



$Z_{TH} = V_{oc}/I_{sc}$

$Z_{TH} = 1 + j1\,\Omega \qquad \boxed{Z_L = Z_{TH}^* = 1 - j1\,\Omega}$

$I = V_{oc}/(Z_{TH} + Z_L) = 8.49\angle 45°\text{ A}$

$P_L = \frac{1}{2} I_m^2 R_L$

9.38 Find the impedance $\mathbf{Z}_L$ for maximum average power transfer and the value of the maximum average power transferred to $\mathbf{Z}_L$ for the circuit shown in Fig. P9.38.

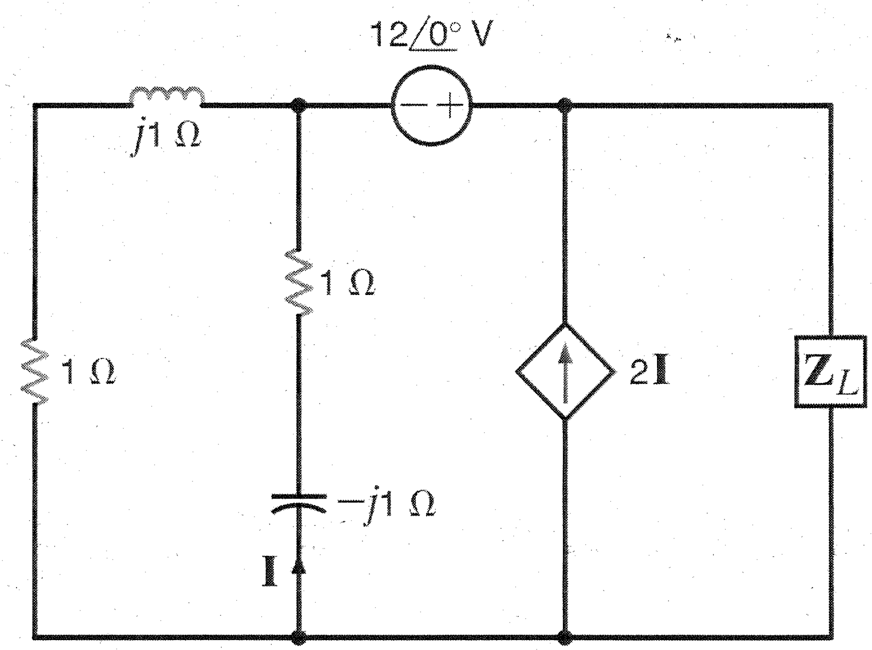
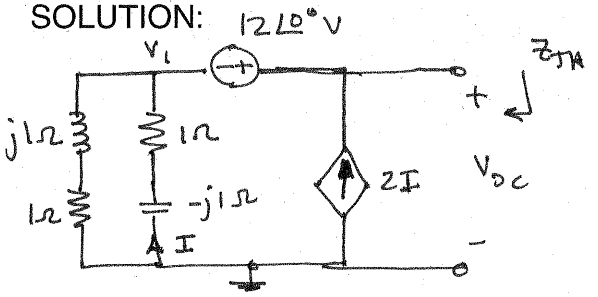


Figure P9.38

SOLUTION:

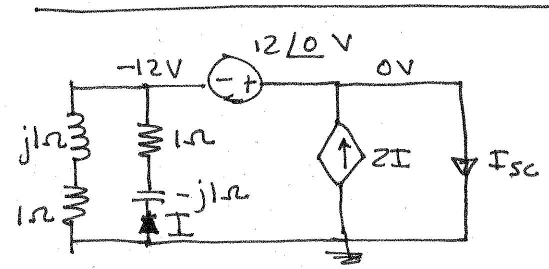


$V_{oc} - V_1 = 12\angle 0° \Rightarrow V_1 = V_{oc} - 12$

$\frac{V_1}{1+j1} + \frac{V_1}{1-j1} = 2I \qquad I = \frac{-V_1}{1-j1}$

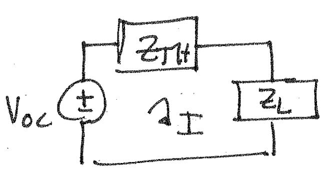
yields $V_1 = 0 \Rightarrow I = 0$

$V_{oc} = 12\angle 0°\ V$



$I_{sc} = 2I + I + \frac{12}{1+j1} \quad \& \quad I = \frac{12}{1-j1}$

$I_{sc} = 24 + j12\ A$



$Z_{TH} = V_{oc}/I_{sc} = 0.4 - j0.2\,\Omega \qquad \boxed{Z_L = Z_{TH}^* = 0.4 + j0.2\,\Omega}$

$I = V_{oc}/(Z_L + Z_{TH}) = 15\angle 0°\ A$

$P_L = \frac{1}{2} I_M^2 R_L \qquad \boxed{P_L = 45\ W}$

9.39 Repeat Problem 9.38 for the network in Fig. P9.39.
PSV

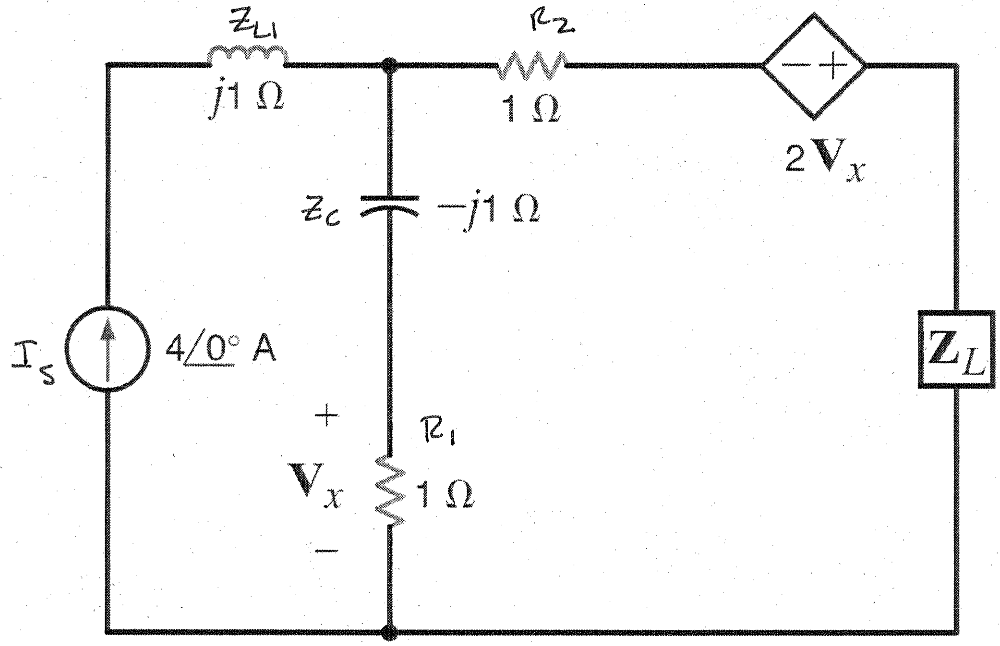
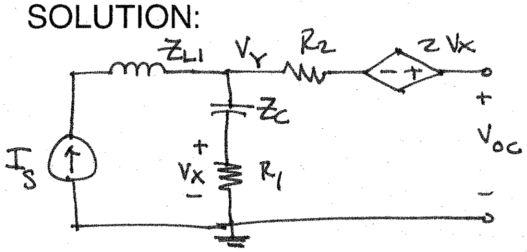


Figure P9.39

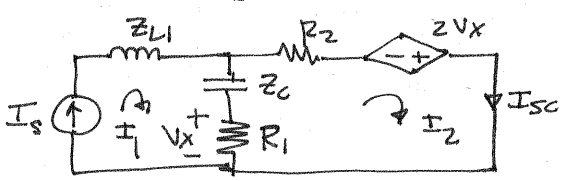
SOLUTION:



$V_Y = I_S\,(R_1 + Z_C) = 4 - j4\ V$

$V_X = I_S R_1 = 4\angle 0\ V$

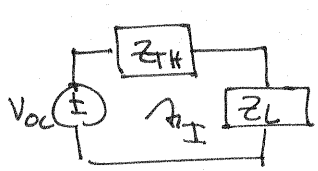
$V_{OC} = V_Y + 2V_X = 12 - j4\ V$



$I_1 = 4\angle 0°$ $\quad I_2 = -I_{SC}$ $\quad V_X = R_1\,(I_1 - I_2)$

$2V_X = I_2\,(2 - j1) - I_1\,(1 - j1)$

yields $I_{SC} = 3.07\angle -4.40°$ A



$Z_{TH} = V_{OC}/I_{SC} = 4 - j1\ \Omega$ $\quad \boxed{Z_L = Z_{TH}^* = 4 + j1\ \Omega}$

$I = V_{OC}\,/\,(Z_{TH} + Z_L) = 1.58\angle -18.4°$ A

$P_L = \frac{1}{2}\, I_M^2\, R_L$ $\quad \boxed{P_L = 5.00\ W}$

9.40 Find the value of $\mathbf{Z}_L$ in the circuit in Fig. P9.40 for maximum average power transfer.

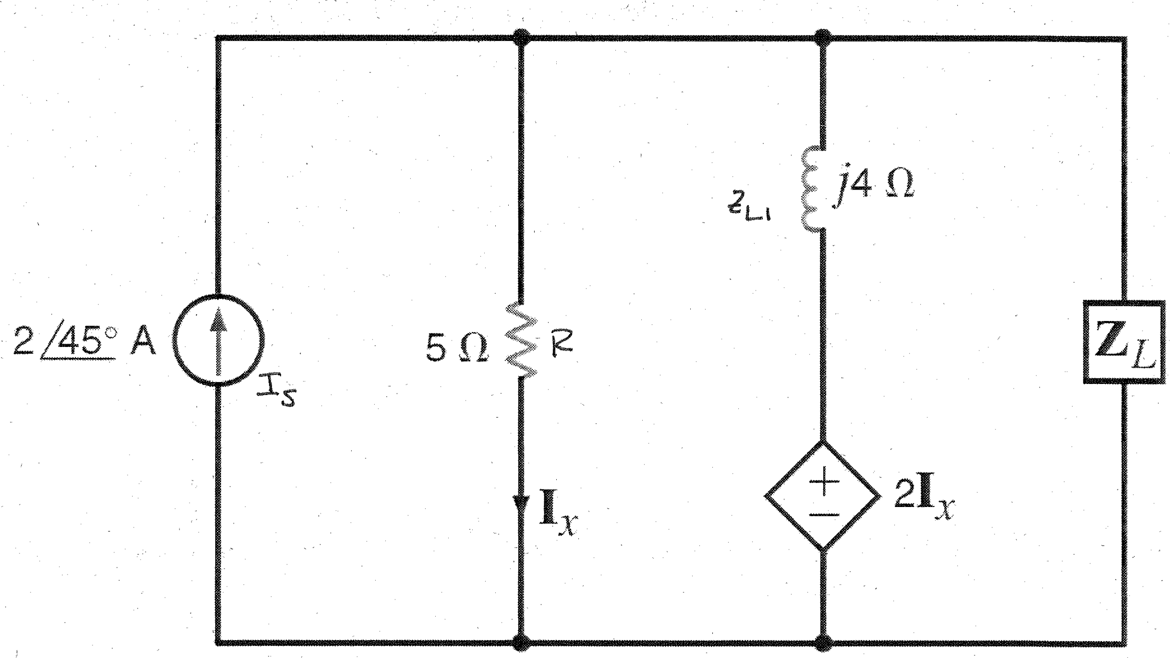
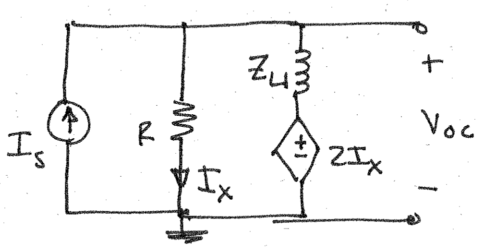


Figure P9.40

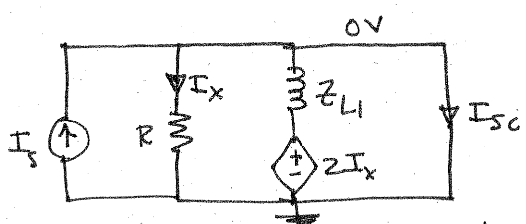
SOLUTION:



$$I_s = \frac{V_{oc}}{R} + \frac{V_{oc} - 2I_x}{Z_{L1}} \qquad I_x = \frac{V_{oc}}{R}$$

$$2\angle 45 = \frac{V_{oc}}{5} + \frac{V_{oc} - 0.4V_{oc}}{j4}$$

$$V_{oc} = 7.98\angle 81.9^\circ \text{ V}$$



$$I_x = 0\text{A} \Rightarrow I_{sc} = I_s = 2\angle 45^\circ \text{A}$$

$$Z_{TH} = \frac{V_{oc}}{I_{sc}} = 3.2 + j2.4\ \Omega$$

$$\boxed{Z_L = Z_{TH}^* = 3.2 - j2.4\ \Omega}$$

9.41 Compute the rms value of the voltage given by the expression $v(t) = 10 + 20\cos(377t + 30°)$ V.

SOLUTION:

$$V_{Rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt} \qquad v^2(t) = 100 + 400\cos(\omega t + \theta) + 400\cos^2(\omega t + \theta)$$

$$\frac{1}{T}\int_0^T 100\,dt = \frac{1}{T}(100t)\Big|_0^T = 100$$

$$\frac{1}{T}\int_0^T 400\cos(\omega t + \theta)\,dt = 0$$

$$\frac{1}{T}\int_0^T 400\cos^2(\omega t + \theta)\,dt = \frac{400}{2} = 200$$

$$V_{Rms} = \sqrt{100 + 0 + 200} \qquad \boxed{V_{Rms} = 17.3\text{ V}}$$

9.42 Compute the rms value of the voltage given by the waveform shown in Fig. P9.42.

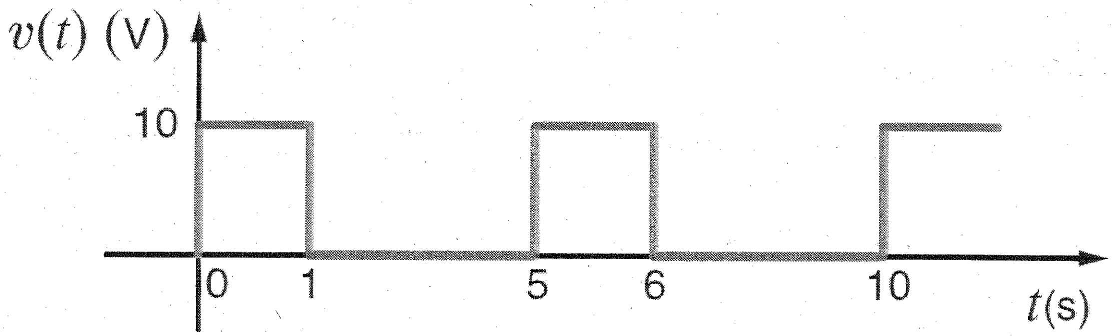


Figure P9.42

SOLUTION:

$T = 5s \qquad v^2(t) = \begin{cases} 100 & t < 1 \\ 0 & 1 \le t < 5 \end{cases}$

$V_{rms} = \sqrt{\frac{1}{T}\int_0^1 100\,dt} = \sqrt{\frac{1}{5}(100t)\Big|_0^1} = \sqrt{20}$

$\boxed{V_{rms} = 4.47\text{ V}}$

9.43 Calculate the rms value of the waveform shown in Fig. P9.43.

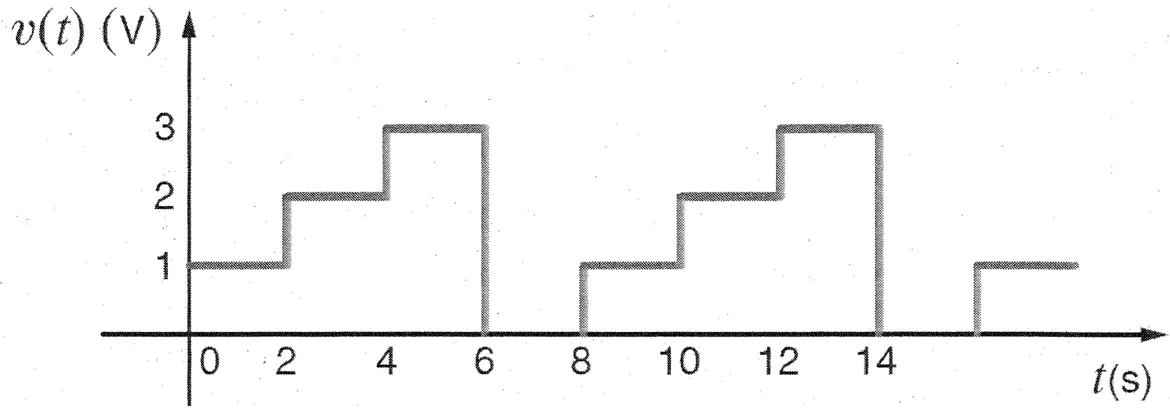


Figure P9.43

SOLUTION:

$T = 8\text{ s}$

$$v^2(t) = \begin{cases} 1 & 0 < t < 2 \\ 4 & 2 < t < 4 \\ 9 & 4 < t < 6 \\ 0 & 6 < t < 8 \end{cases}$$

$$V_{Rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt} = \left[\frac{1}{8}\left\{(1)(2) + (4)(2) + 9(2)\right\}\right]^{1/2}$$

$$\boxed{V_{Rms} = 1.87\text{ V}}$$

9.44 Calculate the rms value of the waveform in Fig. P9.44.

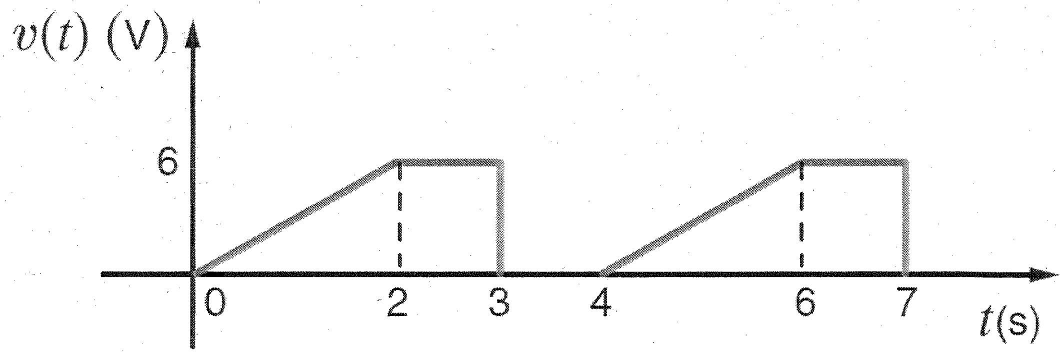


Figure P9.44

SOLUTION:

$T = 4\,s \qquad v(t) = \begin{cases} 3t & 0<t<2 \\ 6 & 2<t<3 \\ 0 & 3<t<4 \end{cases} \qquad v^2(t) = \begin{cases} 9t^2 & 0<t<2 \\ 36 & 2<t<3 \\ 0 & 3<t<4 \end{cases}$

$$V_{rms} = \left[\frac{1}{4}\left\{ \frac{9t^3}{3}\Big|_0^2 + 36t\Big|_2^3 \right\}\right]^{1/2} = \left\{\frac{1}{4}\left[24 + 36\right]\right\}^{1/2}$$

$$\boxed{V_{rms} = 3.87\ V}$$

9.45 Calculate the rms value of the waveform in Fig. P9.45.

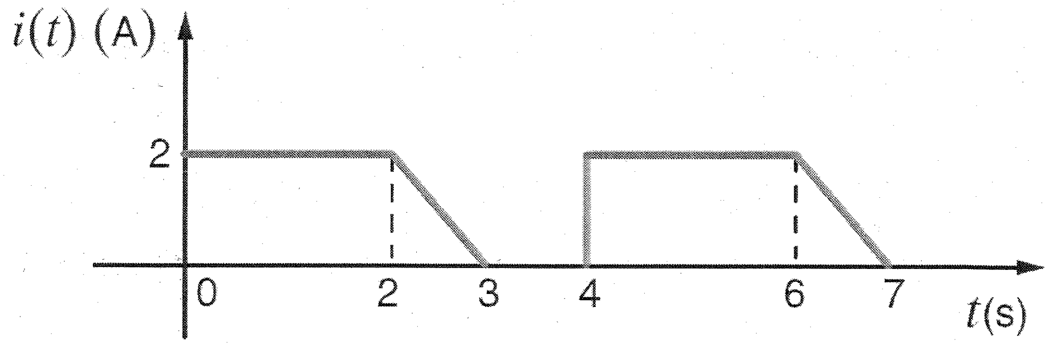


Figure P9.45

SOLUTION:

$T = 4s \qquad i(t) = \begin{cases} 2 & 0<t<2 \\ 6-2t & 2<t<3 \\ 0 & 3<t<4 \end{cases} \Rightarrow i^2(t) = \begin{cases} 4 \\ 36-24t+4t^2 \\ 0 \end{cases}$

$$I_{rms} = \left\{ \frac{1}{4}\left[4t\Big|_0^2 + \left(36t\Big|_2^3 - \frac{24t^2}{2}\Big|_2^3 + \frac{4t^3}{3}\Big|_2^3 \right) \right] \right\}^{1/2}$$

$$I_{rms} = \left\{ \frac{1}{4}\left[8 + 36 - 60 + 25.33 \right] \right\}^{1/2}$$

$$\boxed{I_{rms} = 1.53\ A}$$

9.46 Calculate the rms value of the waveform in Fig. P9.46.

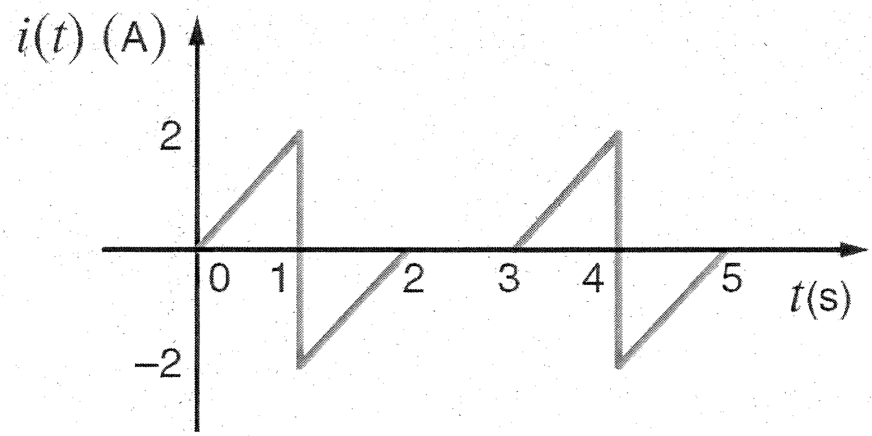


Figure P9.46

SOLUTION:

$T = 3s \qquad i(t) = \begin{cases} 2t & 0<t<1 \\ -4+2t & 1<t<2 \\ 0 & 2<t<3 \end{cases} \qquad i^2(t) = \begin{cases} 4t^2 \\ 16-16t+4t^2 \\ 0 \end{cases}$

$$I_{rms} = \left\{ \frac{1}{T}\int_0^T i^2(t)\,dt \right\}^{1/2} = \left\{ \frac{1}{3}\left[\frac{4t^3}{3}\bigg|_0^1 + 16t\Big|_1^2 - 8t^2\Big|_1^2 + \frac{4}{3}t^3\Big|_1^2 \right] \right\}^{1/2}$$

$$I_{rms} = \left\{ \frac{1}{3}\left[\frac{4}{3} + 16 - 8(4-1) + \frac{4}{3}(8-1) \right] \right\}^{1/2}$$

$$\boxed{I_{rms} = 0.94\ \text{A}}$$

9.47 Calculate the rms value of the waveform shown in Fig. P9.47.

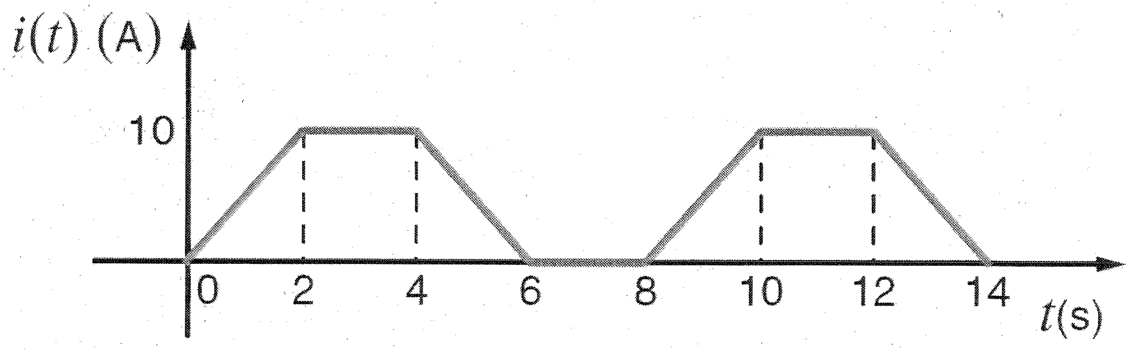


Figure P9.47

SOLUTION:

$i(t)$ consists of 3 parts – 2 identical triangles and a rectangle.

for the triangle: $i_1(t) = 5t$ and $i_1^2(t) = 25t^2$

$$\int_0^2 i_1^2(t)\,dt = \frac{25t^3}{3}\Big|_0^2 = 66.67 \quad \text{(same for 2}^{\text{nd}}\text{ triangle}$$

for the rectangle $i_2(t) = 10$ and $i_2^2(t) = 100$

$$\int_2^4 i_2^2(t)\,dt = 100t\Big|_2^4 = 200$$

$$I_{rms} = \left\{ \frac{1}{T}\left[2(66.67) + 200 \right] \right\}^{1/2} \qquad T = 8s$$

$$\boxed{I_{rms} = 6.45 \text{ A}}$$

9.48 Calculate the rms value of the waveform shown in Fig. P9.48. CS

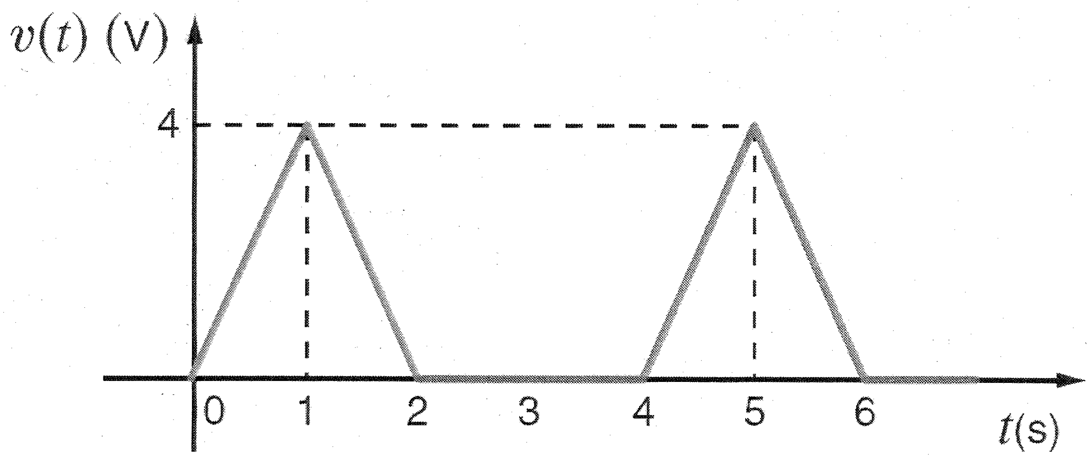


Figure P9.48

SOLUTION:

$v(t)$ consists of 2 identical triangles:

1st triangle : $v_1(t) = 4t$ and $v_1^2(t) = 16t^2$

$$\int_0^1 v_1^2(t)dt = \frac{16t^3}{3}\Big|_0^1 = \frac{16}{3} \quad \text{(same for 2nd triangle)}$$

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)dt} = \left\{\frac{1}{T}\left[2\left(16/3\right)\right]\right\}^{1/2} \qquad T = 4s$$

$$\boxed{V_{rms} = 1.63 \text{ V}}$$

9.49 The current waveform in Fig. P9.49 is flowing through a 5-Ω resistor. Find the average power absorbed by the resistor. **PSV**

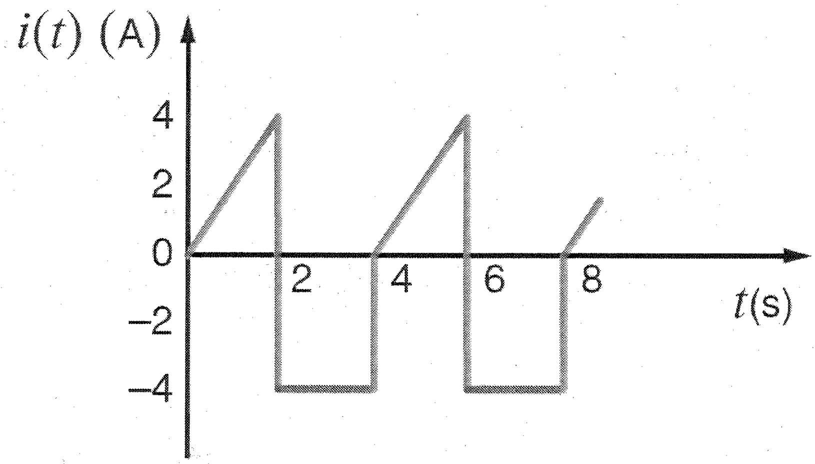


Figure P9.49

SOLUTION: $T = 4\text{s}$

$i(t)$ consists of a triangle and a rectangle

Triangle: $i_1(t) = 2t$ and $i_1^2(t) = 4t^2$

$$\int_0^2 i_1^2(t)\,dt = \frac{4}{3}t^3\Big|_0^2 = 10.67$$

Rectangle: $i_2(t) = -4$ and $i_2^2(t) = 16$

$$\int_2^4 16\,dt = 16t\Big|_2^4 = 32$$

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2(t)\,dt} = \left\{\frac{1}{4}\left[10.67 + 32\right]\right\}^{1/2}$$

$$\boxed{I_{rms} = 3.27\text{ A}}$$

9.50 Calculate the rms value of the waveform shown in Fig. P9.50. CS

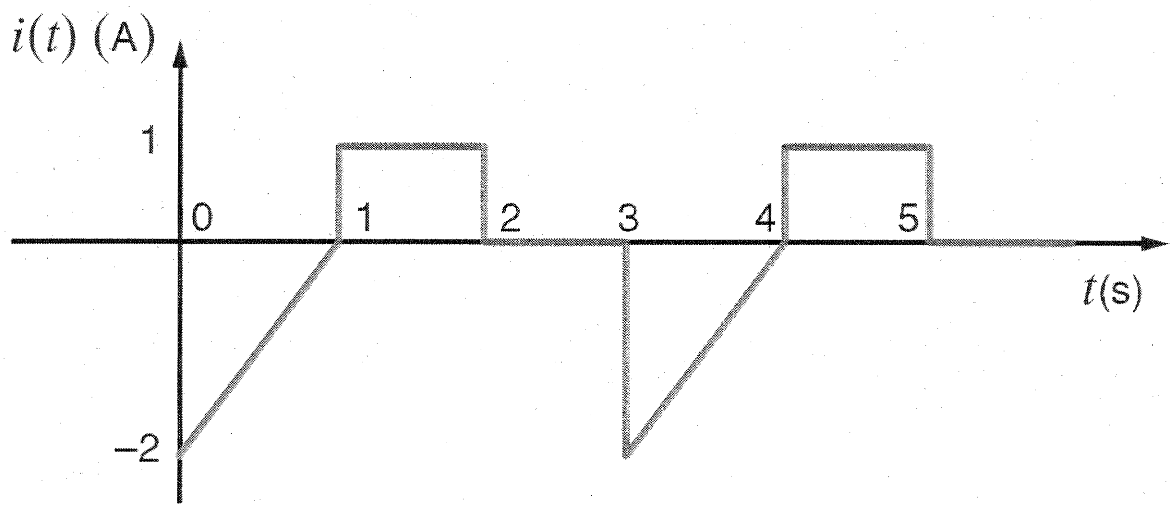


Figure P9.50

SOLUTION: T = 3s

i(t) consists of a triangle of 1 second duration and a rectangle of 1 second duration.

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2(t)\,dt}$$

Triangle: $i_1(t) = 2t - 2$ and $i_1^2(t) = 4t^2 - 8t + 4$

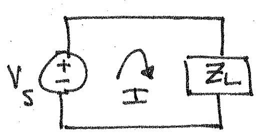
$$\int_0^1 i_1^2(t)\,dt = \frac{4t^3}{3}\Big|_0^1 - 4t^2\Big|_0^1 + 4t\Big|_0^1 = \frac{4}{3}$$

Rectangle: $i_2(t) = 1$ and $i_2^2(t) = 1$ $\int_1^2 i_2^2(t)\,dt = 1$

$$I_{rms} = \left\{\frac{1}{3}\left[\,4/3 + 1\right]\right\}^{1/2}$$ $\boxed{I_{rms} = 0.882\text{ A}}$

9.51 A plant consumes 20 kW of power from a 240-V rms line. If the load power factor is 0.9, what is the angle by which the load voltage leads the load current? What is the load current phasor if the line voltage has a phasor of 240 $\underline{/0^\circ}$ V rms?

SOLUTION:



$V_S = 240 \underline{/0^\circ}\ V_{rms}$

$P_L = 20\ kW \qquad pf = 0.9$ (lagging implied)

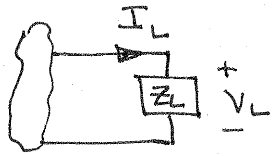
$\theta = \cos^{-1}(pf) = \cos^{-1}(0.9) \Rightarrow \boxed{\theta = 25.8^\circ}$

$P_L = |V_S||I|\,pf \Rightarrow |I| = \dfrac{P_L}{|V_S|\,pf} = \dfrac{2\times10^4}{240\,(.9)} = 92.6\,A$

$\boxed{I = 92.6 \underline{/-25.8^\circ}\ A_{rms}}$

9.52 A plant consumes 100 kW of power at 0.9 pf lagging. If the load current is 200 A rms, find the load voltage.

SOLUTION:



$|I_L| = 200\,A_{rms}$ $\quad pf = 0.9$ lag

$P_L = 100\,kW$ $\quad \theta_{V_L} - \theta_{I_L} = \cos^{-1}(pf) = 25.8°$

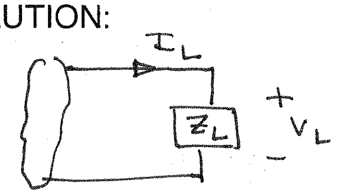
$$P_L = |V_L||I_L|\,pf \Rightarrow |V_L| = \frac{P_L}{|I_L|\,pf} = 556\,V_{rms}$$

assuming $I_L = 200\angle 0°\,A_{rms}$,

$$\boxed{V_L = 556\angle 25.8°\ V_{rms}}$$

9.53 A plant draws 250 A rms from a 240-V rms line to supply a load with 50 kW. What is the power factor of the load?

SOLUTION:

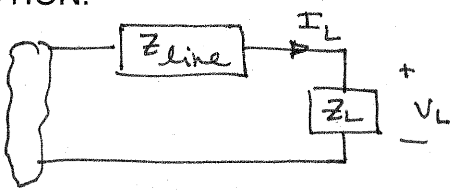


$$P_L = |V_L|\,|I_L|\,pf$$

$$pf = \frac{P_L}{|V_L|\,|I_L|} \qquad \boxed{pf = 0.833}$$

9.54 The power company supplies 80 kW to an industrial load. The load draws 220 A rms from the transmission line. If the load voltage is 440 V rms and the load power factor is 0.8 lagging, find the losses in the transmission line. CS

SOLUTION:



assume $\theta_{V_L} = 0°$

$V_L = 440\angle 0°\ V_{rms}$

$I_L = 220\ \angle -\theta\ A_{rms}$

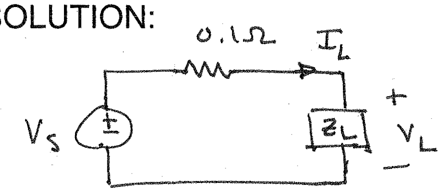
$P_L = |V_L||I_L|\,pf = 77440\text{ W}$

$\theta = \cos^{-1}(0.8) = 36.9°$

$P_{line} = P_S - P_L = 80000 - 77440$ $\boxed{P_{line} = 2560\text{ W}}$

9.55 An industrial load that consumes 40 kW is supplied by the power company through a transmission line with 0.1 Ω resistance, with 44 kW. If the voltage at the load is 240 V rms, find the power factor at the load.

SOLUTION:



$P_L = 4\times10^4\ W \qquad |V_L| = 240\ V_{rms}$

$P_L = |V_L||I_L|\ pf \qquad P_s = 44\ kW$

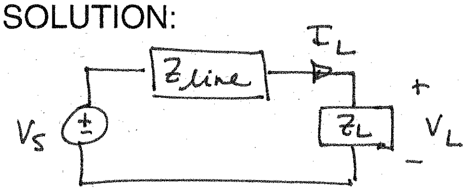
$P_{line} = P_s - P_L = 4\ kW = |I_L|^2(0.1)$

$|I_L| = 200\ A_{rms}$

$pf = \dfrac{P_L}{|V_L||I_L|} \qquad \boxed{pf = 0.83}$

9.56 The power company supplies 40 kW to an industrial load. The load draws 200 A rms from the transmission line. If the load voltage is 240 V rms and the load power factor is 0.8 lagging, find the losses in the transmission line.

SOLUTION:



$P_S = 40\text{ kW}$

$|V_L| = 240\text{ V}_{rms}$ $|I_L| = 200\text{ A}_{rms}$

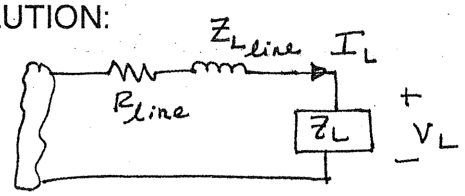
$P_L = |V_L||I_L|pf = 38.4\text{ kW}$

$$\boxed{P_{line} = P_S - P_L = 1600\text{ W}}$$

9.57 A transmission line with impedance $0.08 + j0.25\ \Omega$ is used to deliver power to a load. The load is inductive and the load voltage is $220\,\underline{/0^\circ}$ V rms at 60 Hz. If the load requires 12 kW and the real power loss in the line is 560 W, determine the power factor angle of the load.

CS

SOLUTION:



$R_{line} = 0.08\ \Omega \quad Z_{L_{line}} = j0.25\ \Omega$

$V_L = 220\,\underline{/0^\circ}\ V_{rms}$

$P_{line} = |I_L|^2 R_{line} = 560 \Rightarrow |I_L| = 83.7\ \text{A}$

$P_L = |V_L||I_L|\ pf \Rightarrow pf = \dfrac{12 \times 10^3}{(220)(83.7)} = 0.65$

Since load is inductive, pf is lagging.

$\boxed{pf = 0.65 \text{ lagging}}$ $\boxed{\theta_{Z_L} = \tan^{-1}(pf) = 49.4^\circ}$

9.58 Determine the real power, the reactive power, the complex power, and the power factor for a load having the following characteristics.

(a) $\mathbf{I} = 2\,\underline{/40^\circ}$ A rms, $\mathbf{V} = 450\,\underline{/70^\circ}$ V rms.

(b) $\mathbf{I} = 1.5\,\underline{/-20^\circ}$ A rms, $\mathbf{Z} = 5000\,\underline{/15^\circ}\ \Omega$.

(c) $\mathbf{V} = 200\,\underline{/+35^\circ}$ V rms, $\mathbf{Z} = 1500\,\underline{/-15^\circ}\ \Omega$.

SOLUTION:

a) $P = |I||V|\cos(\theta_V - \theta_I) = (2)(450)\cos(70-40)$

$\boxed{P = 779\text{ W}}$

$Q = |V||I|\sin(\theta_V - \theta_I) \Rightarrow \boxed{Q = 450\text{ VARs}}$

$S = VI^* = (450\,\underline{/70})(2\,\underline{/-40^\circ}) \quad \boxed{S = 900\,\underline{/30^\circ}\text{ VA}}$

$pf = \cos^{-1}(30^\circ) \Rightarrow \boxed{pf = 0.866\text{ lag}}$

b) $V = IZ = 7500\,\underline{/-5^\circ}\ V_{rms} \qquad P = (1.5)(7500)\cos(15) \Rightarrow \boxed{P = 10.9\text{kW}}$

$Q = (1.5)(7500)\sin(15) \Rightarrow \boxed{Q = 2912\text{ VARs}}$

$S = VI^* \Rightarrow \boxed{S = 11250\,\underline{/15^\circ}\text{ VA}} \quad \boxed{pf = 0.966\text{ lagging}}$

c) $I = V/Z = 0.133\,\underline{/50^\circ} \qquad S = VI^* \Rightarrow \boxed{S = 26.7\,\underline{/-15^\circ}\text{ VA}}$

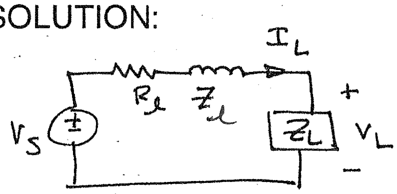
$P = |S|\cos(\theta_V - \theta_I) = 26.7\cos(-15^\circ) \quad \boxed{P = 25.8\text{ W}}$

$Q = |S|\sin(\theta_V - \theta_I) \Rightarrow \boxed{Q = -6.91\text{ VARs}}$

$pf = \cos^{-1}(-15^\circ) \Rightarrow \boxed{pf = 0.966\text{ leading}}$

9.59 An industrial load operates at 30 kW, 0.8 pf lagging. The load voltage is $240\,\underline{/0^\circ}$ V rms. The real and reactive power losses in the transmission-line feeder are 1.8 kW and 2.4 kvar, respectively. Find the impedance of the trasmission line and the input voltage to the line. **PSV**

SOLUTION:



$$|I_L| = \frac{P_L}{|V_L|\,pf} = \frac{3\times10^4}{240(0.8)} = 156\ A_{rms}$$

$$\theta_{Z_L} = \cos^{-1}(pf) = 36.9^\circ$$

$$I_L = 156\,\underline{/-36.9^\circ}\ A_{rms}$$

$$|I_L|^2 R_\ell = 1800$$

$$R_\ell = 74\,m\Omega$$

$$|I_L|^2 Z_\ell = j2400$$

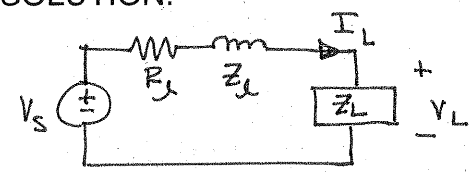
$$Z_\ell = j\,98\,m\Omega$$

$$\boxed{Z_{line} = 74 + j\,98\ m\Omega}$$

$$V_S = Z_{line} I_L + V_L \Rightarrow \boxed{V_S = 259\,\underline{/1.17^\circ}\ V_{rms}}$$

9.60 A transmission line with impedance $0.1 + j0.2\ \Omega$ is used to deliver power to a load. The load is capacitive and the load voltage is $240\angle 0^\circ$ V rms at 60 Hz. If the load requires 15 kW and the real power loss in the line is 660 W, determine the input voltage to the line.

SOLUTION:



$P_\ell = R_\ell |I_L|^2 \Rightarrow |I_L| = 81.2$ Arms

$P_L = 1500 = |V_L||I_L|\, pf_L \Rightarrow pf_L = 0.77$

Since load is capacitive, pf = 0.77 leading

$\theta_{Z_L} = -\cos^{-1}(pf) = -39.7^\circ = \theta_{V_L} - \theta_{I_L}$

$\theta_{I_L} = 39.7^\circ$

$I_L = 81.2\angle 39.7^\circ$ Arms

$V_S = I_L(R_\ell + Z_\ell) + V_L$

$= 81.2\angle 39.7\,(0.1 + j0.2) + 240\angle 0^\circ$

$$\boxed{V_S = 236\angle 4.29^\circ \text{ V}_{rms}}$$

9.61 Find the real and reactive power absorbed by each element in the circuit in Fig. P9.61.

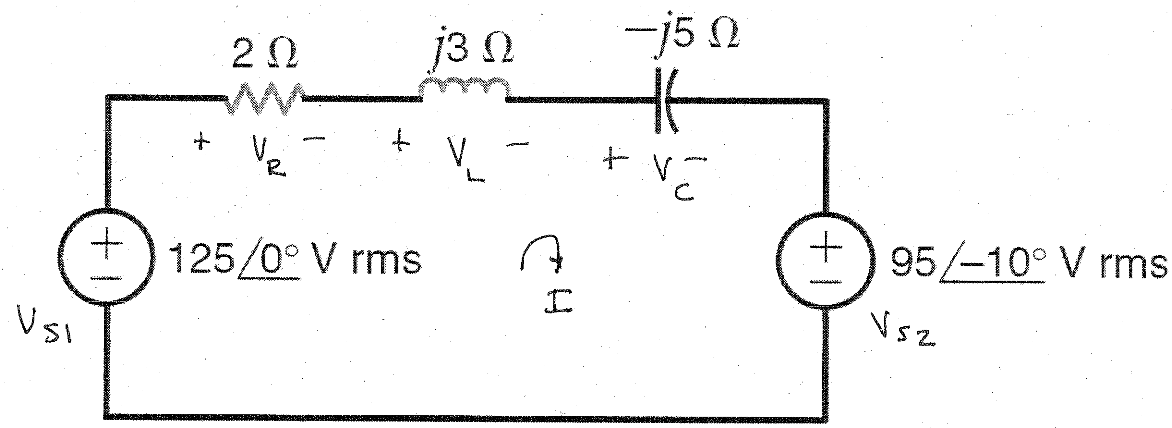


Figure P9.61

SOLUTION:

$$I = \frac{V_{S1} - V_{S2}}{2 + j3 - j5} = \frac{35.5\angle{+27.7^\circ}}{2 - j2} = 12.5\angle{72.7^\circ}\ A_{rms}$$

for V_{S1}: $S = -V_{S1} I^* = -(125\angle 0)(12.5\angle{-72.7^\circ})$

$S_{V_{S1}} = 1563\angle{107.3}$ VA

$\boxed{P_{VS_1} = -463\ W \qquad Q_{VS_1} = 1492\ VAR}$

for V_{S2}: $S_{V_{S2}} = I^* V_{S2} = 1188\angle{-82.7^\circ}$ VA

$\boxed{P_{VS2} = 150\ W \qquad Q_{VS2} = -1178\ VAR}$

$P_R = |I|^2 R$ $\boxed{P_R = 313\ W}$ $\boxed{Q_R = 0}$

$Q_L = |I|^2(3)$ $\boxed{Q_L = 469\ VAR \qquad P_L = 0}$

$Q_C = |I|^2(-5)$ $\boxed{Q_C = -781\ VAR \qquad P_C = 0}$

9.62 Given the network in Fig. P9.62, determine the input voltage $\mathbf{V}_S$.

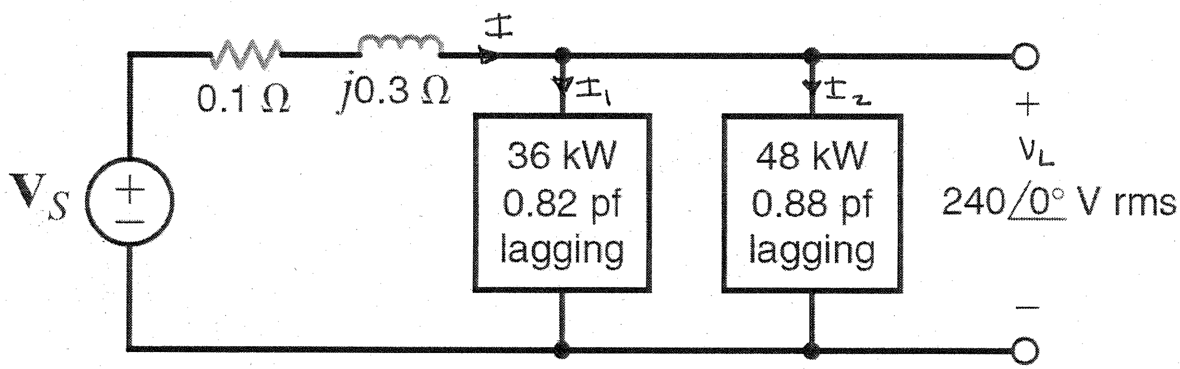


Figure P9.62

SOLUTION:

Load 1 : $P_{L1} = |V_L||I_1|(pf_1) \Rightarrow |I_1| = \dfrac{P_{L1}}{|V_L|\,pf_1} = 183$ Arms

$\angle I_1 = \theta_{V_L} - \cos^{-1}(pf_1) = -34.9°$

$I_1 = 183 \angle -34.9°$ Arms

Load 2: $|I_2| = \dfrac{P_{L2}}{|V_L|\,pf_2} = 227$ Arms $\quad \theta_{I_2} = \theta_{V_L} - \cos^{-1}(pf_2) = -28.4°$

$I_2 = 227 \angle -28.4°$ Arms

$I = I_1 + I_2 = 409 \angle -31.3°$ Arms

$V_s = I(0.1 + j0.3) + V_L$

$$\boxed{V_s = 349 \angle 13.9° \text{ Vrms}}$$

9.63 Given the network in Fig. P9.63, compute the input source voltage and the input power factor.

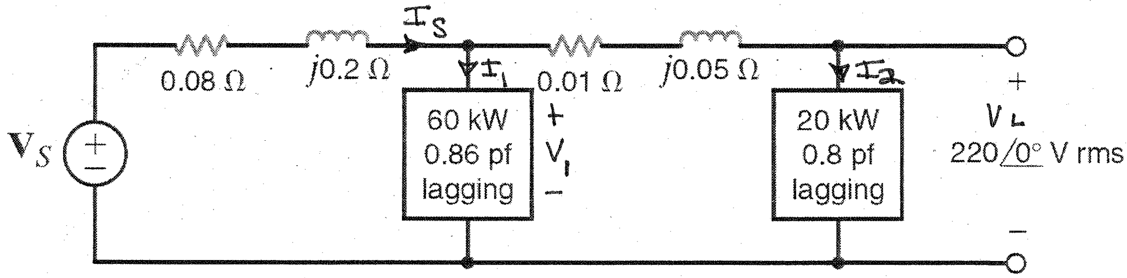


Figure P9.63

SOLUTION:

Load 2: $|I_2| = \frac{P_{L2}}{|V_L|(pf_2)} = \frac{20\times10^3}{220\,(0.8)} = 114$ Arms

$\theta_{I_2} = \theta_{V_L} - \cos^{-1}(pf_2) = -36.9°$

$I_2 = 114\angle{-36.9°}$ Arms

Load 1: $|I_1| = \frac{P_{L1}}{|V_1|(pf_1)}$ $\quad \theta_{I_2} = \theta_{V_1} - \cos^{-1}(pf_1)$

$V_1 = (0.01 + j0.05)I_2 + V_L = 224\angle 1.00°$ Vrms

$I_1 = 311\angle{-29.7°}$ Arms

$I_S = I_1 + I_2 = 424\angle{-31.6°}$ Arms

$V_S = (0.08 + j0.2)\,I_S + V_1$

$$\boxed{V_S = 303\angle 11.1° \text{ Vrms}}$$

$pf_S = \cos(\theta_{V_S} - \theta_{I_S}) = \cos(11.1 - (-31.6))$

$$\boxed{pf_S = 0.73 \text{ lagging}}$$

9.64 Use Kirchhoff's laws to compute the source voltage of the network shown in Fig. P9.64.

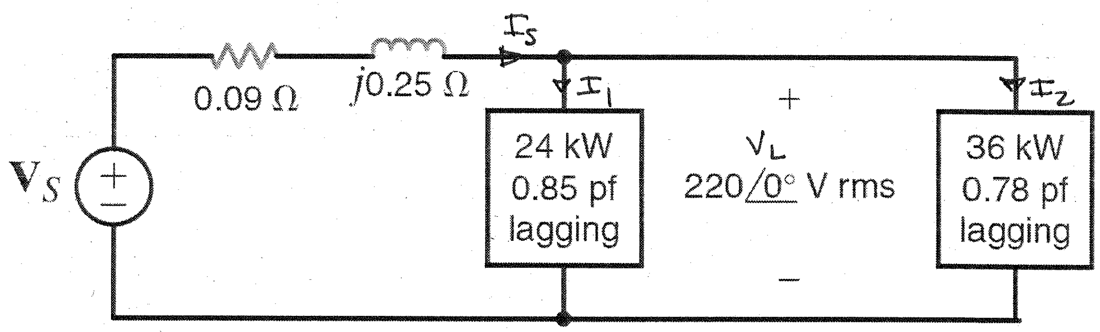


Figure P9.64

SOLUTION:

Load 2: $\left\{ \begin{array}{l} |I_2| = \dfrac{P_{L2}}{|V_L|(pf_2)} = \dfrac{36\times10^3}{220\,(0.78)} = 210\text{ A rms} \\ \theta_{I_2} = \theta_{V_L} - \cos^{-1}(pf_2) = -38.7^\circ \end{array} \right\} I_2 = 210\angle{-38.7^\circ}\text{ Arms}$

Load 1: $\left\{ \begin{array}{l} |I_1| = \dfrac{P_{L1}}{|V_L|(pf_1)} = \dfrac{24\times10^3}{220\,(0.85)} = 128\text{ Arms} \\ \theta_{I_1} = \theta_{V_L} - \cos^{-1}(pf_1) = -31.8^\circ \end{array} \right\} I_1 = 128\angle{-31.8^\circ}\text{ Arms}$

$$I_s = I_1 + I_2 = 337\angle{-36.1^\circ}\text{ Arms}$$

$$V_s = (0.09 + j0.25)\,I_s + V_L \qquad \boxed{V_s = 298\angle{9.70^\circ}\text{ Vrms}}$$

9.65 Given the network in Fig. P9.65, determine the input voltage $\mathbf{V}_S$. **PSV**

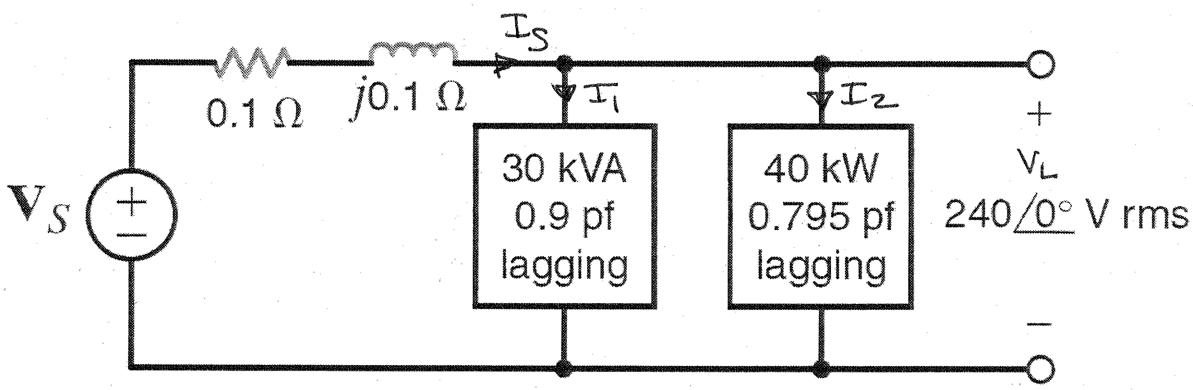


Figure P9.65

SOLUTION:

Load1: $|I_1| = \dfrac{P_{L1}}{|V_L|(pf_1)} = \dfrac{30000}{240(0.9)} = 139 \text{ Arms}$

$\theta_{I_1} = \theta_{V_L} - \cos^{-1}(0.9) = -25.8°$

$I_1 = 139 \angle -25.8° \text{ Arms}$

Load2: $|I_2| = \dfrac{P_{L2}}{|V_L|(pf_2)} = 210 \text{ Arms}$

$\theta_{I_2} = \theta_{V_L} - \cos^{-1}(0.795) = -37.3°$

$I_2 = 210 \angle -37.3° \text{ Arms}$

$I_s = I_1 + I_2 = 347 \angle -32.7° \text{ Arms}$

$V_s = (0.1 + j0.1) I_s + V_L$

$$\boxed{V_s = 288 \angle 2.08° \text{ Vrms}}$$

9.66 Find the input source voltage and the power factor of the source for the network shown in Fig. P9.66. CS

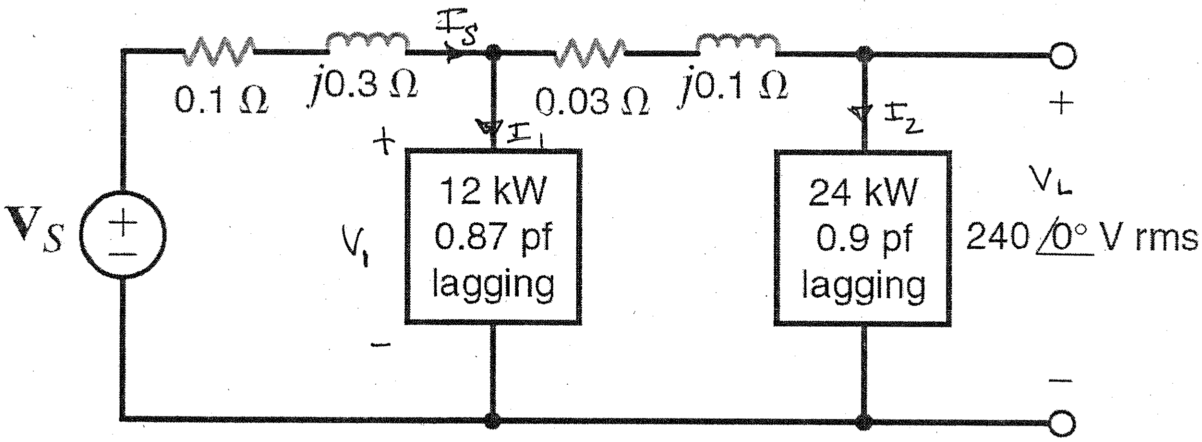


Figure P9.66

SOLUTION:

Load 2: $\left\{ \begin{array}{l} |I_2| = \dfrac{P_{L2}}{|V_L|(pf_2)} = \dfrac{24000}{240\,(0.9)} = 111\ A_{rms} \\ \theta_{I_2} = \theta_{V_L} - \cos^{-1}(pf_2) = -25.8^\circ \end{array} \right\} \; I_2 = 111\,\underline{/-25.8^\circ}\ A_{rms}$

$V_1 = (0.03 + j0.1)I_2 + V_L = 249\ \underline{/1.97^\circ}\ V_{rms}$

Load 1: $\left\{ \begin{array}{l} |I_1| = \dfrac{P_{L1}}{|V_1|(pf_1)} = \dfrac{12000}{249\,(0.87)} = 55.4\ A_{rms} \\ \theta_{I_1} = \theta_{V_1} - \cos^{-1}(pf_1) = -27.6^\circ \end{array} \right\} \; I_1 = 55.4\,\underline{/-27.6^\circ}\ A_{rms}$

$I_S = I_1 + I_2 = 166\ \underline{/-26.4^\circ}\ A_{rms}$

$V_S = (0.1 + j0.3)\, I_S + V_1$ $\qquad \boxed{V_S = 289\ \underline{/9.14^\circ}\ V_{rms}}$

$pf_S = \cos(\theta_{V_S} - \theta_{I_S}) = \cos(35.54^\circ) = 0.814$

$\boxed{pf_S = 0.814 \text{ lagging}}$

9.67 Given the network in Fig. P9.67, find the complex power supplied by the source, the power factor of the source, and the voltage $v_S(t)$. The frequency is 60 Hz.

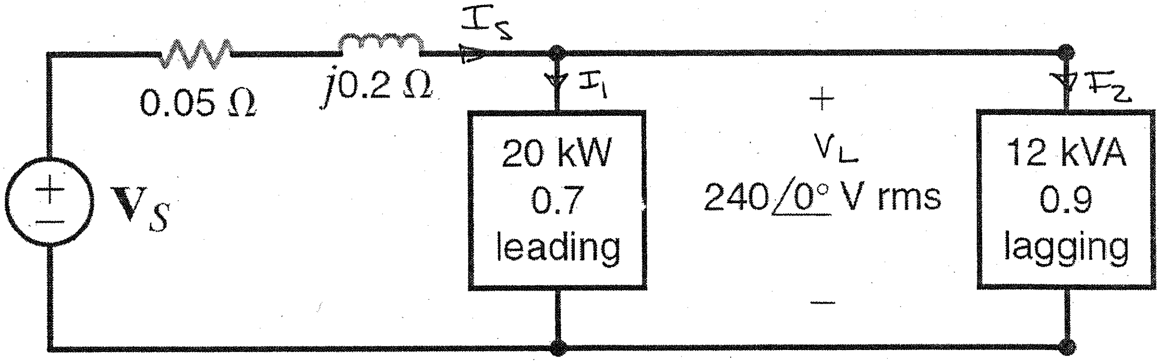


Figure P9.67

SOLUTION:

Load 2: $\begin{cases} |I_2| = \dfrac{P_{L2}}{|V_L|\,(pf_2)} = \dfrac{12000}{240\,(0.9)} = 55.6 \text{ Arms} \\ \theta_{I_2} = \theta_{V_L} - \cos^{-1}(pf_2) = -25.8^\circ \end{cases}$

$I_2 = 55.6 \angle -25.8^\circ$ Arms

Load 1: $\begin{cases} |I_1| = \dfrac{P_{L1}}{|V_L|\,(pf_1)} = \dfrac{20000}{240\,(0.7)} = 119 \text{ Arms} \\ \theta_{I_1} = \theta_{V_L} + \cos^{-1}(pf_1) = +45.6^\circ \end{cases}$

$I_1 = 119 \angle +45.6^\circ$ Arms

$I_S = I_1 + I_2 = 147 \angle 24.5^\circ$ Arms

$V_S = (0.05 + j0.2)\, I_S + V_L = 236 \angle 7.22^\circ$ Vrms

$S_{V_S} = V_S I_S^*$ $\qquad pf_S = \cos(\theta_{S_S}) = \cos(43.8)$ $\qquad \boxed{pf_S = 0.955 \text{ leading}}$

$\boxed{S_{V_S} = 34.6 \angle -17.3^\circ \text{ kVA}}$

$v_S(t) = 334 \cos(377t + 7.22^\circ)$ V

9.68 Given the circuit in Fig. P9.68, find the complex power supplied by the source and the source power factor. If $f = 60$ Hz, find $v_s(t)$.

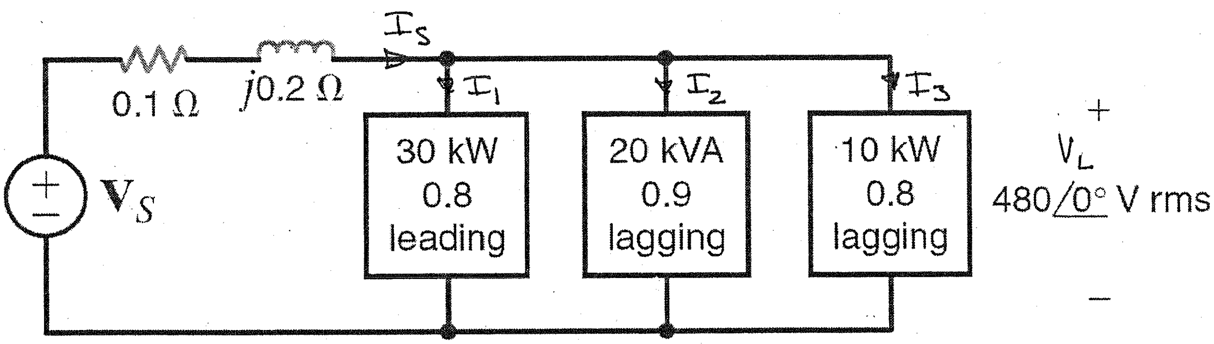


Figure P9.68

SOLUTION:

Load 3: $\left\{\begin{array}{l} |I_3| = \dfrac{P_{L3}}{|V_L|(pf_3)} = \dfrac{10000}{480(0.8)} = 26.0 \text{ Arms} \\ \theta_{I_3} = \theta_{V_L} - \cos^{-1}(pf_3) = -36.8^\circ \end{array}\right\}$ $I_3 = 26.0 \angle -36.8^\circ$ Arms

Load 2: $\left\{\begin{array}{l} |I_2| = \dfrac{20000}{480\,(0.9)} = 46.3 \text{ Arms} \\ \theta_{I_2} = 0 - \cos^{-1}(0.9) = -25.8^\circ \end{array}\right\}$ $I_2 = 46.3 \angle -25.8^\circ$ Arms

Load 1: $\left\{\begin{array}{l} |I_1| = \dfrac{30000}{480\,(0.8)} = 78.1 \text{ Arms} \\ \theta_{I_3} = \theta_{V_L} + \cos^{-1}(pf_1) = 36.9^\circ \end{array}\right\}$ $I_3 = 78.1 \angle 36.9^\circ$ Arms

$I_S = I_1 + I_2 + I_3 = 125 \angle 5.25^\circ$ Arms

$V_S = (0.1 + j0.2) I_S + V_L = 491 \angle 3.06^\circ$

$S_s = V_s I_s^* = 61.4 \angle -2.19^\circ$ kVA $\qquad \boxed{S_S = 61.4 \angle -2.19^\circ \text{ kVA}}$

$pf_s = \cos(\theta_{V_S} - \theta_{I_S}) = 0.999 \qquad \boxed{pf_s = 0.999 \text{ leading}}$

since $\theta_V - \theta_I < 0$, leading

$v_s(t) = \sqrt{2}\,|V_s| \cos(2\pi f t + 3.06^\circ)$ V $\qquad \boxed{v_s = 694 \cos(377t + 3.06^\circ) \text{ V}}$

9.69 In the circuit in Fig. P9.69, the complex power supplied by source $\mathbf{V}_1$ is $2000\,\underline{/-30°}$ VA. If $\mathbf{V}_1 = 200\,\underline{/10°}$ V rms, find $\mathbf{V}_2$.

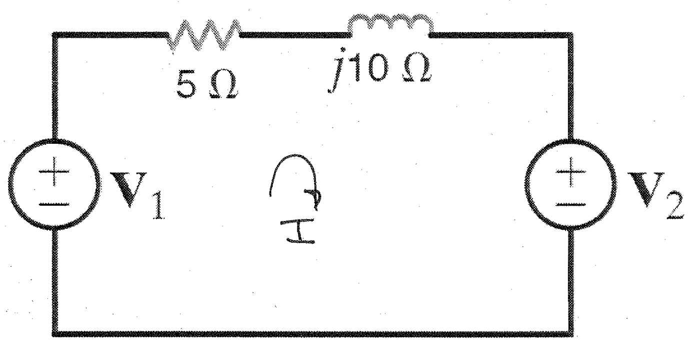


Figure P9.69

SOLUTION:

$S_1 = V_1 I^* \Rightarrow 2000\,\underline{/-30} = 200\,\underline{/10}\, I^*$

$I^* = 10\,\underline{/-40} \qquad I = 10\,\underline{/40°}$ Arms

$V_1 = I(5 + j10) + V_2 \Rightarrow V_2 = V_1 - (5 + j10)\, I$

$\boxed{V_2 = 235\,\underline{/-18.4°} \text{ Vrms}}$

9.70 For the network in Fig. P9.70, the complex power absorbed by the source on the right is $0 + j1582.5$ VA. Find the value of R and the unknown element and its value if $f = 60$ Hz. (If the element is a capacitor, give its capacitance; if the element is an inductor, give its inductance.)

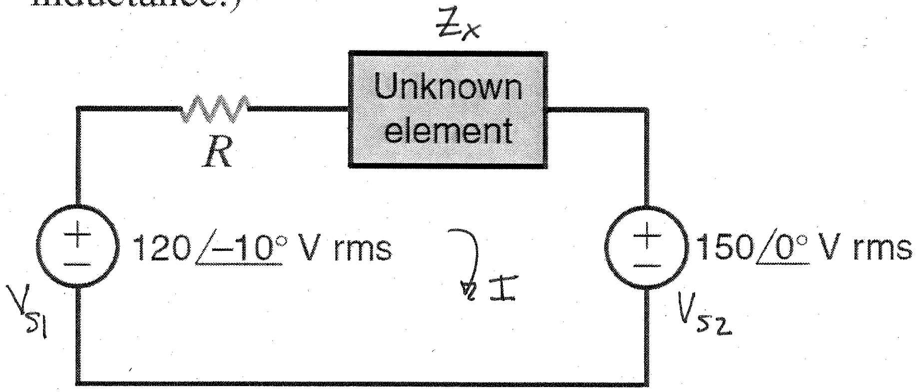


Figure P9.70

SOLUTION:

$$S_2 = j1582.5 \text{ VA} = V_{S2} I^* = 150\angle 0\; I^*$$

$$I^* = \frac{1582.5\angle 90}{150\angle 0°} = 10.6\angle 90° \Rightarrow I = 10.6\angle -90° \text{ Arms}$$

$$\frac{V_{S1} - V_{S2}}{I} = R + Z_X = 1.97 - j3.00\,\Omega$$

$$\boxed{R = 1.97\,\Omega} \qquad Z_X = -j3\,\Omega = -j/\omega C_X \qquad \omega = 377 \text{ r/s}$$

$$\boxed{C_X = 884\,\mu F}$$

9.71 A plant consumes 60 kW at a power factor of 0.75 lagging from a 240-V rms 60-Hz line. Determine the value of the capacitor that when placed in parallel with the load will change the load power factor to 0.9 lagging.

SOLUTION:

$$|S_{old}| = \frac{P_L}{pf_{old}} = \frac{60000}{0.75} = 80\text{kVA} \qquad \theta_{old} = \cos^{-1}(0.75) = 41.4°$$

$$S_{old} = 80\angle 41.4° \text{ kVA} \qquad Q_{old} = |S_{old}| \sin\theta_{old} = 52.9 \text{ kVARs}$$

$$|S_{new}| = \frac{P_L}{pf_{new}} = \frac{60000}{0.9} = 66.7\text{ kVA} \qquad \theta_{new} = 25.8°$$

$$S_{new} = 66.7\angle 25.8° \text{ kVA} \qquad Q_{new} = 29.1 \text{ kVARs}$$

$$Q_C + Q_{old} = Q_{new} \Rightarrow -\omega C|V_L|^2 = (29.1 - 52.9)\times 10^3$$

$$|V_L| = 240\text{ V}_{rms} \qquad \omega = 377 \text{ r/s}$$

$$\boxed{C = 1.1\text{mF}}$$

9.72 A particular load has a pf of 0.8 lagging. The power delivered to the load is 40 kW from a 270-V rms 60-Hz line. What value of capacitance placed in parallel with the load will raise the pf to 0.9 lagging? **PSV**

SOLUTION:

$$|S_{old}| = \frac{P_L}{pf_{old}} = \frac{40000}{0.8} = 50\text{ kVA} \qquad \theta_{old} = \cos^{-1}(0.8) = 36.9^\circ$$

$$Q_{old} = |S_{old}| \sin\theta_{old} = 30\text{ kVAR}$$

$$|S_{new}| = \frac{40000}{0.9} = 44.4\text{ kVA} \qquad \theta_{new} = \cos^{-1}(0.9) = 25.8^\circ$$

$$Q_{new} = (44.4\times10^3)\sin(\theta_{new}) = 19.4\text{ kVAR}$$

$$Q_C + Q_{old} = Q_{new} \Rightarrow -\omega C|V_L|^2 = (19.4 - 30)\times10^3$$

$$\omega = 377\text{ r/s} \qquad |V_L| = 270\text{ V}_{rms}$$

$$\boxed{C = 386\,\mu F}$$

9.73 An industrial load is supplied through a transmission line that has a line impedance of $0.1 + j0.2\ \Omega$. The 60-Hz line voltage at the load is $480\,\underline{/0^\circ}$ V rms. The load consumes 124 kW at 0.75 pf lagging. What value of capacitance when placed in parallel with the load will change the power factor to 0.9 lagging? CS

SOLUTION:

$$|S_{old}| = \frac{P_L}{pf_{old}} = \frac{124\times10^3}{0.75} = 165.3\ kVA \qquad \theta_{old} = \cos^{-1}(pf_{old}) = 41.4^\circ$$

$$S_{old} = 165.3\,\underline{/41.4^\circ}\ kVA \qquad Q_{old} = Im(S_{old}) = 109\ kVAR$$

$$|S_{new}| = \frac{124\times10^3}{0.9} = 137.8\ kVA \qquad \theta_{new} = \cos^{-1}(0.9) = 25.8^\circ$$

$$S_{new} = 137.8\,\underline{/25.8^\circ}\ kVA \qquad Q_{new} = Im(S_{new}) = 60.1\ kVAR$$

$$Q_C = Q_{new} - Q_{old} \Rightarrow -\omega C|V_L|^2 = (60.1 - 109)\times10^3$$

$$\omega = 377\ r/s \qquad |V_L| = 480\ V_{rms}$$

$$\boxed{C = 563\ \mu F}$$

9.74 The 60-Hz line voltage for a 60-kW, 0.76-pf lagging industrial load is 240 $\angle 0°$ V rms. Find the value of capacitance that when placed in parallel with the load will raise the power factor to 0.9 lagging.

SOLUTION:

$$|S_{old}| = \frac{P_L}{pf_{old}} = \frac{60000}{0.76} = 78.9 \text{ kVA}$$

$$\theta_{old} = \cos^{-1}(pf_{old}) = \cos^{-1}(0.76) = 40.5°$$

$S_{old} = 78.9 \angle 40.5°$ kVA

$Q_{old} = 51.3$ kVAR

$$|S_{new}| = \frac{60000}{0.9} = 66.6 \text{ kVA}$$

$$\theta_{new} = \cos^{-1}(0.9) = 25.8°$$

$S_{new} = 66.6 \angle 25.8°$ kVA

$Q_{new} = 29.1$ kVAR

$$Q_C = Q_{new} - Q_{old} \Rightarrow -\omega C |V_L|^2 = (29.1 - 51.3) \times 10^3$$

$\omega = 377$ r/s $\qquad |V_L| = 240$ V rms

$$\boxed{C = 1.02 \text{ mF}}$$

9.75 An industrial load consumes 44 kW at 0.82 pf lagging from a 240 $\angle 0°$-V-rms 60-Hz line. A bank of capacitors totaling 600 μF is available. If these capacitors are placed in parallel with the load, what is the new power factor of the total load?

SOLUTION:

$$|S_{old}| = \frac{P_L}{pf_{old}} = \frac{44\,000}{0.82} = 53.7 \text{ kVA}$$

$$\theta_{old} = \cos^{-1}(pf_{old}) = 34.9°$$

$$S_{old} = 53.7\angle 34.9° \text{ kVA}$$

$$Q_{old} = \text{Im}\{S_{old}\}$$

$$Q_{old} = 30.7 \text{ kVAR}$$

$$Q_C = -\omega C|V_L|^2 = -377(600\times10^{-6})(240)^2 = -13.0 \text{ kVAR}$$

$$Q_C = Q_{new} - Q_{old} \Rightarrow Q_{new} = 17.7 \text{ kVAR}$$

$$\theta_{new} = \tan^{-1}\left(\frac{Q_{new}}{P_L}\right) = 21.9 \quad \text{lagging since } \theta > 0°$$

$$pf_{new} = \cos\theta_{new} \qquad \boxed{pf_{new} = 0.928 \text{ lagging}}$$

9.76 A particular load has a pf of 0.8 lagging. The power delivered to the load is 40 kW from a 220-V rms 60-Hz line. What value of capacitance placed in parallel with the load will raise the pf to 0.9 lagging? CS

SOLUTION:

$$|S_{old}| = \frac{P_L}{pf_{old}} = \frac{40000}{0.8} = 50 \text{kVA} \qquad S_{old} = 50\angle 36.9° \text{ kVA}$$

$$\theta_{old} = \cos^{-1}(pf_{old}) = 36.9° \qquad Q_{old} = \text{Im}\{S_{old}\}$$

$$Q_{old} = 30 \text{ kVAR}$$

$$|S_{new}| = \frac{40000}{0.9} = 44.4 \text{ kVA} \qquad S_{new} = 44.4\angle 25.8° \text{ kVA}$$

$$\theta_{new} = \cos^{-1}(0.9) = 25.8° \qquad Q_{new} = 19.4 \text{kVAR}$$

$$Q_C = -\omega C|V_L|^2 = Q_{new} - Q_{old} = -10.6 \text{ kVAR}$$

$$\omega = 377 \text{ r/s} \qquad |V_L| = 220 \text{ V}_{rms}$$

$$\boxed{C = 581 \mu F}$$

9.77 A single-phase three-wire 60-Hz circuit serves three loads, as shown in Fig. P9.77. Determine $\mathbf{I}_{aA}$, $\mathbf{I}_{nN}$, $\mathbf{I}_c$, and the energy use over a 24-hour period in kilowatt-hours.

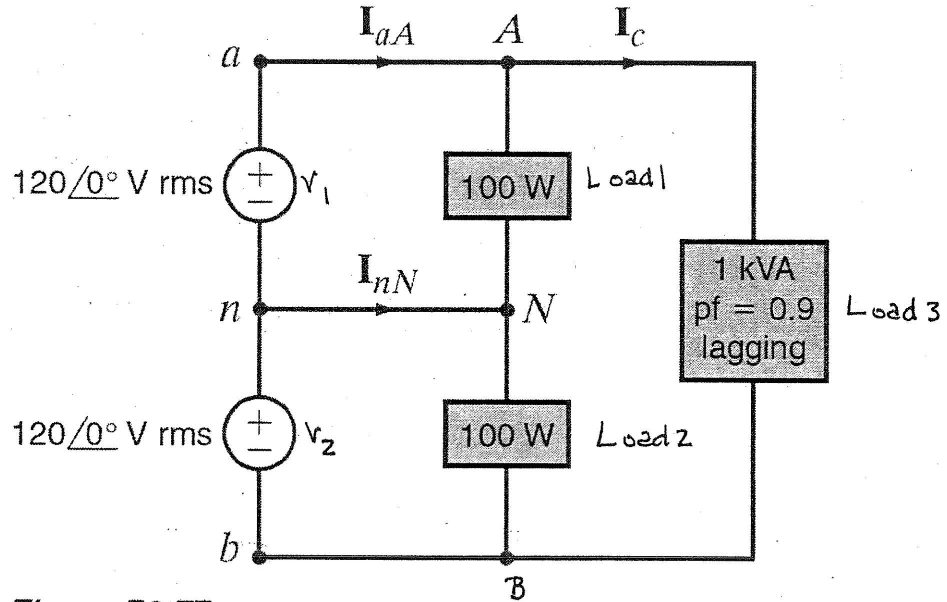


Figure P9.77

SOLUTION:

$S_1 = S_2 = 100 \angle 0° \text{ VA}$ $\quad |S_3| = 1 \text{ kVA}$, $\theta_3 = \cos^{-1}(0.9) = 25.8°$ $\Big\}$ $S_3 = 1000 \angle 25.8 \text{ VA}$

$I_{AN}^* = \dfrac{S_1}{V_1} = 0.833 \angle 0° \text{ Arms} \qquad I_{AN} = 0.833 \angle 0° \text{ Arms}$

$I_{NB}^* = S_2/V_2 = 0.833 \angle 0° \text{ Arms} \qquad I_{NB} = 0.833 \angle 0° \text{ Arms}$

$I_c^* = S_3/(V_1+V_2) = \dfrac{1000\angle 25.8°}{240\angle 0} = 4.17\angle 25.8° \Rightarrow \boxed{I_c = 4.17\angle -25.8° \text{ Arms}}$

$I_{aA} = I_{AN} + I_c \Rightarrow \boxed{I_{aA} = 4.93\angle -21.6° \text{ Arms}}$

$I_{nN} = I_{NB} - I_{AN} \Rightarrow \boxed{I_{nN} = 0}$

Total power $= P = P_1 + P_2 + P_3 = 100 + 100 + 900 = 1100 \text{ W}$

There are 24 hours/day $\quad \boxed{W = P(24) = 26.4 \text{ kW-hrs}}$

9.78 A 5.1-kW household range is designed to operate on a 240-V rms sinusoidal voltage, as shown in Fig. P9.78a. However, the electrician has mistakenly connected the range to 120 V rms, as shown in Fig. P9.78b. What is the effect of this error?

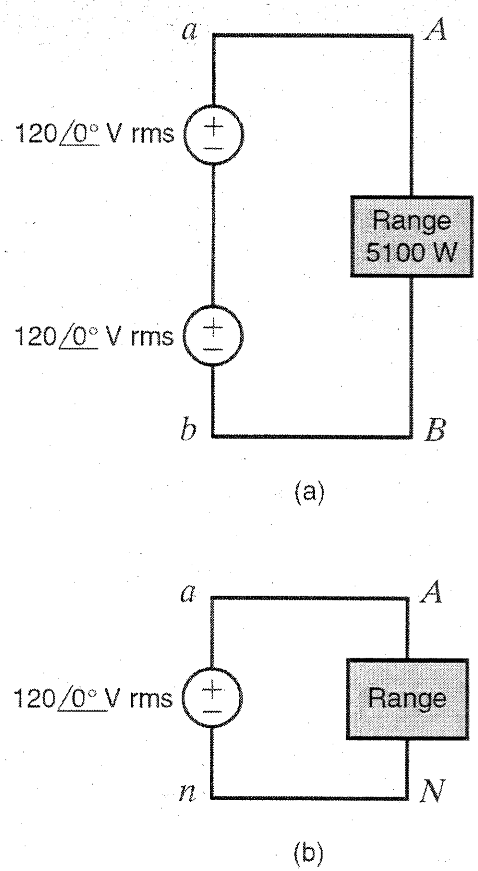


Figure P9.78

SOLUTION: As designed,

$$|I_{range}| = \frac{P_{range}}{|V_{ab}|} = \frac{5100}{|240\angle 0°|} = 21.25 \text{ A}_{rms}$$

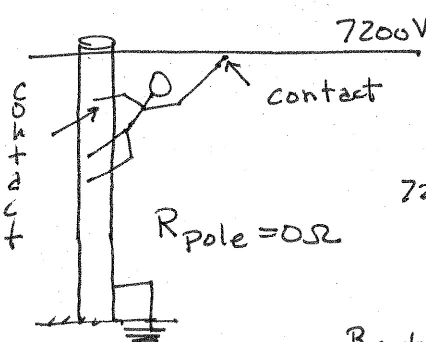
As connected

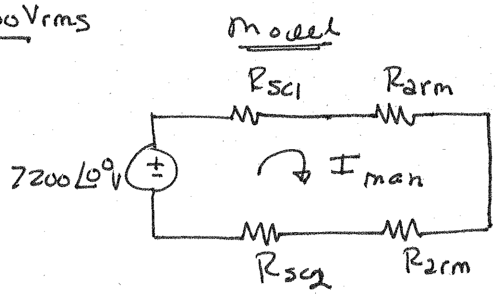
$$|I_{range}| = \frac{5100}{|120\angle 0°|} = 42.5 \text{ A}_{rms}$$

The current drawn by the range is doubled to offset the reduction in voltage by half.

9.79 A man and his son are flying a kite. The kite becomes entangled in a 7200-V rms power line close to a power pole. The man crawls up the pole to remove the kite. While trying to remove the kite, the man accidentally touches the 7200-V rms line. Assuming the power pole is well grounded, what is the potential current through the man's body? CS

SOLUTION:





Current flows from one hand, through both arms, to other hand & down the pole

Best case: skin is dry & $R_{sc} = 15k\Omega$, $R_{arm} = 100\Omega$

$$|I_{man}| = \frac{7200}{R_{sc1} + 2R_{arm} + R_{sc2}} = 238\,mA$$

Worst case: skin is wet & $R_{sc} = 150\Omega$, $R_{arm} = 100\Omega$

$$|I_{man}| = \frac{7200}{2(150) + 2(100)} = 14.4\,A_{rms}$$

$$|I_{man}| = \begin{cases} 238\,mA & \text{dry skin} \\ 14.4\,A & \text{wet skin} \end{cases}$$

9.80 A number of 120-V rms household fixtures are to be used to provide lighting for a large room. The total lighting load is 8 kW. The National Electric Code requires that no circuit breaker be larger than 20 A rms with a 25% safety margin. Determine the number of identical branch circuits needed for this requirement.

SOLUTION:

$$|I_{TOTAL}| = \frac{P}{|V|} = \frac{8000}{120} = 66.7 \text{ A rms}$$

$$|I_{BREAKER}| \leq 0.75(20) = 15 \text{ A rms} \quad (25\% \text{ safety margin})$$

$$\#\text{ of branches} = \frac{|I_{TOTAL}|}{|I_{BREAKER}|} = 4.44$$

$$\boxed{\text{Use 5 branches!}}$$

9.81 To test a light socket, a woman, while standing on cushions that insulate her from the ground, sticks her finger into the socket, as shown in Fig. P9.81. The tip of her finger makes contact with one side of the line, and the side of her finger makes contact with the other side of the line. Assuming that any portion of a limb has a resistance of 95 Ω, is there any current in the body? Is there any current in the vicinity of the heart?

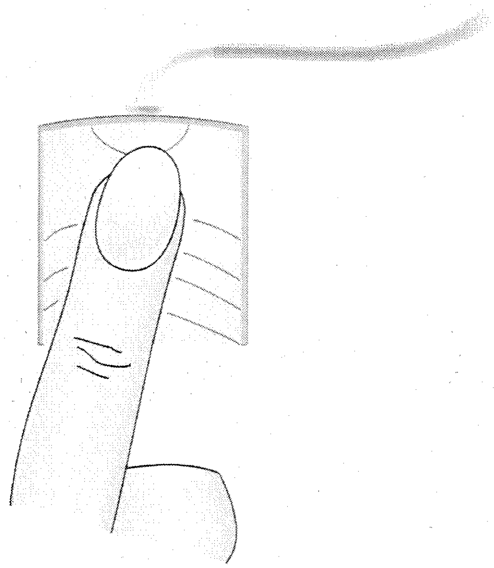
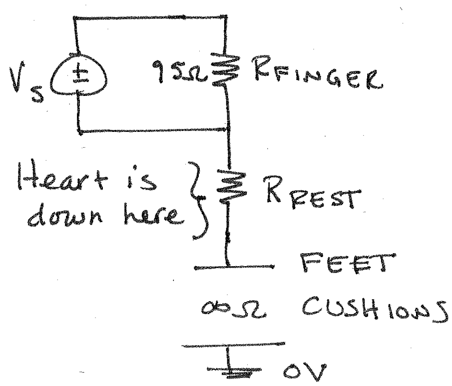


Figure P9.81

SOLUTION:

Model



$R_{FINGER} = 95\Omega$ $|V_S| = 120V$

R_{REST} models rest of body.

cushions have infinite resistance.

So, while $\frac{120}{95} = 1.26$ Arms flows through the fingertip (ouch), there is no current near the heart.

9.82 An inexperienced mechanic is installing a 12-V battery in a car. The negative terminal has been connected. He is currently tightening the bolts on the positive terminal. With a tight grip on the wrench, he turns it so that the gold ring on his finger makes contact with the frame of the car. This situation is modeled in Fig. P9.82, where we assume that the resistance of the wrench is negligible and the resistance of the contact is as follows:

$$R_1 = R_{\text{bolt to wrench}} = 0.012\ \Omega$$
$$R_2 = R_{\text{wrench to ring}} = 0.012\ \Omega$$
$$R_3 = R_{\text{ring}} = 0.012\ \Omega$$
$$R_4 = R_{\text{ring to frame}} = 0.012\ \Omega$$

What power is quickly dissipated in the gold ring, and what is the impact of this power dissipation? CS

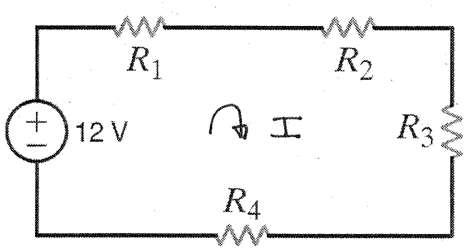


Figure P9.82

SOLUTION:

$$I = \frac{12}{R_1 + R_2 + R_3 + R_4} = \frac{12}{0.048} = 250A$$

$$P_{ring} = I^2 R_3 \Rightarrow \boxed{P_{ring} = 750\,W}$$

The ring will get EXTREMELY hot, burning our mechanic!

9.83 A 5-kW load operates at 60 Hz, 240 V rms and has a power factor of 0.866 lagging. We wish to create a power factor of at least 0.975 lagging using a single capacitor. Can this requirement be met using a single capacitor from Table 6.1?

SOLUTION:

$$|S_{old}| = \frac{P_L}{pf_{old}} = \frac{5000}{0.866} = 5.77 \text{ kVA} \qquad S_{old} = 5.77\angle 30^\circ \text{ kVA}$$

$$\theta_{old} = \cos^{-1}(pf_{old}) = 30^\circ \qquad Q_{old} = 2.88 \text{ kVAR}$$

$$|S_{new}| = \frac{5000}{0.975} = 5.13 \text{ kVA} \qquad S_{new} = 5.13\angle 12.8^\circ \text{ kVA}$$

$$\theta_{new} = \cos^{-1}(0.975) = 12.8^\circ \qquad Q_{new} = 1.14 \text{ kVAR}$$

$$Q_C = -\omega C |V_L|^2 = Q_{new} - Q_{old} = -1.74 \text{ kVAR}$$

$$\omega = 377 \text{ r/s} \qquad |V_L| = 240 \text{ V}$$

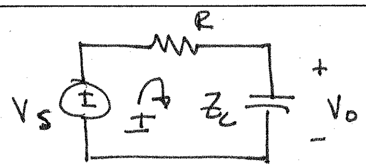
$$\boxed{C = 80.1\,\mu F}$$

$$\boxed{\text{Easily, just use an } 81\,\mu F \text{ capacitor}}$$

9.84 Use an *RC* combination to design a circuit that will reduce a 120-V rms line voltage to a voltage between 75 and 80 V rms while dissipating less than 30 W.

SOLUTION:

This circuit should do



assume $f = 60\,Hz$

$|V_o| = 77.5$ Vrms

$|V_s| = 120$ Vrms

$$\left|\frac{V_o}{V_s}\right| = \frac{77.5}{120} = \frac{1/\omega C}{\sqrt{R^2 + (1/\omega C)^2}}$$

or,

$$\left(\frac{77.5}{120}\right)^2 = \frac{1}{1 + (\omega RC)^2} \Rightarrow 1 + (\omega RC)^2 = \left(\frac{120}{77.5}\right)^2 \qquad (1)$$

Also, $|I|^2 R < 30 \Rightarrow |I| = \dfrac{|V_s|}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}}$

or,

$$\frac{|V_s|^2 (\omega C)^2 R}{1 + (\omega RC)^2} < 30 \qquad (2)$$

Substitute (1) into (2) yields

$$\frac{|V_s|^2 (\omega C)^2 R}{|V_s|^2 / |V_o|^2} = |V_o|^2 (\omega C)^2 R < 30$$

Arbitrarily choose $\boxed{C = 10\,\mu F}$

yields $R < 351\,\Omega$

Quite arbitrarily choose $\boxed{R = 300\,\Omega}$

9FE-1 An industrial load consumes 120 kW at 0.707 pf lagging and is connected to a 480 $\underline{/0°}$ V rms 60-Hz line. Determine the value of the capacitor that, when connected in parallel with the load, will raise the power factor to 0.95 lagging. CS

SOLUTION:

$$|S_{old}| = \frac{P_L}{pf_{old}} = \frac{120\times10^3}{0.707} = 170\ kVA$$

$$\theta_{old} = \cos^{-1}(pf_{old}) = 45°$$

$$S_{old} = 170\ \underline{/45°}\ kVA$$

$$Q_{old} = 120\ kVAR$$

$$|S_{new}| = 120\times10^3/0.95 = 126\ kVA$$

$$\theta_{new} = \cos^{-1}(0.95) = 18.2°$$

$$S_{new} = 126\ \underline{/18.2°}\ kVA$$

$$Q_{new} = 39.3\ kVAR$$

$$Q_c = -\omega C|V_L|^2 = Q_{new} - Q_{old} = -80.7\ kVAR$$

$$\omega = 377\ r/s \qquad |V_L| = 480\ V_{rms}$$

$$\boxed{C = 929\ \mu F}$$

9FE-2 Determine the average and rms values of the following waveform.

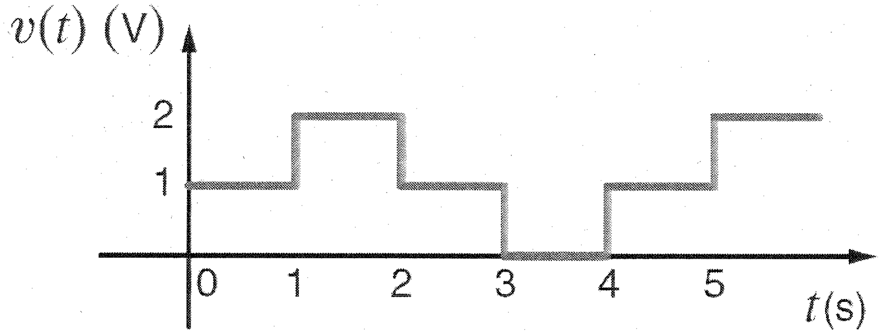


Figure 9PFE-2

SOLUTION:

Average: $\overline{v(t)} = \frac{1}{T}\int_0^T v(t)\,dt$ $\quad T = 4\,s$

$$\left.\begin{aligned} &\int_0^1 v(t)\,dt = \int_0^1 dt = 1 \\ &\int_1^2 v(t)\,dt = \int_1^2 2\,dt = 2 \\ &\int_2^3 v(t)\,dt = 1 \end{aligned}\right\} \quad \overline{v(t)} = \frac{1}{T}\left[1+2+1\right] = 3/4$$

$$\boxed{\overline{v(t)} = 0.75\,V}$$

rms: $V_{rms} = \left\{\frac{1}{T}\int_0^T v^2(t)\,dt\right\}^{1/2}$

for $0<t<1$, $v^2(t) = 1^2 = 1$ & $\int_0^1 v^2 dt = \int_0^1 dt = 1$

for $1<t<2$, $v^2(t) = 2^2 = 4$ & $\int_2^3 v^2 dt = \int_2^3 4\,dt = 4$

for $2<t<3$ same contribution as $0<t<1$, $\int_2^3 v^2 dt = 1$

$$V_{rms} = \left\{\frac{1}{4}\left[1+4+1\right]\right\}^{1/2} \qquad \boxed{V_{rms} = 1.22\,V}$$

9FE-3 Find the impedance $\mathbf{Z}_L$ in the network in Fig. 9PFE-3 for maximum average power transfer. CS

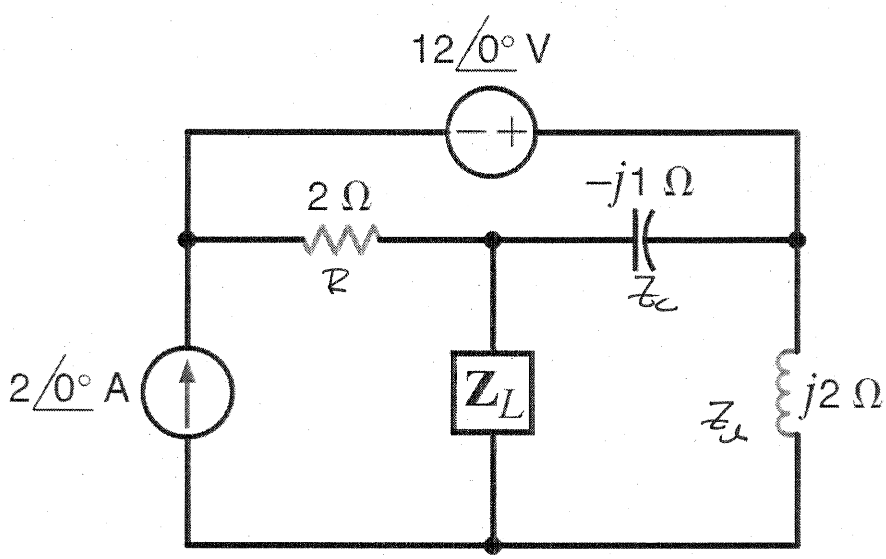
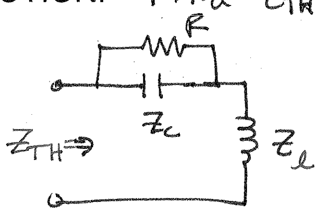


Figure 9PFE-3

SOLUTION: Find Z_{TH}



$$Z_{TH} = Z_l + \frac{RZ_c}{R+Z_c} = j2 + \frac{2(-j1)}{2-j1}$$

$$Z_{TH} = 0.4 + j\,1.2\,\Omega$$

$$Z_L = Z_{TH}^* \qquad \boxed{Z_L = 0.4 - j\,1.2\,\Omega}$$

9FE-4 An rms-reading voltmeter is connected to the output of the op-amp shown in Fig. 9PFE-4. Determine the meter reading.

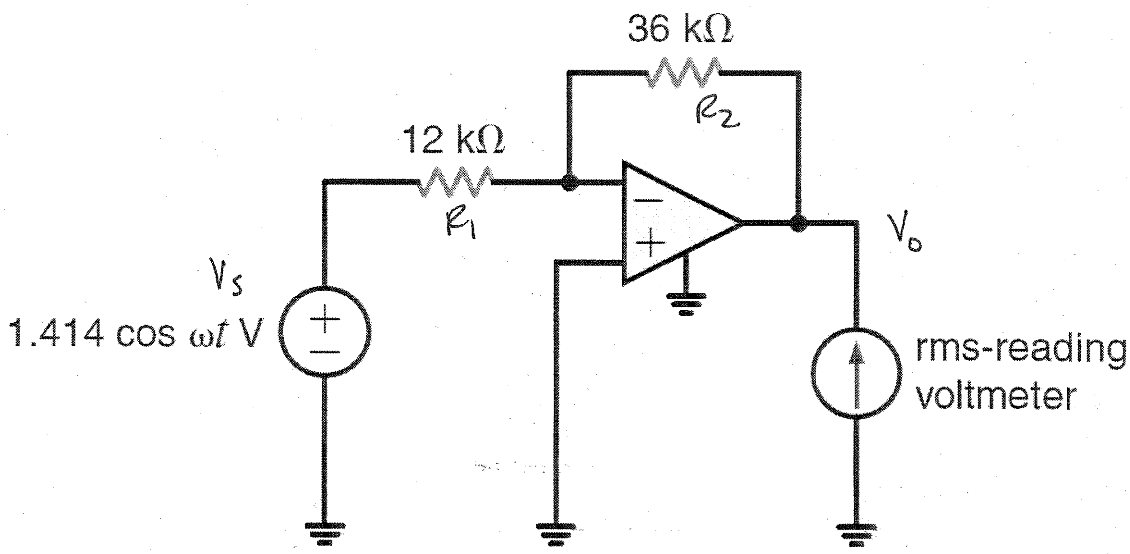


Figure 9PFE-4

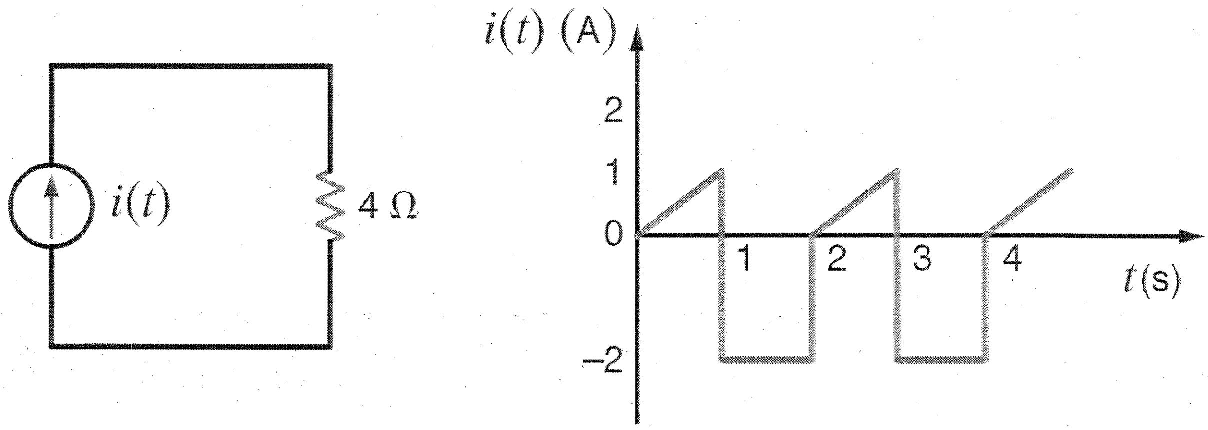
SOLUTION:

$V_o = -\frac{R_2}{R_1} V_s$ $\qquad V_s = 1.414 \angle 0 \text{ V} = 1 \angle 0° \text{ Vrms}$

$V_o = -3V_s = -3 \angle 0° \text{ Vrms}$

$\boxed{\text{meter reads } 3\text{ V}}$

9FE-5 Determine the average power delivered to the resistor in Fig. 9PFE-5a if the current waveform is shown in Fig. 9PFE-5b. CS



(a) (b)

Figure 9PFE-5

SOLUTION:

$P = I_{rms}^2 R$ $\quad T = 2s \quad$ $I_{rms} = \left\{ \frac{1}{T} \int_0^T i^2(t)\,dt \right\}^{1/2}$

for $0 < t \le 1$, $i(t) = t$ & $i^2(t) = t^2$ $\quad \int_0^1 t^2 dt = \frac{t^3}{3}\Big|_0^1 = 1/3$

for $1 < t \le 2$, $t(t) = -2$ & $i^2(t) = 4$ $\quad \int_1^2 4dt = 4$

$I_{rms} = \left\{ \frac{1}{2}\left[1/3 + 4 \right] \right\}^{1/2} = 1.47\,A$

$P = (1.47)^2 (4)$ $\quad \boxed{P = 8.67\ W}$

Chapter Ten:

Magnetically Coupled Networks

10.1 Given the network in Fig. P10.1,

(a) find the equations for $v_a(t)$ and $v_b(t)$.

(b) find the equations for $v_c(t)$ and $v_d(t)$.

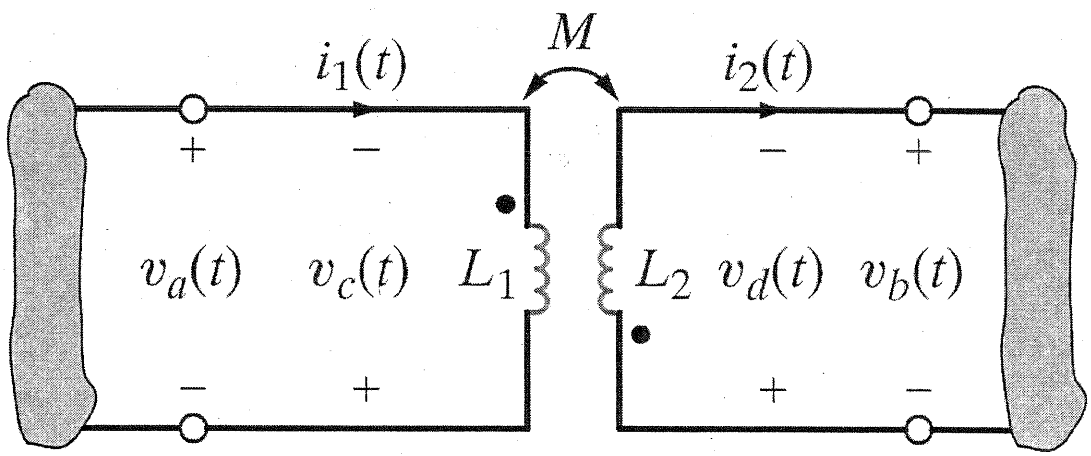


Figure P10.1

SOLUTION:

a) $v_a(t) = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$ $\qquad v_b(t) = -M\frac{di_1}{dt} - L_2\frac{di_2}{dt}$

b) $v_c(t) = -v_a(t) = -L_1\frac{di_1}{dt} - M\frac{di_2}{dt}$

$v_d(t) = -v_b(t) = M\frac{di_1}{dt} + L_2\frac{di_2}{dt}$

10.2 Given the network in Fig. P10.2,

(a) write the equations for $v_a(t)$ and $v_b(t)$.

(b) write the equations for $v_c(t)$ and $v_d(t)$.

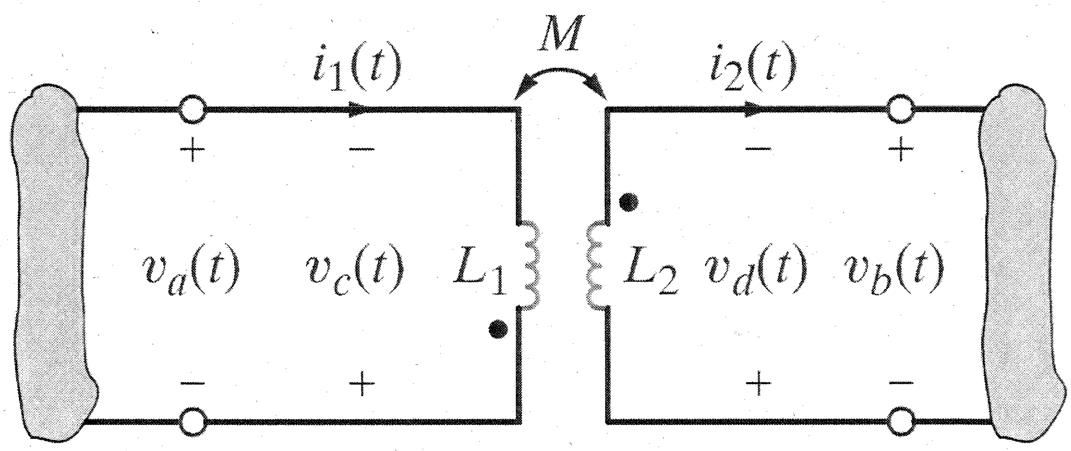


Figure P10.2

SOLUTION:

a) $v_a(t) = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$ $\quad$ $v_b(t) = -M \frac{di_1}{dt} - L_2 \frac{di_2}{dt}$

b) $v_c(t) = -v_a(t) = -L_1 \, di_1/dt - M \, di_2/dt$

$v_d(t) = -v_b(t) = M \frac{di_1}{dt} + L_2 \, di_2/dt$

10.3 Given the network in Fig. P10.3, CS

(a) find the equations for $v_a(t)$ and $v_b(t)$.

(b) find the equations for $v_c(t)$ and $v_d(t)$.

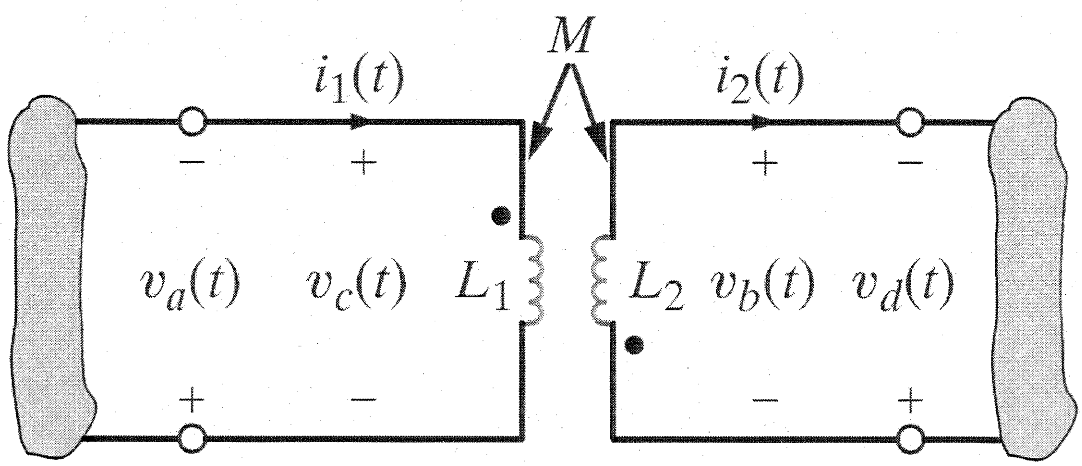


Figure P10.3

SOLUTION:

a) $v_a(t) = -L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$ $\quad$ $v_b(t) = -M\frac{di_1}{dt} - L_2 \frac{di_2}{dt}$

b) $v_c(t) = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$ $\quad$ $v_d(t) = M\frac{di_1}{dt} + L_2 \frac{di_2}{dt}$

10.4 Given the network in Fig. P10.4,

(a) write the equations for $v_a(t)$ and $v_b(t)$.

(b) write the equations for $v_c(t)$ and $v_d(t)$.

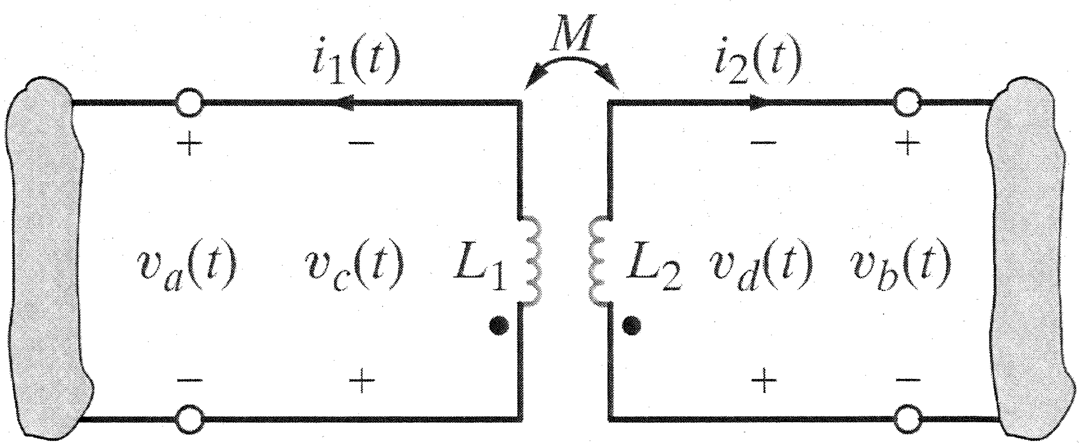


Figure P10.4

SOLUTION:

a) $v_a(t) = -L_1 \frac{di_1}{dt} - M\frac{di_2}{dt}$ $\quad v_b(t) = -\frac{M di_1}{dt} - L_2\frac{di_2}{dt}$

b) $v_c(t) = L_1\, di_1/dt + M\, di_2/dt$ $\quad v_d(t) = M\frac{di_1}{dt} + L_2\frac{di_2}{dt}$

10.5 Find the voltage gain $\mathbf{V}_O/\mathbf{V}_S$ of the network shown in Fig. P10.5. CS

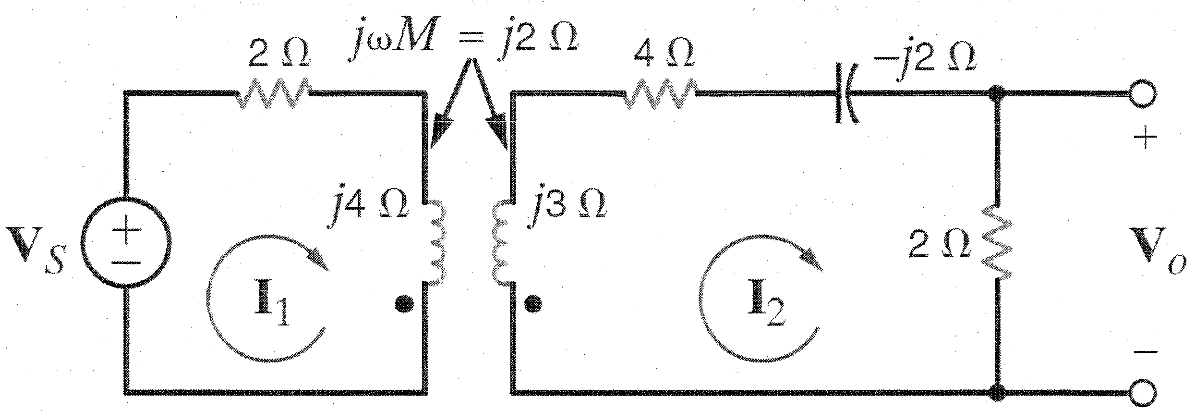


Figure P10.5

SOLUTION:

$V_S = I_1(2+j4) - j2I_2$ & $0 = -j2I_1 + I_2(6+j1)$

$V_o = 2I_2$ $\quad I_1 = \dfrac{V_S + j2I_2}{2+j4}$ $\quad 0 = \dfrac{-j2(V_S + j2I_2)}{2+j4} + I_2(6+j1)$

$I_2 = \dfrac{jV_S}{6+j13}$ $\quad V_o = \dfrac{j2V_S}{6+j13}$ $\quad \dfrac{V_o}{V_S} = \dfrac{j2}{6+j13}$

$$\boxed{\frac{V_o}{V_S} = 0.140\ \angle 24.8^\circ}$$

10.6 Find $\mathbf{V}_o$ in the network in Fig. P10.6.

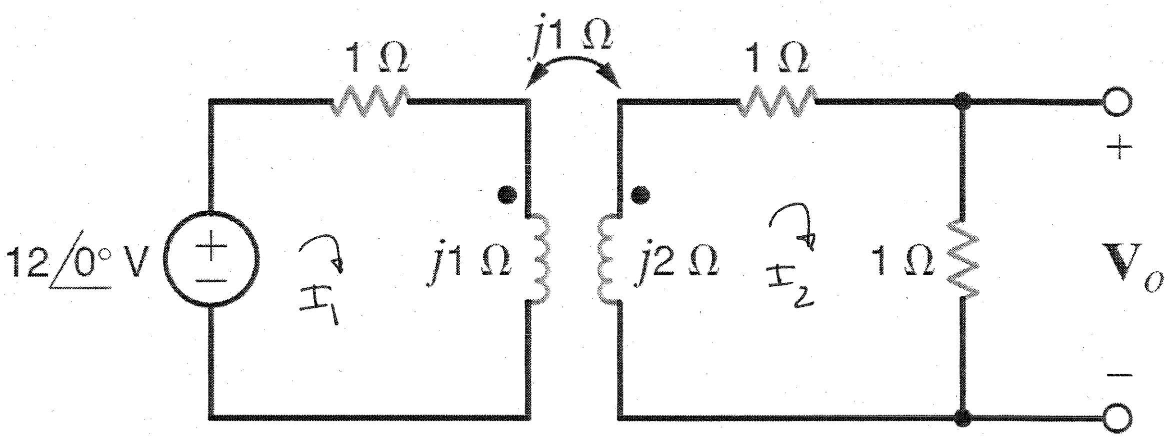


Figure P10.6

SOLUTION:

$12\angle 0 = I_1(1+j1) - j1 I_2$ & $0 = -j1 I_1 + I_2(2+j2)$ $V_o = 1(I_2)$

$$\begin{bmatrix} 1+j1 & -j1 \\ -j1 & 2+j2 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \end{bmatrix} \Rightarrow I_2 = 2.91\angle 14.0^\circ \text{ A}$$

$$\boxed{V_o = 2.91\angle 14.0^\circ \text{ V}}$$

10.7 Given the network in Fig. P10.7, find $\mathbf{V}_o$. **PSV**

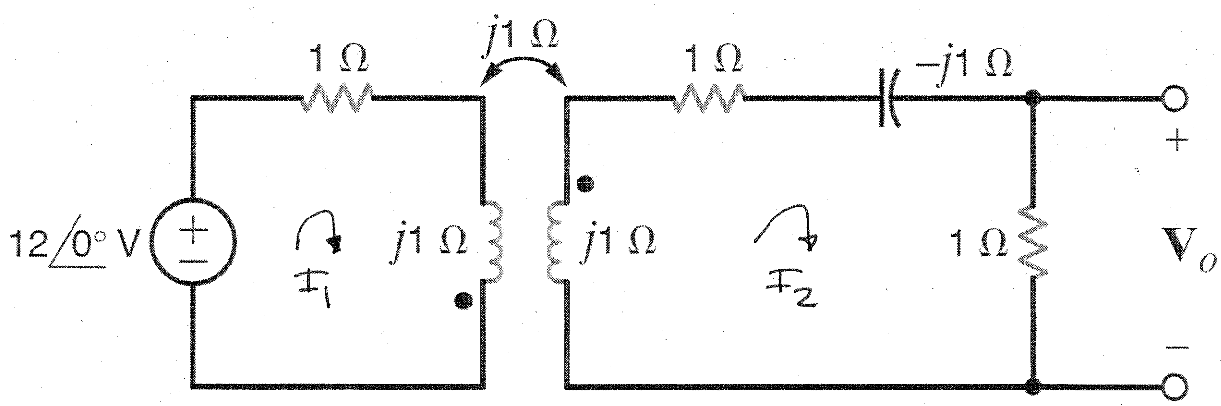


Figure P10.7

SOLUTION:

$12\angle 0^\circ = I_1(1+j1) + j1\,I_2$ & $0 = j1\,I_1 + I_2(2)$ & $V_o = (1)I_2$

$$\begin{bmatrix} 1+j1 & j1 \\ j1 & 2 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \end{bmatrix} \Rightarrow I_2 = 3.32\angle -124^\circ \text{ A}$$

$$\boxed{V_o = 3.32\angle -124^\circ \text{ V}}$$

10.8 Find $\mathbf{V}_o$ in the circuit in Fig. P10.8. CS

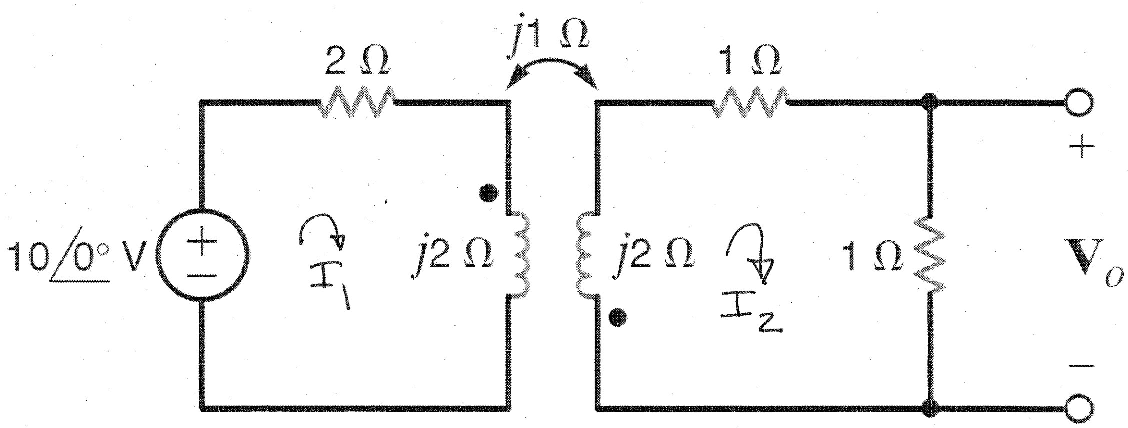


Figure P10.8

SOLUTION:

$10\angle 0° = I_1(2+j2) + j1 I_2$ & $j1 I_1 + I_2(2+j2) = 0$ & $V_o = (1) I_2$

$$\begin{bmatrix} 2+j2 & j1 \\ j1 & 2+j2 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \end{bmatrix} \Rightarrow I_2 = 1.24\angle -173° \text{ A}$$

$$\boxed{V_o = 1.24\angle -173° \text{ V}}$$

10.9 Find the voltage gain $\mathbf{V}_o/\mathbf{V}_S$ of the network shown in Fig. P10.9.

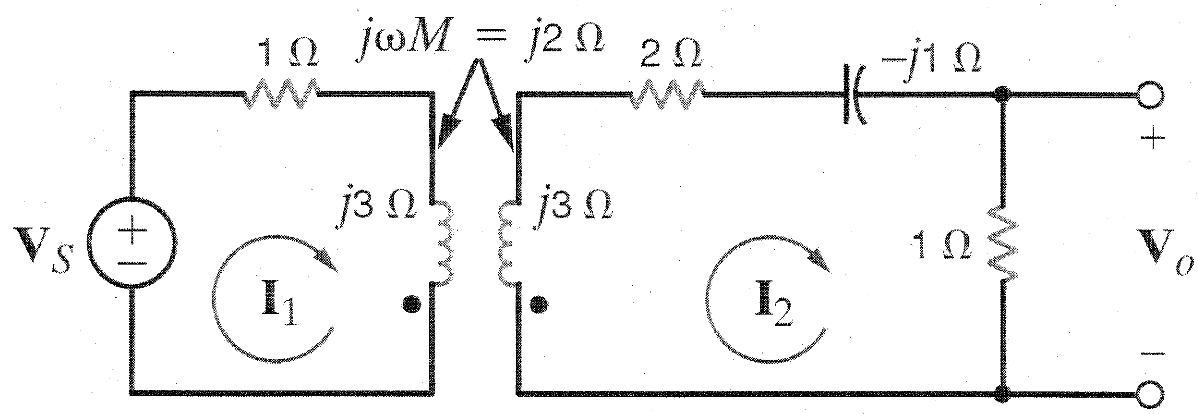


Figure P10.9

SOLUTION:

$V_o = (1)I_2$ & $V_S = I_1(1+j3) - j2I_2$ & $-j2I_1 + I_2(3+j2) = 0$

Let $V_S = 1\angle 0°\,V$. Then $\frac{V_o}{V_S} = V_o$

$$\begin{bmatrix} 1+j3 & -j2 \\ -j2 & 3+j2 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \Rightarrow I_2 = 0.181\angle 5.19°\ A$$

$$\boxed{\frac{V_o}{V_S} = 0.181\angle 5.19°}$$

10.10 Find $\mathbf{V}_o$ in the network in Fig. P10.10.

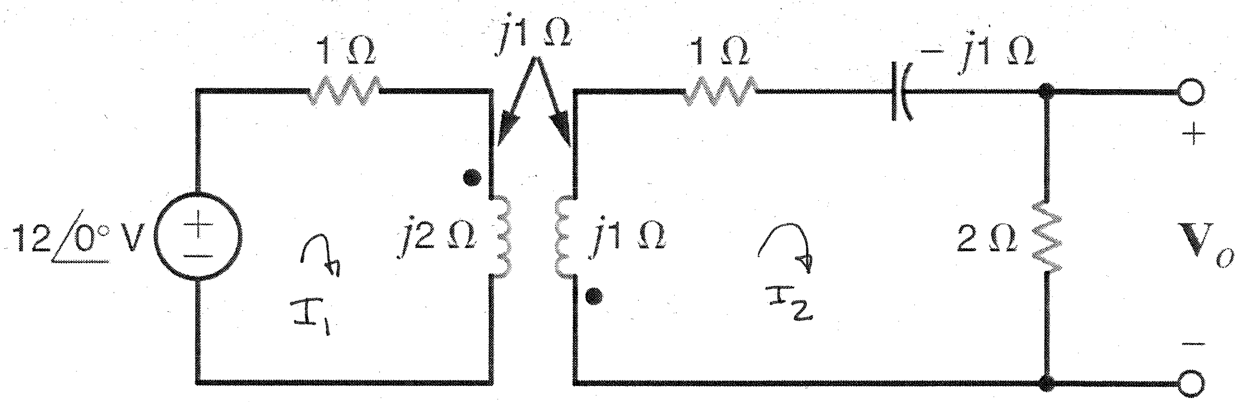


Figure P10.10

SOLUTION:

$12\angle 0° = I_1(1+j2) + j1I_2$ & $j1I_1 + I_2(3) = 0$ & $V_o = 2I_2$

$$\begin{bmatrix} 1+j2 & j1 \\ j1 & 3 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \end{bmatrix} \Rightarrow I_2 = 1.66\angle -146° \text{ A}$$

$$\boxed{V_o = 3.33\angle -146° \text{ V}}$$

10.11 Find $\mathbf{V}_o$ in the network in Fig. P10.11.

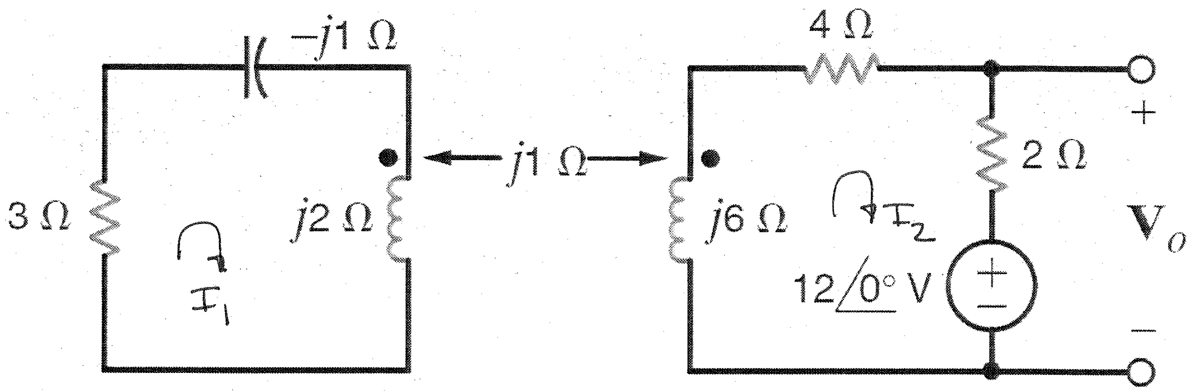


Figure P10.11

SOLUTION:

$I_1(3+j1) - jI_2 = 0$ & $-12\angle 0° = -jI_1 + I_2(6+j6)$ & $V_o = 12\angle 0 + 2I_2$

$$\begin{bmatrix} 3+j1 & -j1 \\ j1 & 6+j6 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0 \\ -12 \end{bmatrix} \Rightarrow I_2 = 1.39\angle 137° \text{ A}$$

$$\boxed{V_o = 10.15\angle 10.8° \text{ V}}$$

10.12 Find $\mathbf{V}_o$ in the circuit in Fig. P10.12.

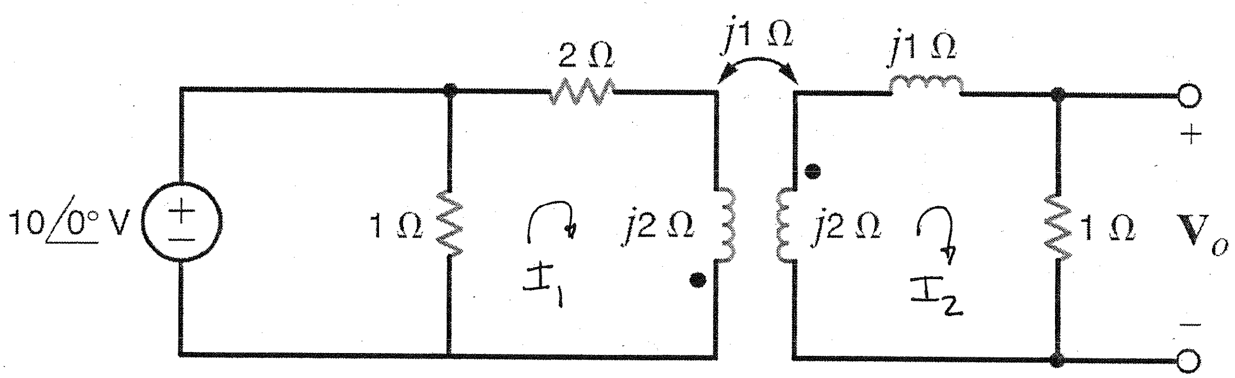


Figure P10.12

SOLUTION:

$10\angle 0^\circ = I_1(2+j2) + j1 I_2$ & $0 = j1 I_1 + I_2(1+j3)$ & $V_o = 1 I_2$

$$\begin{bmatrix} 2+j2 & j1 \\ j1 & 1+j3 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \end{bmatrix} \Rightarrow I_2 = 1.17 \angle 159^\circ \text{ A}$$

$$\boxed{V_o = 1.17 \angle 159^\circ \text{ V}}$$

10.13 Find $\mathbf{V}_o$ in the network in Fig. P10.13.

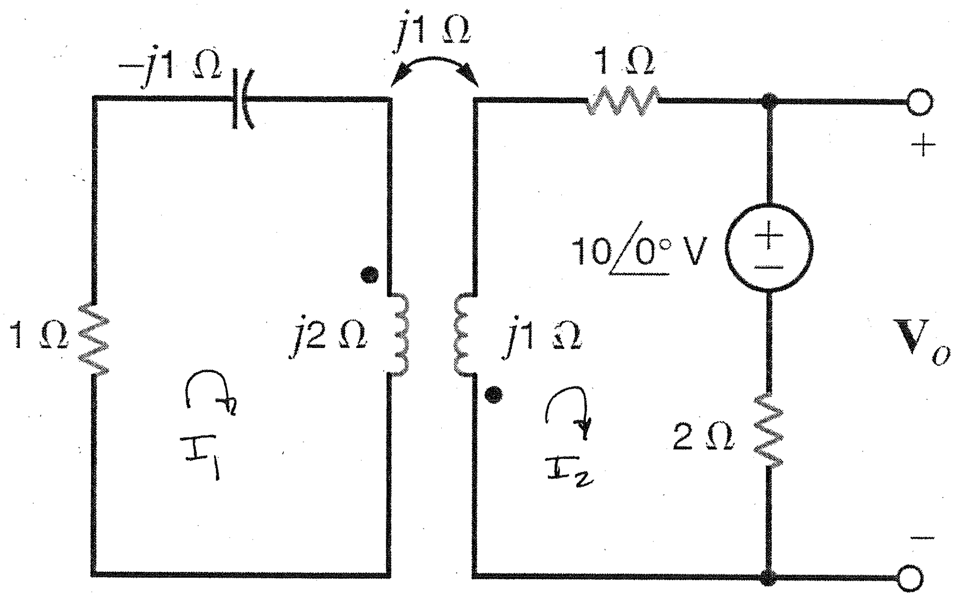


Figure P10.13

SOLUTION:

$I_1(1+j1) + jI_2 = 0$ & $-10\angle 0° = jI_1 + I_2(3+j1)$ & $V_o = 2I_2 + 10\angle 0$

$$\begin{bmatrix} 1+j1 & j1 \\ j1 & 3+j1 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0 \\ -10 \end{bmatrix} \Rightarrow I_2 = 2.83\angle 172° \text{ A}$$

$$\boxed{V_o = 4.47\angle 10.3° \text{ V}}$$

10.14 Find $\mathbf{V}_o$ in the network in Fig. P10.14.

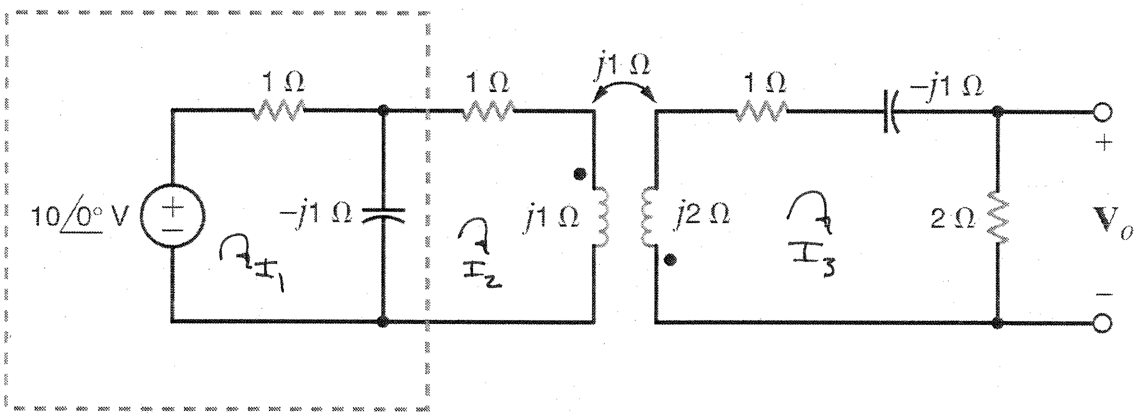


Figure P10.14

SOLUTION:

$10\angle 0 = I_1(1-j1) + j1\, I_2$ & $j1 I_1 + I_2(1) + j I_3 = 0$ & $j I_2 + I_3(3+j1) = 0$

$$\begin{bmatrix} 1-j1 & j1 & 0 \\ j1 & 1 & j1 \\ 0 & j1 & 3+j1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix} \Rightarrow$$

$V_o = 2 I_3$

$I_3 = 1.21 \angle -166° \text{ A}$

$$\boxed{V_o = 2.42 \angle -166° \text{ V}}$$

10.15 Find $\mathbf{V}_o$ in the network in Fig. P10.15.

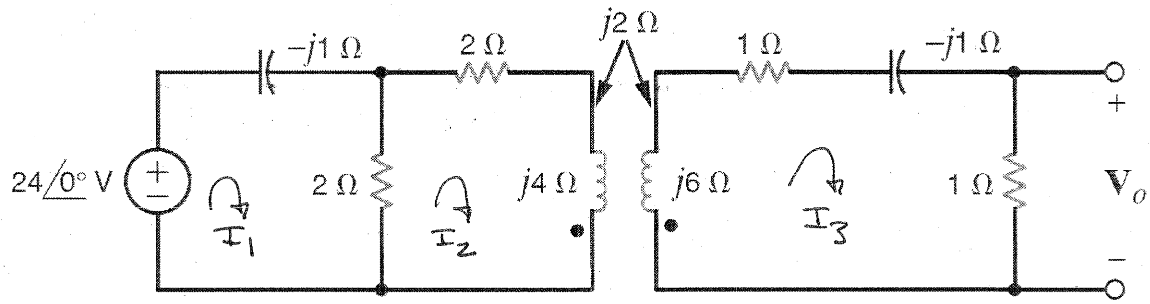


Figure P10.15

SOLUTION:

$24\angle 0° = I_1(2-j1) - 2I_2$ & $-2I_1 + I_2(4+j4) - j2I_3 = 0$ & $-j2I_2 + I_3(2+j5) = 0$

$$\begin{bmatrix} 2-j1 & -2 & 0 \\ -2 & 4+j4 & -j2 \\ 0 & -j2 & 2+j5 \end{bmatrix} \begin{bmatrix} 24 \\ 0 \\ 0 \end{bmatrix} \Rightarrow$$

$V_o = (1)I_3$

$I_3 = 2.17\angle 5.19°$ A

$$\boxed{V_o = 2.17\angle 5.19° \text{ V}}$$

10.16 Find $\mathbf{V}_o$ in the circuit in Fig. P10.16. CS

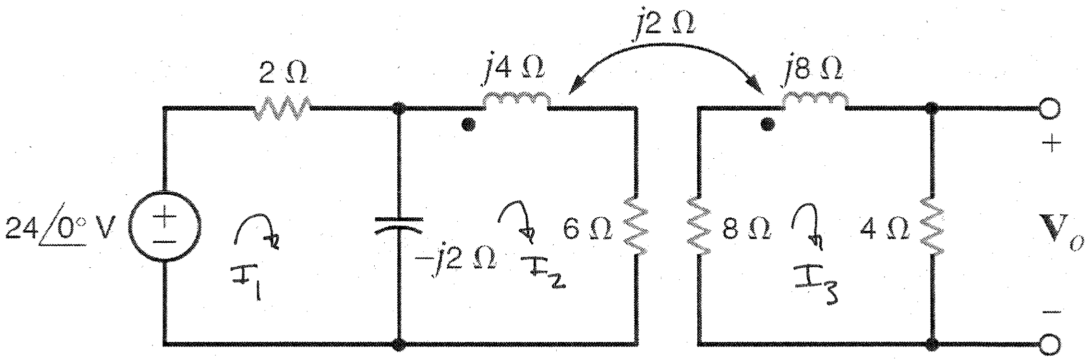


Figure P10.16

SOLUTION:

$24\angle 0° = I_1(2-j2)+j2I_2$ & $j2I_1+I_2(6+j2)+j2I_3=0$ & $j2I_2+I_3(12+j8)=0$

$$\begin{bmatrix} 2-j2 & j2 & 0 \\ j2 & 6+j2 & j2 \\ 0 & j2 & 12+j8 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}=\begin{bmatrix} 24 \\ 0 \\ 0 \end{bmatrix} \Rightarrow$$

$I_3 = 0.303\angle 170°$ A

$V_o = 4I_3$

$\boxed{V_o = 1.21\angle 170° \text{ V}}$

10.17 Find $\mathbf{V}_o$ in the network in Fig. P10.17.

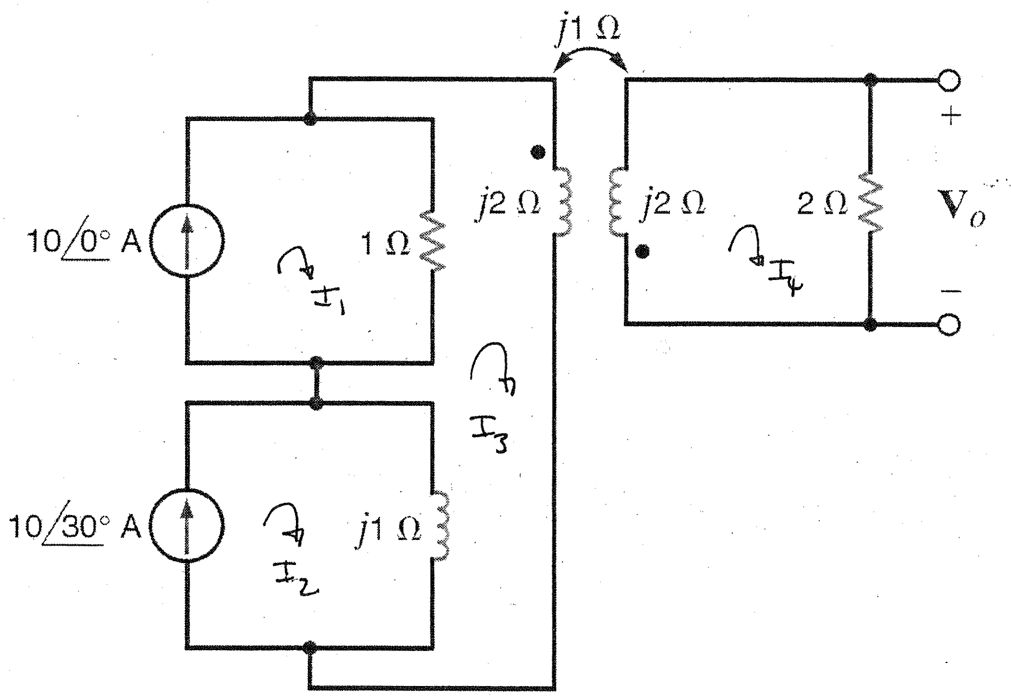


Figure P10.17

SOLUTION:

$I_1 = 10\angle 0°$ A $\quad I_2 = 10\angle 30°$A $\quad I_3(1+j3) - I_1 - jI_2 + jI_4 = 0$

yields, $\quad I_3(1+j3) + jI_4 = I_1 + jI_2 = 5 + j8.66$

and $\quad jI_3 + I_4(2+j2) = 0 \quad$ & $V_o = 2I_4$

$$\begin{bmatrix} 1+j3 & j1 \\ j1 & (2+j2) \end{bmatrix} \begin{bmatrix} I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} 5+j8.66 \\ 0 \end{bmatrix} \Rightarrow \quad I_4 = 1.17\angle -141° \text{ A}$$

$$\boxed{V_o = 2.34\angle -141° \text{ V}}$$

10.18 Find $\mathbf{I}_o$ in the circuit in Fig. P10.18.

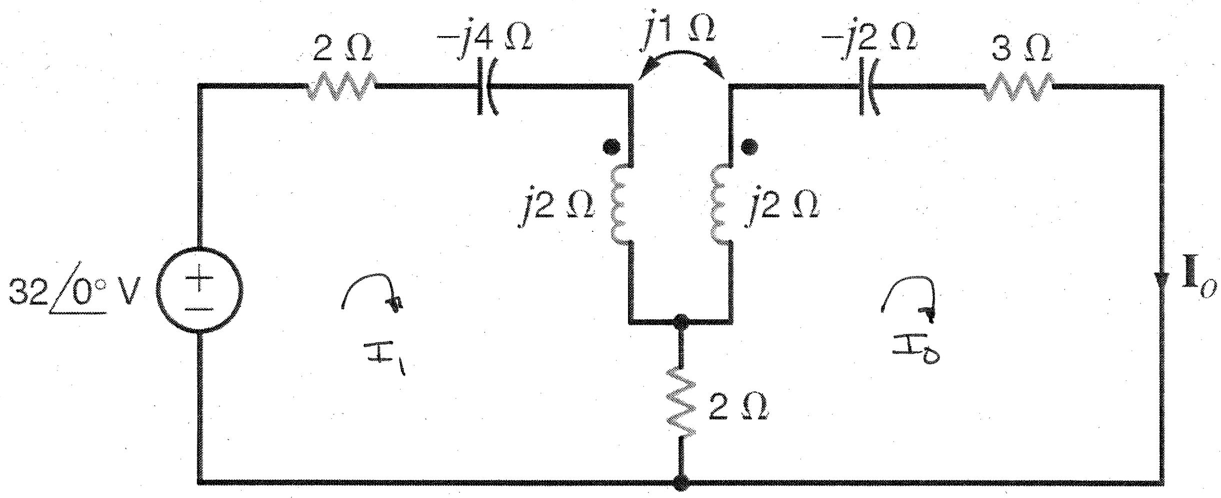


Figure P10.18

SOLUTION:

$$32\angle 0° = I_1(2-j4+j2+2) - jI_o - 2I_o \Rightarrow 32\angle 0 = I_1(4-j2) + I_o(-2-j1)$$

$$0 = -I_1(j1) - 2I_1 + I_2(2+j2-j2+3) \Rightarrow 0 = I_1(-2-j1) + I_2(5)$$

$$\begin{bmatrix} 4-j2 & -2-j1 \\ -2-j1 & 5 \end{bmatrix}\begin{bmatrix} I_1 \\ I_o \end{bmatrix} = \begin{bmatrix} 32 \\ 0 \end{bmatrix} \Rightarrow \boxed{I_o = 3.25\angle 66.0° \text{ A}}$$

10.19 Write the mesh equations for the network in Fig. P10.19.

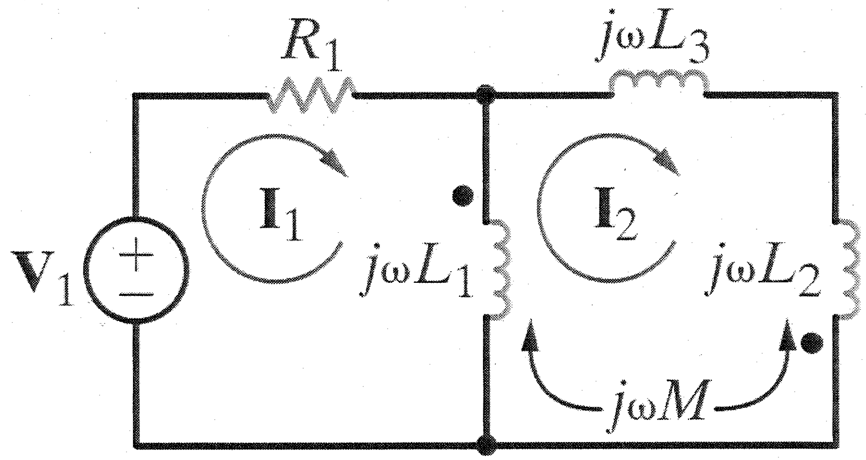


Figure P10.19

SOLUTION:

$$V_1 = I_1 (R_1 + j\omega L_1) - j\omega M I_2$$

$$0 = -j\omega M I_1 + I_2 (j\omega)(L_1 + L_2 + L_3)$$

10.20 Write the mesh equations for the network in Fig. P10.20.

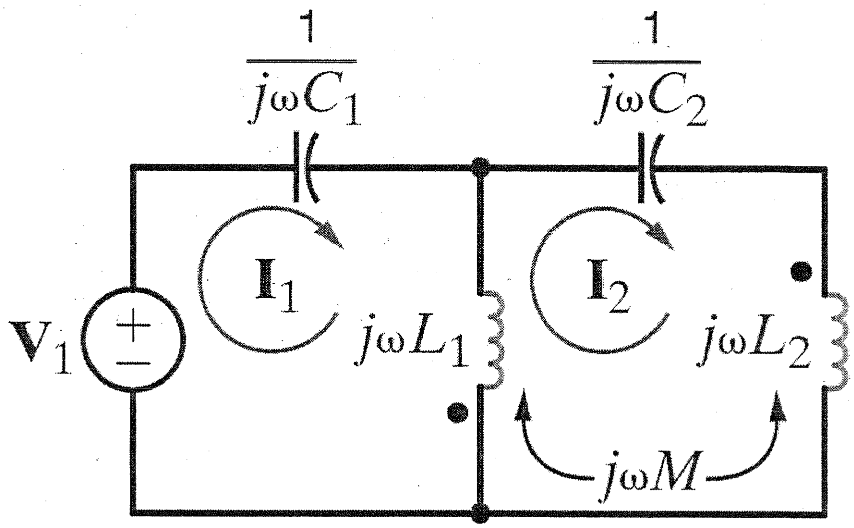


Figure P10.20

SOLUTION:

$$V_1 = I_1\left(j\omega L - j/\omega C_1\right) - j\omega M I_2 - j\omega L_1 I_2$$

$$0 = -j\omega M I_1 + I_2\left(j\omega(L_1 + L_2) - j/\omega C_2\right) - j\omega L_1 I_1$$

10.21 Write the mesh equations for the network shown in Fig. P10.21. CS

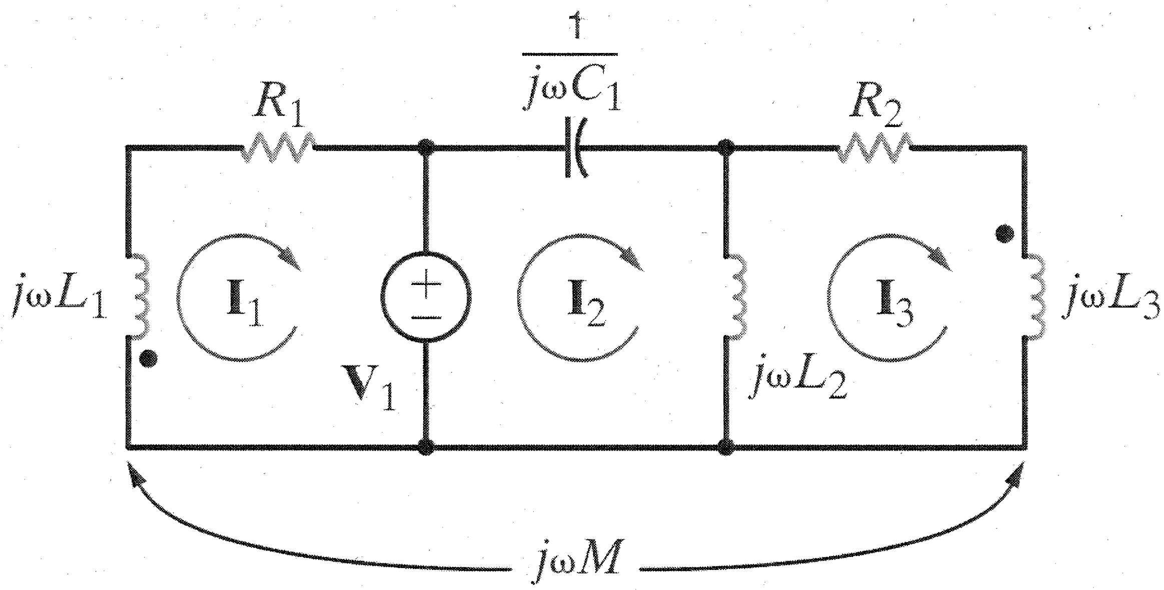


Figure P10.21

SOLUTION:

$$-V_1 = I_1(R_1 + j\omega L_1) + j\omega M I_3$$

$$V_1 = I_2(j\omega L_2 - j/\omega C_1) - j\omega L_2 I_3$$

$$0 = -j\omega L_2 I_2 + I_3(R_2 + j\omega(L_2 + L_3)) + j\omega M I_1$$

10.22 Write the mesh equations for the network shown in Fig. P10.22.

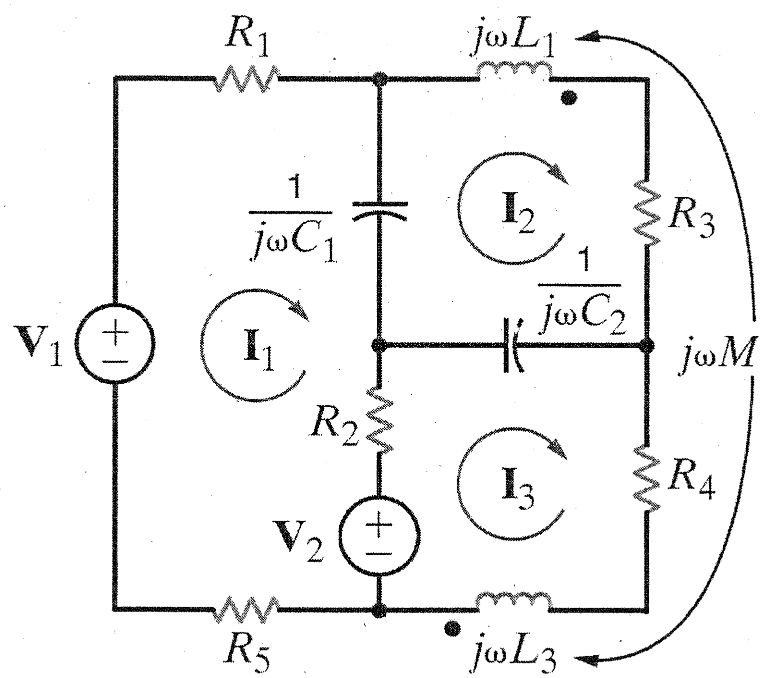


Figure P10.22

SOLUTION:

$$V_1 - V_2 = I_1\left(R_1 + R_2 + R_5 - j/\omega C_1\right) + I_2\left(+ j/\omega_{C_1}\right) - R_2 I_3$$

$$0 = \frac{j}{\omega C_1} I_1 + I_2\left(R_3 + j\left[\omega L_1 - \frac{1}{\omega C_1} - \frac{1}{\omega C_2}\right]\right) + j\omega M I_3 + \frac{j}{\omega C_2} I_3$$

$$V_2 = -R_2 I_1 + I_2\left(j\omega M + \frac{j}{\omega C_2}\right) + I_3\left(R_2 + R_4 + j\omega L_3 - j/\omega C_2\right)$$

10.23 Write the mesh equations for the network in Fig. P10.23.

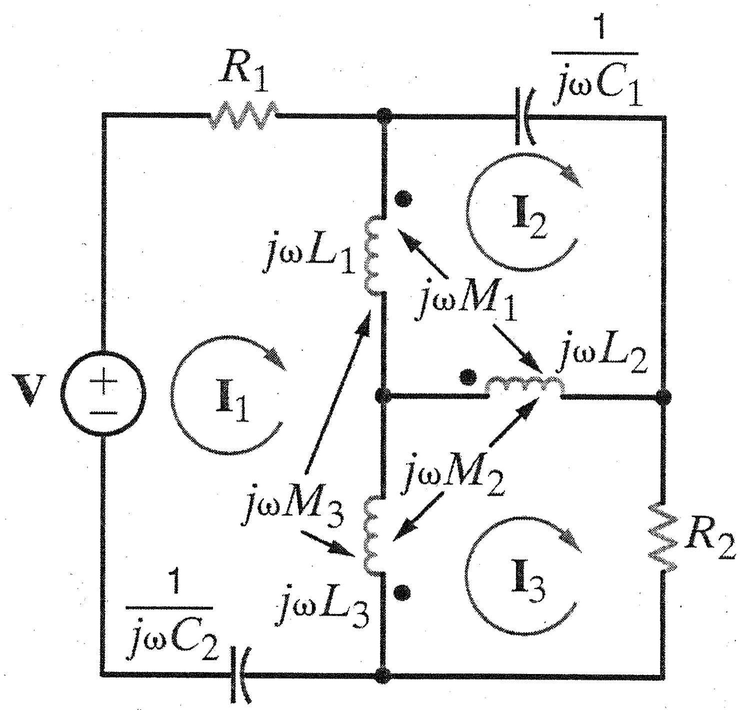


Figure P10.23

SOLUTION:

$$V = I_1\left[R_1 + j\left(\omega L_1 + \omega L_3 - 1/\omega C_2\right)\right] - j\omega L_1 I_2 - j\omega L_3 I_3 \qquad \text{(eq. 1)}$$

$$+ j\omega M_3 (I_3 - I_1) + j\omega M_3 (I_2 - I_1) + j\omega M_1 (I_3 - I_2) + j\omega M_2 (I_2 - I_3)$$

$$0 = I_2\left[j\left(\omega L_1 + \omega L_2 - 1/\omega C_1\right)\right] - j\omega L_1 I_1 - j\omega L_2 I_3 \qquad \text{(eq 2)}$$

$$+ j\omega M_1 (I_2 - I_1) + j\omega M_1 (I_2 - I_3) + j\omega M_2 (I_1 - I_3) + j\omega M_3 (I_1 - I_3)$$

$$0 = I_3\left[R_2 + j\omega(L_2 + L_3)\right] - j\omega L_2 I_2 - j\omega L_3 I_1 \qquad \text{(eq 3)}$$

$$+ j\omega M_1 (I_1 - I_2) + j\omega M_2 (I_3 - I_1) + j\omega M_3 (I_1 - I_2) + j\omega M_2 (I_3 - I_2)$$

$$V = I_1\left[R_1 + j\left(\omega L_1 + \omega L_3 - 2\omega M_3 - \frac{1}{\omega C_2}\right) + I_2\left[j\omega\left(M_3 + M_2 - M_1 - L_1\right)\right]\right.$$
$$+ I_3\left[M_1 - M_2 + M_3 - L_3\right] j\omega$$

$$0 = I_1\left[j\omega\left(M_2 + M_3 - M_1 - L_1\right)\right] + I_2\left[j\left(\omega\left(L_1 + L_2 + 2M_1\right) - 1/\omega C_1\right)\right]$$
$$+ I_3\left[j\omega\left(-M_1 - M_2 - M_3 - L_2\right)\right]$$

$$0 = I_1\left[j\omega\left(M_1 - M_2 + M_3 - L_3\right)\right] + I_2\left[j\omega\left(-M_1 - M_2 - M_3 - L_2\right)\right]$$
$$+ I_3\left[R_2 + j\omega\left(L_2 + L_3 + 2M_2\right)\right]$$

10.24 Find $\mathbf{V}_o$ in the network in Fig. P10.24.

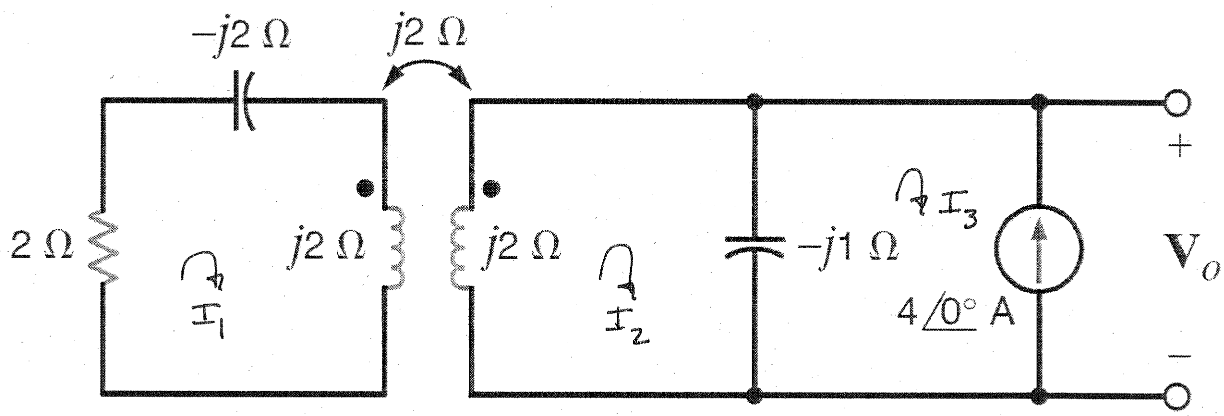


Figure P10.24

SOLUTION:

$0 = I_1(2) - j2 I_2$ & $0 = -j2I_1 + I_2(j1) + j1 I_3$ & $I_3 = -4\angle 0°$ A

$$\begin{bmatrix} 2 & -j2 & 0 \\ -j2 & j1 & j1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -4 \end{bmatrix}$$

$V_o = -j1(I_2 - I_3)$

$I_2 = 0.8 + j1.6$ A

$I_3 = -4 + j0$ A

$$\boxed{V_o = 5.06\angle -71.6°\ \text{V}}$$

10.25 Find $\mathbf{V}_o$ in the network in Fig. P10.25. CS

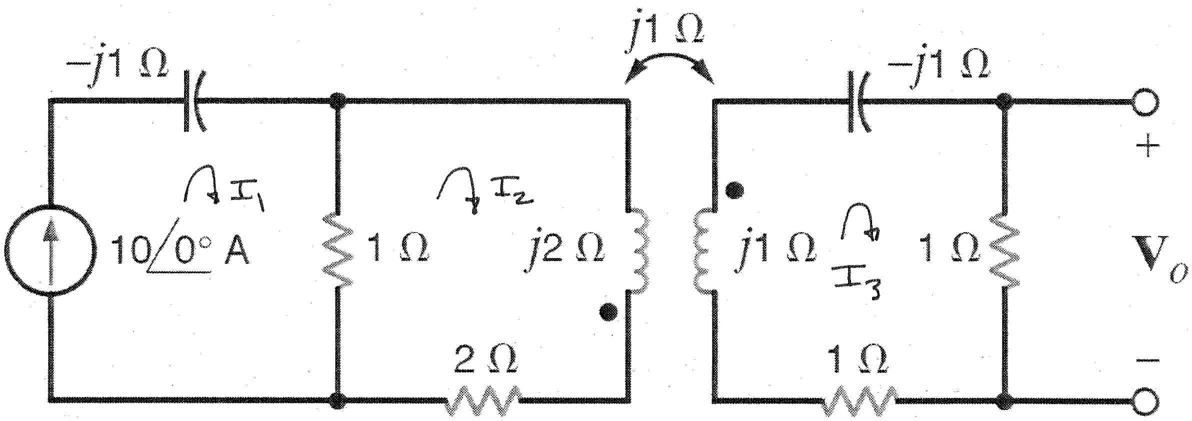


Figure P10.25

SOLUTION:

$I_1 = 10\angle 0°\text{A}$ & $I_2(3+j2) - I_1 + jI_3 = 0$ & $0 = jI_2 + I_3\,[2]$

$$\begin{bmatrix} -1 & 3+j2 & j1 \\ 0 & j1 & 2 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix} \rightarrow$$

$V_o = (1)I_3$

$I_3 = 1.24\angle -120°$ A

$\boxed{V_o = 1.24\angle -120°\ \text{V}}$

10.26 Find $\mathbf{V}_o$ in the network in Fig. P10.26.

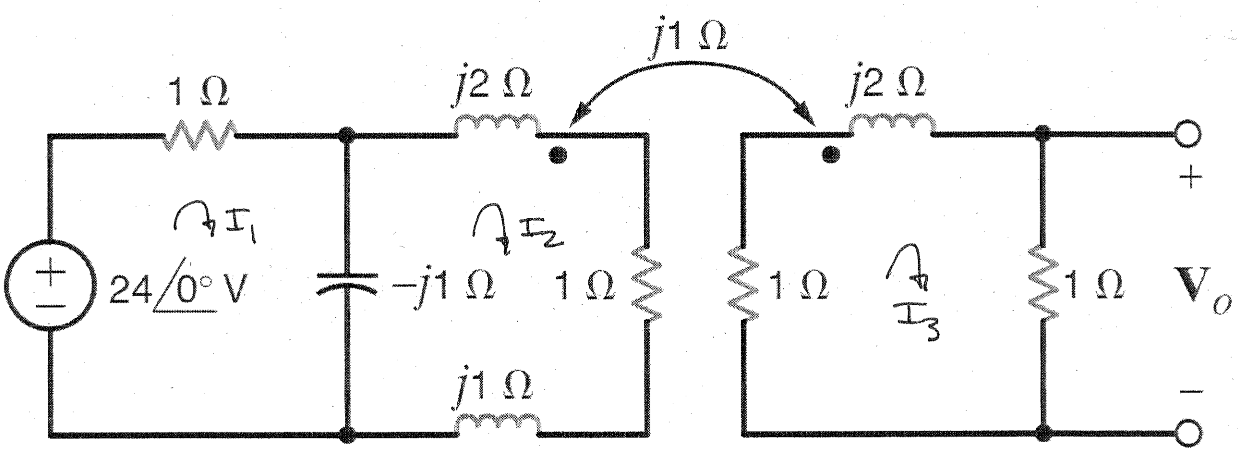


Figure P10.26

SOLUTION:

$24\angle 0° = I_1(1-j1) + jI_2$ & $jI_1 + I_2(1+j2) - j1 I_3 = 0$ $V_o = (1) I_3$

$0 = -jI_2 + I_3(2+j2)$

$$\begin{bmatrix} 1-j1 & j1 & 0 \\ j1 & 1+j2 & -j1 \\ 0 & -j1 & 2+j2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 24 \\ 0 \\ 0 \end{bmatrix} \Rightarrow$$

$I_3 = 2.10 \angle -52.1° \text{ A}$

$$\boxed{V_o = 2.10 \angle -52.1° \text{ V}}$$

10.27 Find $\mathbf{V}_o$ in the network in Fig. P10.27. **PSV**

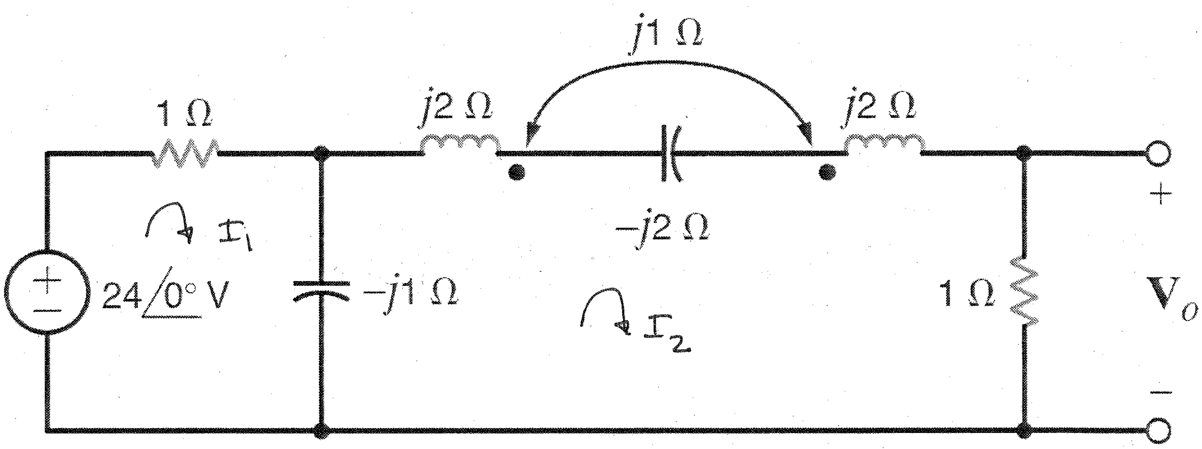


Figure P10.27

SOLUTION:

$24\angle 0^\circ = I_1(1-j1) + j1\,I_2 \quad \& \quad V_o = (1)I_2$

$0 = I_2(1+j1) + j1\,I_1 - j1\,I_2 - j1\,I_2 \Rightarrow jI_1 + I_2(1-j1) = 0$

$$\begin{bmatrix} 1-j1 & j1 \\ j1 & 1-j1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 24 \\ 0 \end{bmatrix}$$

$V_o = (1)I_2$

$I_2 = 10.7\angle -26.6^\circ$ A

$\boxed{V_o = 10.7\angle -26.6^\circ \text{ V}}$

10.28 Find $\mathbf{V}_o$ in the network in Fig. P10.28. CS

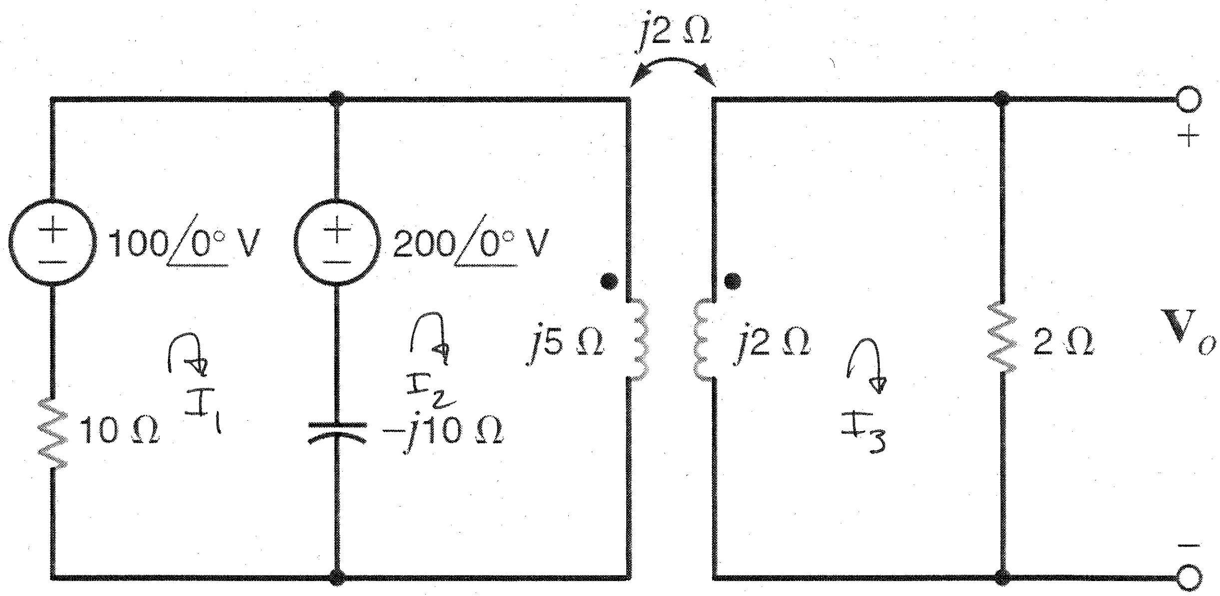


Figure P10.28

SOLUTION:

$$100\angle 0 - 200\angle 0 = I_1(10-j10) + j10 I_2 = -100\angle 0$$

$$200\angle 0 = j10 I_1 + I_2(-j5) - j2 I_3$$

$$0 = -j2 I_2 + I_3(2+j2) \qquad V_o = 2I_3$$

$$\begin{bmatrix} 10-j10 & j10 & 0 \\ j10 & -j5 & -j2 \\ 0 & -j2 & 2+j2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} -100 \\ 200 \\ 0 \end{bmatrix}$$

$$I_3 = 18.4\angle 72.9° \text{ A}$$

$$\boxed{V_o = 36.8\angle 72.9° \text{ V}}$$

10.29 Find $\mathbf{V}_o$ in the network in Fig. P10.29.

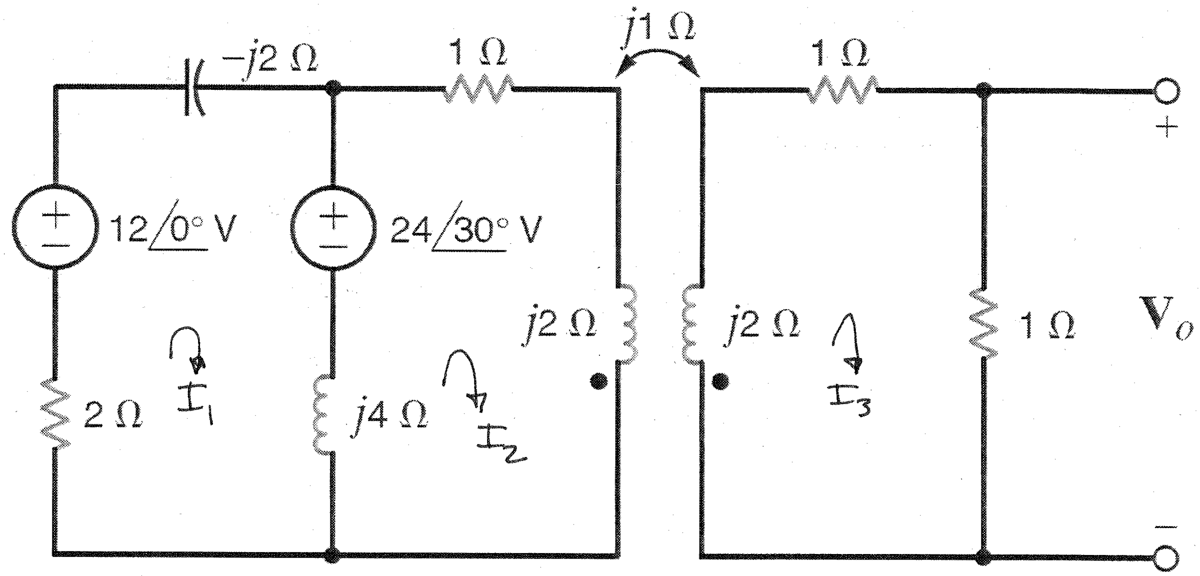


Figure P10.29

SOLUTION:

$$12\angle 0^\circ - 24\angle 30^\circ = I_1(2+j2) - j4I_2 = -8.78 - j12$$

$$24\angle 30^\circ = -j4I_1 + I_2(1+j6) - jI_3 \qquad I_3(1) = V_o$$

$$0 = -jI_2 + I_3(2+j2)$$

$$\begin{bmatrix} 2+j2 & -j4 & 0 \\ -j4 & 1+j6 & -j1 \\ 0 & -j1 & 2+j2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} -8.78 - j12 \\ 20.8 + j12 \\ 0 \end{bmatrix}$$

$$I_3 = 1.63\angle 6.49^\circ \text{ A} \qquad \boxed{V_o = 1.63\angle 6.49^\circ \text{ V}}$$

10.30 Find $\mathbf{V}_o$ in the network in Fig. P10.30.

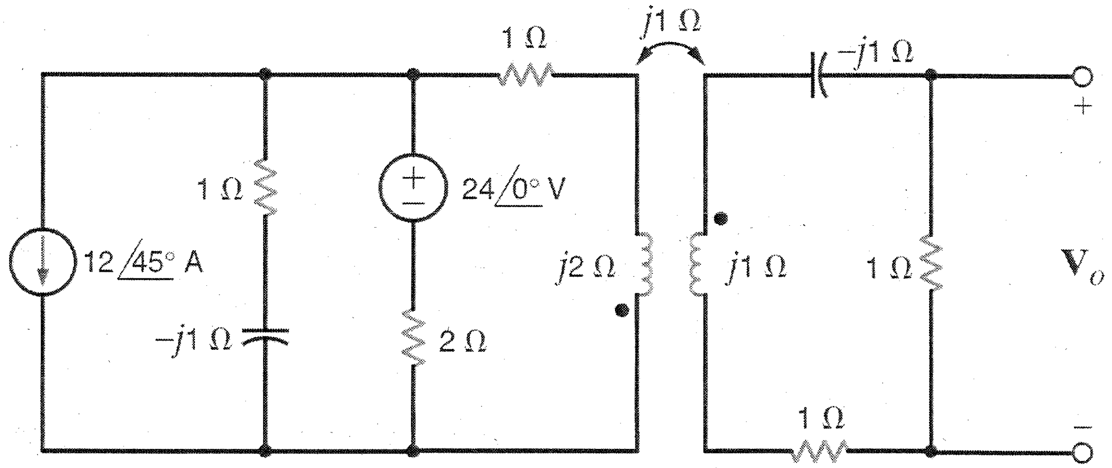
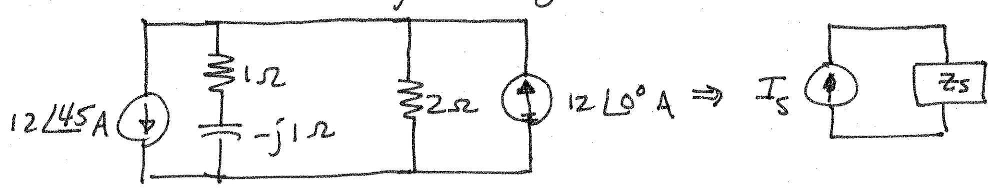


Figure P10.30

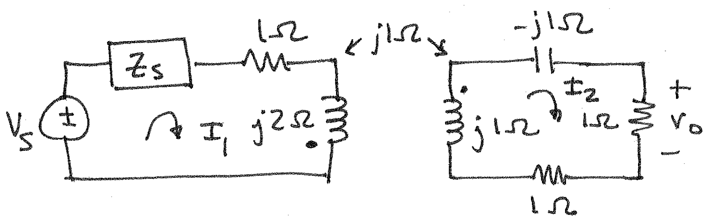
SOLUTION: Simplify through source transformation.



$$I_S = 12\angle 0 - 12\angle 45 = 9.18\angle -67.5^\circ \text{ A}$$

$$Z_S = 2(1-j1)/(3-j1) = 0.8 - j0.4\,\Omega = 0.894\angle -26.6^\circ\,\Omega$$

$$V_S = I_S Z_S = 8.21\angle -94.1^\circ \text{ V}$$



$$V_S = I_1[Z_S + 1 + j2] + j I_2$$

$$0 = j I_1 + I_2(2)$$

$$V_o = (1) I_2$$

$$\begin{bmatrix} 1.8 + j1.6 & j1 \\ j1 & 2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 8.21\angle -94.1^\circ \\ 0 \end{bmatrix} \Rightarrow I_2 = 1.47\angle 141^\circ \text{ A}$$

$$\boxed{V_o = 1.47\angle 141^\circ \text{ V}}$$

10.31 Find $\mathbf{V}_o$ in the network in Fig. P10.31. CS

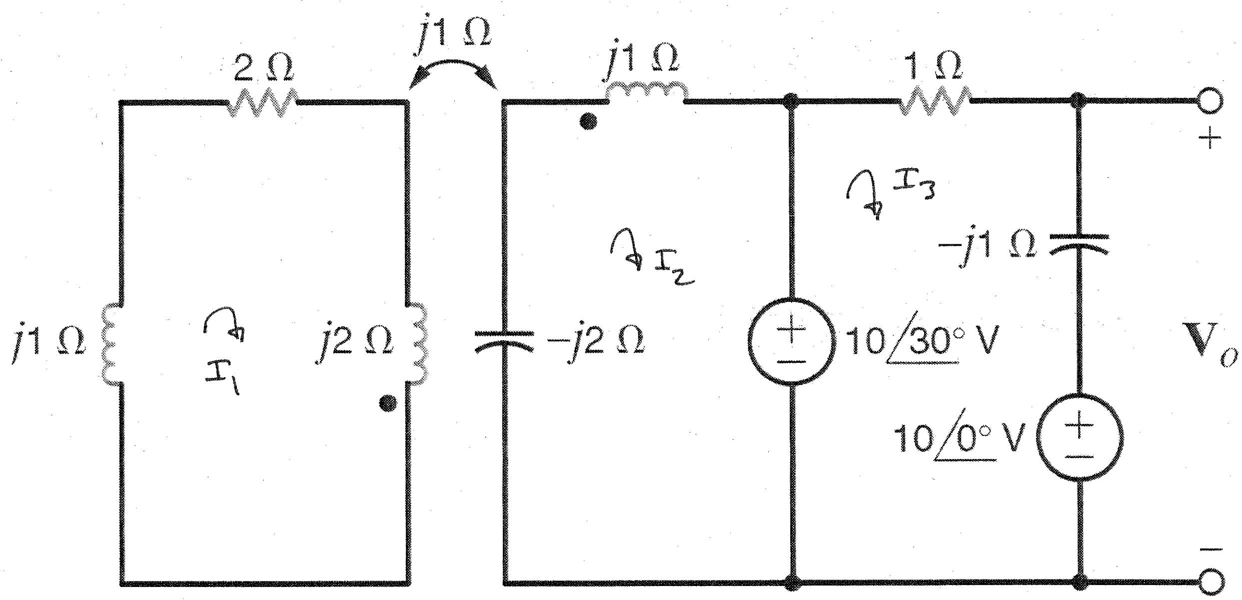


Figure P10.31

SOLUTION:

$I_1(2+j3) - jI_2 = 0$ & $-jI_1 + I_2(-j1) = -10\angle 30°$ & $V_o = -jI_3 + 10\angle 0°$

$10\angle 30 - 10\angle 0 = I_3(1-j1)$

$$\begin{bmatrix} 2+j3 & -j1 & 0 \\ -j1 & -j1 & 0 \\ 0 & 0 & 1-j1 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -10\angle 30° \\ 10\angle 30 - 10\angle 0 \end{bmatrix}$$

$I_3 = 3.66\angle 150°$ A

$$\boxed{V_o = 12.2\angle 15.0° \text{ V}}$$

10.32 Find $\mathbf{I}_o$ in the circuit in Fig. P10.32.

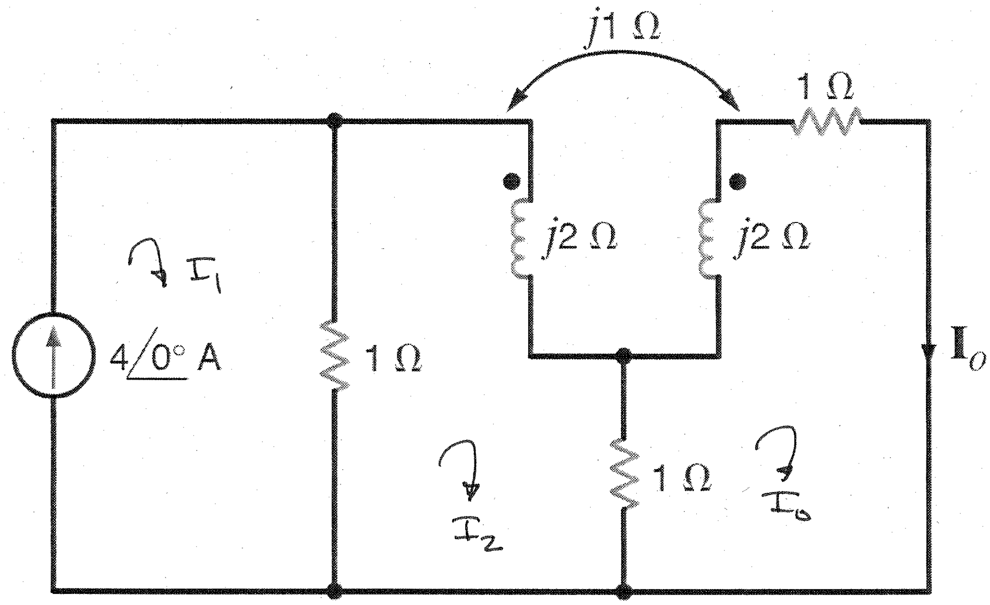


Figure P10.32

SOLUTION:

$I_1 = 4\angle 0°$ & $-I_1 + I_2(2+j2) - jI_o - I_o = 0$

and $0 = I_2(-1-j1) + I_o(2+j2)$

$$\begin{bmatrix} 1 & 0 & 0 \\ -1 & 2+j2 & -1-j1 \\ 0 & -1-j1 & 2+j2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix}$$

$\boxed{I_o = 0.667 - j0.667\text{ A}}$

$\boxed{I_o = 0.943\angle -45°\text{ A}}$

10.33 Find $\mathbf{I}_o$ in the circuit in Fig. P10.33.

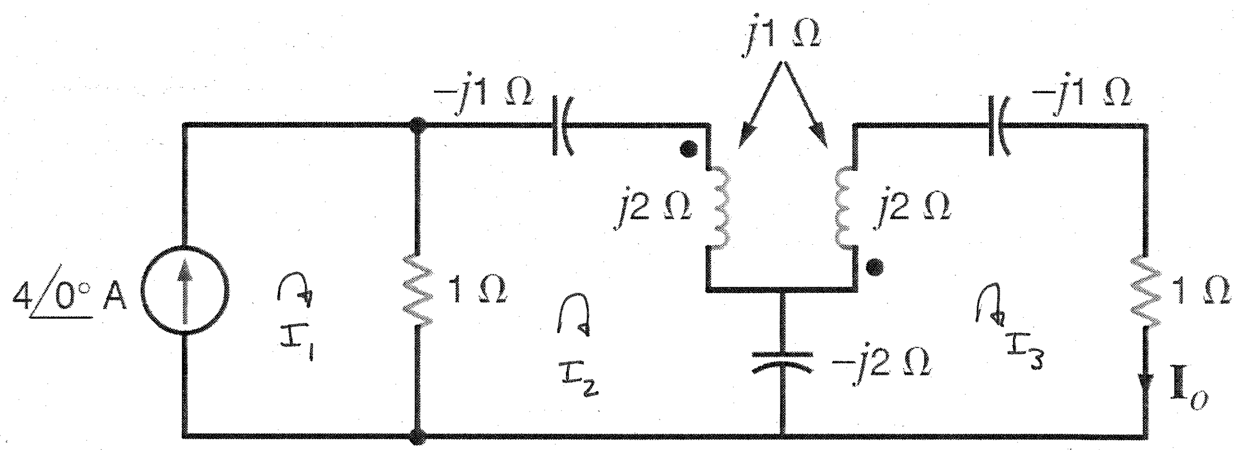


Figure P10.33

SOLUTION:

$I_1 = 4\angle 0° A$ & $0 = -I_1 + I_2(1-j1) + I_3(j1+j2)$ $\quad I_3 = I_o$

and, $\quad 0 = I_2(j2+j1) + I_3(1-j1)$

$$\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1-j1 & j3 \\ 0 & j3 & 1-j1 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \boxed{I_3 = I_o = 1.30\angle -72.5° A}$$

10.34 Find $\mathbf{V}_o$ in the network in Fig. P10.34.

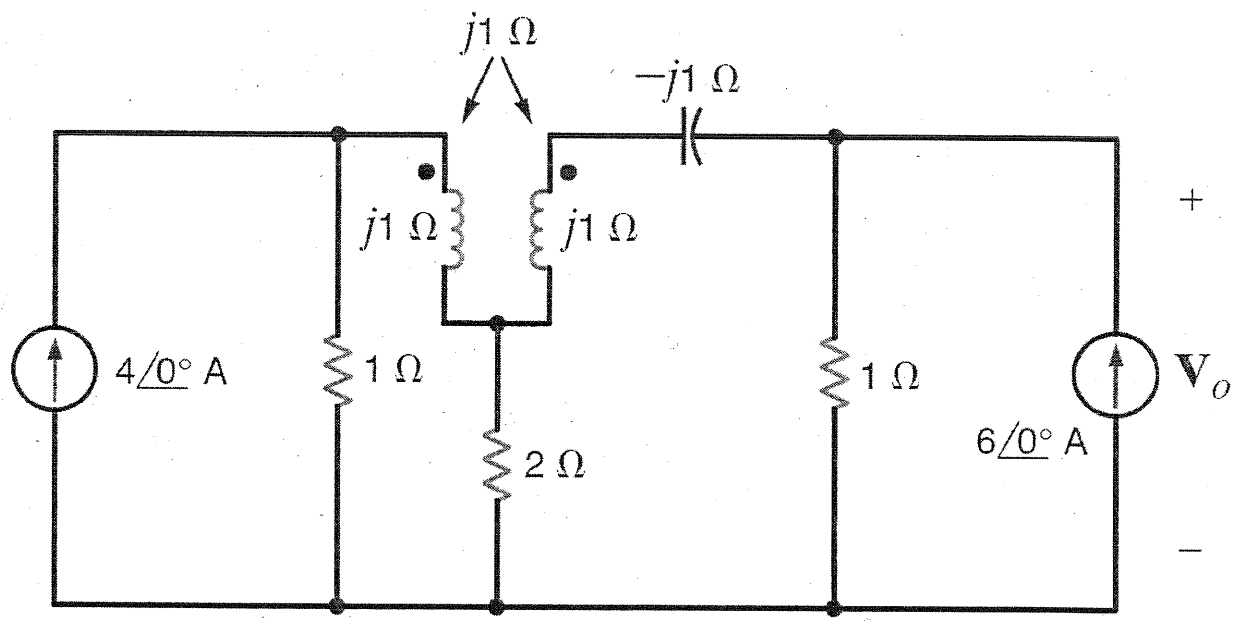
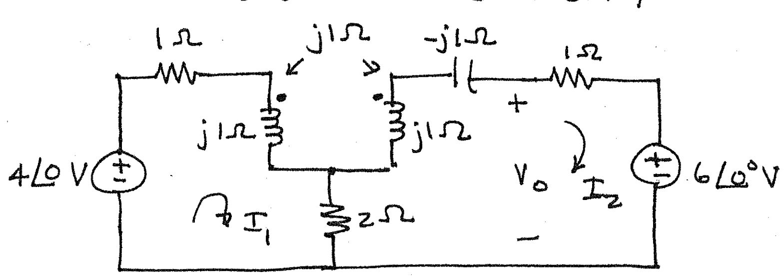


Figure P10.34

SOLUTION: Use source transformations



$$V_o = I_2(1) + 6\angle 0°$$

$$4\angle 0 = I_1(3+j1) - I_2(2+j1) \quad \& \quad -6\angle 0 = -I_1(2+j1) + I_2(3)$$

$$\begin{bmatrix} 3+j1 & -2-j1 \\ -2-j1 & 3 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 4 \\ -6 \end{bmatrix} \Rightarrow I_2 = 1.53\angle -176° \text{ A}$$

$$\boxed{V_o = 4.47\angle -1.51° \text{ V}}$$

10.35 Find $\mathbf{V}_o$ in the circuit in Fig. P10.35. **PSV**

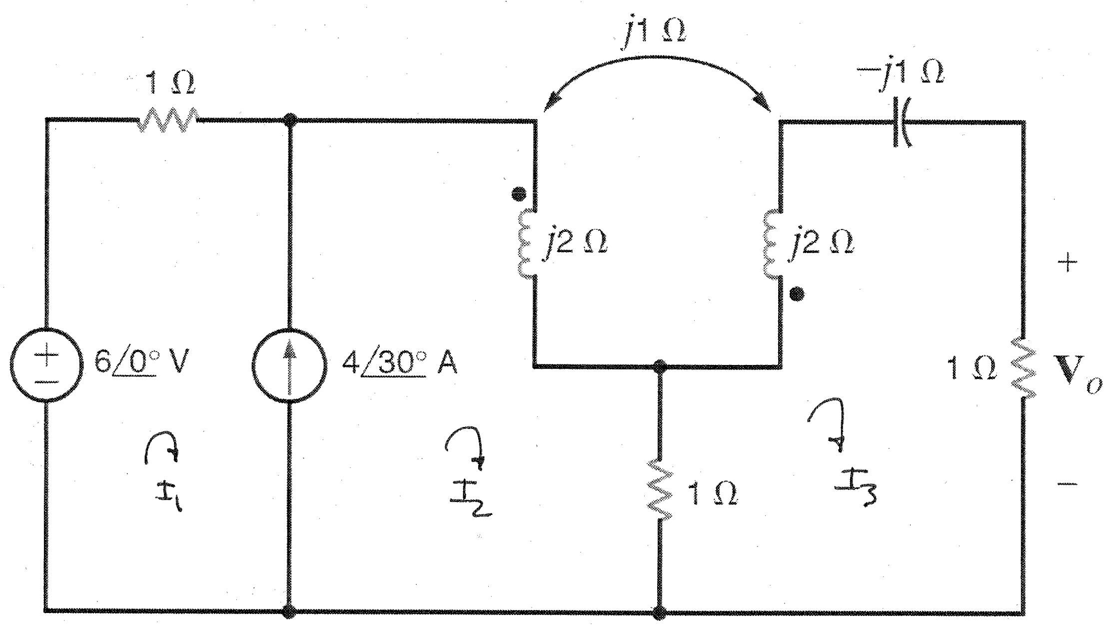


Figure P10.35

SOLUTION:

$6\angle 0 = I_1 + I_2(1+j2) + I_3(-1+j1)$ & $I_2(-1+j1) + I_3(2+j1) = 0$

$I_2 - I_1 = 4\angle 30°$

$V_o = (1)\, I_3$

$$\begin{bmatrix} 1 & 1+j2 & -1+j1 \\ 0 & -1+j1 & 2+j1 \\ -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \\ 4\angle 30° \end{bmatrix} \Rightarrow I_3 = 1.66\angle -109° \text{ A}$$

$$\boxed{V_o = 1.66\angle -109° \text{ V}}$$

10.36 Find $\mathbf{V}_o$ in the network in Fig. P10.36. CS

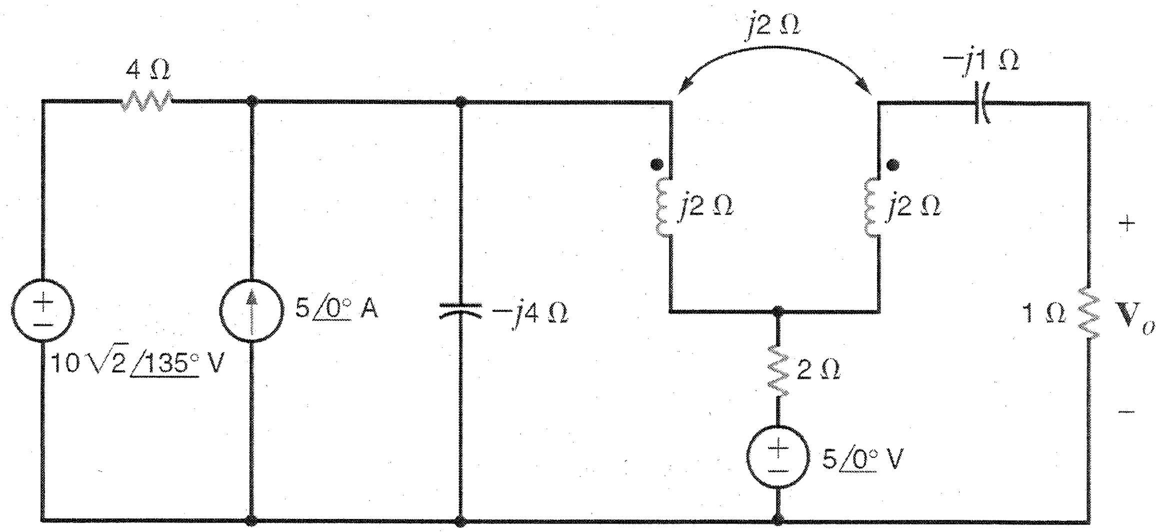
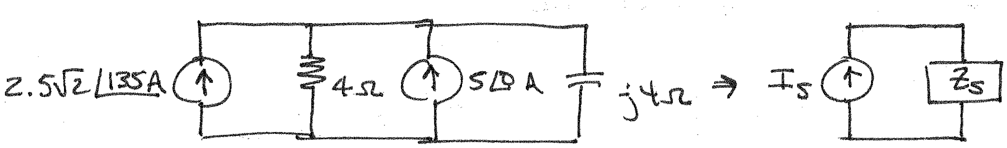


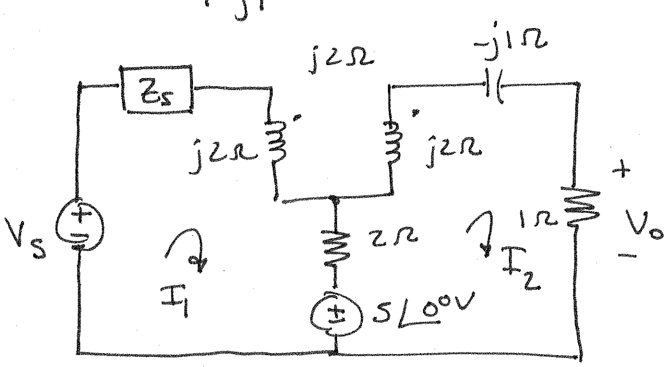
Figure P10.36

SOLUTION: Using source transformation



$$I_S = 5\angle 0 + 2.5\sqrt{2}\angle 135 = 5 - 2.5 + j2.5 = 2.5 + j2.5\,A$$

$$Z_S = \frac{4(-j4)}{4-j4} = 2 - j2\,\Omega \qquad V_S = I_S Z_S = 10\angle 0\ V$$



$$V_S = I_1\left[Z_S + j2 + 2\right] + I_2\left[-2 - j2\right] + 5\angle 0$$

$$5\angle 0° = I_1\left[-2 - j2\right] + I_2\left[3 + j1\right]$$

$$\begin{bmatrix} 4 & -2-j2 \\ -2-j2 & 3+j1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \end{bmatrix}$$

$$I_2 = 2.5\angle 36.9°\ A$$

$$V_o = (1)I_2 \qquad \boxed{V_o = 2.5\angle 36.9°\,V}$$

10.37 Determine the impedance seen by the source in the network shown in Fig. P10.37.

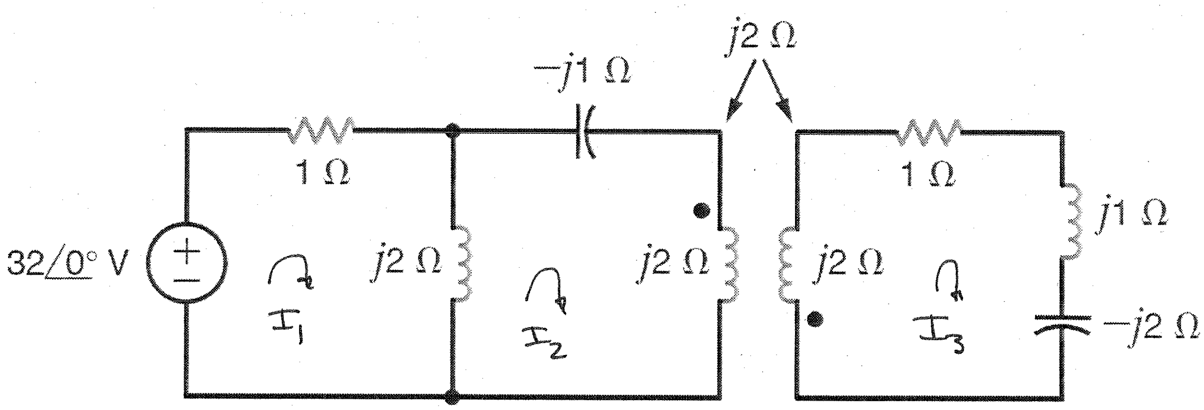


Figure P10.37

SOLUTION:

$32\angle 0 = I_1(1+j2) - j2I_2$ & $0 = -j2I_1 + I_2[j3] + j2\,I_3$

and $j2I_2 + I_3(1+j1) = 0$

$$Z_s = \frac{32\angle 0}{I_1}$$

$$\begin{bmatrix} 1+j2 & -j2 & 0 \\ -j2 & j3 & j2 \\ 0 & j2 & 1+j1 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 32 \\ 0 \\ 0 \end{bmatrix} \Rightarrow I_1 = 11.2\angle -24.8^\circ \text{ A}$$

$$\boxed{Z_s = 2.6 + j\,1.2\,\Omega}$$

10.38 Determine the impedance seen by the source in the network shown in Fig. P10.38.

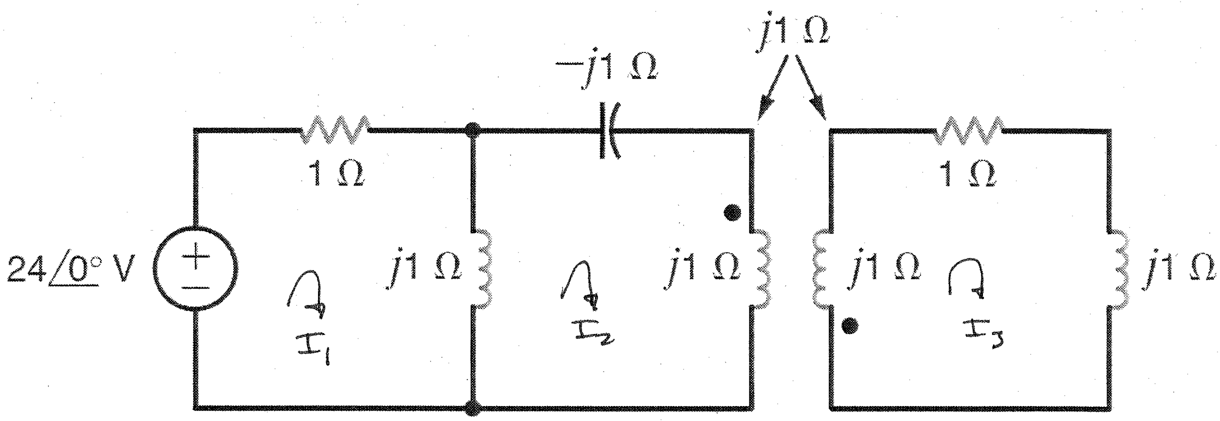


Figure P10.38

SOLUTION:

$24\angle 0 = I_1(1+j1) - jI_2$ & $0 = -jI_1 + I_2(j1) + j1I_3$

and $j1I_2 + I_3(1+j2) = 0$ $\quad Z_S = 24\angle 0 / I_1$

$$\begin{bmatrix} 1+j1 & -j1 & 0 \\ -j1 & j1 & j1 \\ 0 & j1 & 1+j2 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 24 \\ 0 \\ 0 \end{bmatrix} \Rightarrow I_1 = 14.4 + j4.8\text{ A}$$

$$\boxed{Z_S = 1.5 - j0.5\,\Omega}$$

10.39 Determine the input impedance $\mathbf{Z}_{\text{in}}$ in the network in Fig. P10.39.

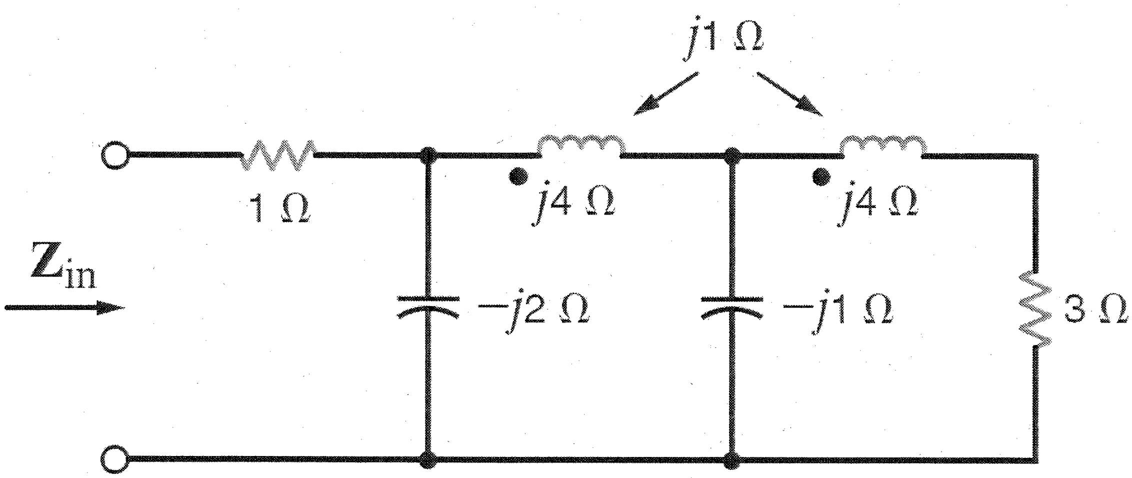
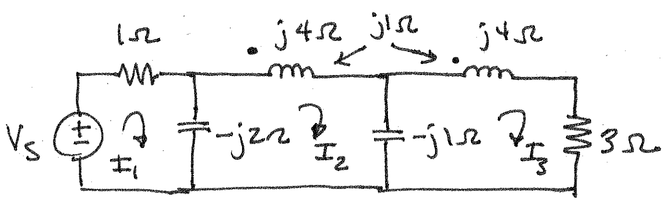


Figure P10.39

SOLUTION:



$Z_{IN} = V_S / I_1$

let $V_S = 1\angle 0°\ V$, so that

$Z_{IN} = 1/I_1$

$$V_S = I_1(1-j2) + j2 I_2$$
$$0 = j2 I_1 + I_2(j1) + j1 I_3 + j1 I_3$$
$$0 = jI_2 + jI_2 + I_3(3+j3)$$

$$\begin{bmatrix} 1-j2 & j2 & 0 \\ j2 & j1 & j2 \\ 0 & j2 & 3+j3 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$I_1 = 0.137\angle 37.2°$

$$\boxed{Z_{IN} = 5.8 - j4.4\ \Omega}$$

10.40 Determine the input impedance $\mathbf{Z}_{in}$ of the circuit in Fig. P10.40. CS

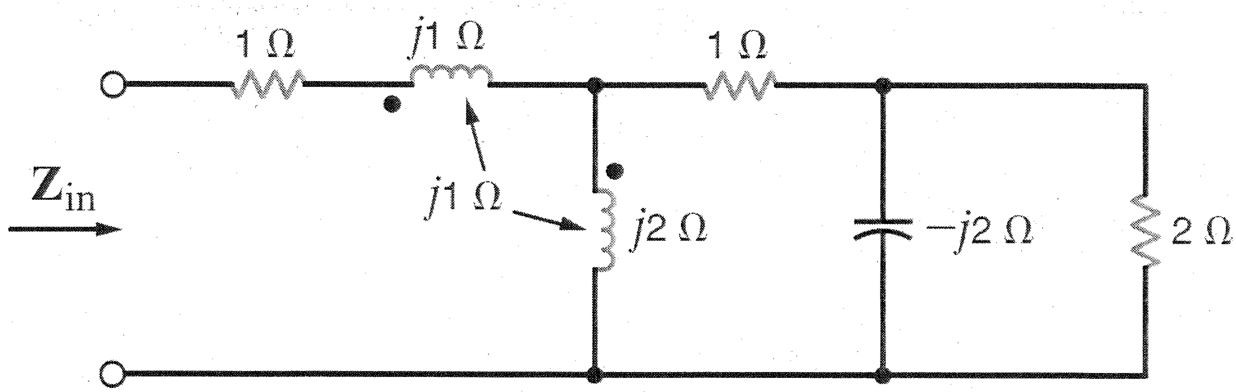
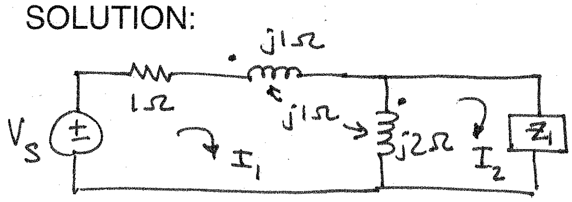


Figure P10.40

SOLUTION:



$Z_{IN} = V_S / I_1$ let $V_S = 1\angle 0^\circ$ V

so that $Z_{IN} = 1/I_1$

$$Z_1 = 1 + \frac{(2)(-j2)}{2-j2} = 2 - j1\,\Omega$$

$$V_S = I_1(1+j3) - j2I_2 + jI_1 + j(I_1 - I_2)$$

$$\left.\begin{aligned} V_S &= I_1(1+j5) + I_2(-j3) \\ 0 &= -j2I_1 + I_2(Z_1 + j2) - j1 I_1 \end{aligned}\right\} \begin{bmatrix} 1+j5 & -j3 \\ -j3 & 2+j1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$I_1 = 0.1785 \angle -34.8^\circ$ A $\boxed{Z_{IN} = 4.6 + j\,3.2\,\Omega}$

10.41 Given the network shown in Fig. P10.41, determine the value of the capacitor C that will cause the impedance seen by the $24\,\underline{/0^\circ}$ V voltage source to be purely resistive. $f = 60$ Hz.

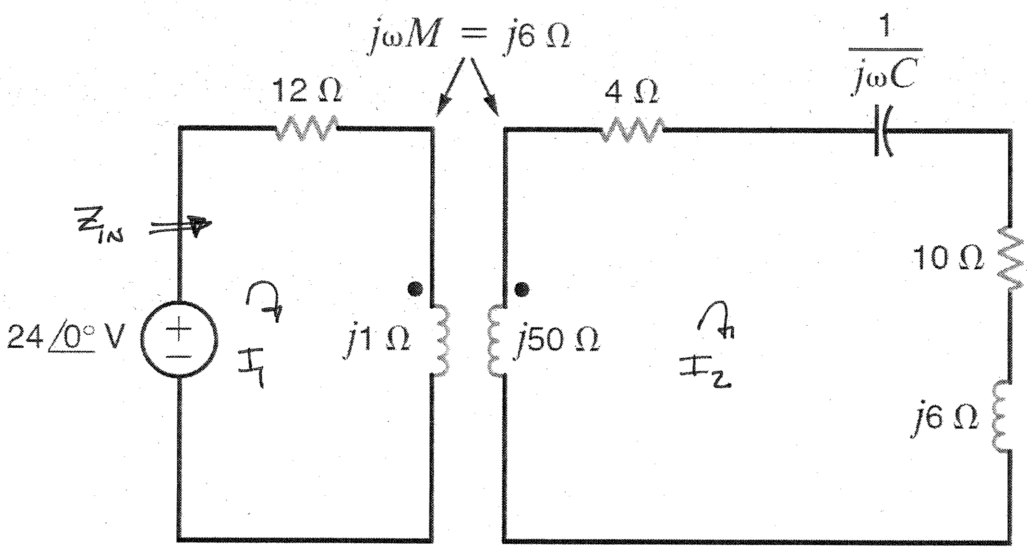


Figure P10.41

SOLUTION: for $Z_{IN} = R_{IN} + j0$, $\angle I_1 = 0°$. $\quad X = 56 - 1/\omega C$

$24\angle 0 = I_1(12+j1) - j6I_2$ & $-j6I_1 + I_2[14 + jX] = 0$

$24\angle 0 = I_1\left[12 + j1 + \frac{36}{14+jX}\right]$ $\Leftarrow$ $I_2 = j6I_1/(14+jX)$

$$\frac{24\angle 0}{I_1} = Z_{IN} = R + j0 = 12 + j1 + \frac{36}{14+jX} = 12 + j1 + \frac{36(14) - j36X}{14^2 + X^2}$$

$$j1 - \frac{j36X}{14^2+X^2} = 0 \Rightarrow X^2 - 36X + 14^2 = 0 \Rightarrow X = \begin{cases} 29.3 \\ 6.69 \end{cases}$$

$$\frac{1}{\omega C} = 56 - X = \begin{Bmatrix} 49.3 \\ 26.7 \end{Bmatrix} \Rightarrow \boxed{C = \begin{Bmatrix} 53.8\mu F \\ 99\mu F \end{Bmatrix} \text{ either will work!}}$$

10.42 Analyze the network in Fig. P10.42 and determine whether a value of X_C can be found such that the output voltage is equal to twice the input voltage.

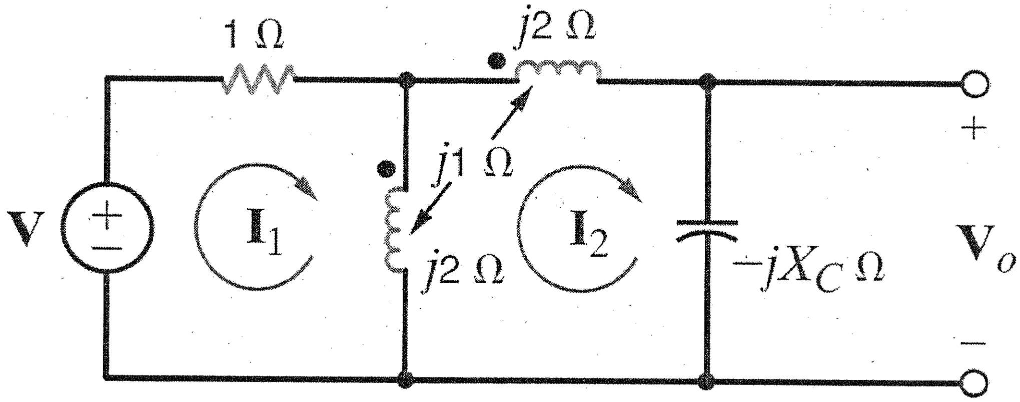


Figure P10.42

SOLUTION:

$$V = I_1(1+j2) - j2I_2 + jI_2 \Rightarrow V = I_1(1+j2) - jI_2$$

$$0 = -j2I_1 + I_2(j(4-X_c)) - jI_2 + j(I_1 - I_2) \Rightarrow 0 = -jI_1 + jI_2(2-X_c)$$

or, $I_1 = I_2(2-X_c)$

now, $V = I_2[(1+j2)(2-X_c) - j1] \Rightarrow V = I_2[2 - X_c + j(3-2X_c)]$

But, $V_o = -jX_c I_2$

So, $\dfrac{V_o}{V} = \dfrac{-jX_c}{2 - X + j(3-2X_c)} = 2 + j0$

requires $2 - X_c = 0 \Rightarrow X = 2$ and $\dfrac{V_o}{V} = \dfrac{-j2}{0 + j(3-4)} = 2 + j0$ ✓

$$\boxed{X_c = 2\Omega}$$

10.43 Two coils in a network are positioned such that there is 100% coupling between them. If the inductance of one coil is 10 mH and the mutual inductance is 6 mH, compute the inductance of the other coil. CS

SOLUTION:

$$k = 1 = \frac{M}{\sqrt{L_1 L_2}} = \frac{6\times10^{-3}}{\sqrt{10^{-2} L_2}} \qquad \boxed{L_2 = 3.6\,mH}$$

10.44 The currents in the network in Fig. P10.44 are known to be $i_1(t) = 10\cos(377t - 30°)$ mA and $i_2(t) = 20\cos(377t - 45°)$ mA. The inductances are $L_1 = 2$ H, $L_2 = 2$ H, and $k = 0.8$. Determine $v_1(t)$ and $v_2(t)$.

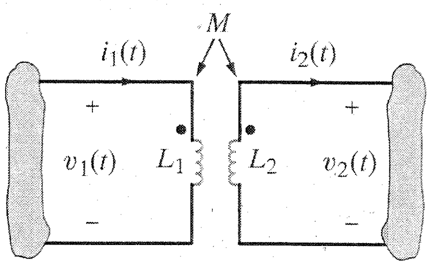
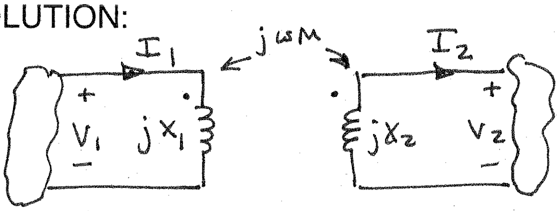


Figure P10.44

SOLUTION:



$X_1 = \omega L_1 = 754\,\Omega$

$X_2 = \omega L_2 = 754\,\Omega$

$M = k\sqrt{L_1 L_2} = 1.6\text{H} \qquad \omega M = 603\,\Omega$

$V_1 = I_1(jX_1) - j\omega M I_2$

$-V_2 = -j\omega M I_1 + I_2\, jX_2$

$I_1 = 10\angle{-30}$ mA

$I_2 = 20\angle{-45}$ mA

$V_1 = 5.16\angle{-157°}$ V $\qquad V_2 = 9.39\angle{-145°}$ V

$$\boxed{\begin{aligned} v_1(t) &= 5.16\cos(377t - 157°)\text{ V} \\ v_2(t) &= 9.39\cos(377t - 145°)\text{ V} \end{aligned}}$$

10.45 Determine the energy stored in the coupled inductors in the circuit in P10.44 at $t = 1$ ms.

SOLUTION:

$w(t) = \frac{1}{2} L_1 i_1^2(t) + \frac{1}{2} L_2 i_2^2(t) - M i_1(t) i_2(t)$

$i_1(t) = 10 \cos(377t - 30°)$ mA $\quad i_2(t) = 20 \cos(377t - 45°)$ mA

$L_1 = 2H \quad L_2 = 2H \quad k = 0.8 \Rightarrow M = 1.6H$

let $t_1 = 1ms$

$i_1(t_1) = 9.89$ mA $\quad i_2(t_1) = 18.4$ mA

$\boxed{w(t_1) = 145 \mu J}$

10.46 The currents in the magnetically-coupled inductors shown in Fig. P10.46 are known to be $i_1(t) = 8\cos(377t - 20°)$ mA and $i_2(t) = 4\cos(377t - 50°)$ mA. The inductor values are $L_1 = 2$ H, $L_2 = 1$ H, and $k = 0.6$. Determine $v_1(t)$ and $v_2(t)$.

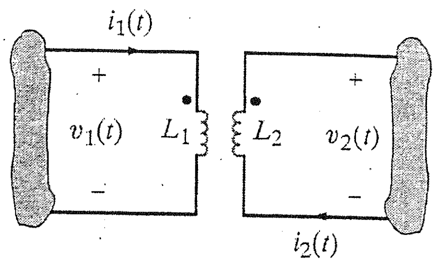


Figure P10.46

SOLUTION: $M = k\sqrt{L_1L_2} \Rightarrow M = 0.849$ H

$$v_1(t) = L_1\frac{di_1}{dt} - M\frac{di_2}{dt} = \frac{-2(8)(377)}{1000}\sin(377t-20°) + \frac{M(4)(377)}{1000}\sin(377t-50°)$$

$M = k\sqrt{L_1L_2} \Rightarrow M = 0.849$ H $\quad I_1 = 8\angle{-20°}$ mA $\quad I_2 = 4\angle{-50°}$ mA

$V_1 = j\omega L_1 I_1 - j\omega M I_2 \quad \omega = 377$ r/s $\quad \omega M = 320\,\Omega$

$-V_2 = -j\omega M I_1 + j\omega L_2 I_2$

$V_1 = 4.96\angle{77.4°}$ V $\quad V_2 = 5.12\angle{70°}$ V

$$\boxed{\begin{aligned} v_1(t) &= 4.96\cos(377t + 77.4°)\ \text{V} \\ v_2(t) &= 1.46\cos(377t + 101°)\ \text{V} \end{aligned}}$$

10.47 Determine the energy stored in the coupled inductors in Problem 10.46 at $t = 1$ ms. CS

SOLUTION: $w(t) = \frac{1}{2} L_1 i_1^2(t) + \frac{1}{2} L_2 i_2^2(t) + M i_1(t) i_2(t)$

$i_1(t) = 8 \cos(377t - 20°)$ mA $\quad i_2(t) = 4 \cos(377t - 50°)$ mA

$L_1 = 2H \quad L_2 = 1H \quad k = 0.6 \Rightarrow M = 0.849\ H$

at $t_1 = 1ms$, $\quad i_1(t_1) = 8.00\ mA \quad i_2(t_1) = 3.52\ mA$

$$\boxed{w(t_1) = 94.1\ \mu J}$$

10.48 Determine $\mathbf{I}_1$, $\mathbf{I}_2$, $\mathbf{V}_1$, and $\mathbf{V}_2$ in the network in Fig. P10.48.

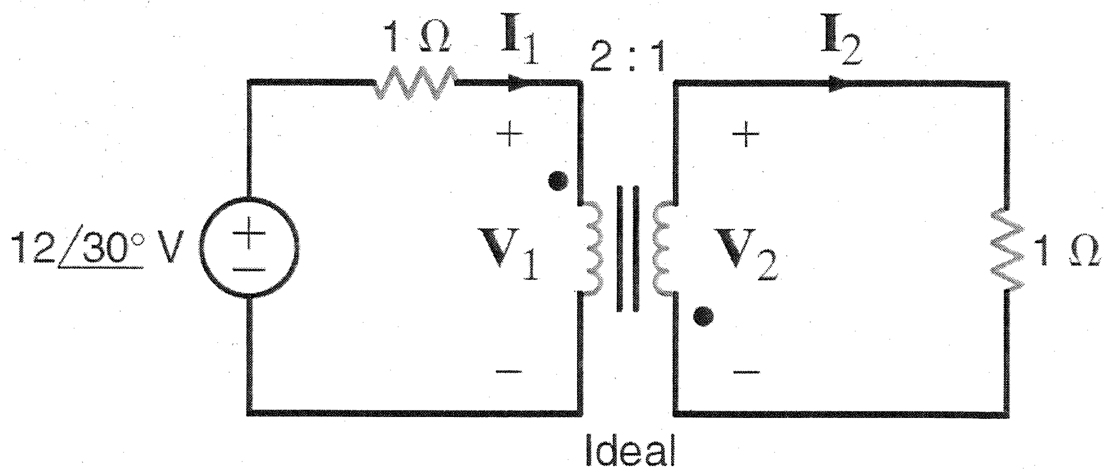


Figure P10.48

SOLUTION:

$12\angle 30 = (1)I_1 + V_1$ & $V_2 = (1)I_2$ & $-V_1 = 2V_2$ & $2I_1 = -I_2$

$12\angle 30 = I_1 - 2V_2 = I_1 - 2I_2 = I_1 + 4I_1 = 5I_1$

$I_1 = \frac{12\angle 30°}{5} = \boxed{2.4\angle 30°\ A = I_1}$

$\boxed{I_2 = 4.8\angle -150°\ A}$

$\boxed{V_2 = 4.8\angle -150°\ V}$ $\boxed{V_1 = 9.6\angle 30°\ V}$

10.49 Find all currents and voltages in the network in Fig. P10.49.

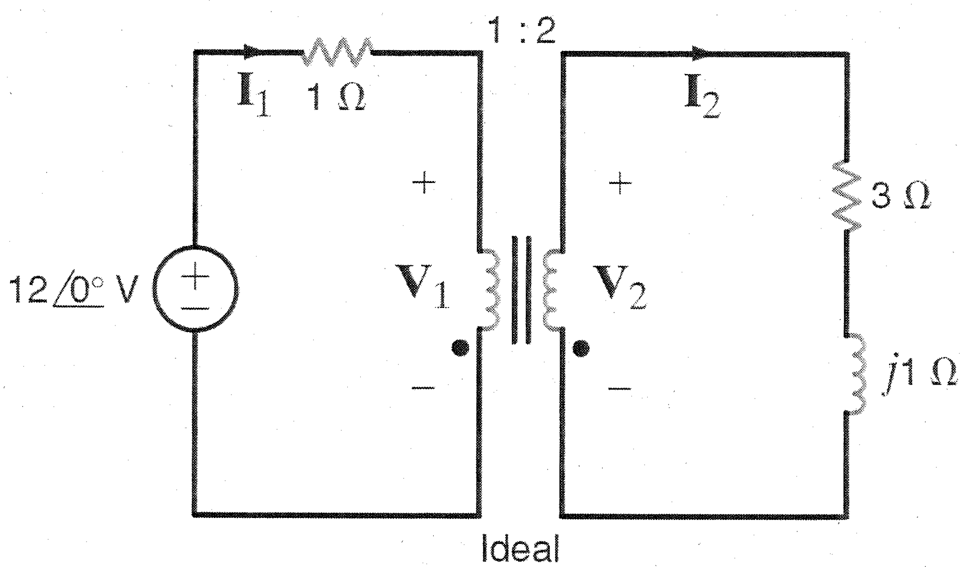
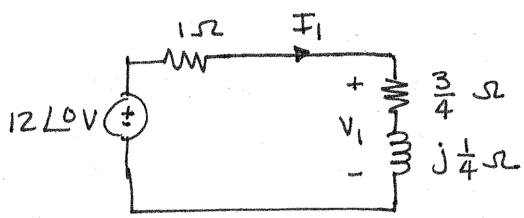


Figure P10.49

SOLUTION: $n = 2$ $\quad n^2 = 4$

$12\angle 0 = (1)I_1 + V_1$ & $V_2 = I_2(3+j1)$ & $V_2 = 2V_1$ & $I_1 = 2I_2$



$$I_1 = \frac{12\angle 0}{1.75 + j0.25}$$

$$V_1 = I_1(0.75 + j0.25)$$

$$\boxed{\begin{array}{ll} I_1 = 6.79\angle -8.13^\circ \text{ A} & I_2 = 3.39\angle -8.13^\circ \text{ A} \\ V_1 = 5.37\angle 10.3^\circ \text{ V} & V_2 = 10.74\angle 10.3^\circ \text{V} \end{array}}$$

10.50 Determine $\mathbf{V}_o$ in the circuit in Fig. P10.50. **PSV**

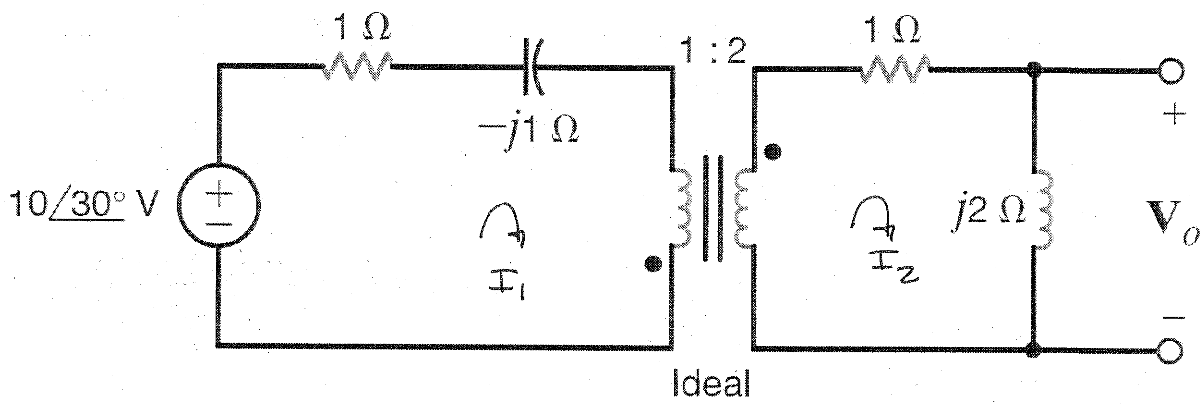
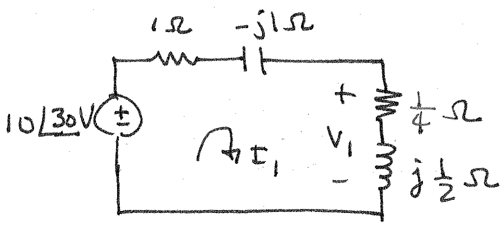


Figure P10.50

SOLUTION: $n = 2 \qquad n^2 = 4$

Use impedance reflection,



$I_2 = -I_1/2 \qquad V_0 = j2\, I_2$

$$I_1 = \frac{10\angle 30°}{1.25 - j0.5} = 7.43\angle 51.8°\text{ A}$$

$I_2 = 3.72\angle -128°$ A

$$\boxed{V_0 = 7.43\angle -38.2°\text{ V}}$$

10.51 Determine $\mathbf{V}_o$ in the circuit in Fig. P10.51.

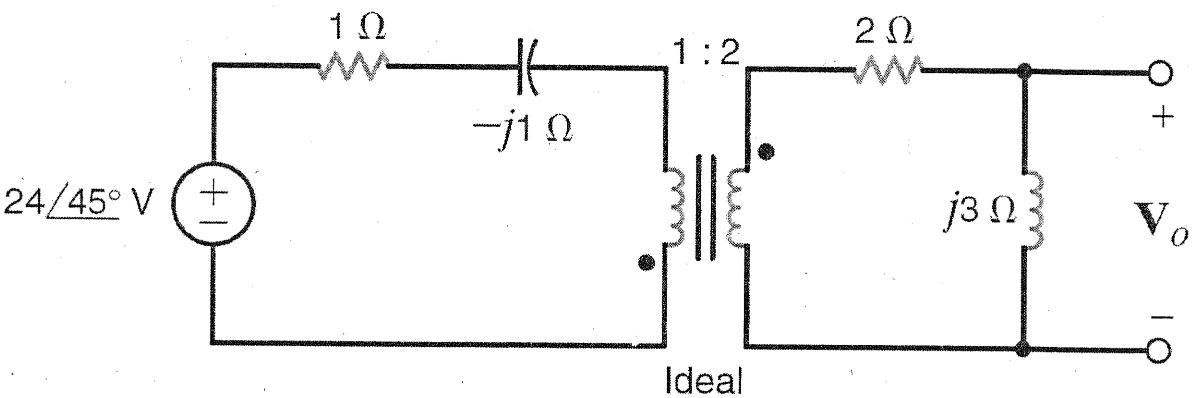
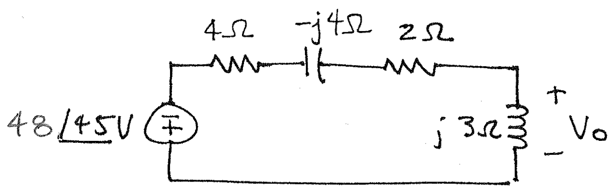


Figure P10.51

SOLUTION: $n = 2$ $n^2 = 4$ Using reflection through transformer



$$V_o = -\frac{48\angle 45\ (j3)}{6 - j4 + j3} = -\frac{48\angle 45\ (j3)}{6 - j1}$$

$$\boxed{V_o = 23.7\angle{-35.5}°\ V}$$

10.52 Determine $\mathbf{I}_1$, $\mathbf{I}_2$, $\mathbf{V}_1$, and $\mathbf{V}_2$ in the network in Fig. P10.52. CS

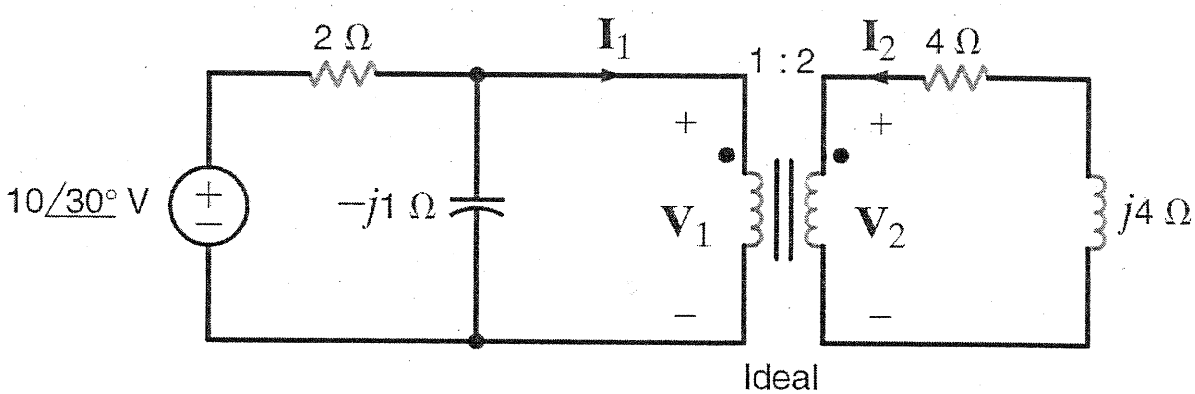
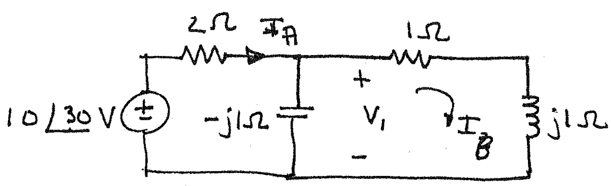


Figure P10.52

SOLUTION: $n = 2 \quad n^2 = 4$



$10\angle 30 = I_A(2-j1) + jI_B$

$0 = jI_A + I_B(1)$

$I_A = jI_B$

$10\angle 30 = I_B(1 + j2 + j1) = I_B(1+j3)$

$I_B = \boxed{I_1 = 3.16\angle -41.6^\circ \text{ A}}$

$V_1 = I_B(1+j1)$

$\boxed{V_1 = 4.47\angle 3.4^\circ \text{ V}}$

$\boxed{V_2 = nV_1 = 8.93\angle 3.4^\circ \text{ V}}$

$\boxed{I_2 = -I_1/n = 1.58\angle 138^\circ \text{ A}}$

10.53 Determine $\mathbf{I}_1$, $\mathbf{I}_2$, $\mathbf{V}_1$, and $\mathbf{V}_2$ in the network in Fig. P10.53.

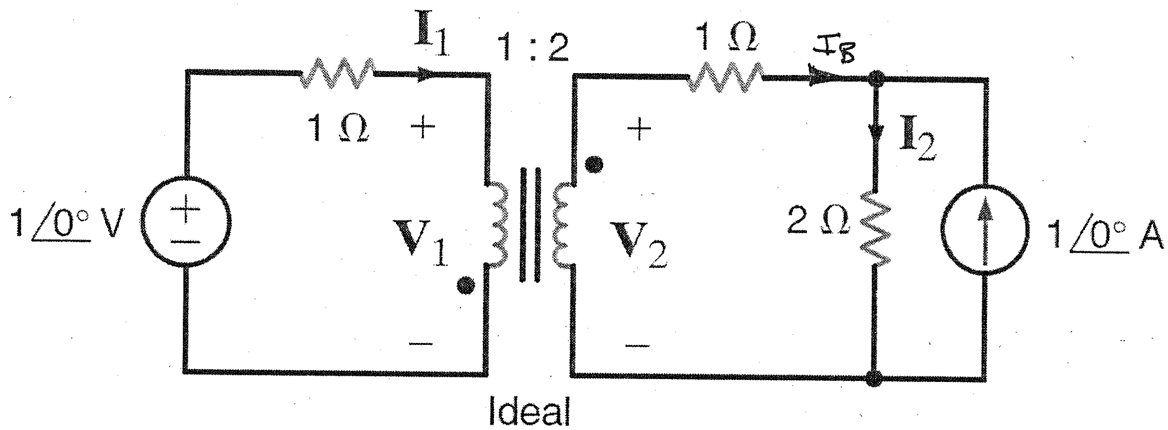
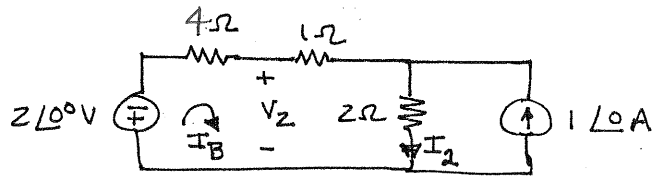


Figure P10.53

SOLUTION: $n = 2 = -\dfrac{V_2}{V_1} = -\dfrac{I_1}{I_B}$



$-2\angle 0 = I_B(4+1+2) + 2(1\angle 0)$

$I_B = 0.571\angle 180°$ A

$I_2 = I_B + 1\angle 0$

$\boxed{I_2 = 0.429\angle 0° \text{ A}}$

$V_2 = 2I_2 + I_B(1)$

$\boxed{V_2 = 0.287\angle 0° \text{ V}}$

$I_1 = -n I_B$ $\quad \boxed{I_1 = 1.14\angle 0° \text{ A}}$

$V_1 = -V_2/n$ $\quad \boxed{V_1 = 0.143\angle 180° \text{ V}}$

10.54 Find **I** in the network in Fig. P10.54.

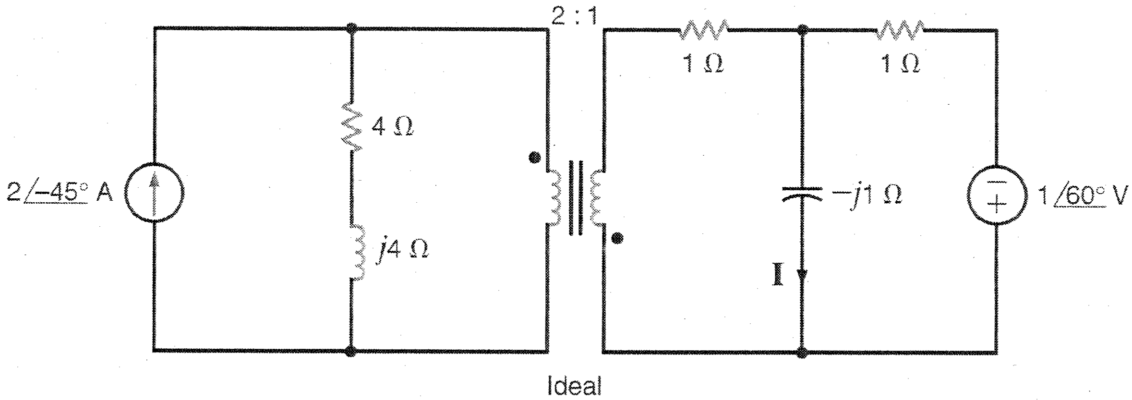
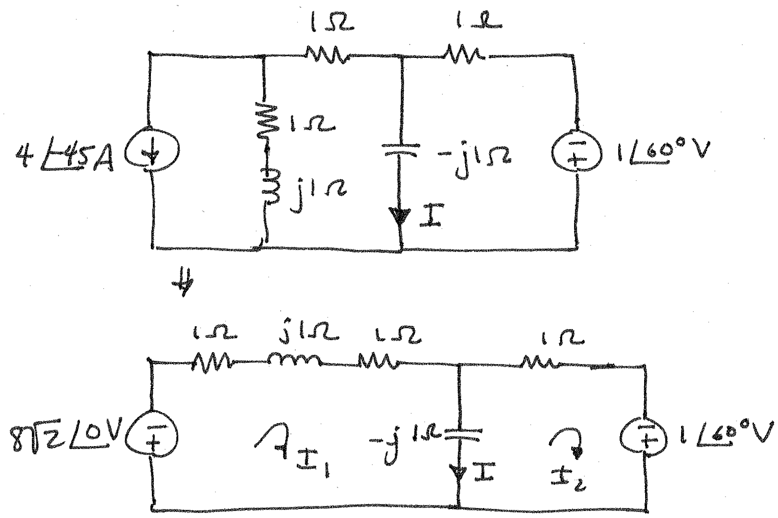


Figure P10.54

SOLUTION: $n = 1/2$



$$-8\sqrt{2}\angle 0 = I_1(2) + j1 I_2$$
$$1\angle 60 = j1 I_1 + (1-j1) I_2$$

$$\begin{bmatrix} 2 & j1 \\ j1 & 1-j1 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} -8\sqrt{2}\angle 0 \\ 1\angle 60 \end{bmatrix}$$

$I_1 = 4.17\angle 168°$ A $\qquad I_2 = 3.63\angle 119°$ A

$I = I_1 - I_2 \qquad \boxed{I = 3.23\angle -135°\text{ A}}$

10.55 Determine $\mathbf{I}_1$, $\mathbf{I}_2$, $\mathbf{V}_1$, and $\mathbf{V}_2$ in the network in Fig. P10.55.

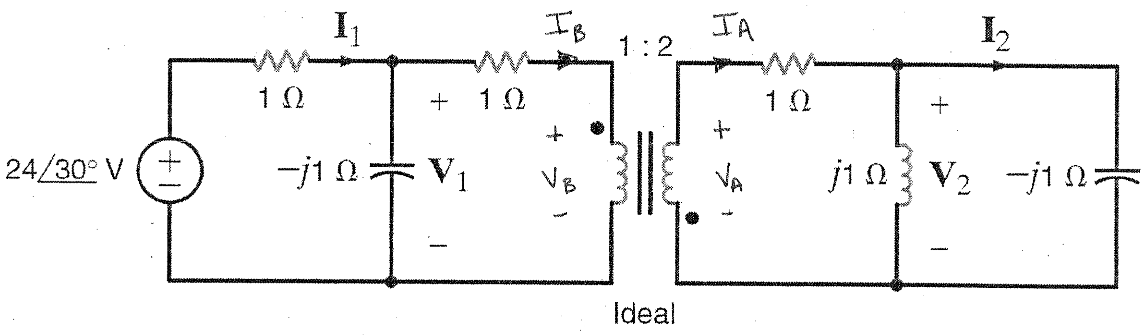
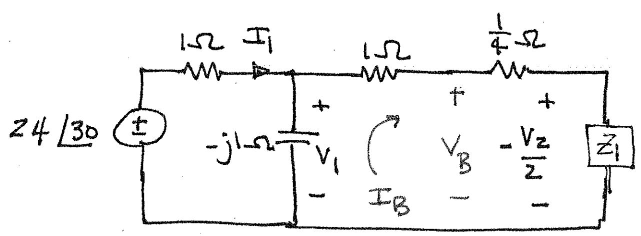


Figure P10.55

SOLUTION: $n = 2$



$$Z_1 = \frac{(j1)(-j1)}{j1 - j1} = \infty$$

$$I_2 = \frac{V_2}{-j1}$$

$$24\angle 30 = I_1(1 - j1) + j I_B \qquad I_B = 0$$

$$V_1 = (I_1 - I_B)(-j1) = 10\sqrt{2}\angle -15$$

$$\boxed{V_1 = 17.0\angle -15°\ \text{V}}$$

$$V_B = -\frac{V_2}{2} = V_1 \Rightarrow \boxed{V_2 = 34.0\angle 165°\ \text{V}} \qquad \boxed{I_1 = 17.0\angle 75°\ \text{A}}$$

$$I_2 = \frac{V_2}{-j1} \Rightarrow \boxed{I_2 = 34.0\angle -105°\ \text{A}}$$

10.56 Find the current **I** in the network in Fig. P10.56.

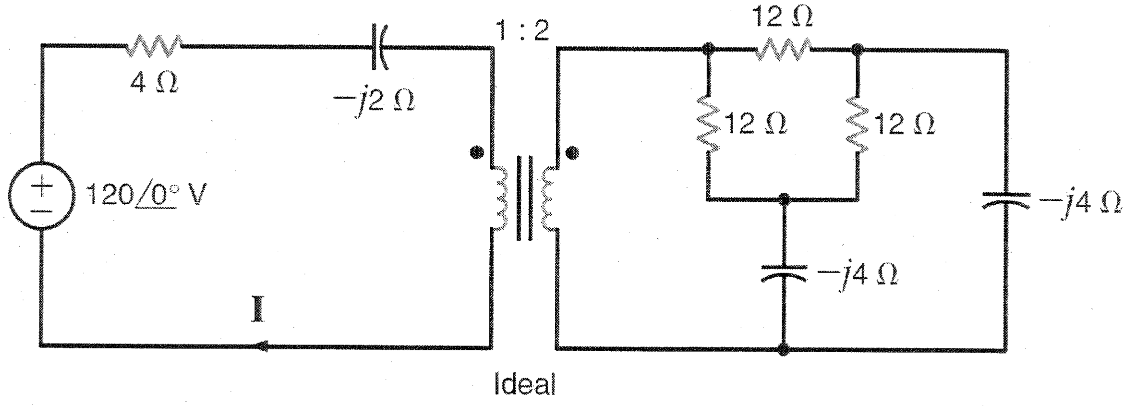
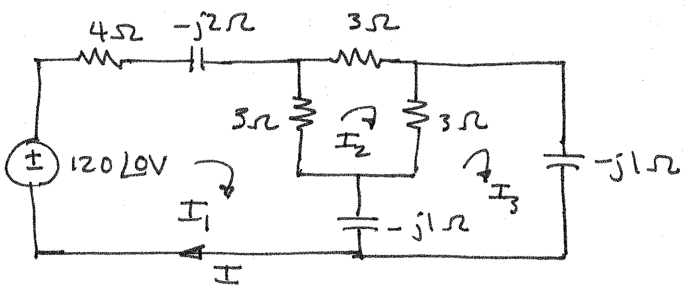


Figure P10.56

SOLUTION: $n = 2$



$$120\angle 0 = I_1(7-j3) - 3I_2 + jI_3$$

$$0 = -3I_1 + 9I_2 - 3I_3$$

$$0 = jI_1 - 3I_2 + I_3(3-j2)$$

$$\begin{bmatrix} 7-j3 & -3 & j1 \\ -3 & 9 & -3 \\ j1 & -3 & 3-j2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 120 \\ 0 \\ 0 \end{bmatrix}$$

$I = I_1$

$I_1 = 19.9\angle 24.4°$ A

$$\boxed{I_o = 19.9\angle 24.4° \text{ A}}$$

10.57 Determine $\mathbf{I}_1$, $\mathbf{I}_2$, $\mathbf{V}_1$, and $\mathbf{V}_2$ in the network in Fig. P10.57.

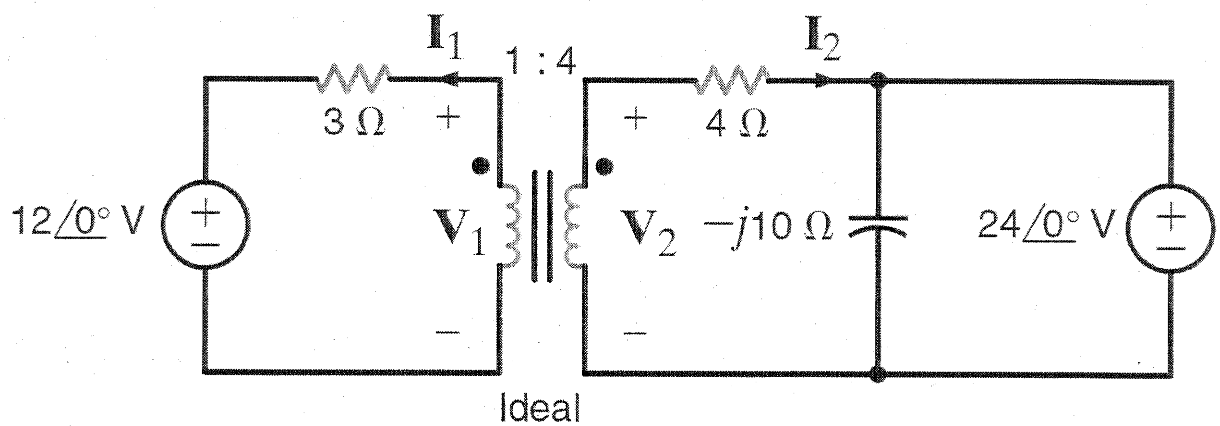
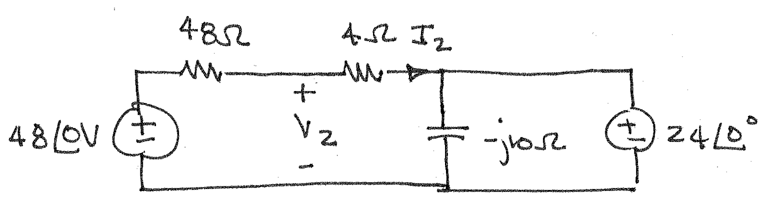


Figure P10.57

SOLUTION:



$n = 4$

$$I_2 = \frac{48\angle 0 - 24\angle 0}{48+4} = \frac{24\angle 0}{52} = 0.46\angle 0^\circ \text{ A}$$

$$V_2 = 48\angle 0 - 48 I_2 \Rightarrow V_2 = 25.9\angle 0^\circ \text{ V}$$

$$V_1 = V_2/n = 6.48\angle 0^\circ \text{ V}$$

$$I_1 = -I_2 n = -1.84\angle 0^\circ \text{ A}$$

$I_1 = -1.84\angle 0^\circ$ A	$V_1 = 6.48\angle 0^\circ$ V
$I_2 = 0.46\angle 0^\circ$ A	$V_2 = 25.9\angle 0^\circ$ V

10.58 Find $\mathbf{V}_o$ in the network in Fig. P10.58.

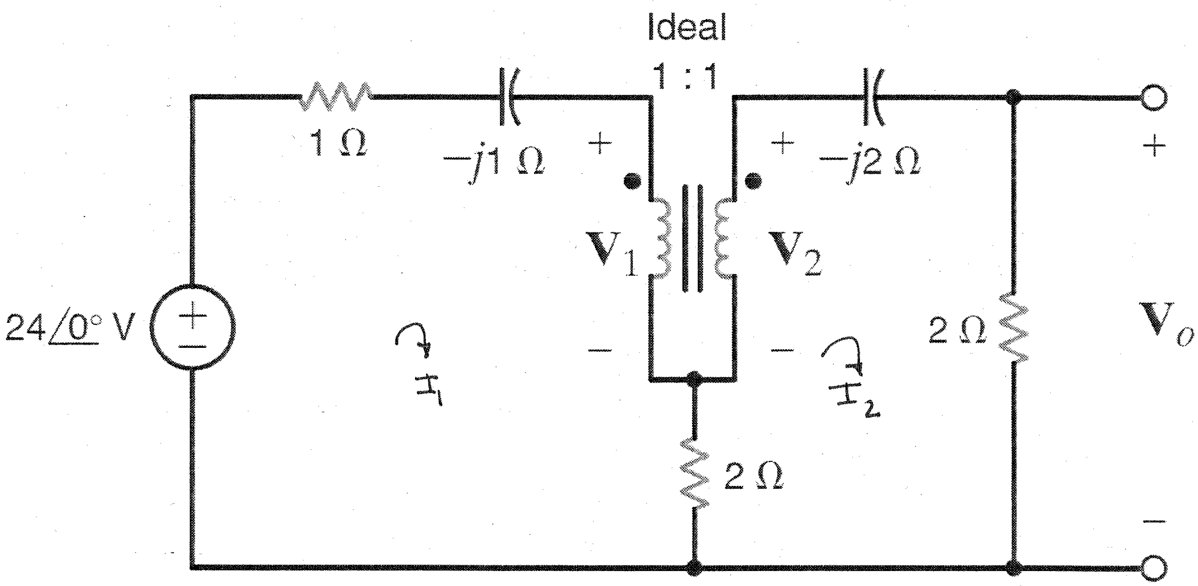


Figure P10.58

SOLUTION: $n = 1$ $\quad I_1 = I_2 \quad V_1 = V_2$

$$\left.\begin{aligned} 24\angle 0^\circ &= I_1(3-j1) - 2I_2 + V_1 \\ V_2 &= -2I_1 + I_2(4-j2) \end{aligned}\right\} \Rightarrow \begin{aligned} 24\angle 0 &= I_1(1-j1) + V_1 \\ V_1 &= I_1(2-j2) \end{aligned}$$

$$24\angle 0 = I_1(1-j1) + I_1(2-j2) = I_1(3-j3) \Rightarrow I_1 = \frac{24\angle 0}{3-j3} = 5.66\angle 45^\circ \text{ A}$$

$$I_2 = I_1 = 5.66\angle 45^\circ \text{ A}$$

$$V_o = 2I_2 \qquad \boxed{V_o = 11.32\angle 45^\circ \text{ V}}$$

10.59 Find the current **I** in the network in Fig. P10.59. CS

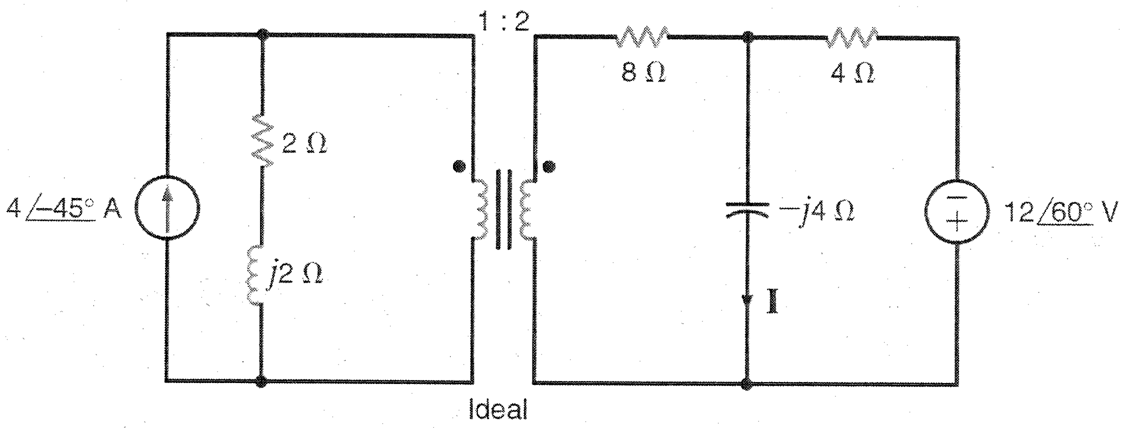
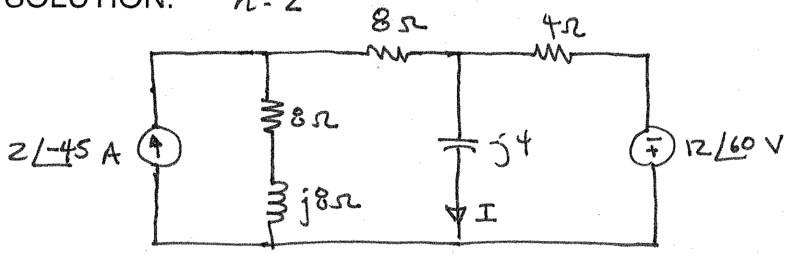
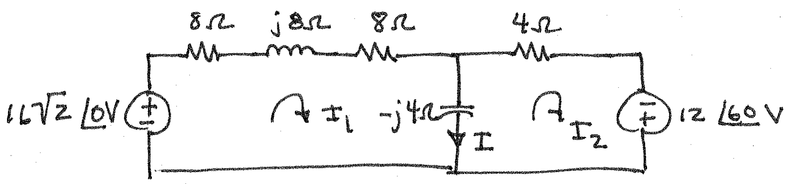


Figure P10.59

SOLUTION: $n = 2$





$I = I_1 - I_2$

$$\left.\begin{aligned} 16\sqrt{2}\angle 0 &= I_1(16+j4) + j4 I_2 \\ 12\angle 60 &= j4 I_1 + I_2(4-j4) \end{aligned}\right\} \quad \begin{bmatrix} 16+j4 & j4 \\ j4 & 4-j4 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 16\sqrt{2} \\ 12\angle 60 \end{bmatrix}$$

$I_1 = 1.63 \angle -14.3° \text{ A} \qquad I_2 = 1.06 \angle 88.0° \text{ A}$

$$\boxed{I = 2.12 \angle -43.5° \text{ A}}$$

10.60 Find the voltage $\mathbf{V}_o$ in the network in Fig. P10.60. PSV

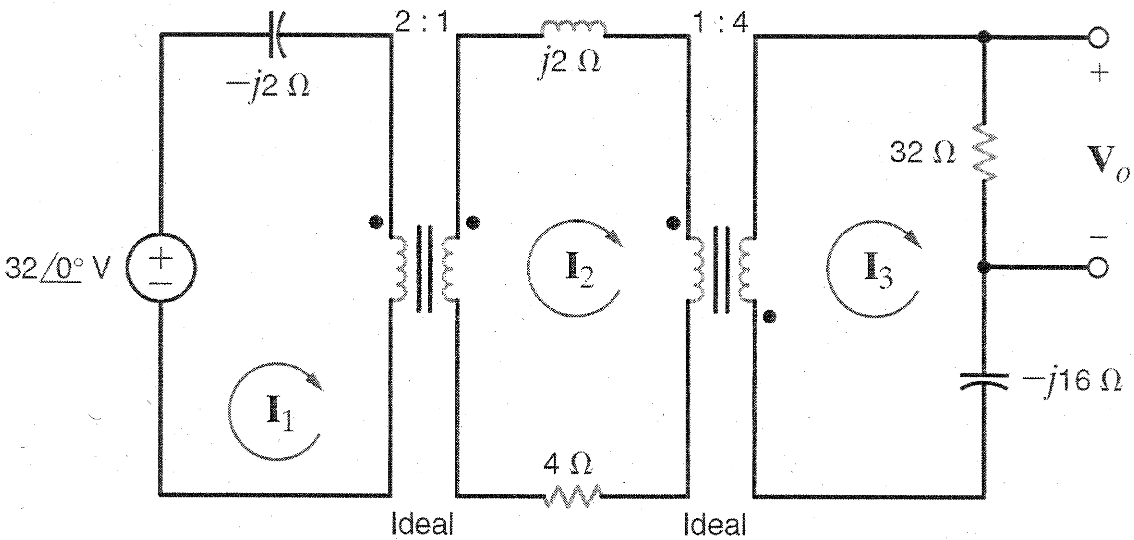
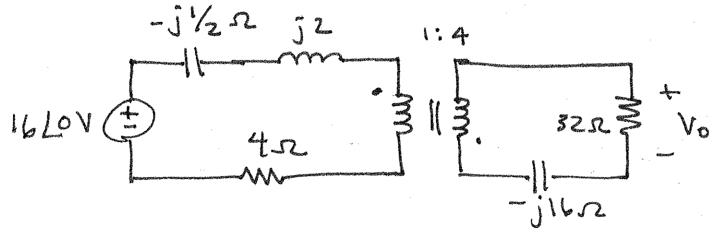
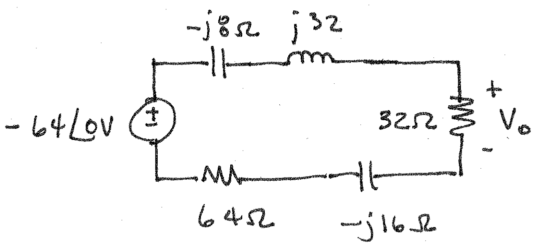


Figure P10.60

SOLUTION: $n_1 = 1/2$ $n_2 = 4$





$$V_o = \frac{-64\angle 0 \,(32)}{32+64+j32-j16-j8}$$

$$\boxed{V_o = 21.3 \angle 175^\circ \text{ V}}$$

10.61 Find $\mathbf{V}_o$ in the circuit in Fig. P10.61.

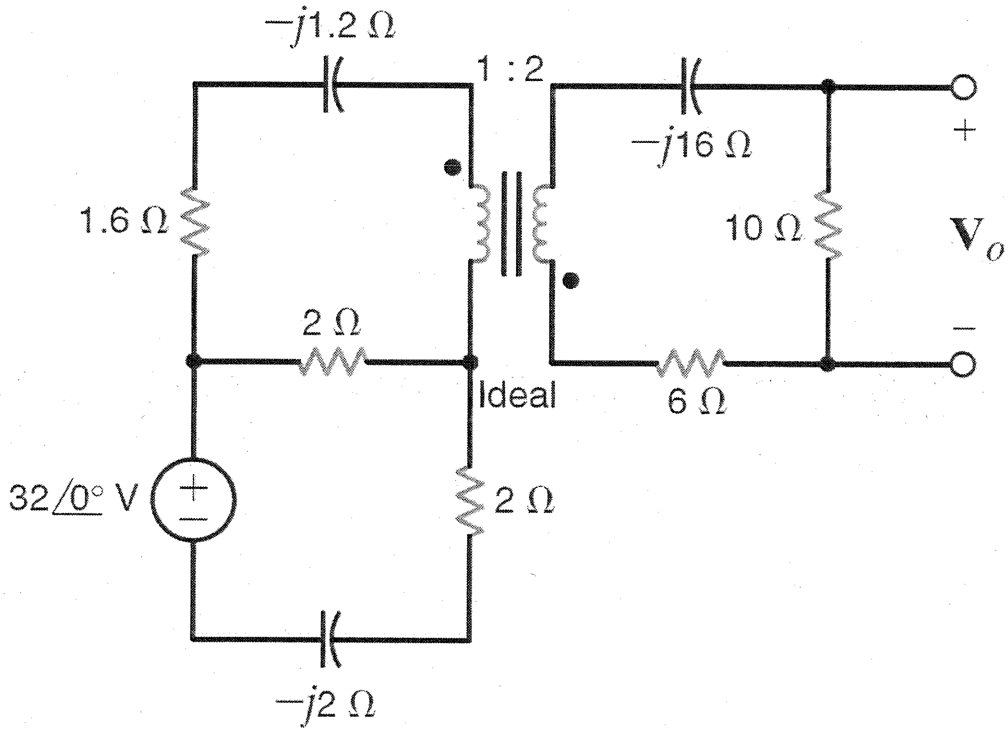
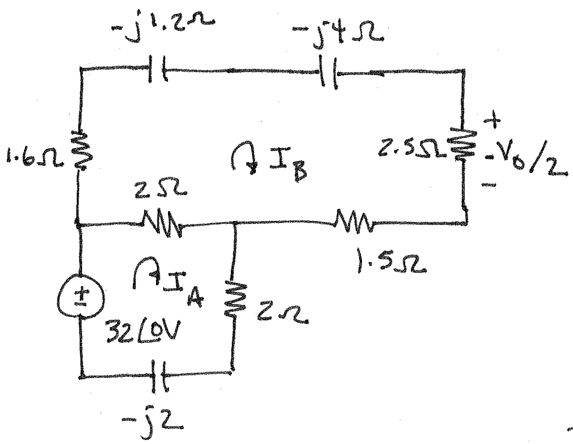


Figure P10.61

SOLUTION: $n = 2$



$$32\angle 0 = I_A(4 - j2) - 2I_B$$

$$0 = -2I_A + I_B(7.6 - j5.2)$$

$$\frac{V_o}{2} = -I_B(2.5) \Rightarrow V_o = -5I_B$$

$$\begin{bmatrix} 4-j2 & -2 \\ -2 & 7.6-j5.2 \end{bmatrix}\begin{bmatrix} I_A \\ I_B \end{bmatrix} = \begin{bmatrix} 32 \\ 0 \end{bmatrix}$$

$$I_B = 1.62\angle 66.0^\circ \text{ A}$$

$$\boxed{V_o = 8.12\angle -114^\circ \text{ V}}$$

10.62 Find $\mathbf{V}_o$ in the network in Fig. P10.62.

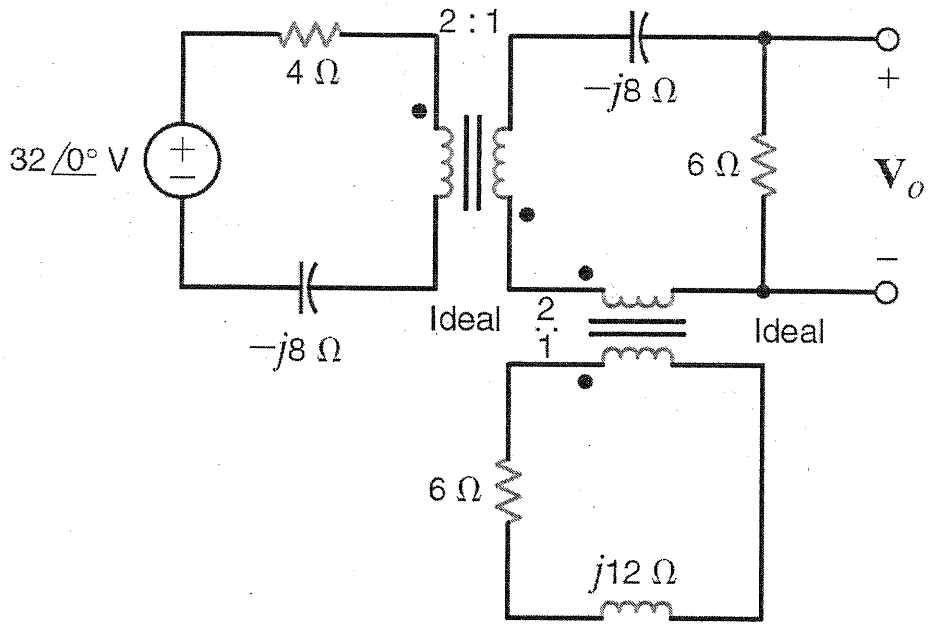
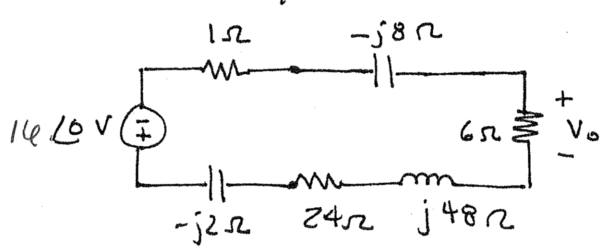


Figure P10.62

SOLUTION: $n_1 = 1/2 \qquad n_2 = 2$



$$V_o = \frac{-16\angle 0\,(6)}{1+6+24+j(48-8-2)}$$

$$\boxed{V_o = 1.96\ \angle 129°\ \text{V}}$$

10.63 Find $\mathbf{V}_o$ in the circuit in Fig. P10.63. CS

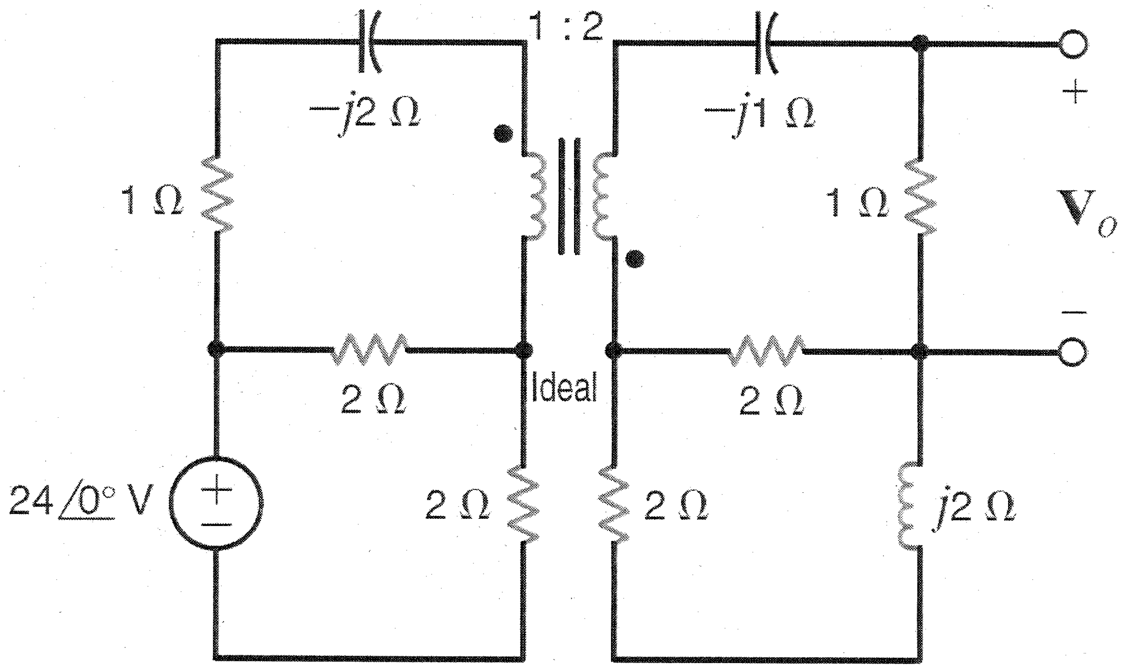
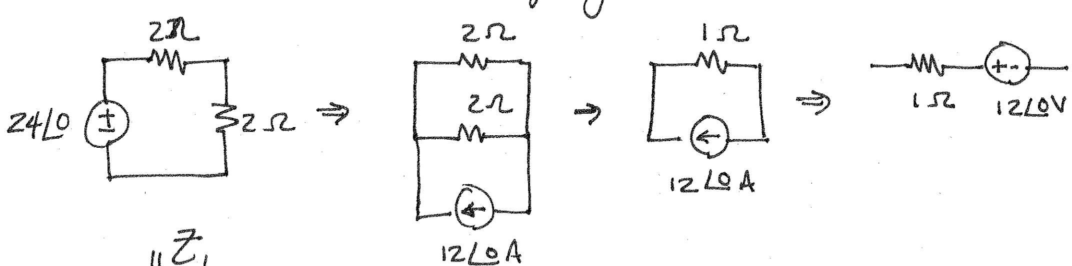
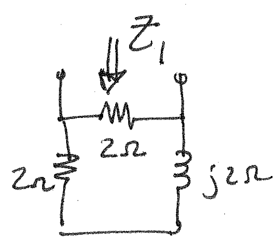


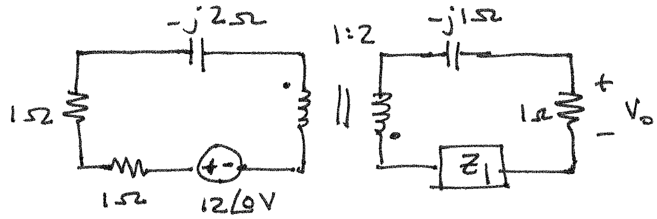
Figure P10.63

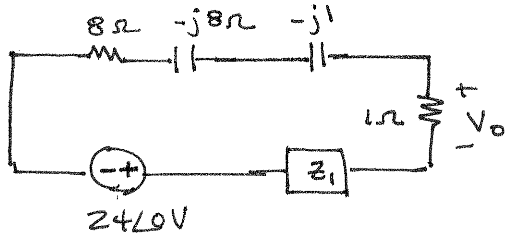
SOLUTION: First, we simplify.





$$Z_1 = \frac{2(2+j2)}{4+j2} = 1.2 + j0.4\,\Omega$$





$$V_o = \frac{-24\angle 0\,(1)}{8+1-j(8+1)+Z_1}$$

$$\boxed{V_o = 1.80\angle -140^\circ\ \text{V}}$$

10.64 Determine the input impedance seen by the source in the circuit in Fig. P10.64.

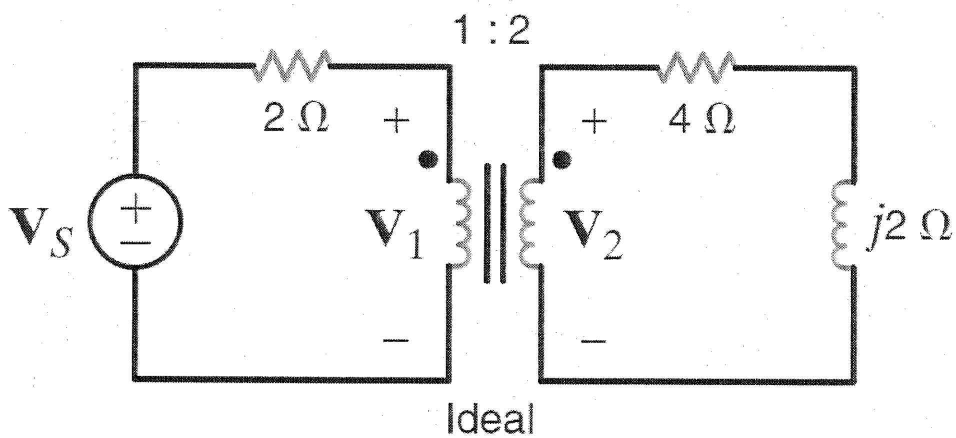
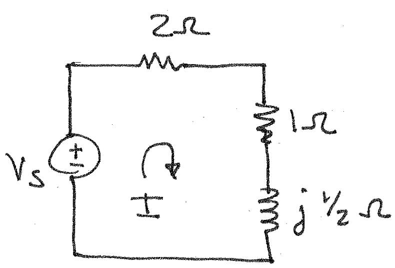


Figure P10.64

SOLUTION:



$Z_{IN} = V_S / I = 3 + j0.5\,\Omega$

$$\boxed{Z_{IN} = 3 + j0.5\,\Omega}$$

10.65 Determine the input impedance seen by the source in the circuit in Fig. P10.65. CS

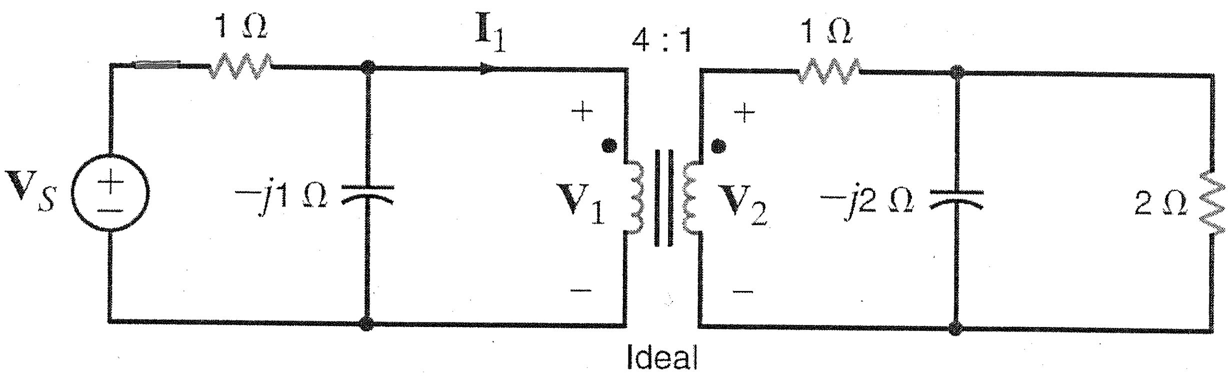
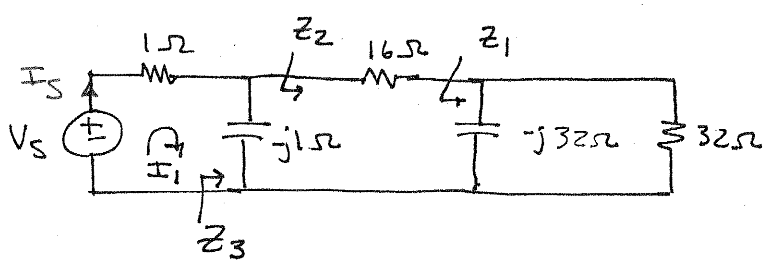


Figure P10.65

SOLUTION:



$$Z_{IN} = V_S / I_S$$

$$Z_1 = \frac{32(-j32)}{32-j32} = 16 - j16\,\Omega \qquad Z_2 = 16 + Z_1 = 32 - j16\,\Omega$$

$$Z_3 = \frac{-j1\,(Z_2)}{-j1 + Z_2} = 0.987\angle{-88.6^\circ}\,\Omega \qquad Z_{IN} = 1 + Z_3$$

$$\boxed{Z_{IN} = 1.42\angle{-43.9}\,\Omega}$$

10.66 Determine the input impedance seen by the source in the network shown in Fig. P10.66.

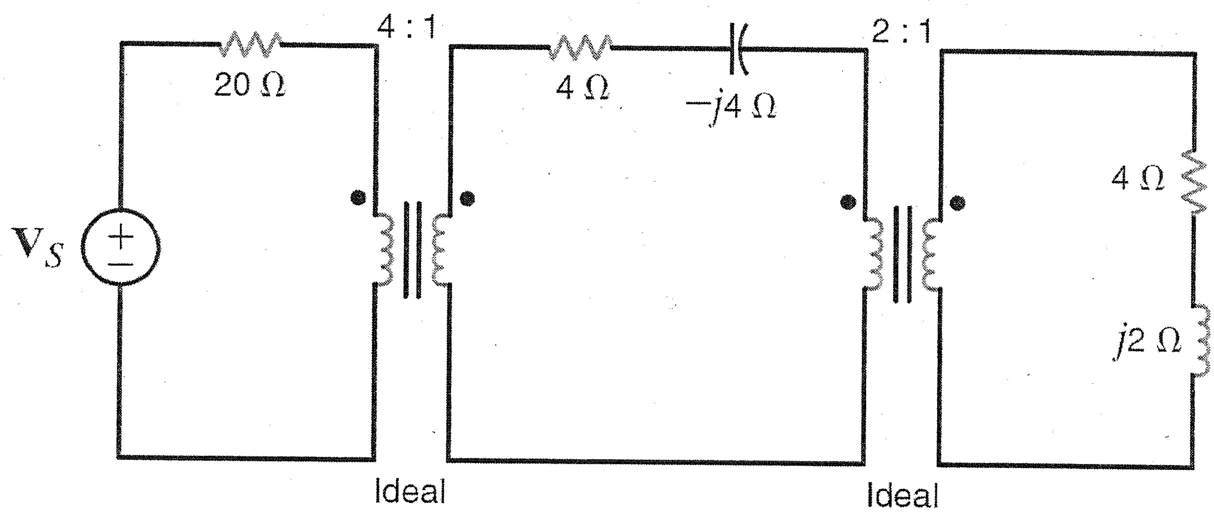
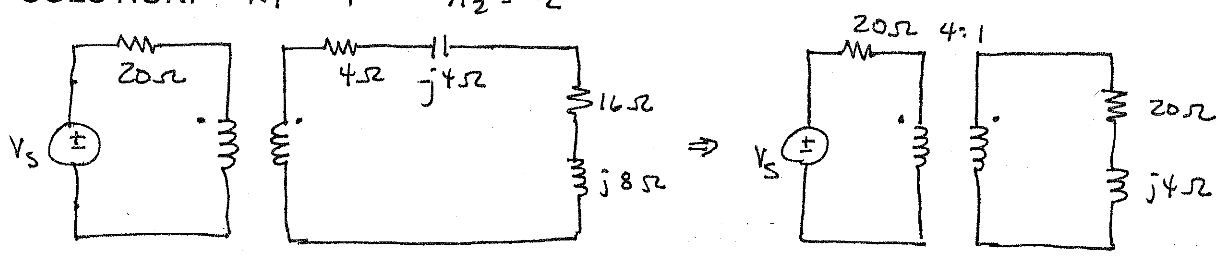
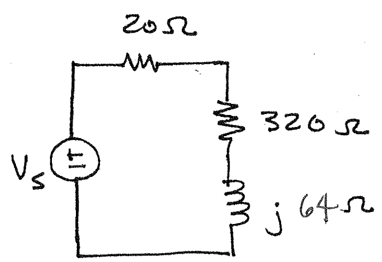


Figure P10.66

SOLUTION: $n_1 = 1/4$ $n_2 = 1/2$





$$\boxed{Z_{IN} = 340 + j\,64\ \Omega}$$

10.67 Determine the input impedance seen by the source in the network shown in Fig. P10.67.

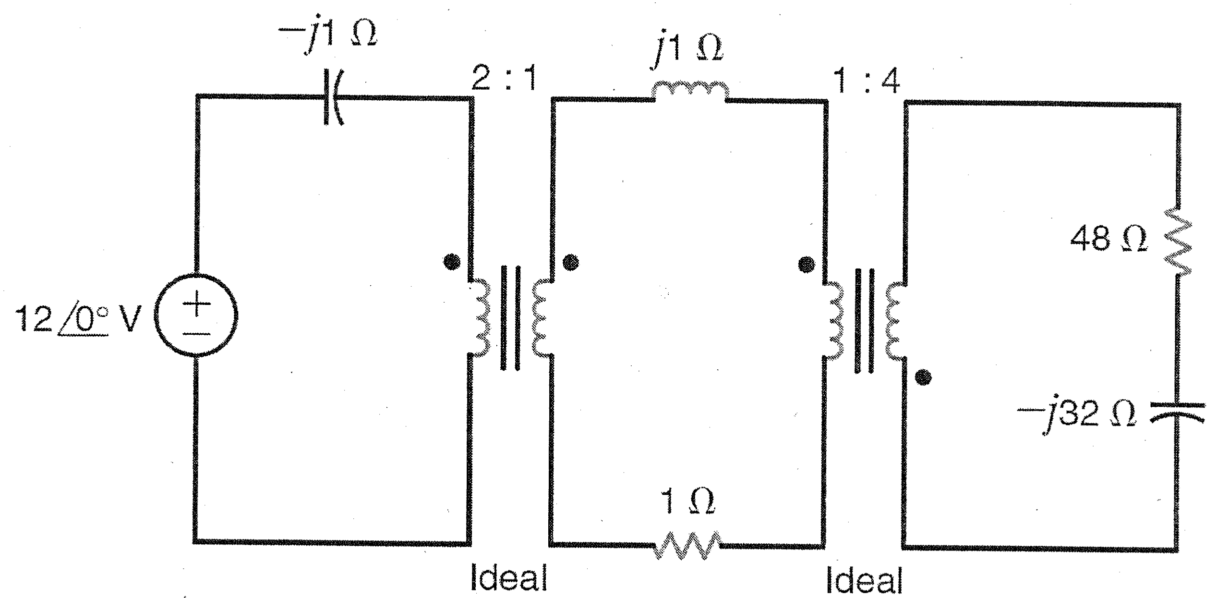
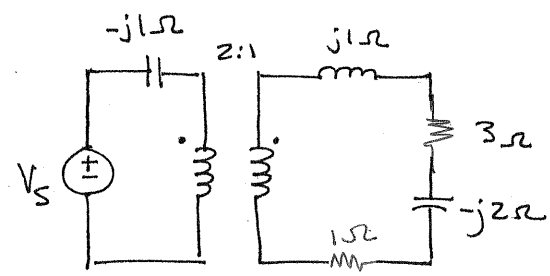
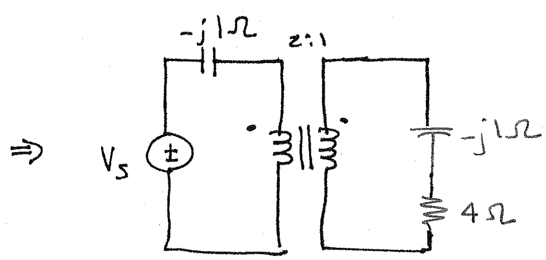


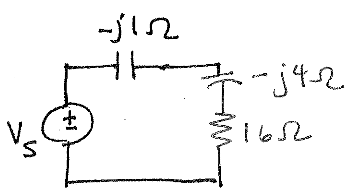
Figure P10.67

SOLUTION:



⇒





$Z_{IN} = 16 - j5\ \Omega$ $\boxed{Z_{IN} = 16 - j5\ \Omega}$

10.68 The output stage of an amplifier in an old radio is to be matched to the impedance of a speaker, as shown in Fig. P10.68. If the impedance of the speaker is 8 Ω and the amplifier requires a load impedance of 3.2 kΩ, determine the turns ratio of the ideal transformer.

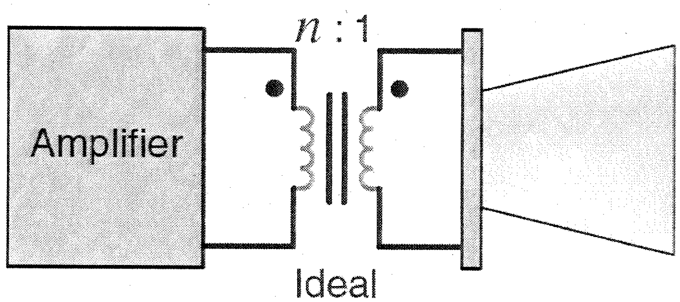


Figure P10.68

SOLUTION:

$Z_{SPEAKER} = 8\Omega \qquad Z_{SPEAKER}\, n^2 = 3200$

$n^2 = \dfrac{3200}{8} = 400 \qquad \boxed{n = 20}$

10.69 Determine $\mathbf{V}_S$ in the circuit in Fig. P10.69.

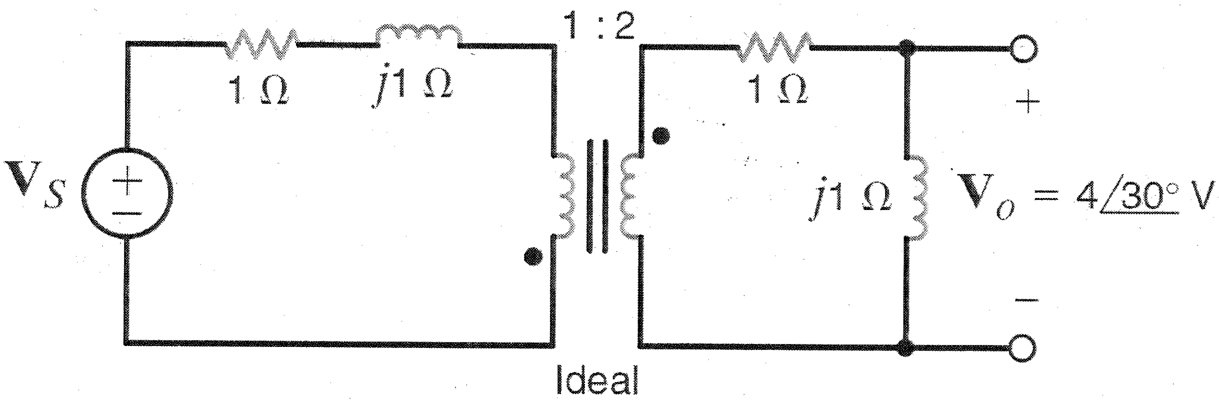
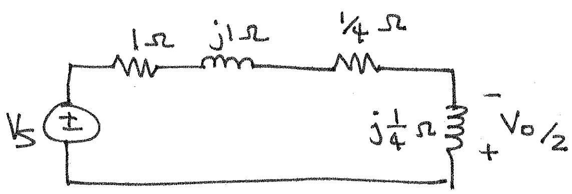


Figure P10.69

SOLUTION:



$$-\frac{V_o/2}{V_s} = \frac{j\,1/4}{1+\frac{1}{4}+j(1+1/4)} = 0.1 + j0.1$$

$$V_s = \frac{-2\angle 30}{0.1+j0.1}$$

$$\boxed{V_s = 14.14\angle 165^\circ \text{ V}}$$

10.70 Determine $\mathbf{I}_S$ in the circuit in Fig. P10.70. PSV

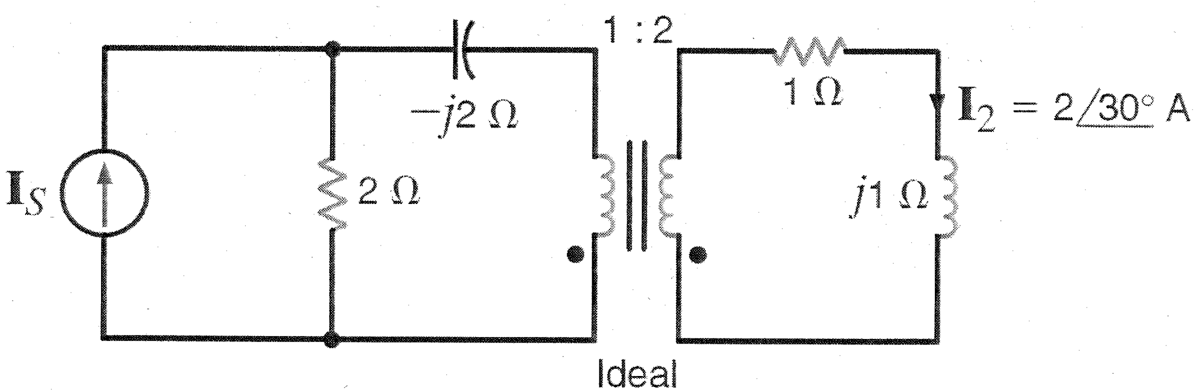
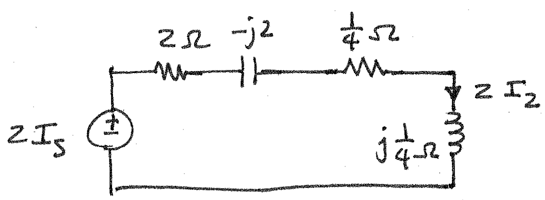


Figure P10.70

SOLUTION:



$$2I_2 = \frac{2I_S}{2.25 - j1.75} = 4\angle 30^\circ$$

$$I_S = 2\angle 30^\circ\,(2.25 - j1.75)$$

$$\boxed{I_S = 5.70\angle -7.87^\circ \text{ A}}$$

10.71 Given that $\mathbf{V}_o = 48\angle 30°$ V in the circuit shown in Fig. P10.71, determine $\mathbf{V}_S$. CS

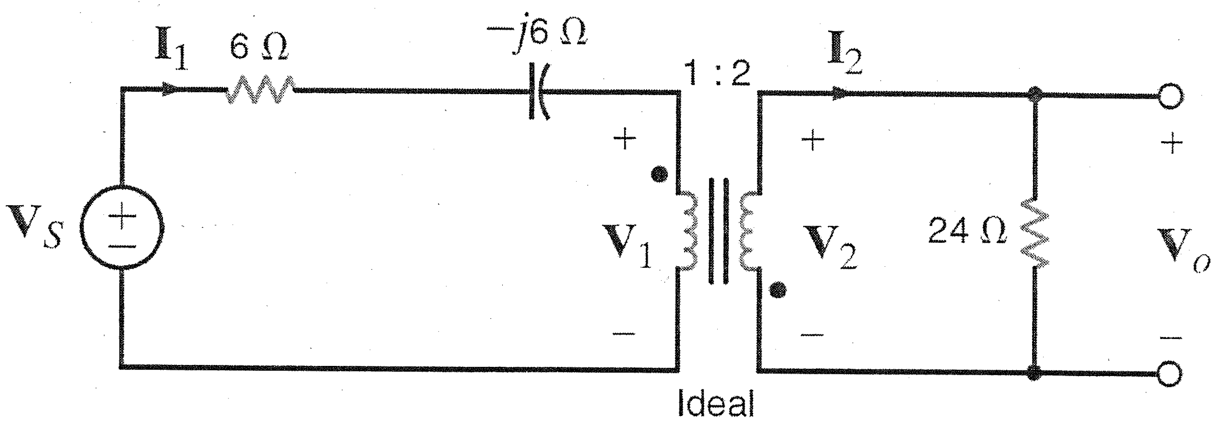


Figure P10.71

SOLUTION:

$V_2 = V_o = 48\angle 30°$ $\quad V_1 = -\dfrac{V_2}{2} = -24\angle 30° = 24\angle -150°$ V

$I_2 = V_o/24 = 2\angle 30°$ A $\quad I_1 = -2I_2 = -4\angle 30°$ A $= 4\angle -150°$ A

$V_S = I_1(6 - j6) + V_1$

$$\boxed{V_S = 53.7\angle -177° \text{ V}}$$

10.72 In the circuit in Fig. P10.72, if $\mathbf{I}_x = 4\underline{/30^\circ}$ A, find $\mathbf{V}_o$.

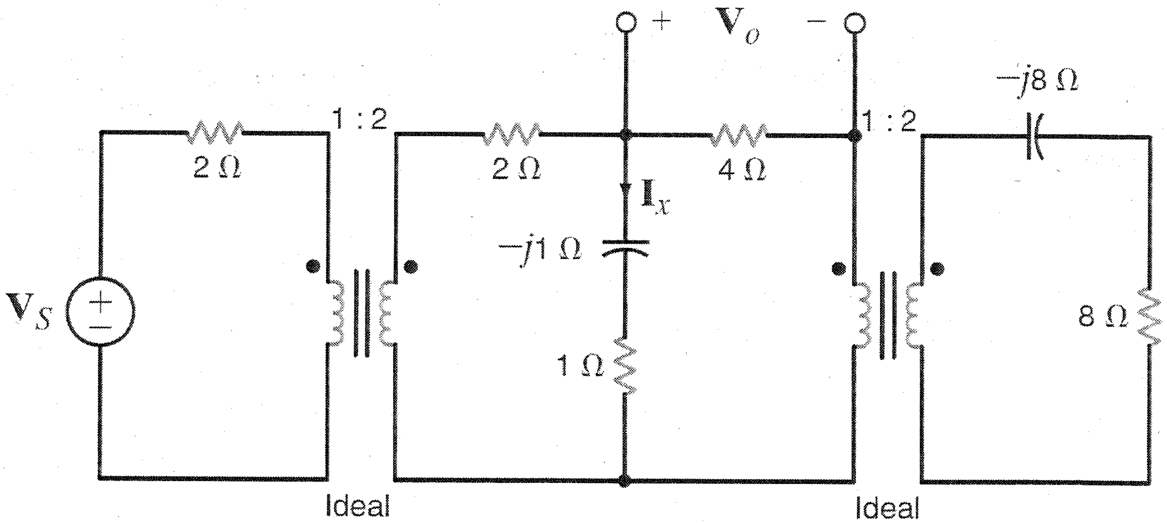
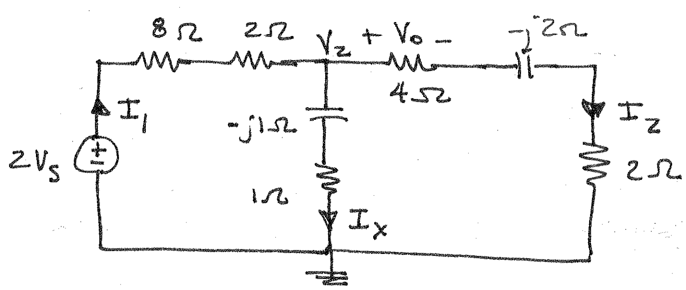


Figure P10.72

SOLUTION:



$$V_2 = I_x(1-j1) = 4\sqrt{2}\,\underline{/-15^\circ}\text{ V}$$

$$I_2 = \frac{V_2}{6-j2} = 0.894\,\underline{/3.43^\circ}\text{ A}$$

$$\boxed{V_o = 4I_2 = 3.58\,\underline{/3.43^\circ}\text{ V}}$$

10.73 In the network in Fig. P10.73, if $\mathbf{I}_1 = 6\angle 0°$ A, find $\mathbf{V}_S$.

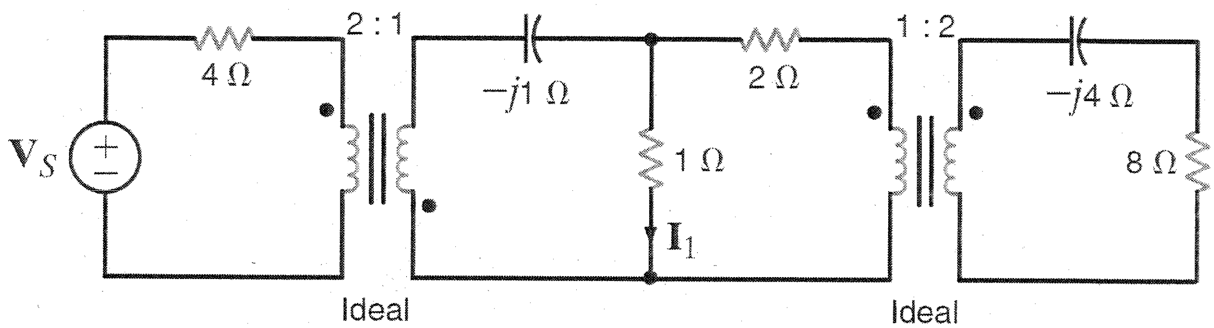
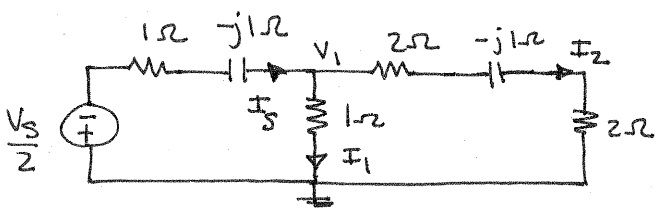


Figure P10.73

SOLUTION:



$V_1 = (1)I_1 = 6\angle 0°$ V

$I_2 = V_1 / (4-j1) = 1.46\angle 14.0°$ A

$I_s = I_2 + I_1 = 7.42\angle 2.73°$ A

$-\frac{V_s}{2} = I_s(1-j1) + V_1 = 15.5\angle -27.1°$ V

$\boxed{V_s = 30.9\angle 153° \text{ V}}$

10.74 For maximum power transfer, we desire to match the impedance of the inverting amplifier stage in Fig. P10.74 to the 50-Ω equivalent resistance of the ac input source. However, standard op-amps perform best when the resistances around them are at least a few hundred ohms. The gain of the op-amp circuit should be −10. Design the complete circuit by selecting resistors no smaller than 1 kΩ and specifying the turns ratio of the ideal transformer to satisfy both the gain and impedance matching requirements.

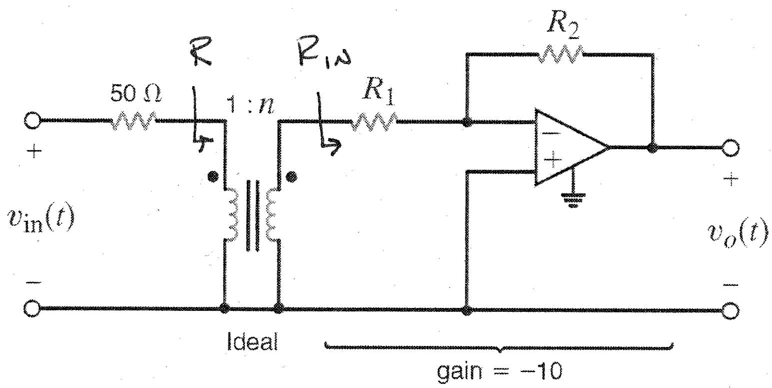


Figure P10.74

SOLUTION:

gain $= -10 = -R_2/R_1$ Arbitrarily select $R_1 = 5\,k\Omega$

yields $R_2 = 50\,k\Omega$

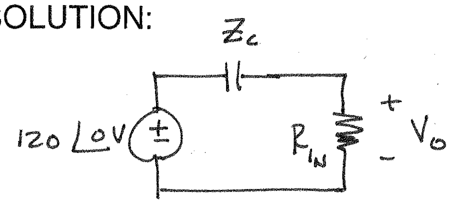
for op amp circuits $R_{IN} = R_1 = 5\,k\Omega$

$R = R_{IN}/n^2 \Rightarrow n^2 = 5000/50 = 100 \Rightarrow n = 10$

$\boxed{R_1 = 5\,k\Omega \quad R_2 = 50\,k\Omega \quad n = 10}$

10.75 Digital clocks often divide a 60-Hz frequency signal to obtain a 1-second, 1-minute, or 1-hour signal. A convenient source of this 60-Hz signal is the power line. However, 120 volts is too high to be used by the low-power electronics. Instead, a 3-V, 60-Hz signal is needed. If a resistive voltage divider is used to drop the voltage from 120 V to 3 V, the heat generated will be unacceptable. In addition, it is costly to use a transformer in this application. Digital clocks are consumer items and must be very inexpensive to be a competitive product. The problem then is to design a circuit that will produce between 2.5 V and 3 V at 60 Hz from the 120-V ac power line without dissipating any heat or the use of a transformer. The design will interface with a circuit that has an input resistance of 1200 ohms.

SOLUTION:



$|V_o| = 3\,V \qquad R_{IN} = 1.2\,k\Omega$

$$V_o = \frac{120\angle 0\, R_{IN}}{R_{IN} + Z_C} \Rightarrow |V_o| = 3 = \frac{120\,(1200)}{\sqrt{1200^2 + Z_C^2}}$$

$$Z_C^2 = 2.30\times 10^9 \Rightarrow Z_C = -j47.98\ k\Omega$$

$$Z_C = -j/\omega C = -j47.98\ k\Omega \qquad \boxed{C = 55.3\ nF}$$

10FE-1 In the network in Fig. 10PFE-1, find the impedance seen by the source. CS

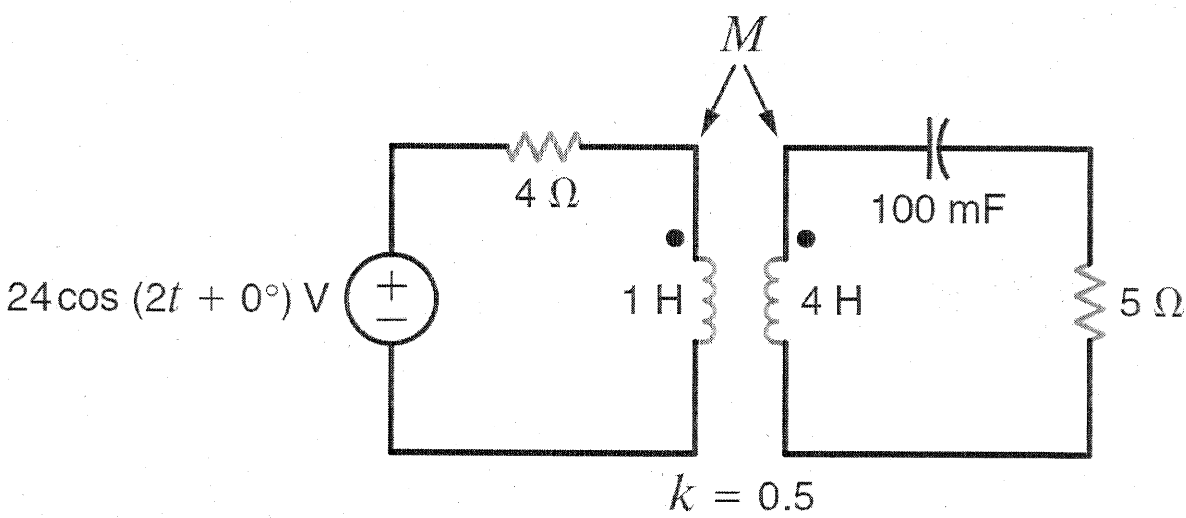
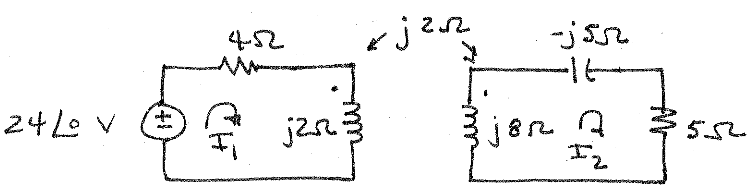


Figure 10PFE-1

SOLUTION: $\omega = 2\ r/s$ $\quad M = k\sqrt{L_1L_2} = 1\,H$



$$24\angle 0 = I_1(4+j2) - j2I_2$$
$$0 = -j2I_1 + I_2(5+j3)$$
$$\Rightarrow \begin{bmatrix} 4+j2 & -j2 \\ -j2 & 5+j3 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 24 \\ 0 \end{bmatrix}$$

$I_1 = 4.92\angle -19.7^\circ$ A

$$Z_{IN} = \frac{24\angle 0}{I_1}$$

$$\boxed{Z_{IN} = 4.87\angle 19.7^\circ\ \Omega}$$

10FE-2 In the circuit in Fig. 10PFE-2, select the value of the transformer's turns ratio $n = N_2/N_1$ to achieve impedance matching for maximum power transfer. Using this value of n, calculate the power absorbed by the 3-Ω resistor.

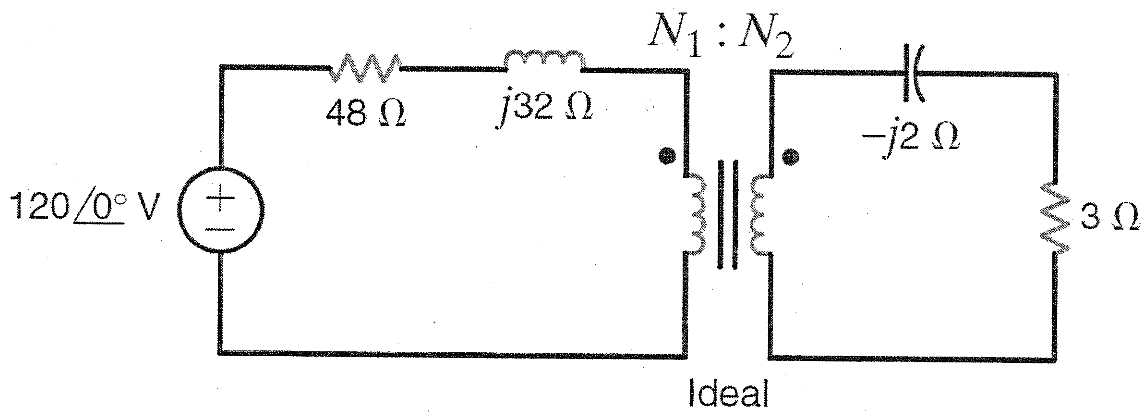
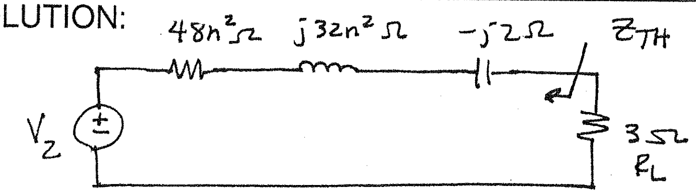


Figure 10PFE-2

SOLUTION:

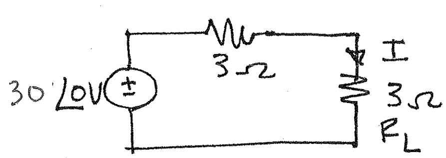


$V_2 = 120n \angle 0°$ V

For max. power transfer, $Z_{TH} = 3\,\Omega$

$Z_{TH} = 48n^2 + j(32n^2 - 2) = 3$

if $n = 1/4$, $Z_{TH} = 48/16 + j(32/16 - 2) = 3\,\Omega$ ✓



$I = \dfrac{30\angle 0}{6} = 5\angle 0$ A

$P_L = \frac{1}{2} I_M^2 R_L$

$\boxed{P_L = 37.5 \text{ W}}$

$\boxed{n = 1/4}$

10FE-3 In the circuit in Fig. 10FE-3, select the turns ratio of the ideal transformer that will match the output of the transistor amplifier to the speaker represented by the 16-Ω load. CS

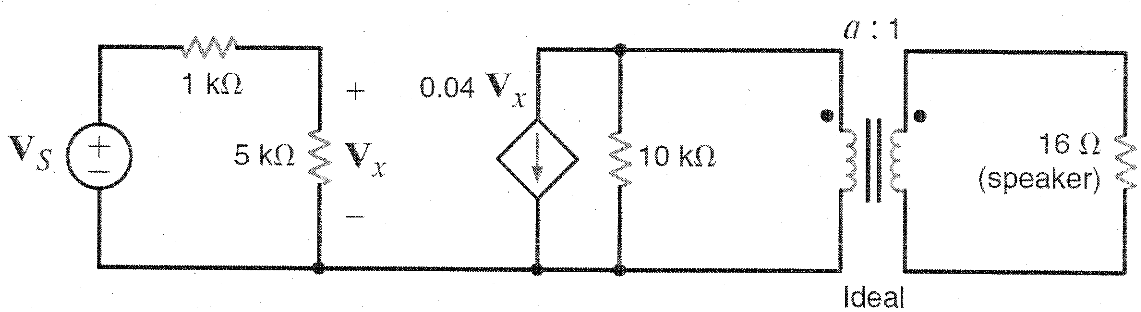
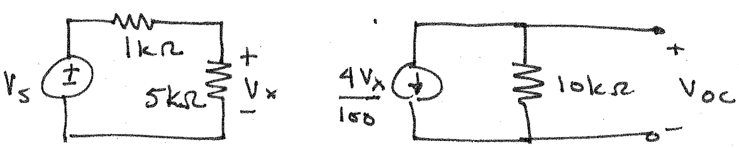


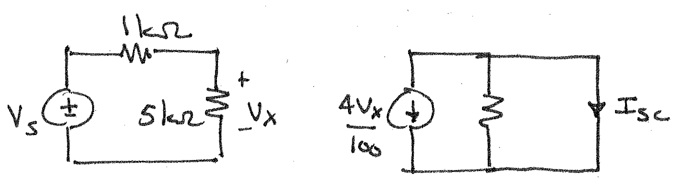
Figure 10PFE-3

SOLUTION:

Find Rout of amplifier



$$V_{oc} = \left(\frac{5}{6} V_s\right)\left(-\frac{4}{100}\right)(10^4)$$



$$I_{sc} = \left(\frac{5}{6} V_s\right)\left(-\frac{4}{100}\right)$$

$$R_{OUT} = V_{oc}/I_{sc} = 10\,k\Omega$$

$$16a^2 = 10^4 \Rightarrow \boxed{a = 25}$$

Chapter Twelve:

Variable-Frequency Network Performance

12.1 Determine the driving point impedance at the input terminals of the network shown in Fig. P12.1 as a function of s. CS

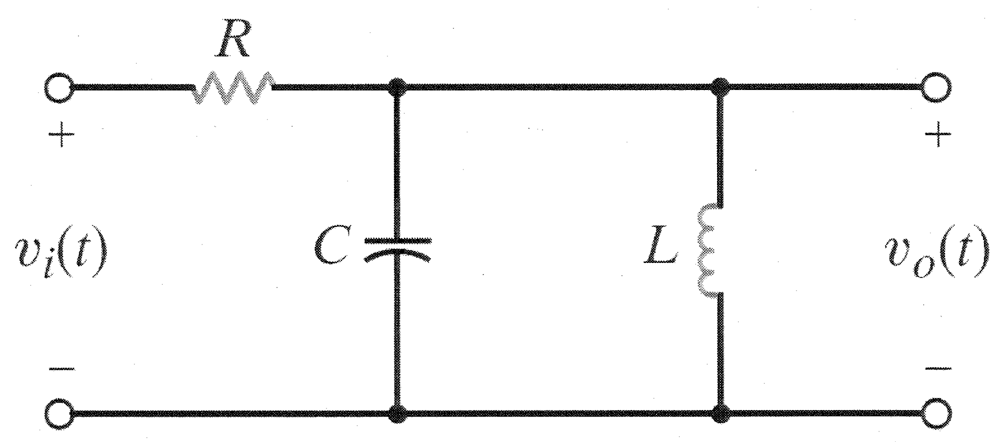
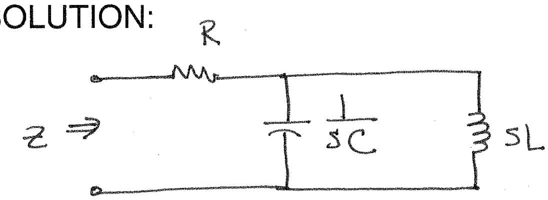


Figure P12.1

SOLUTION:



$$Z = R + \frac{(sL)(1/sC)}{sL + 1/sC}$$

$$= R + \frac{sL}{s^2LC+1} = \frac{s^2LCR+R+sL}{s^2LC+1}$$

$$\boxed{Z = R\left[\frac{s^2 + \frac{s}{RC} + \frac{1}{LC}}{s^2 + \frac{1}{LC}}\right]}$$

12.2 Determine the voltage transfer function $\mathbf{V}_o(s)/\mathbf{V}_i(s)$ as a function of s for the network shown in Fig. P12.2.
PSV

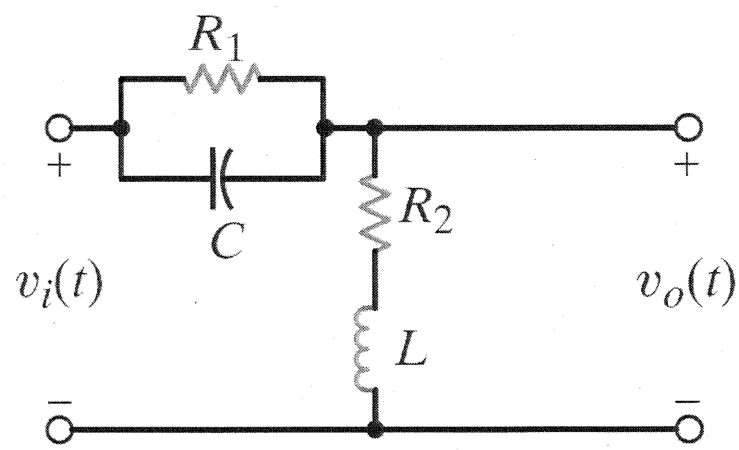
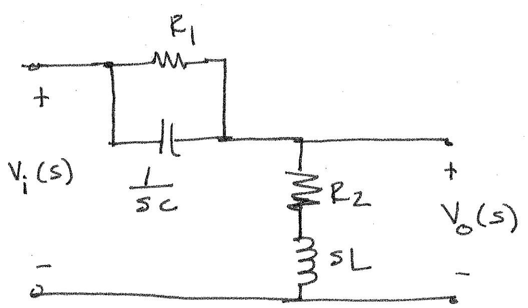


Figure P12.2

SOLUTION:



Let $Z_1 = \dfrac{R_1/sC}{R_1 + 1/sC} = \dfrac{R_1}{sR_1C+1}$

$Z_2 = sL + R_2$

$$\frac{V_o}{V_i}(s) = \frac{Z_2}{Z_1+Z_2} = \frac{sL+R_2}{sL+R_2+\dfrac{R_1}{sR_1C+1}}$$

$$\boxed{\frac{V_o}{V_I}(s) = \frac{s^2 + s\left[\frac{R_2}{L} + \frac{1}{R_1C}\right] + \frac{R_2/R_1}{LC}}{s^2 + s\left[\frac{R_2}{L} + \frac{1}{R_1C}\right] + \frac{(R_1+R_2)/R_1}{LC}}}$$

12.3 Determine the driving point impedance at the input terminals of the network shown in Fig. P12.3 as a function of s.

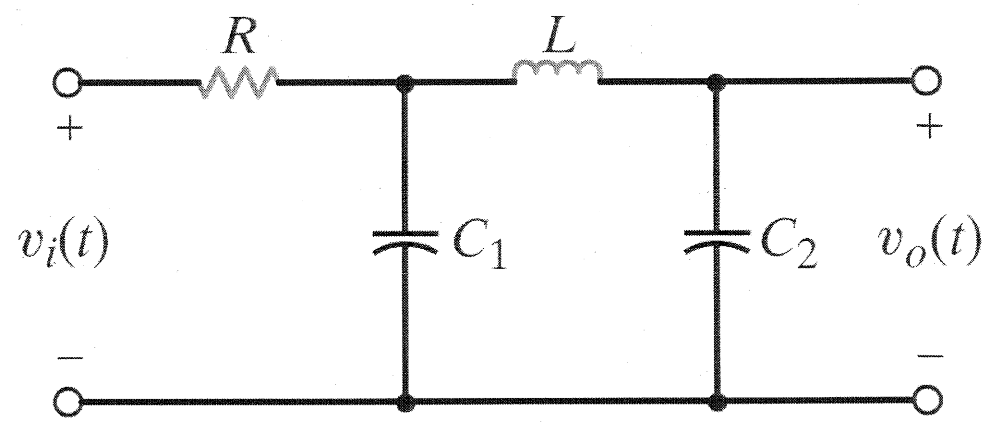
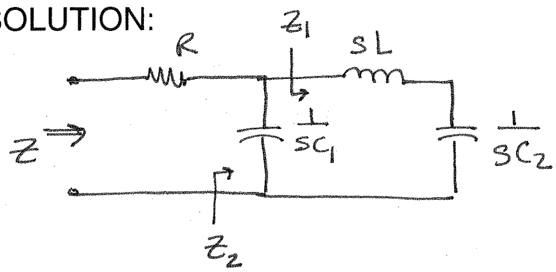


Figure P12.3

SOLUTION:



$$Z_1 = sL + 1/sC_2$$

$$Z_2 = \left(\frac{1}{sC_1}\right) Z_1 \Big/ \left(\frac{1}{sC_1} + Z_1\right)$$

$$Z_2 = \frac{s^2 + \frac{1}{LC_2}}{sC_1\left[s^2 + \frac{C_1 + C_2}{LC_1C_2}\right]}$$

$$Z = R + \frac{s^2 + \frac{1}{LC_2}}{sC_1\left[s^2 + \frac{C_1 + C_2}{LC_1C_2}\right]}$$

$$\boxed{Z = R\left\{\frac{s^3 + \frac{s^2}{C_1R} + \frac{(C_1+C_2)}{LC_1C_2}s + \frac{1}{RLC_1C_2}}{s\left(s^2 + \frac{C_1+C_2}{LC_1C_2}\right)}\right\}}$$

12.4 Find the transfer impedance $\mathbf{V}_o(s)/\mathbf{I}_S(s)$ for the network shown in Fig. P12.4.

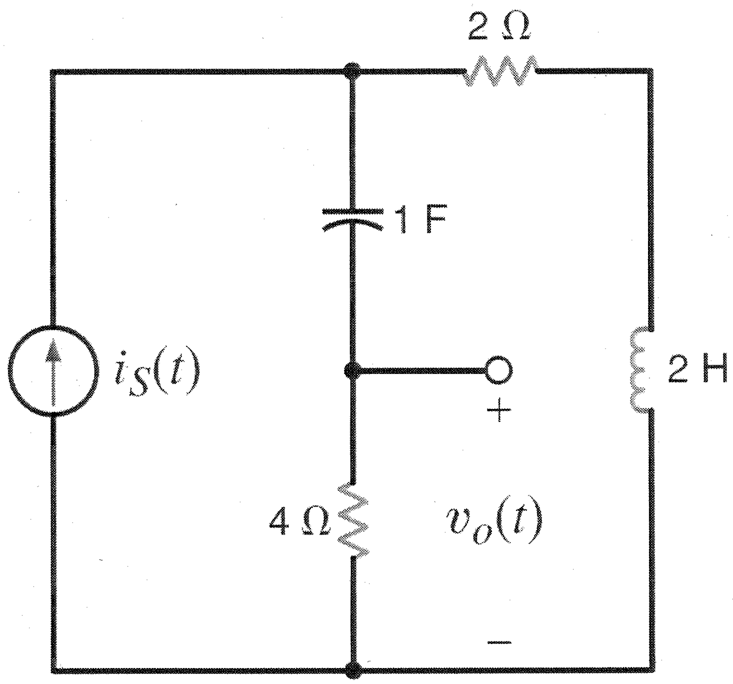
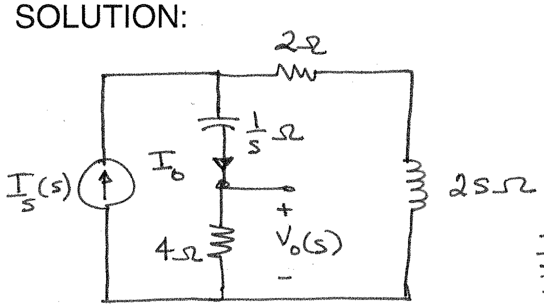


Figure P12.4

SOLUTION:



Let $Z_1 = 4 + 1/s = \frac{4s+1}{s}\ \Omega$

$Z_2 = 2s + 2\ \Omega$

$$\frac{I_o}{I_s} = \frac{Z_2}{Z_1+Z_2} = \frac{2s(s+1)}{2s^2+6s+1}$$

$$\frac{V_o}{I_s} = 4\frac{I_o}{I_s} = \boxed{\frac{8s(s+1)}{2s^2+6s+1} = \frac{V_o}{I_s}}$$

12.5 Find the driving point impedance at the input terminals of the circuit in Fig. P12.5 as a function of s. CS

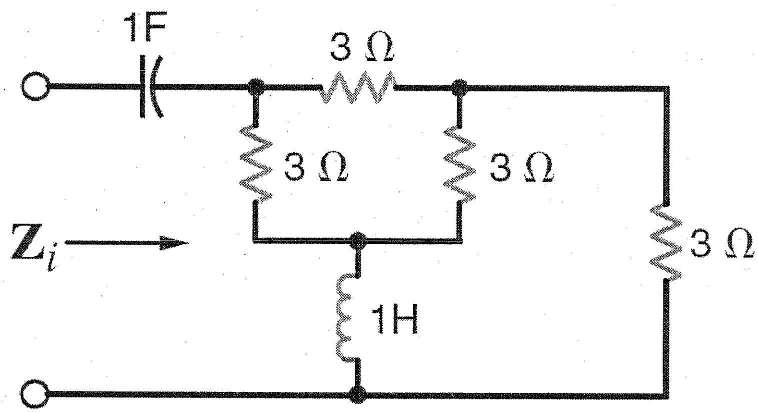
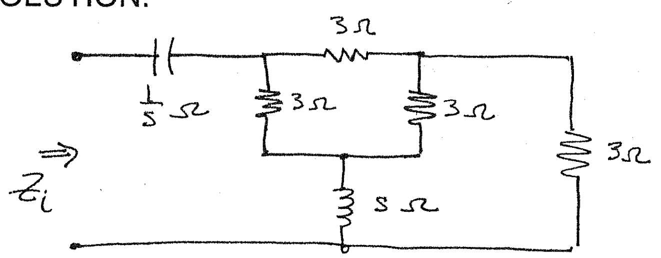
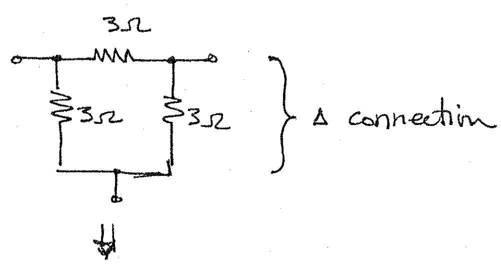
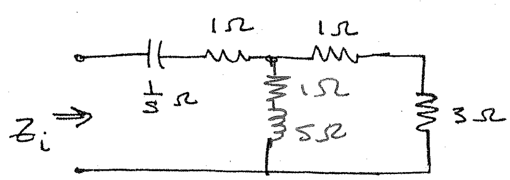


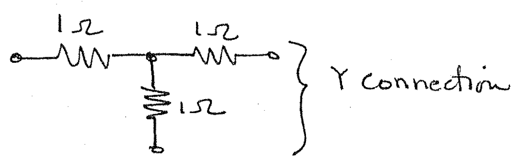
Figure P12.5

SOLUTION:









$Z_1 = 1 + 3 = 4\Omega \qquad Z_2 = (s+1)(Z_1)/(s+1+Z_1) = 4(s+1)/(s+5)$

$$Z_i = Z_2 + 1 + \frac{1}{s} = \frac{5(s^2+2s+1)}{s(s+5)}$$

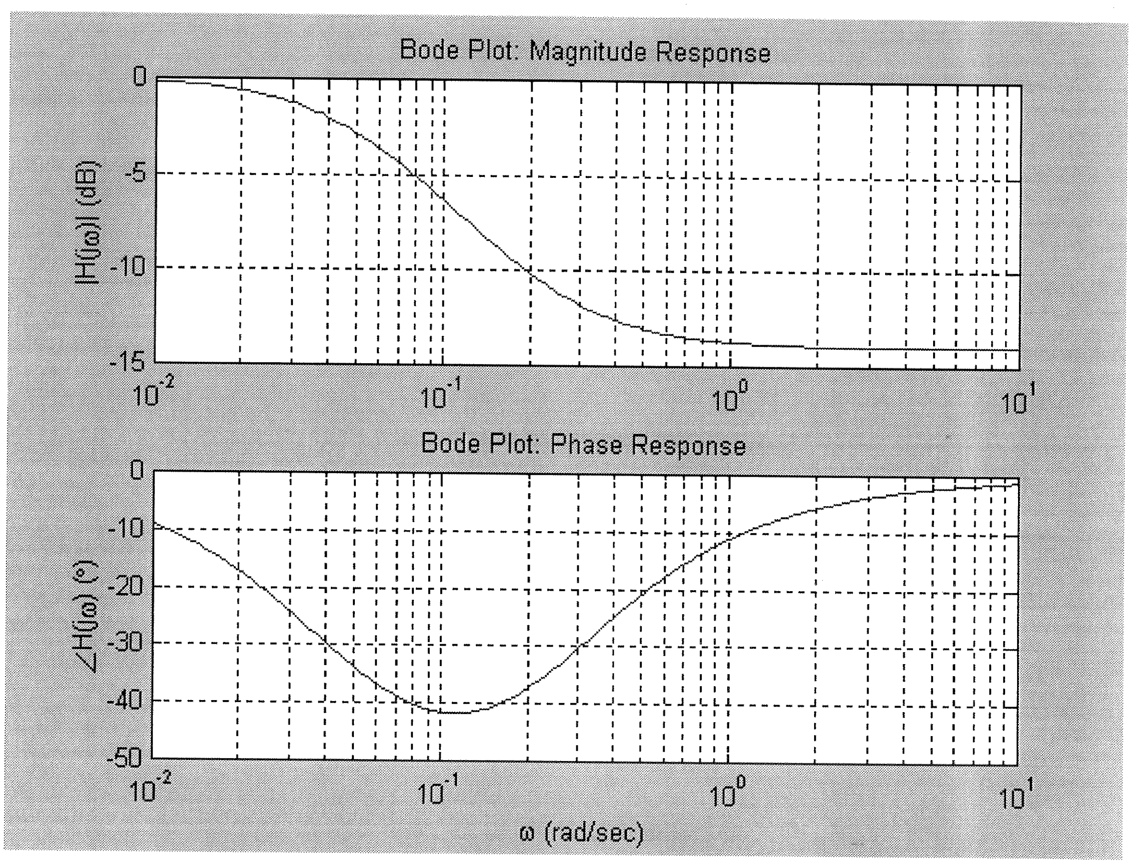
$$\boxed{Z_i = \frac{5(s^2+2s+1)}{s(s+5)}\ \Omega}$$

12.6 Draw the Bode plot for the network function

$$\mathbf{H}(j\omega) = \frac{j\omega 4 + 1}{j\omega 20 + 1}$$

SOLUTION:

$$H(j\omega) = \frac{1}{5}\,\frac{j\omega + 1/4}{j\omega + 1/20}$$

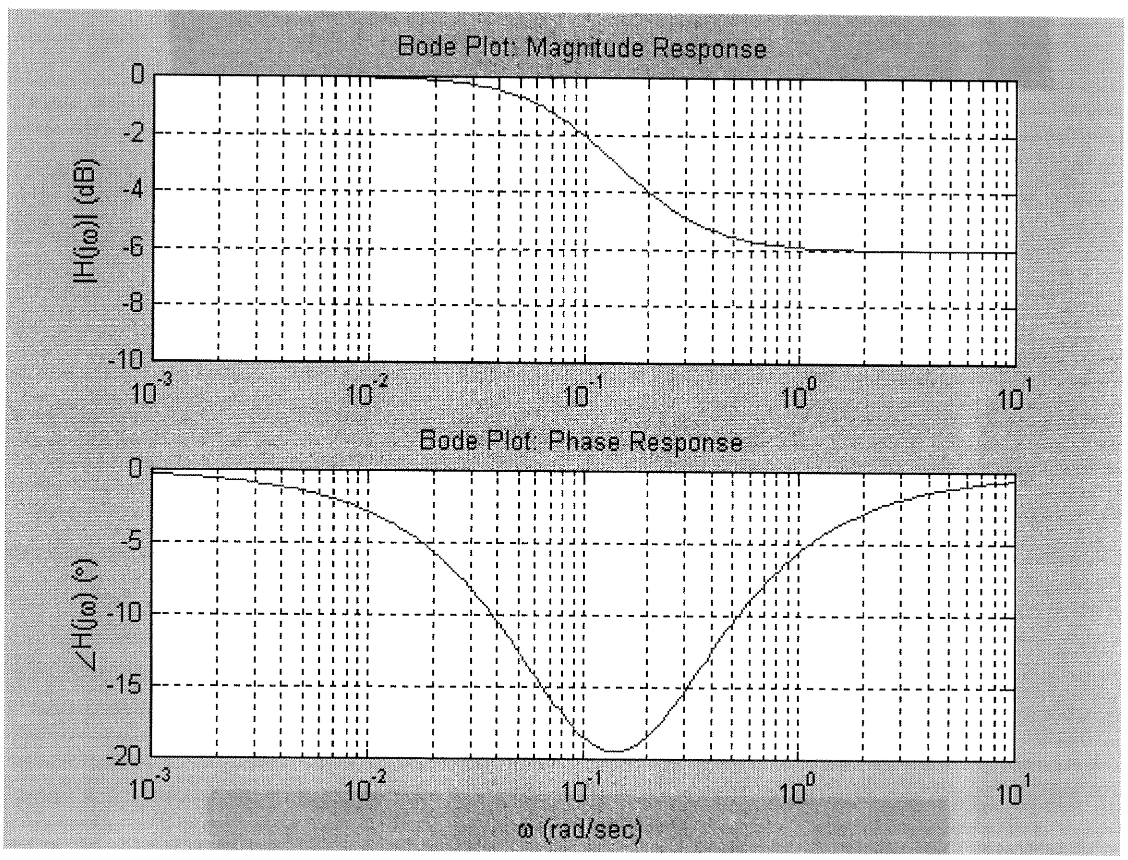


12.7 Draw the Bode plot for the network function

$$\mathbf{H}(j\omega) = \frac{j\omega 5 + 1}{j\omega 10 + 1}$$

SOLUTION:

$$H(j\omega) = \frac{1}{2}\left[\frac{j\omega + 1/5}{j\omega + 1/10}\right]$$

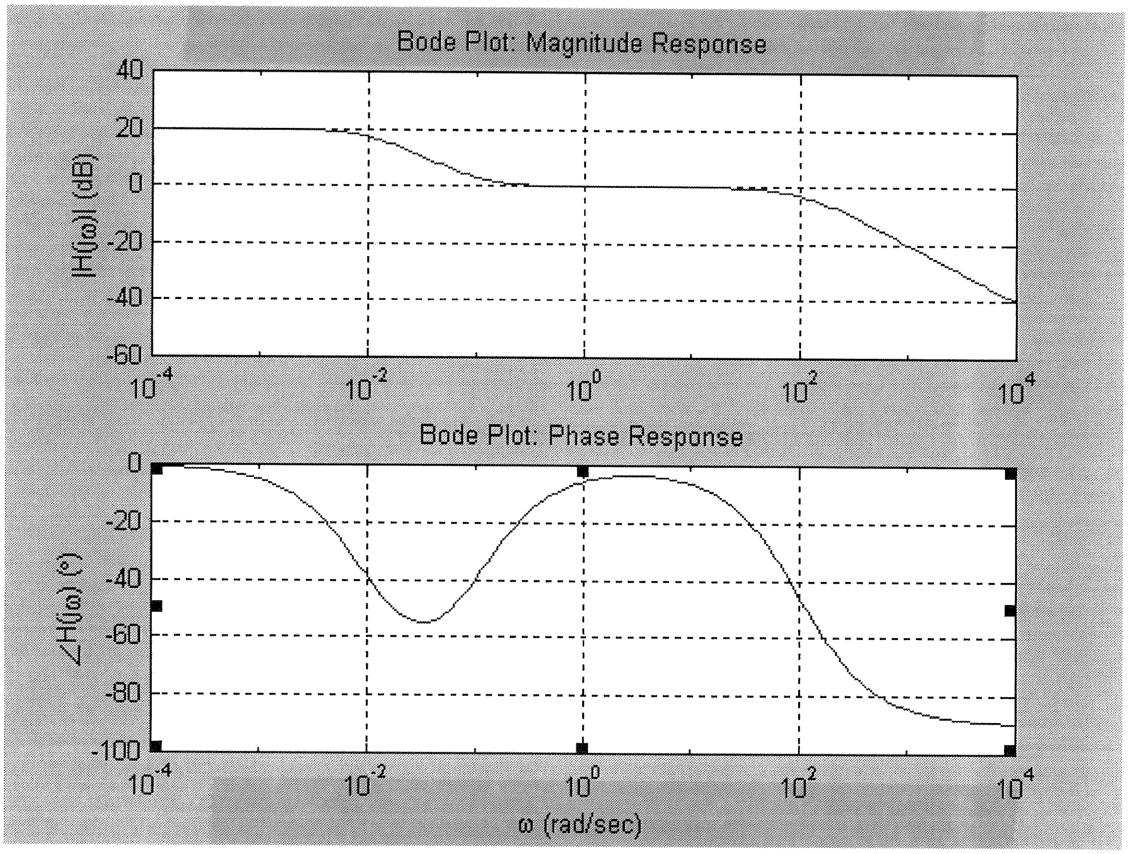


12.8 Draw the Bode plot for the network function

$$\mathbf{H}(j\omega) = \frac{10(10j\omega + 1)}{(100j\omega + 1)(0.01j\omega + 1)}$$

SOLUTION:

$$H(j\omega) = \frac{100\left(j\omega + \frac{1}{10}\right)}{\left(j\omega + \frac{1}{100}\right)\left(j\omega + 100\right)}$$



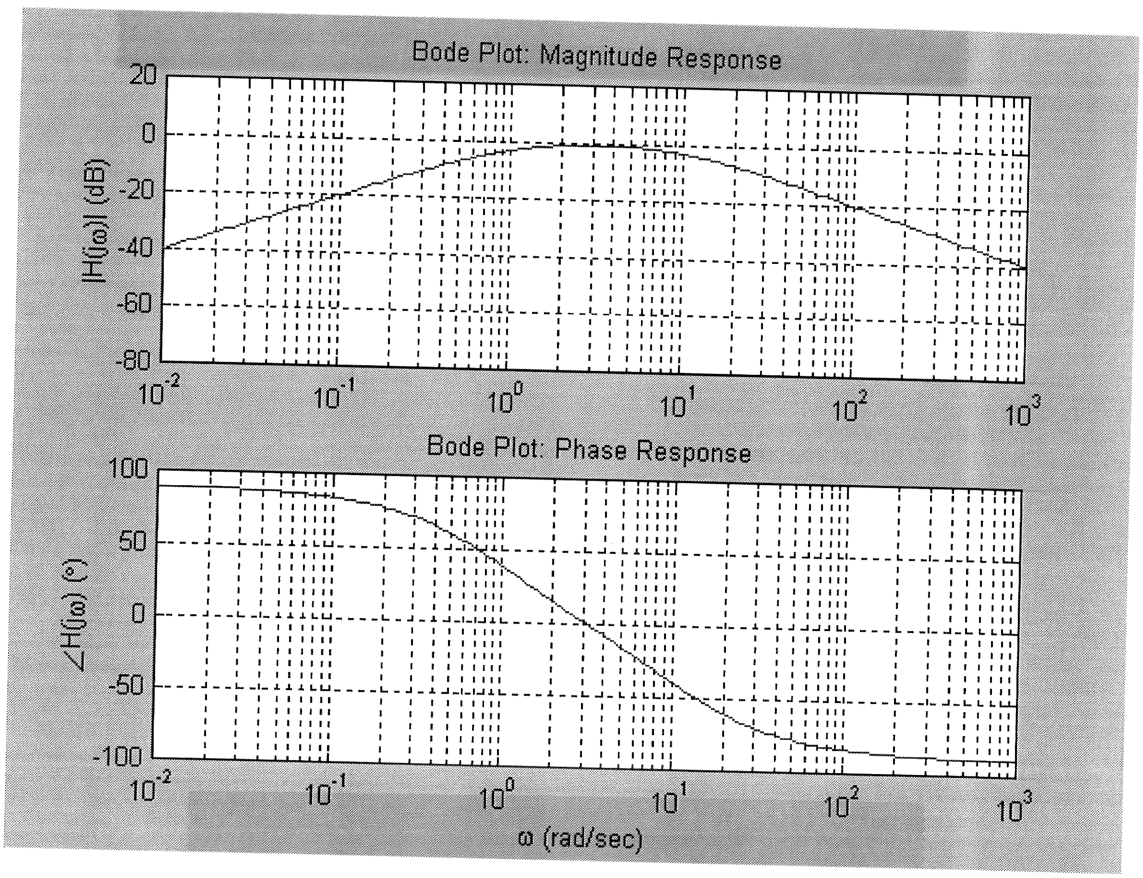
12.9 Draw the Bode plot for the network function

$$\mathbf{H}(j\omega) = \frac{j\omega}{(j\omega + 1)(0.1j\omega + 1)}$$

CS

SOLUTION:

$$H(j\omega) = \frac{10(j\omega)}{(j\omega+1)(j\omega+10)}$$

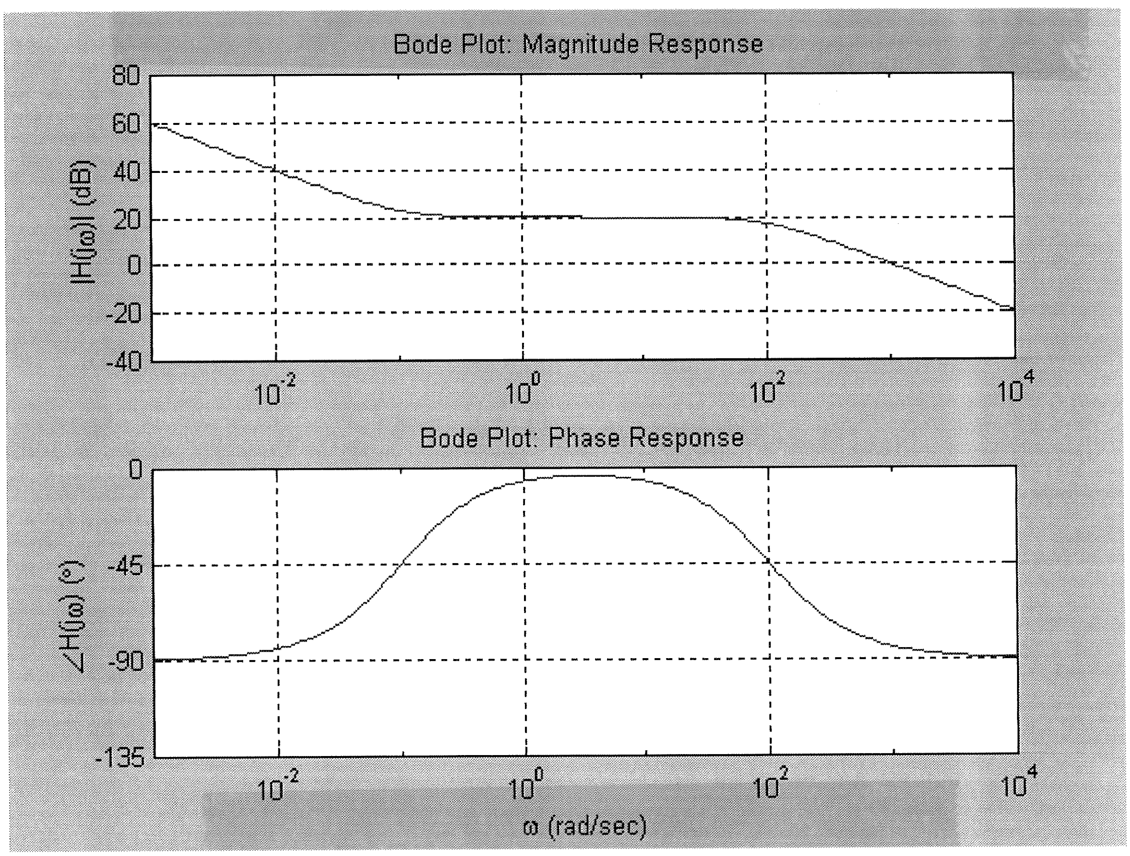


12.10 Draw the Bode plot for the network function

$$\mathbf{H}(j\omega) = \frac{10j\omega + 1}{j\omega(0.01j\omega + 1)}$$

SOLUTION:

$$H(j\omega) = \frac{10(j\omega + 0.1)}{j\omega\frac{(j\omega+100)}{100}} = \frac{1000(j\omega + 0.1)}{j\omega(j\omega+100)}$$



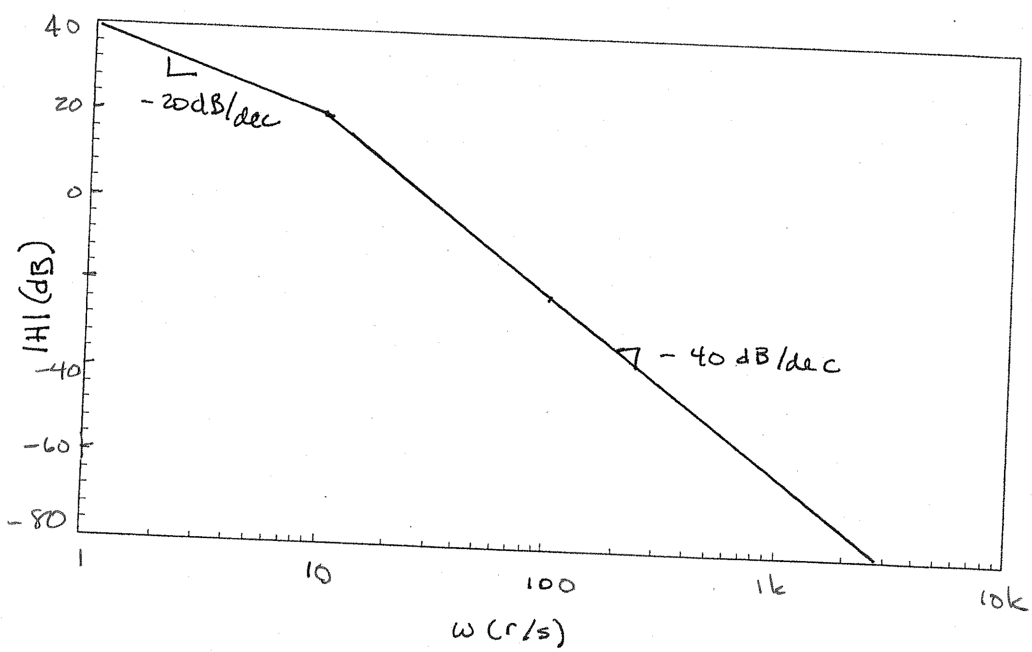
12.11 Sketch the magnitude characteristic of the Bode plot for the transfer function

$$\mathbf{H}(j\omega) = \frac{100}{j\omega(0.1j\omega + 1)}$$ PSV

SOLUTION:

$$H(j\omega) = \frac{100}{j\omega\left(\frac{j\omega + 10}{10}\right)} = \frac{1000}{j\omega(j\omega + 10)}$$

$$|H|\Big|_{\omega=1} \approx \frac{1000}{(1)(10)} = 100 = 40\text{ dB}$$



12.12 Sketch the magnitude characteristic of the Bode plot for the transfer function

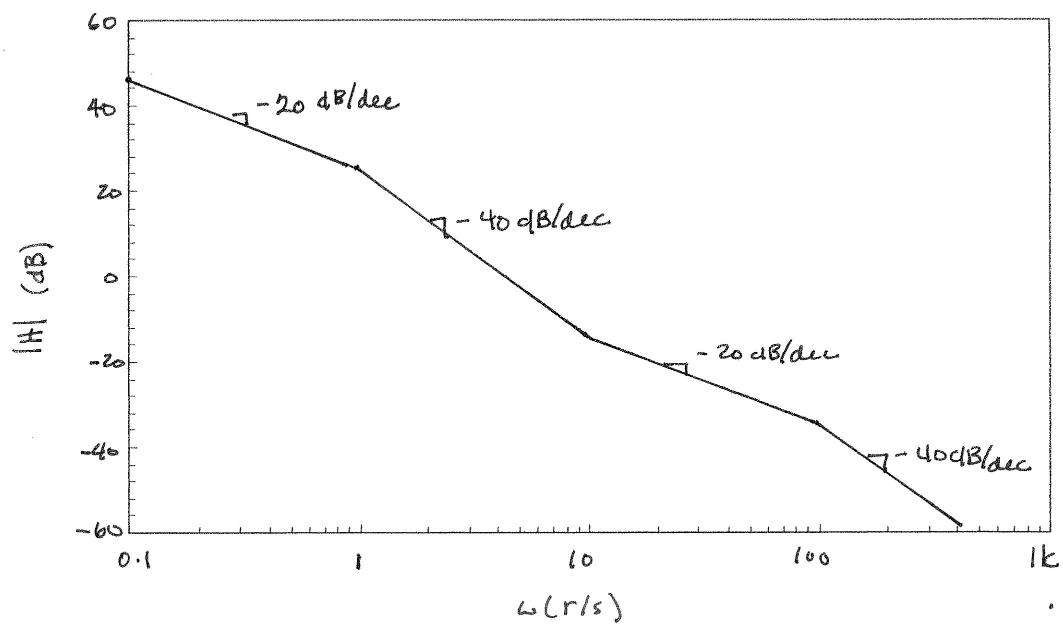
$$\mathbf{H}(j\omega) = \frac{20(0.1j\omega + 1)}{j\omega(j\omega + 1)(0.01j\omega + 1)}$$

SOLUTION:

$$H(j\omega) = \frac{20(0.1)(j\omega+10)}{j\omega(j\omega+1)(0.01)(j\omega+100)} = \frac{200(j\omega+10)}{j\omega(j\omega+1)(j\omega+100)}$$

As $\omega \rightarrow 0$, $|H| \rightarrow 20/\omega$

$$|H|\Big|_{\omega=0.1} \simeq \frac{200(10)}{(0.1)(1)(100)} = 200 = 46\,dB$$

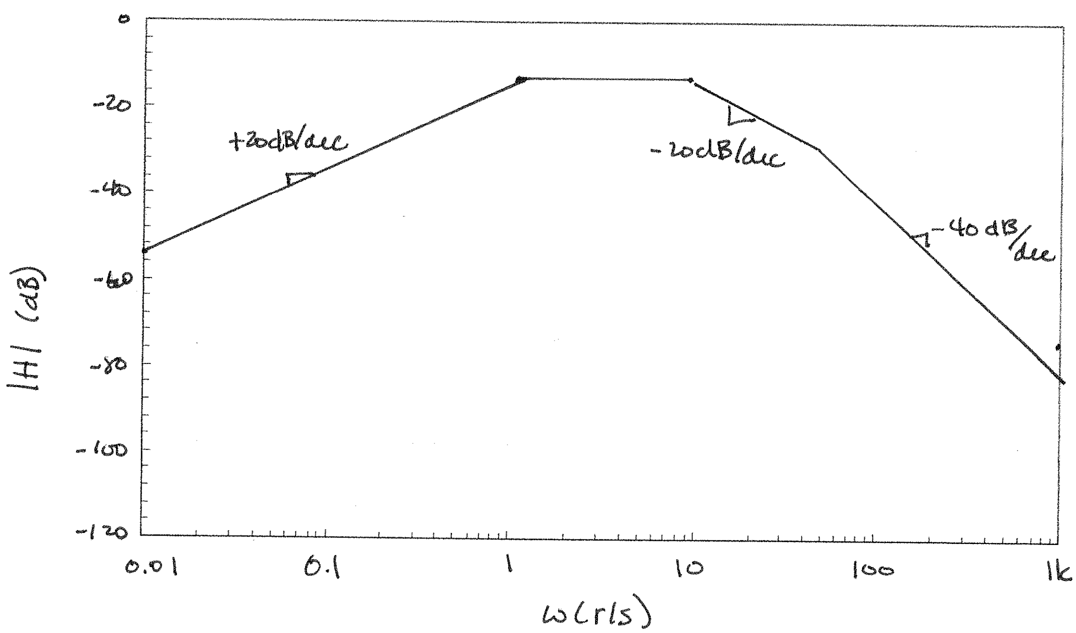


12.13 Sketch the magnitude characteristic of the Bode plot for the transfer function

$$\mathbf{H}(j\omega) = \frac{100(j\omega)}{(j\omega + 1)(j\omega + 10)(j\omega + 50)}$$ CS

SOLUTION:

$$H(j\omega) = \frac{100(j\omega)}{(j\omega+1)(j\omega+10)(j\omega+50)} \qquad |H|\Big|_{\omega=\frac{1}{100}} \simeq 2\times10^{-3} = -54\,dB$$

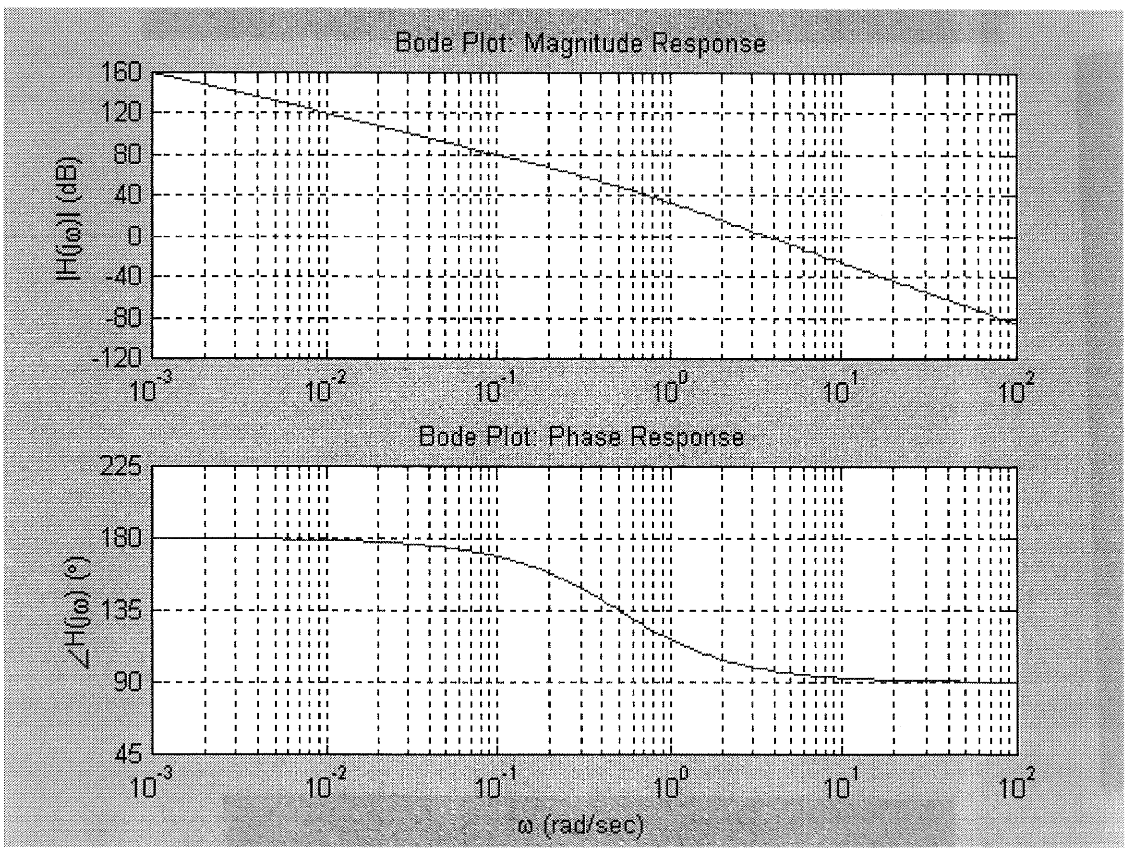


12.14 Draw the Bode plot for the network function

$$\mathbf{H}(j\omega) = \frac{100}{(j\omega)^2(j\omega 2 + 1)}$$

SOLUTION:

$$H(j\omega) = \frac{100}{(j\omega)^2 (j\omega + 1/2)2} = \frac{50}{(j\omega)^2 (j\omega + 0.5)}$$



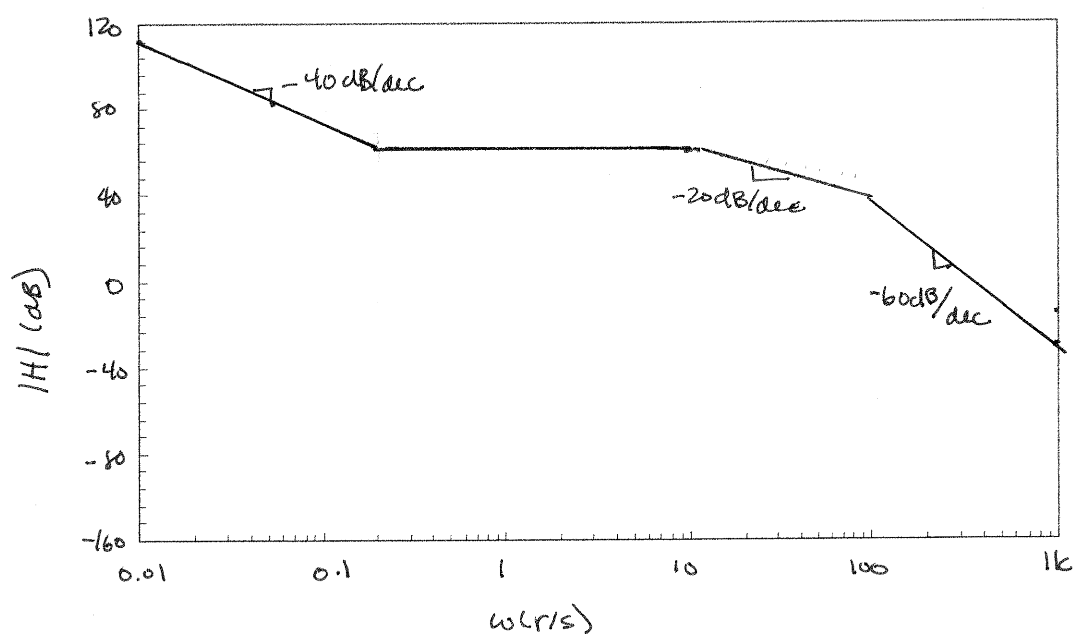
12.15 Sketch the magnitude characteristic of the Bode plot for the transfer function

$$\mathbf{H}(j\omega) = \frac{36 \times 10^5(5j\omega + 1)^2}{(j\omega)^2(j\omega + 10)(j\omega + 100)^2}$$

SOLUTION:

$$H(j\omega) = \frac{36\times10^5(25)(j\omega+0.2)^2}{(j\omega)^2(j\omega+10)(j\omega+100)^2} = \frac{9\times10^7(j\omega+0.2)^2}{(j\omega)^2(j\omega+10)(j\omega+100)^2}$$

$$|H|\Big|_{\omega=0.01} \simeq \frac{9\times10^7(0.2)^2}{(0.01)^2(10)(100)^2} = 3.60\times10^5 = 111\,dB$$



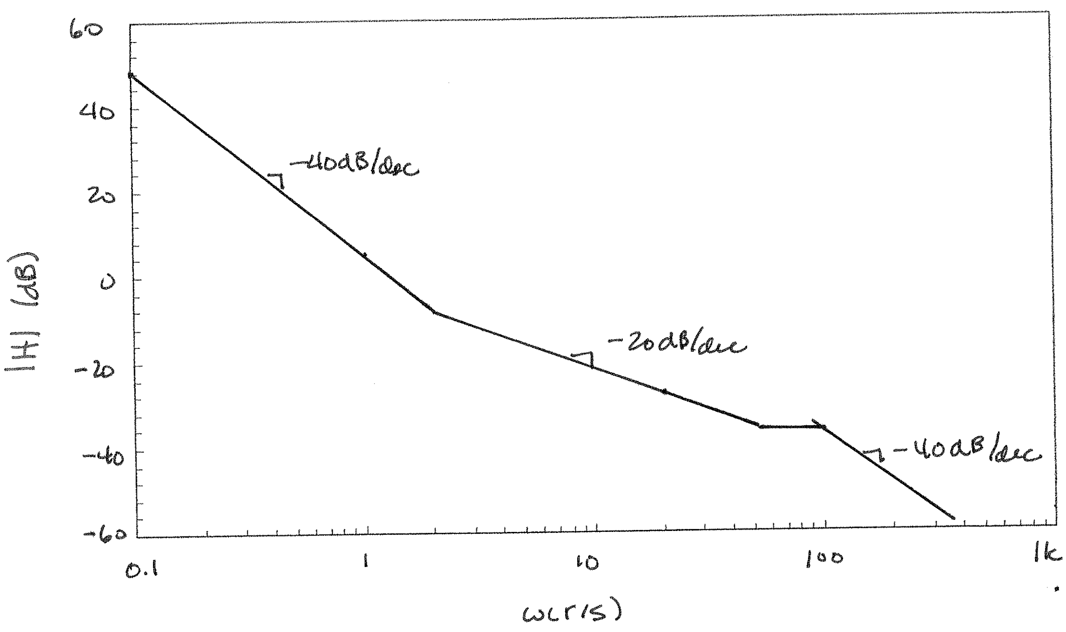
12.16 Sketch the magnitude characteristic of the Bode plot for the transfer function

$$\mathbf{G}(j\omega) = \frac{400(j\omega + 2)(j\omega + 50)}{-\omega^2(j\omega + 100)^2}$$

SOLUTION:

$$G(j\omega) = \frac{400(j\omega+2)(j\omega+50)}{-\omega^2(j\omega+100)^2} \qquad |G|\Big|_{\omega=0.1} \approx \frac{400(2)(50)}{(0.1)^2(100)^2} = 400$$

$$= 52.0\,dB$$



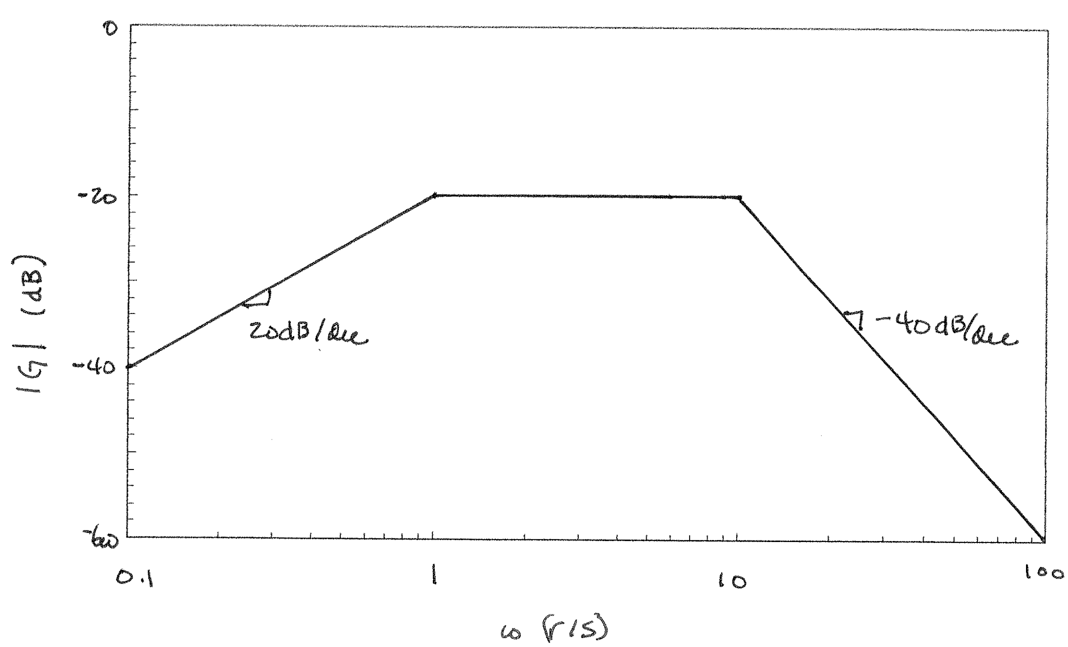
12.17 Sketch the magnitude characteristic of the Bode plot for the transfer function

$$\mathbf{G}(j\omega) = \frac{10j\omega}{(j\omega + 1)(j\omega + 10)^2}$$

CS

SOLUTION:

$$|G|\Big|_{\omega=0.1} \approx \frac{10(0.1)}{1(10)^2} = \frac{1}{100} = -40\,\text{dB}$$

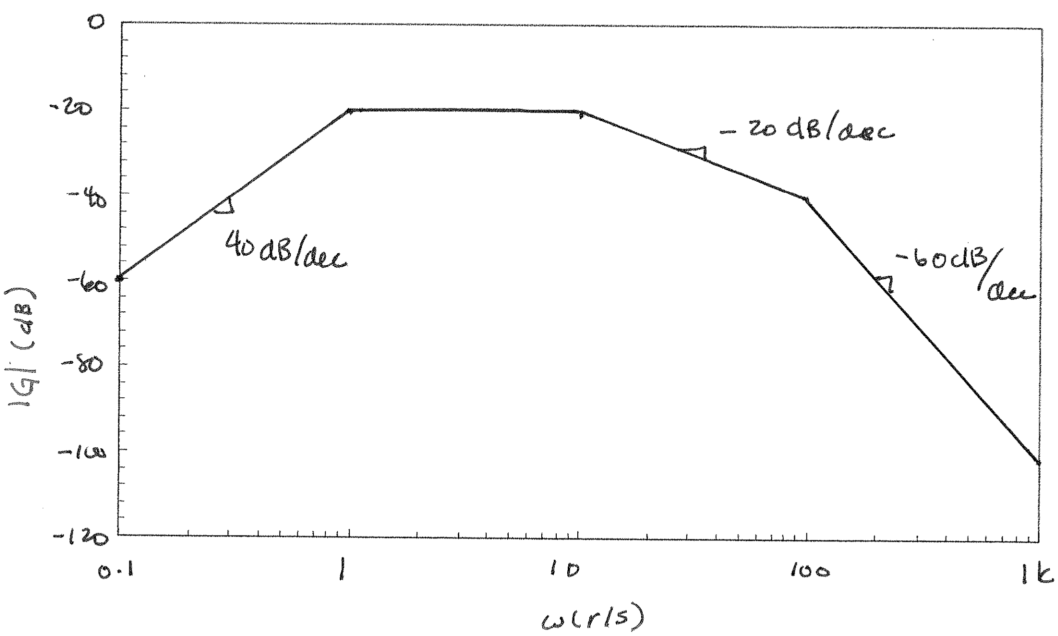


12.18 Sketch the magnitude characteristic of the Bode plot for the transfer function

$$\mathbf{G}(j\omega) = \frac{-\omega^2 10^4}{(j\omega + 1)^2(j\omega + 10)(j\omega + 100)^2}$$

SOLUTION:

$$|G|\Big|_{\omega=0.1} \approx \frac{(0.1)^2(10^4)}{(1)^2(10)(100)^2} = 10^{-3} = -60\text{ dB}$$



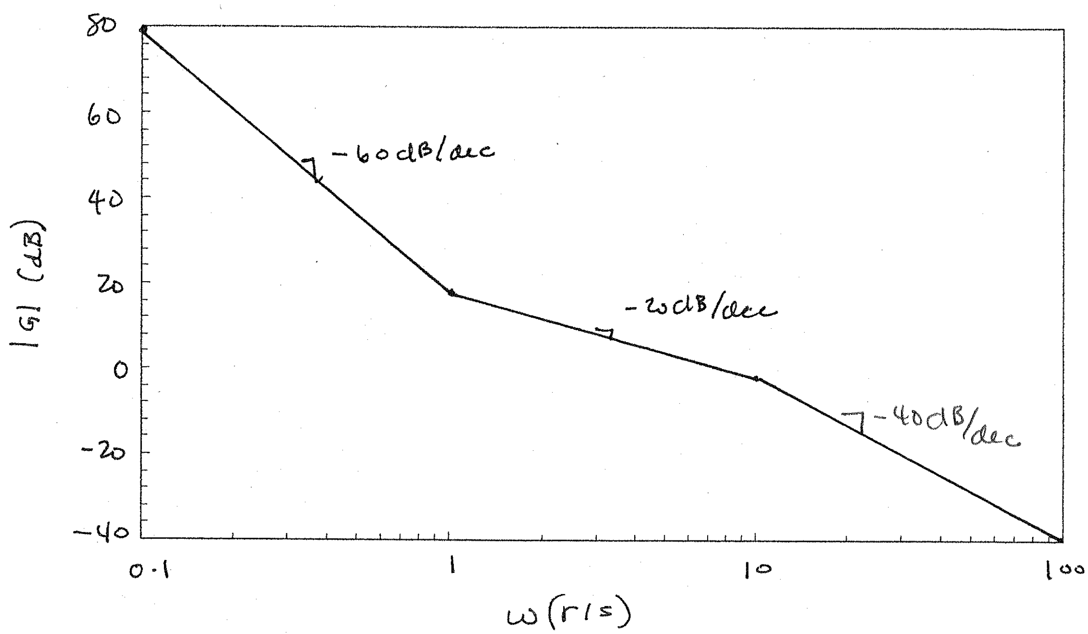
12.19 Sketch the magnitude characteristic of the Bode plot for the transfer function

$$\mathbf{G}(j\omega) = \frac{8(j\omega + 1)^2}{-j\omega^3(0.1j\omega + 1)}$$

SOLUTION:

$$|G(j\omega)|\Big|_{\omega=0.1} \approx \frac{8(1)^2}{(0.1)^3(1)} = 8000 = 78\text{dB}$$

$$G(j\omega) = \frac{80(j\omega+1)^2}{-j\omega^3(\omega+10)}$$



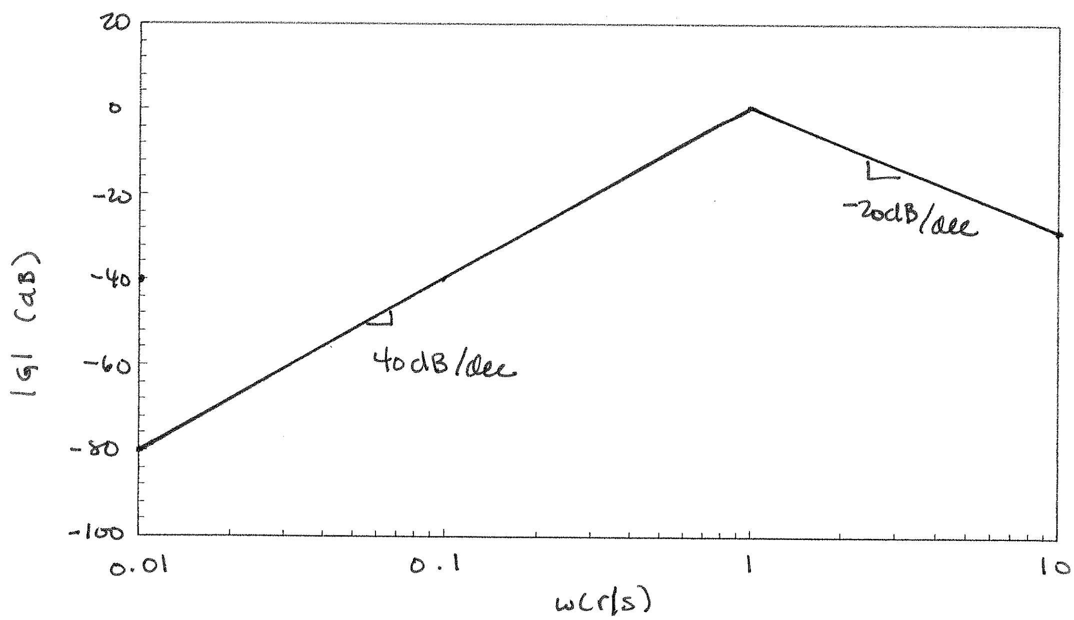
12.20 Sketch the magnitude characteristic of the Bode plot for the transfer function

$$\mathbf{G}(j\omega) = \frac{-\omega^2}{(j\omega + 1)^3}$$

CS

SOLUTION:

$$|G|\Big|_{\omega=0.1} \approx \frac{(0.1)^2}{(1)^3} = 10^{-2} = -40\,\text{dB}$$



12.21 Sketch the magnitude characteristic of the Bode plot for the transfer function

$$\mathbf{G}(j\omega) = \frac{10^4(j\omega + 1)(-\omega^2 + 6j\omega + 225)}{j\omega(j\omega + 50)^2(j\omega + 450)}$$

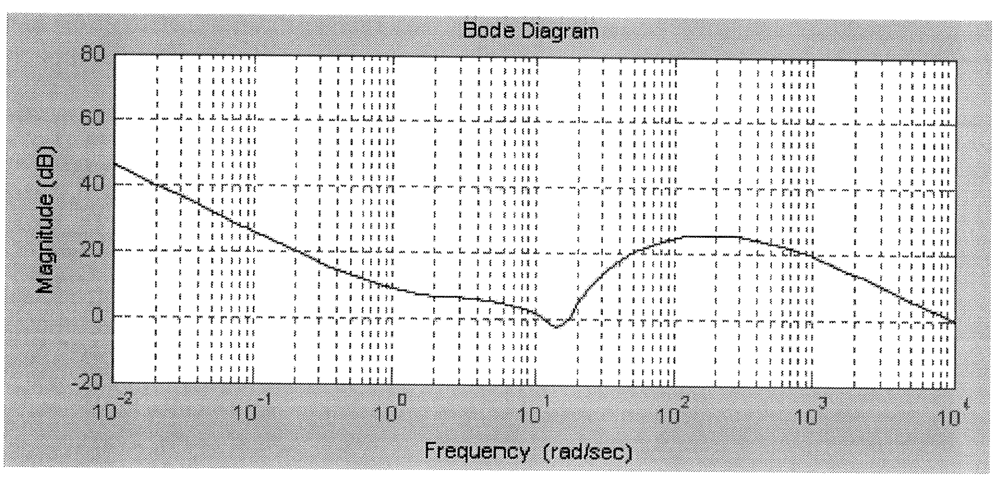
SOLUTION:

$$|G|\Big|_{\omega=0.1} \approx \frac{10^4(1)(225)}{(0.1)(50)^2(450)} = 20 = 26\text{dB}$$

$$-\omega^2 + 6j\omega + 225 = (j\omega)^2 + 6j\omega + 225 = (j\omega + 3 + j14.7)(j\omega + 3 - j14.7)$$

$$\text{or,} \quad \frac{1}{225}\left[1 - \frac{\omega^2}{225} + \frac{6j\omega}{225}\right] = \frac{1}{225}\left[1 - (\omega\tau)^2 + 2j\xi(\omega\tau)\right]$$

$$\text{So,} \quad \tau = 1/\sqrt{225} = 1/15 \qquad \xi = \frac{6}{225(2)\tau} = 0.2$$



12.22 Sketch the magnitude characteristic of the Bode plot for the transfer function

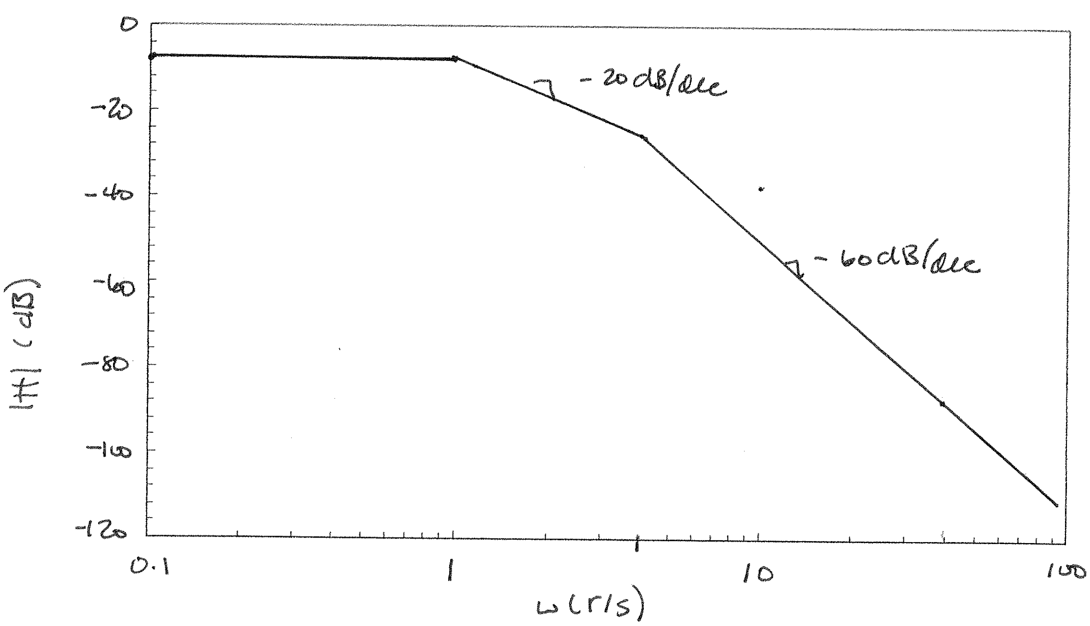
$$\mathbf{H}(j\omega) = \frac{+6.4}{(j\omega + 1)(-\omega^2 + 8j\omega + 16)}$$

SOLUTION:

$$-\omega^2 + 8j\omega + 16 = 16\left\{\left(1 - \left(\frac{\omega}{4}\right)^2\right) + j\,\omega/2\right\} = 16\left\{1 - (\omega\tau)^2 + 2j\,\zeta\omega\tau\right\}$$

$$\tau = 1/4 \text{ s} \qquad \zeta = \frac{1}{2(2)\tau} = 1$$

$$H(j\omega) = \frac{6.4}{(j\omega+1)(j\omega+4)^2} \qquad |H|\Big|_{\omega=0.1} \cong \frac{6.4}{(1)(16)} = 0.4 = -8\text{dB}$$



12.23 Sketch the magnitude characteristic of the Bode plot for the transfer function

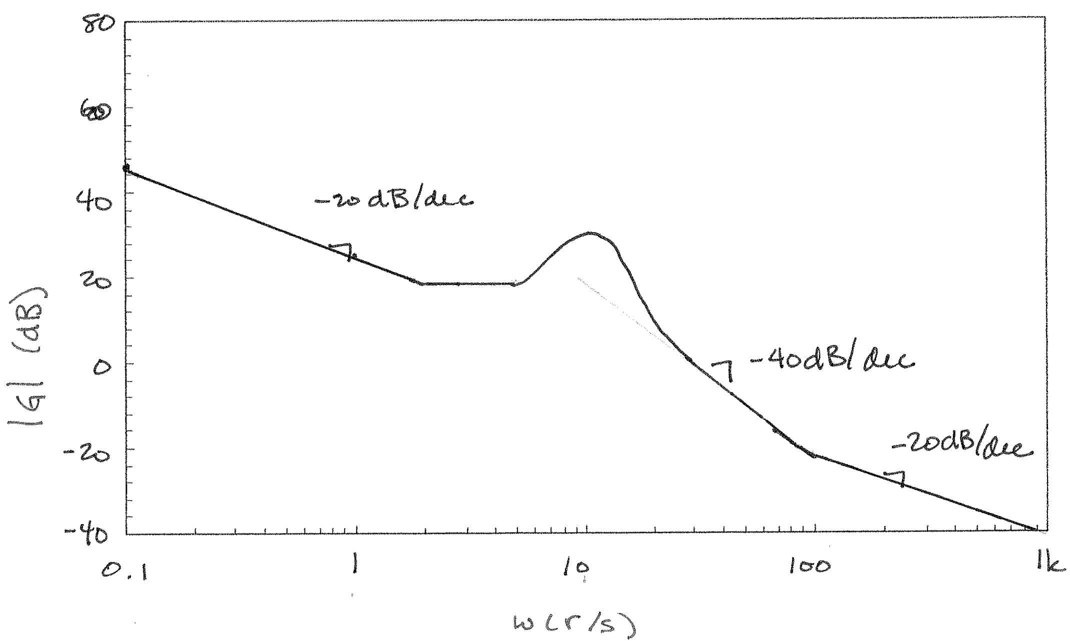
$$\mathbf{G}(j\omega) = \frac{10(j\omega + 2)(j\omega + 100)}{j\omega(-\omega^2 + 4j\omega + 100)}$$

SOLUTION:

$$100 - \omega^2 + 4j\omega = 100\left[1 - \left(\frac{\omega}{10}\right)^2 + \frac{4}{100}j\omega\right] = 100\left[1 - (\omega\tau)^2 + j2\xi(\omega\tau)\right]$$

$$\tau = \frac{1}{10}\text{ s} \qquad \xi = \frac{4}{100}\,\frac{1}{2}\,\frac{1}{\tau} = 0.2$$

$$|G|\Big|_{\omega=0.1} \simeq \frac{10(2)(100)}{(0.1)(100)} = 200 = 46\text{dB}$$



12.24 Sketch the magnitude characteristic of the Bode plot for the transfer function

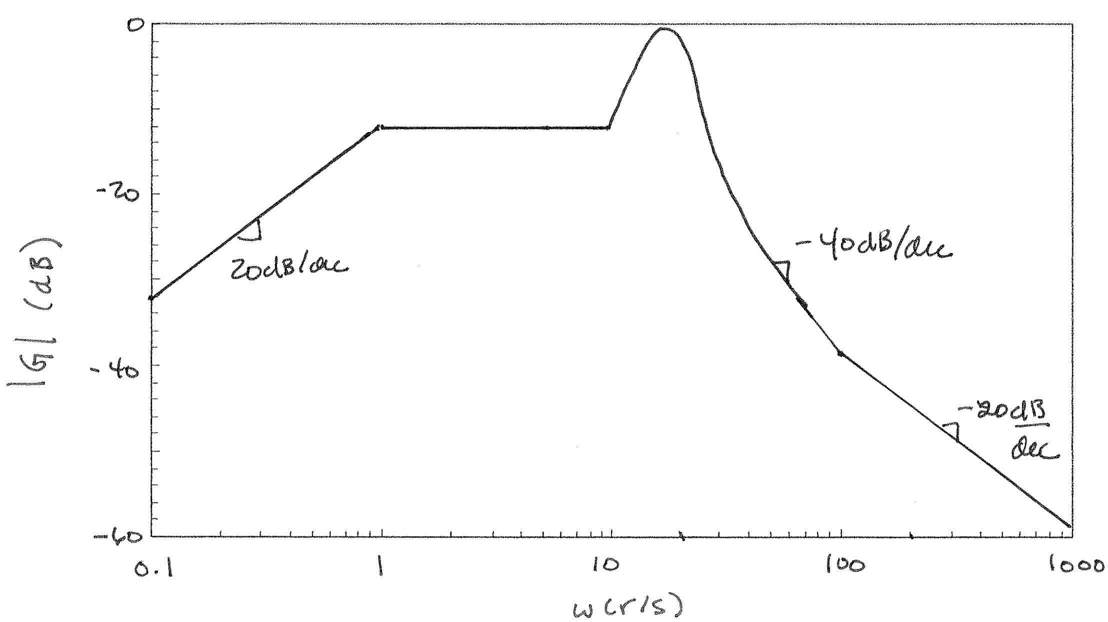
$$\mathbf{G}(j\omega) = \frac{j\omega(j\omega + 100)}{(j\omega + 1)(-\omega^2 + 6j\omega + 400)}$$

SOLUTION:

$$400 - \omega^2 + 6j\omega = 400\left[1 - \left(\frac{\omega}{20}\right)^2 + \frac{6}{400}(j\omega)\right] = 400\left[1 - (\omega\tau)^2 + 2j\xi\omega\tau\right]$$

$$\tau = \frac{1}{20} \qquad \xi = \frac{6}{400}\,\frac{1}{2}\,\frac{1}{\tau} = 0.15$$

$$|G|\Big|_{\omega=0.1} \approx \frac{(0.1)(100)}{(1)(400)} = \frac{1}{40} = -32\,\text{dB}$$



12.25 Sketch the magnitude characteristic of the Bode plot for the transfer function

$$\mathbf{H}(j\omega) = \frac{+6.4(j\omega)}{(j\omega + 1)(-\omega^2 + 8j\omega + 64)}$$

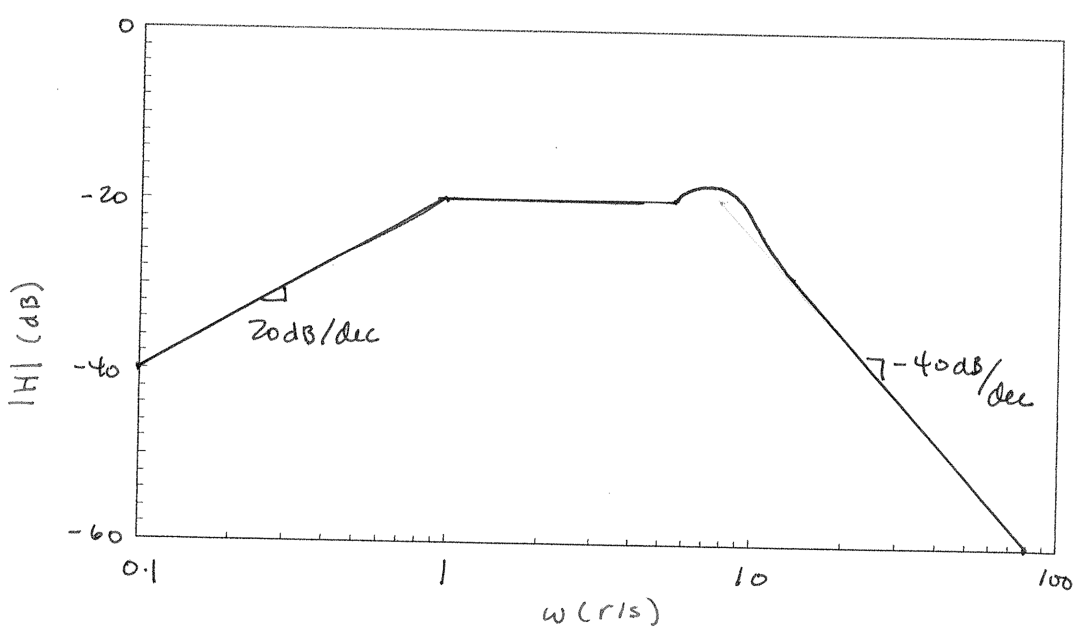
CS

SOLUTION:

$$64 - \omega^2 + 8j\omega = 64\left[1 - \left(\frac{\omega}{8}\right)^2 + \left(\frac{1}{8}\right)j\omega\right] = 64\left[1 - (\omega\tau)^2 + 2j\xi\omega\tau\right]$$

$$\tau = 1/8 \text{ s} \qquad \xi = \frac{\left(\frac{1}{8}\right)}{2\tau} = 0.5$$

$$|G|\Big|_{\omega=0.1} \approx \frac{6.4(0.1)}{(1)(64)} = 10^{-2} = -40\text{dB}$$



12.26 Use MATLAB to generate the Bode plot for the following transfer function over the frequency range from $\omega = 0.01$ to 1000 rad/s.

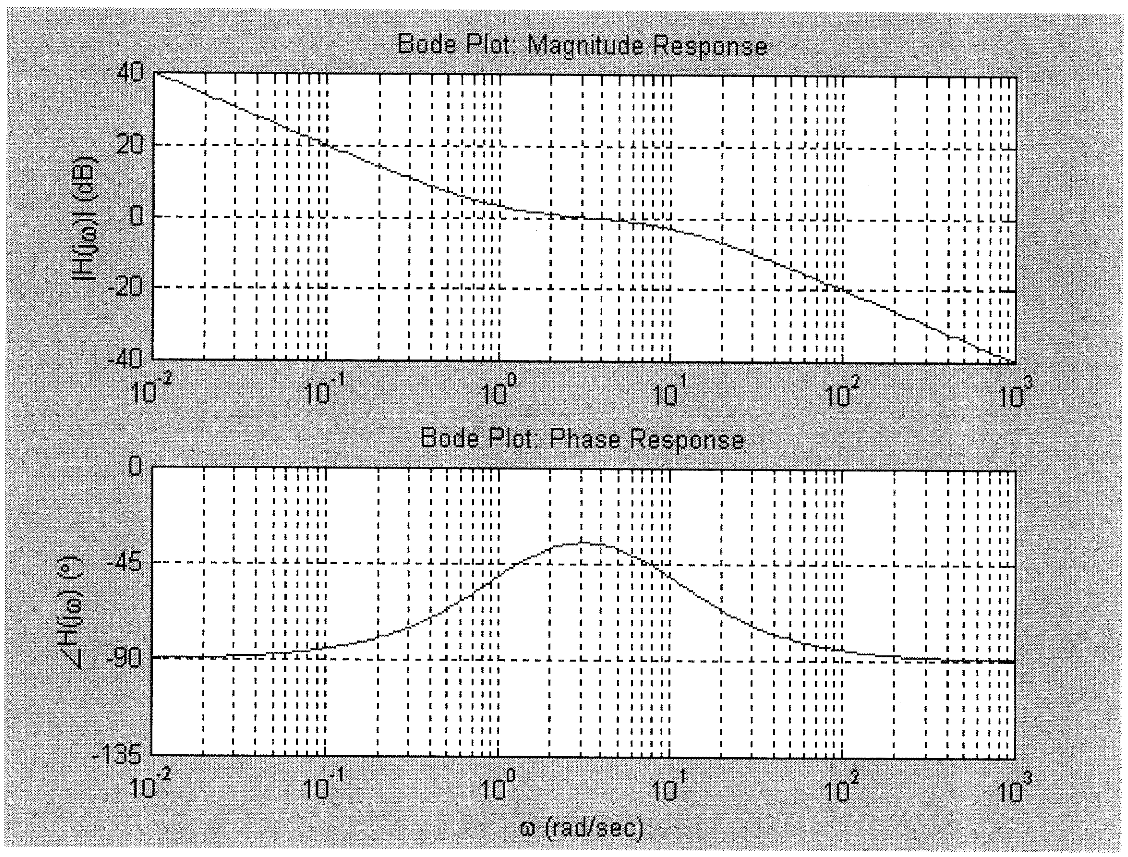
$$\mathbf{G}(j\omega) = \frac{10(j\omega + 1)}{(j\omega)(j\omega + 10)}$$

SOLUTION:

12.26

This sequence of commands works in both the commercial and student versions of MATLAB.

```
» figure(1);
» w = logspace(-2,3,200);
» H = 10*(j*w+1)./((10+j*w).*(j*w));
» subplot(2,1,1);
» semilogx(w,20*log10(abs(H)));
» grid; ylabel('|H(j\omega)| (dB)');
» title('Bode Plot: Magnitude Response');
» subplot(2,1,2);
» semilogx(w,unwrap(angle(H))*180/pi);
» grid; xlabel('\omega (rad/sec)'); ylabel('\angleH(j\omega) (\circ)');
» title('Bode Plot: Phase Response');
```



12.27 Use MATLAB to generate the Bode plot for the following transfer function over the frequency range from $\omega = 0.1$ to 10,000 rad/s.

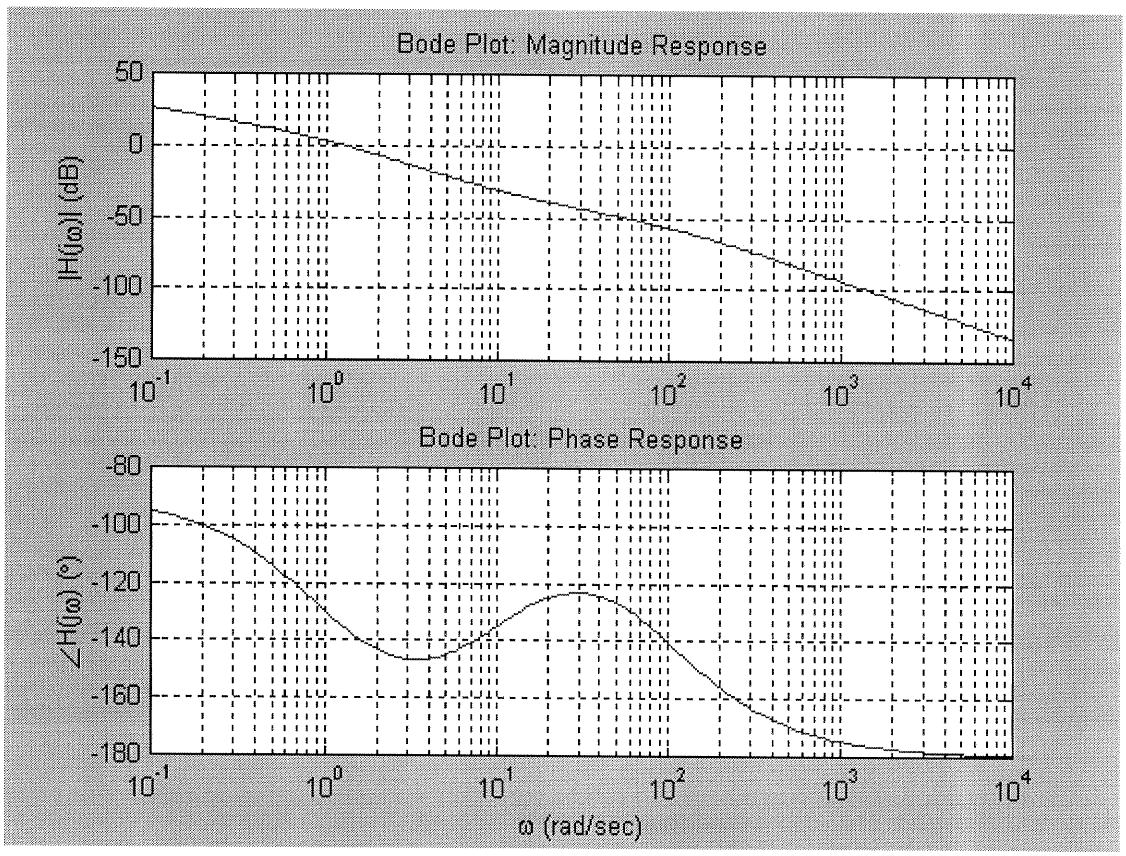
$$\mathbf{G}(j\omega) = \frac{20(j\omega + 10)}{(j\omega)(j\omega + 1)(j\omega + 100)}$$

SOLUTION:

12.27

This sequence of commands works in both the commercial and student versions of MATLAB.

```
EDU» figure(1);
EDU» w = logspace(-2,3,200);
EDU» H = 20*(j*w+10)./((1+j*w).*(j*w).*(100+j*w));
EDU» figure(1);
EDU» w = logspace(-1,4,200);
EDU» H = 20*(j*w+10)./((1+j*w).*(j*w).*(100+j*w));
EDU» subplot(2,1,1);
EDU» semilogx(w,20*log10(abs(H)));
EDU» grid; ylabel('|H(j\omega)| (dB)');
EDU» title('Bode Plot: Magnitude Response');
EDU» subplot(2,1,2);
EDU» semilogx(w,unwrap(angle(H))*180/pi);
EDU» grid; xlabel('\omega (rad/sec)'); ylabel('\angleH(j\omega) (\circ)');
EDU» title('Bode Plot: Phase Response');
```



12.28 The magnitude characteristic of a band-elimination filter is shown in Fig. P12.28. Determine $\mathbf{H}(j\omega)$.

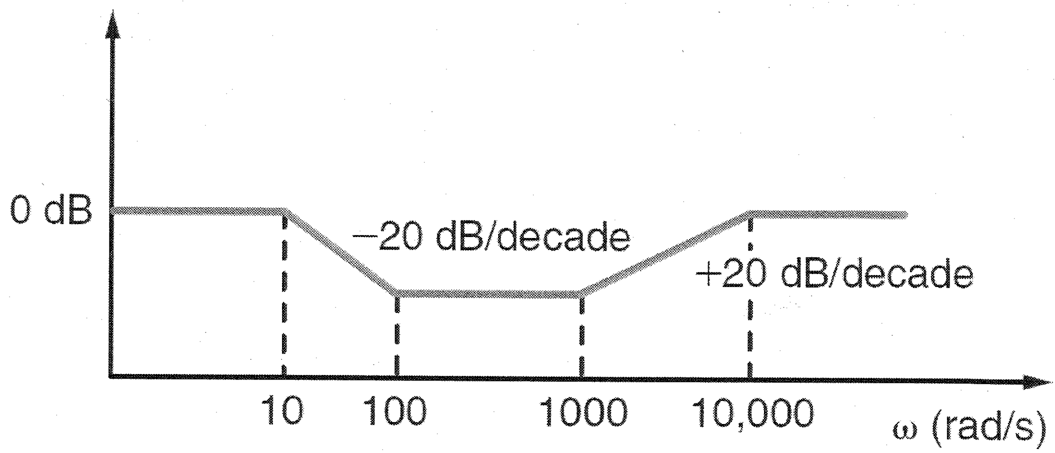


Figure P12.28

SOLUTION:

poles at 10 and 10,000 r/s zeros at 100 and 1000 r/s

$$H(j\omega) = \frac{K(j\omega+100)(j\omega+1000)}{(j\omega+10)(j\omega+10000)} \qquad 0\,dB = 1$$

$$|H|\Big|_{\omega \ll 10} = \frac{K(100)(1000)}{10(10^4)} = 1 \Rightarrow K = 1$$

$$\boxed{H(j\omega) = \frac{(j\omega+100)(j\omega+10^3)}{(j\omega+10)(j\omega+10^4)}}$$

12.29 Find $\mathbf{H}(j\omega)$ if its magnitude characteristic is shown in Fig. P12.29. **PSV**

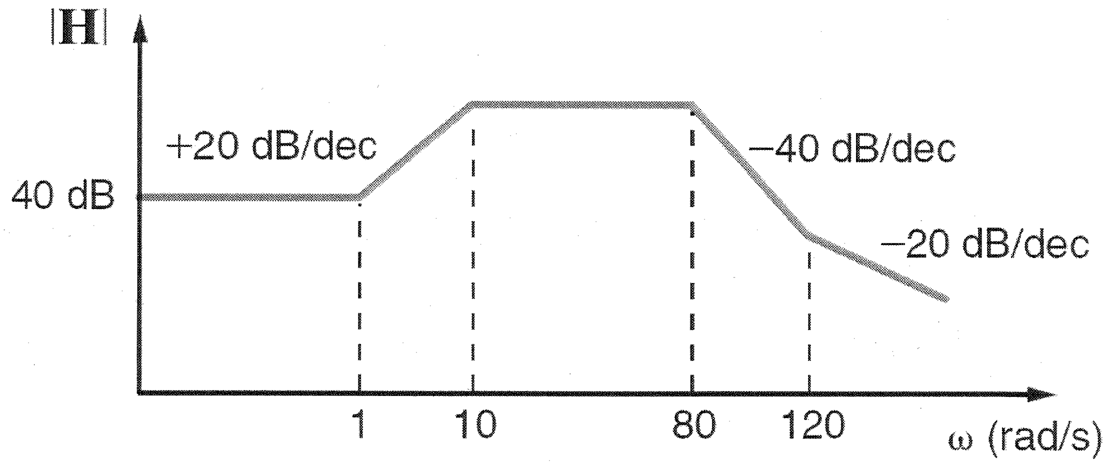


Figure P12.29

SOLUTION:

poles at $\left\{ \begin{array}{l} 10 \text{ r/s} \\ 80 \text{ r/s (double)} \end{array} \right\}$ zeros at $\left\{ \begin{array}{l} 1 \text{ r/s} \\ 120 \text{ r/s} \end{array} \right.$

$$H(j\omega) = \frac{K(j\omega+1)(j\omega+120)}{(j\omega+10)(j\omega+80)^2}$$

for $\omega << 1$, $|H| = 40\text{dB} = 100 = \dfrac{K(1)(120)}{10(80)^2} \Rightarrow K = 5.33\times10^4$

$$\boxed{H(j\omega) = \frac{5.33\times10^4(j\omega+1)(j\omega+120)}{(j\omega+10)(j\omega+80)^2}}$$

12.30 Find $\mathbf{H}(j\omega)$ if its magnitude characteristic is shown in Fig. P12.30. CS

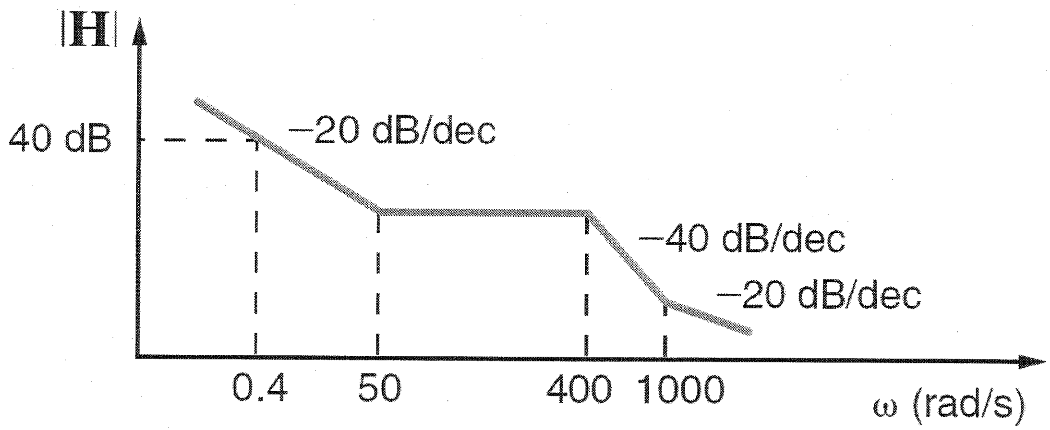


Figure P12.30

SOLUTION:

poles at: $\left\{ \begin{array}{l} \text{origin } (\omega = 0) \\ 400 \text{ r/s (double} \end{array} \right\}$ zeros at: $\left\{ \begin{array}{l} 50 \text{ r/s} \\ 1000 \text{ r/s} \end{array} \right\}$

$$H(j\omega) = \frac{K(j\omega + 50)(j\omega + 1000)}{j\omega(j\omega + 400)^2}$$

at $\omega = 0.4$ r/s, $|H| = 40\text{dB} = 100 \cong \dfrac{K(50)(1000)}{0.4(400)^2} \Rightarrow K = 128$

$$\boxed{H(j\omega) = \frac{128(j\omega + 50)(j\omega + 1000)}{j\omega(j\omega + 400)^2}}$$

12.31 Find $\mathbf{H}(j\omega)$ if its amplitude characteristic is shown in Fig. P12.31.

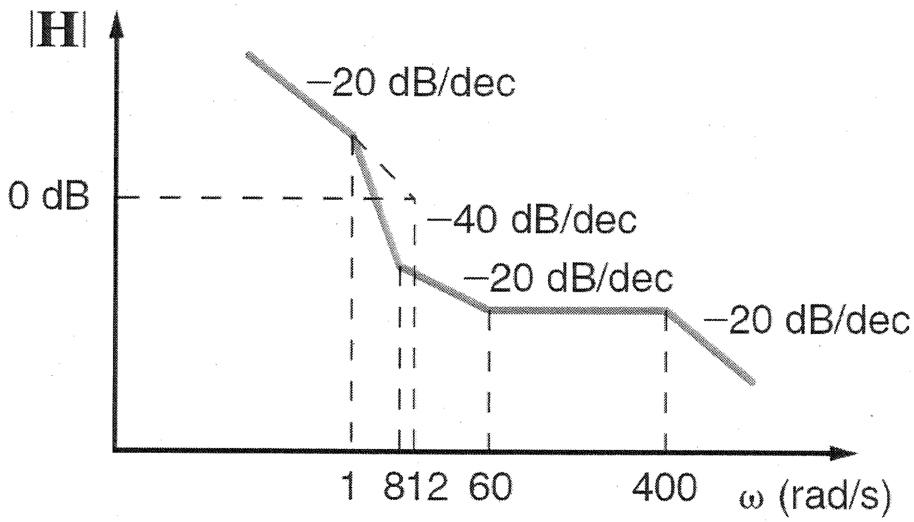


Figure P12.31

SOLUTION:

Poles at $\left\{ \begin{array}{l} \omega = 0 \\ \omega = 1 \text{ r/s} \\ \omega = 400 \text{ r/s} \end{array} \right\}$ Zeros at : $\left\{ \begin{array}{l} \omega = 8 \text{ r/s} \\ \omega = 60 \text{ r/s} \end{array} \right\}$

$$H(j\omega) = \frac{K(j\omega + 8)(j\omega + 60)}{j\omega(j\omega + 1)(j\omega + 400)}$$

At $\omega \simeq 2$ r/s , $|H| = 0\text{dB} = 1 \cong \dfrac{K(8)(60)}{2\sqrt{2^2+1^2}\,(400)} \Rightarrow K = 3.73$

$$\boxed{H(j\omega) = \frac{3.73(j\omega + 8)(j\omega + 60)}{j\omega(j\omega + 1)(j\omega + 400)}}$$

12.32 Given the magnitude characteristic in Fig. P12.32, find $\mathbf{H}(j\omega)$.

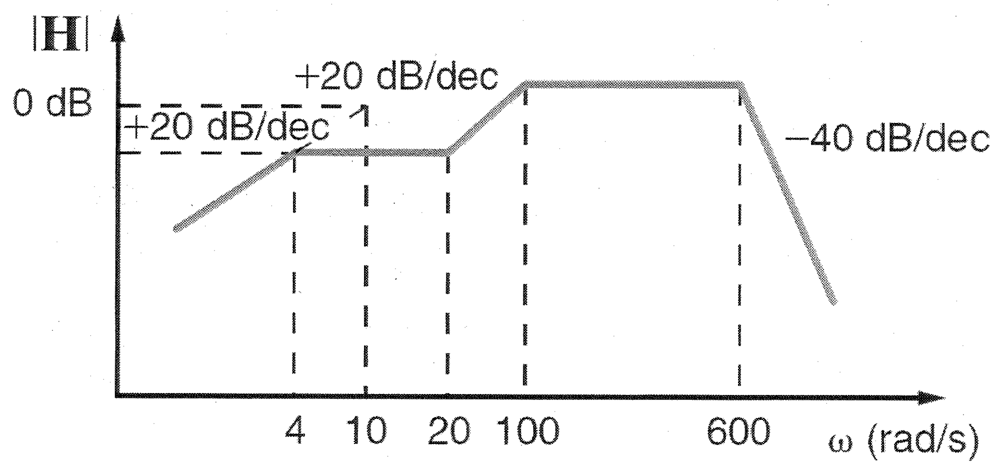


Figure P12.32

SOLUTION:

poles at: $\left\{ \begin{array}{l} \omega = 4 \text{ r/s} \\ 100 \text{ r/s} \\ 600 \text{ r/s (double)} \end{array} \right\}$ zeros at: $\left\{ \begin{array}{l} \omega = 0 \text{ r/s} \\ 20 \text{ r/s} \end{array} \right\}$

$$H(j\omega) = \frac{K(j\omega)(j\omega+20)}{(j\omega+4)(j\omega+100)(j\omega+600)^2}$$

at $\omega = 1$ r/s, $|H| = -20\text{dB} = 0.1 = \dfrac{K(1)(20)}{4(100)(600)^2} \Rightarrow K = 7.2\times10^5$

$$\boxed{H(j\omega) = \frac{7.2\times10^5 (j\omega)(j\omega+20)}{(j\omega+4)(j\omega+100)(j\omega+600)^2}}$$

12.33 Determine $\mathbf{H}(j\omega)$ if its magnitude characteristic is shown in Fig. P12.33.

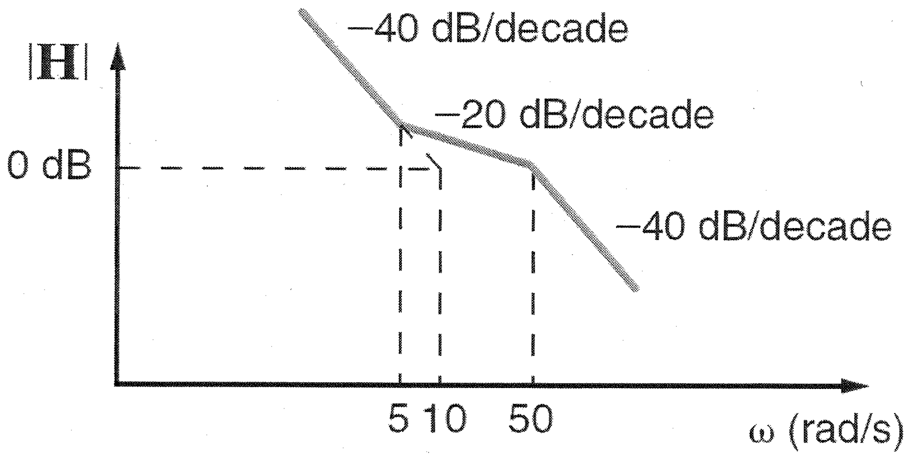


Figure P12.33

SOLUTION:

poles at: $\left\{ \begin{matrix} \omega = 0 \text{ r/s} \quad (\text{double}) \\ \omega = 50 \text{ r/s} \end{matrix} \right\}$ zero at: $\{ 5 \text{ r/s} \}$

$$H(j\omega) = \frac{K(j\omega+5)}{(j\omega)^2(j\omega+50)}$$

at $\omega = 1$ r/s, $|H| = 40\text{dB} = 100 = \dfrac{K(5)}{(1)^2(50)} \Rightarrow K = 1000$

$$\boxed{H(j\omega) = \frac{1000(j\omega+5)}{(j\omega)^2(j\omega+50)}}$$

12.34 Find $\mathbf{G}(j\omega)$ for the magnitude characteristic shown in Fig. P12.34. CS

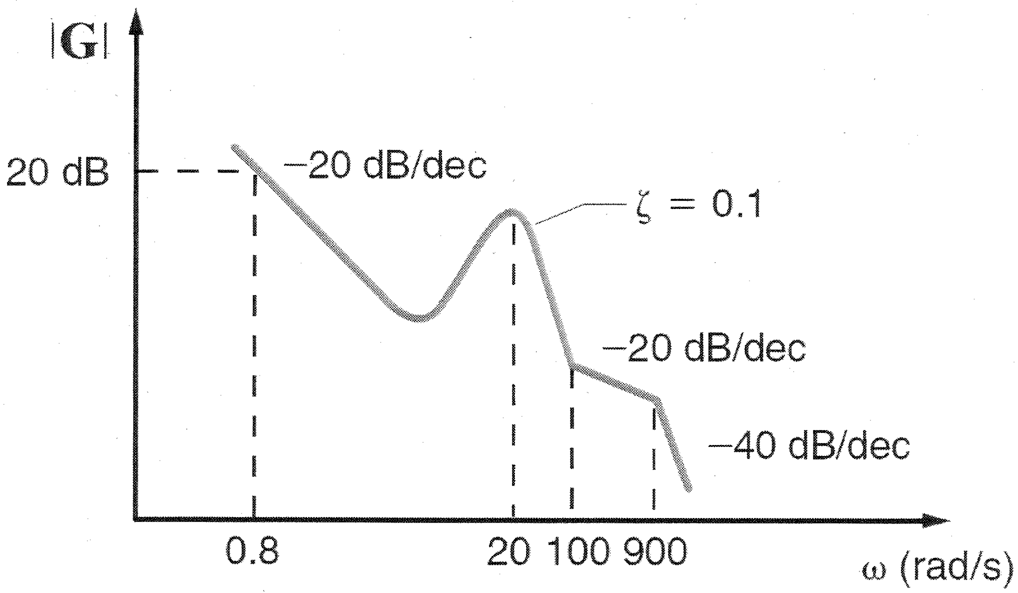


Figure P12.34

SOLUTION:

Simple poles: $\left\{ \begin{array}{l} \omega = 0 \text{ r/s} \\ \omega = 900 \text{ r/s} \end{array} \right\}$ zeros at 100 r/s (double)

Complex conjugate poles at $\omega = 20$ r/s with $\zeta = 0.1$

or $1 - \left(\frac{\omega}{20}\right)^2 + 2j(0.1)/20 \rightarrow \left((j\omega)^2 + j4\omega + 400\right)/400$

$$G(j\omega) = \frac{K(j\omega + 100)^2}{j\omega(j\omega + 900)\left[(j\omega)^2 + j4\omega + 400\right]}$$

at $\omega = 0.8$ r/s, $|G| = 20\text{dB} = 10 = K(100)^2 / \{(0.8)(900)(400)\}$

$K = 288$

$$\boxed{G(j\omega) = \frac{288(j\omega + 100)^2}{j\omega(j\omega + 900)\left[(j\omega)^2 + j4\omega + 400\right]}}$$

12.35 A series *RLC* circuit resonates at 1000 rad/s. If $C = 20\ \mu\text{F}$, and it is known that the impedance at resonance is 2.4 Ω, compute the value of L, the Q of the circuit, and the bandwidth.

SOLUTION:

$$Z(j\omega) = R + j\omega L + \frac{1}{j\omega C}$$

at resonance, $\omega_o = \dfrac{1}{\sqrt{LC}} = 1000 \Rightarrow \boxed{L = 50\text{ mH}}$

also $Z(j\omega_o) = R = 2.4\,\Omega$

$$Q = \frac{\omega_o L}{R} = \frac{1000\,(50\times10^{-3})}{2.4} \qquad \boxed{Q = 20.8}$$

$$BW = \frac{\omega_o}{Q} \qquad \boxed{BW = 48\ r/s}$$

12.36 A series resonant circuit has a Q of 120 and a resonant frequency of 10,000 rad/s. Determine the half-power frequencies and the bandwidth of the circuit.

SOLUTION: $\omega_o = 10k \ r/s$

$BW = \omega_o / Q$ $\quad \boxed{BW = 83.3 \ r/s}$

$\omega_{LO} = \omega_o \left[-\frac{1}{2Q} + \sqrt{\left(\frac{1}{2Q}\right)^2 + 1} \right]$ $\quad \boxed{\omega_{LO} = 9958 \ r/s}$

$\omega_{HI} = \omega_o^2 / \omega_{LO} \Rightarrow$ $\quad \boxed{\omega_{HI} = 10.04 \ kr/s}$

12.37 The series *RLC* circuit in Fig. P12.37 is driven by a variable-frequency source. If the resonant frequency of the network is selected as $\omega_0 = 1600$ rad/s, find the value of C. In addition, compute the current at resonance and at $\omega_0/4$ and $4\,\omega_0$. **PSV**

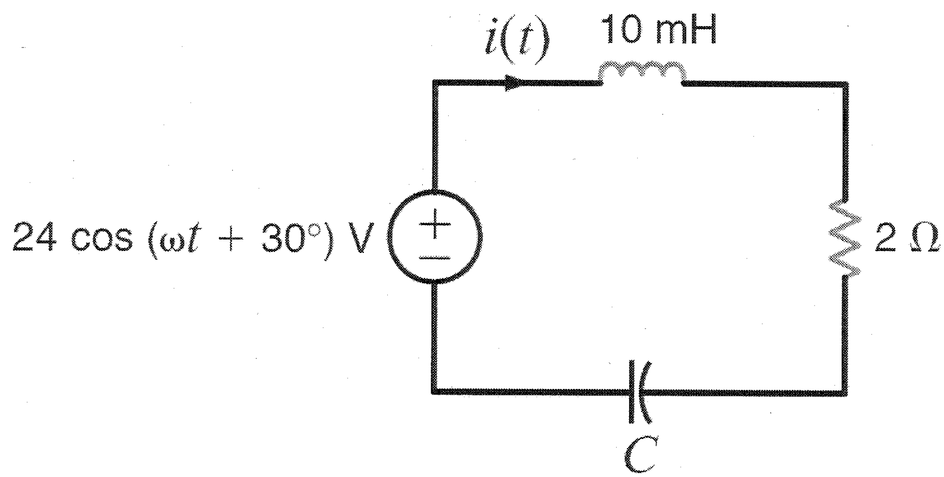
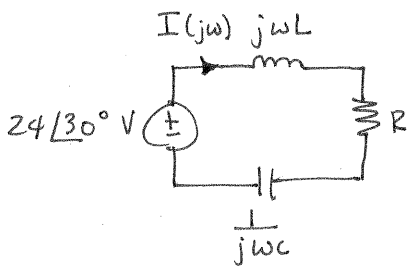


Figure P12.37

SOLUTION:

$$\omega_0 = 1600 = \frac{1}{\sqrt{LC}} \Rightarrow \boxed{C = 39\,\mu F}$$



$$I(j\omega) = \frac{24\angle 30^\circ}{j\left(\omega L - \frac{1}{\omega C}\right) + R}$$

ω/ω_0	$I(j\omega)$
1	$12\angle 30^\circ$ A
1/4	$0.400\angle 118^\circ$ A
4	$0.400\angle -58^\circ$ A

12.38 Given the series *RLC* circuit in Fig. P12.38, (a) derive the expression for the half-power frequencies, the resonant frequency, the bandwidth, and the quality factor for the transfer characteristic $\mathbf{I}/\mathbf{V}_{in}$ in R, L, C, (b) Compute the quantities in part (a) if $R = 10\ \Omega$, $L = 50$ mH, and $C = 10\ \mu$F.

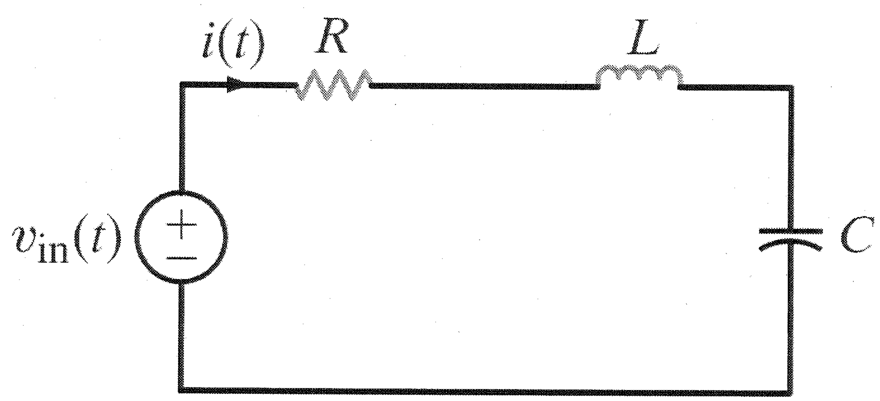


Figure P12.38

SOLUTION: a. Let $Z = R + j\omega L + \frac{1}{j\omega C}$

at resonance, $Z = R$. So $\omega_o L + \frac{1}{\omega_o C} = 0 \Rightarrow \boxed{\omega_o = \frac{1}{\sqrt{LC}}}$

Q is defined as $\frac{\omega_o L}{R} \Rightarrow \boxed{Q = \frac{1}{R}\sqrt{\frac{L}{C}}}$

$\frac{I}{V_{IN}} = \frac{1}{Z(j\omega)}$ At half power, $\left|\frac{I}{V_{IN}}\right| = \frac{1}{\sqrt{2}R} = \frac{1}{|Z(j\omega)|}$

so, $|R + j(\omega L - 1/\omega C)| = \sqrt{2}R = R\left|1 + j\left(\omega L/R - \frac{1}{\omega RC}\right)\right|$

$\sqrt{2} = \left|1 + jQ(\omega/\omega_o - \omega_o/\omega)\right|$ or $Q(\omega/\omega_o - \omega_o/\omega) = \pm 1$

Quadratic equation yields $\omega_{LO} = \omega_o\left[-\frac{1}{2Q} + \sqrt{\left(\frac{1}{2Q}\right)^2 + 1}\right]$

$\omega_{HI} = \omega_o\left[\frac{1}{2Q} + \sqrt{\left(\frac{1}{2Q}\right)^2 + 1}\right]$

In terms of R, L, C $\rightarrow \frac{\omega_0}{Q} = R/L$

$$\boxed{\begin{aligned} \omega_{LO} &= \left[-\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \right] \\ \omega_{HI} &= \left[\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \right] \end{aligned}}$$

$BW = \omega_{HI} - \omega_{LO} = \omega_0/Q$ $\quad \boxed{BW = R/L}$

b. $R = 10\,\Omega \quad L = 50\,mH \quad C = 10\,\mu F$

$\boxed{\omega_0 = 1414 \text{ r/s}}$ $\boxed{Q = 7.07}$ $\boxed{BW = 200 \text{ r/s}}$

$\boxed{\omega_{HI} = 1518 \text{ r/s}}$ $\boxed{\omega_{LO} = 1318 \text{ r/s}}$

12.39 Given the network in Fig. P12.39, find ω_0, Q, ω_{max}, and $|\mathbf{V}_o|_{max}$. CS

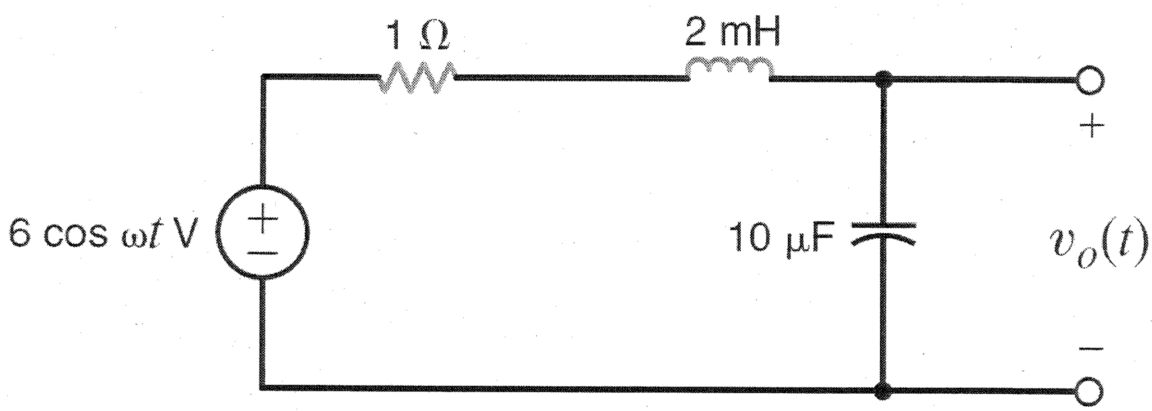


Figure P12.39

SOLUTION:

$\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow$ $\boxed{\omega_0 = 7.07 \text{ kr/s}}$

$Q = \omega_0 L/R \rightarrow$ $\boxed{Q = 14.14}$

$\omega_{max} = \omega_0 \sqrt{1 - \frac{1}{2Q^2}} \Rightarrow$ $\boxed{\omega_{max} = 7.06 \text{ kr/s}}$

$|V_{omax}| = \frac{Q|V_s|}{\sqrt{1 - \frac{1}{4Q^2}}}$ $|V_s| = 6\text{V}$

$\boxed{|V_{omax}| = 84.9 \text{ V}}$

12.40 A series *RLC* circuit is driven by a signal generator. The resonant frequency of the network is known to be 1600 rad/s, and at that frequency the impedance seen by the signal generator is 5 Ω. If $C = 20\ \mu F$, find L, Q, and the bandwidth.

SOLUTION:

$Z = R + j(\omega L - 1/\omega C)$ at $\omega = 1600$ r/s, $Z = 5 = R$

So, $1600 \text{ r/s} = \omega_o = \dfrac{1}{\sqrt{LC}} \Rightarrow L = \dfrac{1}{\omega_o^2 C}$ $\boxed{L = 19.5\text{mH}}$

$Q = \dfrac{\omega_o L}{R} \Rightarrow \boxed{Q = 6.24}$

$BW = \dfrac{\omega_o}{Q} \Rightarrow \boxed{BW = 256 \text{ r/s}}$

12.41 A variable-frequency voltage source drives the network in Fig. P12.41. Determine the resonant frequency, Q, BW, and the average power dissipated by the network at resonance.

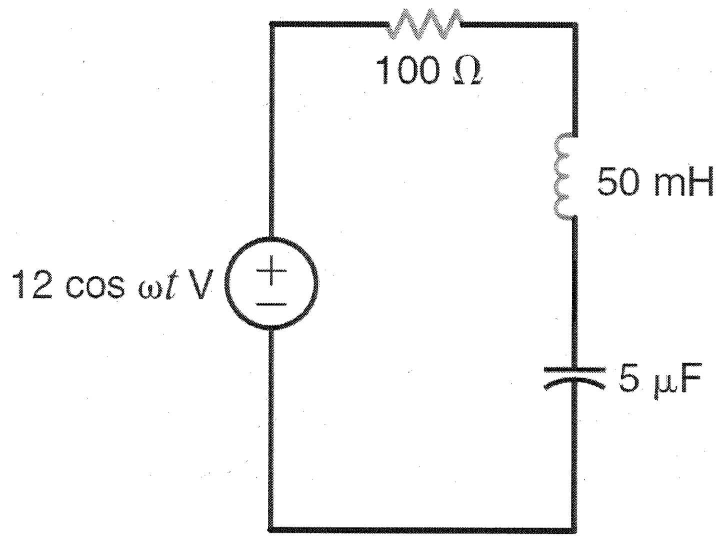
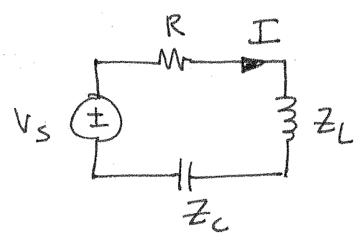


Figure P12.41

SOLUTION:

$\omega_o = \dfrac{1}{\sqrt{LC}} \Rightarrow \boxed{\omega_o = 2\ kr/s}$



$Q = \omega_o L / R \Rightarrow \boxed{Q = 1}$

$BW = \omega_o / Q \Rightarrow \boxed{BW = 2kr/s}$

at resonance, $Z = R = 100\Omega$ $\qquad I = \dfrac{V_s}{R} = \dfrac{12\angle 0}{100} = 0.12\angle 0°A$

$P = \dfrac{|I|^2 R}{2} \qquad \boxed{P = 720\,mW}$

12.42 A parallel *RLC* resonant circuit with a resonant frequency of 20,000 rad/s has an admittance at resonance of 1 mS. If the capacitance of the network is 2 μF, find the values of *R* and *L*. **PSV**

SOLUTION: $\omega_0 = 20\ \text{kr/s}$ $\quad Y(j\omega_0) = 10^{-3}\ S = 1/R \Rightarrow \boxed{R = 1\,k\Omega}$

$$\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow L = \frac{1}{\omega_0^2 C} \Rightarrow \boxed{L = 1.25\ mH}$$

12.43 A parallel *RLC* resonant circuit has a resistance of 200 Ω. If it is known that the bandwidth is 80 rad/s and the lower half-power frequency is 800 rad/s, find the values of the parameters *L* and *C*.

SOLUTION: $R = 200\,\Omega$ $\quad BW = 80\ r/s$ $\quad \omega_{LO} = 800\ r/s$

$$BW = \frac{1}{RC} \Rightarrow \boxed{C = 62.5\,\mu F}$$

$$\omega_{HI} = \omega_{LO} + BW = 880\ r/s \qquad \omega_0 = \sqrt{\omega_{HI}\,\omega_{LO}} = 839\ r/s$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow L = \frac{1}{\omega_0^2 C} \Rightarrow \boxed{L = 22.7\,mH}$$

12.44 In the network in Fig. P12.44, the inductor value is 10 mH, and the circuit is driven by a variable-frequency source. If the magnitude of the current at resonance is 12 A, $\omega_0 = 1000$ rad/s, and $L = 10$ mH, find C, Q, and the bandwidth of the circuit. CS

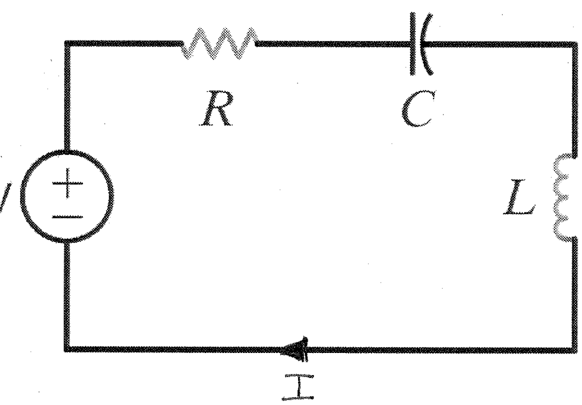


Figure P12.44

SOLUTION: $Z = R + j(\omega L - 1/\omega C)$

at $\omega = \omega_0$, $Z = R$ and $|I| = 36/R = 12 \Rightarrow R = 3\,\Omega$

$Q = \omega_0 L / R \Rightarrow \boxed{Q = 3.33}$

$\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow C = \frac{1}{\omega_0^2 L} \Rightarrow \boxed{C = 100\,\mu F}$

$BW = \omega_0 / Q \Rightarrow \boxed{BW = 300\ r/s}$

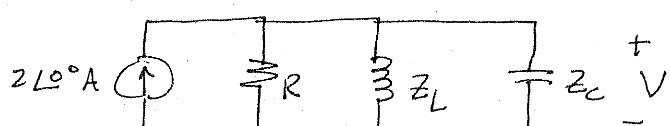
12.45 A parallel *RLC* circuit, which is driven by a variable-frequency 2-A current source, has the following values: $R = 1\ \text{k}\Omega$, $L = 400$ mH, and $C = 10\ \mu$F. Find the bandwidth of the network, the half-power frequencies, and the voltage across the network at the half-power frequencies. CS

SOLUTION:

$BW = \dfrac{1}{RC} \Rightarrow \boxed{BW = 100\ r/s}$ $\qquad \omega_0 = \dfrac{1}{\sqrt{LC}} = 500\ r/s$

$Q = \omega_0 / BW = 5$ $\qquad \omega_{HI} = \omega_0\left[\dfrac{1}{2Q} + \sqrt{\left(\dfrac{1}{2Q}\right)^2 + 1}\right] = 552\ r/s$

$\boxed{\omega_{HI} = 552\ r/s}$ $\qquad \omega_{LO} = \omega_0^2/\omega_{HI} \Rightarrow \boxed{\omega_{LO} = 452\ r/s}$



$Y = \dfrac{1}{R} + \dfrac{1}{Z_L} + \dfrac{1}{Z_C}$ $\qquad V = I/Y = 2\angle 0 / Y$

$Y(j\omega_{HI}) = 1 + j1\ \text{mS}$ $\qquad \boxed{V(j\omega_{HI}) = 1414\ \angle{-45^\circ}\ V}$

$Y(j\omega_{LO}) = 1 - j1\ \text{mS}$ $\qquad \boxed{V(j\omega_{LO}) = 1414\ \angle{+45^\circ}\ V}$

12.46 A parallel *RLC* circuit, which is driven by a variable-frequency 2-A current source, has the following values: $R = 1\ \text{k}\Omega$, $L = 100$ mH, and $C = 10\ \mu$F. Find the bandwidth of the network, the half-power frequencies, and the voltage across the network at the half-power frequencies. **CS**

SOLUTION:

$BW = 1/RC \Rightarrow \boxed{BW = 100\ r/s}$ $\omega_0 = \frac{1}{\sqrt{LC}} = 1000\ r/s$

$Q = \omega_0 / BW = 10$

$\omega_{HI} = \omega_0 \left[\frac{1}{2Q} + \sqrt{\left(\frac{1}{2Q}\right)^2 + 1} \right] \Rightarrow \boxed{\omega_{HI} = 1051\ r/s}$

$\omega_{LO} = \omega_0^2 / \omega_{HI} \Rightarrow \boxed{\omega_{LO} = 951\ r/s}$

$Y = \frac{1}{R} + \frac{1}{Z_L} + Z_C$ $V = I/Y$ $I = 2\angle 0°\text{A}$

$Y(j\omega_{HI}) = 1 + j1\ \text{mS}$

$Y(j\omega_{LO}) = 1 - j1\ \text{mS}$

$$\boxed{\begin{array}{l} V = \sqrt{2}\angle -45°\ \text{kV} \ @\ \omega_{HI} \\ V = \sqrt{2}\angle +45°\ \text{kV} \ @\ \omega_{LO} \end{array}}$$

12.47 Consider the network in Fig. P12.47. If $R = 1\ \text{k}\Omega$, $L = 20$ mH, $C = 50\ \mu\text{F}$, and $R_S = \infty$, determine the resonant frequency ω_0, the Q of the network, and the bandwidth of the network. What impact does an R_S of $10\ \text{k}\Omega$ have on the quantities determined?

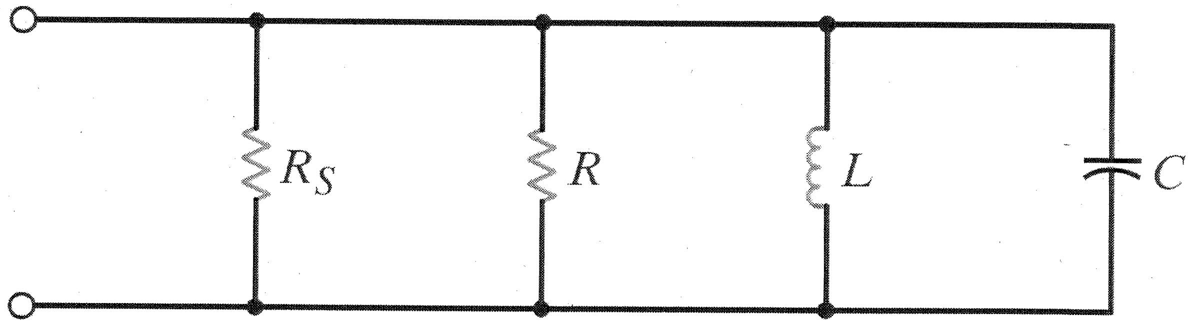


Figure P12.47

SOLUTION: $\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow \boxed{\omega_0 = 1000\ r/s}$ $\quad R_{eq} = R_S // R = 1\text{k}\Omega$

$Q = \omega_0 R_{eq} C = 50 \Rightarrow \boxed{Q = 50}$ $\quad BW = \omega_0/Q \Rightarrow \boxed{BW = 20\ r/s}$

If $R_S = 10\text{k}\Omega$, $R_{eq} = 909\Omega$.

ω_0 is unchanged. $\boxed{\omega_0 = 1000\ r/s}$

Q changes $\boxed{Q = 45.5}$

BW changes $\boxed{BW = 22.0\ r/s}$

12.48 The source in the network in Fig. P12.48 is $i_S(t) = \cos 1000t + \cos 1500t$ A. $R = 200\ \Omega$ and $C = 500\ \mu$F. If $\omega_0 = 1000$ rad/s, find L, Q, and the BW. Compute the output voltage $v_o(t)$ and discuss the magnitude of the output voltage at the two input frequencies.

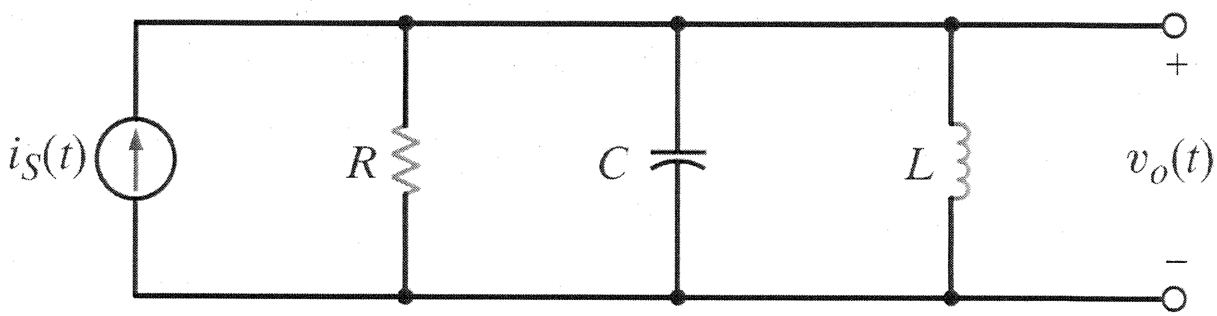


Figure P12.48

SOLUTION:

$$\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow L = \frac{1}{\omega_0^2 C} \Rightarrow \boxed{L = 2\text{mH}}$$

$$BW = \frac{1}{RC} \Rightarrow \boxed{BW = 10\ r/s} \qquad Q = \omega_0/BW \Rightarrow \boxed{Q = 100}$$

Use superposition. Let $I_{S1} = 1\angle 0°$ at 1000 r/s and $I_{S2} = 1\angle 0°$ A @ 1500 r/s.

$$Y = \frac{1}{R} + \frac{1}{Z_L} + \frac{1}{Z_C} \qquad Y(j1000) = 1/R = 5\text{ mS} = Y_1$$

$$V_{o1} = I_{S1}/Y_1 = 1\angle 0° / 5\times 10^{-3} = 200\angle 0°\ V$$

$$Y(j1500) = Y_2 = 5 + j417\text{ mS} \qquad V_{o2} = 1\angle 0°/Y_2 = 2.4\angle -89.3°\ V$$

$$v_{o1}(t) = 200\cos 1000t\ V \qquad v_{o2}(t) = 2.4\cos(1500t - 89.3°)\ V$$

$$v_o(t) = v_{o1} + v_{o2} \Rightarrow \boxed{v_o(t) = 200\cos 1000t + 2.4\cos(1500t - 89.3°)\ V}$$

Magnitude difference (200V versus 2.4V) is due to $|Y|$ at the two frequencies!

12.49 Determine the parameters of a parallel resonant circuit that has the following properties: $\omega_0 = 2$ Mrad/s, BW = 20 rad/s, and an impedance at resonance of 2000 Ω. CS

SOLUTION:

at Resonance, $Z = R = 2000\Omega$ $\boxed{R = 2000\,\Omega}$

$BW = \frac{1}{RC} \Rightarrow C = \frac{1}{(BW)R} \Rightarrow \boxed{C = 25\mu F}$

$\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow L = \frac{1}{\omega_0^2 C} \Rightarrow \boxed{L = 10\,nH}$

12.50 Determine the value of C in the network shown in Fig. P12.50 for the circuit to be in resonance.

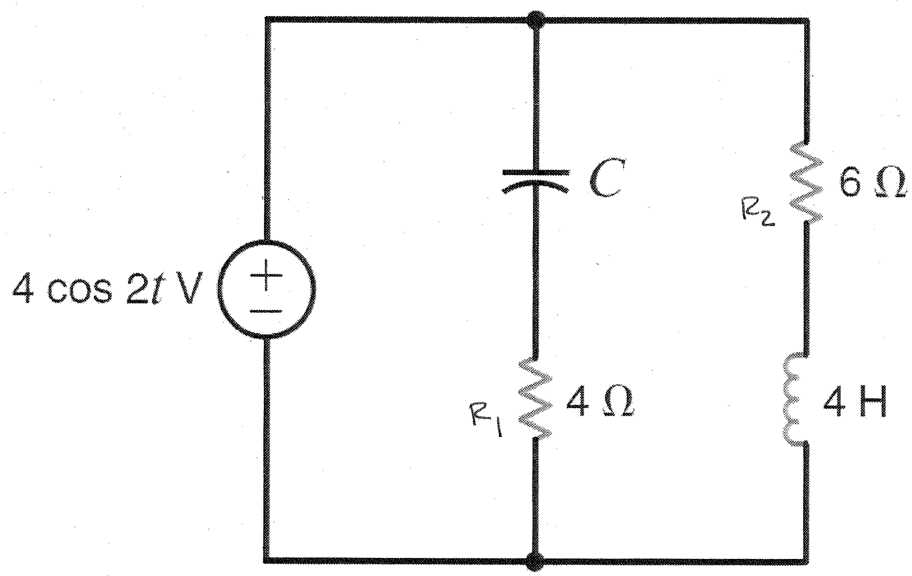
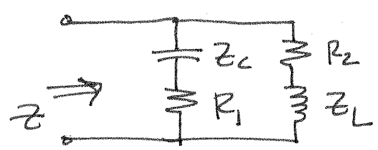


Figure P12.50

SOLUTION: $\omega = 2$ r/s $\quad Z_L = j8\,\Omega \quad Z_C = -j/2C\ \Omega$



Let $Z_1 = Z_C + R_1$ & $Z_2 = R_2 + Z_L = 6 + j8\,\Omega$

$$Z = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{R_1R_2 + L/C + j(\omega L R_1 - R_2/\omega C)}{R_1 + R_2 + j(\omega L - 1/\omega C)}$$

At resonance, Z is real. So, phase angle of numerator and denominator are equal.

$$\frac{\omega L R_1 - R_2/\omega C}{R_1R_2 + L/C} = \frac{\omega L - 1/\omega C}{R_1 + R_2} \Rightarrow \frac{32 - 3/C}{24 + 4/C} = \frac{8 - 1/2C}{10}$$

$$\frac{32C-3}{24C+4} = \frac{16C-1}{20C} \Rightarrow \frac{32C-3}{6C+1} = \frac{16C-1}{5C} \Rightarrow 64C^2 - 25C + 1 = 0$$

$$\boxed{C = \begin{cases} 345\text{ mF} \\ 46\text{ mF} \end{cases}}$$

12.51 Determine the equation for the nonzero resonant frequency of the impedance shown in Fig. P12.51.

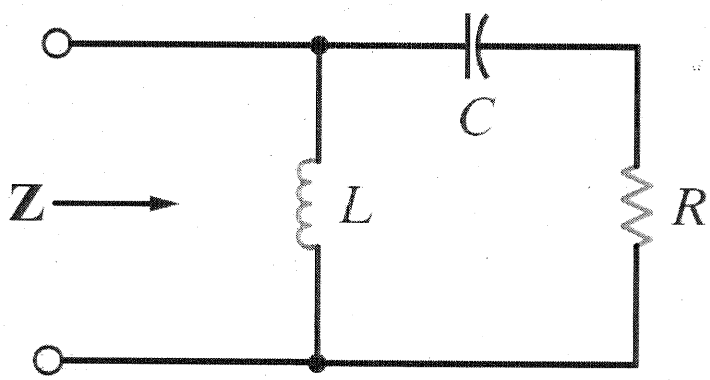


Figure P12.51

SOLUTION:

$$Z = \frac{j\omega L\,(R - j/\omega C)}{R + j(\omega L - 1/\omega C)} = \frac{L/C + j\omega L R}{R + j(\omega L - 1/\omega C)} = \frac{L\left[1/C + j\omega R\right]}{R + j(\omega L - 1/\omega C)}$$

at resonance, $\angle Z = 0$. So,

$$\frac{\omega_0 R}{1/C} = \frac{\omega_0 L - 1/\omega_0 C}{R} \Rightarrow \omega_0 R^2 C = \omega_0 L - \frac{1}{\omega_0 C} \Rightarrow \omega_0^2 (R^2C^2 - LC) = -1$$

$$\boxed{\omega_0 = \frac{1}{\sqrt{LC - (RC)^2}}}$$

12.52 Determine the new parameters of the network shown in Fig. P12.52 if $\mathbf{Z}_{\text{new}} = 10^4 \mathbf{Z}_{\text{old}}$.

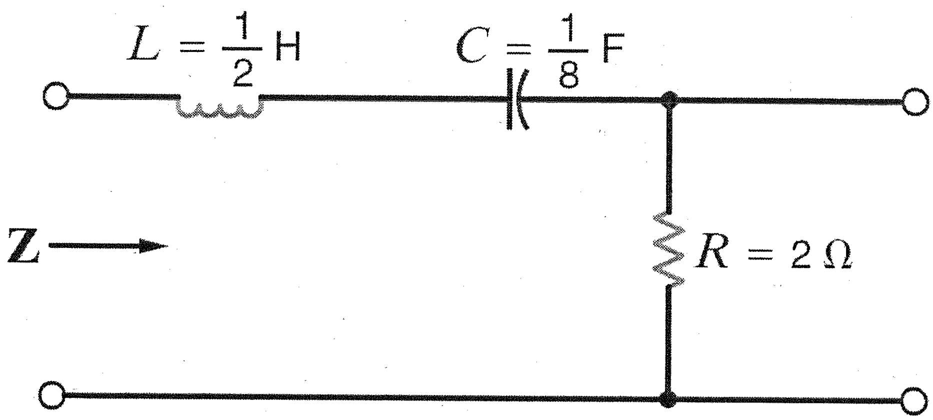


Figure P12.52

SOLUTION:

scale factor $= K_M = Z_{new} / Z_{old} = 10^4$

$R_{new} = K_M R_{old}$	$R_{new} = 20\,k\Omega$
$L_{new} = K_M L_{old}$	$L_{new} = 5\,kH$
$C_{new} = C_{old} / K_m$	$C_{new} = 12.5\,\mu F$

12.53 Determine the new parameters of the network in Problem 12.52 if $\omega_{new} = 10^4\omega_{old}$. CS

SOLUTION:

$L_{old} = 0.5\,H \qquad C_{old} = 0.125\,F \qquad R_{old} = 2\Omega$

Let $K_F = \omega_{new}/\omega_{old} = 10^4$

$L_{new} = L_{old}/K_F$

$C_{new} = C_{old}/K_F$

$R_{new} = R_{old}$

$$\boxed{\begin{array}{c} L_{new} = 50\,\mu H \\ C_{new} = 12.5\,\mu F \\ R = 2\Omega \end{array}}$$

12.54 Given the network in Fig. P12.54, sketch the magnitude characteristic of the transfer function

$$\mathbf{G}_v(j\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_1}(j\omega)$$

Identify the type of filter.

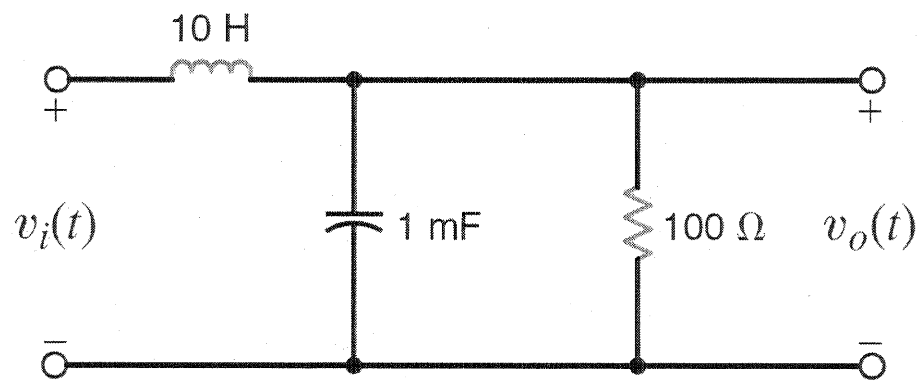
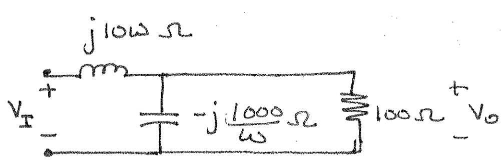


Figure P12.54

SOLUTION:



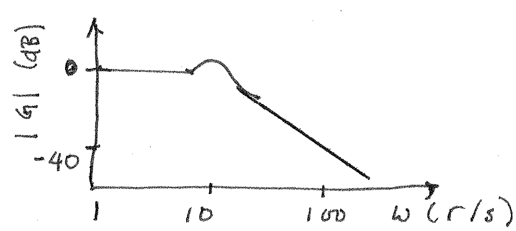
Let $Z = \dfrac{100\left(-j1000/\omega\right)}{100 - j\dfrac{1000}{\omega}} = \dfrac{10^3}{10 + j\omega}$

$$\frac{V_o}{V_s} = \frac{Z}{Z + j10\omega} = \frac{10^3}{10^3 + j10\omega(10 + j\omega)} = \frac{10^3}{10^3 - 10\omega^2 + j100\omega} = \frac{100}{100 - \omega^2 + j10\omega}$$

$$\frac{V_o}{V_s} = \frac{100}{(j\omega)^2 + j\omega(10) + 100}$$

complex conjugate poles: $\tau = 1/10$ $\quad \frac{10}{100} = 2\zeta\tau \Rightarrow \zeta = 0.5$

$\boxed{\text{Filter is low-pass}}$



12.55 Given the network in Fig. P12.55, sketch the magnitude characteristic of the transfer function

$$\mathbf{G}_v(j\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_1}(j\omega)$$

Identify the type of filter.

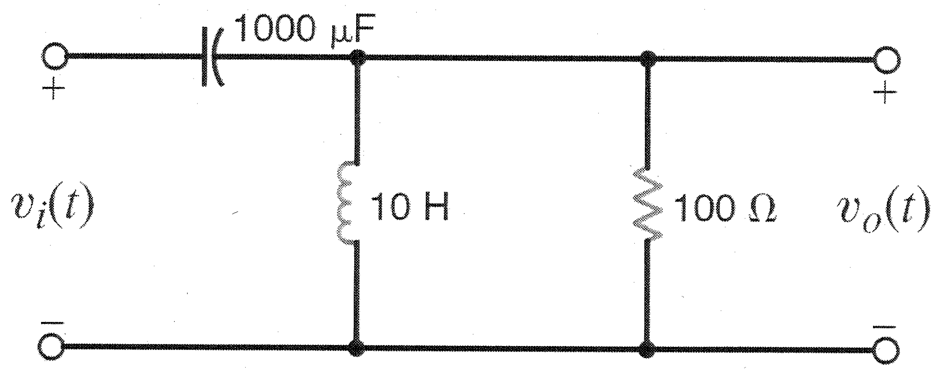
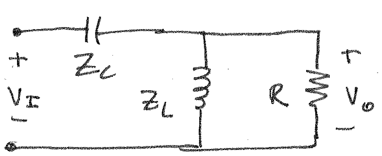


Figure P12.55

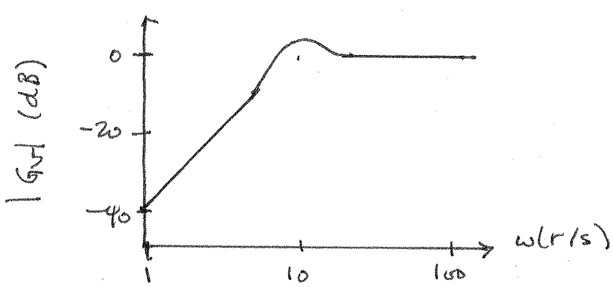
SOLUTION: $Z_L = j\omega(10)\ \Omega$ $Z_C = 1000/(j\omega)\ \Omega$ $R = 100\Omega$



Let $Z = RZ_L/(R+Z_L) = 100(j\omega)/(10+j\omega)$

$$V_o/I_I = G_v(j\omega) = Z/(Z+Z_C) = \frac{(j\omega)^2}{(j\omega)^2 + 10(j\omega) + 100}$$

Complex conjugate poles at: $\tau = 1/10$ $2\xi\tau = 0.1 \Rightarrow \xi = 0.5$



$\boxed{\text{Filter is high pass}}$

12.56 Determine what type of filter the network shown in Fig. P12.56 represents by determining the voltage transfer function. **CS**

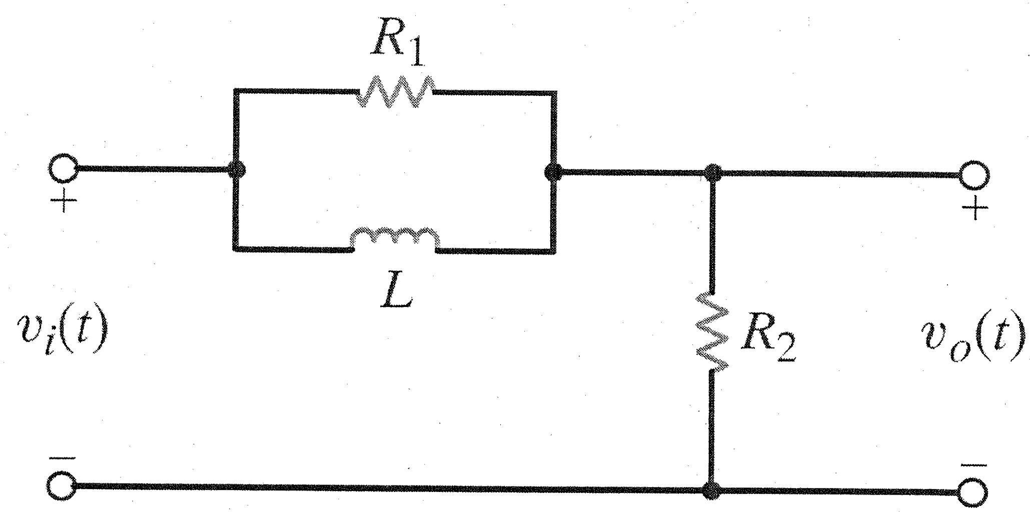
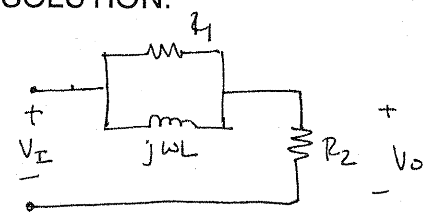


Figure P12.56

SOLUTION:

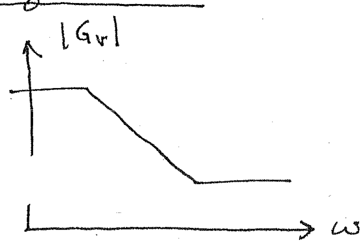


Let $Z = R_1(j\omega L)/(R_1 + j\omega L)$

$$\frac{V_o}{V_I} = G_v(j\omega) = \frac{R_2}{R_2 + Z}$$

$$G_v(j\omega) = \frac{j\omega L R_1 + R_1 R_2}{j\omega L R_1 + R_1 R_2 + j\omega L R_2} = \frac{(j\omega L + R_1) R_2}{R_1 R_2 + j\omega L (R_1 + R_2)} = \frac{R_2}{R_1 + R_2}\left[\frac{j\omega + R_1/L}{j\omega + \left(\frac{R_1 R_2}{R_1 + R_2}\right)/L}\right]$$

Rough sketch



filter is lowpass

12.57 Determine what type of filter the network shown in Fig. P12.57 represents by determining the voltage transfer function. **PSV**

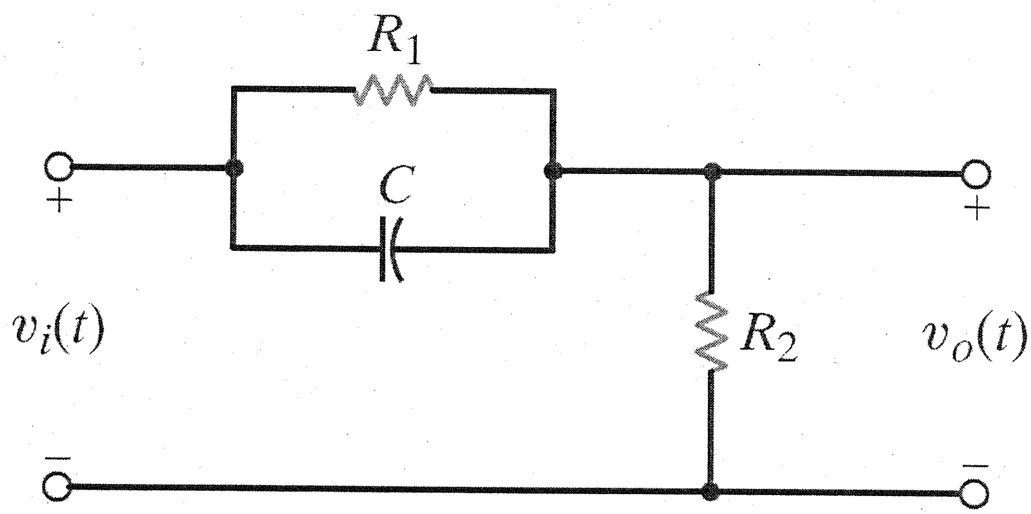
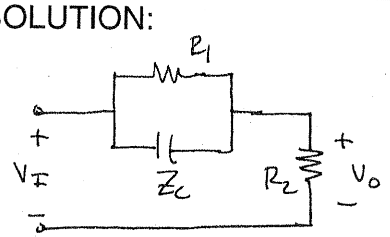


Figure P12.57

SOLUTION:

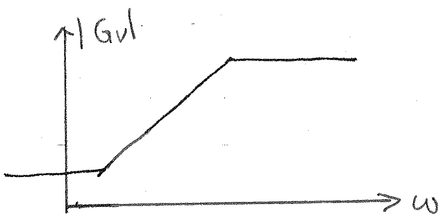


$Z_C = \frac{1}{j\omega C}$ $Z = R_1 Z_C / (R_1 + Z_C) = \frac{R_1}{j\omega R_1 C + 1}$

$G_v(j\omega) = \frac{V_o}{V_I} = \frac{R_2}{R_2 + Z}$

$$G_v(j\omega) = \frac{R_2}{R_2 + \frac{R_1}{j\omega R_1 C + 1}} = \frac{R_2(j\omega R_1 C + 1)}{j\omega R_1 R_2 C + R_1 + R_2} = \frac{j\omega + 1/R_1 C}{j\omega + 1/R_p C} \qquad R_p = R_1 // R_2$$

Rough sketch



$\boxed{\text{filter is highpass}}$

12.58 Given the lattice network shown in Fig. P12.58, determine what type of filter this network represents by determining the voltage transfer function.

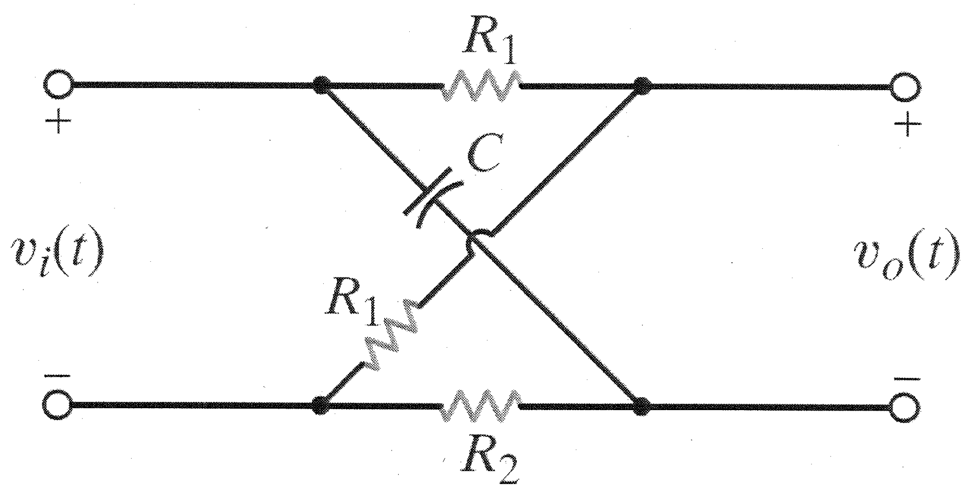
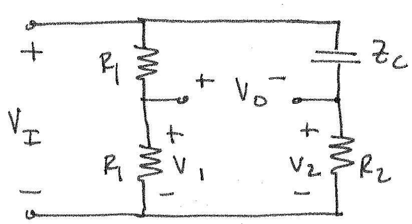


Figure P12.58

SOLUTION: $Z_C = 1/j\omega C$



$$V_1 = \frac{V_I R_1}{R_1 + R_1} = V_I/2$$

$$V_2 = \frac{V_I R_2}{R_2 + Z_C} = \frac{V_I\,(j\omega R_2 C)}{j\omega R_2 C + 1}$$

$$V_o = V_1 - V_2 = V_I\left[\frac{1}{2} - \frac{j\omega R_2 C}{j\omega R_2 C + 1}\right] = V_I\left\{\frac{1}{2}\left[\frac{1 - j\omega C R_2}{1 + j\omega C R_2}\right]\right\}$$

$$G_v = V_o/V_I = \frac{1}{2}\left[\frac{1 - j\omega C R_2}{1 + j\omega C R_2}\right]$$

$|G_v|$ is independent of ω! $\boxed{\text{All pass filter}}$

12.59 Given the network in Fig. P12.59, and employing the voltage follower analyzed in Chapter 4, determine the voltage transfer function and its magnitude characteristic. What type of filter does the network represent?

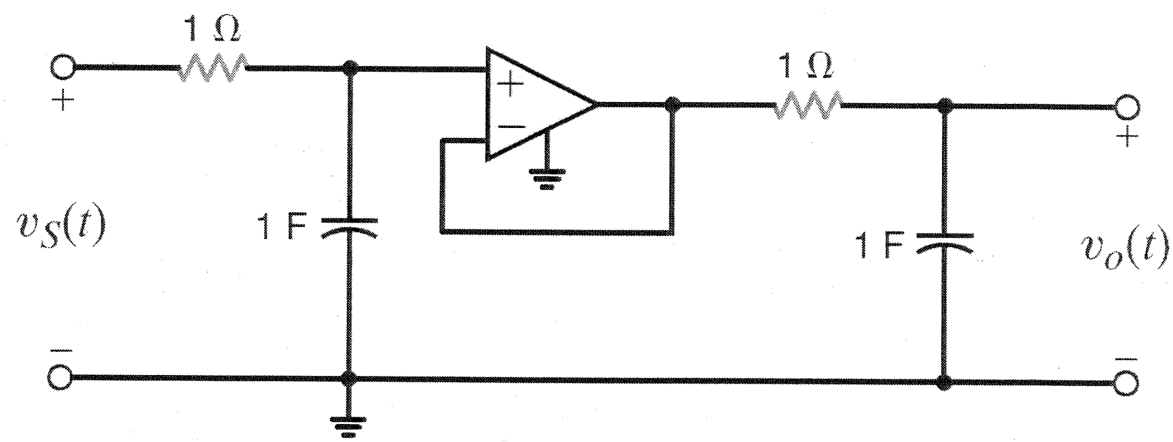
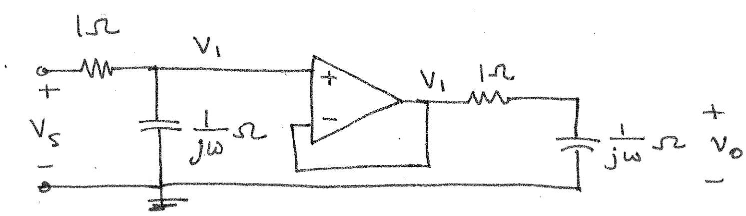


Figure P12.59

SOLUTION:

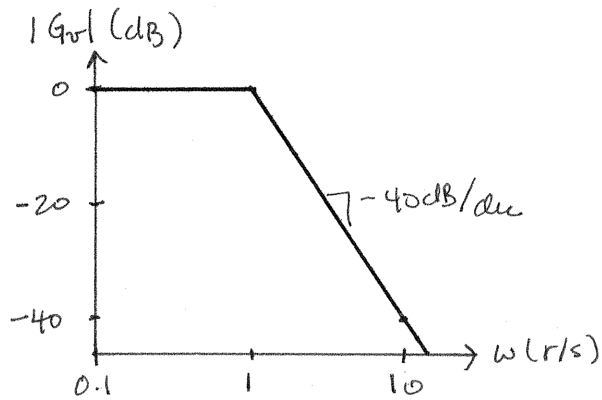


$$\frac{V_1}{V_s} = \frac{1/j\omega}{1 + 1/j\omega} = \frac{1}{j\omega + 1}$$

$$\frac{V_o}{V_1} = \frac{1/j\omega}{1 + 1/j\omega} = \frac{1}{j\omega + 1}$$

$$V_o/V_s = \frac{1}{(j\omega + 1)^2} = G_v$$

$\boxed{\text{low pass filter}}$



12.60 Given the network in Fig. P12.60, find the transfer function

$$\frac{\mathbf{V}_o}{\mathbf{V}_1}(j\omega)$$

and determine what type of filter the network represents.

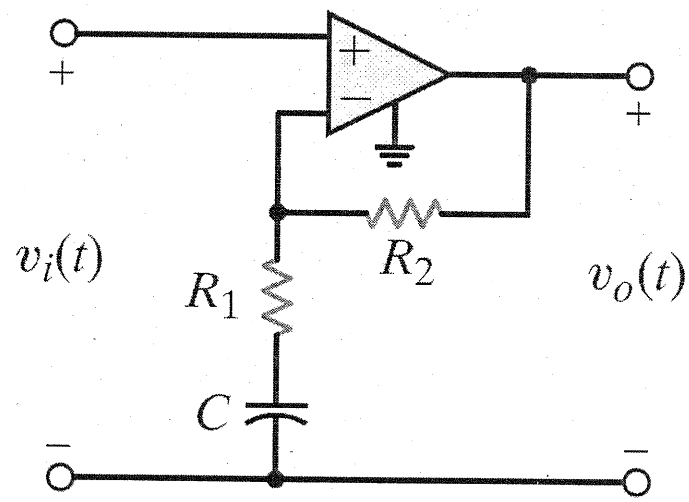


Figure P12.60

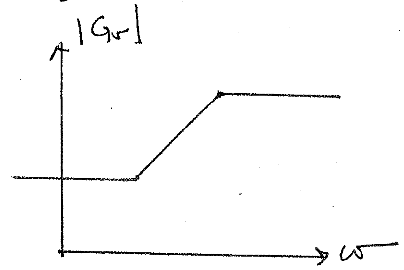
SOLUTION: Let $Z_1 = R_1 + \frac{1}{j\omega C}$

$$\frac{V_o}{V_I} = G_v(j\omega) = 1 + \frac{R_2}{Z_1} = 1 + \frac{j\omega C R_2}{j\omega C R_1 + 1} = \frac{j\omega C(R_1+R_2)+1}{j\omega C R_1 + 1} = G_v(j\omega)$$

$$\boxed{G_v = \left(1+\frac{R_2}{R_1}\right)\frac{j\omega + \frac{1}{C(R_1+R_2)}}{j\omega + \frac{1}{CR_1}}}$$

$\boxed{\text{High pass filter}}$

Rough sketch



12.61 Repeat Problem 12.55 for the network shown in Fig. P12.61.

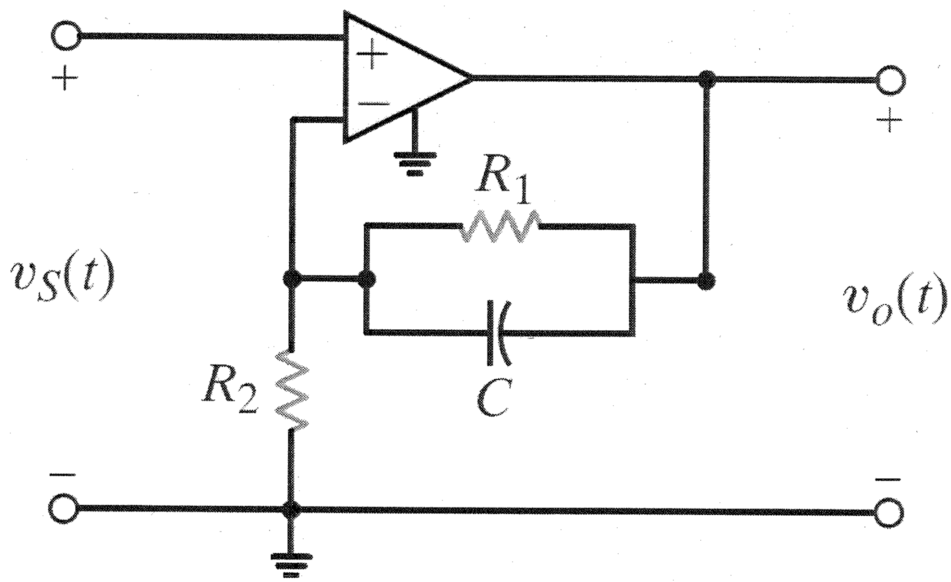


Figure P12.61

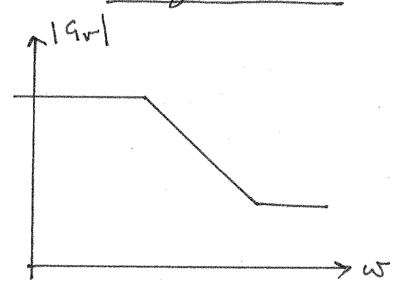
SOLUTION:

Let $Z_2 = R_1(1/j\omega C)/(R_1 + 1/j\omega C) = R_1/(j\omega C R_1 + 1)$

$$\frac{V_o}{V_I} = G_v(j\omega) = 1 + \frac{Z_2}{R_2} = 1 + \frac{R_1/R_2}{j\omega C R_1 + 1} = \frac{j\omega C R_1 + 1 + R_1/R_2}{j\omega C R_1 + 1}$$

$$\boxed{G_v(j\omega) = \frac{j\omega + \frac{1}{R_P C}}{j\omega + \frac{1}{R_1 C}} \qquad R_P = R_1 \,//\, R_2}$$

Rough sketch



$\boxed{\text{Lowpass Filter}}$

12.62 Determine the voltage transfer function and its magnitude characteristic for the network shown in Fig. P12.62 and identify the filter properties.

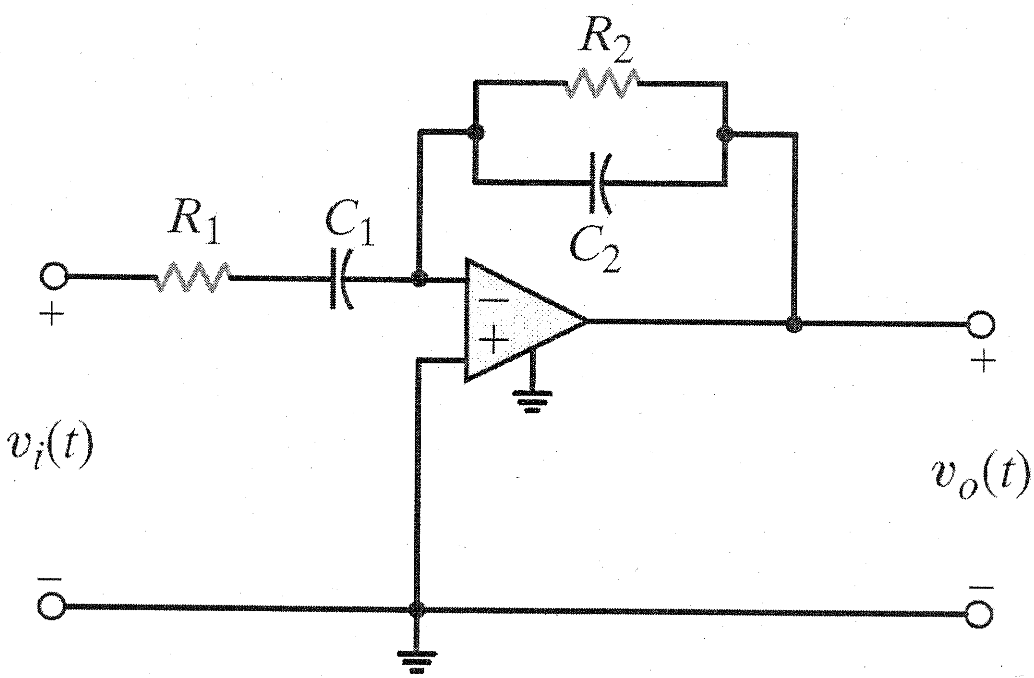


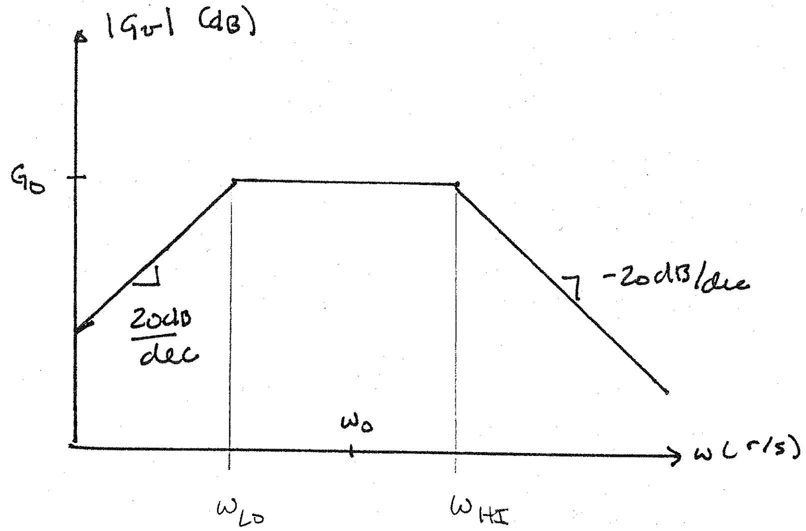
Figure P12.62

SOLUTION: Let $Z_1 = R_1 + 1/j\omega C_1 = \dfrac{j\omega C_1 R_1 + 1}{j\omega C_1}$

Let $Z_2 = \dfrac{R_2(1/j\omega C_2)}{R_2 + 1/j\omega C_2} = \dfrac{R_2}{j\omega C_2 R_2 + 1}$

$$G_v(j\omega) = \frac{V_o}{V_I} = -\frac{Z_2}{Z_1} = \boxed{-\frac{j\omega C_1 R_2}{(j\omega C_2 R_2+1)(j\omega C_1 R_1+1)} = G_v(j\omega)}$$

Bandpass filter



$$G_v(j\omega) = \frac{j\omega C_1 R_2 / C_1 C_2 R_1 R_2}{(j\omega)^2 + j\omega\left(\frac{1}{R_1C_1} + \frac{1}{R_2C_2}\right) + \frac{1}{R_1R_2C_1C_2}}$$

center freq $= \boxed{\omega_o = \frac{1}{\sqrt{R_1R_2C_1C_2}}}$

$$\frac{\omega_o}{Q} = \frac{1}{R_1C_1} + \frac{1}{R_2C_2} \Rightarrow \boxed{Q = \frac{\sqrt{R_1R_2C_1C_2}}{R_1C_1 + R_2C_2}}$$

$$Bw = \frac{\omega_o}{Q} \Rightarrow \boxed{Bw = \frac{1}{R_1C_1} + \frac{1}{R_2C_2}}$$

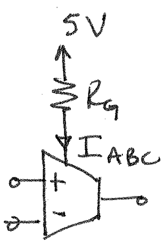
12.63 An OTA with a transconductance of 1 mS is required. A 5-V supply is available, and the sensitivity of g_m to I_{ABC} is 20.

(a) What values of I_{ABC} and R_G do you recommend?

(b) If R_G has a tolerance of +5%, what is the possible range of g_m in the final circuit?

SOLUTION:

a)



$I_{ABC} = 5/R_G \qquad g_m = 20\, I_{ABC} = 1\text{mS}$

$I_{ABC} = \frac{10^{-3}}{20} \Rightarrow \boxed{I_{ABC} = 50\mu A}$

$R_G = 5/I_{ABC} \rightarrow \boxed{R_G = 100\text{k}\Omega}$

b) $g_m = 20\left(\frac{5}{R_G}\right) = \frac{100}{R_G}$

$95\text{k}\Omega \le R_G \le 105\text{k}\Omega \qquad \boxed{0.952\text{mS} \le g_m \le 1.053\text{mS}}$

12.64 A particular OTA has a maximum transconductance of 5 mS with a range of 6 decades.

(a) What is the minimum possible transconductance?

(b) What is the range of I_{ABC}?

(c) Using a 5-V power supply and resistor to set I_{ABC}, what is the range of values for the resistor and the power it consumes?

SOLUTION:

a) $\dfrac{G_{max}}{G_{min}} = 10^6 \Rightarrow G_{min} = \dfrac{G_{max}}{10^6} = 5\,nS$ $\boxed{G_{min} = 5\,nS}$

b) $G_m = 20\,I_{ABC} \Rightarrow$ $\boxed{250\,pA \leq I_{ABC} < 250\,\mu A}$

c) $5 = R\,I_{ABC} \Rightarrow R = \dfrac{5}{I_{ABC}}$ $\boxed{20\,k\Omega \leq R \leq 20\,G\Omega}$

$P_R = I_{ABC}^2 R$ $\boxed{1.25\,nW \leq P \leq 1.25\,mW}$

12.65 The OTA and 5-V source described in Problem 12.64 are used to create a transconductance of 2.5 mS.

(a) What resistor value is required?

(b) If the input voltage to the amplifier is $v_{in}(t) = 1.5\cos(\omega t)$V, what is the output current function?

SOLUTION:

$G_{max} = 5\,mS \qquad g_m = 20 I_{ABC} \qquad I_{ABC} = 5/R_G$

a) $g_m = \dfrac{100}{R_G} = 2.5\,mS \Rightarrow \boxed{R_G = 40\,k\Omega}$

b) $\dfrac{i_o(t)}{v_{in}(t)} = g_m = 2.5\,mS \qquad \boxed{i_o(t) = 3.75\cos(\omega t)\ mA}$

12.66 A fluid level sensor, used to measure water level in a reservoir, outputs a voltage directly proportional to fluid level. Unfortunately, the sensitivity of the sensor drifts about 10% over time. Some means for tuning the sensitivity is required. Your engineering team produces the simple OTA circuit in Fig. P12.66.

(a) Show that either V_G or R_G can be used to vary the sensitivity.

(b) List all pros and cons you can think of for these two options.

(c) What's your recommendation?

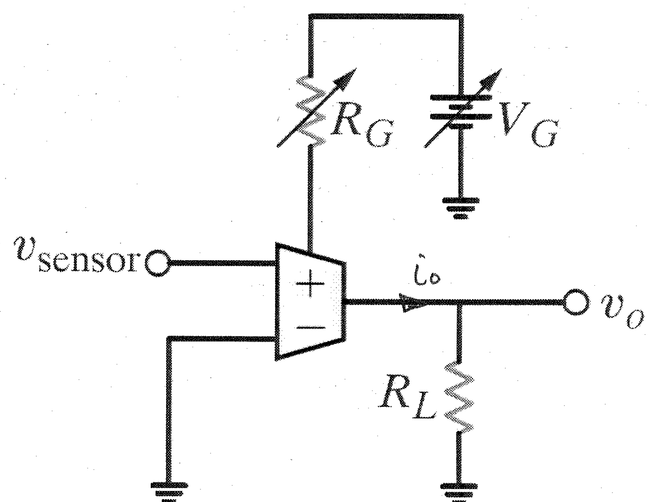


Figure P12.66

SOLUTION:

a) $v_o = i_o R_L \qquad i_o = G_m v_{sensor} \qquad G_m = 20\, I_{ABC} \qquad I_{ABC} = V_G / R_G$

So, $\quad v_o = \dfrac{20\, V_G R_L}{R_G} v_{sensor} \qquad$ changing V_G or R_G will change $V_O(t)$!

b) Tuning with V_G requires a variable voltage source. These are much more costly than a fixed source. On the plus side, V_G and v_o are directly related - double V_G and double v_o. Tuning with R_G requires only an inexpensive potentiometer, but the $R_G - v_o$ relation is indirect.

c) Based primarily on cost considerations, recommend tuning with R_G !

12.67 A circuit is required that can double the frequency of a sinusoidal voltage.

(a) If $v_{in}(t) = 1\sin(\omega t)$, show that the multiplier circuit in Fig. P12.67 can produce an output that contains a sinusoid at frequency 2ω.

(b) We want the magnitude of the double-frequency sinusoid to be 1 V. Determine values for R_G and R_L if the transconductance range is limited between 10 μS and 10 mS.

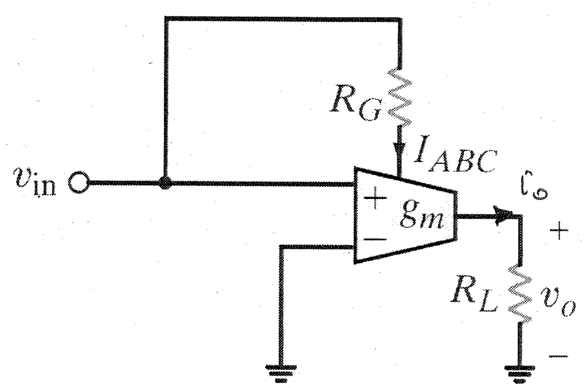


Figure P12.67

SOLUTION:

a) $v_o = i_o R_L$ $\quad i_o = g_m v_{in}$ $\quad g_m = 20 I_{ABC}$ $\quad I_{ABC} = v_{in}/R_G$

So, $v_o = R_L \frac{20}{R_G} v_{in}^2 = R_L \frac{20}{R_G} \sin^2(\omega t) = \boxed{\frac{10 R_L}{R_G}\left[1 - \cos(2\omega t)\right] = v_o}$

b) $|v_{in}| = 1V$ $\quad g_{max} = 10\,mS = 20\,|v_{in}|/R_G = 20\,(1)/R_G$

$R_G = 20/10^{-2} \Rightarrow \boxed{R_G = 2k\Omega}$

$|v_o| = 1 = \frac{10 R_L}{R_G} \Rightarrow \boxed{R_L = 200\Omega}$ (applies to $\sin(2\omega t)$ signal only)

12.68 The frequency doubler in Problem 12.67 uses a two-quadrant multiplier.

(a) What effect does this have on the output signal?

(b) The circuit in Fig. P12.68 is one solution. Show that v_o has a double-frequency term.

(c) How would you propose to eliminate the other terms?

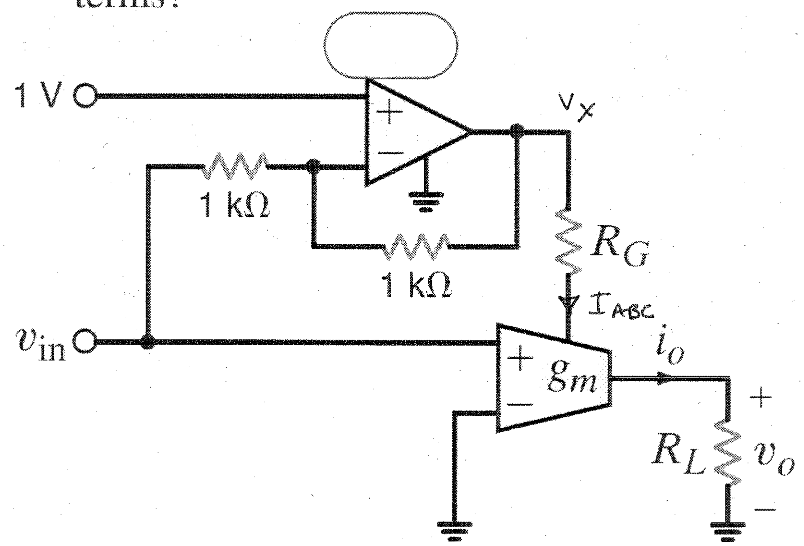


Figure P12.68

SOLUTION:

a) In problem 12.67, the negative half-cycles of $v_{in}(t)$ "try" to create a $I_{ABC} < 0$. This is not possible. The multiplier works only when $v_{in}(t) > 0$.

b) $v_X = 1\left(1 + \frac{1000}{1000}\right) - v_{in}\left(\frac{1000}{1000}\right) = 2 - v_{in}$

If as in 12.67, $v_{in}(t) = 1\sin(\omega t)$ V, v_X is always > 0 and so is I_{ABC}.

$$v_o = i_o R_L = g_m v_{in} R_L = 20 I_{ABC} v_{in} R_L = 20 v_X R_L v_{in} / R_G$$

$$\boxed{v_o = 20(R_L/R_G)\left[2\sin(\omega t) - 1/2 + \tfrac{1}{2}\cos(2\omega t)\right]}$$

c) use a highpass filter to reject the dc & $\sin(\omega t)$ terms!

12.69 In Fig. P12.69, V_x is a dc voltage. The circuit is intended to be a dc wattmeter where the output voltage value equals the power consumed by R_L in watts.

(a) The $g_m - I_{ABC}$ sensitivity is 20 S/A. Find R_G such that $I_x/I_1 = 10^4$.

(b) Choose R such that 1 V at V_o corresponds to 1 W dissipated in R_L.

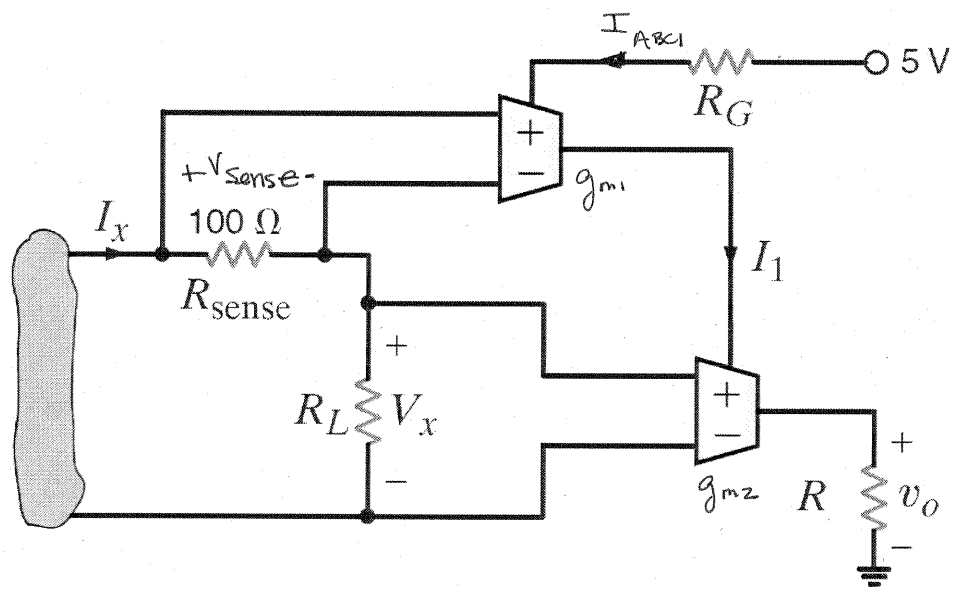


Figure P12.69

SOLUTION:

a) $V_{sense} = 100\, I_x \qquad I_1 = g_{m1} v_{sense} = \frac{20(5)}{R_G} v_{sense} = \frac{10^4 I_x}{R_G}$

$$\frac{I_x}{I_1} = 10^4 = \frac{R_G}{10^4} \Rightarrow \boxed{R_G = 100\text{M}\Omega}$$

b) $v_o = V_x\, g_{m2} R = V_x (20 I_1) R = V_x \frac{I_x}{10^4}(20R) = P_L\left(\frac{20R}{10^4}\right)$

$$1\text{V} = (1\text{W})\left(^{20R}/_{10^4}\right) \Rightarrow \boxed{R = 500\Omega}$$

12.70 The automatic gain control circuit in Fig. P12.70 is used to limit the transconductance, i_o/v_{in}.

(a) Find an expression for v_o in terms of v_{in}, R_G, and R_L.

(b) Express the asymptotic transconductance, i_o/v_{in}, in terms of R_G and R_L at $v_{in} = 0$ and as v_{in} approaches infinity. Given R_L and R_G values in the circuit diagram, what are the values of the asymptotic transconductance?

(c) What are the consequences of your results in (b)?

(d) If v_{in} must be no more than V_{CC} for proper operation, what is the minimum transconductance for the functional circuit?

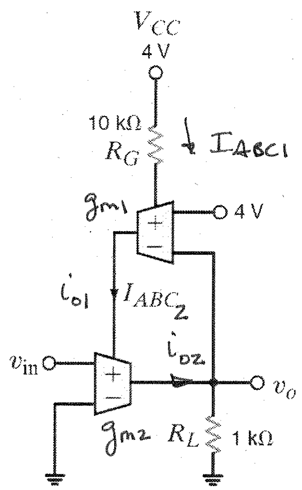


Figure P12.70

SOLUTION: a) $v_o = i_{o2} R_L = v_{in} g_{m2} R_L = 20 I_{ABC2} v_{in} R_L = 20 i_{o1} v_{in} R_L$

$i_{o1} = g_{m1}(4 - v_o) = 20 I_{ABC1}(4 - v_o) = 20(4/R_G)(4 - v_o)$

So, $v_o = 20 v_{in} R_L \left[\frac{80}{R_G}(4 - v_o)\right] \Rightarrow 6400 v_{in} \frac{R_L}{R_G} - 1600 v_{in} v_o \frac{R_L}{R_G}$

yields $v_o(1 + 160 v_{in}) = 640 v_{in} \Rightarrow$ $\boxed{v_o = \frac{640 v_{in}}{1 + 160 v_{in}}}$

b) $G_m = i_o/v_{in} = \frac{v_o}{v_{in}}(1/R_L) = \frac{0.64}{1 + 160 v_{in}}$

as $v_{in} \to 0$, $G_m \to 0.64$ S	$G_m \to 6400/R_G$
as $v_{in} \to \infty$, $G_m \to \frac{1}{250 v_{in}}$	$G_m \to \frac{4}{R_L v_{in}}$

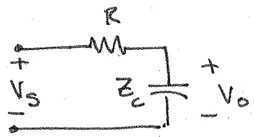
c) When v_{in} is small, G_m is high at 0.64 S.

But as v_{in} increases, G_m decreases. Eventually, i_o is limited to $4/R_L$.

d) $G_m \rightarrow \frac{0.64}{1+160(4)} = \boxed{\text{minimum } G_m = 998 \mu S}$

12.71 Design a low-pass filter using one resistor and one capacitor that will produce a 4.24-volt output at 159 Hz when 6 volts at 159 Hz is applied at the input.

SOLUTION:



$V_S = 6\angle 0^\circ$ V $\quad f = 159$ Hz $\quad \omega = 2\pi f = 1$ kr/s

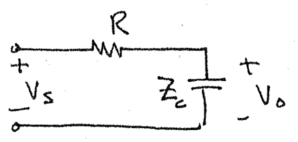
$$V_o = \frac{V_S Z_C}{Z_C + R} = \frac{6\angle 0}{j\omega CR + 1}$$

$$|V_o| = \frac{6}{\sqrt{(\omega RC)^2 + 1}} = 4.24 \Rightarrow RC = 10^{-3}\text{ s}$$

$$\boxed{\text{Arbitrarily select } C = 1\,\mu F \Rightarrow R = 1\,k\Omega}$$

12.72 Design a low-pass filter with a cutoff frequency between 15 and 16 kHz.

SOLUTION:



$$\frac{V_o}{V_s} = \frac{Z_C}{R+Z_C} = \frac{1}{j\omega RC+1} = \frac{1/RC}{j\omega + 1/RC}$$

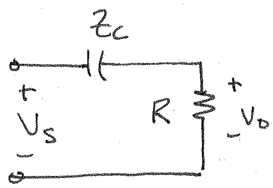
$$\omega_{cutoff} = \frac{1}{RC}.$$

Arbitrarily select $\omega_{cutoff} = 2\pi(15.5)\,\text{kr/s}$ & $C = 10\,\text{nF}$

yields, $R = 1.03\,\text{k}\Omega$

12.73 Design a high-pass filter with a half-power frequency between 159 and 161 Hz.

SOLUTION:



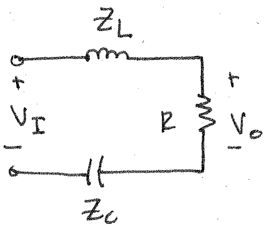
$$\frac{V_o}{V_S} = \frac{R}{R+Z_C} = \frac{j\omega}{j\omega + \frac{1}{RC}} \qquad \omega_{cutoff} = \frac{1}{RC}$$

Arbitrarily select $\omega_{cutoff} = 160\,\text{Hz}$
and $C = 1\,\mu\text{F}$

yields, $R = 6.25\,\text{k}\Omega$

12.74 Design a band-pass filter with a low cutoff frequency of approximately 4535 Hz and a high cutoff frequency of approximately 5535 Hz.

SOLUTION:



$$\frac{V_o}{V_I} = \frac{R}{j\omega L + R + \frac{1}{j\omega C}} = \frac{j\omega\,(R/L)}{(j\omega)^2 + j\omega\,(R/L) + \frac{1}{LC}}$$

Series RLC circut, $BW = \frac{R}{L} = 2\pi\,(1000)$

$$\omega_0 = \sqrt{\omega_{HI}\,\omega_{LO}} = 2\pi\sqrt{f_{HI} f_{LO}} = 31.5\ \text{kr/s} = \frac{1}{\sqrt{LC}}$$

Arbitrarily select $C = 100\ \text{nF}$

yields $L = 10\ \text{mH}$

and $R = 62.8\ \Omega$

12.75 An engineer has proposed the circuit shown in Fig. P12.75 to filter out high-frequency noise. Determine the values of the capacitor and resistor to achieve a 3-dB voltage drop at 23.16 kHz.

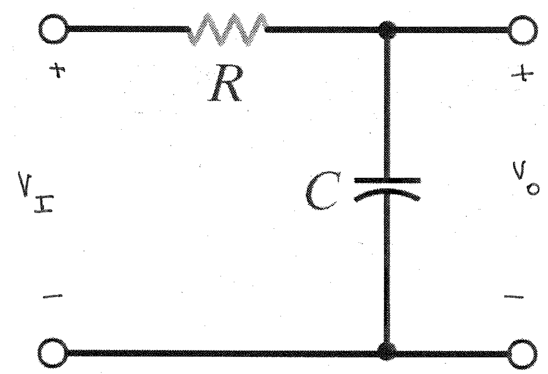


Figure P12.75

SOLUTION: $Z_C = 1/j\omega C$

$$\frac{V_o}{V_I} = \frac{Z_C}{Z_C + R} = \frac{1/RC}{j\omega + 1/RC}$$

3dB down at $\omega = \frac{1}{RC} = 2\pi(23.16\times10^3)$

Arbitrarily select $C = 1\,nF$, yields $R = 6.87\,k\Omega$

12.76 For the low-pass active filter in Fig. P12.76, choose R_2 and C such that $H_o = -7$ and $f_c = 10$ kHz.

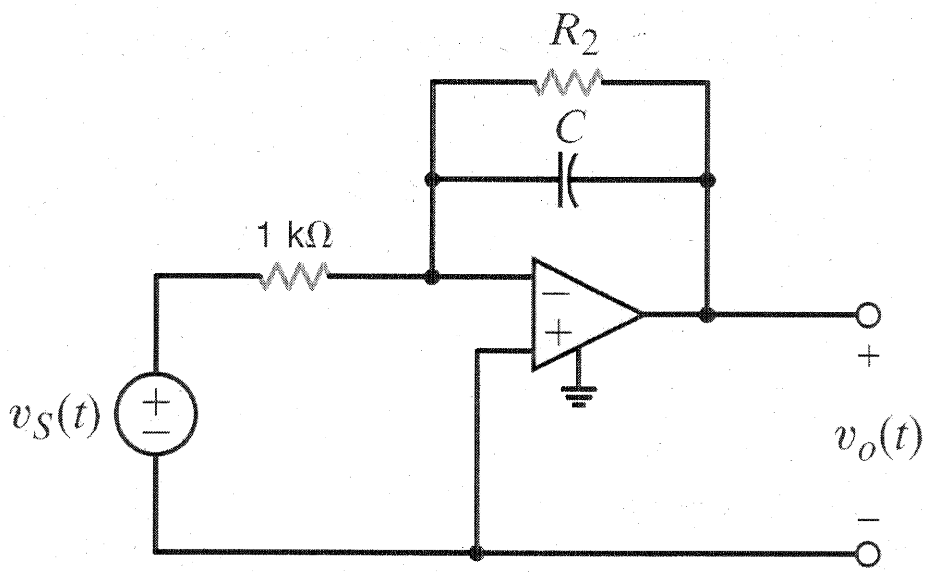


Figure P12.76

SOLUTION: Let $Z_2 = \dfrac{R_2 / j\omega C}{R_2 + \frac{1}{j\omega C}} = \dfrac{R_2}{j\omega C R_2 + 1}$ & $Z_1 = 1\,\text{k}\Omega$

$$H(j\omega) = \frac{V_o}{V_S} = -\frac{Z_2}{Z_1} = \frac{-R_2\,(10^{-3})}{j\omega C R_2 + 1} = -\left(\frac{1}{10^3 C}\right)\left(\frac{1}{j\omega + \frac{1}{CR_2}}\right)$$

$$H_o = -7 = -\frac{1\,(CR_2)}{CR_1} = -\frac{R_2}{R_1} \Rightarrow \boxed{R_2 = 7\,\text{k}\Omega}$$

$$\omega_c = \frac{1}{CR_2} = 2\pi f_c \Rightarrow \boxed{C = 2.27\,\text{nF}}$$

12.77 For the high-pass active filter in Fig. P12.77, choose C, R_1, and R_2 such that $H_o = 5$ and $f_c = 3$ kHz.

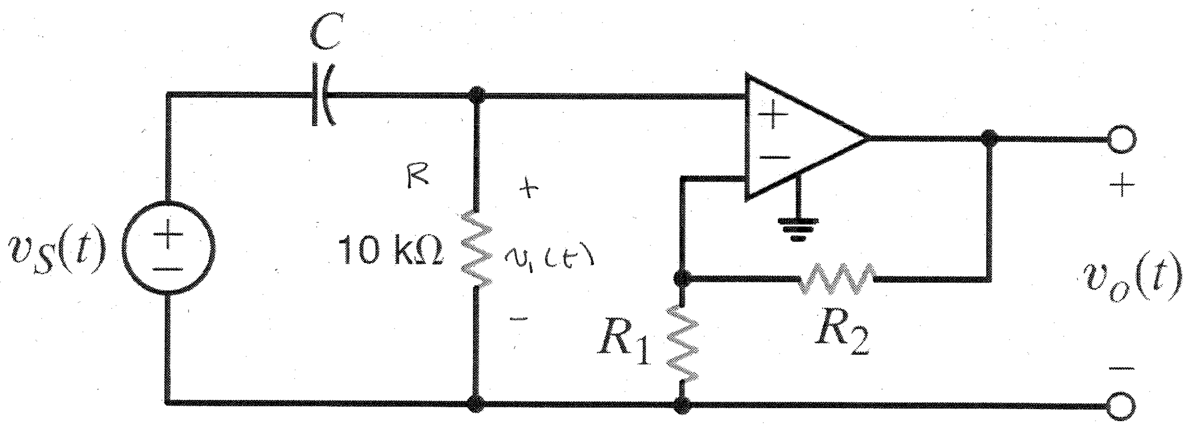


Figure P12.77

SOLUTION:

$$\frac{V_1}{V_S} = \frac{R}{R + \frac{1}{j\omega C}} = \frac{j\omega}{j\omega + \frac{1}{RC}} \qquad \frac{V_o}{V_1} = 1 + R_2/R_1$$

$$\frac{V_o}{V_S} = \left(1 + \frac{R_2}{R_1}\right)\frac{j\omega}{j\omega + \frac{1}{RC}} \qquad H_o = 1 + \frac{R_2}{R_1} = 5 \qquad \omega_c = 2\pi f_c = \frac{1}{RC}$$

$$C = \frac{1}{2\pi f_c R} \Rightarrow \boxed{C = 5.31\text{ nF}}$$

$$\boxed{\text{Arbitrarily select } R_1 = 10\text{k}\Omega\text{, yields } R_2 = 40\text{k}\Omega}$$

12.78 Given the second-order low-pass filter in Fig. P12.78, design a filter that has $H_o = 100$ and $f_c = 5$ kHz. Set $R_1 = R_3 = 1$ kΩ, and let $R_2 = R_4$ and $C_1 = C_2$. Use an op-amp model with $R_i = \infty$, $R_o = 0$, and $A = (2)10^5$.

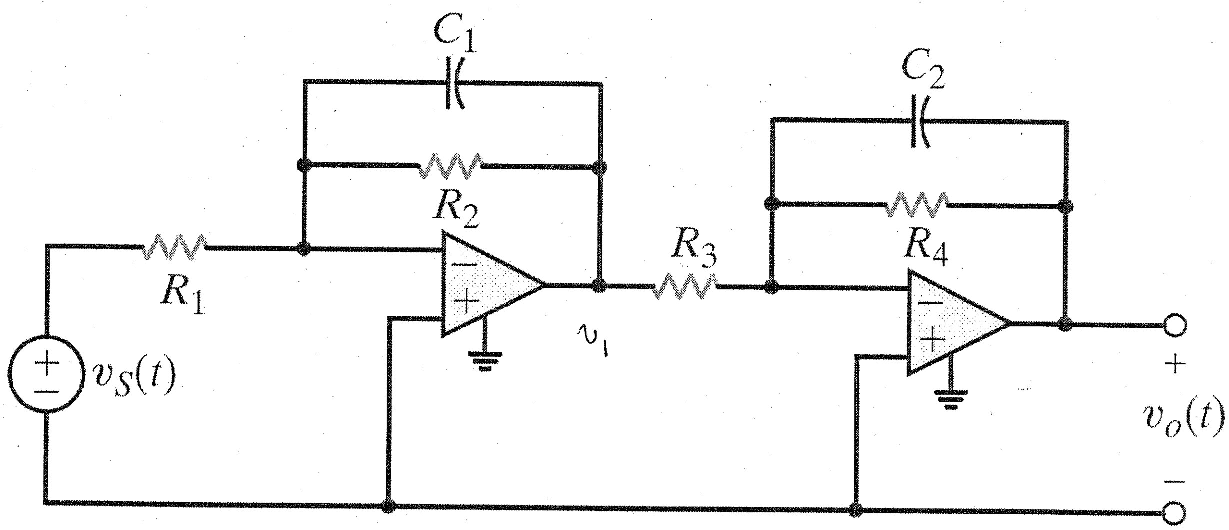


Figure P12.78

SOLUTION: Constraints yield

$$\frac{V_1}{V_s} = G_1(j\omega) = \frac{V_o}{V_1} = G_2(j\omega)$$

Let $$Z = \frac{R_2/j\omega C}{R_2 + \frac{1}{j\omega C}} = \frac{R_2}{j\omega C R_2 + 1} = \frac{1/C}{j\omega + 1/R_2C}$$

$$^{*}G_1(j\omega) = -Z/R_1 = \frac{-1/R_1C}{j\omega + 1/R_2C}$$

$$G(j\omega) = G_1 G_2 = \frac{(1/R_1C)^2}{\left(j\omega + \frac{1}{R_2C}\right)^2}$$

$$\begin{cases} H_o = (R_2/R_1)^2 = 100 \Rightarrow R_2/R_1 = 10 \\ \omega_c = 2\pi f_c = \frac{1}{R_2C} = \frac{1}{10^4 C} \end{cases}$$

$C = 3.18$nF, yields $R_2 = 10$ kΩ and $R_4 = 10$ kΩ

* For each op amp, $R_2/R_1 = 10$ which is << A for op amp (2×10^5). Including A in the analysis would only affect beyond 4 digits!

12.79 The second-order low-pass filter shown in Fig. P12.79 has the transfer function

$$\frac{\mathbf{V}_o}{\mathbf{V}_1}(s) = \frac{\dfrac{-R_3}{R_1}\left(\dfrac{1}{R_2R_3C_1C_2}\right)}{s^2 + \dfrac{s}{C_1}\left(\dfrac{1}{R_1}+\dfrac{1}{R_2}+\dfrac{1}{R_3}\right) + \dfrac{1}{R_2R_3C_1C_2}}$$

Design a filter with $H_o = -10$ and $f_c = 5$ kHz, assuming that $C_1 = C_2 = 10$ nF and $R_1 = 1$ kΩ.

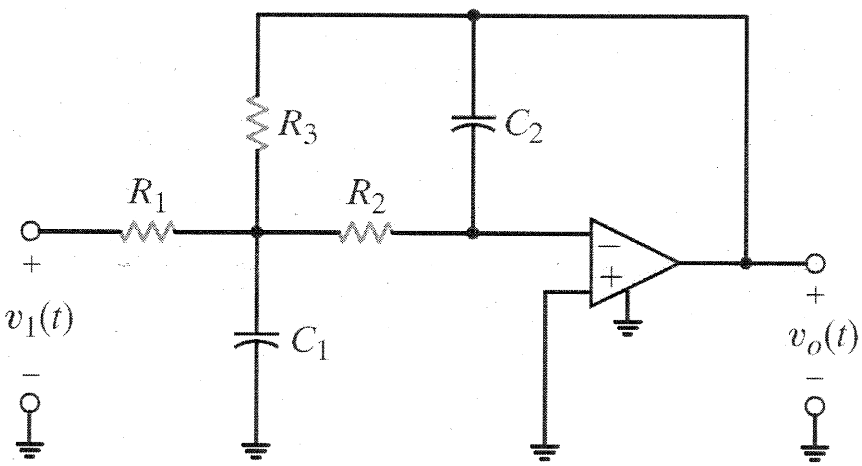


Figure P12.79

SOLUTION:

$$H_o = -\frac{R_3}{R_1}\left(\frac{1}{R_2R_3C_1C_2}\right) \Big/ \frac{1}{R_2R_3C_1C_2} = -R_3/R_1 = -10 \Rightarrow \boxed{R_3 = 10\,k\Omega}$$

R_2 affects f_c. Look at characteristic equation

$$s^2 + Bs + C = 0 \qquad B = \frac{1}{C_1}\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right) \qquad C = \frac{1}{R_2R_3C_1C_2}$$

Roots at

$$\left(s + \frac{B}{2} - \sqrt{\frac{B^2}{4} - C}\right)\left(s + B/2 + \sqrt{\frac{B^2}{4} - C}\right) = 0 = (s+\omega_{P1})(s+\omega_{P2})$$

where ω_{P1} & ω_{P2} are the poles. Since $\omega_{P1} < \omega_{P2}$,

$\omega_{P1} = 2\pi f_c = \omega_c$.

$$B/2 - \sqrt{B^2/4 - C} = \omega_c \Rightarrow \left(\frac{B}{2} - \omega_c\right)^2 = B^2/4 - C$$

$$\frac{B^2}{4} - B\omega_c + \omega_c^2 = B^2/4 - C \Rightarrow B\omega_c - C = \omega_c^2$$

Using R & C values and $f_c = 500\text{Hz}$, $B = 1.1\times10^5 + 10^8/R_2$

and $C = 10^{12}/R_2$ & $\omega_c = 1000\pi$

$$\left(1.1\times10^5 + \frac{10^8}{R_2}\right)1000\pi - \frac{10^{12}}{R_2} = 10^6\pi^2$$

$$1.1\times10^8\pi + \frac{10^{11}\pi}{R_2} - \frac{10^{12}}{R_2} = 10^6\pi^2$$

$$110\pi - \pi^2 = \frac{10^6 - 10^5\pi}{R_2} \Rightarrow R_2 = \frac{10^5(10-\pi)}{\pi(110-\pi)}$$

$$\boxed{R_2 = 2043\,\Omega}$$

As an aside: $\omega_{p1} = 500(2\pi)$ r/s

$\omega_{p2} = 2\pi\,(24.8)$ kr/s

12.80 Given the circuit in Figure 12.57, design a second-order bandpass filter with a center frequency gain of -5, $\omega_0 = 50$ krad/s, and a BW $= 10$ krad/s. Let $C_1 = C_2 = C$ and $R_1 = 1\ \text{k}\Omega$. What is the Q of this filter? Sketch the Bode plot for the filter. Use the ideal op-amp model.

SOLUTION:

From the text,

$$\omega_0 = \left(\frac{1 + R_1/R_3}{R_1 R_2 C^2}\right)^{1/2} = 50\ \text{kr/s}$$

$$\frac{\omega_0}{Q} = BW = \frac{2C}{R_2 C^2} = \frac{2}{R_2 C} = BW = 10\ \text{kr/s}$$

$$\text{center freq gain} = \frac{-\frac{1}{R_1 C}}{\frac{1}{R_2 C} + \frac{1}{R_2 C}} = -\frac{R_2}{2R_1} = -5 = H_0$$

$R_2 = 10 R_1$ $\boxed{R_2 = 10\,\text{k}\Omega}$

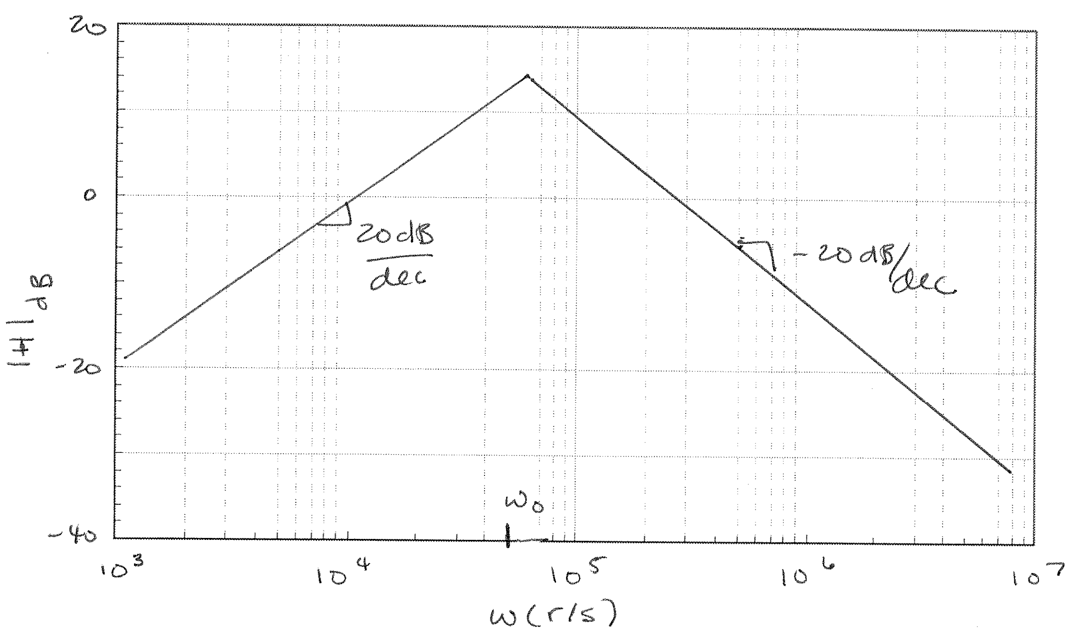
$C = \dfrac{2}{R_2 (BW)}$ $\boxed{C = 20\,\text{nF}}$

$R_3 = \dfrac{R_1}{\omega_0^2 R_1 R_2 C^2 - 1}$ $\boxed{R_3 = 111\,\Omega}$

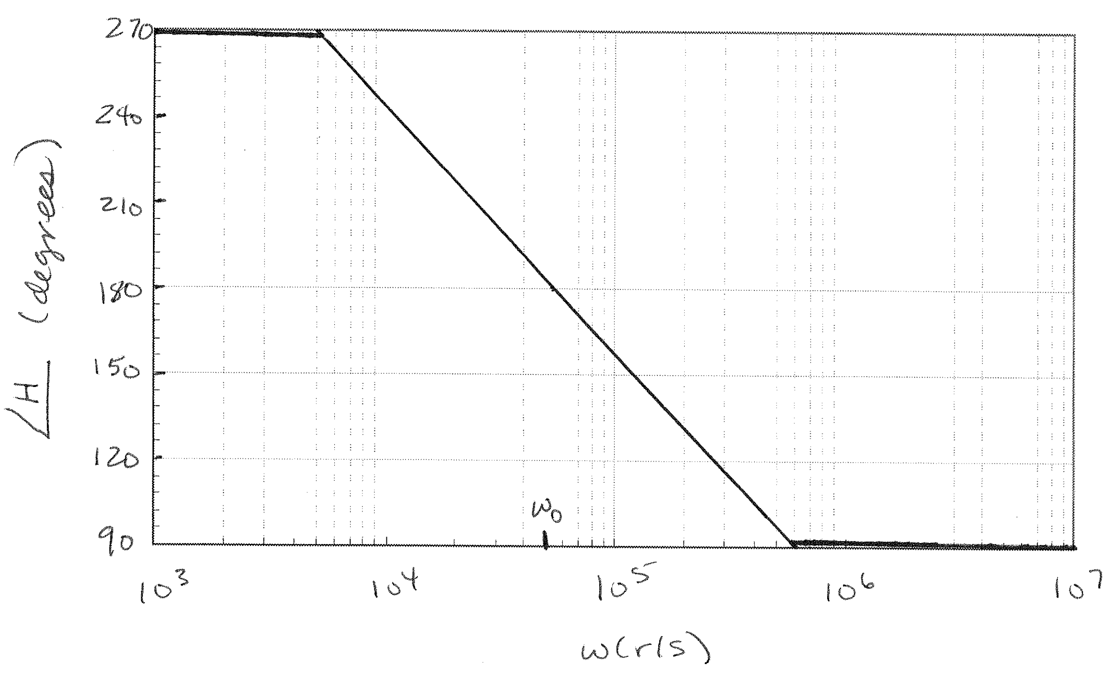
$Q = \omega_0 / BW$ $\boxed{Q = 5}$

$$H(s) = \frac{H_0 (2BW) s}{s^2 + 2BW s + \omega_0^2} = \frac{-10^5 s}{s^2 + 2\times10^4 s + 2.5\times10^9}$$

poles at $s = -10^4 \pm j4.90\times10^4$ r/s



20
0
-20
-40
|H|dB
20dB/dec
-20dB/dec
ω0
10^3
10^4
10^5
10^6
10^7
ω(r/s)



270
240
210
180
150
120
90
∠H (degrees)
ω0
10^3
10^4
10^5
10^6
10^7
ω(r/s)

12.81 Referring to Example 12.38, design a notch filter for the tape deck for use in Europe, where power utilities generate at 50 Hz.

SOLUTION:

From Ex. 12.38, $\omega_z = \frac{1}{\sqrt{LC}}$

We need $\omega_z = 2\pi(50) = 100\pi$ r/s

$\boxed{\text{Arbitrarily, we select } C = 1000\,\mu F \text{ yielding } L = 10\,mH}$

12FE-1 Determine the resonant frequency of the circuit in Fig. 12PFE-1, and find the voltage $\mathbf{V}_o$ at resonance. **CS**

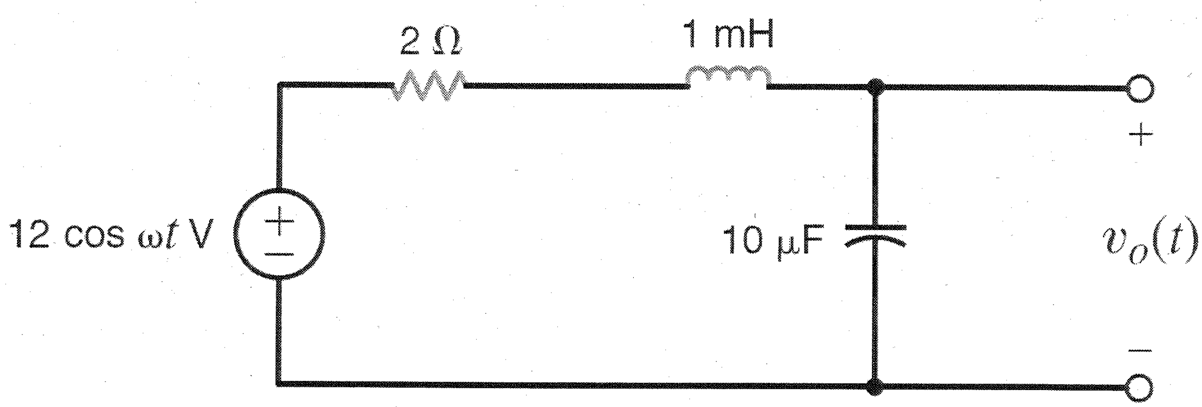


Figure 12PFE-1

SOLUTION:

$$\omega_o = \frac{1}{\sqrt{LC}} \Rightarrow \boxed{\omega_o = 10k\ r/s}$$

$$V_o(j\omega_o) = 12\angle 0 \left[\frac{Z_C(j\omega_o)}{2}\right] = 6\angle 0 \left(\frac{1}{\omega_o C \angle 90}\right)$$

$$\boxed{V_o(j\omega_o) = 60\angle -90^\circ\ V}$$

12FE-2 Given the series circuit in Fig. 12PFE-2, determine the resonant frequency, and find the value of R so that the BW of the network about the resonant frequency is 200 rad/s.

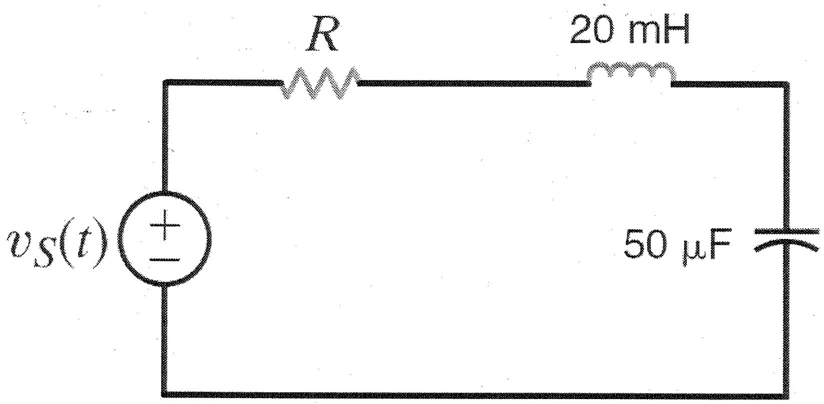


Figure 12PFE-2

SOLUTION:

$$\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow \boxed{\omega_0 = 1000 \text{ r/s}}$$

$$BW = R/L = 200 \text{ r/s} \Rightarrow \boxed{R = 4\Omega}$$

12FE-3 Given the low-pass filter circuit shown in Fig. 12PFE-3, find the frequency in Hz at which the output is down 3 dB from the dc, or very low frequency, output. CS

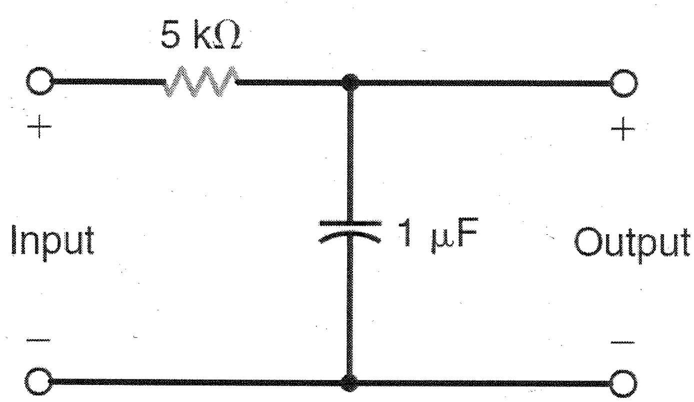


Figure 12PFE-3

SOLUTION:

$$\frac{V_o}{V_I} = \frac{Z_C}{Z_C + R} = \frac{1/j\omega C}{1/j\omega C + R} = \frac{1/RC}{j\omega + 1/RC} = G_v(j\omega)$$

at dc, $G_v = 1 = 0\text{dB}$

at 3dB down, $\omega = \frac{1}{RC} \Rightarrow \boxed{\omega = 200 \text{ r/s}}$

12FE-4 Given the band-pass filter shown in Fig. 12PFE-4, find the components L and R necessary to provide a resonant frequency of 1000 rad/s and a BW of 100 rad/s.

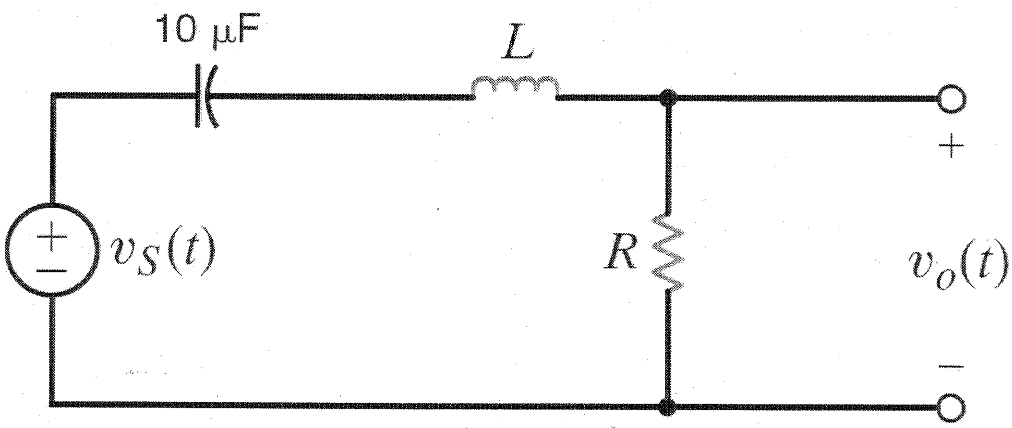


Figure 12PFE-4

SOLUTION:

$$\omega_0 = \frac{1}{\sqrt{LC}} = 1000 \rightarrow L = \frac{1}{\omega_0^2 C} \Rightarrow \boxed{L = 100\,mH}$$

$$BW = R/L = 100\ r/s \Rightarrow \boxed{R = 10\,\Omega}$$

12FE-5 Given the low-pass filter shown in Fig. 12PFE-5, find the half-power frequency and the gain of this circuit, if the source frequency is 8 Hz. CS

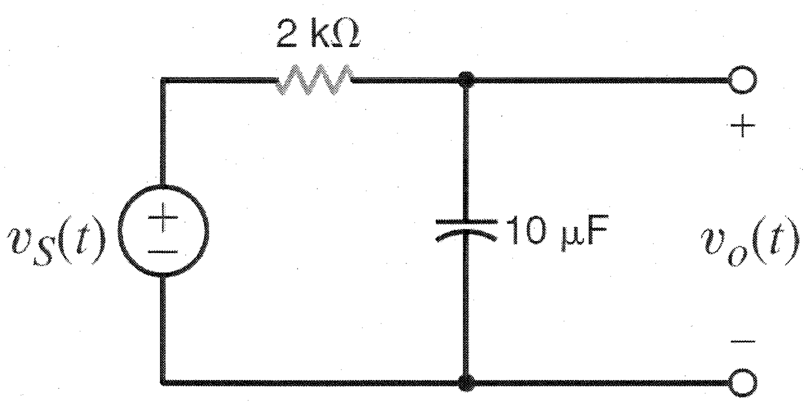


Figure 12PFE-5

SOLUTION:

$f = 8\ Hz \quad \omega_1 = 16\pi\ r/s \quad Z_C = \frac{1}{j\omega C} = -j2.0\ k\Omega$

$\frac{V_o}{V_S} = G_V(j\omega_1) = \frac{Z_C}{Z_C + R} = \frac{-j2000}{2000 - j2000} \Rightarrow \boxed{G_V(j\omega_1) = 0.707 \angle -45°}$

In general, $G_V(j\omega) = \frac{1/j\omega C}{R + 1/j\omega C} = \frac{1/RC}{j\omega + 1/RC}$

Half power freq $= f_C = \frac{1}{2\pi RC} \Rightarrow \boxed{f_C = 8.0\ Hz}$

Chapter Fourteen: Application of the LaPlace Transform to Circuit Analysis

14.1 Find the input impedance $\mathbf{Z}(s)$ of the network in Fig. P14.1. CS

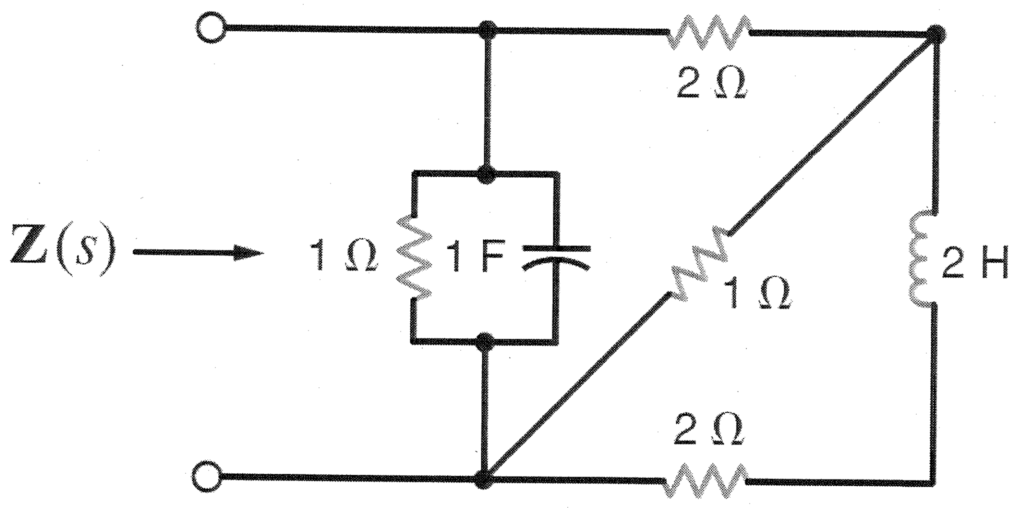
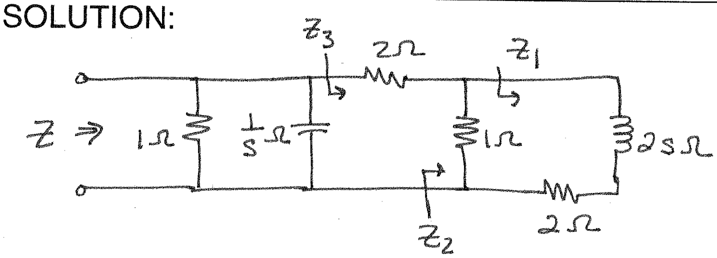


Figure P14.1

SOLUTION:



$Z_1 = 2 + 2s\ \Omega$

$Z_2 = \dfrac{Z_1}{1+Z_1} = \dfrac{2s+2}{2s+3}$

$Z_3 = 2 + Z_2 = 2 + \dfrac{2s+2}{2s+3} = \dfrac{4s+6+2s+2}{2s+3} = \dfrac{6s+8}{2s+3}$

$Z = \dfrac{1}{\frac{1}{1} + s + \frac{1}{Z_3}}$ $\qquad 1 + s + \dfrac{1}{Z_3} = s+1+\dfrac{2s+3}{6s+8} = \dfrac{6s^2+16s+11}{6s+8}$

$$\boxed{Z = \frac{6s+8}{6s^2+16s+11}}$$

14.2 Given the network in Fig. P14.2, determine the value of the output voltage as $t \to \infty$.

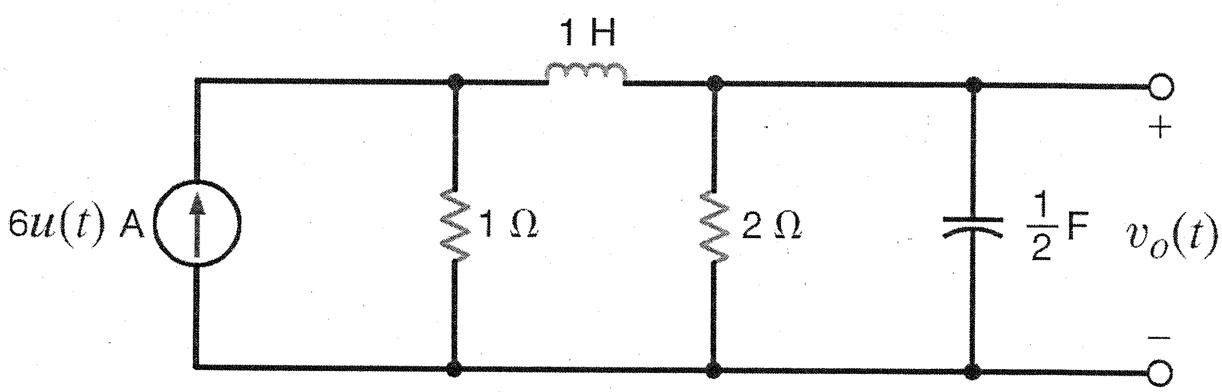


Figure P14.2

SOLUTION:

Since input is dc for $t > 0$, all voltages & currents will eventually become dc as well. thus $v_L \to 0$ & $i_C \to 0$ as $t \to \infty$.

$$v_o(\infty) = \frac{6\left((1)(2)\right)}{1+2} = \frac{6(2)}{3} = 4 \qquad \boxed{v_o(\infty) = 4V}$$

14.3 For the network shown in Fig. P14.3, determine the value of the output voltage as $t \to \infty$.

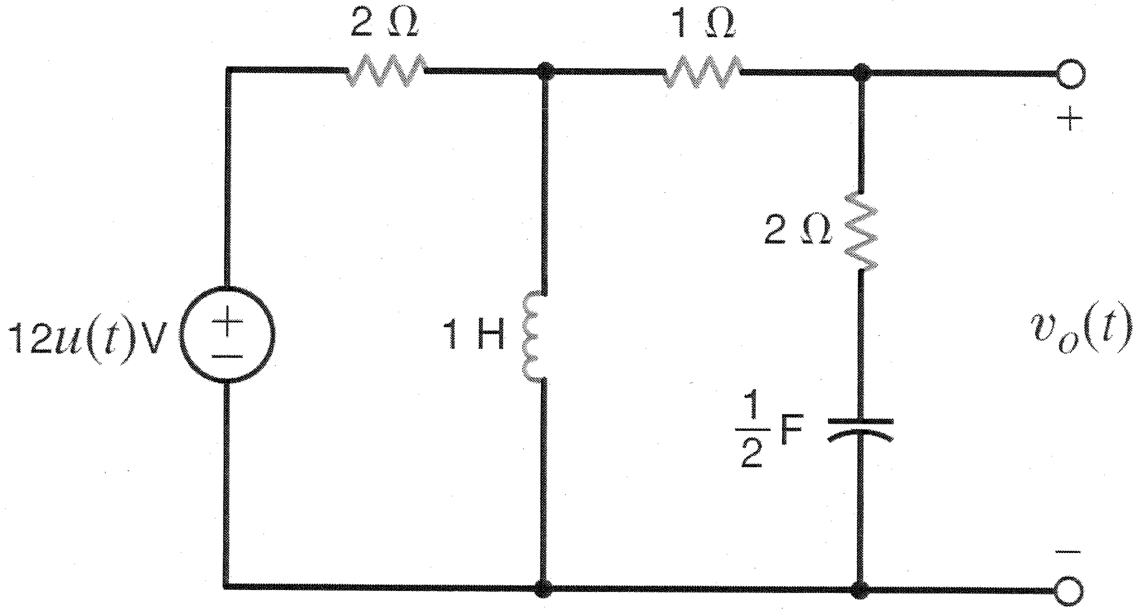
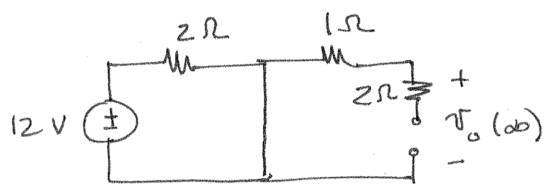


Figure P14.3

SOLUTION:

Since input is dc for $t>0$, all voltages & currents will eventually go to dc as well. Thus $v_L \to 0$ & $i_c \to 0$ as $t \to 0$.



$\boxed{v_o(\infty) = 0\text{ V}}$

14.4 Use Laplace transforms to find $v(t)$ for $t > 0$ in the network shown in Fig. P14.4. Assume zero initial conditions.

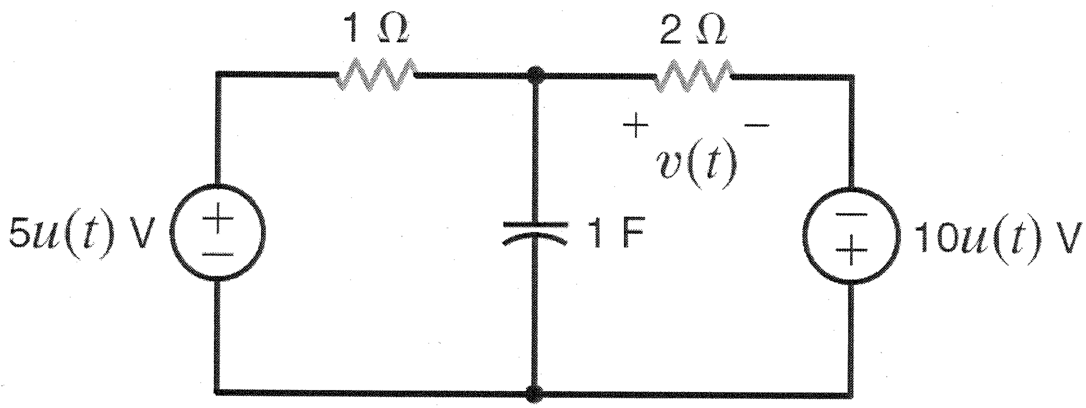
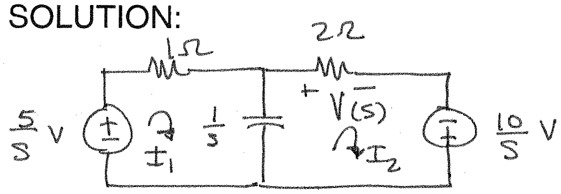


Figure P14.4

SOLUTION:



$$\frac{5}{s} = I_1(s)\left[1+\frac{1}{s}\right] - I_2(s)\left(\frac{1}{s}\right)$$

$$\frac{10}{s} = -I_1(s)\left(\frac{1}{s}\right) + I_2(s)\left[2+\frac{1}{s}\right]$$

or,

$$\left.\begin{aligned} 5 &= I_1(s+1) - I_2 \\ 10 &= -I_1 + I_2(2s+1) \end{aligned}\right\} \quad \begin{aligned} I_1 &= 5/s \\ I_2 &= 5/s \end{aligned}$$

$$V(s) = 2I_2(s) = \frac{10}{s}$$

$$\boxed{v(t) = 10\,u(t)}$$

14.5 Use Laplace transforms and nodal analysis to find $i_1(t)$ for $t > 0$ in the network shown in Fig. P14.5. Assume zero initial conditions.

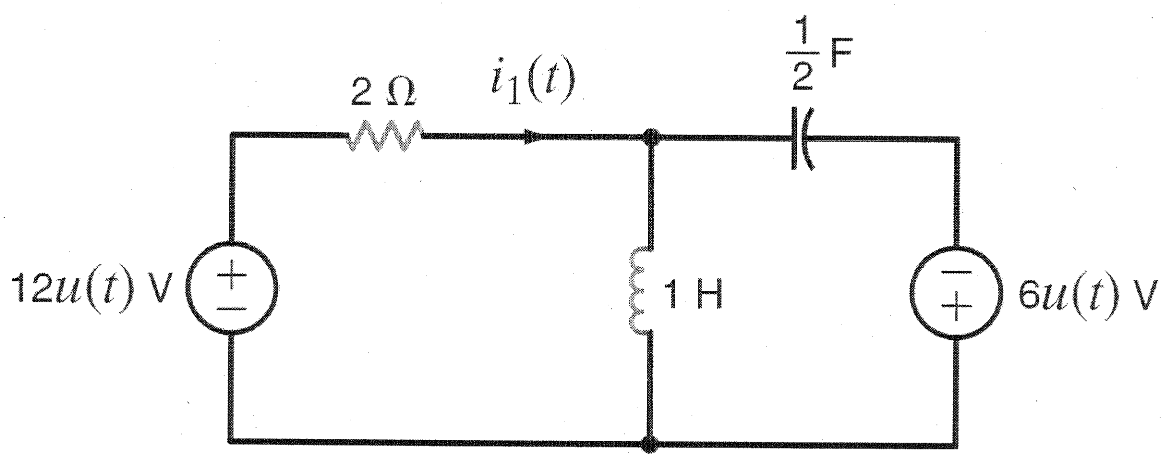
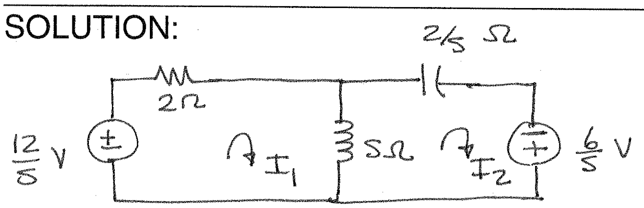


Figure P14.5

SOLUTION:



$$\frac{12}{s} = I_1[s+2] - sI_2$$

$$\frac{6}{s} = -sI_1 + I_2\left[s + \frac{2}{s}\right]$$

or, $\frac{12}{s} = I_1[s+2] - sI_2$ & $\frac{6}{s} = -sI_1 + I_2\left[\frac{s^2+2}{s}\right]$

Solve for I_1 yields $I_1(s) = \dfrac{3(3s^2+4)}{s(s^2+s+2)}$

$$I_1(s) = \frac{K_1}{s} + \frac{K_2}{s+\frac{1}{2}-j\frac{\sqrt{7}}{2}} + \frac{K_2^*}{s+\frac{1}{2}+j\sqrt{7}/2} \qquad K_1 = 6$$

$$K_2 = \left.\frac{3(3s^2+4)}{s(s+\frac{1}{2}+j\sqrt{7}/2)}\right|_{s=-\frac{1}{2}+j\sqrt{7}/2} = 3.21\ \angle 62.1^\circ$$

$$i_1(t) = \left[6 + 6.42e^{-t/2}\cos\left(\sqrt{7}t/2 + 62.1^\circ\right)\right]u(t)\ \text{V}$$

14.6 For the network shown in Fig. P14.6, find $v_o(t)$, $t > 0$, using node equations. **PSV**

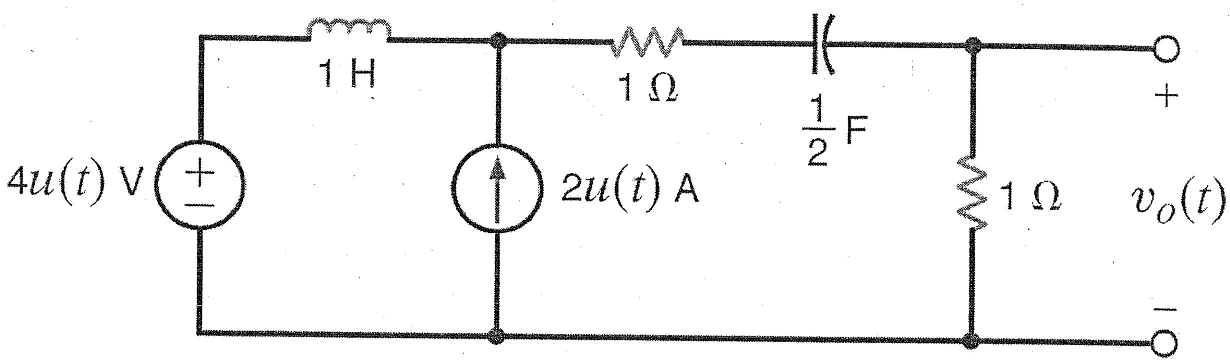
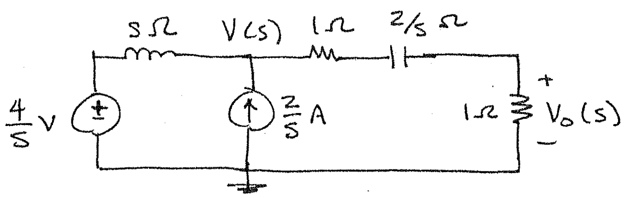


Figure P14.6

SOLUTION: at $t=0^-$, no excitation. So initial conditions $=0$.



$$\frac{V-4/s}{s}+\frac{V}{2+\frac{2}{s}}=\frac{2}{s}$$

$$\frac{V}{s}+\frac{Vs}{2(s+1)}=\frac{2}{s}+\frac{4}{s^2}$$

$$V\left[s(s+1)+\frac{s^3}{2}\right]=2s(s+1)+4(s+1)=2s^2+6s+4$$

$$\frac{V}{2}\left[s^3+2s^2+2s\right]=2(s^2+3s+2)\Rightarrow V=\frac{4(s+1)(s+2)}{s(s^2+2s+2)}$$

$$V_o=V\left[\frac{1}{2+2/s}\right]=V\left[\frac{s}{2(s+1)}\right]=\frac{2(s+2)}{(s+1-j1)(s+1+j1)}$$

$$V_o=\frac{K_1}{s+1-j1}+\frac{K_1^*}{s+1+j1}\qquad K_1=\frac{2(1+j1)}{j2}=\sqrt{2}\,\angle{-45^\circ}$$

$$v_o(t)=\left[\sqrt{2}\,e^{-t}\cos(t-45^\circ)\right]u(t)\ \checkmark$$

14.7 For the network shown in Fig. P14.7, find $i_o(t)$, $t > 0$.
CS

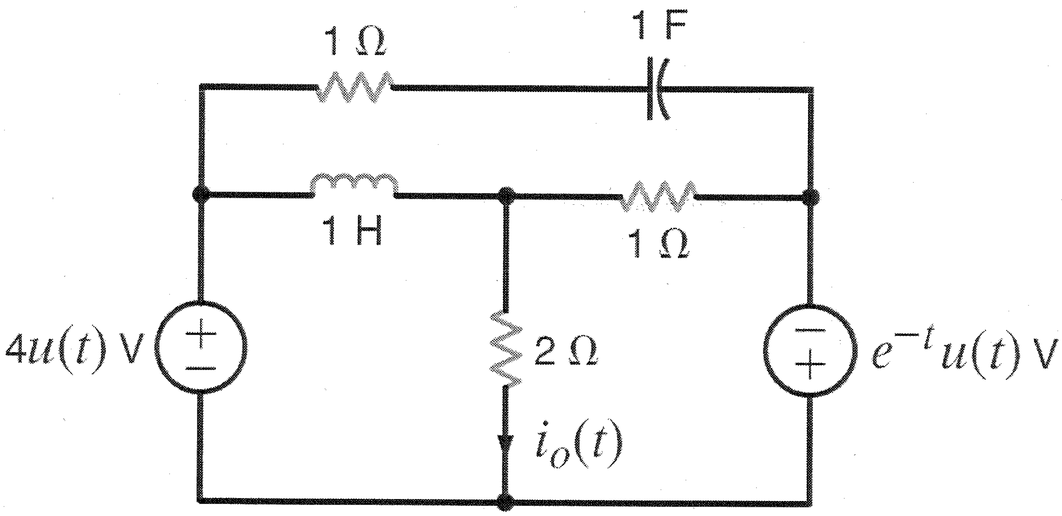
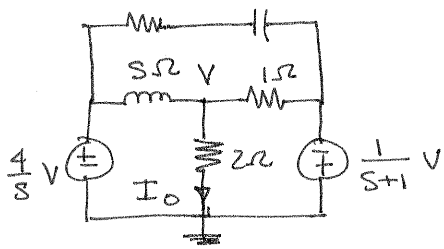


Figure P14.7

SOLUTION: $t = 0^-$: No excitation. So, all initial conditions $= 0$.



$$\frac{V - 4/s}{s} + \frac{V + \frac{1}{s+1}}{1} + \frac{V}{2} = 0$$

$$V\left(\frac{1}{s} + \frac{3}{2}\right) = \frac{4}{s^2} - \frac{1}{s+1}$$

$$V = \frac{2(-s^2 + 4s + 4)}{s(s+1)(3s+2)}$$

$$I_o = V/2 = \frac{1/3\,(-s^2+4s+4)}{s(s+1)(s+2/3)} = \frac{K_1}{s} + \frac{K_2}{s+1} + \frac{K_3}{s+2/3}$$

$K_1 = 2$ $\qquad K_2 = -1$ $\qquad K_3 = -4/3$

$$i_o(t) = \left[2 - e^{-t} - \frac{4}{3}e^{-(2/3)t}\right]u(t)$$

14.8 Find $v_o(t)$, $t > 0$, in the network in Fig. P14.8 using node equations.

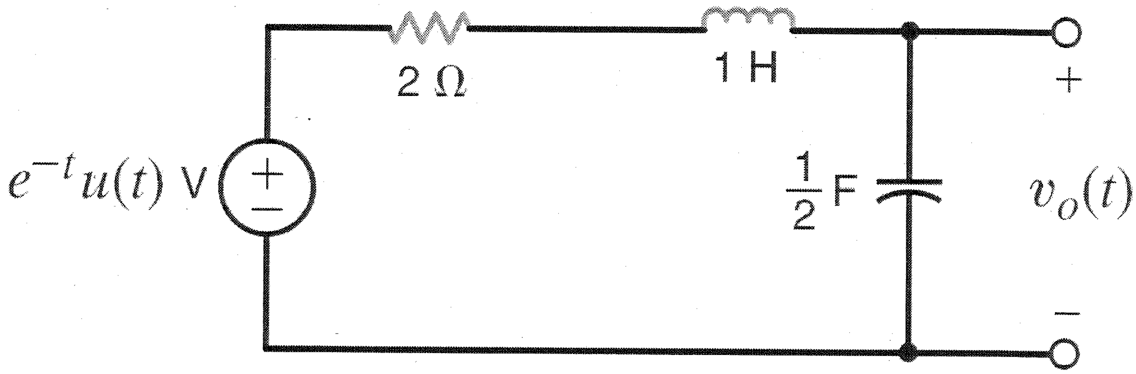
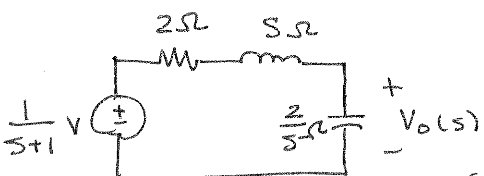


Figure P14.8

SOLUTION: $v_c(0^-) = 0V$



$$\frac{V_o(s)}{2/s} + \frac{V_o(s) - \frac{1}{s+1}}{s+2} = 0$$

$$\frac{s V_o}{2} + \frac{V_o}{s+2} - \frac{1}{(s+2)(s+1)} = 0$$

$$s(s+2)V_o + 2V_o = \frac{2}{s+1} = V_o\left[s^2+2s+2\right]$$

$$V_o = \frac{2}{(s+1)(s^2+2s+2)} = \frac{K_1}{s+1} + \frac{K_2}{s+1-j1} + \frac{K_2^*}{s+1+j1}$$

$$K_1 = 2 \qquad K_2 = \frac{2}{(j1)(j2)} = -1 \qquad K_2^* = -1$$

$$V_o = \frac{2}{s+1} - \frac{1}{s+1-j1} - \frac{1}{s+1+j1}$$

$$\boxed{v_o(t) = \left[2e^{-t} - 2e^{-t}\cos(t)\right]u(t)}$$

14.9 Find $v_o(t)$, $t > 0$, in the network shown in Fig. P14.9 using nodal analysis. CS

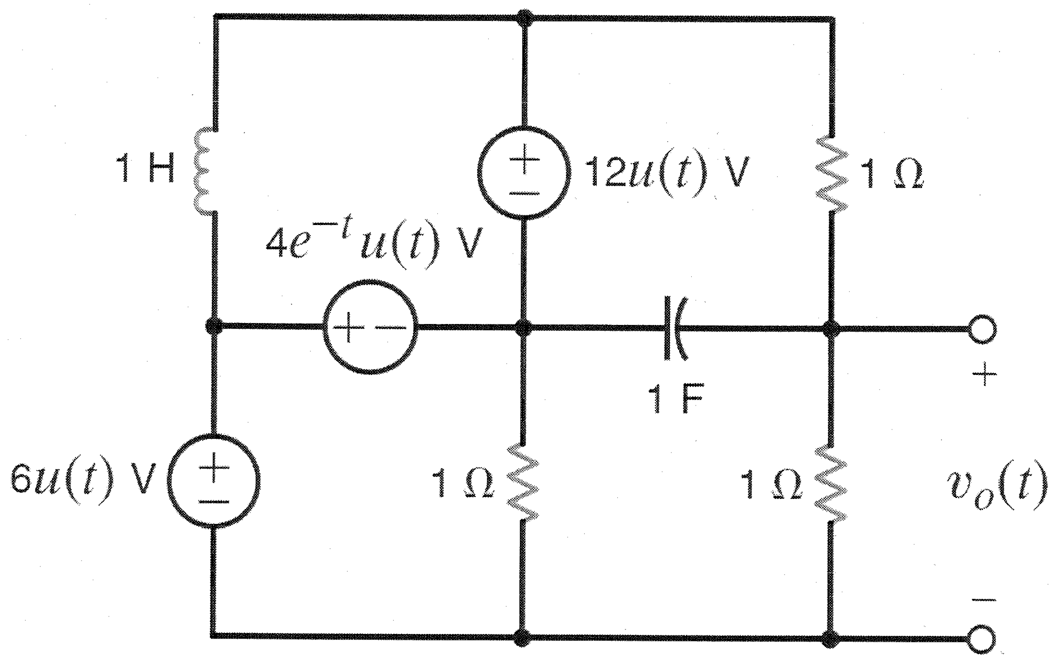
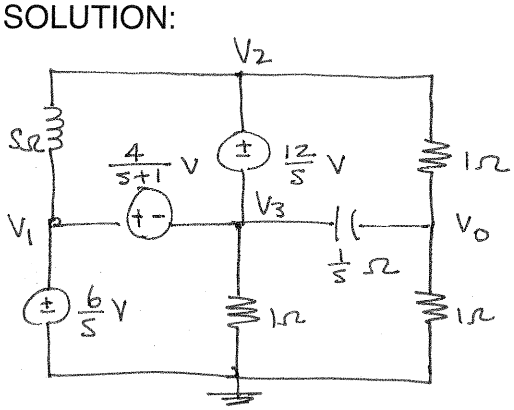


Figure P14.9

SOLUTION:



$V_1 = \frac{6}{s}$ $\quad V_1 - V_3 = \frac{4}{s+1} \Rightarrow V_3 = \frac{6}{s} - \frac{4}{s+1}$

$V_2 - V_3 = \frac{12}{s} \Rightarrow V_2 = \frac{12}{s} + V_3 = \frac{18}{s} - \frac{4}{s+1}$

$\frac{V_0 - V_2}{1} + s(V_0 - V_3) + \frac{V_0}{1} = 0$

$V_0(s+2) = V_2 + sV_3$

$V_0(s+2) = \frac{2(s+9)}{s} \Rightarrow V_0(s) = \frac{2(s+9)}{s(s+2)} = \frac{K_1}{s} + \frac{K_2}{s+2}$

$K_1 = 9 \qquad K_2 = -7$

$$\boxed{v_0(t) = [9 - 7e^{-2t}]u(t)}$$

14.10 Use nodal analysis to find $v_o(t)$, $t > 0$, in the network in Fig. P14.10. **PSV**

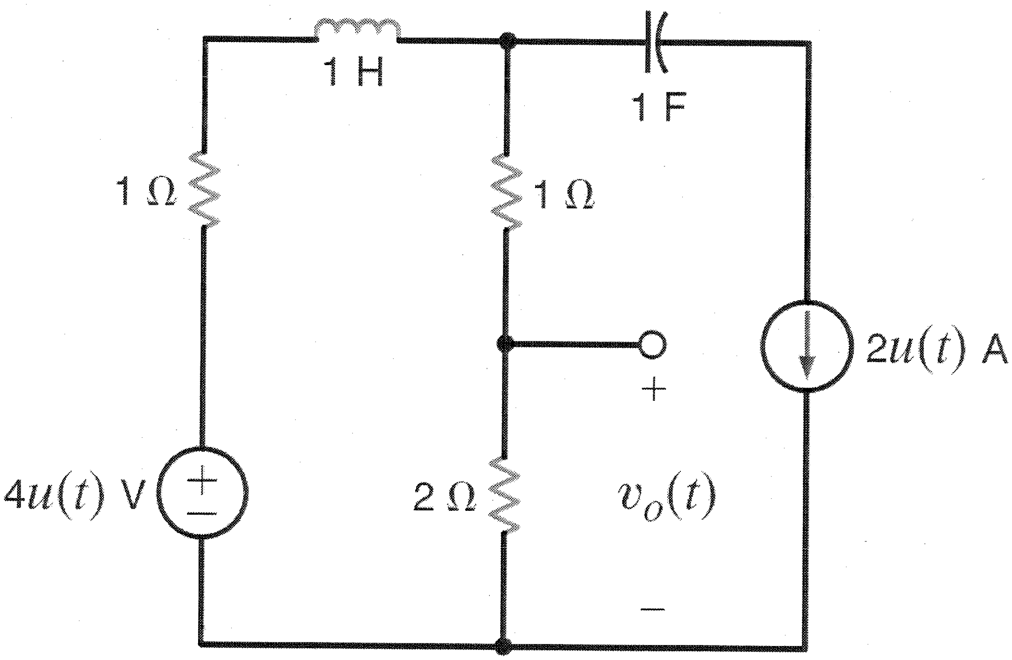
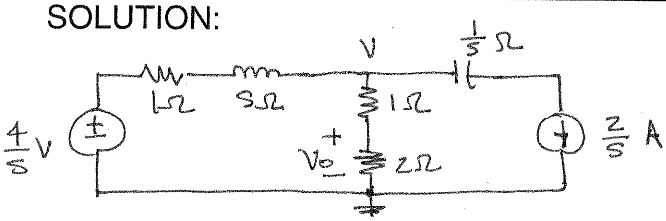


Figure P14.10

SOLUTION:



$$\frac{V - 4/s}{s+1} + \frac{V}{3} + \frac{2}{s} = 0$$

$$V\left[\frac{1}{s+1} + \frac{1}{3}\right] = \frac{4}{s(s+1)} - \frac{2}{s}$$

$$V\left[\frac{3 + s+1}{3(s+1)}\right] = \frac{-2s+2}{s(s+1)} \Rightarrow V(s) = \frac{6(-s+1)}{s(s+4)} \qquad V_o = \frac{2}{3}V$$

$$V_o(s) = \frac{4(-s+1)}{s(s+4)} = \frac{1}{s} - \frac{5}{s+4}$$

$$\boxed{v_o(t) = [1 - 5e^{-4t}]u(t)}$$

14.11 For the network shown in Fig. P14.11, find $v_o(t)$, $t > 0$, using loop equations.

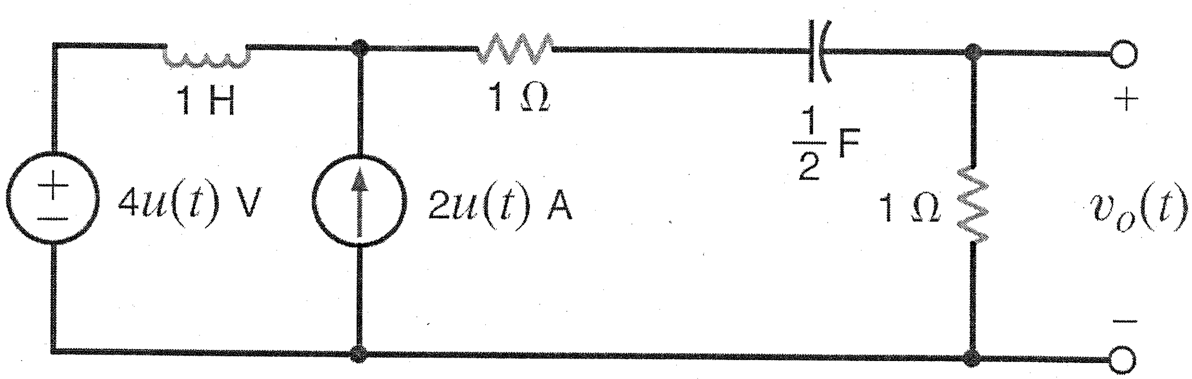
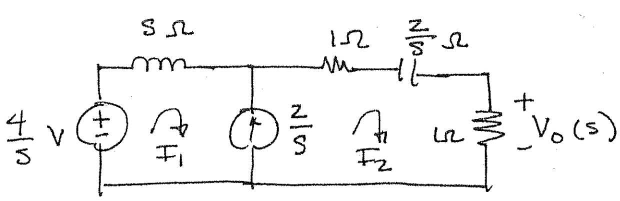


Figure P14.11

SOLUTION:



$\frac{4}{s} = sI_1 + \left(2 + \frac{2}{s}\right) I_2$

or, $4 = s^2 I_1 + (2s+2) I_2$

and, $I_2 - I_1 = 2/s \Rightarrow I_1 = I_2 - \frac{2}{s}$

$4 = s^2 I_2 - 2s + (2s+2)I_2 = I_2(s^2+2s+2) - 2s$

$I_2 = \dfrac{2s+4}{s^2+2s+2}$ $\qquad V_o = (1)I_2 = \dfrac{2(s+2)}{(s+1-j1)(s+1+j1)} = \dfrac{K_1}{s+1-j1} + \dfrac{K_1^*}{s+1+j1}$

$K_1 = \dfrac{2(-1+j1+2)}{j2} = \sqrt{2}\,\angle -45°$

$$\boxed{v_o(t) = \left[2\sqrt{2}\, e^{-t} \cos(t - 45°)\right] u(t) \text{ V}}$$

14.12 For the network shown in Fig. P14.12, find $v_o(t)$, $t > 0$, using mesh equations.

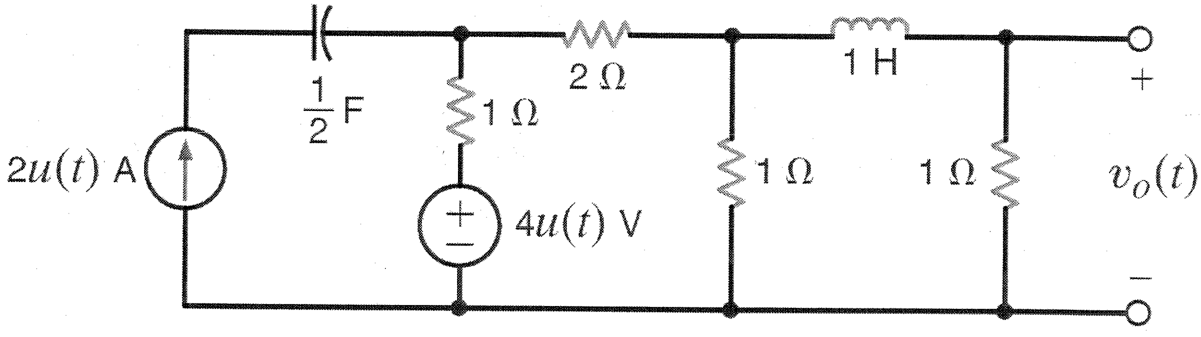
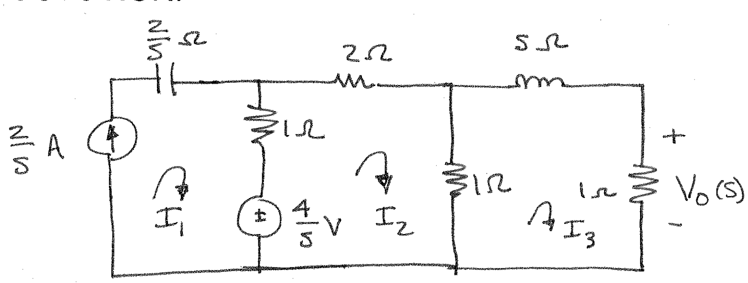


Figure P14.12

SOLUTION:



$$I_1 = \frac{2}{s}\text{ A}$$

$$\frac{4}{s} = -I_1 + I_2(4) - I_3$$

$$0 = -I_2 + I_3(s+2)$$

$$\frac{4}{s} = -\frac{2}{s} + 4I_2 - I_3 \Rightarrow \frac{6}{s} = 4I_2 - I_3$$

and

$$0 = -I_2 + I_3(s+2)$$

$$\left.\right\} \quad I_3(s)\left[4(s+2)-1\right] = 6/s$$

$$I_3(s) = \frac{6}{s(4s+7)} \qquad V_o = (1)\,I_3 = \frac{3/2}{s(s+7/4)}$$

$$V_o(s) = \frac{K_1}{s} + \frac{K_2}{s+7/4} \qquad K_1 = \left(\frac{3}{2}\right)\left(\frac{4}{7}\right) = \frac{6}{7} \qquad K_2 = \frac{3}{2}\left(-\frac{4}{7}\right) = -\frac{6}{7}$$

$$V_o(s) = \frac{6}{7}\left[\frac{1}{s} - \frac{1}{s+7/4}\right] \qquad \boxed{v_o(t) = \left[\frac{6}{7}\left(1 - e^{-1.75t}\right)\right]u(t)}$$

14.13 Use mesh equations to find $v_o(t)$, $t > 0$, in the network in Fig. P14.13. **cs**

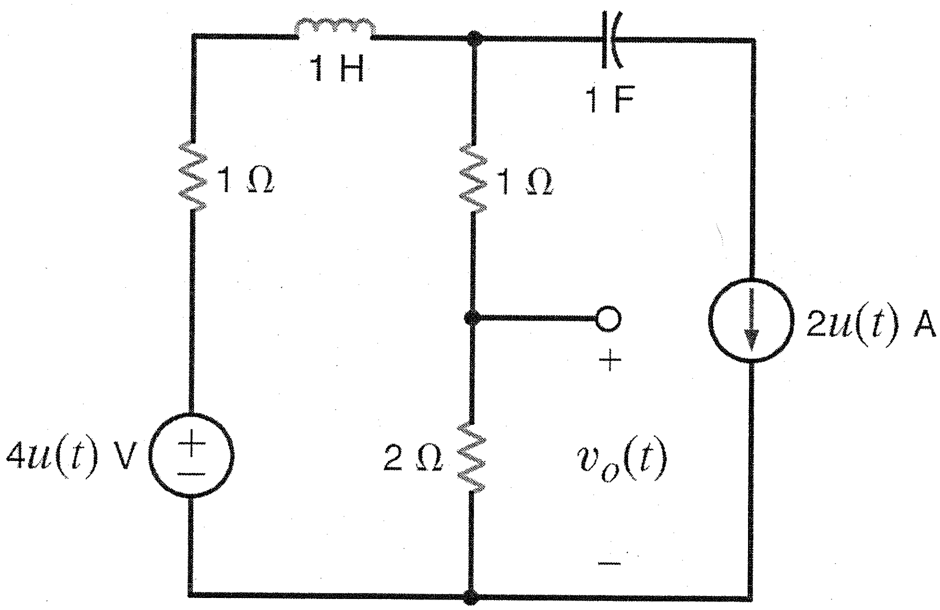
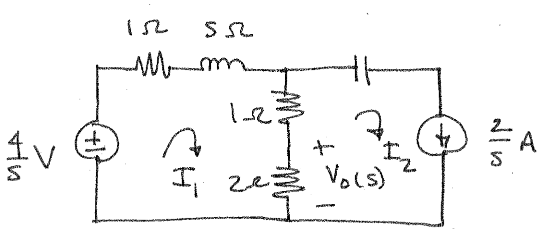


Figure P14.13

SOLUTION:



$$\frac{4}{s} = I_1(s+4) - 3I_2 \qquad I_2 = 2/s$$

$$\frac{4}{s} = I_1(s+4) - \frac{6}{s}$$

$$\frac{10}{s} = I_1(s+4)$$

$$I_1 = \frac{10}{s(s+4)}$$

$$V_o = 2(I_1 - I_2) = 2\left[\frac{10}{s(s+4)} - \frac{2}{s}\right] = \frac{4(-s+1)}{s(s+4)} = \frac{1}{s} - \frac{5}{s+4}$$

$$\boxed{v_o(t) = \left[1 - 5e^{-4t}\right]u(t) \text{ V}}$$

14.14 Use loop equations to find $i_o(t)$, $t > 0$, in the network shown in Fig. P14.14.

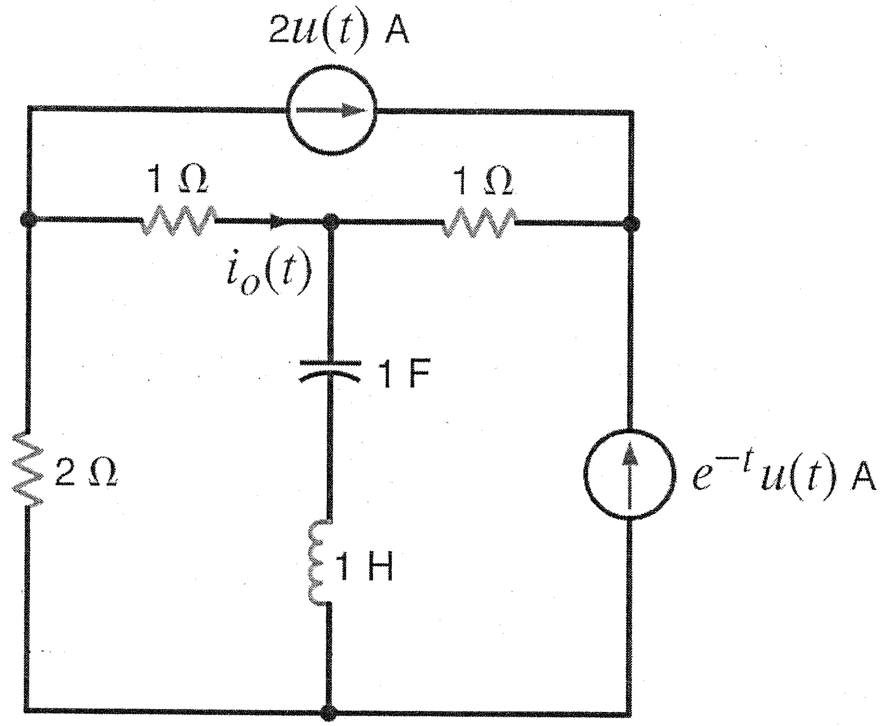
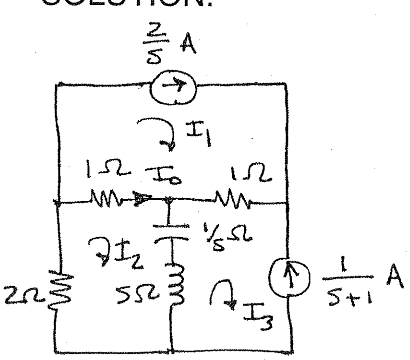


Figure P14.14

SOLUTION:



$$I_1 = \frac{2}{s}\text{ A} \quad \& \quad I_3 = -\frac{1}{s+1}\text{ A}$$

$$I_2\left(s + 3 + 1/s\right) - I_1(1) - I_3\left(s + \frac{1}{s}\right) = 0$$

$$\text{or, } I_2\left(s^2+3s+1\right) = sI_1 + \left(s^2+1\right)I_3$$

$$I_2\left(s^2+3s+1\right) = 2 - \frac{s^2+1}{s+1} = \frac{-s^2+2s+1}{s+1}$$

$$I_2 = \frac{-s^2+2s+1}{\left(s^2+3s+1\right)(s+1)}$$

$$I_o = I_2 - I_1 = \frac{-s^2+2s+1}{s^2+3s+1} - \frac{2}{s} = \frac{-\left(3s^3+6s^2+7s+2\right)}{s(s+1)\left(s^2+3s+1\right)}$$

$$I_o(s) = \frac{-\left(3s^3+6s^2+7s+2\right)}{s(s+1)(s+0.382)(s+2.62)} = \frac{K_1}{s} + \frac{K_2}{s+1} + \frac{K_3}{s+0.382} + \frac{K_4}{s+2.62}$$

$$K_1 = \frac{-2}{(1)(1)} = -2$$

$$K_2 = \frac{-(-3+6-7+2)}{(-1)(1-3+1)} = 2$$

$$K_3 = \left.\frac{-(3s^3+6s^2+7s+2)}{s(s+1)(s+2.62)}\right|_{s=-0.382} = 0.065$$

$$K_4 = \left.\frac{-(3s^2+6s^2+7s+2)}{s(s+1)(s+0.382)}\right|_{s=2.62} = -3.065$$

$$i_o(t) = \left[2 + 2e^{-t} + 0.065e^{-0.382t} - 3.065e^{-2.62t}\right]u(t)$$

14.15 Use loop analysis to find $v_o(t)$ for $t > 0$ in the network in Fig. P14.15.

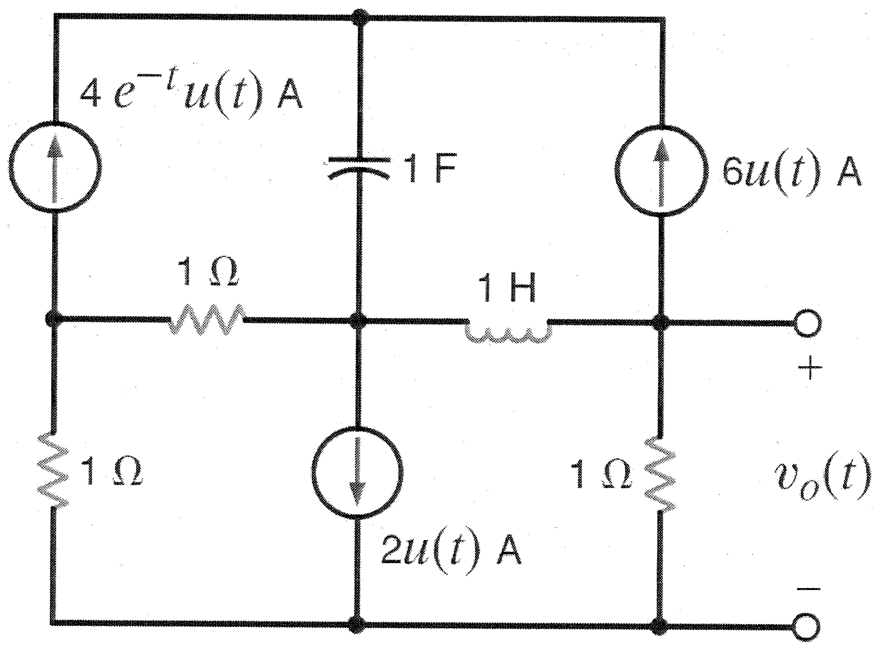
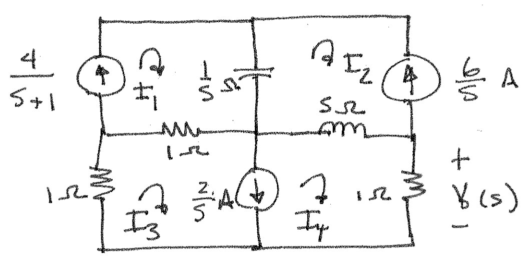


Figure P14.15

SOLUTION:



$I_1 = \frac{4}{s+1} \qquad I_2 = -\frac{6}{s} \qquad I_3 - I_4 = \frac{2}{s}$

$I_3(2) - I_1 + I_4(s+1) - s I_2 = 0$

$V_o = (1) I_4 \qquad I_3 = \frac{2}{s} + I_4$

$$2\left[\frac{2}{s} + I_4\right] - \frac{4}{s+1} + I_4(s+1) + 6 = 0 \Rightarrow I_4(s+3) = \frac{4}{s+1} - \frac{4}{s} - 6$$

$$I_4(s+3) = \frac{4s - 4s - 4 - 6s^2 - 6s}{s(s+1)} = -\frac{(6s^2+6s+4)}{s(s+1)}$$

$$V_o = \frac{-(6s^2+6s+4)}{s(s+1)(s+3)} = \frac{-4/3}{s} + \frac{2}{s+1} - \frac{20/3}{s+3}$$

$$v_o(t) = \left[2e^{-t} - 4/3 - \tfrac{20}{3} e^{-3t}\right] u(t)$$

14.16 Use mesh analysis to find $v_o(t)$, $t > 0$, in the network in Fig. P14.16. CS

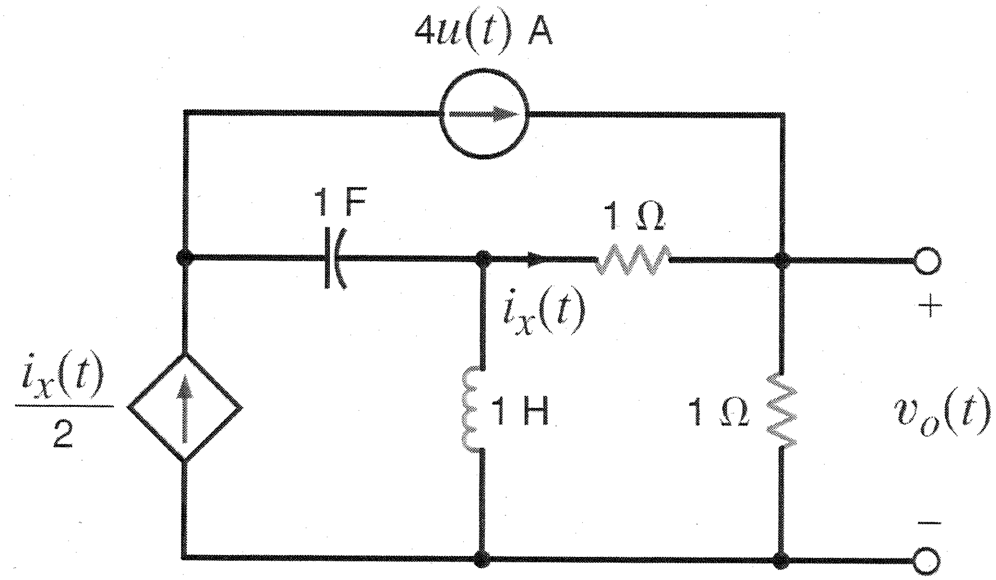
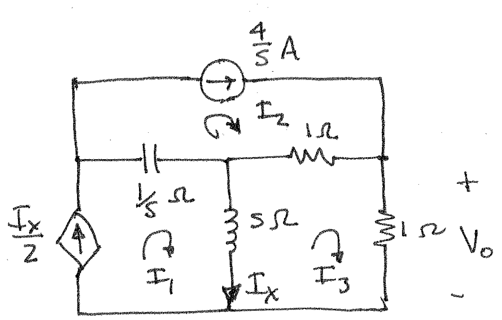


Figure P14.16

SOLUTION:



$$I_1 = \frac{I_x}{2} = \frac{I_1 - I_3}{2} \Rightarrow I_1 = -I_3$$

$$I_2 = 4/s$$

$$I_3(s+2) - sI_1 - I_2 = 0$$

$$V_o = (1)I_3$$

$$I_3(s+2) + sI_3 = 4/s \Rightarrow I_3 = \frac{4}{s(2s+2)} = \frac{2}{s(s+1)}$$

$$V_o = \frac{2}{s(s+1)} = \frac{2}{s} - \frac{2}{s+1}$$

$$\boxed{v_o(t) = [2(1-e^{-t})]u(t)}$$

14.17 Use superposition to find $v_o(t)$, $t > 0$, in the network shown in Fig. P14.17.

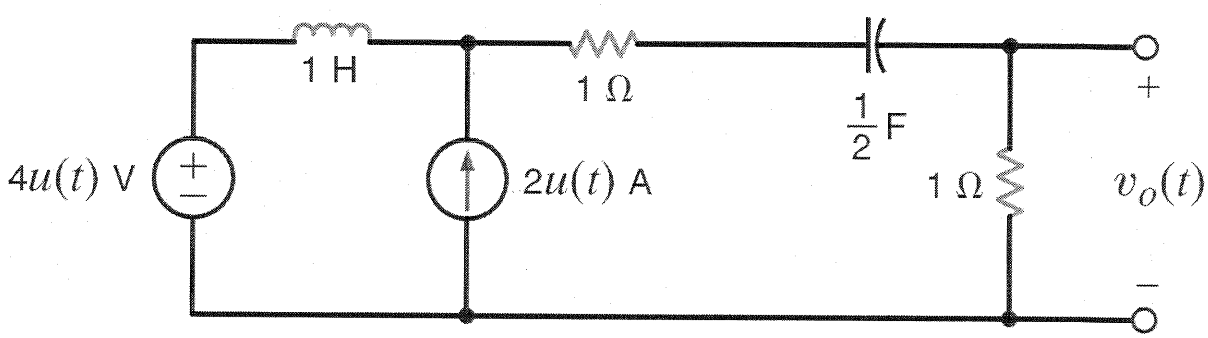
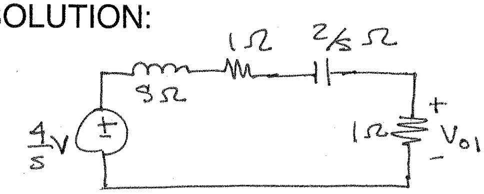
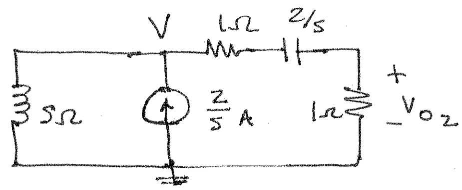


Figure P14.17

SOLUTION:





$$V_{01} = \frac{4}{s}\left[\frac{1}{s+1+\frac{2}{s}+1}\right]$$

$$\frac{V}{s} + \frac{V}{2+2/s} = \frac{2}{s} \Rightarrow V\left[1 + \frac{s^2}{2(s+1)}\right] = 2$$

$$V_{01} = \frac{4}{s^2+2s+2}$$

$$V = \frac{4(s+1)}{s^2+2s+2} \qquad \frac{V_{o2}}{V} = \frac{1}{2+\frac{2}{s}} = \frac{s}{2(s+1)}$$

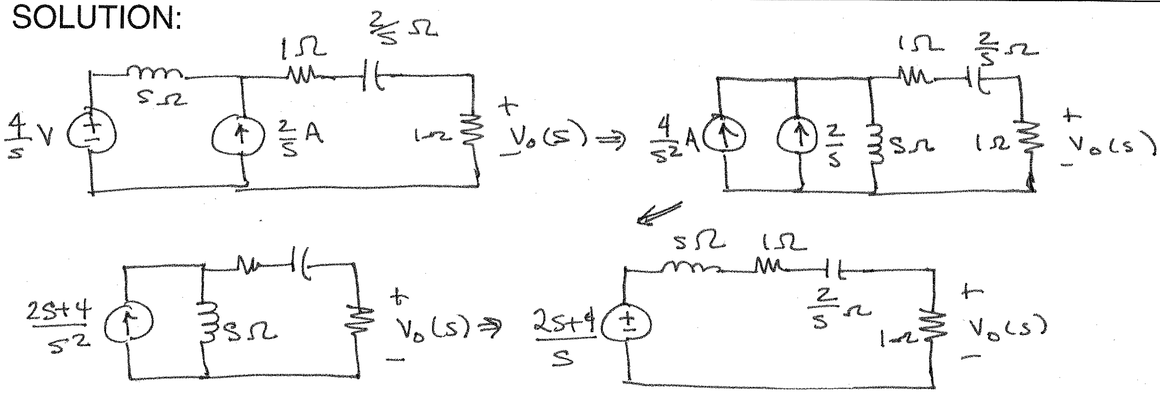
$$V_{o2} = \frac{2s}{s^2+2s+2}$$

$$V_o = V_{01}+V_{o2} = \frac{2(s+2)}{s^2+2s+2} = \frac{K_1^*}{s+1+j1} + \frac{K_1}{s+1-j1} \qquad K_1 = \sqrt{2}\,\angle -45^\circ$$

$$\boxed{v_o(t) = \left[2\sqrt{2}\, e^{-t} \cos(t-45^\circ)\, V\right] u(t)}$$

14.18 Use source transformation to solve Problem 14.17.

SOLUTION:



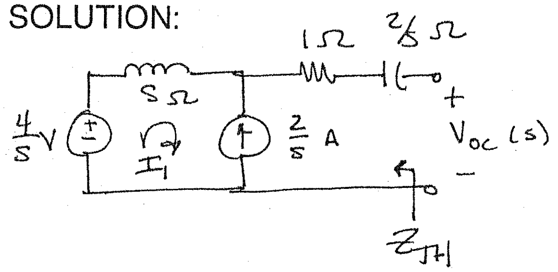
$$V_o(s) = \frac{2s+4}{s}\left[\frac{1}{s+1+\frac{2}{s}+1}\right] = \frac{2s+4}{s}\left(\frac{s}{s^2+2s+2}\right) = \frac{2(s+2)}{s^2+2s+2}$$

$$V_o = \frac{K_1}{s+1-j1} + \frac{K_1^*}{s+1+j1} \qquad K_1 = \sqrt{2}\angle{-45^\circ}$$

$$\boxed{v_o(t) = \left[2\sqrt{2}\, e^{-t}\cos(t-45^\circ)\right]u(t) \ \text{V}}$$

14.19 Use Thévenin's theorem to solve Problem 14.17.
CS

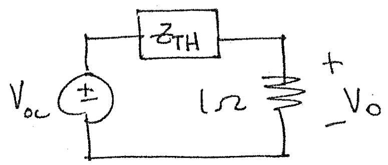
SOLUTION:



$$\frac{4}{s} = sI_1 + V_{oc} \qquad I_1 = -2/s$$

$$V_{oc} = \frac{4}{s} + 2 = \frac{2s+4}{s}$$

$$Z_{TH} = s + 1 + \frac{2}{s} = \frac{s^2+s+2}{s}$$



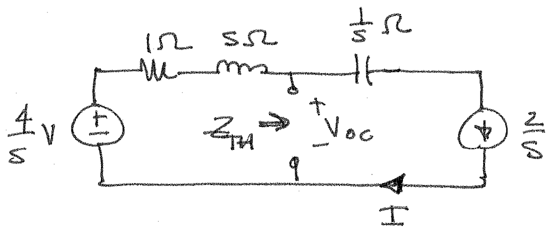
$$V_o = V_{oc}\left[\frac{1}{1+Z_{TH}}\right] = \frac{2(s+2)}{s^2+2s+2}$$

$$V_o = \frac{K_1}{s+1-j1} + \frac{K_1^*}{s+1+j1} \qquad K_1 = \sqrt{2}\angle{-45^\circ}$$

$$\boxed{v_o(t) = \left[2\sqrt{2}\, e^{-t}\cos(t-45^\circ)\right]u(t) \quad \text{V}}$$

14.20 Use Thévenin's theorem to solve Problem 14.13.

SOLUTION:

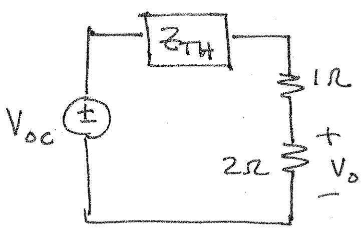


$$I = \frac{2}{s}$$

$$\frac{4}{s} = (1)I + sI + V_{oc}$$

$$V_{oc} = \frac{4}{s} - \frac{2}{s} - 2 = \frac{2}{s} - 2 = \frac{(-s+1)2}{s}$$

$$Z_{TH} = s+1\ \Omega$$



$$V_o = \frac{V_{oc}(2)}{2+1+Z_{TH}} = \frac{4(-s+1)}{s\left[3+s+1\right]} = \frac{4(-s+1)}{s(s+4)}$$

$$V_o = \frac{1}{s} - \frac{5}{s+4}$$

$$\boxed{v_o(t) = \left[1 - 5e^{-4t}\right]u(t)\ \text{V}}$$

14.21 Use Thévenin's theorem to find $v_o(t)$, $t > 0$, in the network in Fig. P14.21. CS

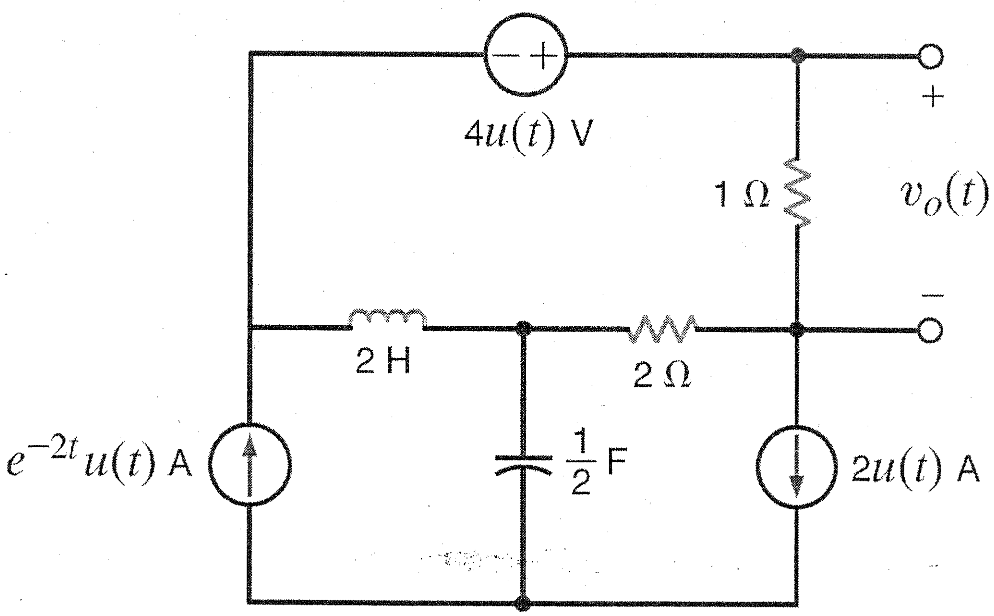
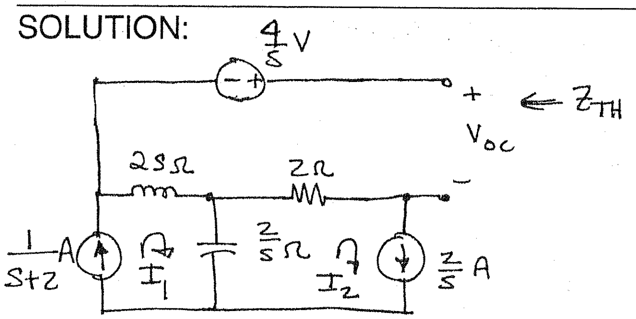


Figure P14.21

SOLUTION:



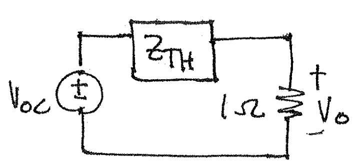
$I_1 = \frac{1}{s+2}$ $I_2 = \frac{2}{s}$

$\frac{4}{s} = V_{oc} - 2I_2 - 2sI_1$

$V_{oc} = \frac{4}{s} + \frac{4}{s} + \frac{2s}{s+2} = \frac{8}{s} + \frac{2s}{s+2}$

$V_{oc} = \frac{2s^2+8s+16}{s(s+2)}$

$Z_{TH} = 2s+2$



$$V_o = \frac{V_{oc}(1)}{1+Z_{TH}} = \frac{2(s^2+4s+8)}{s(s+2)(s+1.5)2} = \frac{s^2+4s+8}{s(s+1.5)(s+2)} = \frac{8/3}{s} - \frac{17/3}{s+1.5} + \frac{4}{s+2}$$

$$\boxed{v_o(t) = \left[\frac{8}{3} - \frac{17}{3}e^{-1.5t} + 4e^{-2t}\right]u(t) \text{ V}}$$

14.22 Find $v_o(t)$, for $t > 0$, in the network in Fig. P14.22 using Thévenin's theorem.

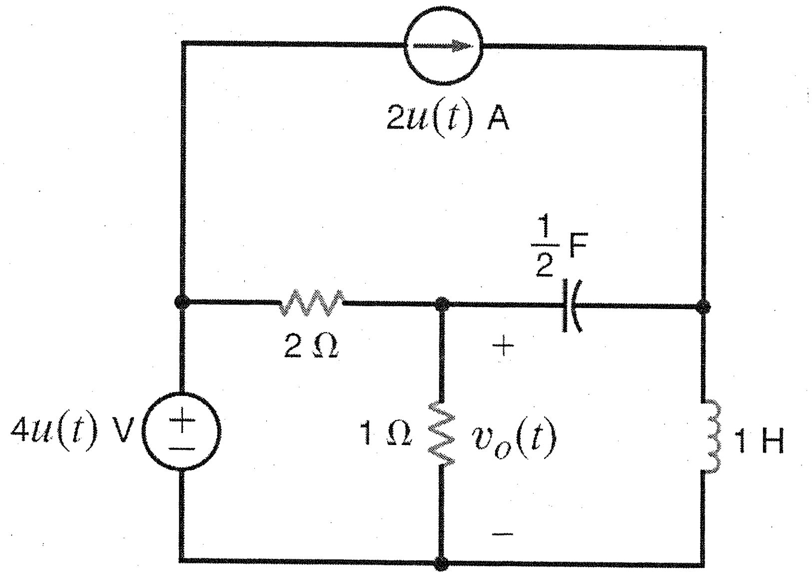
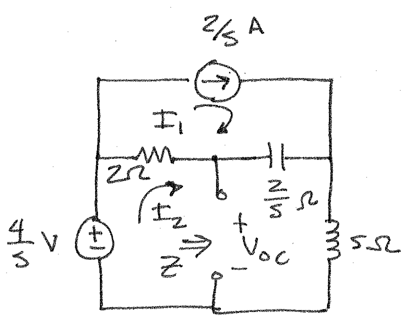


Figure P14.22

SOLUTION:



$$I_1 = \frac{2}{s} \quad ; \quad \frac{4}{s} = I_2\left[2 + \frac{2}{s} + s\right] - I_1\left[2 + \frac{2}{s}\right]$$

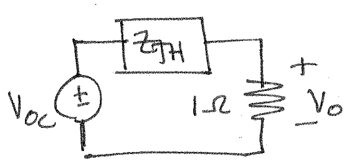
or, $$\frac{4}{s} = I_2\left(\frac{s^2+2s+2}{s}\right) - \frac{2}{s}\left(\frac{2s+2}{s}\right)$$

$$I_2 = \frac{8s+4}{s(s^2+2s+2)}$$

$$V_{oc} = \frac{4}{s} - 2(I_2 - I_1) = \frac{8s^2+8}{s(s^2+2s+2)}$$

$$Z = \frac{2\left(s+\frac{2}{s}\right)}{2+s+2/s} = \frac{2s^2+4}{s^2+2s+2}$$

$$V_o = \frac{V_{oc}(1)}{1+Z_{TH}} = \frac{(8/3)(s^2+1)}{s(s^2+\frac{2}{3}s+2)}$$



$$V_o = \frac{4/3}{s} + \frac{K_1}{s+\frac{1}{3}-j\frac{\sqrt{17}}{3}} + \frac{K_1^*}{s+\frac{1}{3}+j\sqrt{17}/3}$$

$$K_1 = 0.825 \angle 36.0°$$

$$v_o(t) = \left[\frac{4}{3} + 1.65\, e^{-t/3} \cos\left(\frac{\sqrt{17}}{3}t + 36°\right)\right]u(t) \text{ V}$$

14.23 Use Thévenin's theorem to determine $i_o(t)$, $t > 0$, in the circuit shown in Fig. P14.23. **PSV**

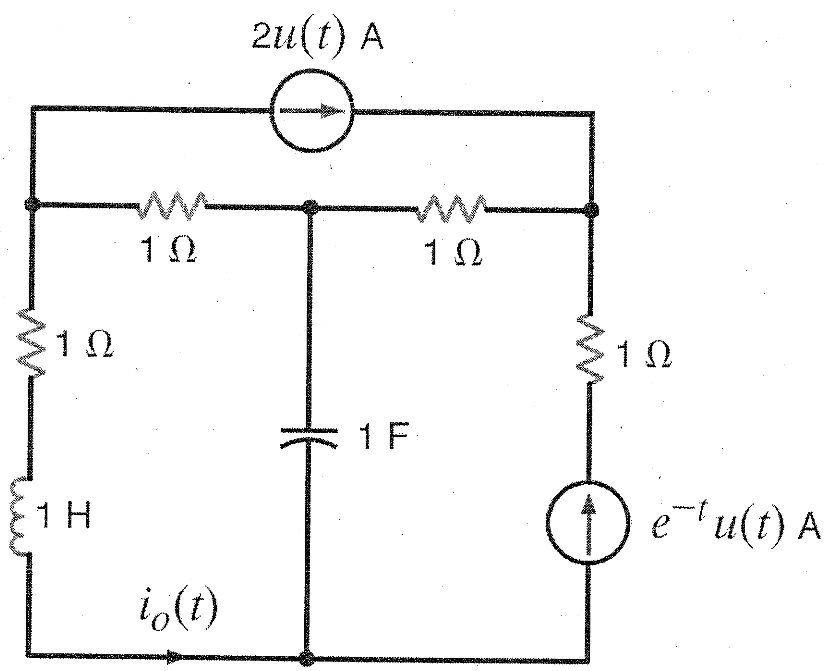
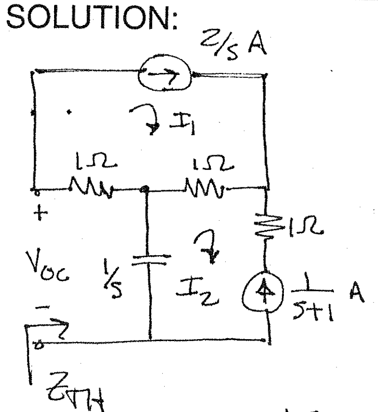


Figure P14.23

SOLUTION:



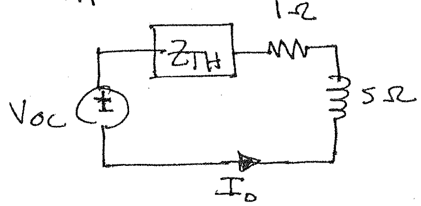
$I_1 = \frac{2}{s}$ $\qquad I_2 = -\frac{1}{s+1}$

$$V_{oc} = (1)(-I_1) - \frac{1}{s} I_2 = -\frac{2}{s} + \frac{1}{s(s+1)} = \frac{-(2s+1)}{s(s+1)}$$

$$Z = 1 + 1/s = (s+1)/s$$

$$I_o = \frac{-V_{oc}}{Z+1+s} = \frac{(2s+1)}{(s+1)^3}$$

$$I_o = \frac{K_1}{(s+1)^3} + \frac{K_2}{(s+1)^2} + \frac{K_3}{s+1} = \frac{-1}{(s+1)^3} + \frac{2}{(s+1)^2}$$



$$\boxed{i_o(t) = \left[2te^{-t} - \frac{1}{2}t^2 e^{-t}\right] u(t) \text{ A}}$$

14.24 Use Thévenin's theorem to find $v_o(t)$, $t > 0$, in the network in Fig. P14.24.

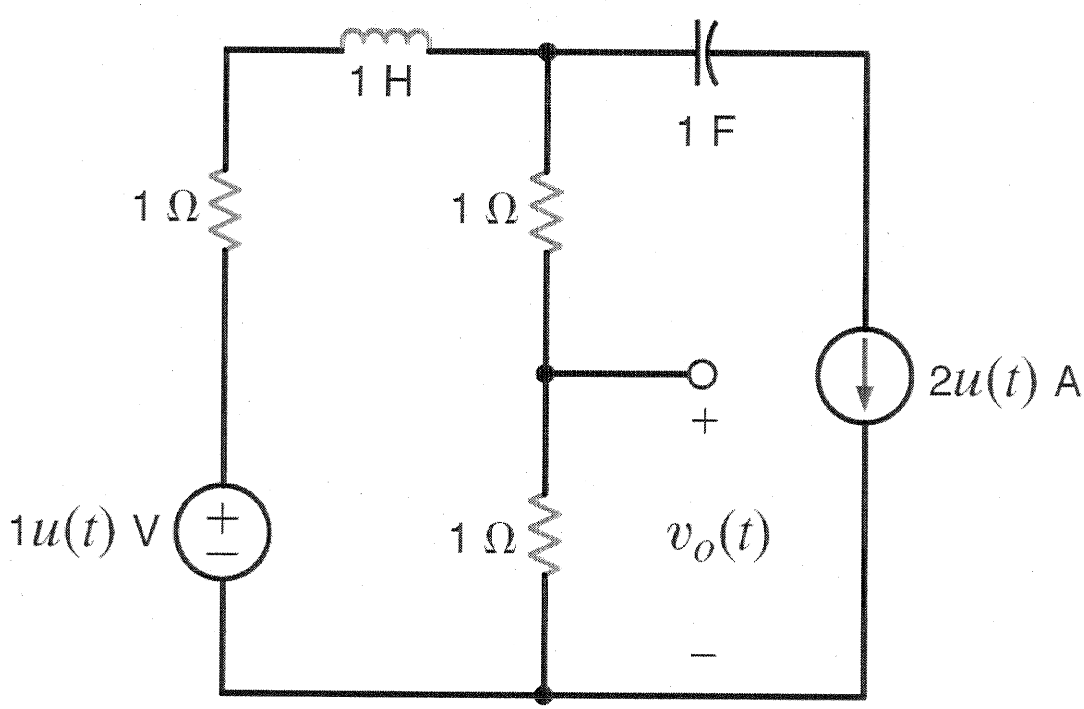
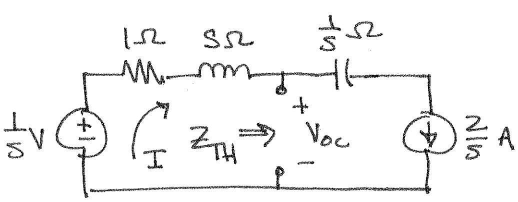


Figure P14.24

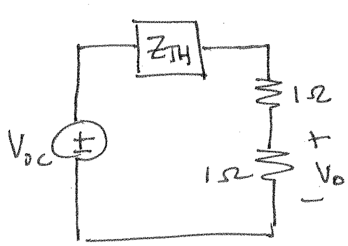
SOLUTION:



$I = \frac{2}{s} A \qquad \frac{1}{s} = (s+1)I + V_{OC}$

So, $V_{OC} = \frac{-(2s+1)}{s}$

$Z_{TH} = s + 1\Omega$



$$V_o = \frac{V_{OC}(1)}{2 + Z_{TH}} = \frac{-(2s+1)}{s} \cdot \frac{1}{s+3} = \frac{-(2s+1)}{s(s+3)}$$

$$V_o = \frac{-1/3}{s} - \frac{5/3}{s+3}$$

$$\boxed{v_o(t) = \left[-\frac{1}{3} - \frac{5}{3}e^{-3t}\right]u(t)\ V}$$

14.25 Use Thévenin's theorem to find $v_o(t)$, $t > 0$, in the network shown in Fig. P14.25. **PSV**

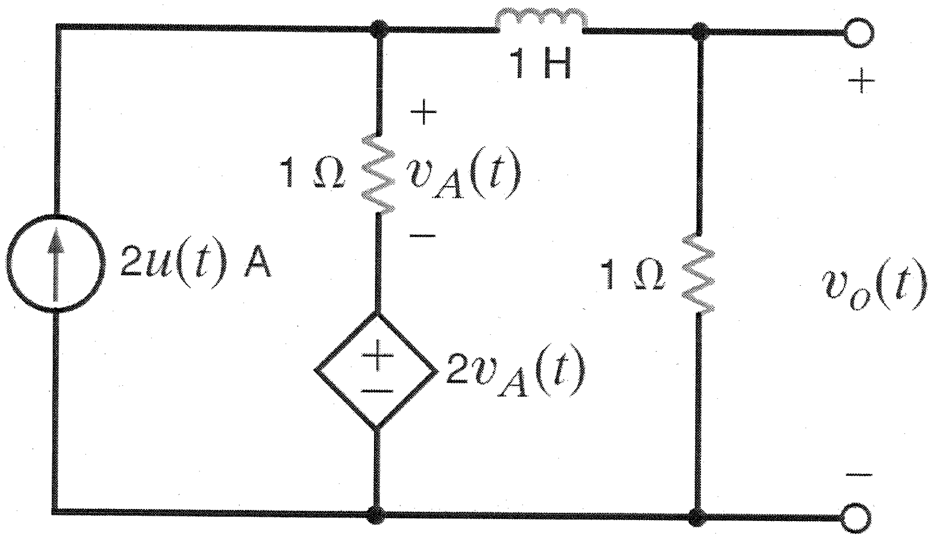
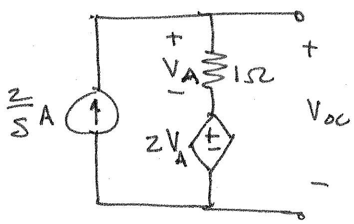
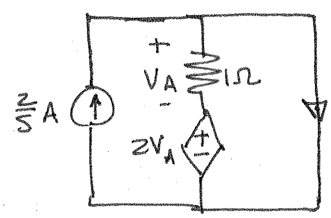


Figure P14.25

SOLUTION:





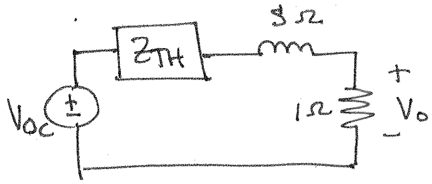
$Z_{TH} = V_{oc}/I_{sc}$

$Z_{TH} = 3\,\Omega$

$V_{oc} = 3V_A$ & $V_A = (1)(2/s)$

$V_{oc} = 6/s$

$I_{sc} = \frac{2}{s}$



$$V_o = \frac{V_{oc}\,(1)}{s+1+Z_{TH}} = \frac{6}{s(s+4)} = \frac{3/2}{s} - \frac{3/2}{s+4}$$

$$\boxed{v_o(t) = [1.5(1-e^{-4t})]u(t)\ \text{V}}$$

14.26 Find $v_o(t)$, $t > 0$, in the network shown in Fig. P14.26 using Laplace transforms. Assume that the circuit has reached steady state at $t = 0-$.

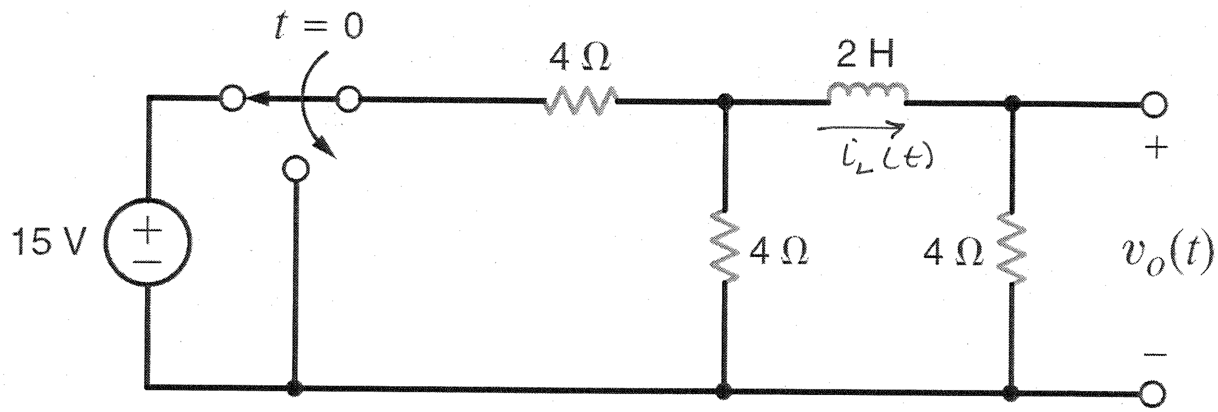
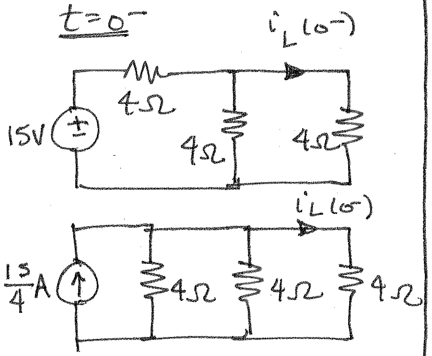


Figure P14.26

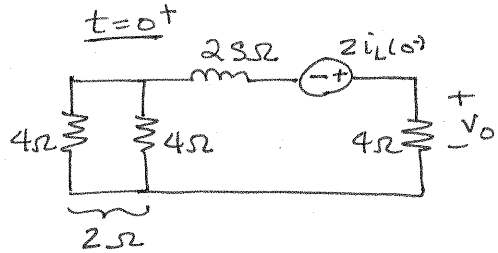
SOLUTION:

$t=0^-$



$i_L(0^-) = \left(\frac{15}{4}\right)\frac{1}{3} = 1.25\text{A}$

$t=0^+$



$$V_o = 2\, i_L(0^-)\left[\frac{4}{4+2+2s}\right] = \frac{5}{s+3}$$

$$\boxed{v_o(t) = 5e^{-3t} \text{ V}}$$

14.27 Find $i_o(t)$, $t > 0$, in the network shown in Fig. P14.27.

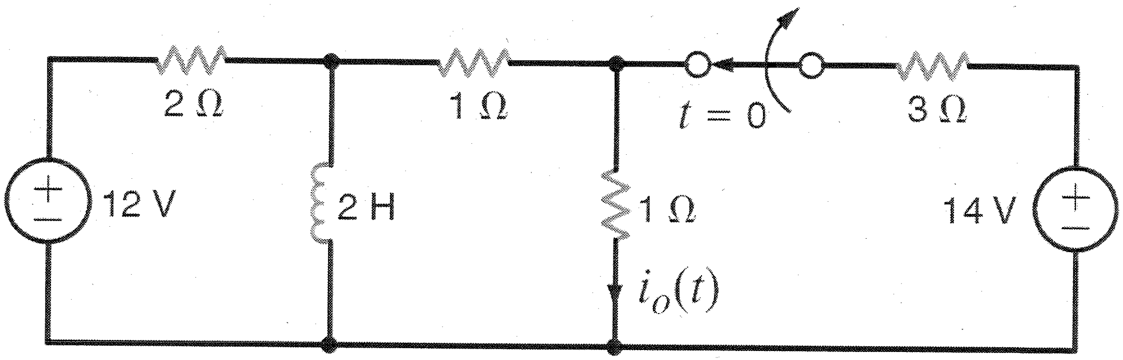
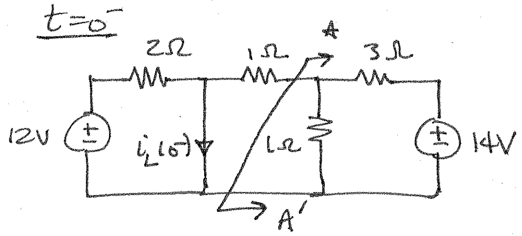


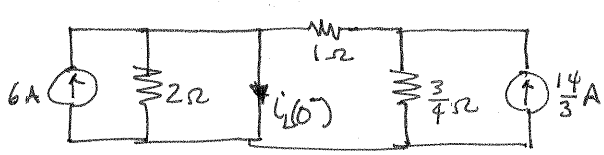
Figure P14.27

SOLUTION:

$t = 0^-$



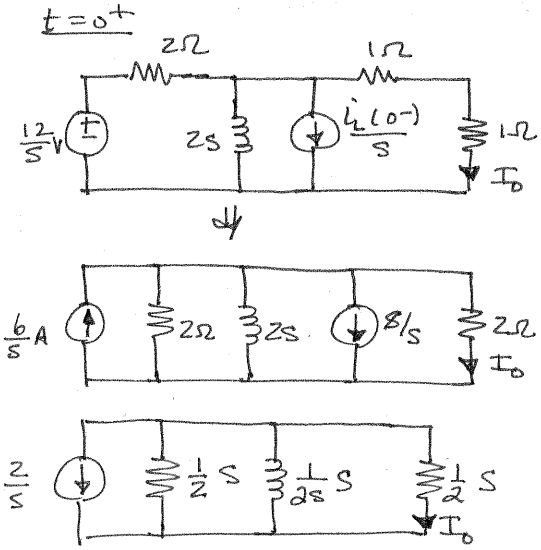
Use source transform & Norton's



By superposition:

$$i_L(0^-) = 6 + \frac{14}{3}\left[\frac{3/4}{1+3/4}\right] = 8A$$

$t = 0^+$



$$I_o = -\frac{2}{s}\left[\frac{1/2}{1/2 + 1/2 + 1/2s}\right] = \frac{-1}{s + 1/2}$$

$$\boxed{i_o(t) = -e^{-t/2}u(t)\ A}$$

14.28 Find $i_o(t)$, $t > 0$, in the network shown in Fig. P14.28.
CS

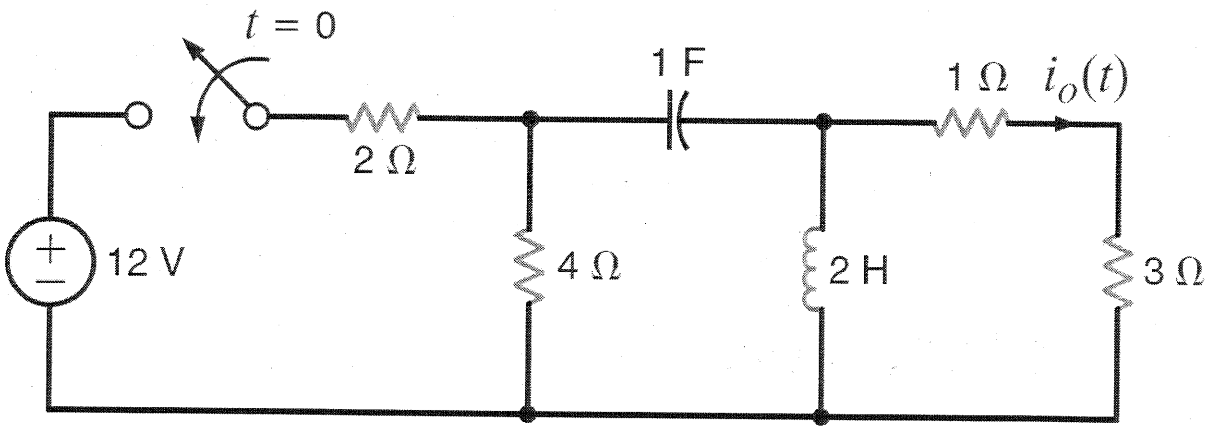


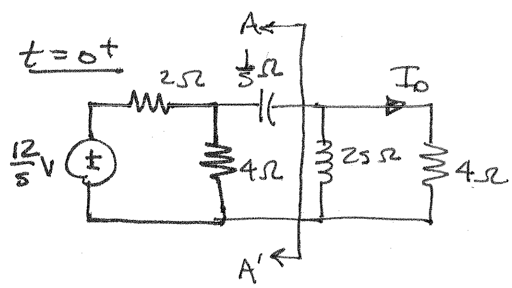
Figure P14.28

SOLUTION:

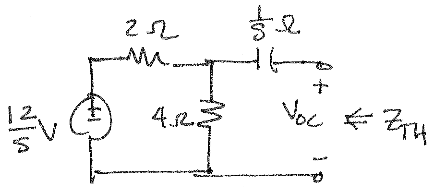
$t = 0^-$, $i_L(0^-) = 0$

$v_c(0^-) = 0$

$t = 0^+$

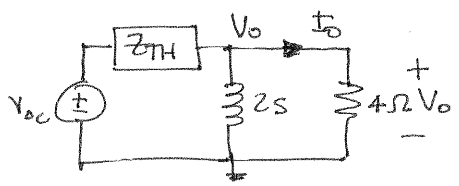


Thevenin at A-A'



$$V_{oc} = \frac{12}{s}\left(\frac{4}{6}\right) = 8/s$$

$$Z_{TH} = \frac{1}{s} + \frac{2(4)}{6} = \frac{1}{s} + \frac{4}{3} = \frac{4s+3}{3s}$$



$$\frac{V_o - V_{oc}}{Z_{TH}} + \frac{V_o}{4} + \frac{V_o}{2s} = 0 \qquad I_o = \frac{V_o}{4}$$

Let $Z_1 = 2s(4)/(2s+4) = 4s/(s+2)$

$$V_o = V_{oc}\, Z_1 / (Z_1 + Z_{TH}) \Rightarrow I_o = V_o/4 = \frac{1.5s}{s^2 + \left(\frac{11}{16}\right)s + \frac{6}{16}}$$

$$I_o = \frac{K}{s + \frac{11}{32} - j\frac{\sqrt{263}}{32}} + \frac{K^*}{s + \frac{11}{32} + j\frac{\sqrt{263}}{32}} \qquad K = 0.906\,\angle 34.2^\circ$$

$$i_o(t) = 1.81e^{-0.344t}\cos(0.507t + 34.2^\circ)\,u(t)\ \text{A}$$

14.29 Find $v_o(t)$, $t > 0$, in the circuit shown in Fig. P14.29.

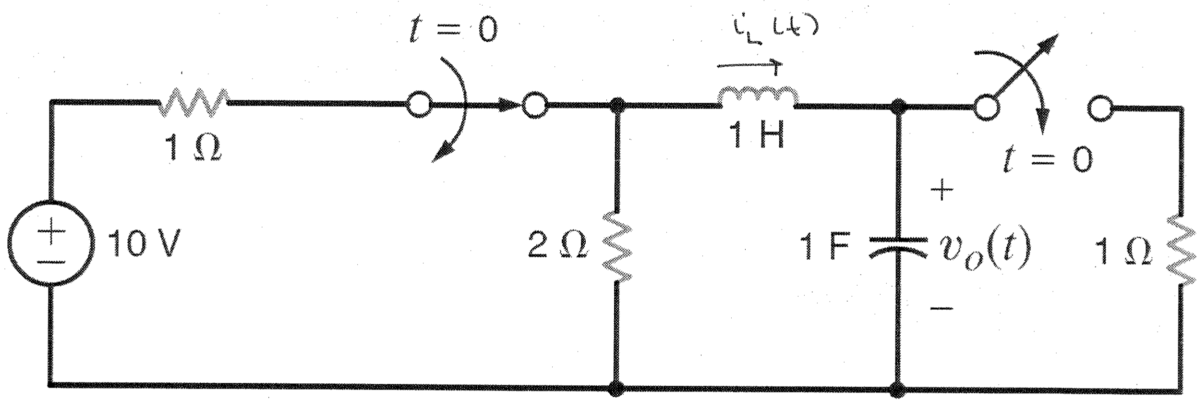
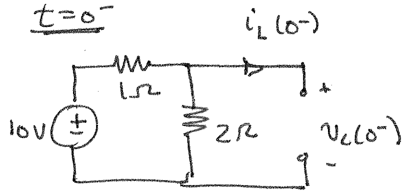


Figure P14.29

SOLUTION:

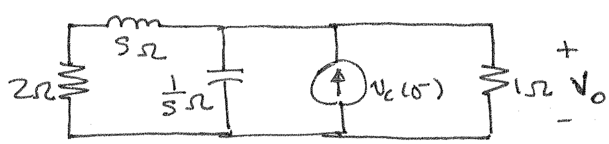
$t=0^-$



$i_L(0^-) = 0$

$v_C(0^-) = \frac{10(2)}{3} = \frac{20}{3}\,V$

$t=0^+$



$$\frac{V_o}{s+2} + V_o(s) + \frac{V_o}{1} = \frac{20}{3} = V_o\left[s+1+\frac{1}{s+2}\right]$$

$$V_o\left[\frac{s^2+3s+3}{s+2}\right] = 20/3$$

$$V_o = \frac{20/3\,(s+2)}{s^2+3s+3} = \frac{K}{s+\frac{3}{2}-j\frac{\sqrt{3}}{2}} + \frac{K^*}{s+\frac{3}{2}+j\frac{\sqrt{3}}{2}}; \quad K = 3.85\angle -30^\circ$$

$$\boxed{v_o(t) = 7.7e^{-(3/2)t}\cos\left[(\sqrt{3}/2)t - 30^\circ\right]u(t)\ \ V}$$

14.30 Find $v_o(t)$, $t > 0$, in the circuit in Fig. P14.30. CS

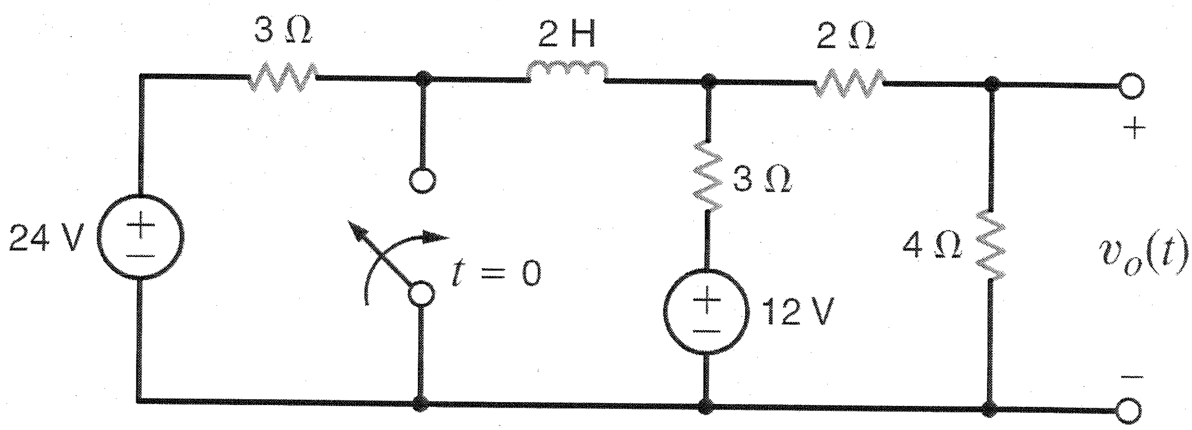
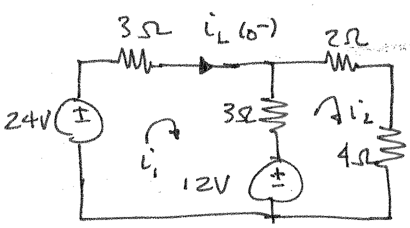


Figure P14.30

SOLUTION:

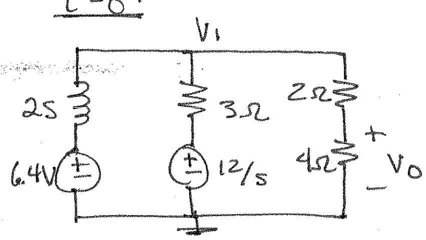
$\underline{t=0^-}$



$$12 = 6i_1 - 3i_2$$
$$12 = -3i_1 + 9i_2$$

yields $i_L(0^-) = i_1 = 3.2\text{A}$

$\underline{t=0^+}$



$$\frac{V_1 - 6.4}{2s} + \frac{V_1 - 12/s}{3} + \frac{V_1}{6} = 0$$

$$V_1(3+3s) = 6(3.2) + 24 = 43.2$$

$$V_1 = 14.4/(s+1)$$

$$V_o = V_1(4/6) = 9.6/(s+1)$$

$$\boxed{v_o(t) = 9.6e^{-t}u(t)\ \text{V}}$$

14.31 Find $i_o(t)$, $t > 0$, in the network in Fig. P14.31.
PSV

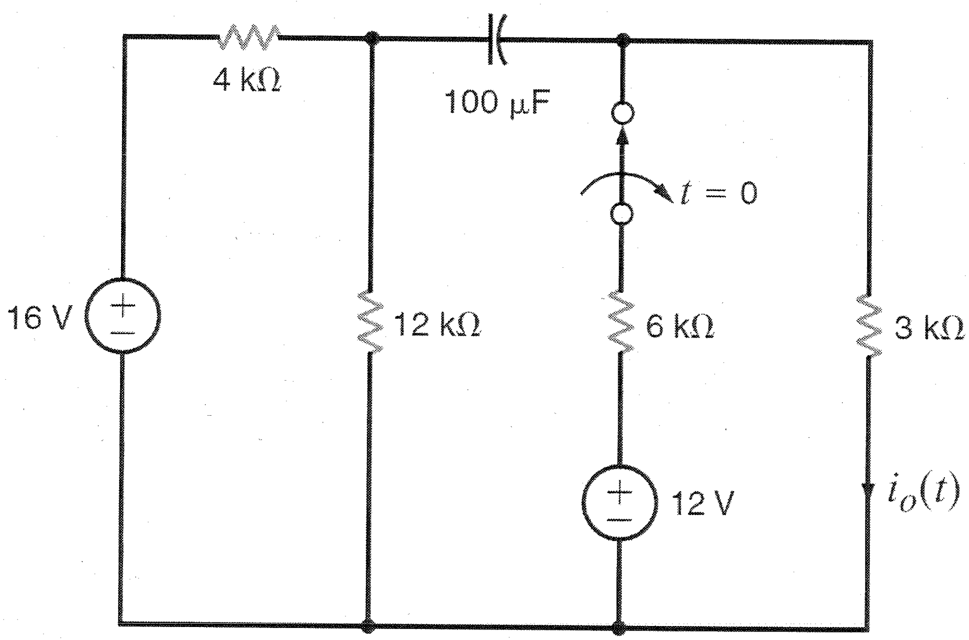
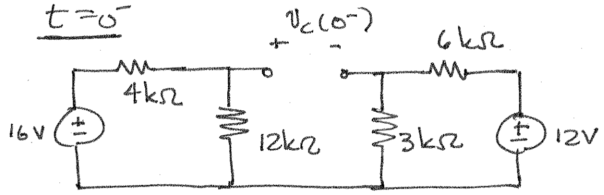


Figure P14.31

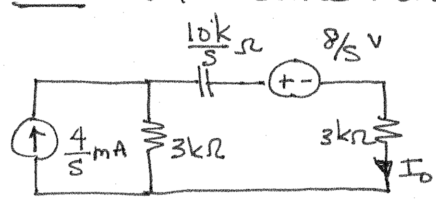
SOLUTION:

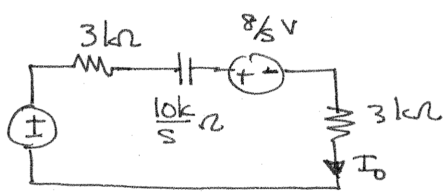
$t=0^-$



$$v_c(0^-) = 16\left(\frac{12}{16}\right) - 12\left(\frac{3}{9}\right) = 8V$$

$t=0^+$ (w/ a source transformation)





$$\frac{12}{s} = \left(3000 + 3000 + \frac{10^4}{s}\right) I_o + \frac{8}{s} \Leftarrow$$

$$I_o = \frac{4}{6s+10} \text{ mA} = \frac{2/3}{s + 5/3}$$

$$\boxed{i_o(t) = \frac{2}{3} e^{-(5/3)t} u(t) \text{ mA}}$$

14.32 Find $v_o(t)$, $t > 0$, in the network in Fig. P14.32.

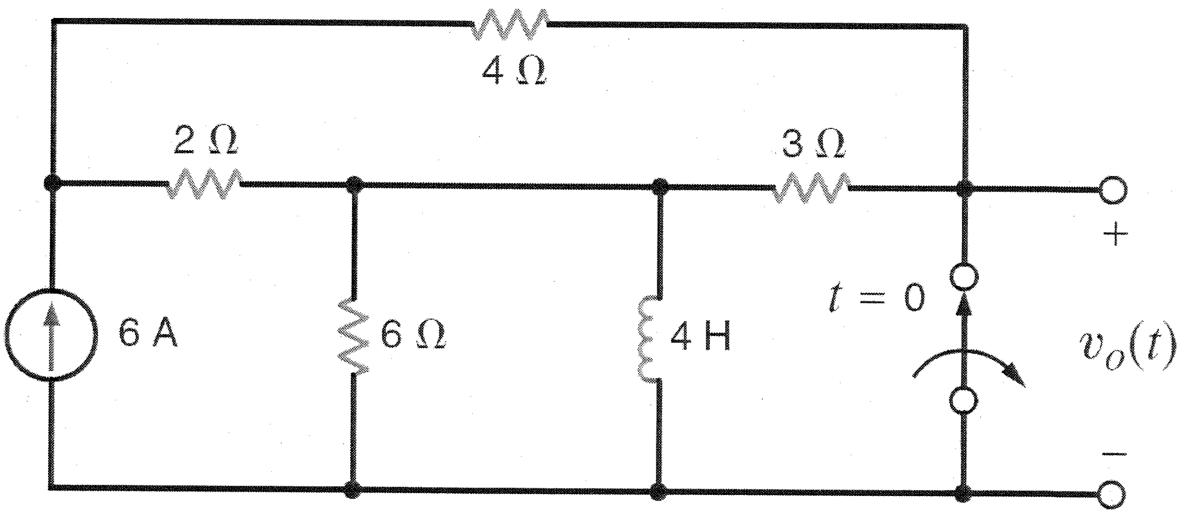
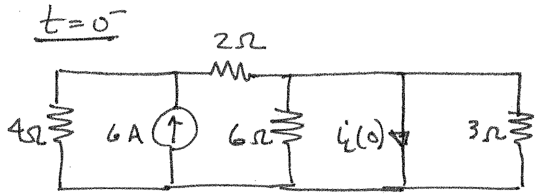


Figure P14.32

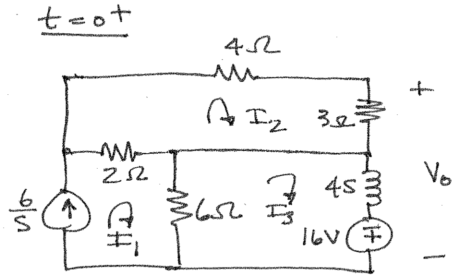
SOLUTION:

$t = 0^-$



$$i_L(0) = 6\left[\frac{4}{4+2}\right] = 4A$$

$t = 0^+$



$I_1 = 6/s \qquad I_2(9) - 2I_1 = 0$

$16 = I_3(4S+6) - 6I_1$

$V_0 = 3I_2 + 4SI_3 - 16$

$V_0 = \frac{4}{S} + \frac{32S+72}{2S+3} - 16 \Leftarrow$ yields $I_2 = \frac{4/3}{S} \qquad I_3 = \frac{8S+18}{S(2S+3)}$

$V_0 = \frac{16S+6}{S(S+1.5)} = \frac{4}{S} + \frac{12}{S+1.5} \Rightarrow$ $\boxed{v_0(t) = \left[4 + 12e^{-1.5t}\right]u(t)\ V}$

14.33 Find $v_o(t)$, for $t > 0$, in the network in Fig. P14.33.

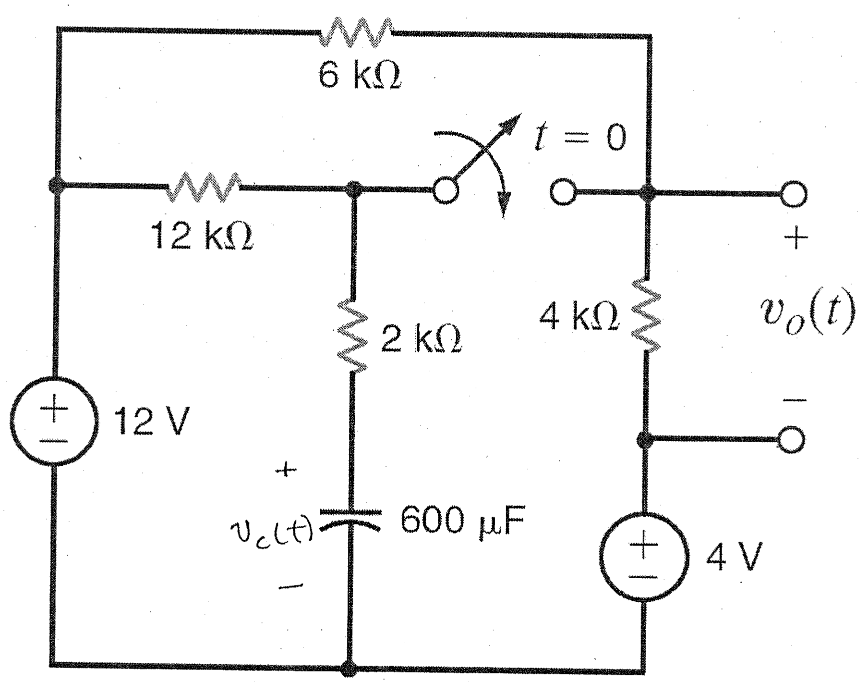
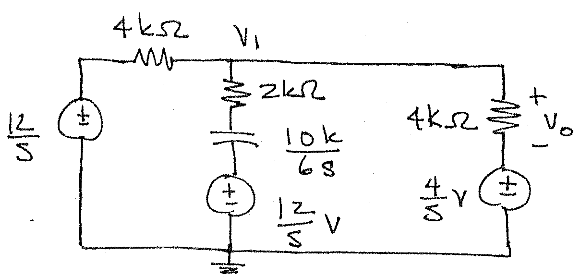


Figure P14.33

SOLUTION: $v_C(0^-) = 12V$

$t=0^+$ (12k & 6k in parallel!)



$$\frac{V_1 - 12/s}{4\times10^3} + \frac{V_1 - 12/s}{\left(2+\frac{10}{6s}\right)\times10^3} + \frac{V_1 - 4/s}{4\times10^3} = 0$$

or, $\frac{V_1}{4} + \frac{V_1}{2+\frac{10}{6s}} + \frac{V_1}{4} = \frac{3}{s} + \frac{1}{s} + \frac{12}{s\left(2+\frac{10}{6s}\right)}$

$$V_1\left[\frac{12s+5}{12s+10}\right] = \frac{120s+40}{s(12s+10)}$$

$$V_1 = \frac{120s+40}{s(12s+5)}$$

$$V_o = V_1 - \frac{4}{s} = \frac{72s+20}{s(12s+5)}$$

$$V_o = \frac{(72s+20)/12}{s(s+5/12)} = \frac{4}{s} + \frac{2}{s+5/12}$$

$$\boxed{v_o(t) = \left[4 + 2e^{-(5/12)t}\right]u(t)\ V}$$

14.34 Find $v_o(t)$, for $t > 0$, in the network in Fig. P14.34.

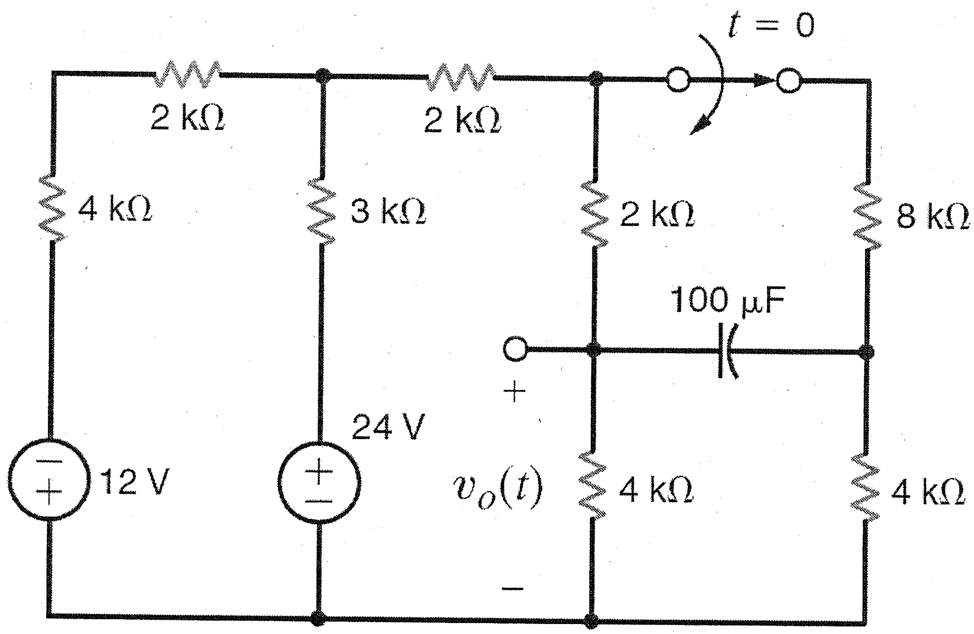
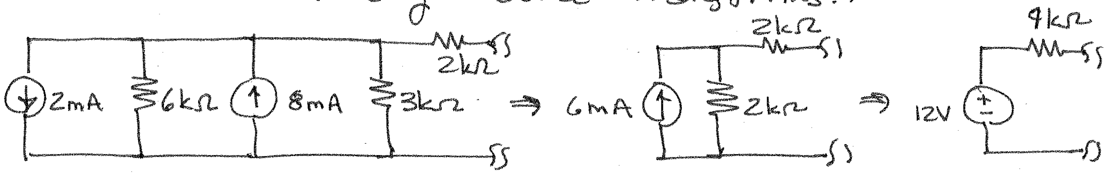
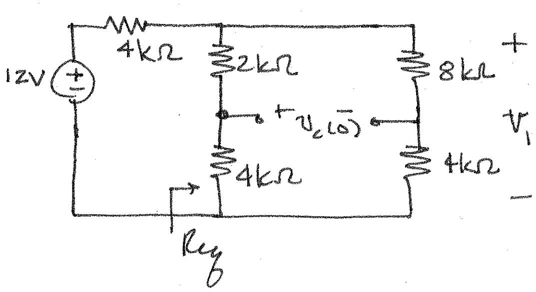


Figure P14.34

SOLUTION: $t = 0^-$ (using source transforms!)



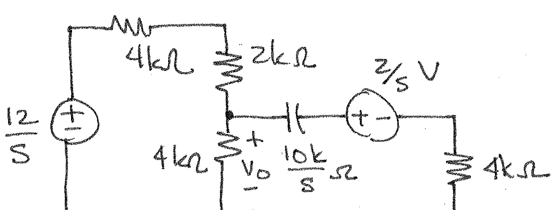


$R_{eq} = (6000)(12{,}000)/18{,}000 = 4k\Omega$

$v_1 = 12\,R_{eq}/(4000 + R_{eq}) = 6\,V$

$v_c(0) = v_1\left[\dfrac{4000}{6000} - \dfrac{4000}{12{,}000}\right] = 2\,V$

$t = 0^+$



$$\frac{V_o - 12/s}{6000} + \frac{V_o}{4000} + \frac{V_o - 2/s}{4000 + \frac{10{,}000}{s}} = 0$$

$$\frac{V_o}{6} + \frac{V_o}{4} + \frac{V_o s}{4s + 10} = \frac{2}{s} + \frac{2}{4s+10}$$

$$V_o\left[\frac{s}{12}+\frac{s}{4s+10}\right]=\frac{10s+20}{s(4s+10)}=V_o\left[\frac{32s+50}{12(4s+10)}\right]$$

$$V_o=\frac{\frac{15}{4}(s+2)}{s\left(s+\frac{25}{16}\right)}=\frac{24/5}{s}-\frac{21/20}{s+25/16}$$

$$\boxed{v_o(t)=\left[\frac{24}{5}-\frac{21}{20}e^{-(25/16)t}\right]u(t)\ \mathrm{V}}$$

14.35 Find $v_o(t)$, for $t > 0$, in the network in Fig. P14.35.
CS

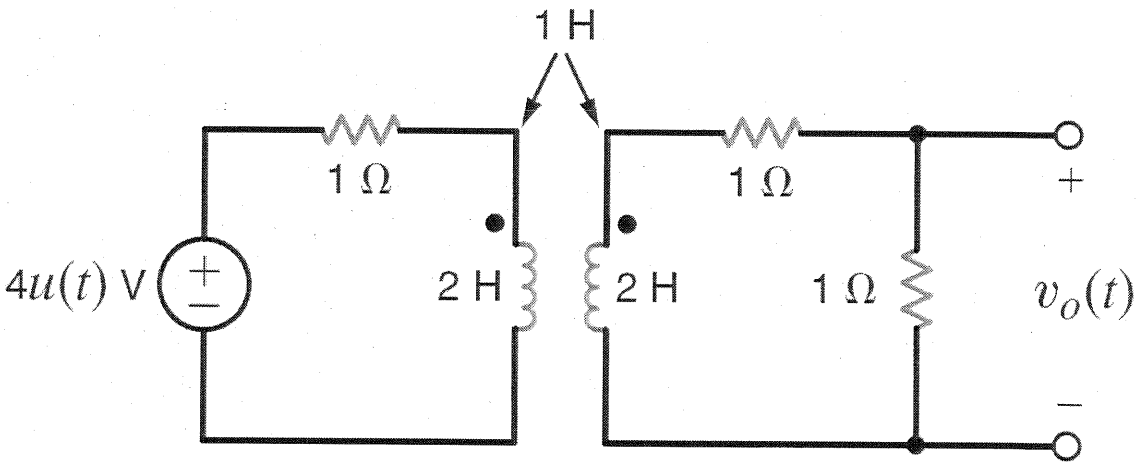
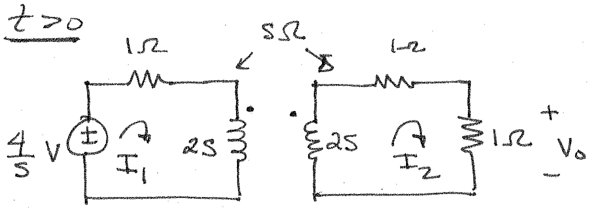


Figure P14.35

SOLUTION: $t = 0^-$: no excitation → ∅ initial conditions

$\underline{t > 0}$



$$\frac{4}{s} = I_1(2s+1) - sI_2$$

$$0 = -sI_1 + I_2(2s+2)$$

or, $I_1 = I_2(2s+2)/s$

yields, $I_2 = \dfrac{4/3}{s^2 + 2s + 2/3}$

$$V_o = (1)I_2 = \frac{4/3}{(s+0.42)(s+1.58)}$$

$$V_o = \frac{1.15}{s+0.42} - \frac{1.15}{s+1.58}$$

$$\boxed{v_o(t) = 1.15\left[e^{-0.42t} - e^{-1.58t}\right]u(t)\ \text{V}}$$

14.36 Find $v_o(t)$, for $t > 0$, in the network in Fig. P14.36.
PSV

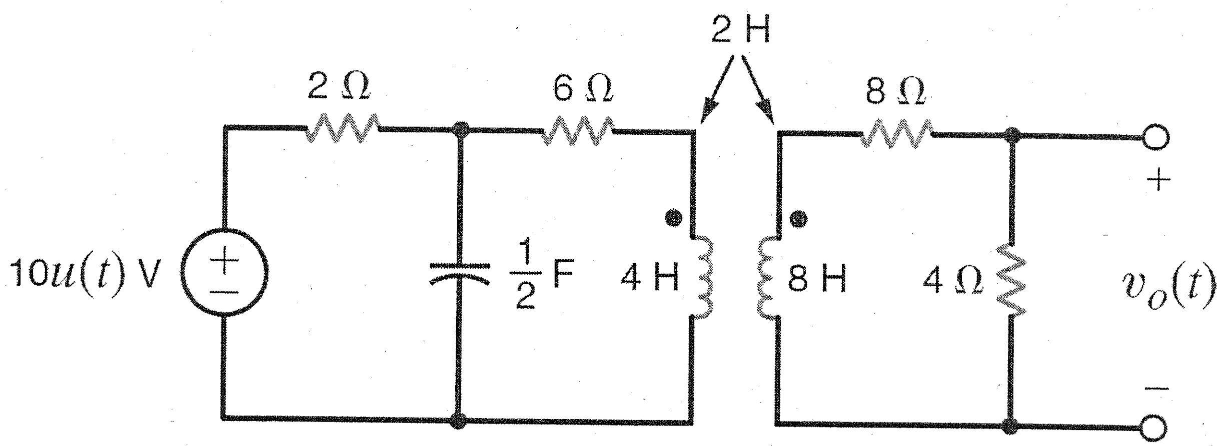
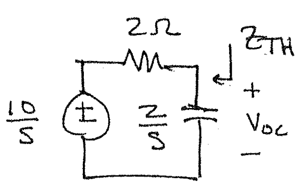


Figure P14.36

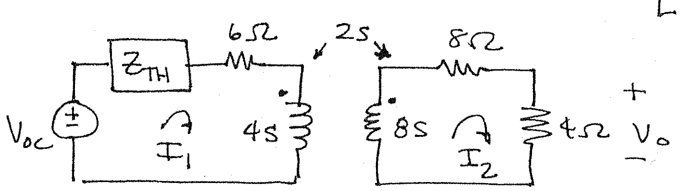
SOLUTION: $t=0^-$: no excitation $\Rightarrow$ $\emptyset$ initial conditions

$t=0^+$ (use Thevenin 1st)



$$V_{oc} = \frac{10}{s}\left[\frac{2/s}{2/s+2}\right] = \frac{10}{s(s+1)} \text{ V}$$

$$Z_{TH} = 2(2/s)\,/\left[2+2/s\right] = \frac{2}{s+1} \ \Omega$$



$$V_{oc} = I_1(4s+6+Z_{TH}) - 2sI_2$$

$$0 = -2sI_1 + I_2(8s+12)$$

yields, $I_1 = I_2(4s+6)/s \Rightarrow V_{oc} = I_2\left[(4s+6+Z_{TH})(4s+6) - 2s\right]$

solve for I_2 and use $V_o = 4I_2$

$$V_o = \frac{20/7}{s^3 + \left(\frac{31}{7}\right)s^2 + \left(\frac{46}{7}\right)s + \frac{24}{7}}$$

Using the ROOTS function in MATLAB yields

$$V_0 = \frac{20/7}{(s+2)(s+1.21-j0.5)(s+1.21+j0.5)} = \frac{A}{s+2} + \frac{K}{s+1.21-j/2} + \frac{K^*}{s+1.21+j/2}$$

$A = 3.33$ $\qquad K = 3.15\angle -122$

$$v_0(t) = \left[3.33e^{-2t} + 6.30\, e^{-1.21t} \cos\left(t/2 - 122°\right)\right] u(t)\ V$$

14.37 Find $v_o(t)$, for $t > 0$, in the network in Fig. P14.37.

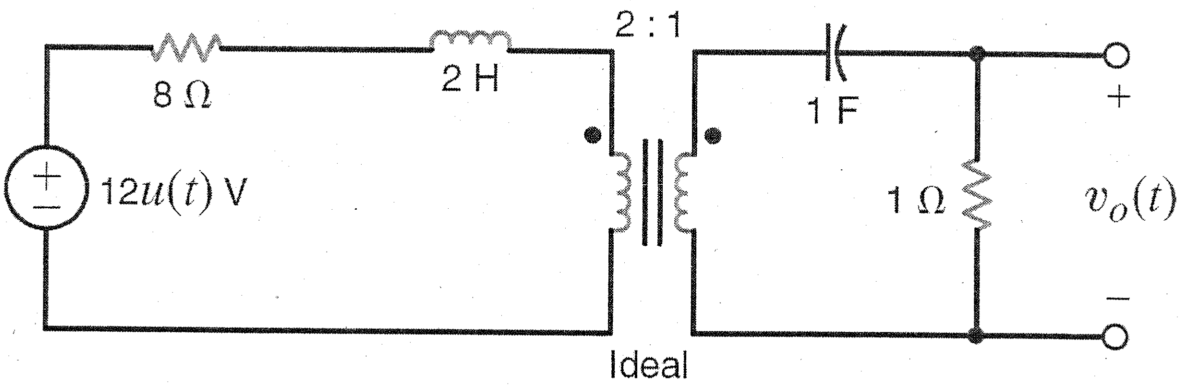
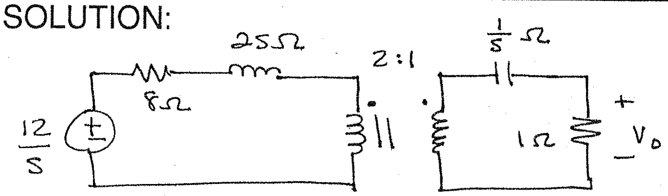
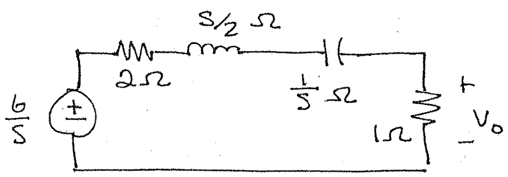


Figure P14.37

SOLUTION:



$n = 1/2$



$$V_o = \frac{6}{s}\left[\frac{1}{2 + s/2 + \frac{1}{s} + 1}\right] = \frac{12}{s^2 + 6s + 2}$$

$$V_o = \frac{A}{s + 0.35} + \frac{B}{s + 5.65}$$

$A = 2.28$ & $B = -2.28$

$$\boxed{v_o(t) = 2.28\left[e^{-0.35t} - e^{-5.65t}\right]u(t)\ \text{V}}$$

14.38 Find $v_o(t)$, for $t > 0$, in the network in Fig. P14.38.
CS

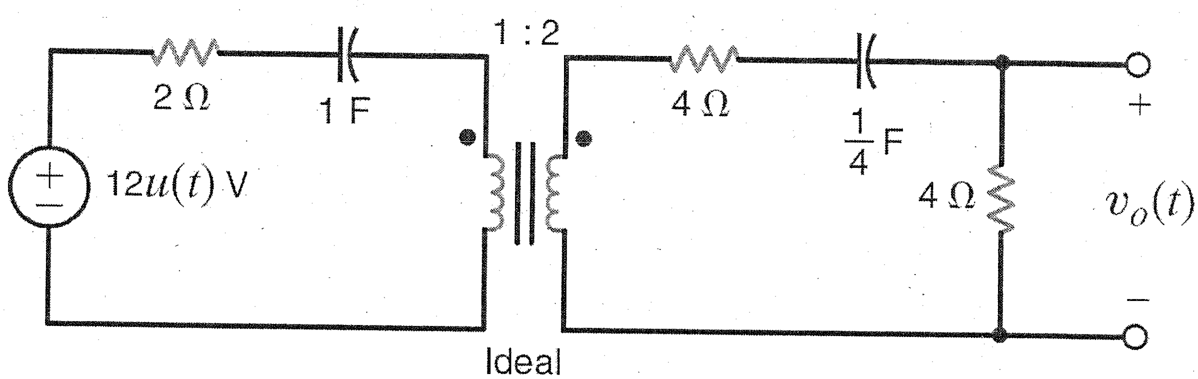
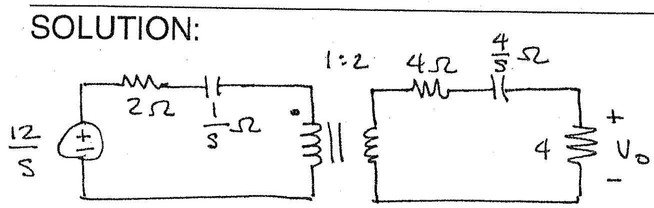
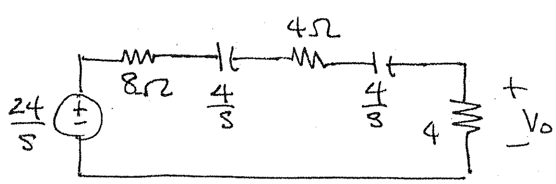


Figure P14.38

SOLUTION:



$n = 2/1$



$$V_o = \frac{24}{s}\left[\frac{4}{(8/s) + 16}\right] \Rightarrow V_o = \frac{6}{s + 1/2}$$

$$\boxed{v_o(t) = 6e^{-t/2}u(t)\ \text{V}}$$

14.39 Determine the initial and final values of the voltage $v_o(t)$ in the network in Fig. P14.39.

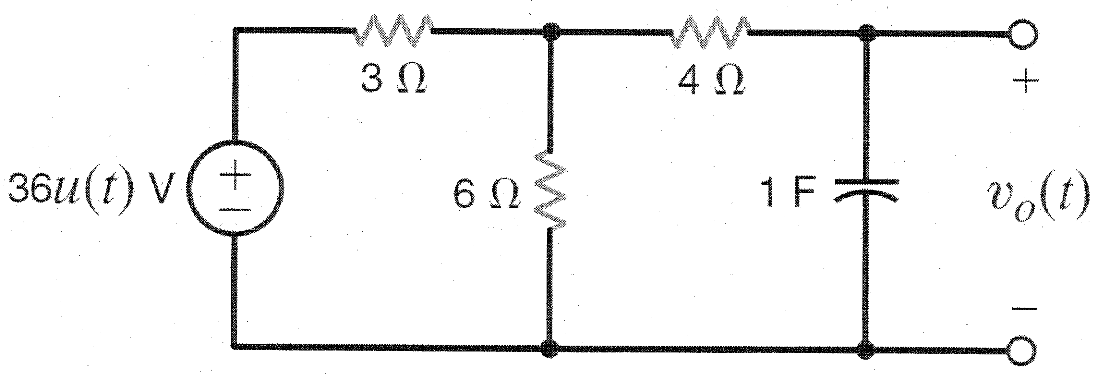
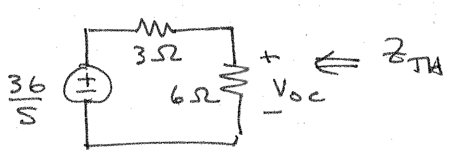


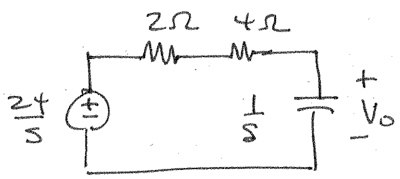
Figure P14.39

SOLUTION: Use Thevenin's



$V_{oc} = \frac{6}{9}\left(36/s\right) = 24/s$ V

$Z_{TH} = 3(6)/9 = 2\Omega$



$V_o = \frac{24}{s}\left[\frac{1/s}{6+1/s}\right] = \frac{24}{s(6s+1)}$

$\lim_{t\to 0} v_o(t) = \lim_{s\to\infty} sV_o(s) = \frac{24}{6(\infty)} = 0$ $\boxed{v_o(0) \to 0\text{ V}}$

$\lim_{t\to\infty} v_o(t) = \lim_{s\to 0} sV_o(s) = \frac{24}{1} = 24\text{V}$ $\boxed{v_o(\infty) \to 24\text{V}}$

14.40 Determine the initial and final values of the voltage $v_o(t)$ in the network in Fig. P14.40.

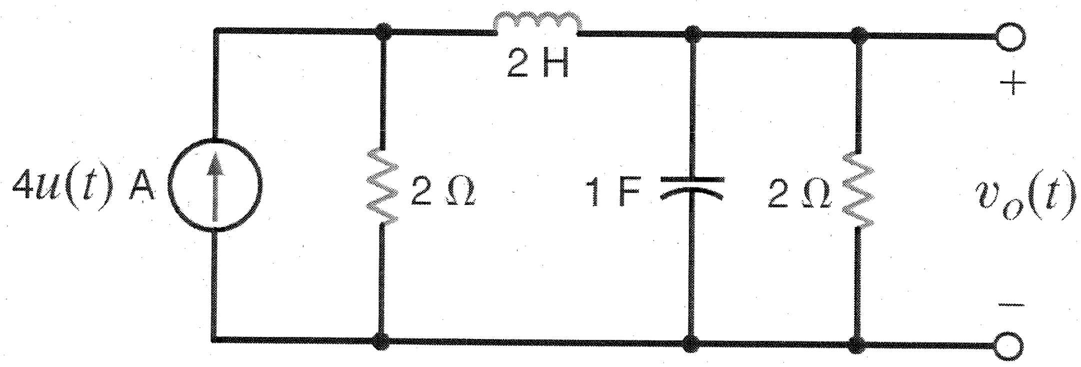
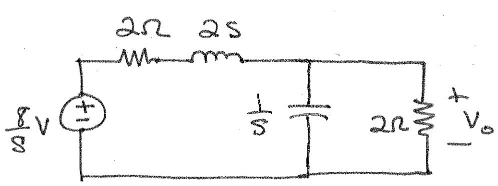


Figure P14.40

SOLUTION: Use source transformation,



Let $Z_1 = 2s + 2\,\Omega$ and

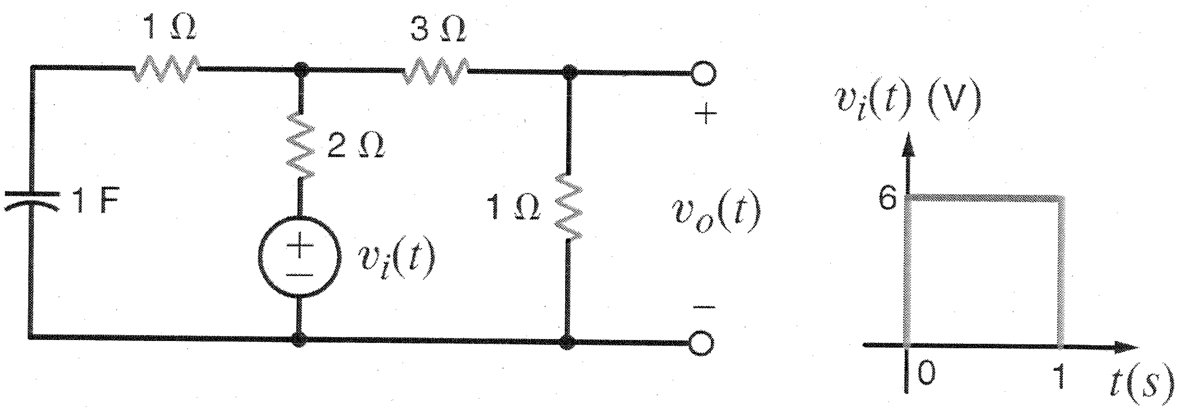
$Z_2 = 2(1/s) / (2 + 1/s) = \dfrac{2}{2s+1}\ \Omega$

$$V_o = \frac{8}{s}\left[\frac{Z_2}{Z_1+Z_2}\right] = \frac{8}{s}\left[\frac{2}{2+(2s+2)(2s+1)}\right] = \frac{16}{s(4s^2+6s+4)}$$

$$\lim_{t\to 0} v_o(t) = \lim_{s\to\infty} sV_o(s) = \frac{16}{4\infty^2} = 0 \qquad \boxed{v_o(0) \to 0}$$

$$\lim_{t\to\infty} v_o(t) = \lim_{s\to 0} sV_o(s) = \frac{16}{4} = 4 \qquad \boxed{v_o(\infty) \to 4V}$$

14.41 Determine the output voltage $v_o(t)$ in the network in Fig. P14.41a if the input is given by the source in Fig. P14.41b. **PSV**

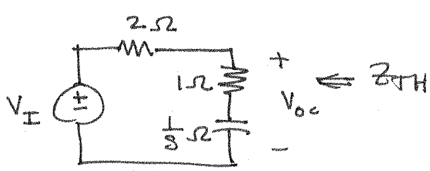


(a) (b)

Figure P14.41

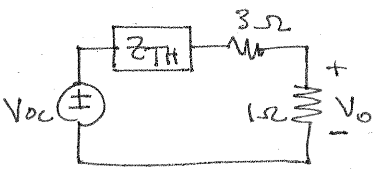
SOLUTION: $v_i(t) = 6u(t) - 6u(t-1)$ $\quad V_I(s) = \frac{6}{s}(1-e^{-s})$

Use Thevenin eq.



$$V_{oc} = V_I\left[\frac{1+1/s}{3+1/s}\right] = V_I\left(\frac{s+1}{3s+1}\right)$$

$$Z_{TH} = \frac{(1+1/s)(2)}{3+1/s} = \frac{2(s+1)}{3s+1}$$



$$V_o = V_{oc}\left[\frac{1}{4+Z_{TH}}\right] = V_I\left(\frac{s+1}{3s+1}\right)\left(\frac{3s+1}{4(3s+1)+2s+2}\right)$$

$$V_o = \frac{(6/14)(s+1)}{s(s+6/14)}(1-e^{-s}) = \left[\frac{1}{s} - \frac{4/7}{s+6/14}\right](1-e^{-s})$$

$$v_o(t) = \left[1 - \tfrac{4}{7}e^{-(6/14)t}\right]u(t) - \left[1 - \tfrac{4}{7}e^{-(6/14)(t-1)}\right]u(t-1) \text{ V}$$

14.42 Find the output voltage, $v_o(t)$, $t > 0$, in the network in Fig. P14.42a if the input is represented by the waveform shown in Fig. P14.42b.

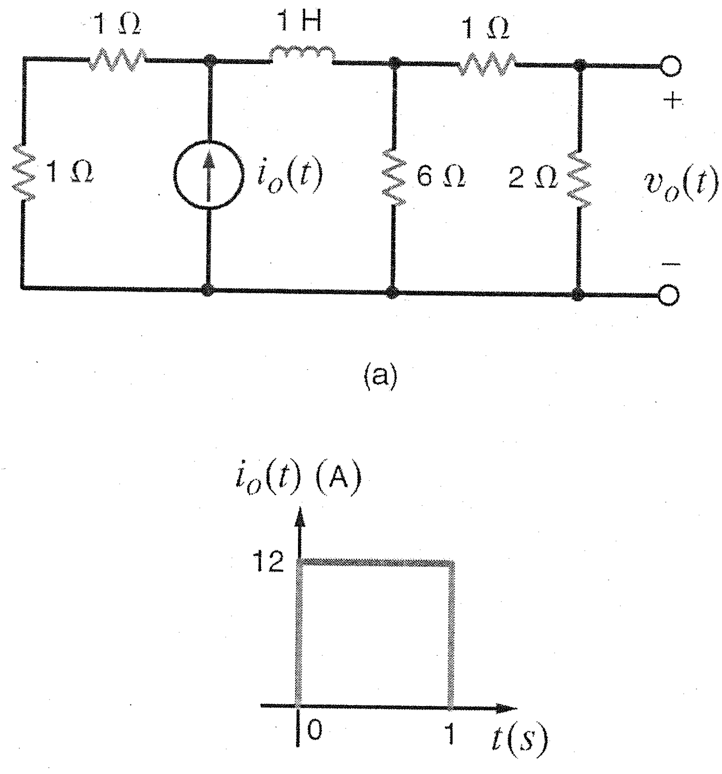
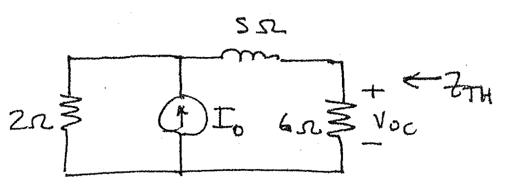


Figure P14.42

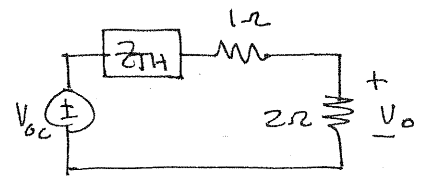
SOLUTION: $i_o(t) = 12u(t) - 12u(t-1)$ A $\Rightarrow$ $I_o(s) = \frac{12}{s}(1-e^{-s})$ A

Use Thevenin Eq.



$$V_{oc} = I_o\left[\frac{2(6)}{s+8}\right] = I_o\left(\frac{12}{s+8}\right)$$

$$Z_{TH} = \frac{6(s+2)}{s+8}$$



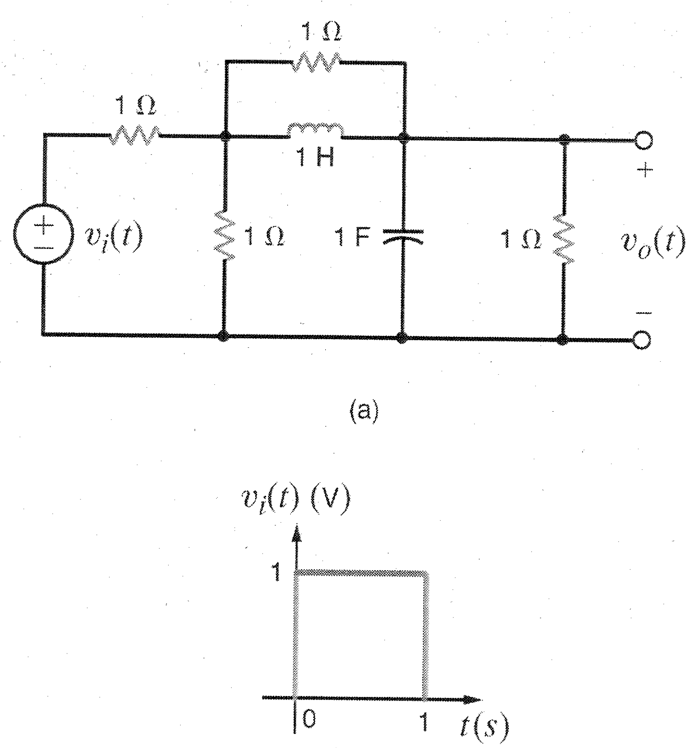
$$V_o = V_{oc}\left(\frac{2}{3+Z_{TH}}\right) = I_o\left[\frac{24}{3(s+8)+6(s+2)}\right]$$

$$V_o = I_o\left[\frac{24}{9s+36}\right] = \frac{(8/3)(12)}{s(s+4)}\,(1-e^{-s})$$

$$V_o = \left(\frac{8}{s} - \frac{8}{s+4}\right)(1-e^{-s})$$

$$v_o(t) = \left[8 - 8e^{-4t}\right]u(t) - \left[8 - 8e^{-4(t-1)}\right]u(t-1) \text{ V}$$

14.43 Determine the output voltage, $v_o(t)$, in the circuit in Fig. P14.43a if the input is given by the source described in Fig. P14.43b.



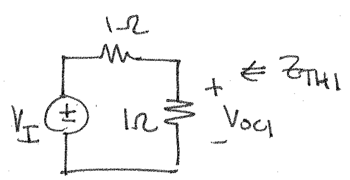
(a)

(b)

Figure P14.43

SOLUTION: $v_i(t) = u(t) - u(t-1) \Rightarrow \quad V_I(s) = \frac{1}{s}(1 - e^{-s})$ V

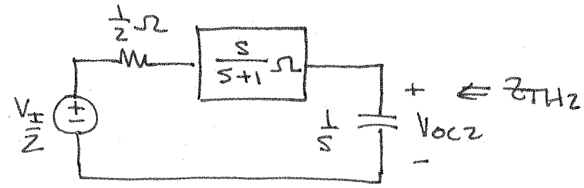
Use Thevenins twice!



$V_{oc1} = \frac{V_I}{2}$

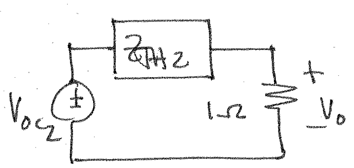
$Z_{TH1} = \frac{1}{2}\,\Omega$

$\Rightarrow$



$$V_{oc2} = \frac{V_I}{2}\left[\frac{1/s}{\frac{1}{2} + \frac{s}{s+1} + \frac{1}{s}}\right] = \frac{V_I(s+1)}{3s^2+3s+2}$$

$$Z_{TH2} = \frac{\frac{1}{s}\left(\frac{1}{2} + \frac{s}{s+1}\right)}{\frac{1}{2} + \frac{1}{s} + \frac{s}{s+1}}$$



$$V_o = V_{oc2}\left(\frac{1}{1+Z_{TH2}}\right)$$

$$Z_{TH2} = \frac{3s+1}{3s^2+3s+2}$$

$$V_o = \frac{(1/3)}{s(s+1)}(1-e^{-s}) = \left(\frac{1/3}{s} - \frac{1/3}{s+1}\right)(1-e^{-s})$$

$$v_o(t) = \frac{1}{3}\left[1-e^{-t}\right]u(t) - \frac{1}{3}\left[1-e^{-(t-1)}\right]u(t-1)\ \text{V}$$

14.44 Determine the transfer function $\mathbf{I}_o(s)/\mathbf{I}_i(s)$ for the network shown in Fig. P14.44.

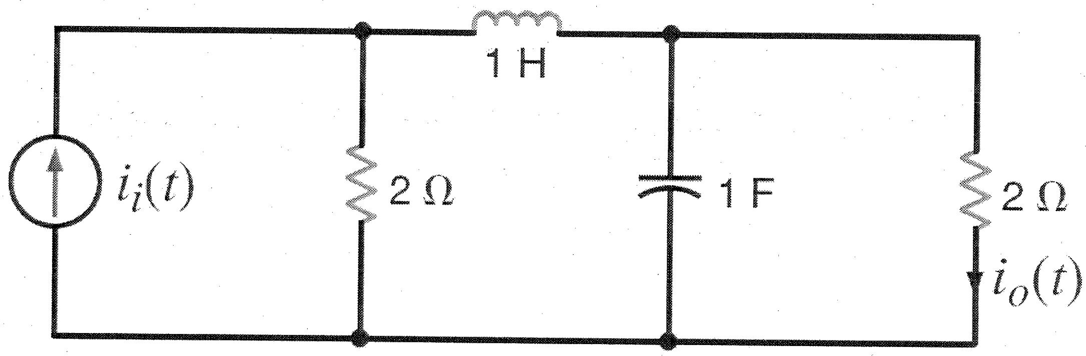
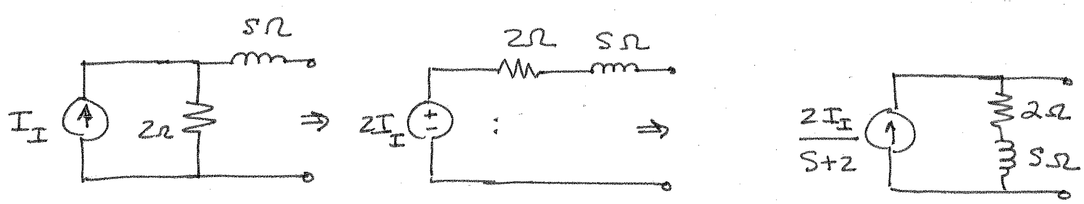
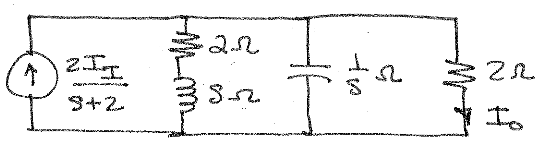


Figure P14.44

SOLUTION: Use source transformations





Current Division: $I_o = \dfrac{2I_I}{s+2}\left[\dfrac{1/2}{1/2 + s + \frac{1}{s+2}}\right] = \dfrac{I_I}{(s+2)(s+1/2)+1}$

$$\boxed{\frac{I_o}{I_I} = \frac{1}{s^2+2.5s+2}}$$

14.45 Find the transfer function $\mathbf{V}_o(s)/\mathbf{V}_i(s)$ for the network shown in Fig. P14.45. CS

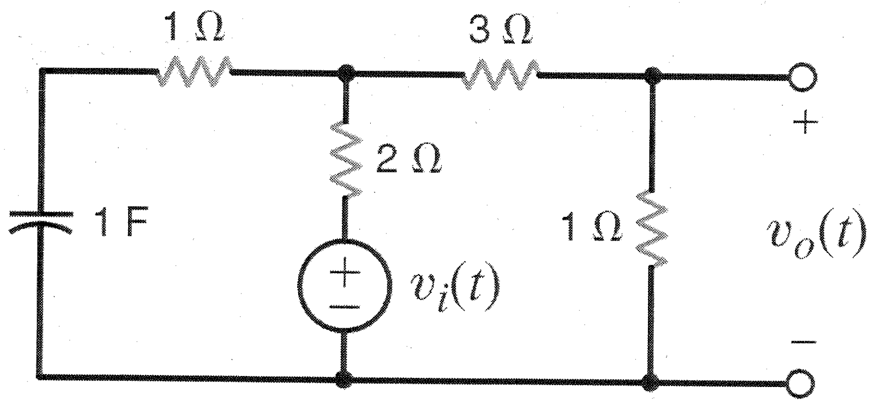
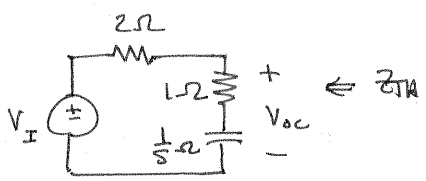


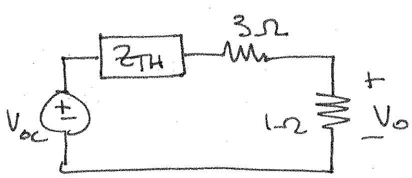
Figure P14.45

SOLUTION: Use Thevenin eq



$$V_{oc} = \frac{V_I\left(1+1/s\right)}{3+1/s} = \frac{V_I\,(s+1)}{3s+1}$$

$$Z_{TH} = \frac{2\left(1+1/s\right)}{3+1/s} = \frac{2(s+1)}{3s+1}$$



$$V_o = V_{oc}\left[\frac{1}{4+Z_{TH}}\right] = \frac{V_I\,(s+1)}{3s+1}\left(\frac{3s+1}{4(3s+1)+2(s+1)}\right)$$

$$V_o = V_I\left[\frac{s+1}{14s+6}\right]$$

$$\boxed{\frac{V_o}{V_I} = \frac{s+1}{14s+6}}$$

14.46 Find the transfer function for the network shown in Fig. P14.46.

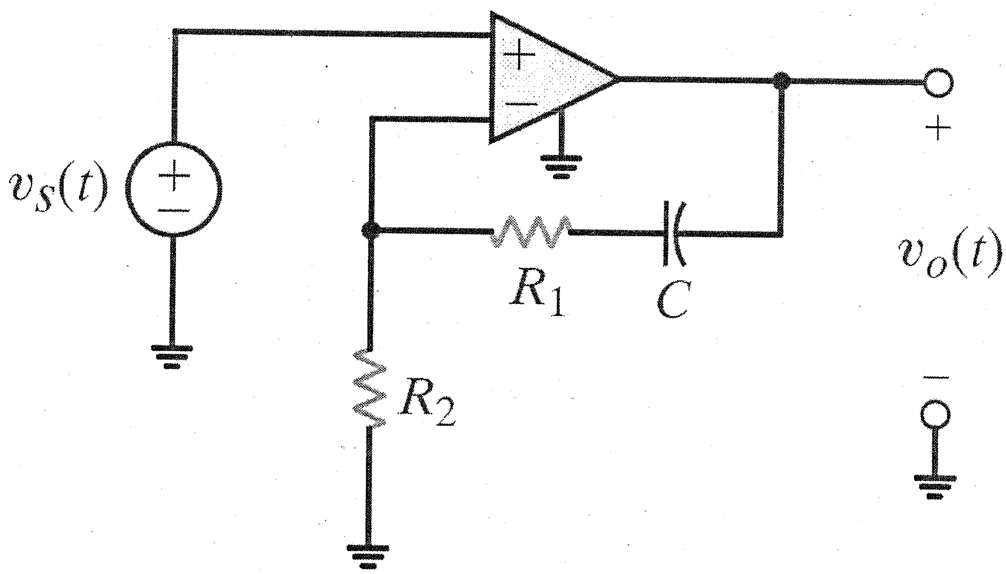


Figure P14.46

SOLUTION:

Let $Z_2 = R_1 + \frac{1}{sC}$ & $Z_1 = R_2$ $\quad \frac{V_o}{V_s} = 1 + \frac{Z_2}{Z_1}$

$$\frac{V_o}{V_s} = 1 + \frac{R_1 + 1/sC}{R_2} = 1 + \frac{R_1Cs+1}{R_2Cs} = \frac{(R_1+R_2)Cs+1}{R_2Cs}$$

$$\boxed{\frac{V_o}{V_s} = \left(1 + \frac{R_1}{R_2}\right)\left[\frac{s + \frac{1}{C(R_1+R_2)}}{s}\right]}$$

14.47 Find the transfer function for the network shown in Fig. P14.47.

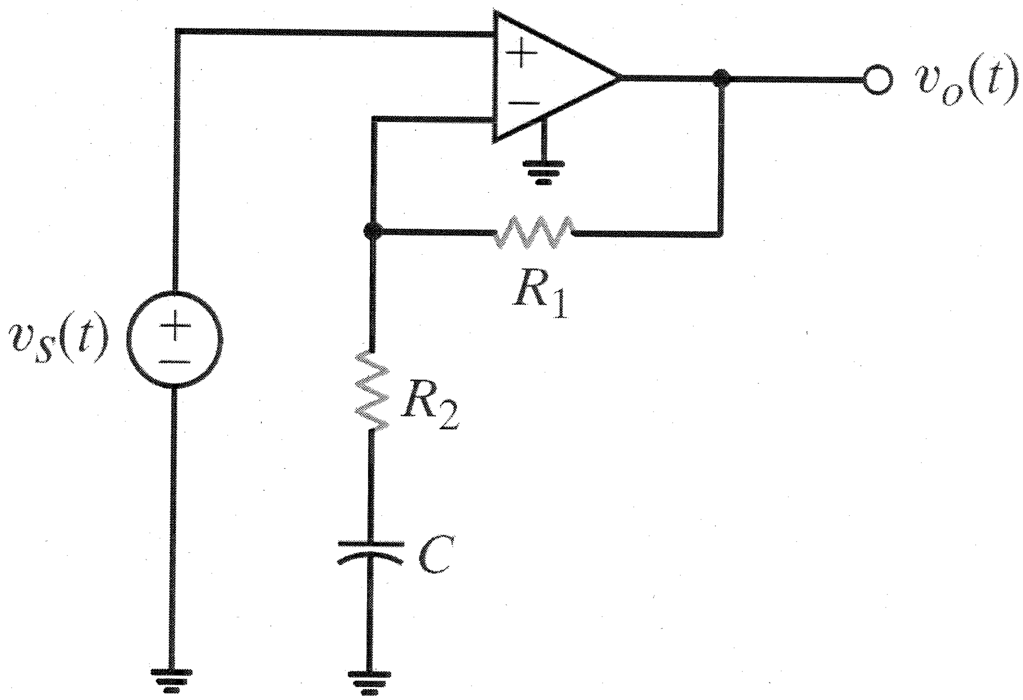


Figure P14.47

SOLUTION: Let $Z_2 = R_1$ & $Z_1 = R_2 + \frac{1}{sC} = \frac{R_2Cs+1}{sC}$

$$\frac{V_o}{V_s} = 1 + \frac{Z_2}{Z_1} = 1 + \frac{R_1Cs}{R_2Cs+1} = \frac{(R_1+R_2)Cs+1}{R_2Cs+1}$$

$$\boxed{\frac{V_o}{V_s} = \left(1 + \frac{R_1}{R_2}\right)\left(\frac{s+\frac{1}{CR}}{s+\frac{1}{CR_2}}\right) \quad R = R_1 + R_2}$$

14.48 Find the transfer function for the network in Fig. P14.48. **PSV**

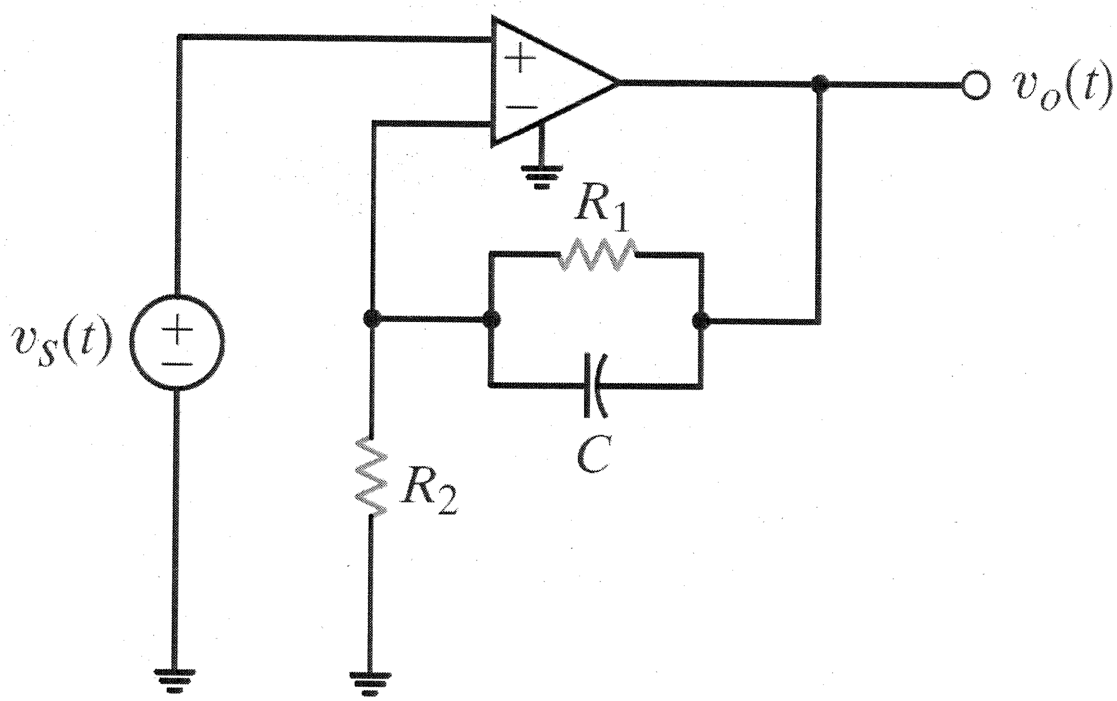


Figure P14.48

SOLUTION: Let $Z_1 = R_2$ & $Z_2 = \dfrac{R_1/sC}{R_1 + 1/sC} = \dfrac{R_1}{sCR_1+1}$

$$\frac{V_o}{V_s} = 1 + \frac{Z_2}{Z_1} = 1 + \frac{R_1/R_2}{sCR_1+1} = \frac{sCR_1 + 1 + R_1/R_2}{sCR_1+1} = \frac{1}{R_2}\left[\frac{sCR_1R_2 + R_1 + R_2}{sCR_1+1}\right]$$

$$\boxed{\frac{V_o}{V_s} = \left(1 + \frac{R_1}{R_2}\right)\left(\frac{sCR_p+1}{sCR_1+1}\right) \qquad R_p = \frac{R_1R_2}{R_1+R_2}}$$

14.49 Find the transfer function for the network in Fig. P14.49. If a step function is applied to the network, will the response be overdamped, underdamped, or critically damped?

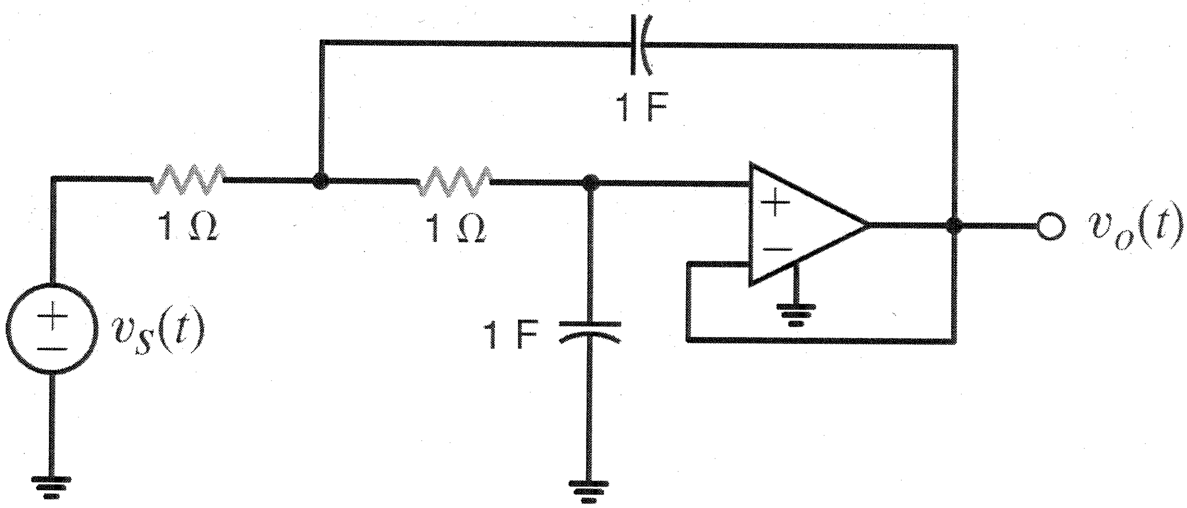
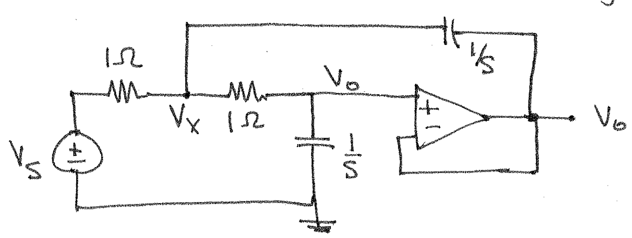


Figure P14.49

SOLUTION: Op amp is in unity gain configuration.



$$\frac{V_X - V_O}{1} = \frac{V_O}{1/s} \Rightarrow V_X = V_O(s+1)$$

$$\frac{V_S - V_X}{1} = \frac{V_X - V_O}{1} + \frac{V_X - V_O}{1/s}$$

yields $V_S = V_O(s+1)^2$

$$\frac{V_O}{V_S} = \frac{1}{(s+1)^2}$$

poles are real & identical, so,

CRITICALLY DAMPED !

14.50 Find the transfer function for the network in Fig. P14.50.

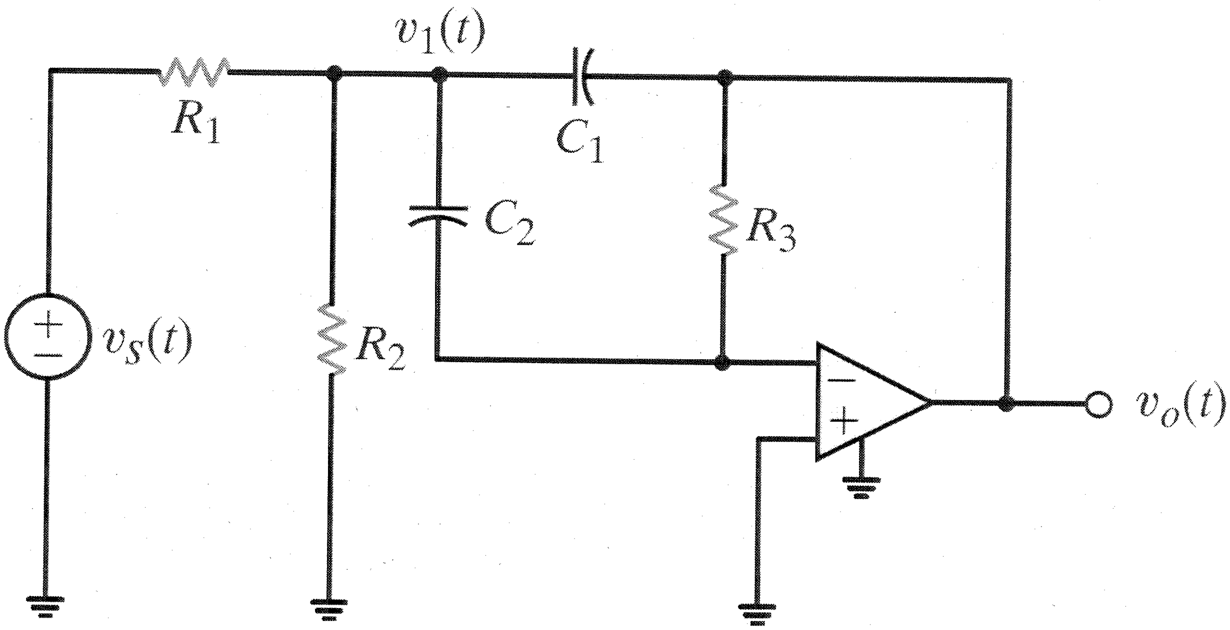
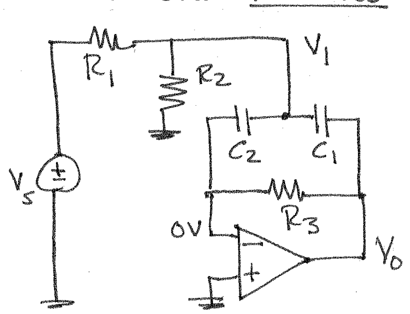


Figure P14.50

SOLUTION: Redraw



Nodal analysis

$$V_1 sC_2 + V_o/R_3 = 0 \Rightarrow V_1 = -V_o/sC_2R_3$$

$$\frac{V_s - V_1}{R_1} = \frac{V_1}{R_2} + V_1 sC_2 + (V_1 - V_o)sC_1$$

$$\frac{V_s}{R_1} = -V_o\left[sC_1 + \frac{1}{sC_2R_3}\left(\frac{1}{R_1} + \frac{1}{R_2} + s(C_2 + C_1)\right)\right]$$

$$\frac{V_o}{V_s} = \frac{-1/R_1}{sC_1 + \frac{C_1 + C_2}{C_2R_3} + \frac{R_1 + R_2}{sR_1R_2R_3C_2}}$$

$$\boxed{\frac{V_o}{V_s} = \frac{-(1/C_1R_1)s}{s^2 + s\left(\frac{C_1 + C_2}{C_1C_2R_3}\right) + \frac{R_1 + R_2}{C_1C_2R_1R_2R_3}}}$$

14.51 Determine the transfer function for the network shown in Fig. P14.51. If a step function is applied to the network, what type of damping will the network exhibit?

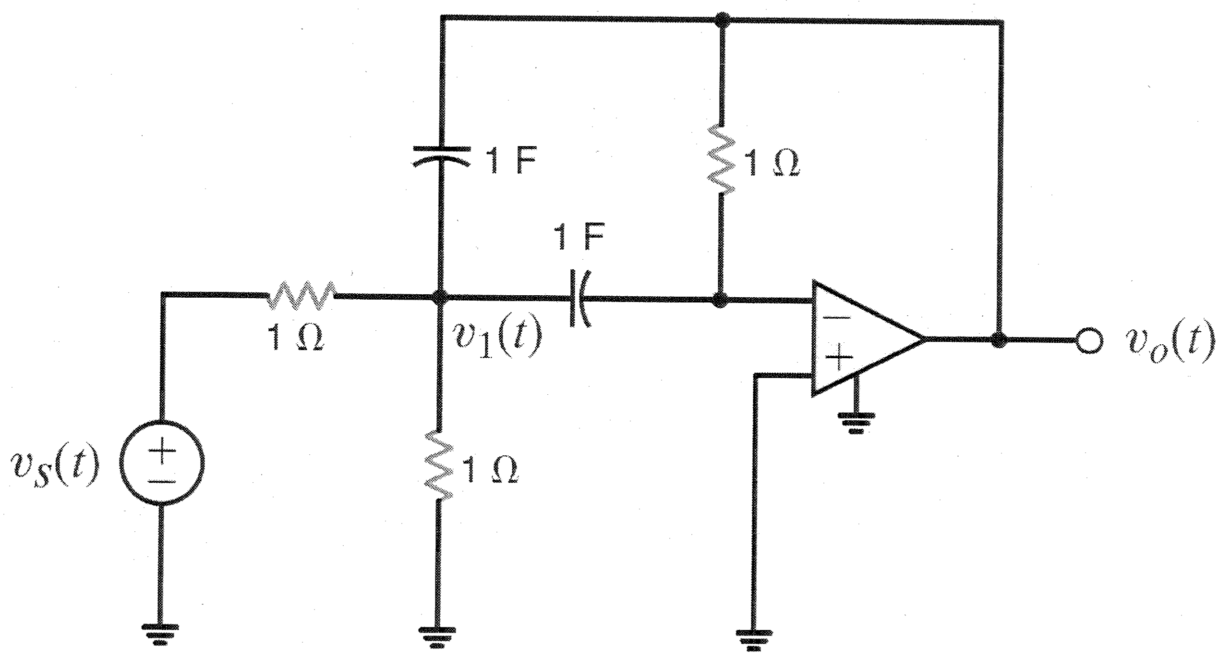
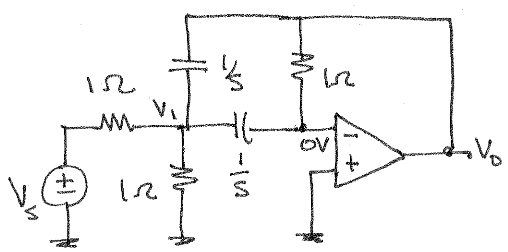


Figure P14.51

SOLUTION:



$$\frac{V_s - V_1}{1} = \frac{V_1}{1} + V_1 s + (V_1 - V_o)s$$

$$\& \quad \frac{V_o}{1} + V_1 s = 0 \Rightarrow \quad V_1 = -V_o/s$$

$$V_s = V_1(2s+2) - sV_o = -V_o\left(\frac{s^2+2s+2}{s}\right)$$

$$\boxed{\frac{V_o}{V_s} = \frac{-s}{s^2+2s+2}}$$

Roots at $s = \frac{-2 \pm \sqrt{4-8}}{2} = -1 \pm j1$ {complex conjugate poles. Network is UNDERDAMPED!}

14.52 The voltage response of the network to a unit step input is

$$\mathbf{V}_o(s) = \frac{2(s + 1)}{s(s^2 + 10s + 25)}$$

Is the response overdamped?

SOLUTION:

3 poles at $s = \begin{cases} 0 \\ -\frac{10}{2} \pm \frac{\sqrt{100-100}}{2} = -5 \end{cases} \Leftarrow$ [These poles are real & equal]

$\boxed{\text{System is critically damped, not overdamped}}$

14.53 The transfer function of the network is given by the expression

$$\mathbf{G}(s) = \frac{100s}{s^2 + 13s + 40}$$

Determine the damping ratio, the undamped natural frequency, and the type of response that will be exhibited by the network.

SOLUTION:

Char. eq. is $s^2 + 13s + 40 = s^2 + 2\zeta\omega_0 s + \omega_0^2 = 0$

$$\boxed{\omega_0 = \sqrt{40} \text{ r/s}}$$

$2\zeta\omega_0 = 13 \Rightarrow \zeta = \frac{13}{2\sqrt{40}} \Rightarrow \boxed{\zeta = 1.03}$

$\boxed{\zeta > 1 \text{ (barely), so system is overdamped}}$

14.54 The transfer function of the network is given by the expression

$$\mathbf{G}(s) = \frac{100s}{s^2 + 22s + 40}$$

Determine the damping ratio, the undamped natural frequency, and the type of response that will be exhibited by the network. CS

SOLUTION:

Char. eq. is: $s^2 + 22s + 40 = s^2 + 2\zeta\omega_0 s + \omega_0^2$

$\boxed{\omega_0 = \sqrt{40} \text{ r/s}}$

$2\zeta\omega_0 = 22 \Rightarrow \boxed{\zeta = 1.74}$ $\boxed{\text{overdamped}}$

14.55 The voltage response of a network to a unit step input is

$$V_o(s) = \frac{10}{s(s^2 + 8s + 18)}$$

Is the response critically damped?

SOLUTION:

$V_I(s) = \frac{1}{s}$ $\qquad H(s) = \frac{V_o(s)}{V_I(s)} = \frac{10}{s^2+8s+18}$

Char. eq. is: $s^2 + 8s + 18 = s^2 + 2\zeta\omega_o s + \omega_o^2 = 0$

$\omega_o = \sqrt{18}$ r/s

$\zeta = \frac{8}{2\sqrt{18}} = 0.94$ $\qquad \boxed{\text{Underdamped! Not critically damped!}}$

14.56 For the network in Fig. P14.56, choose the value of C for critical damping.

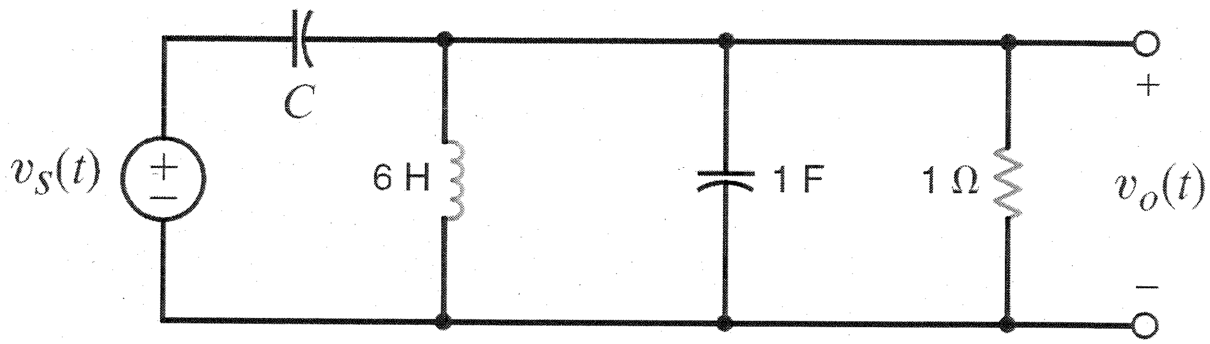
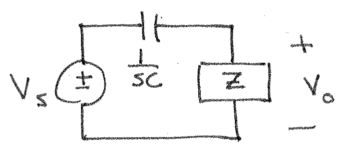


Figure P14.56

SOLUTION:



$$H(s) = \frac{V_o}{V_s} = \frac{Z}{Z + \frac{1}{sC}} \qquad \frac{1}{Z} = \frac{1}{6s} + s + 1 = \frac{6s^2+6s+1}{6s}$$

$$H(s) = \frac{C6s^2}{6Cs^2 + 6s^2 + 6s + 1} = \frac{6Cs^2}{6(C+1)s^2 + 6s + 1}$$

$$H(s) = \frac{\left(\frac{C}{C+1}\right)s}{s^2 + \frac{s}{C+1} + \frac{1}{6(C+1)}}$$

$$\omega_o = \frac{1}{\sqrt{6(C+1)}} \qquad \zeta = 1$$

$$2\zeta\omega_o = \frac{1}{C+1} = \frac{2}{\sqrt{6(C+1)}}$$

$$\sqrt{C+1} = \sqrt{6}/2 \Rightarrow \boxed{C = 0.5F}$$

14.57 For the filter in Fig. P14.57, choose the values of C_1 and C_2 to place poles at $s = -2$ and $s = -5$ rad/s.

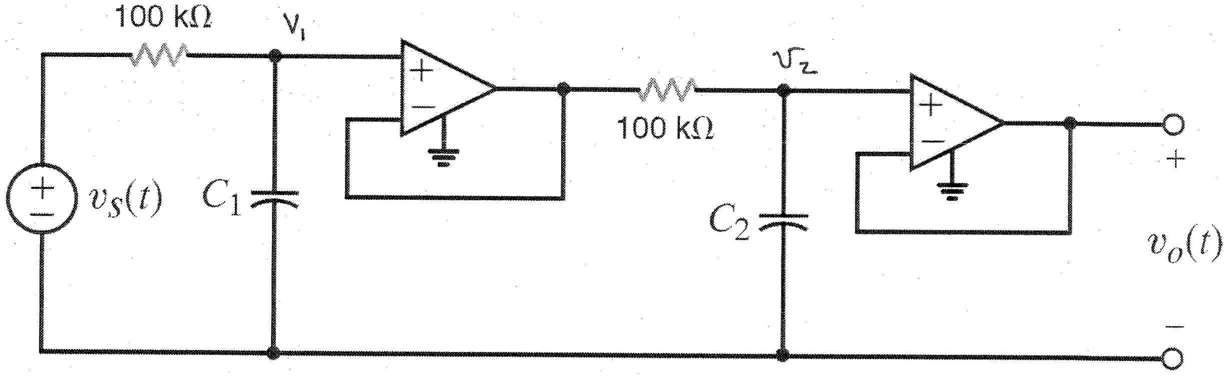
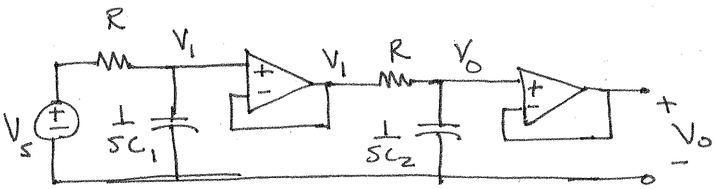


Figure P14.57

SOLUTION:



$R = 100\,k\Omega$

Both op-amps in unity gain configuration!

$$\frac{V_1}{V_s} = \frac{1/sC_1}{R + 1/sC_1} = \frac{1}{sC_1R+1} = \frac{1/RC_1}{s + 1/RC_1}$$

$$\frac{V_o}{V_1} = \frac{1/RC_2}{s + 1/RC_2} \qquad \frac{V_o}{V_s} = \frac{\frac{1}{R^2C_1C_2}}{\left(s + \frac{1}{RC_1}\right)\left(s + \frac{1}{RC_2}\right)}$$

$$\frac{1}{RC_1} = 2 \quad \& \quad \frac{1}{RC_2} = 5 \quad \Rightarrow \quad \boxed{\begin{array}{l} C_1 = 5\,\mu F \\ C_2 = 2\,\mu F \end{array}}$$

14.58 Find the steady-state response $v_o(t)$ for the network in Fig. P14.58.

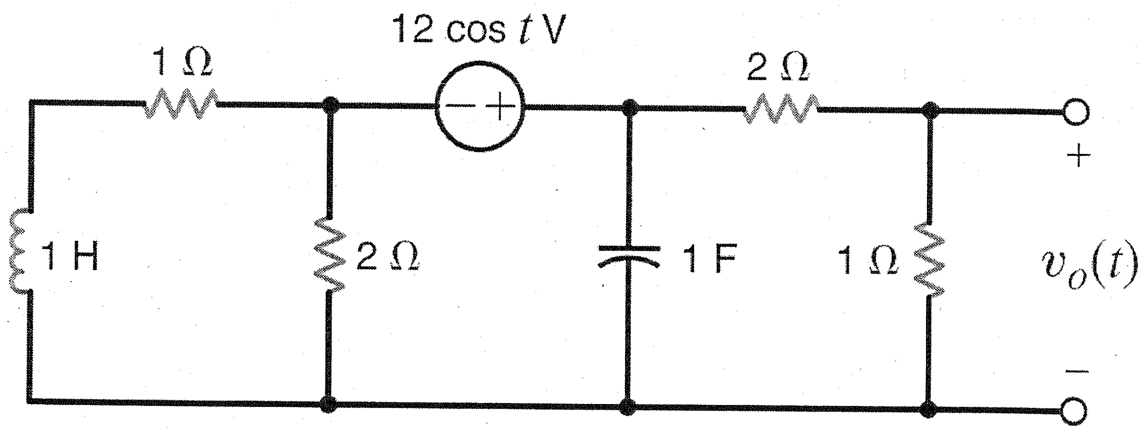
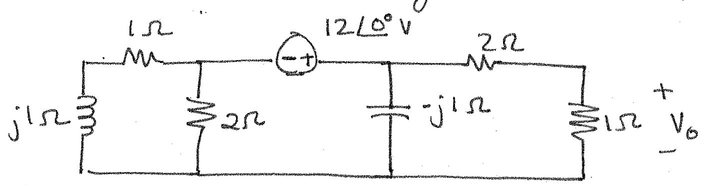


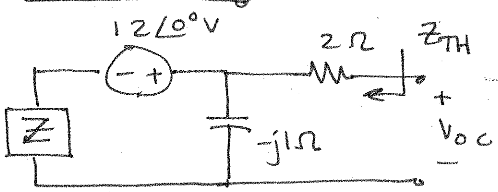
Figure P14.58

SOLUTION: In steady state $s \to j\omega$ and $\cos\omega t \Rightarrow$ phasor.



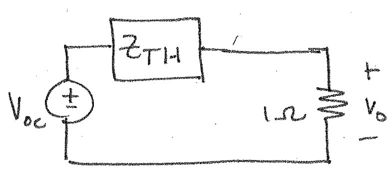
$Z = 2(1+j1)/(3+j1)$

Thevenin eq



$Z_{TH} = 2 + \dfrac{Z(-j1)}{Z-j1} = \dfrac{8-j4}{-j1+3}\ \Omega$

$V_{oc} = 12\angle 0\left(\dfrac{-j1}{-j1+Z}\right) = \dfrac{12(1-j3)}{3-j1}$



$V_o = \dfrac{V_{oc}}{1+Z_{TH}} = \dfrac{12(1-j3)}{17-j7} = \dfrac{12(1-j3)}{11-j5}$

$V_o = 3.13\angle -47.2^\circ\,V$

$$\boxed{V_o(t) = 3.13\cos(t - 47.2^\circ)\,V}$$

14.59 Find the steady-state response $v_o(t)$ for the circuit shown in Fig. P14.59. **PSV**

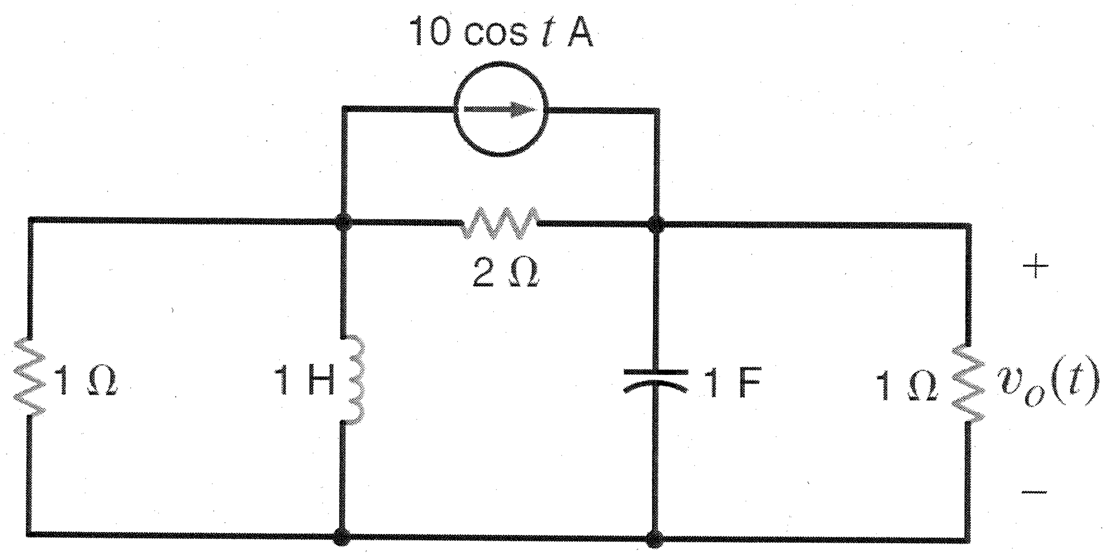
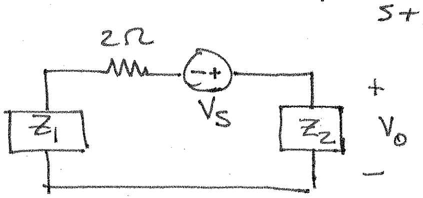


Figure P14.59

SOLUTION: Let $Z_1 = \frac{s}{s+1}$ & $Z_2 = \frac{1}{s+1}$



Eventually V_s is a phasor $\Rightarrow 20\angle 0°$ V

$$V_o = V_S\left(\frac{Z_2}{Z_1+Z_2+2}\right) = V_S\left(\frac{1}{s+1+2s+2}\right) = V_S\left(\frac{1/3}{s+1}\right)$$

In steady state, $s \to j\omega$

$$V_o = 20\angle 0°\left[\frac{1/3}{1+j\omega}\right] \qquad V_o = 4.17\angle -45° \text{ V}$$

$$\boxed{v_o(t) = 4.71 \cos(t - 45°) \text{ V}}$$

14.60 Determine the steady-state response $i_o(t)$ for the network in Fig. P14.60.

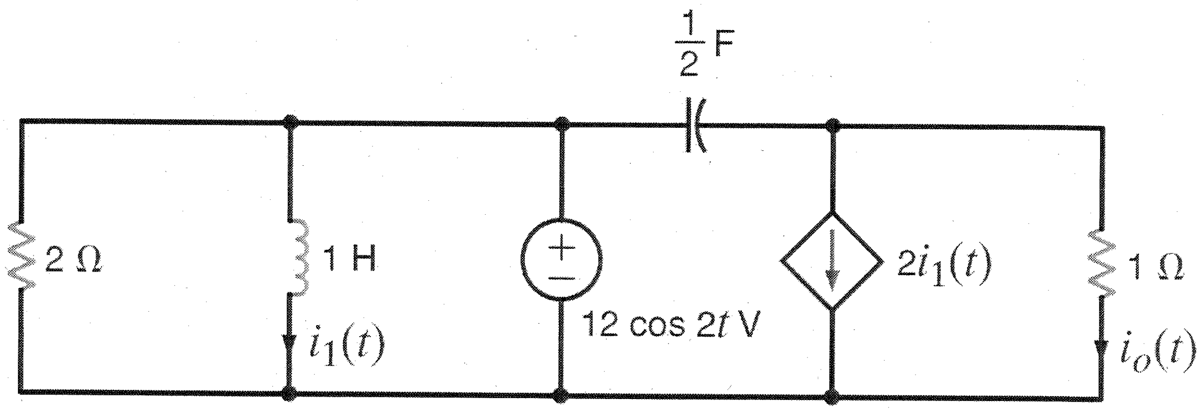
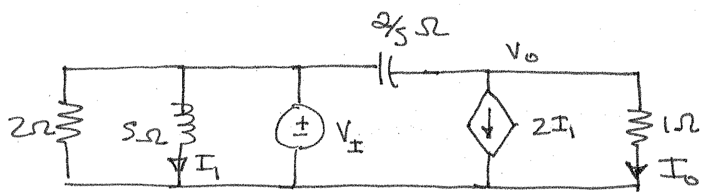


Figure P14.60

SOLUTION: Use KCL,



$I_1 = \frac{V_I}{s}$ $\qquad$ $I_o = \frac{V_o}{1}$

$$\frac{V_o - V_I}{2/s} + 2I_1 + \frac{V_o}{1} = 0 \Rightarrow V_o\left(\frac{s}{2}+1\right) = V_I\left(\frac{s}{2} - \frac{2}{s}\right)$$

$$\frac{V_o}{V_I} = \frac{s-2}{s} \Rightarrow I_o = \frac{V_I\,(s-2)}{s}$$

At steady-state, $V_I = 12\angle 0°$ V & $s = j2$

$I_o = 12\sqrt{2}\,\angle 45°$ A

$$\boxed{i_o(t) = 12\sqrt{2}\cos(2t + 45°)\ A}$$

14.61 Find the steady-state response $i_o(t)$ for the network shown in Fig. P14.61. **CS**

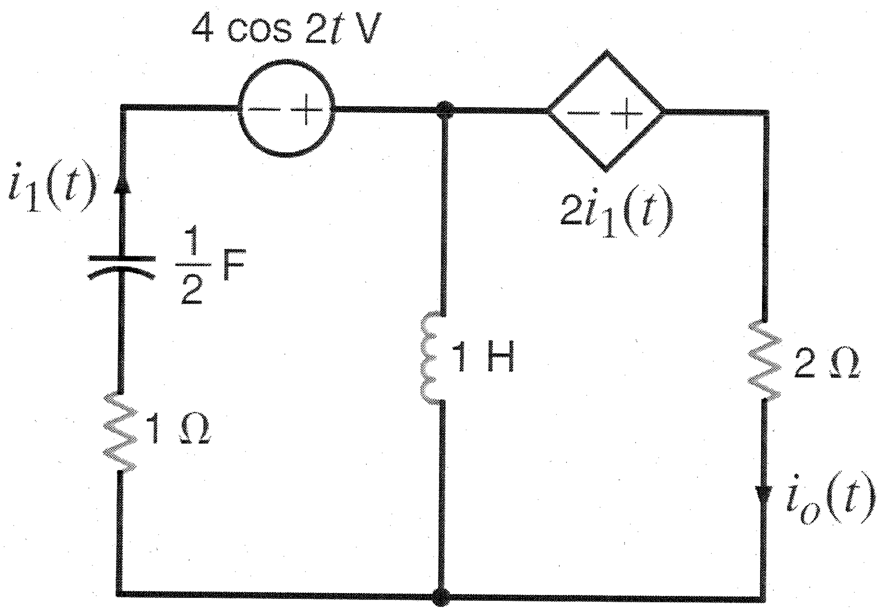
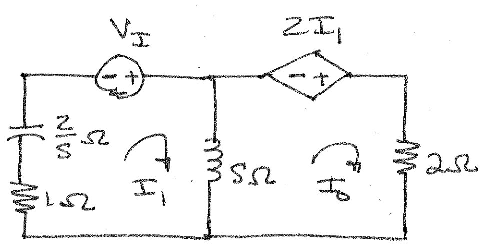


Figure P14.61

SOLUTION:



$$V_I = I_1\left(s+1+2/s\right) - sI_o$$

$$= I_1\left(\frac{s^2+s+2}{s}\right) - sI_o$$

and, $2I_1 = -sI_1 + (s+2)I_o$

or, $0 = -I_1(s+2) + I_o(s+2)$

$\Leftarrow$ yields $I_1 = I_o$

$$V_I = I_o\left[\frac{s^2+s+2-s^2}{s}\right]$$

$$I_o = \frac{V_I(s)}{s+2}$$

In steady state, $V_I = 4\angle 0° \text{ V}$ $\quad s \to j2$

$$I_o = \frac{4\angle 0°\,(j2)}{2+j2} = 2\sqrt{2}\angle 45° \text{ A}$$

$$\boxed{i_o(t) = 2\sqrt{2}\cos(2t+45°) \text{ A}}$$

14.62 Find the steady-state response $v_o(t)$, for $t > 0$, in the network in Fig. P14.62.

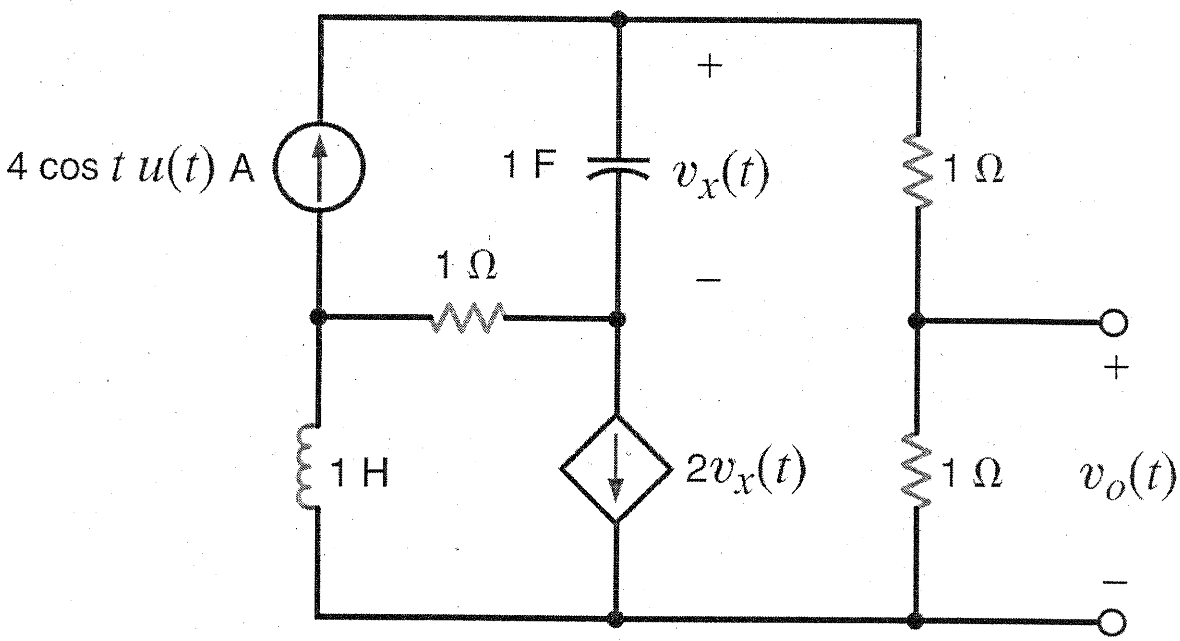
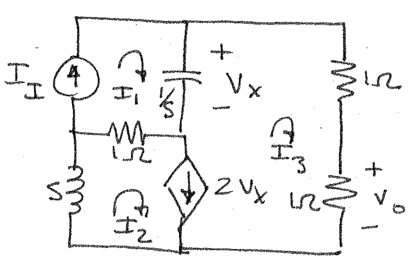


Figure P14.62

SOLUTION:



$I_1 = I_I \qquad I_2 - I_3 = 2V_x = 2\,(I_1 - I_3)/s$

yields $\quad I_2 = \dfrac{2I_1}{s} + I_3\left(1 - \dfrac{2}{s}\right)$

and

$I_2(s+1) + I_3(2 + 1/s) - I_1\,(1 + 1/s) = 0$

$\Leftarrow$

$$I_1\left[\frac{2}{s}(s+1) - \frac{s+1}{s}\right] + I_3\left[\frac{s-2}{s}(s+1) + \frac{2s+1}{s}\right] = 0 \qquad V_o = (1)\,I_3$$

$$I_I\left[2s + 2 - s - 1\right] + V_o\left[s^2 - s - 2 + 2s + 1\right] = 0$$

$$V_o = -I_I\,(s+1)/(s^2 + s - 1)$$

In steady state, $I_I = 4\angle 0°\,A$ & $s = j1 \Rightarrow V_o = 2.53\angle 71.6°\ V$

$$\boxed{v_o(t) = 2.53\cos(t + 71.6°)\ V}$$

14.63 Find the steady-state response $v_o(t)$, for $t > 0$, in the network in Fig. P14.63.

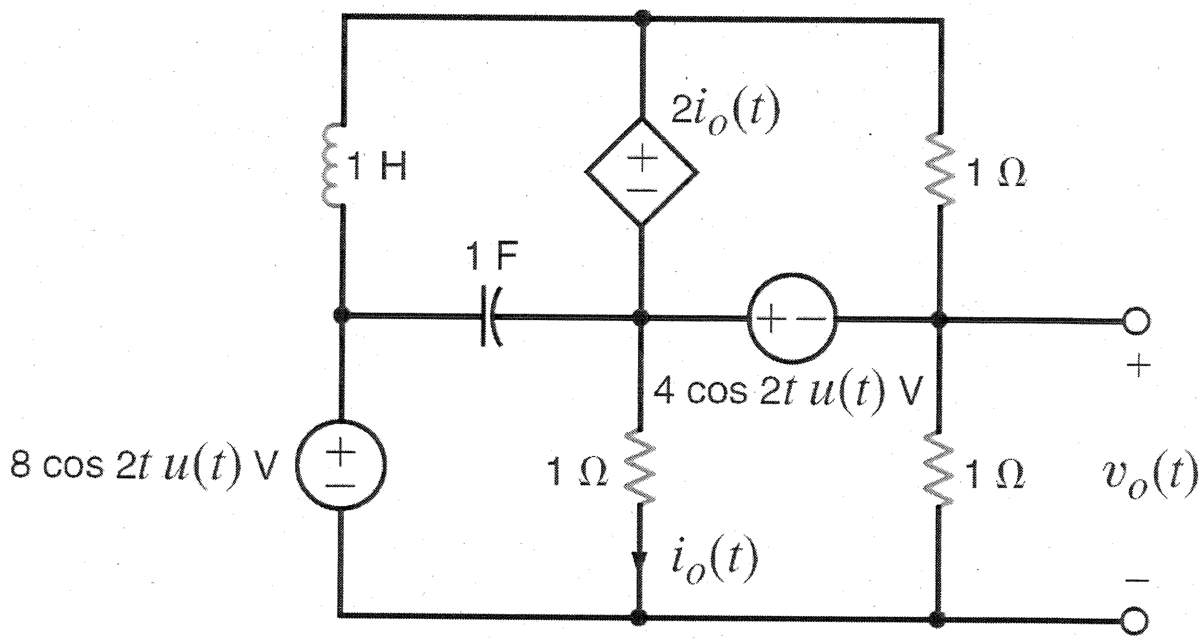
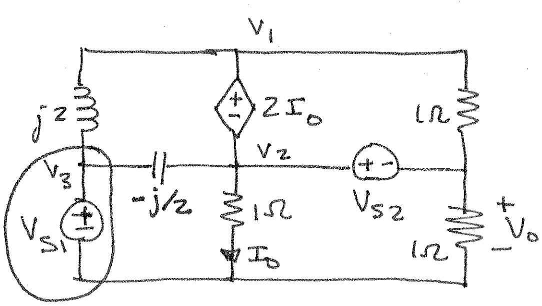


Figure P14.63

SOLUTION: Go straight to freq domain. $s \to j2$ & sources $\to$ phasors.



$V_{S1} = 8\angle 0° \text{ V}$ $\quad V_{S2} = 4\angle 0° \text{ V}$

$V_3 = 8\angle 0° \text{ V} \qquad V_2 - V_o = 4\angle 0° \text{ V}$

$V_1 - V_2 = 2I_o = 2(V_2/1) \Rightarrow V_1 = 3V_2$

At super node:

$$\frac{V_1 - V_3}{j2} + \frac{V_2 - V_3}{-j/2} + \frac{V_2}{1} + \frac{V_o}{1} = 0$$

yields $V_o = 5.22\angle 97.8° \text{ V}$

$$\boxed{v_o(t) = 5.22 \cos(2t + 97.8°) \text{ V}}$$

14FE-1 A single loop, second-order circuit is described by the following differential equation.

$$2\frac{dv^2(t)}{dt^2} + 4\frac{dv(t)}{dt} + 4v(t) = 12u(t) \qquad t > 0$$

Which is the correct form of the total (natural plus forced) response? **CS**

(a) $v(t) = K_1 + K_2e^{-t}$

(b) $v(t) = K_1 \cos t + K_2 \sin t$

(c) $v(t) = K_1 + K_2te^{-t}$

(d) $v(t) = K_1 + K_2e^{-t} \cos t + K_3e^{-t} \sin t$

SOLUTION:

Natural response has char eq: $s^2+2s+2=0$

roots are at $s=-1\pm j1$ ⇒ natural response is sinusodial!

Forced response is constant $= K_1$

$\boxed{\text{Answer is (d)}}$

14FE-2 If all initial conditions are zero in the network in Fig. 14PFE-2, find the transfer function $\mathbf{V}_o(s)/\mathbf{V}_S(s)$, and determine the type of damping exhibited by the network.

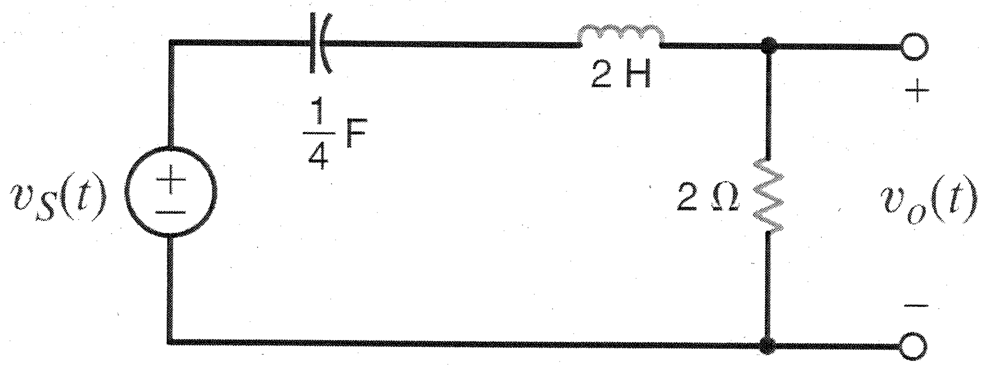


Figure 14PFE-2

SOLUTION:

$$\frac{V_o(s)}{V_s(s)} = \frac{2}{2+2s+4/s} = \frac{2s}{2s^2+2s+4} = \frac{s}{s^2+s+2}$$

$$\boxed{\frac{V_o}{V_s} = \frac{s}{s^2+s+2}}$$

Poles at $s = -\frac{1}{2} \pm j\frac{\sqrt{7}}{2}$

since poles are complex, the circuit is underdamped

14FE-3 The initial conditions in the circuit in Fig. 14PFE-3 are zero. Find the transfer function $\mathbf{I}_o(s)/\mathbf{I}_s(s)$, and determine the type of damping exhibited by the circuit.

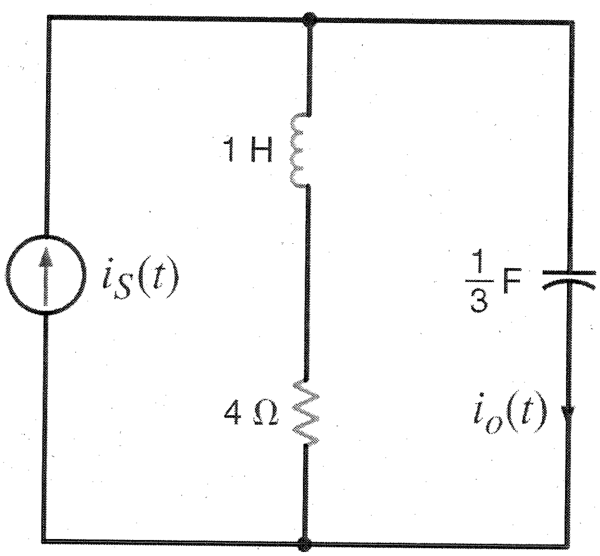
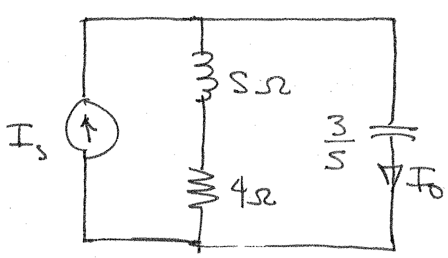


Figure 14PFE-3

SOLUTION:



$$\frac{I_o}{I_s} = \frac{3/s}{3/s + 4 + s} = \frac{3}{s^2+4s+3}$$

$$\boxed{\frac{I_o}{I_s} = \frac{3}{s^2+4s+3}}$$

Char equation: s^2+4s+3

Poles at $s = -2 \pm 1 = \begin{cases} -1 \\ -3 \end{cases}$

$\boxed{\text{Poles are real and unequal, network is overdamped}}$

Chapter Fifteen:

Fourier Analysis Techniques

15.1 Find the exponential Fourier series for the periodic signal shown in Fig. P15.1.

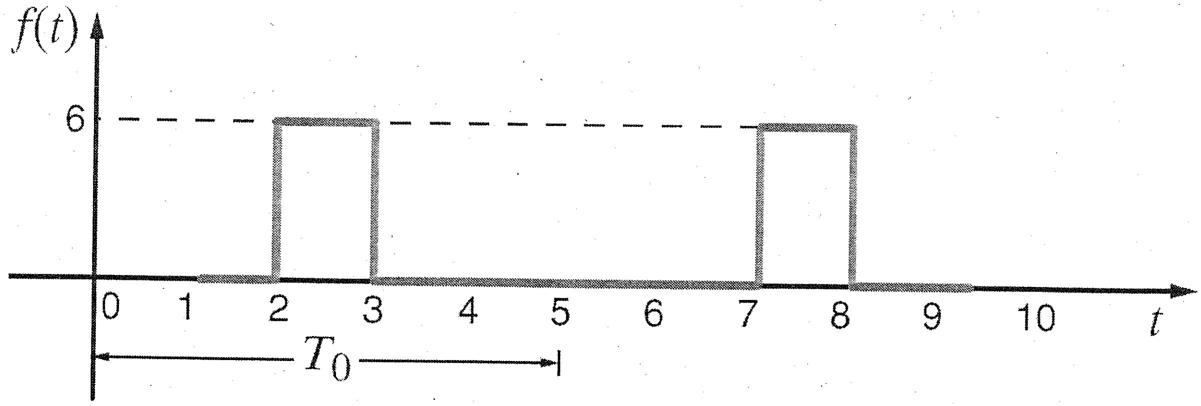


Figure P15.1

SOLUTION: $T_o = 5$ sec. $\omega_o T_o = 2\pi$

$$C_n = \frac{1}{T_o}\int_0^{T_o/5} 6\, e^{-jn\omega_o t}\,dt = \frac{6}{jn2\pi}\left[e^{jn(\pi/5)} - e^{-jn(\pi/5)}\right]$$

$$C_n = \frac{6\, e^{-jn(\pi/5)}}{n\pi} \sin\left(n\pi/5\right) \quad \therefore \quad f(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_o t}$$

$$\boxed{f(t) = \sum_{n=-\infty}^{\infty} \frac{6}{n\pi} e^{-jn\pi} \sin\left(n\pi/5\right) e^{j0.4\pi t}}$$

15.2 Find the exponential Fourier series for the periodic pulse train shown in Fig. P15.2. **PSV**

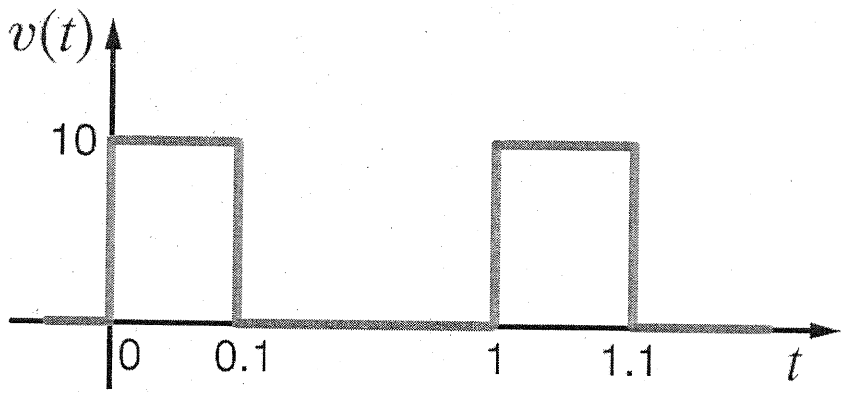


Figure P15.2

SOLUTION:

$$C_0 = \frac{10(0.1)}{1} = 1 \qquad T_0 = 1 \qquad \omega_0 = 2\pi$$

$$C_n = \frac{1}{T_0}\int_0^{T_0/10} 10\, e^{-jn\omega_0 t}\, dt = \frac{10 e^{-jn\pi/10}}{n\pi}\left[\frac{e^{jn\pi/10} - e^{-jn\pi/10}}{j2}\right]$$

$$C_n = \frac{10}{n\pi} e^{-jn\pi/10} \sin(n\pi/10)$$

$$\boxed{f(t) = \frac{10}{\pi}\sum_{n=-\infty}^{\infty} \frac{1}{n} e^{-jn\pi/10} \sin(n\pi/10)\, e^{jn\omega_0 t}}$$

15.3 Find the exponential Fourier series for the signal shown in Fig. P15.3. CS

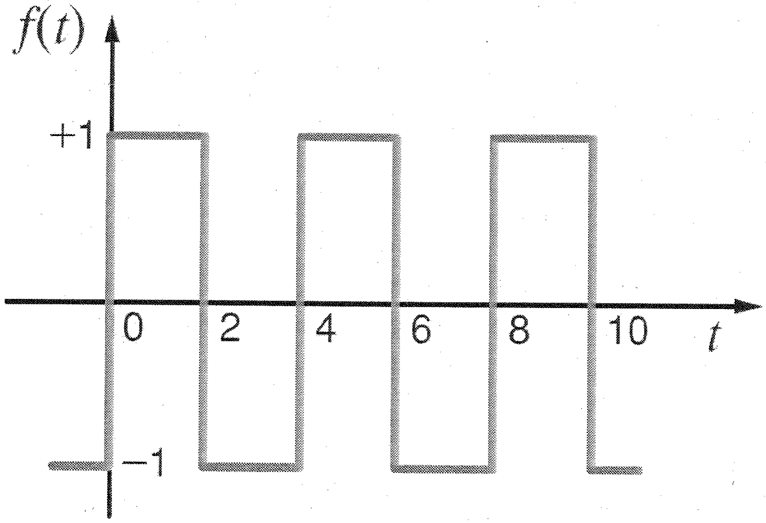


Figure P15.3

SOLUTION:

$$C_n = \frac{1}{T_0}\int_{-T_0/2}^{0} -e^{-jn\omega_0 t}\,dt + \frac{1}{T_0}\int_{0}^{T_0/2} e^{-jn\omega_0 t}\,dt \qquad \omega_0 = 2\pi/T_0$$

$$C_n = \frac{1}{T_0 jn\omega_0}\left[1 - e^{-jn\omega_0 T_0/2} + 1 - e^{jn\omega_0 T_0/2}\right] = \frac{1}{2jn\pi}\left[2 - e^{jn\pi} - e^{-jn\pi}\right]$$

$$C_n = \frac{1}{2jn\pi}\left[2 - 2\cos(n\pi)\right] = \frac{1 - \cos(n\pi)}{jn\pi} = \begin{cases} 0 & n \text{ even} \\ \frac{2}{jn\pi} & n \text{ odd} \end{cases}$$

$$\boxed{f(t) = \sum_{\substack{n=-\infty \\ n \text{ odd}}}^{\infty} \frac{2}{jn\pi} e^{jn\omega_0 t}}$$

15.4 Find the exponential Fourier series for the signal shown in Fig. P15.4. CS

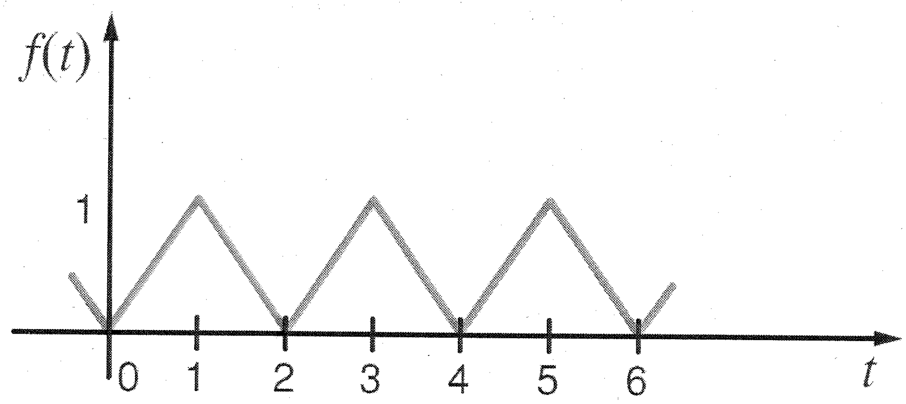


Figure P15.4

SOLUTION: $T_0 = 2\text{ sec}$ $\quad \omega_0 T_0 = 2\pi \Rightarrow \omega_0 = \pi$ $\quad$ let $a = jn\pi$

$$C_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} f(t)\,dt = \frac{1}{2}\int_{-1}^{0} -t\,e^{-jn\omega_0 t}\,dt + \frac{1}{2}\int_{0}^{1} t\,e^{-jn\omega_0 t}\,dt$$

$$C_n = \frac{1}{2}\left\{\int_{-1}^{0} -t\,e^{-at}\,dt + \int_{0}^{1} t\,e^{-at}\,dt\right\} = \frac{1}{2}\left\{\left(\frac{1}{a^2}+\frac{t}{a}\right)e^{-at}\Big|_{-1}^{0} + \left(\frac{1}{a^2}+\frac{t}{a}\right)e^{-at}\Big|_{1}^{0}\right\}$$

$$C_n = \frac{1}{2}\left\{\frac{1}{a^2} - \left(\frac{1}{a^2}-\frac{1}{a}\right)e^{a} + \left(\frac{1}{a^2}\right) - \left(\frac{1}{a^2}+\frac{1}{a}\right)e^{-a}\right\}$$

$$C_n = \frac{1}{2a^2}\left\{2 + e^{a}(a-1) - e^{-a}(a+1)\right\} = -\frac{1}{2n^2\pi^2}\left\{2 + e^{jn\pi}(jn\pi-1) - e^{-jn\pi}(jn\pi+1)\right\}$$

$C_n = \dfrac{-2}{n^2\pi^2}$ if n is odd $\qquad C_n = 0$ if n is even

$C_0 = 1/2$

$$\boxed{f(t) = \frac{1}{2} + \sum_{\substack{n=-\infty \\ n\neq 0 \\ n\ \text{odd}}}^{\infty} \frac{-2}{n^2\pi^2}\, e^{jn\pi t}}$$

15.5 Compute the exponential Fourier series for the waveform that is the sum of the two waveforms in Fig. P15.5 by computing the exponential Fourier series of the two waveforms and adding them.

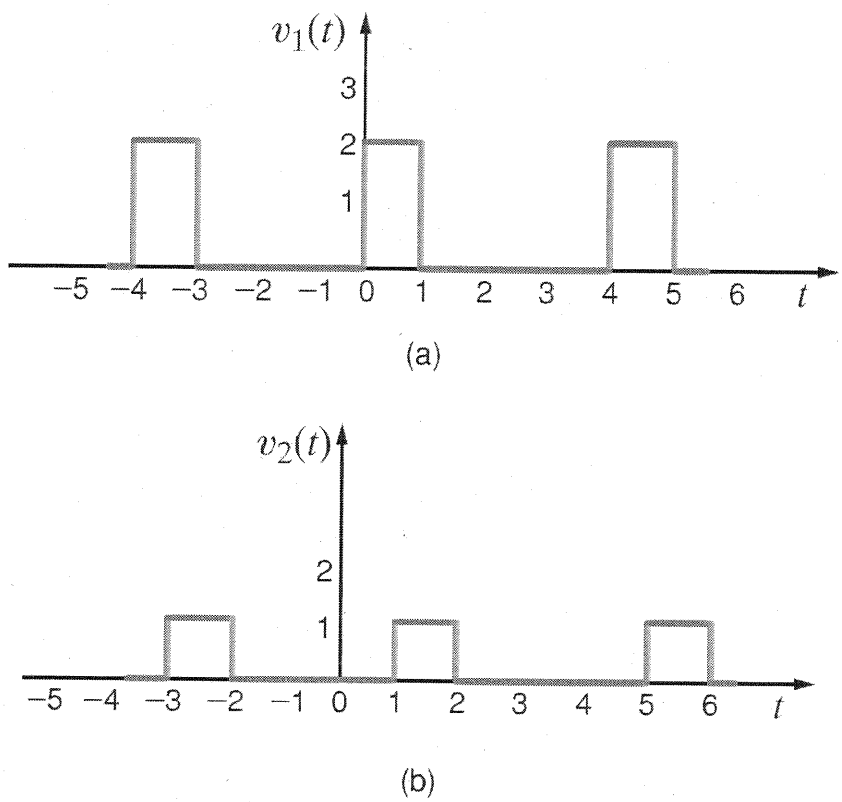


Figure P15.5

SOLUTION: $T_0 = 4 \qquad \omega_0 T_0 = 2\pi \quad \Rightarrow \quad \omega_0 = \pi/2$

$$C_{n_1} = \frac{2}{T_0}\int_0^{T_0/4} e^{-jn\omega_0 t}\,dt = \frac{2}{jn2\pi}\left[1 - e^{-jn\pi/2}\right]$$

$$C_{n2} = \frac{C_{n1}}{2} e^{-jn\omega_0}$$ Since $v_2(t)$ is half $v_1(t)$ shifted by 1 sec.

$$C_n = C_{n1} + C_{n2} = \frac{1}{jn2\pi}\left\{2 - 2e^{-jn\pi/2} + e^{-jn\pi/2} - e^{-jn\pi}\right\}$$

$$C_n = \frac{1}{jn\pi}\left\{1 - e^{-jn3\pi/4}\left[\frac{e^{jn\pi/4} + e^{-jn\pi/4}}{2}\right]\right\} = \frac{1}{jn\pi}\left(1 - e^{-jn3\pi/4}\cos(n\pi/4)\right)$$

$$v(t) = \sum_{n=-\infty}^{\infty} \frac{1}{jn\pi}\left\{1 - e^{-jn3\pi/4}\cos(n\pi/4)\right\} e^{jn\pi t/2} \text{ V}$$

15.6 Given the waveform in Fig. P15.6, determine the type of symmetry that exists if the origin is selected at (a) l_1 and (b) l_2.

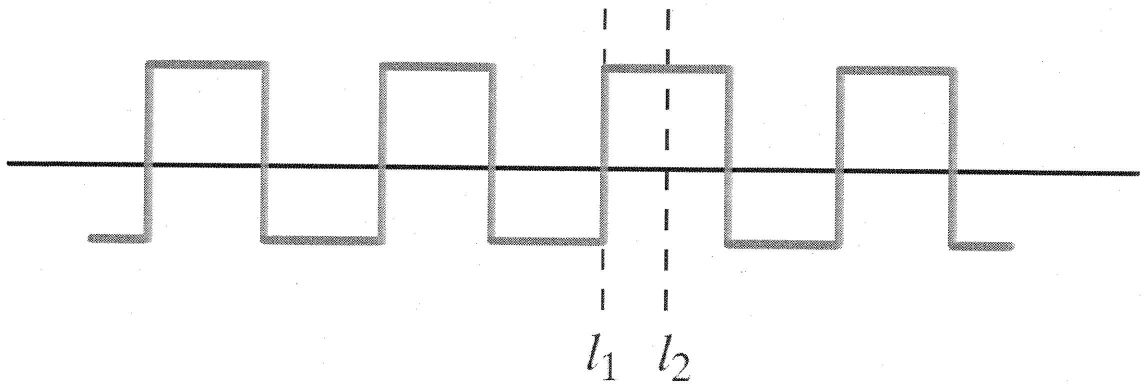


Figure P15.6

SOLUTION:

If origin is at l_1, $v(t) = -v(-t)$ odd symmetry

If origin is at l_2, $v(t) = v(-t)$ even symmetry

15.7 What type of symmetry is exhibited by the two waveforms in Fig. P15.7?

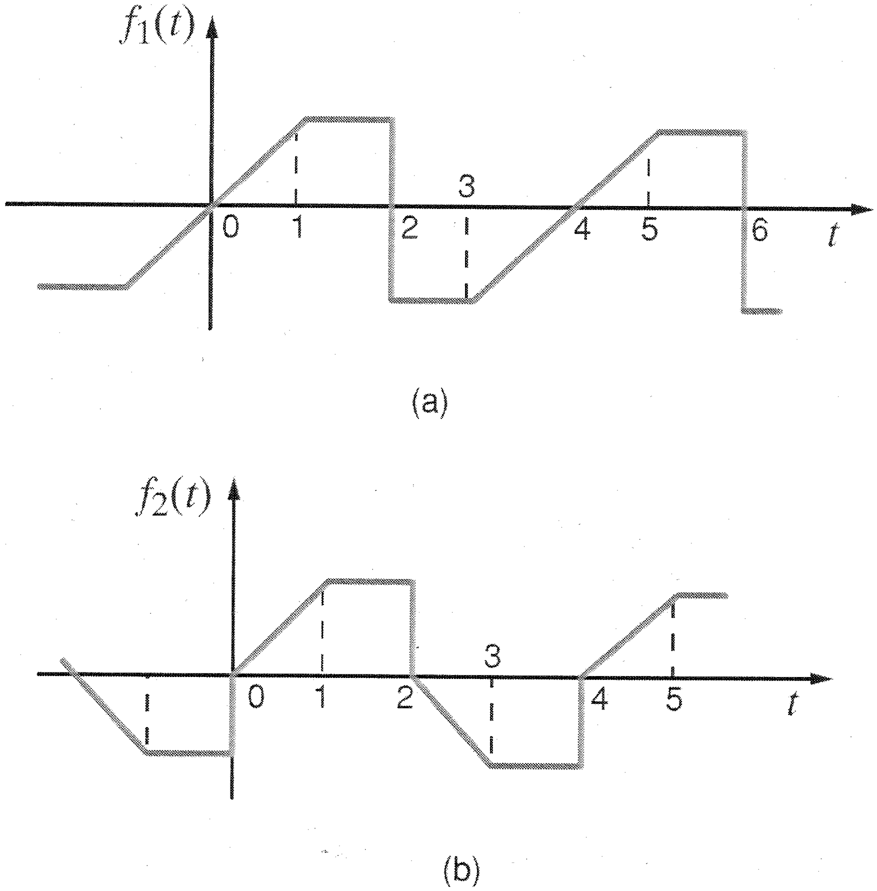


Figure P15.7

SOLUTION:

(a) $f_1(t) = -f_1(-t)$ odd symmetry

(b) $f_2(t) = -f_2(t - T_0/2)$ half wave symmetry

15.8 Find the trigonometric Fourier series for the waveform shown in Fig. P15.8.

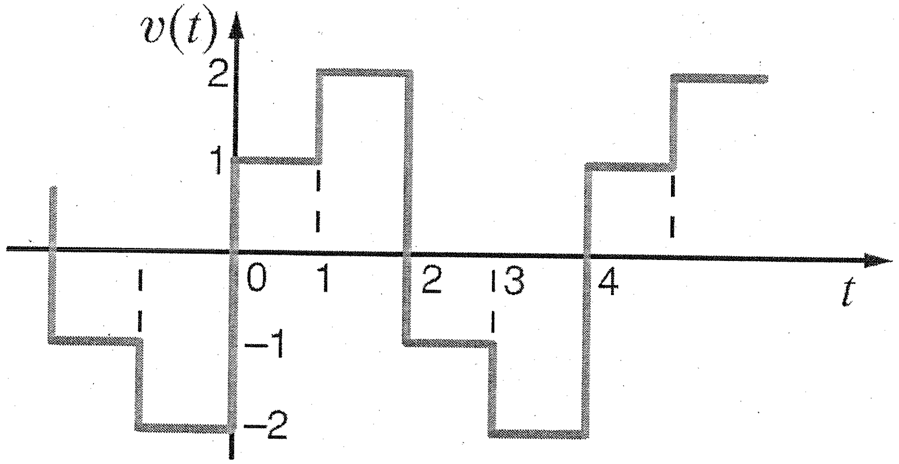


Figure P15.8

SOLUTION: $v(t)$ has half wave symmetry!

$a_0 = 0 \quad T_0 = 4\text{ sec} \quad \omega_0 = \pi/2 \quad a_n = b_n = 0$ for n even

$$a_n = \frac{4}{T_0}\int_0^{T_0/2} f(t)\cos(n\omega_0 t)\,dt = \int_0^1 \cos(n\omega_0 t\,dt) + 2\int_1^2 \cos(n\omega_0 t)\,dt$$

$$a_n = \frac{\sin(n\omega_0 t)}{n\omega_0}\bigg|_0^1 + \frac{2\sin(n\omega_0 t)}{n\omega_0}\bigg|_1^2 = \frac{\sin(n\pi/2) + 2\sin(n\pi) - 2\sin(n\pi/2)}{n\pi/2}$$

for n odd, $a_n = \dfrac{-2}{n\pi}\sin(n\pi/2)$

$$b_n = \int_0^1 \sin(n\omega_0 t)\,dt + 2\int_1^2 \sin(n\omega_0 t)\,dt = \frac{\cos(n\omega_0 t)}{n\omega_0}\bigg|_1^0 + \frac{2\cos(n\omega_0 t)}{n\omega_0}\bigg|_2^1$$

$$b_n = \frac{1 - \cos(n\pi/2) + 2\cos(n\pi/2) - 2\cos(n\pi)}{n\pi/2}$$

for n odd, $b_n = \dfrac{2}{n\pi}\left[1 \mp \cos(\pi/2) \pm 2\cos(\pi/2) + 2\cos(0)\right] = \dfrac{6}{n\pi}$

$$v(t) = \frac{2}{\pi}\sum_{\substack{n=1 \\ n\,\text{odd}}}^{\infty}\left[\frac{3}{n}\sin(n\pi t/2) - \frac{\sin(n\pi/2)}{n}\cos(n\pi/2)\right] \text{V}$$

15.9 Find the trigonometric Fourier series for the periodic waveform shown in Fig. P15.9.

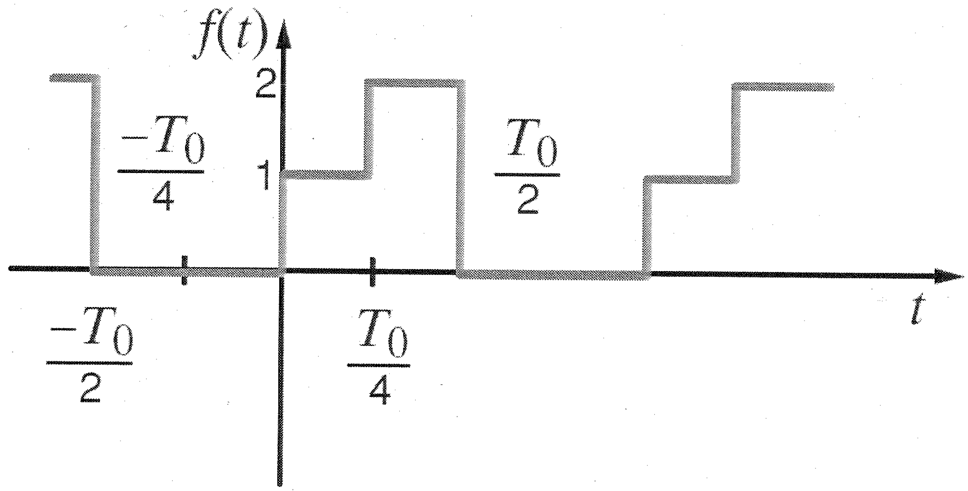


Figure P15.9

SOLUTION: $a_0 = \dfrac{1(T_0/4) + 2(T_0/4)}{T_0} = 3/4$

$$a_n = \frac{2}{T_0}\left\{\int_0^{T_0/4} \cos(n\omega_0 t)\,dt + 2\int_{T_0/4}^{T_0/2} \cos(n\omega_0 t)\,dt\right\}$$

$$a_n = \frac{2}{n\omega_0 T_0}\left\{\sin(n\omega_0 t)\Big|_0^{T_0/4} + 2\sin(n\omega_0 t)\Big|_{T_0/4}^{T_0/2}\right\} = -\frac{\sin(n\pi/2)}{n\pi}$$

$$b_n = \frac{2}{T_0}\left\{\int_0^{T_0/4} \sin(n\omega_0 t)\,dt + 2\int_{T_0/4}^{T_0/2} \sin(n\omega_0 t)\,dt\right.$$

$$b_n = \frac{2}{n\omega_0 T_0}\left\{\cos(n\omega_0 t)\Big|_{T_0/4}^{0} + 2\cos(n\omega_0 t)\Big|_{T_0/2}^{T_0/4}\right. = \frac{1}{n\pi}\Big[1 + \cos(n\pi/2) - 2\cos(n\pi)\Big]$$

$$f(t) = \frac{3}{4} + \sum_{n=1}^{\infty} \frac{1}{n\pi}\Big[\big(1 + \cos(n\pi/2) - 2\cos(n\pi)\big)\sin(n\omega_0 t) - \sin(n\pi/2)\cos(n\omega_0 t)\Big] \checkmark$$

15.10 Given the waveform in Fig. P15.10 show that

$$f(t) = \frac{A}{2} + \sum_{n=1}^{\infty} \frac{-A}{n\pi} \sin \frac{2n\pi}{T_0} t \qquad \textbf{PSV}$$

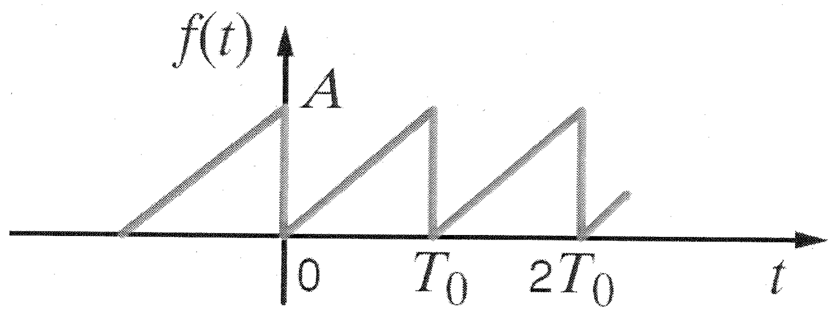


Figure P15.10

SOLUTION: $a_0 = A/2$ except for a_0, odd symmetry, so $a_n = 0$ $n \neq 0$

$$b_n = \frac{2A}{T_0} \int \frac{t}{T_0} \sin(n\omega_0 t)\, dt = \frac{2A}{T_0^2} \left[\frac{\sin(n\omega_0 t)}{(n\omega_0)^2} - \frac{t\cos(n\omega_0 t)}{n\omega_0} \right]_0^{T_0}$$

$$b_n = \frac{2A}{T_0^2} \left[\frac{\sin(n2\pi)}{(n\omega_0)^2} - \frac{T_0\cos(n2\pi)}{n\omega_0} \right] = \frac{-2A}{T_0} \cdot \left(\frac{1}{n\omega_0} \right) = \frac{-2A}{n2\pi} = -\frac{A}{n\pi}$$

$$\boxed{f(t) = \frac{A}{2} + \sum_{n=1}^{\infty} \frac{-A}{n\pi} \sin\left(\frac{2n\pi}{T_0} t \right)}$$

15.11 Find the trigonometric Fourier series coefficients for the waveform in Fig. P15.11. CS

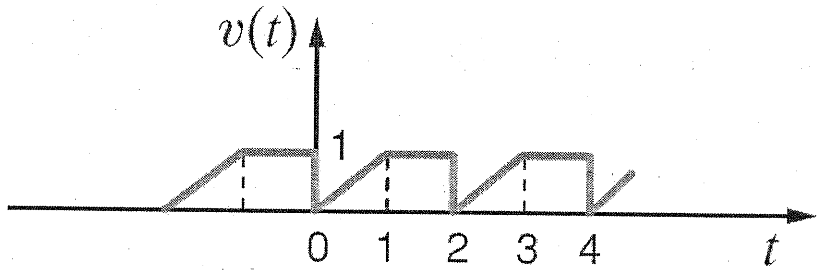


Figure P15.11

SOLUTION: $a_0 = \dfrac{(1/2)(1)(1) + (1)(1)}{2} = \dfrac{3}{4}$ $\quad T_0 = 2\text{ sec} \quad \omega_0 = \pi$

$$a_n = \frac{2}{T_0}\int_0^1 t\cos(n\omega_0 t)\,dt + \frac{2}{T_0}\int_1^2 \cos(n\omega_0 t)\,dt = \int_0^1 t\cos(n\pi t)\,dt + \int_1^2 \cos(n\pi t)\,dt$$

$$a_n = \frac{\sin(n\pi)}{n\pi} + \frac{\cos(n\pi)}{(n\pi)^2} - \frac{1}{(n\pi)^2} + \frac{\sin(2n\pi)}{n\pi} - \frac{\sin(n\pi)}{n\pi} = \frac{\cos(n\pi) - 1}{(n\pi)^2}$$

$$b_n = \frac{2}{T_0}\left\{\int_0^1 t\sin(n\omega_0 t)\,dt + \int_1^2 \sin(n\omega_0 t)\,dt\right\} = \int_0^1 t\sin(n\pi t)\,dt + \int_1^2 \sin(n\pi)\,dt$$

$$b_n = \frac{\sin(n\pi)}{(n\pi)^2} - \frac{\cos(n\pi)}{n\pi} + \frac{\cos(n\pi)}{n\pi} - \frac{\cos(2n\pi)}{n\pi} = -\frac{1}{n\pi}$$

$$\boxed{a_0 = 3/4 \qquad a_n = \frac{\cos(n\pi) - 1}{(n\pi)^2} \qquad b_n = -\frac{1}{n\pi}}$$

15.12 Find the trigonometric Fourier series coefficients for the waveform in Fig. P15.12.

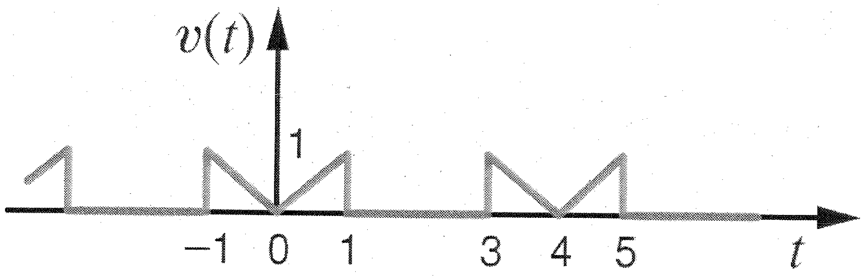


Figure P15.12

SOLUTION: $T_0 = 4 \qquad \omega_0 = \pi/2 \qquad a_0 = \frac{1}{4}\left[\frac{1}{2}(1)(1)+\frac{1}{2}(1)(1)\right] = 1/4$

Even symmetry, $b_n = 0$

$$a_n = \frac{4}{T_0}\int_0^{T_0/2} v(t)\cos(n\omega_0 t)\,dt = \int_0^1 t\cos(n\omega_0 t)\,dt = \left[\frac{\cos(n\omega_0 t)}{(n\omega_0)^2} + \frac{t\sin(n\omega_0 t)}{n\omega_0}\right]_0^1$$

$$a_n = \frac{\cos(n\pi/2)-1}{(n\pi/2)^2} + \frac{\sin(n\pi/2)}{n\pi/2} = \frac{4}{(n\pi)^2}\left(\cos(n\pi/2)-1\right) + \frac{2}{n\pi}\sin(n\pi/2)$$

$$\boxed{a_0 = 1/4 \qquad b_n = 0 \qquad a_n = \frac{4}{(n\pi)^2}\left(\cos(n\pi/2)-1\right) + \frac{2}{n\pi}\sin(n\pi/2)}$$

15.13 Find the trigonometric Fourier series coefficients for the waveform in Fig. P15.13.

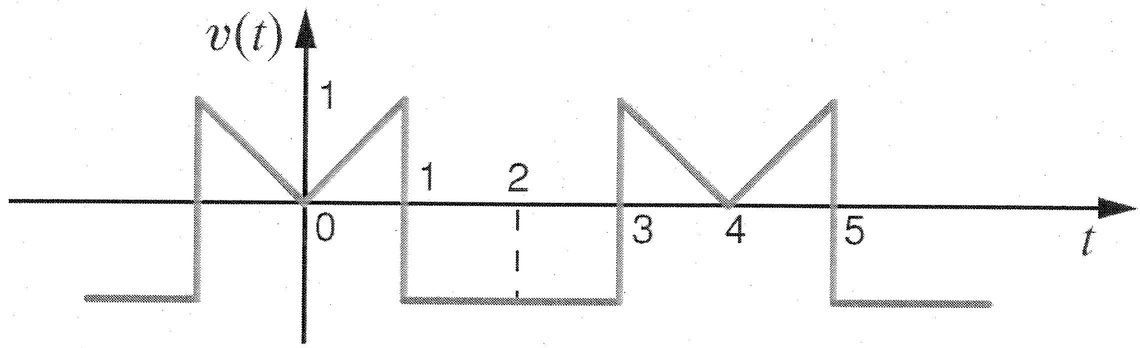


Figure P15.13

SOLUTION: $T_0 = 4 \quad \omega_0 = \pi/2 \quad a_0 = \dfrac{\frac{1}{2}(1)(1) - 2(1) + \frac{1}{2}(1)(1)}{4} = -\dfrac{1}{4}$

even symmetry, so $b_n = 0$

$$a_n = \frac{4}{T_0}\int_0^{T_0/2} v(t)\cos(n\omega_0 t)\,dt = \int_0^1 t\cos(n\omega_0 t)\,dt - \int_1^2 \cos(n\omega_0 t)\,dt$$

$$a_n = \left[\frac{\cos(n\omega_0 t)}{(n\omega_0)^2} + \frac{t\sin(n\omega_0 t)}{n\omega_0}\right]_0^1 - \frac{\sin(n\omega_0 t)}{n\omega_0}\Bigg|_1^2$$

$$a_n = \frac{\cos(n\omega_0) - 1}{(n\omega_0)^2} + \frac{\sin(n\omega_0)}{n\omega_0} - \cancel{\frac{\sin(2n\omega_0)}{n\omega_0}}^{\,0} + \frac{\sin(n\omega_0)}{n\omega_0}$$

$$\boxed{a_n = \frac{4}{(n\pi)^2}\left(\cos(n\pi/2) - 1\right) + \frac{4}{n\pi}\sin(n\pi/2) \qquad a_0 = -\frac{1}{4} \qquad b_n = 0}$$

15.14 Find the trigonometric Fourier series coefficients for the waveform in Fig. P15.14.

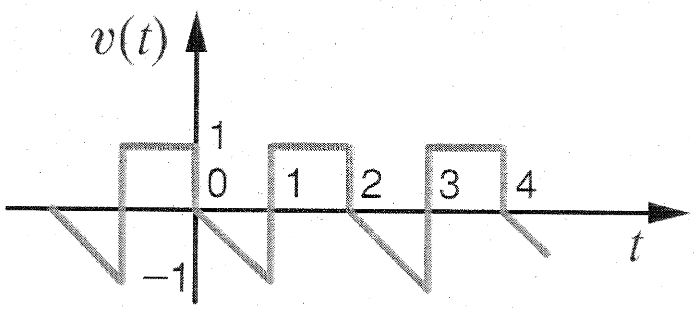


Figure P15.14

SOLUTION: $T_0 = 2\text{ sec}$ $\quad \omega_0 = \pi \text{ r/s}$ $\quad a_0 = \dfrac{-(1/2)(1)(1) + (1)(1)}{2} = \dfrac{1}{4}$

$$a_n = \frac{2}{T_0}\left\{\int_0^1 -t\cos(n\omega_0 t)\,dt + \int_1^2 \cos(n\omega_0 t)\,dt\right\} = \frac{\cos(n\omega_0 t)}{(n\pi)^2} + \frac{t\sin(n\omega_0 t)}{n\omega_0}\Bigg|_1^0 + \frac{\sin(n\omega_0 t)}{n\omega_0}\Bigg|_1^2$$

$$a_n = \frac{1-\cos(n\pi)}{(n\pi)^2} - \frac{\sin(n\pi)}{n\pi} + \frac{\sin(2n\pi)}{n\pi} - \frac{\sin(n\pi)}{n\pi} = \frac{1-\cos(n\pi)}{(n\pi)^2}$$

$$b_n = \frac{2}{T_0}\left\{\int_0^1 -t\sin(n\omega_0 t)\,dt + \int_1^2 \sin(n\omega_0 t)\,dt\right\} = \frac{t\cos(n\pi t)}{n\pi} - \frac{\sin(n\pi t)}{(n\pi)^2}\Bigg|_0^1 + \frac{\cos(n\pi t)}{n\pi}\Bigg|_2^1$$

$$b_n = \frac{\cos(n\pi)}{n\pi} - \frac{\sin(n\pi)}{(n\pi)^2} + \frac{\cos(n\pi)}{n\pi} - \frac{\cos(2n\pi)}{n\pi} = \frac{2\cos(n\pi) - 1}{n\pi}$$

$$\boxed{a_0 = 1/4 \qquad a_n = \frac{1-\cos(n\pi)}{(n\pi)^2} \qquad b_n = \frac{2\cos(n\pi)-1}{n\pi}}$$

15.15 Find the trigonometric Fourier series coefficients for the waveform in Fig. P15.15.

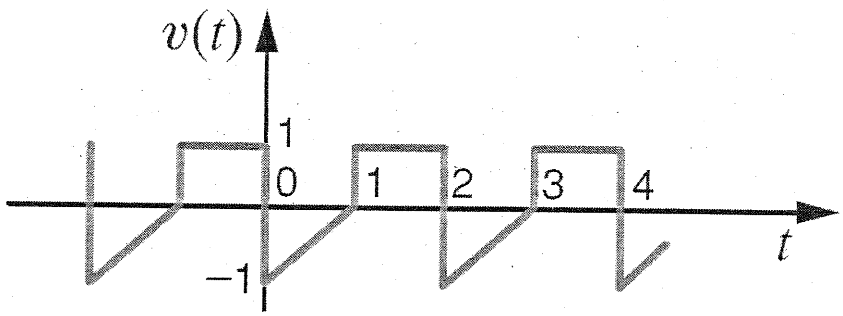


Figure P15.15

SOLUTION: $T_0 = 2\,\text{sec}$ $\quad \omega_0 = \pi\ r/s$ $\quad a_0 = \frac{1}{2}\left[-\frac{1}{2}(1)(1) + (1)(1)\right] = 1/4$

$$a_n = \frac{2}{T_0}\left\{\int_0^1 (t-1)\cos(n\pi t)\,dt + \int_1^2 \cos(n\pi t)\,dt\right\}$$

$$a_n = \left(\frac{\cos(n\pi t)}{(n\pi)^2} + \frac{t\sin(n\pi t)}{n\pi}\right)\Bigg|_0^1 - \frac{\sin(n\pi t)}{n\pi}\Bigg|_0^1 + \frac{\sin(n\pi t)}{n\pi}\Bigg|_1^2$$

$$a_n = \frac{\cos(n\pi)-1}{(n\pi)^2} + \frac{\sin(n\pi)}{n\pi} - \frac{\sin(n\pi)}{n\pi} + \frac{\sin(2n\pi)}{n\pi} - \frac{\sin(n\pi)}{n\pi} = \frac{\cos(n\pi)-1}{(n\pi)^2}$$

$$b_n = \int_0^1 (t-1)\sin(n\pi t)\,dt + \int_1^2 \sin(n\pi t)\,dt$$

$$b_n = \left(\frac{\sin(n\omega_0 t)}{(n\omega_0)^2} - \frac{t\cos(n\omega_0 t)}{n\omega_0}\right)\Bigg|_0^1 + \frac{\cos(n\omega_0 t)}{n\omega_0}\Bigg|_2^1 + \frac{\cos(n\omega_0 t)}{n\omega_0}\Bigg|_0^1$$

$$b_n = \frac{\sin(n\pi)}{(n\pi)^2} - \frac{\cos(n\pi)}{n\pi} + \frac{\cos(n\pi)}{n\pi} - \frac{\cos(2n\pi)}{n\pi} + \frac{\cos(n\pi)}{n\pi} - \frac{1}{n\pi} = \frac{\cos(n\pi)-2}{n\pi}$$

$$\boxed{a_0 = 1/4 \qquad a_n = \frac{\cos(n\pi)-1}{(n\pi)^2} \qquad b_n = \frac{\cos(n\pi)-2}{n\pi}}$$

15.16 Derive the trigonometric Fourier series for the waveform shown in Fig. P15.16.

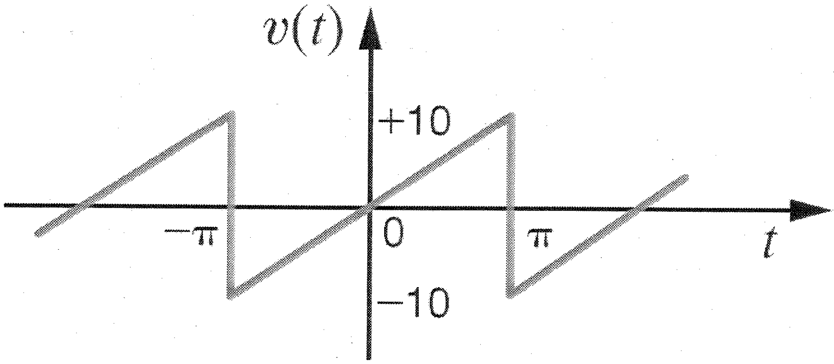


Figure P15.16

SOLUTION: $a_0 = 0$ $\quad T_0 = 2\pi$ $\quad \omega_0 = 1\ r/s$

odd symmetry $\Rightarrow a_n = 0$

$$b_n = \frac{4}{T_0}\int_0^{T_0/2} \frac{10}{\pi} t \sin(nt)\,dt = \frac{40}{\pi^2}\int_0^{\pi/2} t \sin(nt)\,dt$$

$$b_n = \frac{40}{\pi^2}\left[\frac{\sin(nt)}{n^2} - \frac{t\cos(nt)}{n}\right]_0^{\pi} = \frac{40}{\pi^2}\left[\frac{\sin(n\pi)}{n^2} - \frac{\pi\cos(n\pi)}{2n}\right]$$

$$b_n = \frac{40}{\pi^2}\left[-\frac{\pi\cos(n\pi)}{2n}\right] = -\frac{20}{n\pi}\cos(n\pi) = (-1)^{n+1}\left(\frac{20}{n\pi}\right)$$

$$\boxed{v(t) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{20}{n\pi}\sin(nt)}$$

15.17 Find the trigonometric Fourier series coefficients for the waveform in Fig. P15.17.

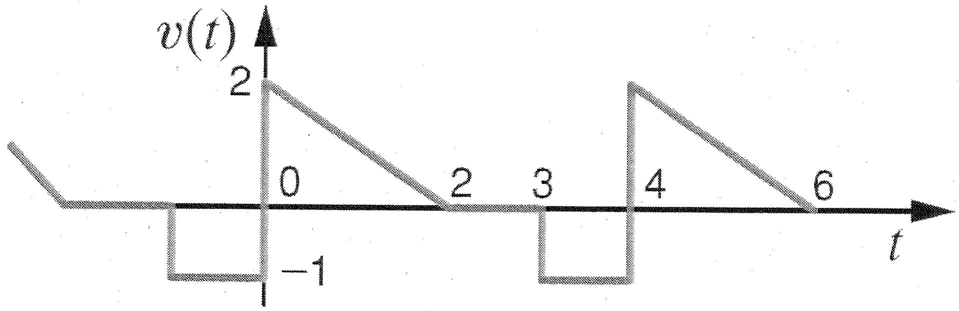


Figure P15.17

SOLUTION: $T_0 = 4 \qquad \omega_0 = \pi/2 \qquad a_0 = \frac{1}{4}\left[\frac{1}{2}(2)(2) - (1)(1)\right] = 1/4$

$$a_n = \frac{2}{T_0}\left\{\int_0^2 (2-t)\cos(n\omega_0 t)\,dt - \int_3^4 \cos(n\omega_0 t)\,dt\right\}$$

$$a_n = \frac{\sin(n\omega_0 t)}{n\omega_0}\bigg|_0^2 + \left(\frac{t\sin(n\omega_0 t)}{2n\omega_0} + \frac{\cos(n\omega_0 t)}{2(n\omega_0)^2}\right)\bigg|_2^0 + \frac{\sin(n\omega_0 t)}{2n\omega_0}\bigg|_4^3$$

$$a_n = \frac{\sin(n\pi)}{n\omega_0} + \frac{1}{2(n\omega_0)^2} - \frac{\sin(n\pi)}{n\omega_0} - \frac{\cos(n\pi)}{2(n\omega_0)^2} + \frac{\sin(n3\pi/2)}{2n\omega_0} - \frac{\sin(n2\pi)}{2n\omega_0}$$

$$\boxed{a_n = \frac{2}{(n\pi)^2}\left(1 - \cos(n\pi)\right) - \frac{\sin(n\pi/2)}{n\pi}} \qquad \boxed{a_0 = 1/4}$$

$$b_n = \frac{2}{T_0}\left\{\int_0^2 (2-t)\sin(n\omega_0 t)\,dt + \int_3^4 -\sin(n\omega_0 t)\,dt\right\}$$

$$b_n = \frac{\cos(n\omega_0 t)}{n\omega_0}\bigg|_2^0 + \left(\frac{t\cos(n\omega_0 t)}{2n\omega_0} - \frac{\sin(n\omega_0 t)}{2(n\omega_0)^2}\right)\bigg|_0^2 + \frac{\cos(n\omega_0 t)}{2n\omega_0}\bigg|_3^4$$

$$b_n = \frac{1-\cos(n\pi)}{n\omega_0} + \frac{\cos(n\pi)}{n\omega_0} - \frac{\sin(n\pi)}{2(n\omega_0)^2} + \frac{\cos(n2\pi) - \cos(3n\pi/2)}{2n\omega_0}$$

$$\boxed{b_n = \frac{3 - \cos(n\pi/2)}{n\pi}}$$

15.18 Find the trigonometric Fourier series for the waveform shown in Fig. P15.18. CS

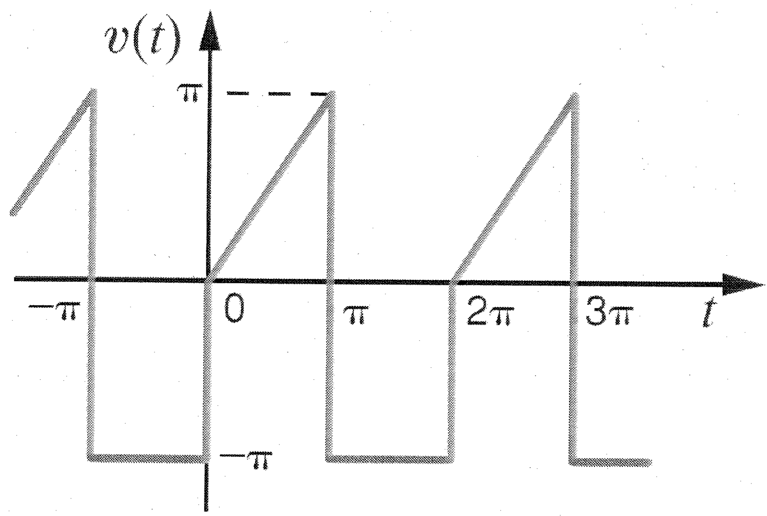


Figure P15.18

SOLUTION: $T_0 = 2\pi$ $\quad \omega_0 = 1$ r/s $\quad a_0 = \frac{1}{2\pi}\left[\pi^2/2 - \pi^2\right] = -\pi/4$

$$a_n = \frac{1}{\pi}\int_0^{\pi} t\cos(n\omega_0 t)\,dt - \frac{1}{\pi}\int_{\pi}^{2\pi} \pi\cos n\omega_0 t\,dt$$

$$a_n = \frac{1}{\pi}\left[\frac{\cos(n\omega_0 t)}{(n\omega_0)^2} + \frac{t\sin(n\omega_0 t)}{n\omega_0}\right]\Bigg|_0^{\pi} - \left[\frac{\sin n\omega_0 t}{n\omega_0}\right]\Bigg|_{\pi}^{2\pi}$$

$$a_n = \frac{1}{\pi}\left[\frac{\cos(n\pi)}{n^2} - \frac{1}{n^2} + \frac{\pi\sin(n\pi)}{n}\right] - \frac{\sin(n2\pi)}{n} + \frac{\sin(n\pi)}{n} = \frac{1}{\pi n^2}\left[\cos(n\pi) - 1\right]$$

$$b_n = \frac{1}{\pi}\int_0^{\pi} t\sin(n\omega_0 t)\,dt - \int_{\pi}^{2\pi}\sin(n\omega_0 t)\,dt$$

$$b_n = \frac{1}{\pi}\left[\frac{\sin(n\omega_0 t)}{(n\omega_0)^2} - \frac{t\cos(n\omega_0 t)}{n\omega_0}\right]_0^{\pi} + \frac{\cos(n\omega_0 t)}{n\omega_0}\Bigg|_{\pi}^{2\pi}$$

$$b_n = \frac{1}{\pi}\left[\frac{\sin(n\pi)}{n^2\pi^2} - \frac{\pi\cos(n\pi)}{n\omega_0}\right] + \frac{\cos(2n\pi) - \cos(n\pi)}{n\omega_0} = \frac{1}{n}\left(1 - 2\cos(n\pi)\right)$$

$$v(t) = -\frac{\pi}{4} + \sum_{n=1}^{\infty} \frac{1}{\pi n^2}\left(\cos(n\pi) - 1\right)\cos(nt) + \frac{1}{n}\left(1 - 2\cos(n\pi)\right)\sin(nt) \text{ V}$$

15.19 Derive the trigonometric Fourier series for the function shown in Fig. P15.19. **CS**

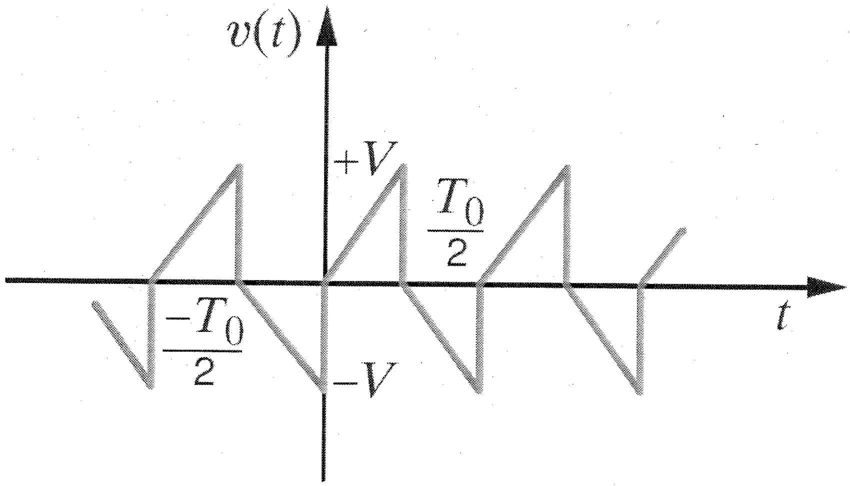


Figure P15.19

SOLUTION: Half wave symmetry $\Rightarrow a_n = b_n = 0$ for n even & $a_0 = 0$

$$a_n = \frac{4}{T_0}\int_0^{T_0/2} v(t)\cos(n\omega_0 t)\,dt = \frac{4}{T_0}\int_0^{T_0/2} \frac{2V}{T_0} t\cos(n\omega_0 t)\,dt = \frac{8V}{T_0^2}\int_0^{T_0/2} t\cos(n\omega_0 t)\,dt$$

$$a_n = \frac{8V}{T_0^2}\left[\frac{t\sin(n\omega_0 t)}{n\omega_0} + \frac{\cos(n\omega_0 t)}{(n\omega_0)^2}\right]\Bigg|_0^{T_0/2} = \frac{8V}{T_0^2}\left[\frac{T_0}{2}\left(\frac{\sin(n\pi)}{n\omega_0}\right) + \frac{\cos(n\pi)-1}{(n\omega_0)^2}\right]$$

$$a_n = 8V\left(\frac{\cos(n\pi)-1}{4(n\pi)^2}\right) \quad \text{for } n \text{ odd}, \quad a_n = \frac{-4V}{n^2\pi^2}$$

$$b_n = \frac{4}{T_0}\int_0^{T_0/2} v(t)\sin(n\omega_0 t)\,dt = \frac{8V}{T_0^2}\int_0^{T_0/2} t\sin(n\omega_0 t)\,dt$$

$$b_n = \frac{8V}{T_0^2}\left[\frac{\sin(n\omega_0 t)}{(n\omega_0)^2} - \frac{t\cos(n\omega_0 t)}{n\omega_0}\right]\Bigg|_0^{T_0/2} = \frac{8V}{T_0^2}\left\{\frac{\sin(n\pi)}{(n\omega_0)^2} - \frac{T_0}{2}\frac{\cos(n\pi)}{n\omega_0}\right\}$$

$$b_n = \frac{-2V}{n\pi}\cos(n\pi) \quad \text{for } n \text{ odd}, \quad b_n = \frac{2V}{n\pi}$$

$$\boxed{v(t) = \sum_{\substack{n=1 \\ n \text{ odd}}}^{\infty} \frac{2V}{n\pi}\sin(n\omega_0 t) - \frac{4V}{n^2\pi^2}\cos(n\omega_0 t)\ \text{V}}$$

15.20 Derive the trigonometric Fourier series for the function $v(t) = A|\sin t|$ as shown in Fig. P15.20.

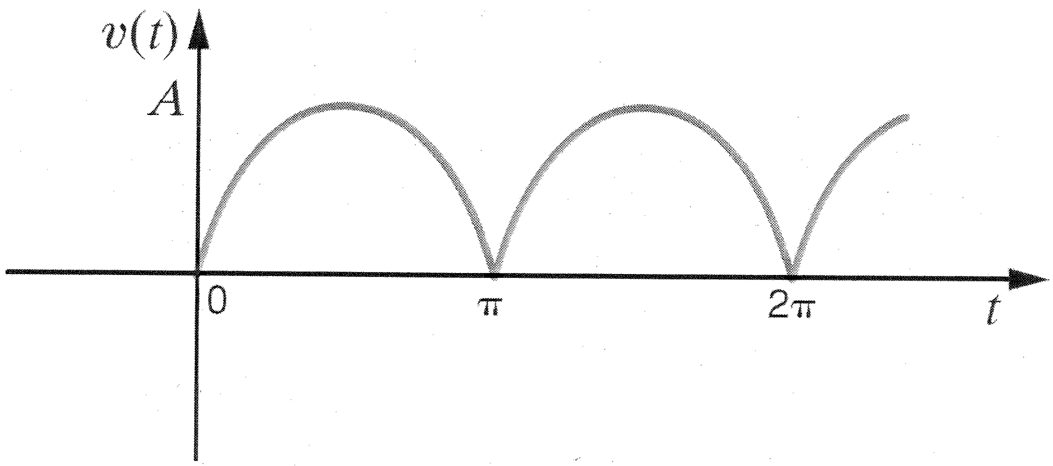


Figure P15.20

SOLUTION: $T_0 = \pi \qquad \omega_0 = 2 \qquad$ even function $\Rightarrow b_n = 0$

$$a_0 = \frac{1}{T_0}\int_0^{T_0} A\sin(t)\,dt = \frac{A}{T_0}\Big[\cos(t)\Big]\Big|_{T_0}^{0} = \frac{2A}{T_0} = \frac{2A}{\pi}$$

$$a_n = \frac{4}{T_0}\int_0^{T_0/2} A\sin(t)\cos(n\omega_0 t)\,dt = \frac{4A}{\pi}\int_0^{\pi/2} \sin(t)\cos(n2t)\,dt$$

$$\sin(t)\cos(2nt) = \Big(\sin\big[(2n+1)t\big] + \sin\big[(1-2n)t\big]\Big)/2$$

$$a_n = \frac{2A}{\pi}\left[\frac{\cos(1+2n)t}{1+2n} + \frac{\cos(1-2n)t}{1-2n}\right]\Bigg|_{T_0/2}^{0}$$

$$a_n = \frac{2A}{\pi}\left[\frac{1-\cos\big[(1+2n)\pi/2\big]}{1+2n} + \frac{1-\cos\big[(1-2n)\pi/2\big]}{1-2n}\right] = \frac{2A}{\pi}\left[\frac{1}{1+2n} + \frac{1}{1-2n}\right]$$

$$a_n = \frac{4A}{\pi(1-4n^2)}$$

$$\boxed{v(t) = \frac{2A}{\pi}\left[1 + \sum_{n=1}^{\infty}\left(\frac{2}{1-4n^2}\right)\cos(n2t)\right]\ \text{V}}$$

15.21 Derive the trigonometric Fourier series for the waveform shown in Fig. P15.21.

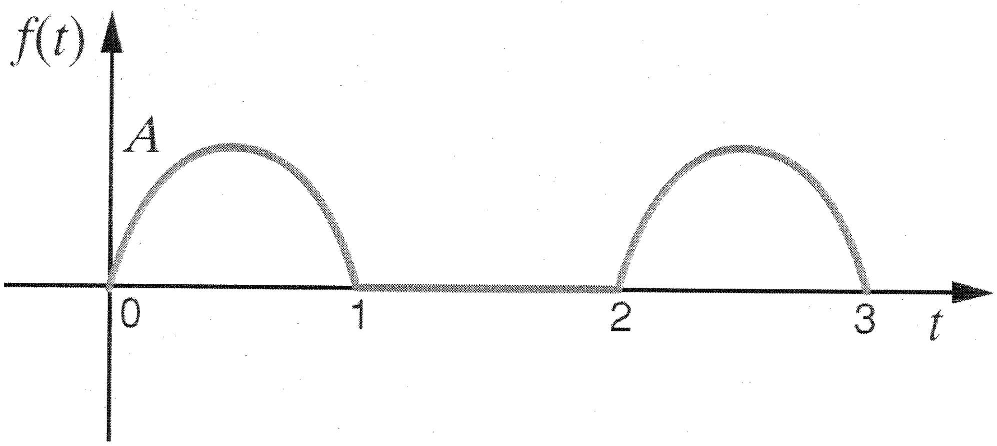


Figure P15.21

SOLUTION: $T_0 = 2 \qquad \omega_0 = \pi$

$$a_0 = \frac{1}{2}\int_0^1 A\sin(\pi t)\,dt = \frac{A}{2\pi}\cos(\pi t)\Big|_1^0 = A/\pi$$

$$a_n = A\int_0^1 \sin(\pi t)\cos(n\pi t)\,dt = \frac{A}{2}\int_0^1 \left[\sin(1-n)\pi t + \sin(1+n)\pi t\right]dt$$

$$a_n = \frac{A}{2}\left[\frac{\cos(1-n)\pi t}{(1-n)\pi} + \frac{\cos(1+n)\pi t}{(1+n)\pi}\right]\Bigg|_1^0 = \frac{A}{2}\left[\frac{1-\cos(1-n)\pi}{(1-n)\pi} + \frac{1-\cos(1+n)\pi}{(1+n)\pi}\right]$$

for n odd, $a_n = 0 \qquad a_n = \dfrac{2A}{\pi(1-n^2)}$ for n even

$$b_n = A\int_0^1 \sin(\pi t)\sin(n\pi t)\,dt = \begin{cases} 0 & n \neq 1 \\ A\int_0^1 \sin^2(\pi t)\,dt & \text{for } n = 1 \end{cases}$$

$$b_1 = \frac{A}{2}\int_0^1 (1-\cos(2\pi t))\,dt = \frac{A}{2}\left[t - \frac{\sin(2\pi t)}{2\pi}\right]\Bigg|_0^1 = \frac{A}{2}[1] = A/2$$

$$\boxed{f(t) = \frac{A}{\pi} + \frac{A}{2}\sin(\pi t) + \sum_{\substack{n=2 \\ n \text{ even}}}^{\infty} \frac{2A}{\pi(1-n^2)}\cos(n\pi t)}$$

15.22 Use PSPICE to determine the Fourier series of the waveform in Fig. P15.22 in the form

$$v_S(t) = a_0 + \sum_{n=1}^{\infty} b_n \sin(n\omega_t + \theta_n)$$

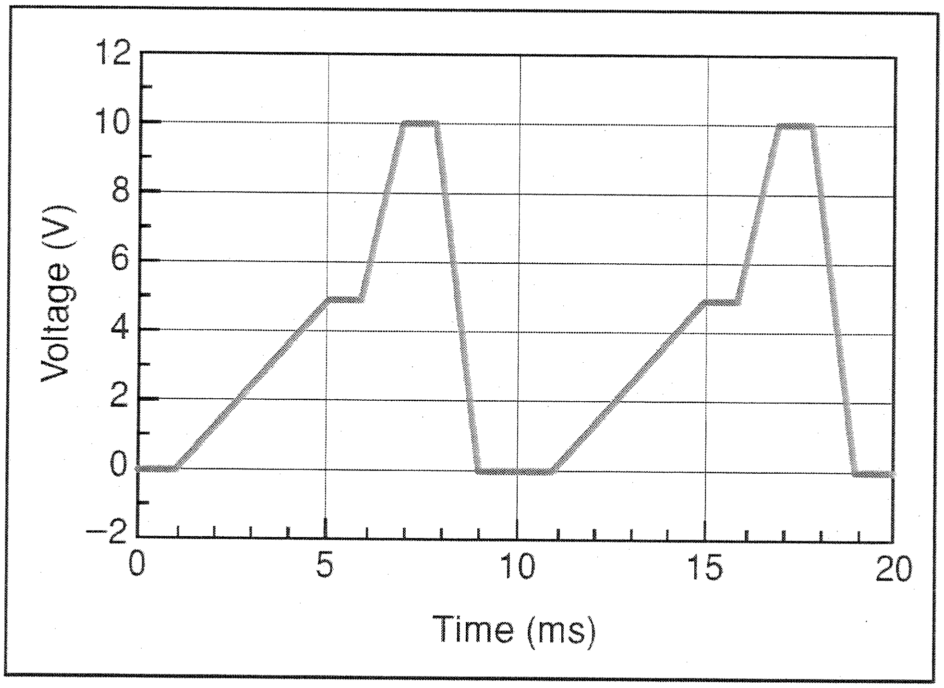


Figure P15.22

SOLUTION: Final time = 10 ms Step ceiling = 10 μs Center freq = 100 Hz
Number of harmonics = 10.

Use VPWL source to produce $v_s(t)$

15.22

FOURIER COMPONENTS OF TRANSIENT RESPONSE V(Vs)

DC COMPONENT = 3.750000E+00

HARMONIC NO	FREQUENCY (HZ)	FOURIER COMPONENT	NORMALIZED COMPONENT	PHASE (DEG)	NORMALIZED PHASE (DEG)
1	1.000E+02	3.954E+00	1.000E+00	-1.424E+02	0.000E+00
2	2.000E+02	2.016E+00	5.099E-01	-8.774E+01	1.971E+02
3	3.000E+02	1.247E+00	3.154E-01	-8.735E+00	4.185E+02
4	4.000E+02	6.417E-01	1.623E-01	6.991E+01	6.395E+02
5	5.000E+02	2.027E-01	5.126E-02	9.000E+01	8.020E+02
6	6.000E+02	2.852E-01	7.213E-02	1.101E+02	9.645E+02
7	7.000E+02	2.291E-01	5.793E-02	-1.713E+02	8.256E+02
8	8.000E+02	1.260E-01	3.188E-02	-9.226E+01	1.047E+03
9	9.000E+02	4.883E-02	1.235E-02	-3.759E+01	1.244E+03
10	1.000E+03	1.993E-08	5.040E-09	-1.206E+02	1.303E+03

TOTAL HARMONIC DISTORTION = 6.310255E+01 PERCENT

15.23 Use PSPICE to determine the Fourier series of the waveform in Fig. P15.23 in the form

$$i_S(t) = a_0 + \sum_{n=1}^{\infty} b_n \sin(n\omega_0 t + \theta_n) \quad \text{CS}$$

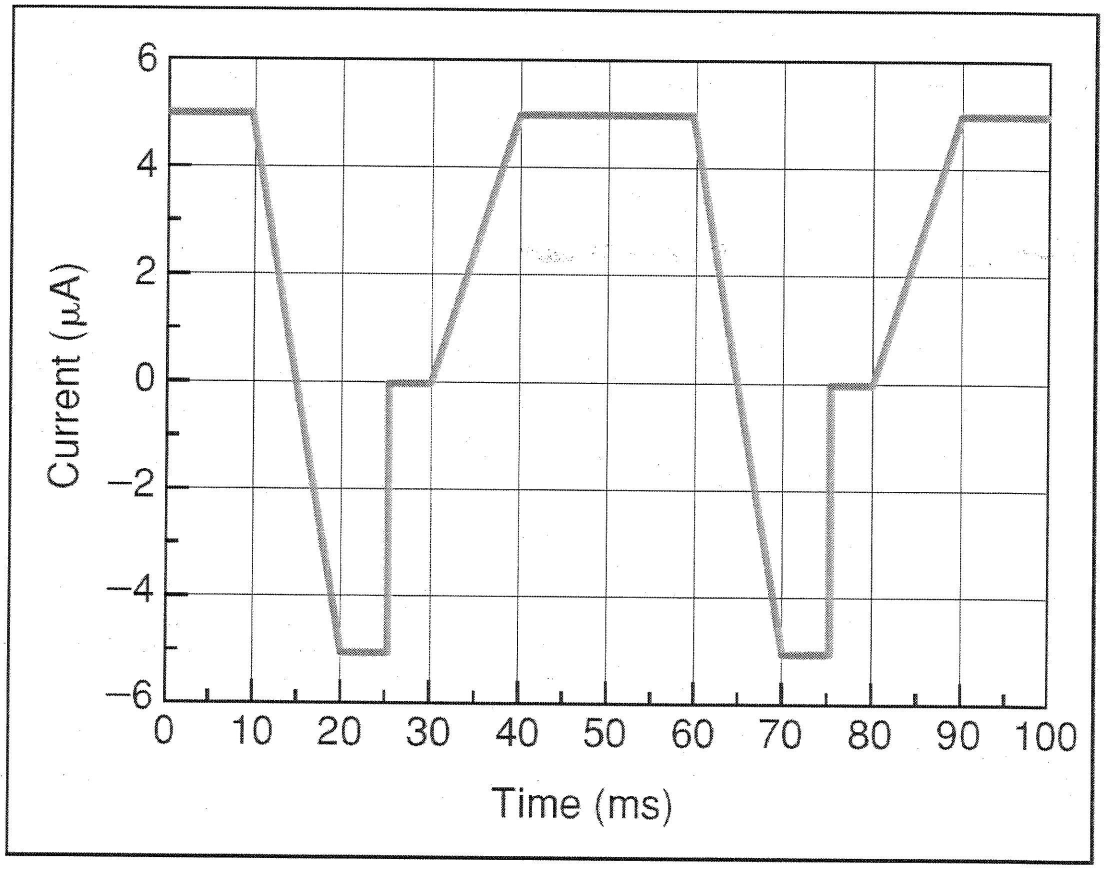


Figure P15.23

SOLUTION:

Final time = 50 ms   Step Ceiling = 50 μs   # harmonics = 10

Use IPWL to create $i_s(t)$

15.23

FOURIER COMPONENTS OF TRANSIENT RESPONSE I(I_Is)

DC COMPONENT = 1.997500E-06

HARMONIC NO	FREQUENCY (HZ)	FOURIER COMPONENT	NORMALIZED COMPONENT	PHASE (DEG)	NORMALIZED PHASE (DEG)
1	2.000E+01	4.401E-06	1.000E+00	1.049E+02	0.000E+00
2	4.000E+01	1.669E-06	3.792E-01	-3.975E+01	-2.495E+02
3	6.000E+01	8.810E-07	2.002E-01	-1.480E+02	-4.627E+02
4	8.000E+01	4.518E-07	1.027E-01	3.521E+01	-3.844E+02
5	1.000E+02	3.183E-07	7.233E-02	1.791E+02	-3.454E+02
6	1.200E+02	3.001E-07	6.819E-02	2.212E+01	-6.073E+02
7	1.400E+02	2.046E-07	4.648E-02	-1.565E+02	-8.907E+02
8	1.600E+02	1.829E-07	4.157E-02	-2.297E+01	-8.621E+02
9	1.800E+02	1.913E-07	4.347E-02	1.625E+02	-7.816E+02
10	2.000E+02	1.592E-07	3.617E-02	-1.800E+00	-1.051E+03

TOTAL HARMONIC DISTORTION = 4.597326E+01 PERCENT

15.24 The discrete line spectrum for a periodic function $f(t)$ is shown in Fig. P15.24. Determine the expression for $f(t)$.
PSV

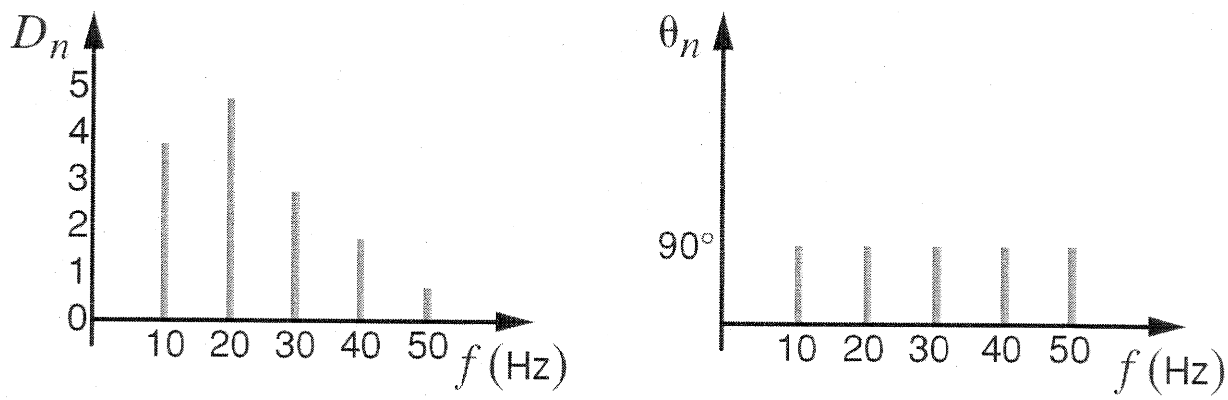


Figure P15.24

SOLUTION: $f_0 = 10\,Hz$ $\omega_0 = 20\pi$ r/s $D_n = a_n - j b_n$

all $\theta_n = 90°$, so $a_n = 0$ & $b_n = -|D_n|$

$f(t) = -4\sin(20\pi t) - 5\sin(40\pi t) - 3\sin(60\pi t) - 2\sin(80\pi t) - \sin(100\pi t)$ ✓

15.25 The amplitude and phase spectra for a periodic function $v(t)$ that has only a small number of terms is shown in Fig. P15.25. Determine the expression for $v(t)$ if $T_0 = 0.1$ s.

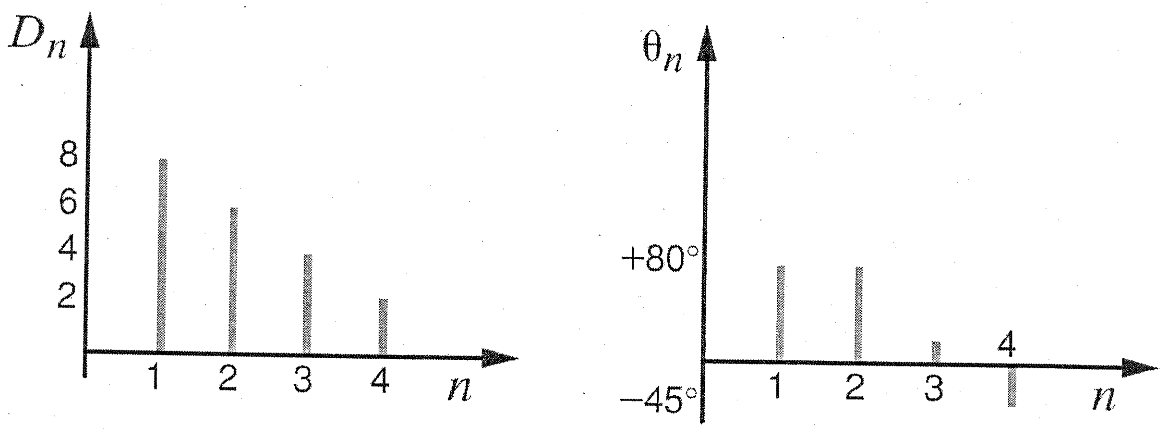


Figure P15.25

SOLUTION: $T_0 = 0.1\,s$ $\quad f_0 = 10$ Hz $\quad \omega_0 = 20\pi$ r/s

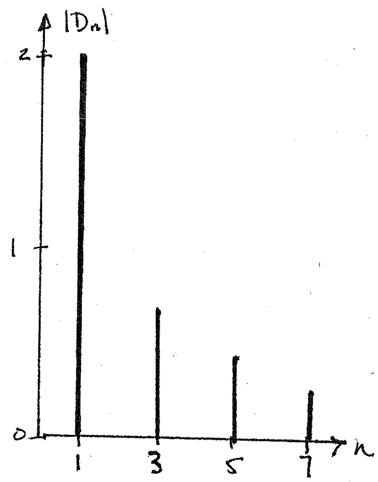
$v(t) = 8\cos(20\pi t + 80°) + 6\cos(40\pi t + 80°) + 4\cos(60\pi t + 15°) + 2\cos(80\pi t - 45°)$

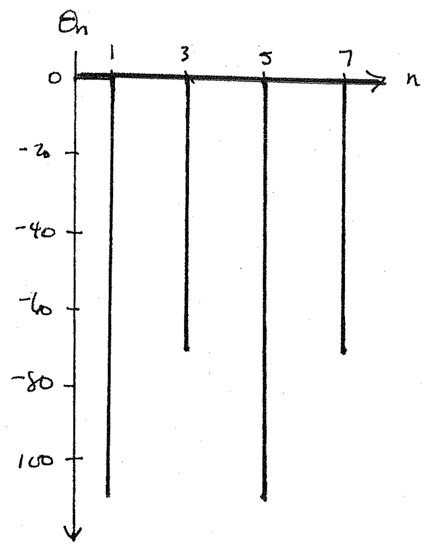
15.26 Plot the first four terms of the amplitude and phase spectra for the signal

$$f(t) = \sum_{\substack{n=1 \\ n \text{ odd}}}^{\infty} \frac{-2}{n\pi} \sin \frac{n\pi}{2} \cos n\omega_0 t + \frac{6}{n\pi} \sin n\omega_0 t$$

SOLUTION: $D_n = a_n - jb_n$

n	a_n	b_n	$\lvert D_n \rvert$	θ_n (°)
1	-0.637	1.91	2.01	-108
3	0.212	0.637	0.671	-71.6
5	-0.127	0.382	0.403	-108
7	0.091	0.273	0.288	-71.6





15.27 Determine the steady-state response of the current $i_o(t)$ in the circuit shown in Fig. P15.27 if the input voltage is described by the waveform shown in Problem 15.16. CS

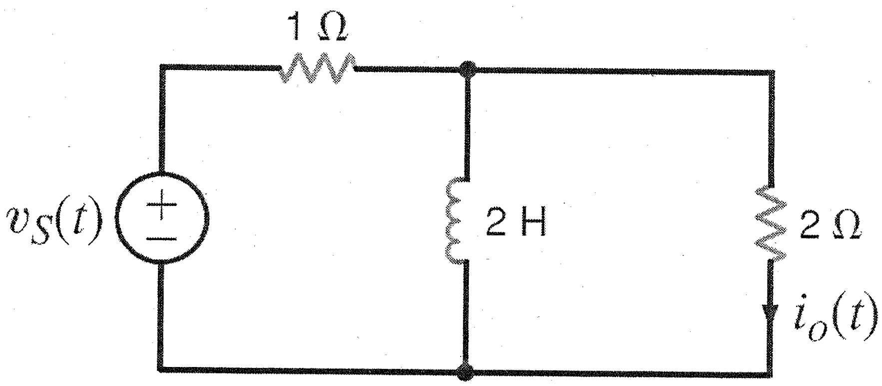
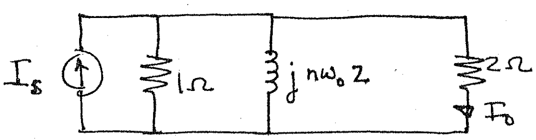


Figure P15.27

SOLUTION:



From problem 15.16 $\omega_o = 1$ r/s

$$V_S = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{20}{n\pi} \cos(nt - 90°)\ V$$

$$I_S = V_s / 1\Omega = V_s$$

$$\frac{I_o}{I_s} = \frac{I_o}{V_s} = \frac{1/2}{1/2 + 1 + \frac{1}{j2n}} = \frac{jn}{1 + j3n}$$

Let $G(n) = \dfrac{jn}{1+j3n}$ $\Rightarrow I_o(n) = G(n)\, V_s(n)$

$$i_o(t) = \sum_{n=1}^{\infty} (-1)^{n+1} \left(\frac{20}{n\pi}\right) |G(n)| \cos(nt - 90° + \theta_{G(n)})\ A$$

$$G(n) = \frac{n}{\sqrt{1+9n^2}} \angle 90° - \tan^{-1}(3n)$$

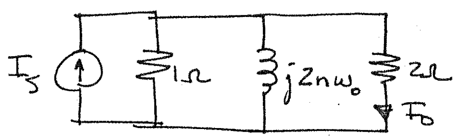
$$\boxed{i_o(t) = \sum_{n=1}^{\infty} \left[(-1)^{n+1} (20/n\pi) \frac{n}{\sqrt{1+9n^2}} \cos(nt - \tan^{-1}(3n)) \right] A}$$

15.28 If the input voltage in Problem 15.27 is

$$v_S(t) = 1 - \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin 0.2\pi n t \text{ V}$$

find the expression for the steady-state current $i_o(t)$.

SOLUTION:



$$I_S = V_S/1 = V_S$$

$$\frac{I_o}{I_S} = \frac{I_o}{V_S} = G(n) = |G(n)|\angle\theta(n)$$

$$G(n) = \frac{1/2}{1/2 + 1 + \frac{1}{j2n\omega_o}} = \frac{jn\omega_o}{1 + j3n\omega_o} \qquad I_o(n) = G(n)\,V_s(n)$$

$$v_s(t) = 1 - \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\cos(0.2\pi n t - 90^\circ) \qquad \omega_o = 0.2\pi = \pi/5$$

$$G(n) = \frac{jn\pi}{5 + j3n\pi} = |G(n)|\angle\theta(n)$$

for $n=0$, inductor is a short & $i_o \to 0$.

$$\boxed{i_o = \sum_{n=1}^{\infty} -\frac{2}{\pi n}|G(n)|\cos\left(\frac{\pi n t}{5} - 90^\circ + \theta(n)\right) \text{ A}}$$

15.29 Determine the first three terms of the steady-state voltage $v_o(t)$ in Fig. P15.29 if the input voltage is a periodic signal of the form

$$v(t) = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{1}{n\pi}(\cos n\pi - 1)\sin nt \text{ V}$$ **PSV**

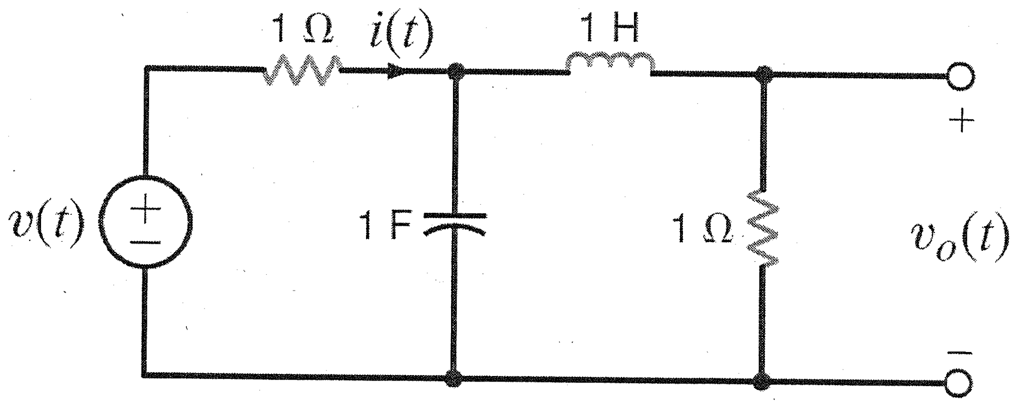
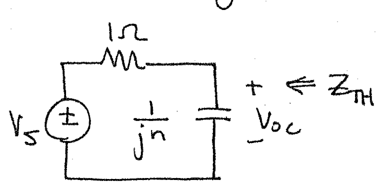


Figure P15.29

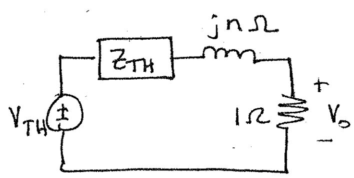
SOLUTION: $\omega_o = 1$ r/s

Thevenin eq:



$V_{OC} = \dfrac{V_S}{1+jn}$

$Z_{TH} = \dfrac{1}{1+jn}$



$$\frac{V_o}{V_{OC}} = \frac{1}{1+jn+\frac{1}{1+jn}} = \frac{1+jn}{(1+jn)^2+1}$$

$$\frac{V_o}{V_S} = \frac{V_o}{V_{OC}}\frac{V_{OC}}{V_S} = \frac{1}{(1+jn)^2+1} = \frac{1}{2-n^2+j2n} = H(n) = |H(n)|\angle\theta(n)$$

for $n=0$, cap → open & inductor → short

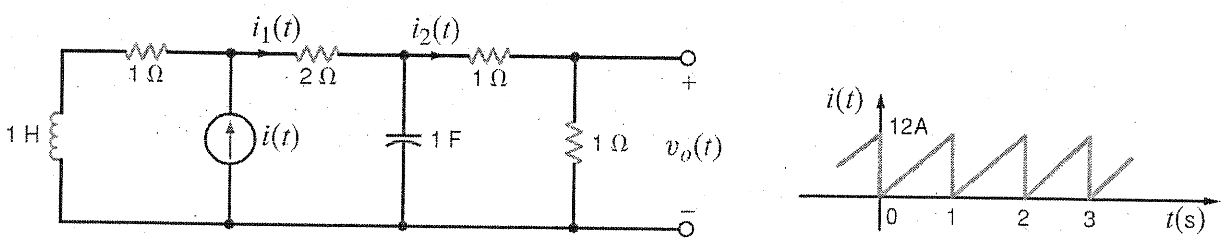
$v_o(0) = v(0)\left[\left(\tfrac{1}{2}\right)\right] = \tfrac{1}{4}$ V

for $n=1$, $V_o(1) = V(1)\,H(1) = \dfrac{\cos\pi - 1}{\pi}\,\dfrac{1}{1+j2} = 0.285\angle 116°$

for $n=2$, $V_o(2) = V(2)\,H(2) = 0$ for $n=3$, $V_o(3) = 0.023\angle 41.0°$

$$\boxed{v_o(t) = \frac{1}{4} + 0.285\cos(t+26.6) + 0.023\cos(3t-49°) \text{ V}}$$

15.30 Determine the steady-state voltage $v_o(t)$ in the network in Fig. P15.30a if the input current is given in Fig. P15.30b.



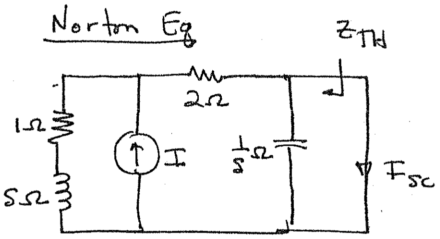
(a) (b)

Figure P15.30

SOLUTION: $T_o = 1\,s$ $\quad \omega_o = 2\pi\ r/s$

From Table 15.2, $\quad i(t) = 6 + \sum_{n=1}^{\infty} \frac{-12}{n\pi} \sin n2\pi t = 6 + \sum_{n=1}^{\infty} \frac{-12}{n\pi} \cos(n2\pi t - 90°)\ A$

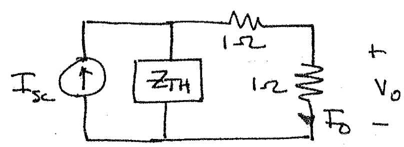
Norton Eq



$I_{sc} = I \left\{ \frac{s+1}{s+3} \right\}$

$Z_{TH} = \frac{(1/s)(s+3)}{s+3+1/s}$

$Z_{TH} = \frac{s+3}{s^2+3s+1}$



$\frac{I_o}{I_{sc}} = \frac{Z_{TH}}{2+Z_{TH}}$

$V_o = (1)\,I_o$

$\frac{V_o}{I_{sc}} = \frac{s+3}{s+3+2(s^2+3s+1)} = \frac{3+s}{(2s+5)(s+1)}$

$\frac{V_o}{I} = \frac{s+3}{(2s+5)(s+1)} \cdot \frac{s+1}{s+3} = \frac{1}{2s+5}$

let $s = j2\pi n$, $\quad \frac{V_o}{I} = \frac{1}{5+j4\pi n} = |G(n)| \angle \theta(n)$

for $n=0$, $\quad i(t) = 6 \quad \& \quad G(0) = 1/5$

$v_o(t) = \frac{6}{5} + \sum_{n=1}^{\infty} \frac{-12}{n\pi} |G(n)| \cos(2\pi n t - 90° + \theta(n))\ V$ ✓

15.31 Find the average power absorbed by the network in Fig. P15.31 if

$$v(t) = 12 + 6\cos(377t - 10^\circ) + 4\cos(754t - 60^\circ)\text{ V}$$

$$i(t) = 0.2 + 0.4\cos(377t - 150^\circ)$$
$$-0.2\cos(754t - 80^\circ) + 0.1\cos(1131t - 60^\circ)\text{ A}$$

PSV

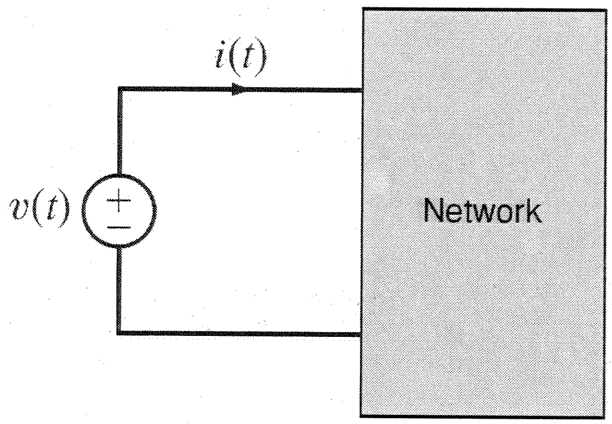


Figure P15.31

SOLUTION:

$$P = V_{DC}I_{DC} + \frac{1}{2}\sum_{n=1}^{\infty} |V(n)||I(n)| \cos(\theta_V - \theta_I)$$

$$P = 12(0.2) + \frac{1}{2}\left\{(6)(0.4)\cos(140^\circ) + 4(0.2)\cos(-160^\circ)\right\}$$

$$\boxed{P = 1.1\text{ W}}$$

15.32 Find the average power absorbed by the network in Fig. P15.32 if $v(t) = 60 + 36\cos(377t + 45°) + 24\cos(754t - 60°)$ V. CS

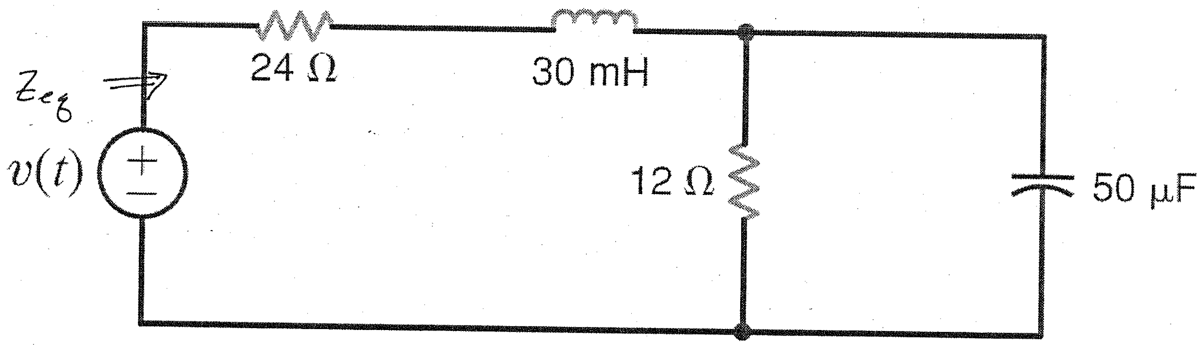


Figure P15.32

SOLUTION: $\omega_0 = 377\ r/s$

$$Z_{eq} = 24 + sL + \frac{12/sC}{12 + 1/sC} = 24 + sL + \frac{12}{1+12sC} = \frac{12LCs^2 + s(288C + L) + 36}{12sC + 1}$$

Let $s \to jn\omega_0 = jn377$ $\quad Z_{eq} = \dfrac{36 - 2.56n^2 + jn(16.7)}{1 + jn(0.226)}$

$I(n) = V(n)/Z_{eq}(n)$

$$P = \frac{V^2(0)}{Z_{eq}(0)} + \frac{1}{2}\sum_{n=1}^{\infty} \frac{|V(n)|^2}{|Z(n)|} \cos\theta_{Z(n)}$$

$$P = \frac{60^2}{36} + \frac{1}{2}\left\{\frac{36^2}{36.4}\cos(13.8°) + \frac{24^2}{38.4}\cos(28.1)\right\}$$

$\boxed{P = 123.8\ W}$

15.33 Determine the Fourier transform of the waveform shown in Fig. P15.33.

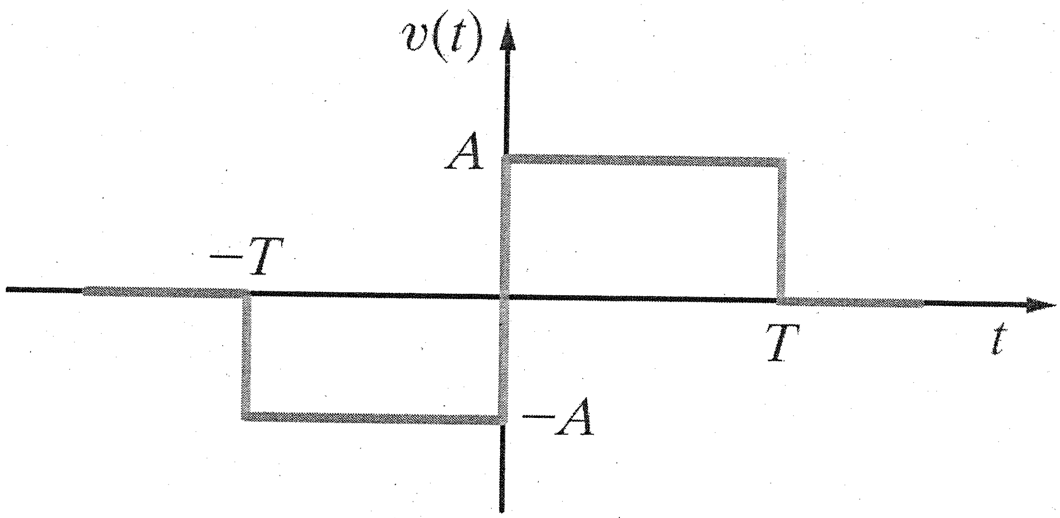


Figure P15.33

SOLUTION:

$$F(\omega) = \int_{-\infty}^{\infty} f(t)\,e^{-j\omega t}dt = \int_{-T}^{0} -A\,e^{-j\omega t}dt + \int_{0}^{T} A\,e^{-j\omega t}dt$$

$$F(\omega) = \frac{A}{j\omega}\left[e^{-j\omega t}\Big|_{-T}^{0} - e^{-j\omega t}\Big|_{0}^{T}\right] = \frac{A}{j\omega}\left[1 - e^{j\omega T} + 1 - e^{-j\omega T}\right]$$

$$F(\omega) = \frac{A}{j\omega}\left[2 - \left(e^{j\omega T} + e^{-j\omega T}\right)\right] = \frac{2A}{j\omega}\left[1 - \frac{e^{j\omega T} + e^{-j\omega T}}{2}\right]$$

$$\boxed{F(\omega) = \frac{2A}{j\omega}\left[1 - \cos(\omega T)\right]}$$

15.34 Derive the Fourier transform for the following functions:

(a) $f(t) = e^{-2t}\cos 4t u(t)$

(b) $f(t) = e^{-2t}\sin 4t u(t)$

SOLUTION:

a) $F(\omega) = \int_0^\infty e^{-2t}\cos(4t)\, e^{-j\omega t}\,dt = \frac{1}{2}\int_0^\infty \left(e^{-(2-j4+j\omega)t} + e^{-(2+j4+j\omega)t}\right)dt$

$$F(\omega) = \frac{1}{2}\left[\frac{e^{-(2-j4+j\omega)t}}{2-j4+j\omega}\Bigg|_\infty^0 + \frac{e^{-(2+j4+j\omega)t}}{2+j4+j\omega}\Bigg|_\infty^0\right] = \frac{1}{2}\left[\frac{1}{2-j4+j\omega} + \frac{1}{2+j4+j\omega}\right]$$

$$\boxed{F(\omega) = \frac{2+j\omega}{(2+j\omega)^2+16}}$$

b) $F(\omega) = \int_0^\infty e^{-2t}\sin(4t)e^{-j\omega t}dt = \frac{1}{2j}\int_0^\infty \left[e^{-(2-j4+j\omega)t} - e^{-(2+j4+j\omega)t}\right]dt$

$$F(\omega) = \frac{1}{2j}\left[\frac{e^{-(2-j4+j\omega)t}}{2-j4+j\omega}\Bigg|_\infty^0 - \frac{e^{-(2+j4+j\omega)t}}{2+j4+j\omega}\Bigg|_\infty^0\right] = \frac{1}{2j}\left[\frac{1}{2-j4+j\omega} - \frac{1}{2+j4+j\omega}\right]$$

$$F(\omega) = \frac{1}{j2}\left[\frac{j8}{4+(j\omega)^2+4j\omega+16}\right] \qquad \boxed{F(\omega) = \frac{4}{(2+j\omega)^2+16}}$$

15.35 Show that

$$\mathcal{F}[f_1(t)f_2(t)] = \frac{1}{2\pi}\int_{-\infty}^{\infty}\mathbf{F}_1(x)\mathbf{F}_2(\omega - x)\,dx$$ CS

SOLUTION:

Let $G = \frac{1}{2\pi}\int_{-\infty}^{\infty} F_1(x) F_2(\omega - x)\,dx$

$$F^{-1}[G] = \frac{1}{(2\pi)^2}\int_{x=-\infty}^{\infty} F_1(x)\int_{\omega=-\infty}^{\infty} F_2(\omega - x)\,e^{j\omega t}\,d\omega\,dx$$

let $u = \omega - x \Rightarrow du = d\omega$

$$F^{-1}[G] = \frac{1}{(2\pi)^2}\int_{x=-\infty}^{\infty} F_1(x)\int_{u=-\infty}^{\infty} F_2(u)\,e^{jut}\,e^{jxt}\,du\,dx$$

$$F^{-1}[G] = \frac{1}{(2\pi)^2}\int_{x=-\infty}^{\infty} F_1(x)\,e^{jxt}\,dt\int_{-\infty}^{\infty} F_2(u)\,e^{jut}\,du = f_1(t)\,f_2(t)$$

Thus,

$$F[f_1(t)\,f_2(t)] = G = \frac{1}{2\pi}\int_{-\infty}^{\infty} F_1(x)\,F_2(\omega - x)\,dx \quad \checkmark$$

15.36 Find the Fourier transform of the function $f(t) = 12e^{-2|t|}\cos 4t$.

SOLUTION: Let $g(t) = 12e^{-2|t|}$

From Table 15.3, $G(\omega) = \frac{48}{4+\omega^2}$

$\cos 4t = \frac{e^{j4t} + e^{-j4t}}{2} \Rightarrow \mathcal{F}[g(t)\cos 4t] = \mathcal{F}\left[\frac{48}{4+\omega^2}\left(\frac{e^{j4t}+e^{-j4t}}{2}\right)\right]$

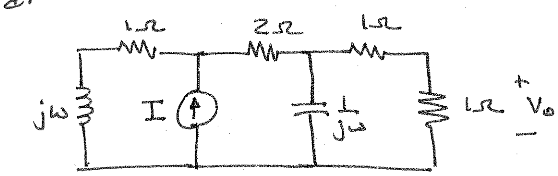
From Table 15.4, $F(\omega) = \frac{1}{2}\left[\frac{48}{4+(\omega-4)^2} + \frac{48}{4+(\omega+4)^2}\right]$

$$\boxed{F(\omega) = \frac{24}{4+(\omega-4)^2} + \frac{24}{4+(\omega+4)^2}}$$

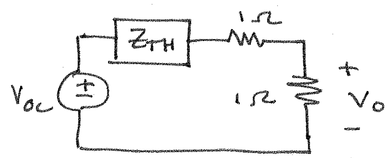
15.37 Use the transform technique to find $v_o(t)$ in the network in Fig. P15.30a if (a) $i(t) = 4(e^{-t} - e^{-2t})u(t)$ A and (b) $i(t) = 12\cos 4t$ A. **PSV**

SOLUTION:

a)



$\Rightarrow$



$$I(\omega) = 4\left[\frac{1}{1+j\omega} - \frac{1}{2+j\omega}\right]$$

$$I(\omega) = \frac{4}{(1+j\omega)(2+j\omega)}$$

$$V_{oc} = I\left[\frac{1+j\omega}{3+j\omega+1/j\omega}\right]1/j\omega$$

$$V_{oc} = I\left[\frac{1+j\omega}{1-\omega^2+j3\omega}\right]$$

$$Z_{TH} = \frac{1}{j\omega}//(3+j\omega) = \frac{3+j\omega}{1-\omega^2+j3\omega}$$

$$V_o = V_{oc}\left[\frac{1}{2+Z_{TH}}\right] = \frac{I(1+j\omega)}{5-2\omega^2+j7\omega} = \frac{I}{5+j2\omega}$$

$$H(\omega) = \frac{V_o(\omega)}{I(\omega)} = \frac{1/2}{2.5+j\omega} \qquad V_o(\omega) = \frac{2}{(1+j\omega)(2+j\omega)(2.5+j\omega)}$$

$$V_o(\omega) = \frac{4/3}{1+j\omega} - \frac{4}{2+j\omega} + \frac{8/3}{2.5+j\omega}$$

$$\boxed{v_o(t) = \frac{4}{3}e^{-t} - 4e^{-2t} + \frac{8}{3}e^{-2.5t} \quad u(t)}$$

b) $I(\omega) = 12\pi\left[\delta(\omega-4) + \delta(\omega+4)\right] \qquad H(\omega) = \dfrac{1}{5+2\omega}$

$$V_o(\omega) = 6(2\pi)\left[\frac{\delta(\omega-4)}{5+j8} + \frac{\delta(\omega+4)}{5-j8}\right] = \frac{6(2\pi)}{K}\left[e^{-j\theta}\delta(\omega-4) + e^{j\theta}\delta(\omega+4)\right]$$

$K = 9.43 \qquad \theta = 58^\circ \qquad v_o(t) = \dfrac{6}{K}\left[e^{j(4t-\theta)} + e^{-j(4t-\theta)}\right]$

$v_o(t) = \dfrac{12}{K}\cos(4t-\theta)$ V $\qquad \boxed{v_o(t) = 1.27\cos(4t - 58^\circ)\text{ V}}$

15.38 The input signal to a network is $v_i(t) = e^{-3t}u(t)$ V. The transfer function of the network is $\mathbf{H}(j\omega) = 1/(j\omega + 4)$. Find the output of the network $v_o(t)$ if the initial conditions are zero. CS

SOLUTION:

$V_i(\omega) = \dfrac{1}{3+j\omega}$ $\quad$ $V_o(\omega) = V_i(\omega)\,H(\omega) = \dfrac{1}{(3+j\omega)(4+j\omega)} = \dfrac{1}{3+j\omega} - \dfrac{1}{4+j\omega}$

$$\boxed{v_o(t) = \left(e^{-3t} - e^{-4t}\right)u(t)\ \text{V}}$$

15.39 Determine $v_o(t)$ in the circuit shown in Fig. P15.39 using the Fourier transform if the input signal is $i_S(t) = (e^{-2t} + \cos t)u(t)$ A.

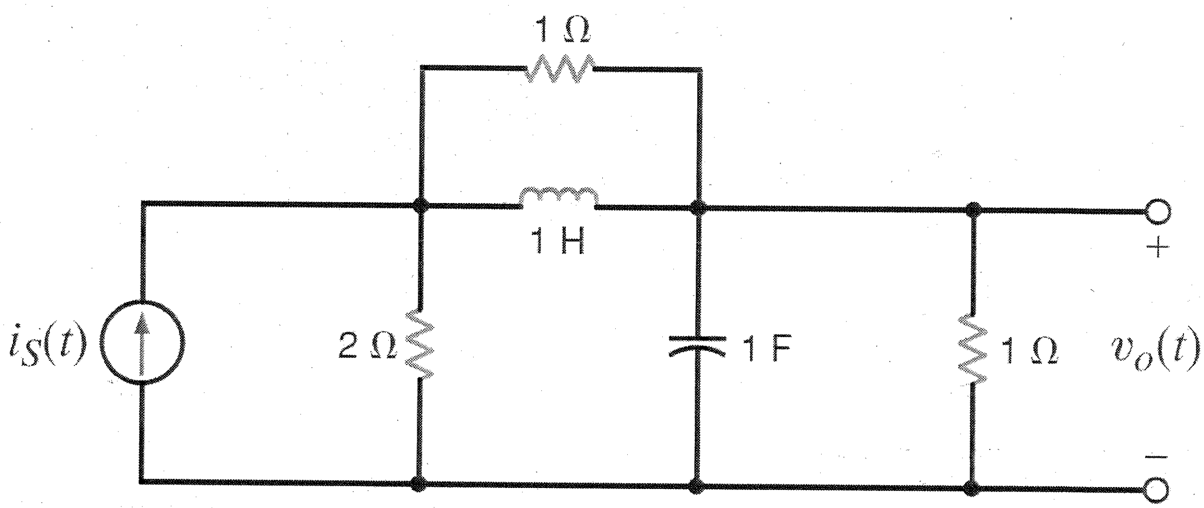
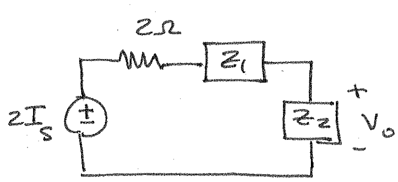


Figure P15.39

SOLUTION: Use source transformation!



$$Z_1 = 1 // j\omega = \frac{j\omega}{1+j\omega} \qquad Z_2 = 1 // \frac{1}{j\omega} = \frac{1}{1+j\omega}$$

$$\frac{V_o}{2I_S} = \frac{Z_2}{2+Z_1+Z_2} = \frac{1}{3(1+j\omega)} \qquad H(\omega) = \frac{V_o}{I_S} = \frac{2/3}{j\omega+1}$$

$$I_S = \frac{1}{2+j\omega} + \pi\delta(\omega-1) + \pi\delta(\omega+1)$$

$$V_o(\omega) = \frac{2/3}{(2+j\omega)(1+j\omega)} + \frac{\pi(2/3)\delta(\omega-1)}{1+j1} + \frac{\pi(2/3)\delta(\omega+1)}{1-j1}$$

$$V_o(\omega) = \frac{-2/3}{2+j\omega} + \frac{2/3}{1+j\omega} + \frac{2}{3}\pi\left\{\frac{\delta(\omega-1)}{1+j1} + \frac{\delta(\omega+1)}{1-j1}\right\}$$

$$\boxed{v_o(t) = \frac{2}{3}\left[e^{-t} - e^{-2t} + \frac{1}{\sqrt{2}}\cos(t-45°)\right]u(t)\ \text{V}}$$

15.40 The input signal for the network in Fig. P15.40 is $v_i(t) = 10e^{-5t}u(t)$ V. Determine the total 1-Ω energy content of the output $v_o(t)$.

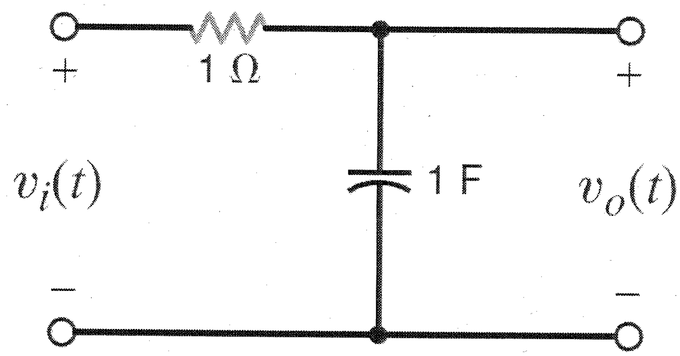


Figure P15.40

SOLUTION:

$$V_i(\omega) = \frac{10}{5+j\omega} \qquad H(\omega) = \frac{1/j\omega}{1+1/j\omega} = \frac{1}{1+j\omega} \qquad V_o = \frac{10}{(1+j\omega)(5+j\omega)}$$

$$|V_o(\omega)|^2 = \frac{100}{(1+\omega^2)(25+\omega^2)} = \frac{100}{24}\left[\frac{1}{1+\omega^2} - \frac{1}{25+\omega^2}\right] = \frac{25}{6}\left[\frac{1}{1+\omega^2} - \frac{1}{25+\omega^2}\right]$$

$$W = \frac{1}{2\pi}\int_{-\infty}^{\infty} |V_o(\omega)|^2 d\omega = \frac{25}{12\pi}\left\{\int_{-\infty}^{\infty} \frac{d\omega}{1+\omega^2} - \int_{-\infty}^{\infty} \frac{d\omega}{25+\omega^2}\right\}$$

$$W = \frac{25}{12\pi}\left\{\tan^{-1}(\omega)\Big|_{-\infty}^{\infty} - \frac{1}{5}\tan^{-1}\left(\frac{\omega}{5}\right)\Big|_{-\infty}^{\infty}\right\} = \frac{25}{12\pi}\left[\pi - \frac{\pi}{5}\right] = \frac{25}{12\pi}\left(\frac{4\pi}{5}\right)$$

$$\boxed{W = 5/3 \text{ J}}$$

15.41 Compute the 1-Ω energy content of the signal $v_o(t)$ in Fig. P15.40 in the frequency range from $\omega = 2$ to $\omega = 4$ rad/s. **CS**

SOLUTION:

From problem 15.40,

$$|V_o|^2 = \frac{25}{6}\left[\frac{1}{1+\omega^2} - \frac{1}{25+\omega^2}\right]$$

$$W = \frac{25}{6\pi}\left[\int_2^4 \frac{d\omega}{1+\omega^2} - \int_2^4 \frac{d\omega}{25+\omega^2}\right] = \frac{25}{6\pi}\left\{\tan^{-1}(\omega)\Big|_2^4 - \frac{\tan^{-1}}{5}(\omega/5)\Big|_2^4\right\}$$

$$W = \frac{25}{6\pi}\left\{1.326 - 1.107 - \left(\frac{0.675 - 0.381}{5}\right)\right\}$$

$$\boxed{W = 0.212\,\text{J}}$$

15.42 Determine the 1-Ω energy content of the signal $v_o(t)$ in Fig. P15.40 in the frequency range from 0 to 1 rad/s.

SOLUTION:

From problem 15.40

$$|V_o|^2 = \frac{25}{6}\left[\frac{1}{1+\omega^2} - \frac{1}{25+\omega^2}\right]$$

$$W = \frac{2}{2\pi}\int_0^1 \frac{25}{6}\left(\frac{d\omega}{1+\omega^2}\right) - \frac{2}{2\pi}\int_0^1 \frac{25}{6}\left(\frac{d\omega}{25+\omega^2}\right)$$

$$W = \frac{25}{6\pi}\left[\tan^{-1}(\omega)\Big|_0^1 - \frac{\tan^{-1}(\omega/5)}{5}\Big|_0^1\right] = \frac{25}{6\pi}\left[0.785 - 0.039\right]$$

$$\boxed{W = 0.990\ J}$$

15.43 Compare the 1-Ω energy at both the input and output of the network in Fig. P15.43 for the given input forcing function $i_i(t) = 2e^{-4t}u(t)$ A.

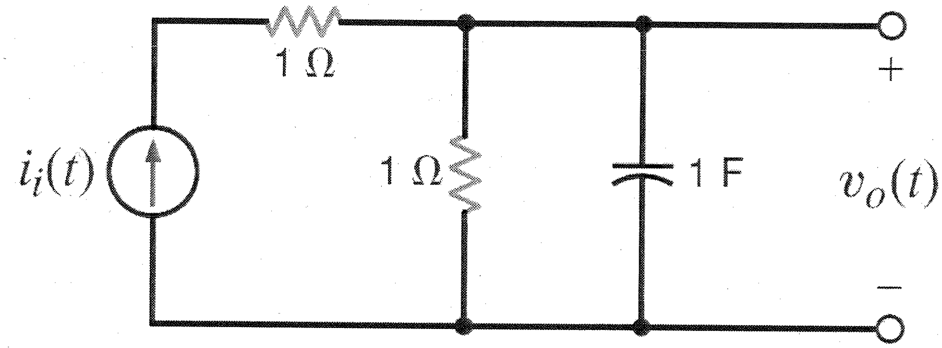
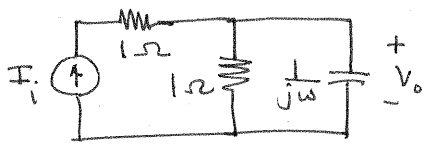


Figure P15.43

SOLUTION:



$$H(\omega) = \frac{V_o(\omega)}{I_i(\omega)} = 1 \,//\, \left(\frac{1}{j\omega}\right) = \frac{1}{1+j\omega}$$

$$I_i = \frac{2}{4+j\omega} \qquad V_o = \frac{2}{(1+j\omega)(4+j\omega)}$$

$$V_o = \frac{2/3}{1+j\omega} - \frac{2/3}{4+j\omega} \qquad |I_i|^2 = \frac{4}{16+\omega^2}$$

$$W_{in} = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{4}{16+\omega^2}\,d\omega = \frac{1}{2\pi}\left[\tan^{-1}(\omega/4)\right]\Big|_{-\infty}^{\infty} = \frac{1}{2}\ \text{J}$$

$$|V_o|^2 = \frac{4}{(1+\omega^2)(16+\omega^2)} = \frac{4/15}{1+\omega^2} - \frac{4/15}{16+\omega^2}$$

$$W_{OUT} = \frac{1}{2\pi}\left(\frac{4}{15}\right)\int_{-\infty}^{\infty}\left(\frac{1}{1+\omega^2} - \frac{1}{16+\omega^2}\right)d\omega = \frac{2}{15\pi}\left[\tan^{-1}(\omega) - \frac{\tan^{-1}(\omega/4)}{4}\right]\Big|_{-\infty}^{\infty}$$

$$W_{OUT} = \frac{2}{15\pi}\left[\pi - \pi/4\right]$$

$$\boxed{W_{OUT} = 0.1\ \text{J}}$$

$$\boxed{W_{IN} = 0.5\ \text{J}}$$

15.44 The waveform shown in Fig. P15.44 demonstrates what is called the duty cycle; that is, D illustrates the fraction of the total period that is occupied by the pulse. Determine the average value of this waveform.

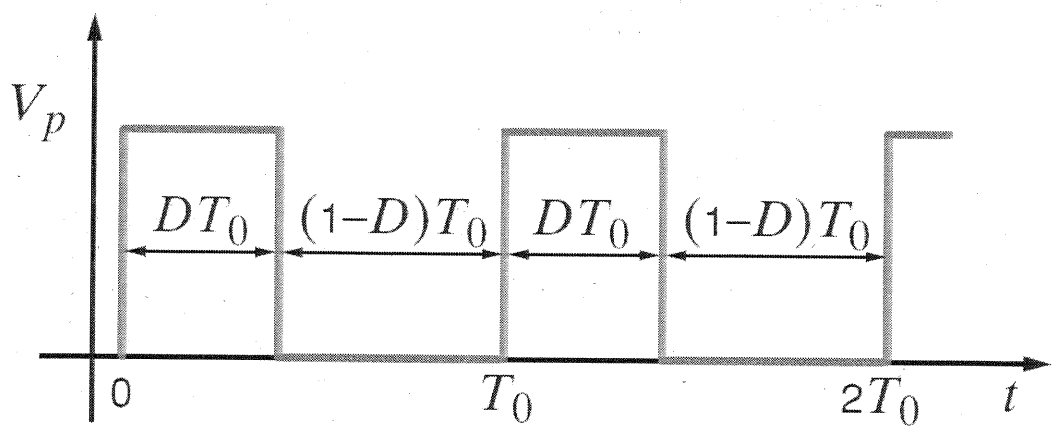


Figure P15.44

SOLUTION:

$$\text{average value} = a_0 = \frac{1}{T_0}\int_0^{T_0} v(t)\,dt = \frac{1}{T_0}\int_0^{DT_0} V_p\,dt$$

$$a_0 = \frac{V_P}{T_0}\, t\Big|_0^{DT_0} \qquad \boxed{a_0 = V_P D}$$

15FE-1 Given the waveform in Fig. 15PFE-1, determine which of the trigonometric Fourier coefficients have zero value, which have nonzero value, and why.

CS

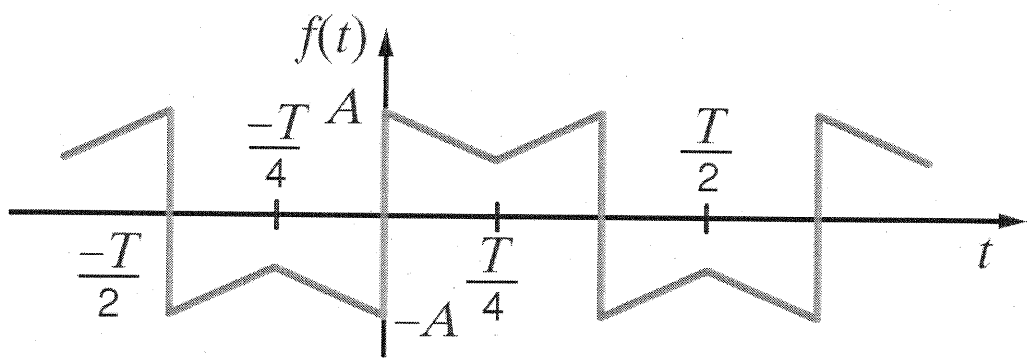


Figure 15PFE-1

SOLUTION:

Average value = 0 $\Rightarrow a_0 = 0$

$a_n = 0$ for all n since waveform has odd symmetry

$b_n = 0$ for n even since waveform has halfwave symmetry

b_n is finite & non zero for n odd.

15FE-2 Given the waveform in Fig. 15PFE-2, describe the type of symmetry and its impact on the trigonometric coefficients in the Fourier series—that is, a_0, a_n, and b_n.

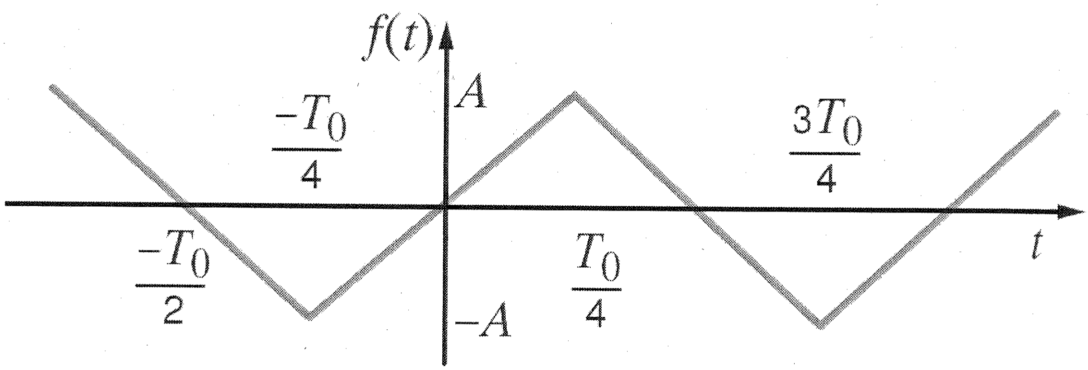


Figure 15PFE-2

SOLUTION:

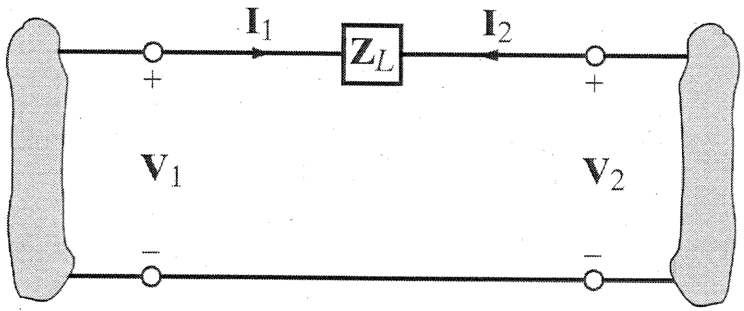
Average value = 0 $\Rightarrow a_0 = 0$

Odd symmetry $\Rightarrow$ all $a_n = 0$

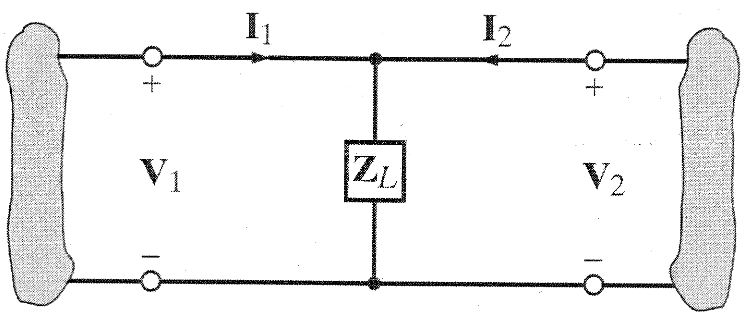
Halfwave symmetry $\Rightarrow b_n = 0$ for n even

b_n is non zero only for n-odd

16.1 Given the two networks in Fig. P16.1, find the Y parameters for the circuit in (a) and the Z parameters for the circuit in (b). CS



(a)



(b)

Figure P16.1

SOLUTION:

a) $Y_{11} = I_1/V_1\big|_{V_2=0} = 1/Z_L$ $\quad Y_{21} = -\frac{1}{Z_L}$ $\quad Y_{12} = -\frac{1}{Z_L}$ $\quad Y_{22} = 1/Z_L$

b) $Z_{11} = V_1/I_1\big|_{I_2=0} = Z_L$ $\quad Z_{21} = V_2/I_1\big|_{I_2=0} = Z_L$

$Z_{12} = Z_L$ $\quad Z_{22} = Z_L$

16.2 Find the Y parameters for the two-port network shown in Fig. P16.2.

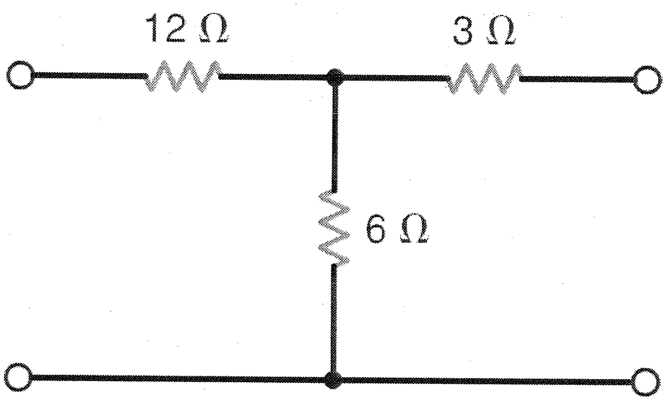
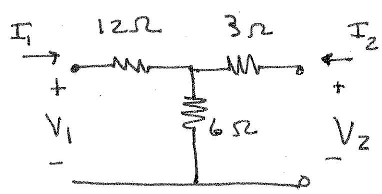


Figure P16.2

SOLUTION:



$$Y_{11} = I_1/V_T\big|_{V_2=0} = \frac{1}{12+(6//3)} = \frac{1}{14}\ S$$

$$Y_{22} = I_2/V_2\big|_{V_1=0} = \frac{1}{3+(12//6)} = \frac{1}{7}\ S$$

$$Y_{12} = I_1/V_2\big|_{V_1=0} = \left[\frac{12//6}{(12//6)+3}\right]\left(\frac{-1}{12}\right) = -\frac{1}{21}\ S$$

$$Y_{21} = I_2/V_1\big|_{V_2=0} = \left[\frac{3//6}{(3//6)+12}\right]\left(\frac{-1}{3}\right) = -\frac{1}{21}\ S$$

16.3 Find the Y parameters for the two-port network shown in Fig. P16.3. **PSV**

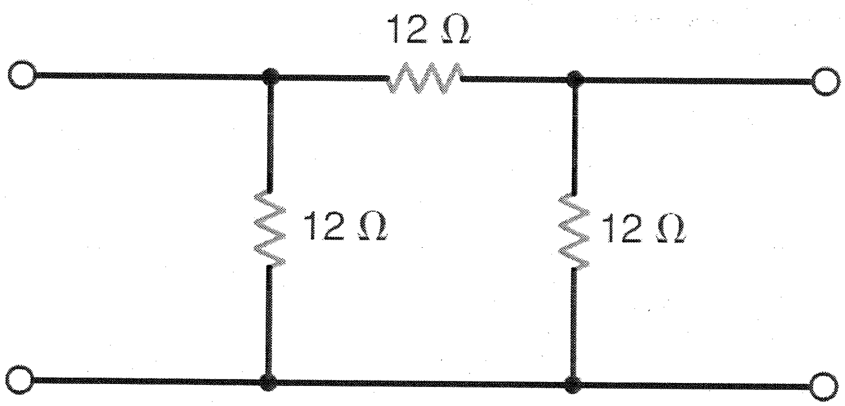
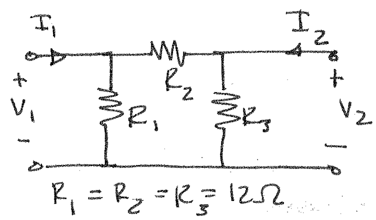


Figure P16.3

SOLUTION:



$$Y_{11} = I_1/V_1\big|_{V_2=0} = \frac{1}{R_1 // R_2} = \frac{1}{6}\ S$$

$$Y_{22} = \frac{1}{R_2 // R_3} = \frac{1}{6}\ S$$

$$Y_{21} = I_2/V_1\big|_{V_2=0} = \frac{-1}{R_2} = -\frac{1}{12}\ S$$

$$Y_{12} = I_1/V_2\big|_{V_1=0} = -\frac{1}{R_2} = -\frac{1}{12}\ S$$

16.4 Determine the Y parameters for the network shown in Fig. P16.4.

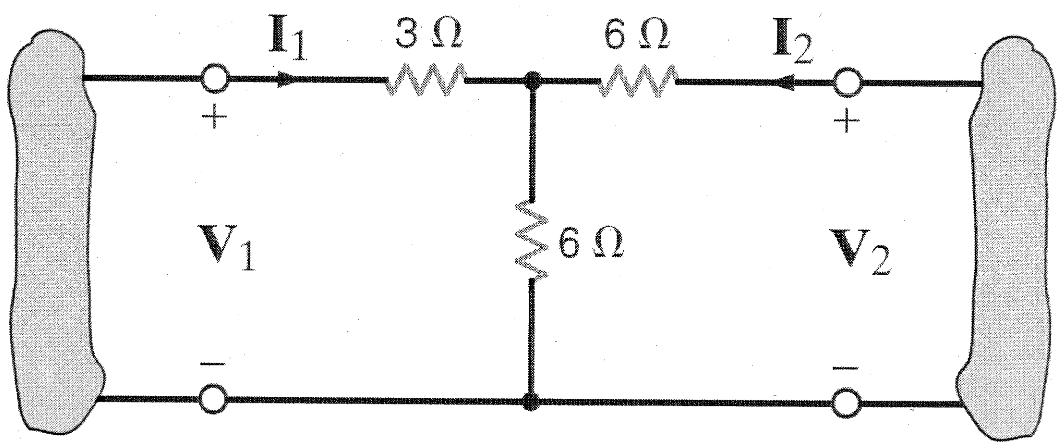


Figure P16.4

SOLUTION:

$$Y_{11} = I_1/V_1\Big|_{V_2=0} = \frac{1}{3+(6\,//\,6)} = \frac{1}{6}\ S$$

$$Y_{22} = I_2/V_2\Big|_{V_1=0} = \frac{1}{6+(6//3)} = \frac{1}{8}\ S$$

$$Y_{21} = I_2/V_1\Big|_{V_2=0} = \frac{6//6}{(6//6)+3}\left(-\frac{1}{6}\right) = -\frac{1}{12}\ S$$

$$Y_{12} = I_1/V_2\Big|_{V_1=0} = \frac{3//6}{(3//6)+6}\left(-\frac{1}{3}\right) = -\frac{1}{12}\ S$$

16.5 Find the Z parameters for the two-port network in Fig. P16.5.

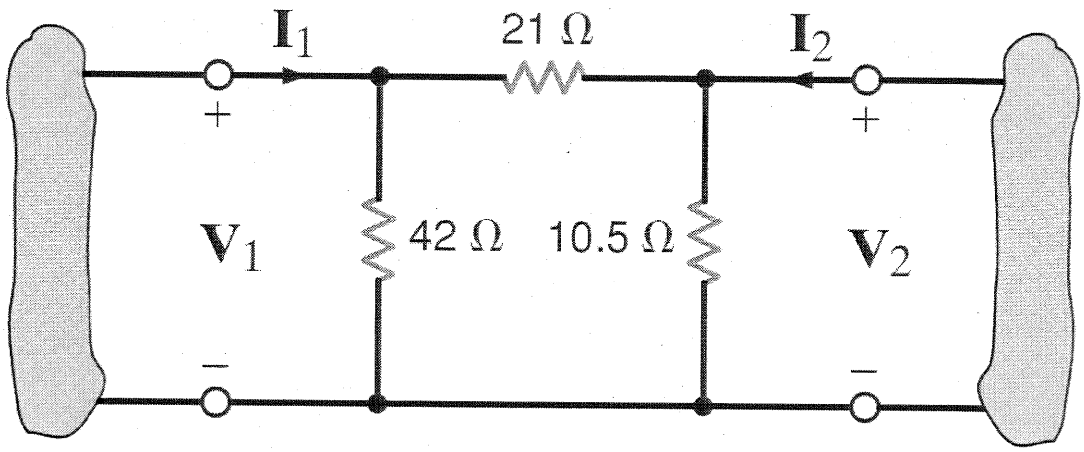


Figure P16.5

SOLUTION:

$$Z_{11} = V_1/I_1\Big|_{I_2=0} = 42//(21+10.5) = 18\,\Omega$$

$$Z_{22} = V_2/I_2\Big|_{I_1=0} = 10.5//(21+42) = 9\,\Omega$$

$$Z_{21} = V_2/I_1\Big|_{I_2=0} = \frac{42}{42+21+10.5}(10.5) = 6\,\Omega$$

$$Z_{12} = V_1/I_2\Big|_{I_1=0} = \frac{10.5}{10.5+21+42}(42) = 6\,\Omega$$

16.6 Determine the admittance parameters for the network shown in Fig. P16.6.

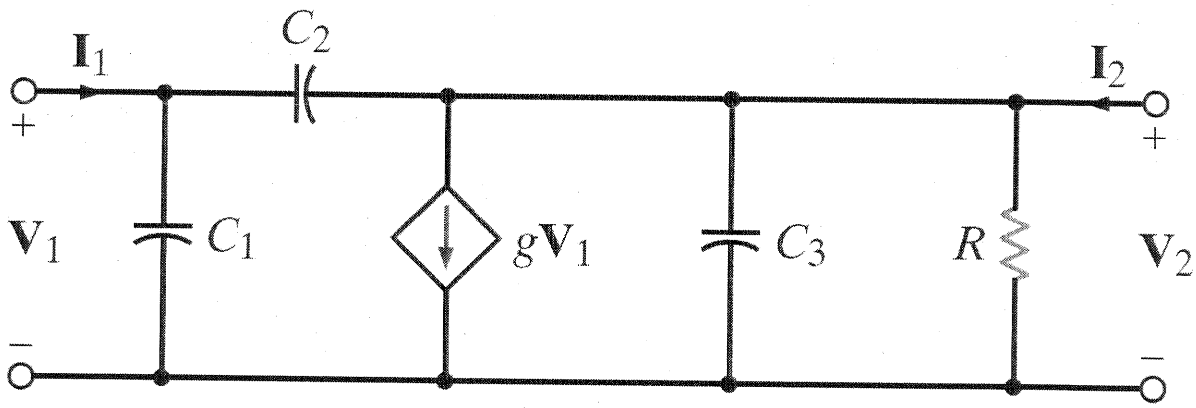


Figure P16.6

SOLUTION:

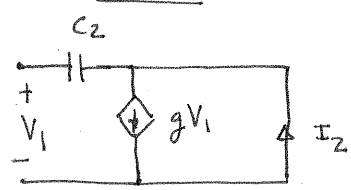
$Y_{11} = I_1/V_1 \big|_{V_2=0} = j\omega(C_1 + C_2)$

$Y_{22} = I_2/V_2 \big|_{V_1=0} = \frac{1}{R} + j\omega(C_2 + C_3)$

$Y_{21} = I_2/V_1 \big|_{V_2=0} = g - j\omega C_2$

$Y_{12} = I_1/V_2 \big|_{V_1=0} = -j\omega C_2$

For Y_{21}



$I_2 + V_1(j\omega C_2) = gV_1$

$I_2/V_1 = g - j\omega C_2$

16.7 Find the Y parameters for the two-port network in Fig. P16.7. **CS**

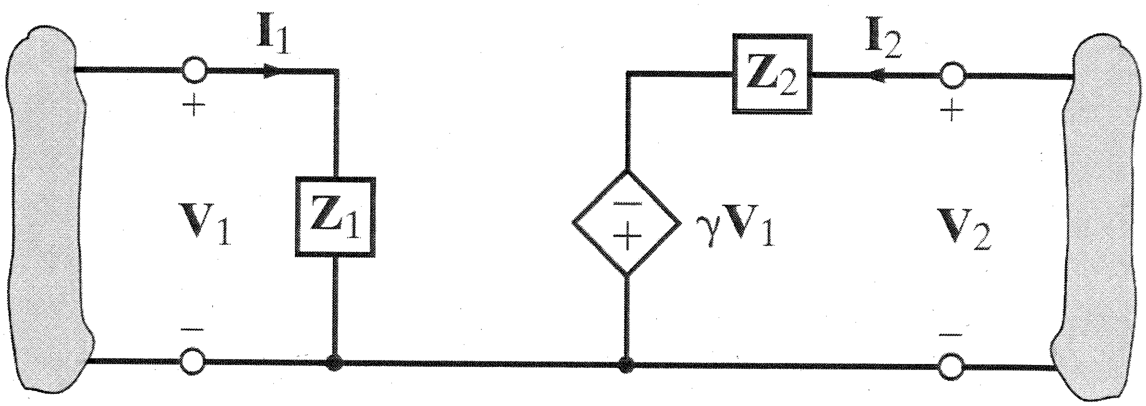


Figure P16.7

SOLUTION:

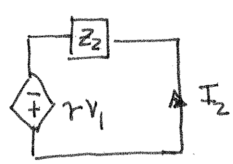
$Y_{11} = I_1 / V_1 \Big|_{V_2=0} = \frac{1}{Z_1}$

$Y_{22} = I_2 / V_2 \Big|_{V_1=0} = \frac{1}{Z_2}$

$Y_{21} = I_2 / V_1 \Big|_{V_2=0} = \gamma / Z_2$

$Y_{12} = I_1 / V_2 \Big|_{V_1=0} = 0$

For Y_{21}



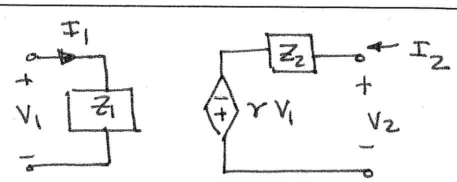
$I_2 = \frac{\gamma V_1}{Z_2}$

$\frac{I_2}{V_1} = \frac{\gamma}{Z_2}$

16.8 Find the Z parameters for the network in Fig. P16.7.

CS

SOLUTION:



$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = Z_1$$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = Z_2$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = \frac{-\gamma V_1}{V_1/Z_1} = -\gamma Z_1$$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = 0$$

16.9 Find the Z parameters for the two-port network shown in Fig. P16.9 and determine the voltage gain of the entire circuit with a 4-kΩ load attached to the output. **PSV**

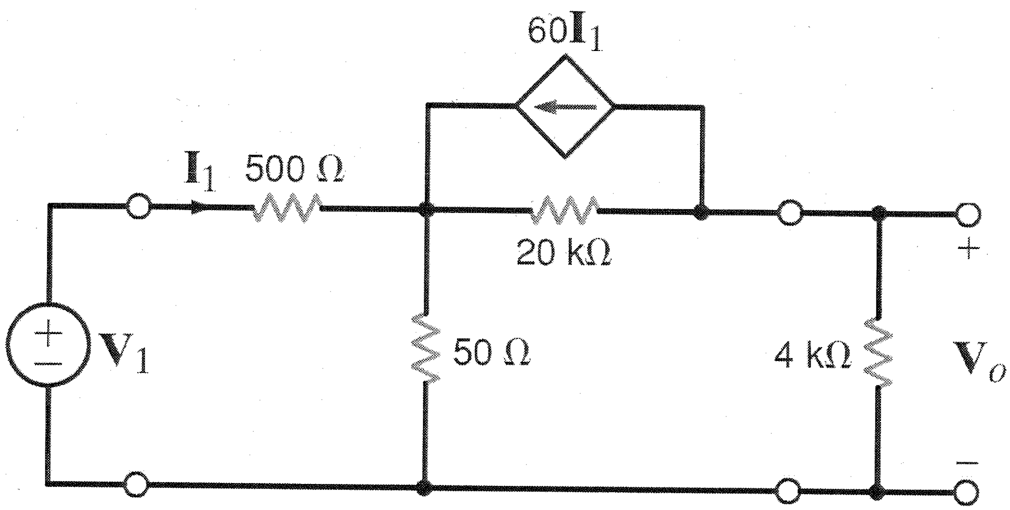
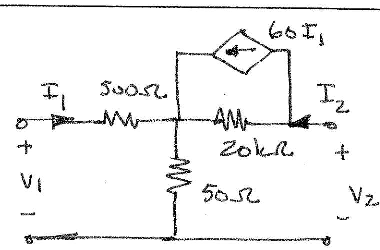


Figure P16.9

SOLUTION:



$$Z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0} = 550\,\Omega$$

$$Z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0} = 20.05\,k\Omega$$

$$Z_{12} = V_1/I_2\big|_{I_1=0} = 50\,\Omega \qquad Z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0} = \frac{50I_1 - 60I_1(20\times10^3)}{I_1} = -1.2M\,\Omega$$

w/ load

$$\left.\begin{aligned} V_1 &= 550I_1 + 50I_2 \\ V_2 &= -1.2\times10^6 I_1 + 20.05\times10^3 I_2 \end{aligned}\right\} \quad V_2 = -4000I_2$$

Solving for I_2/V_1 yields $\quad I_2/V_1 = 16.39\times10^{-3}$

$$\boxed{V_o/V_1 = 65.5}$$

16.10 Find the Z parameters for the two-port network shown in Fig. P16.10.

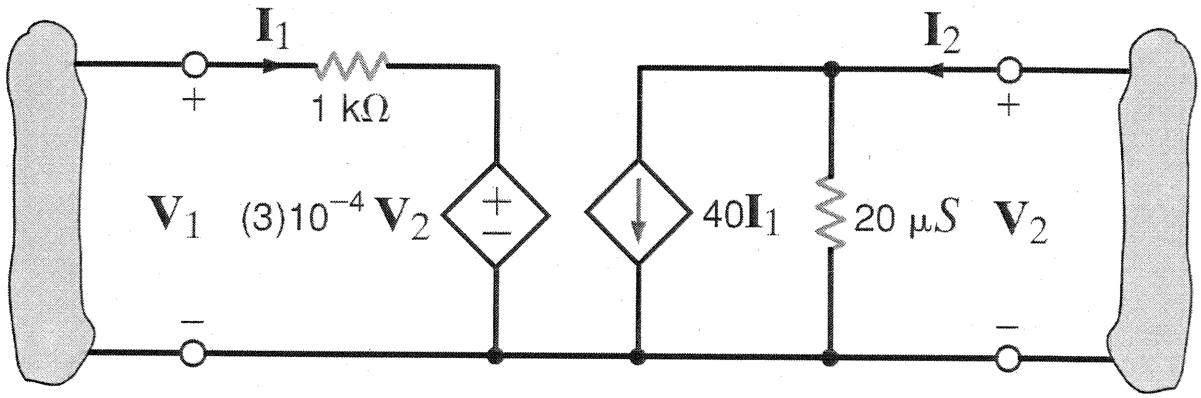


Figure P16.10

SOLUTION: Using h parameters,

$h_{11} = V_1/I_1\big|_{V_2=0} = 1k\Omega$ $\qquad h_{22} = I_2/V_2\big|_{I_1=0} = 20\mu S$

$h_{12} = V_1/V_2\big|_{I_1=0} = 3\times10^{-4}$ $\qquad h_{21} = I_2/I_1\big|_{V_2=0} = 40$

$\Delta h = h_{11}h_{22} - h_{12}h_{21} = 8\times10^{-3}$

$Z_{11} = \dfrac{\Delta h}{h_{22}} = 400\,\Omega$ $\qquad Z_{12} = \dfrac{h_{12}}{h_{22}} = 15\,\Omega$

$Z_{21} = \dfrac{-h_{21}}{h_{22}} = -2\times10^{6} = -2M\Omega$ $\qquad Z_{22} = \dfrac{1}{h_{22}} = 50\,k\Omega$

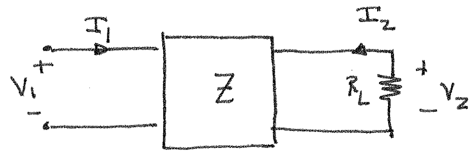
16.11 Find the voltage gain of the two-port network in Fig. P16.10 if a 12-kΩ load is connected to the output port. CS

SOLUTION:

From P16.10, $Z_{11} = 400\,\Omega$ $\quad Z_{12} = 15\,\Omega$ $\quad Z_{21} = -2\,M\Omega$ $\quad Z_{22} = 50\,k\Omega$

$V_1 = Z_{11} I_1 + Z_{12} I_2$

$V_2 = Z_{21} I_1 + Z_{22} I_2$



$R_L = 12\,k\Omega$

$V_2 = -I_2 R_L$

3 equations + unknowns. Eliminate I_1 + find V_2/V_1.

$$\boxed{\frac{V_2}{V_1} = -438}$$

16.12 Find the input impedance of the network in Fig. P16.10.

SOLUTION:

$$Z_{IN} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = Z_{11}$$

from P16.10, $Z_{11} = 400\,\Omega$

$$\boxed{Z_{IN} = 400\,\Omega}$$

16.13 Find the Z parameters of the two-port network in Fig. P16.13.

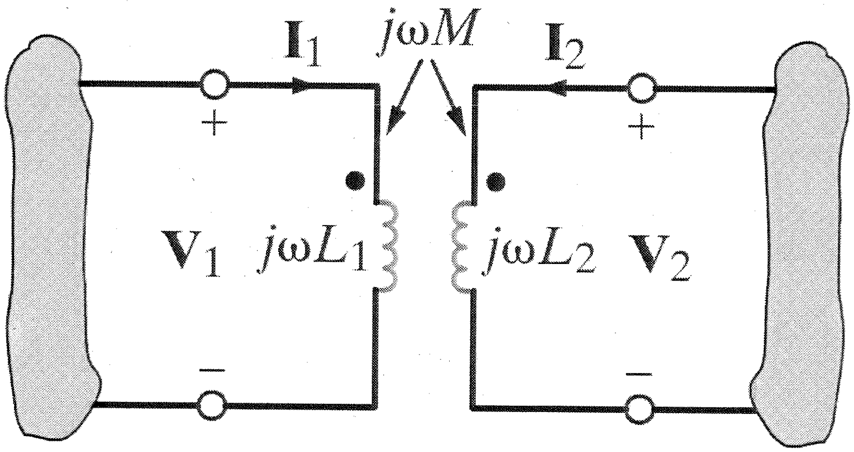


Figure P16.13

SOLUTION:

$V_1 = I_1(j\omega L_1) + I_2(j\omega M)$ $\quad$ $V_2 = I_1(j\omega M) + I_2(j\omega L_2)$

$Z_{11} = j\omega L_1$ $\quad$ $Z_{12} = j\omega M$ $\quad$ $Z_{21} = j\omega M$ $\quad$ $Z_{22} = j\omega L_2$

16.14 Determine the Z parameters for the two-port network in Fig. P16.14.

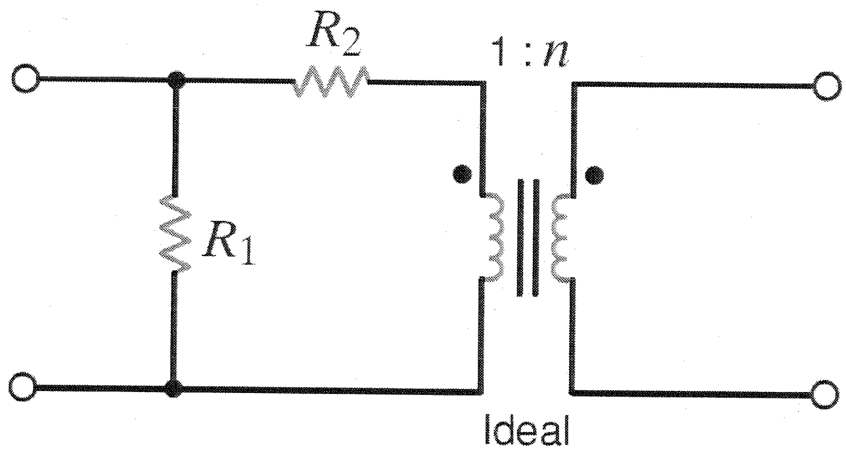
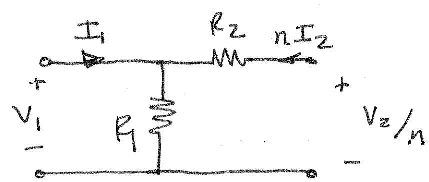


Figure P16.14

SOLUTION:



$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = R_1$$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$

$$V_1 = nI_2R_1 \Rightarrow Z_{12} = nR_1$$

$$Z_{21} = V_2/I_1\Big|_{I_2=0} \Rightarrow \frac{V_2}{n} = I_1R_1 \Rightarrow Z_{21} = nR_1$$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} \Rightarrow \frac{V_2}{n} = I_2n(R_1+R_2) \Rightarrow Z_{22} = n^2(R_1+R_2)$$

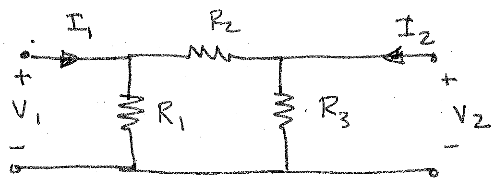
16.15 Draw the circuit diagram (with all passive elements in ohms) for a network that has the following Y parameters:

$$[Y] = \begin{bmatrix} \frac{3}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{5}{6} \end{bmatrix}$$

SOLUTION:

$I_1 = V_1 Y_{11} + V_2 Y_{12}$

$I_2 = V_1 Y_{21} + V_2 Y_{22}$



$$I_1 = V_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right) - V_2\left(\frac{1}{R_2}\right)$$

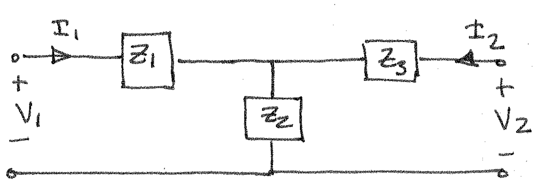
$$I_2 = -\frac{1}{R_2}V_1 + V_2\left(\frac{1}{R_2} + \frac{1}{R_3}\right)$$

$\boxed{R_2 = 2\Omega \qquad R_1 = 1\Omega \qquad R_3 = 3\Omega}$

16.16 Draw the circuit diagram for a network that has the following Z parameters:

$$[Z] = \begin{bmatrix} 6 - j2 & 4 - j6 \\ 4 - j6 & 7 + j2 \end{bmatrix}$$

SOLUTION:



$V_1 = I_1(Z_1 + Z_2) + Z_2 I_2$

$V_2 = +Z_2 I_1 + I_2(Z_2 + Z_3)$

$Z_2 = 4 - j6\,\Omega \qquad Z_1 = 2 + j4\,\Omega$

$Z_3 = 3 + j8\,\Omega$

16.17 Show that the network in Fig. P16.17 does not have a set of Y parameters unless the source has an internal impedance.

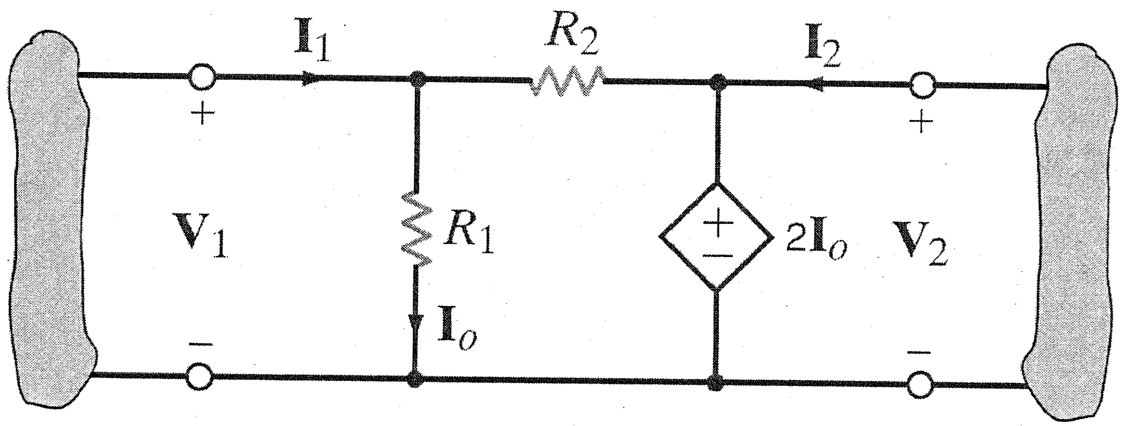
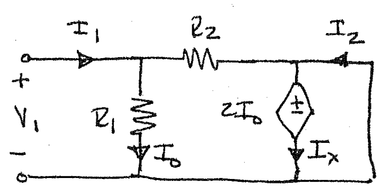


Figure P16.17

SOLUTION:

with $V_2 = 0$



$2I_o = V_2 = 0 \Rightarrow I_o = 0$

$V_1 = I_o R_1 = 0 \Rightarrow V_1 = 0$

$I_1 = \frac{V_1}{R_1} + \frac{V_1 - V_2}{R_2} = 0$

No power supplied by input port or consumed by R_1 & R_2

thus, $I_x = 0$ & $I_2 = 0$.

$Y_{11} = I_1/V_1 \big|_{V_2=0} = \frac{0}{0}$ undefined! $\quad Y_{21} = I_2/V_1 \big|_{V_2=0} = \frac{0}{0}$ undefined!

with $V_1 = 0$: $\quad I_o = V_1/R_1 = 0 \quad V_2 = 2I_o = 0$

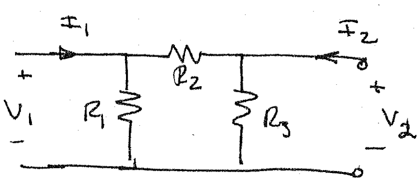
$I_1 = V_1/R_1 + (V_1 - V_2)/R_2 = 0$

no power at input port or consumed by R_1 & $R_2 \Rightarrow I_2 = 0$

$Y_{12} = I_1/V_2 \big|_{V_1=0} = \frac{0}{0}$ undefined $\quad Y_{22} = I_2/V_2 \big|_{V_1=0} = \frac{0}{0}$ undefined!

16.18 Compute the hybrid parameters for the network in Fig. E16.1.

SOLUTION:



$R_1 = 42\Omega$ $R_2 = 21\Omega$ $R_3 = 10.5\Omega$

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = R_1 /\!/ R_2 = 14\Omega$$

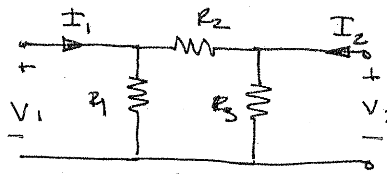
$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} = \frac{-R_1}{R_1+R_2} = -\frac{2}{3}$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} = \frac{R_1}{R_1+R_2} = \frac{2}{3}$$

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} = \frac{1}{R_3 /\!/ (R_1+R_2)} = \frac{1}{9}\ S$$

16.19 Find the hybrid parameters for the network in Fig. 16.3. **PSV**

SOLUTION:



$R_1 = 1\Omega$ $R_2 = 2\Omega$ $R_3 = 3\Omega$

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = R_1 // R_2 = 2/3\,\Omega$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} = -\frac{R_1}{R_1+R_2} = -\frac{1}{3}$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} = +\frac{R_1}{R_1+R_2} = \frac{1}{3}$$

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} = \frac{1}{R_3 // (R_1+R_2)} = \frac{2}{3}\ \text{S}$$

16.20 Consider the network in Fig. P16.20. The two-port network is a hybrid model for a basic transistor. Determine the voltage gain of the entire network, $\mathbf{V}_2/\mathbf{V}_S$, if a source $\mathbf{V}_S$ with internal resistance R_1 is applied at the input to the two-port network and a load R_L is connected at the output port.

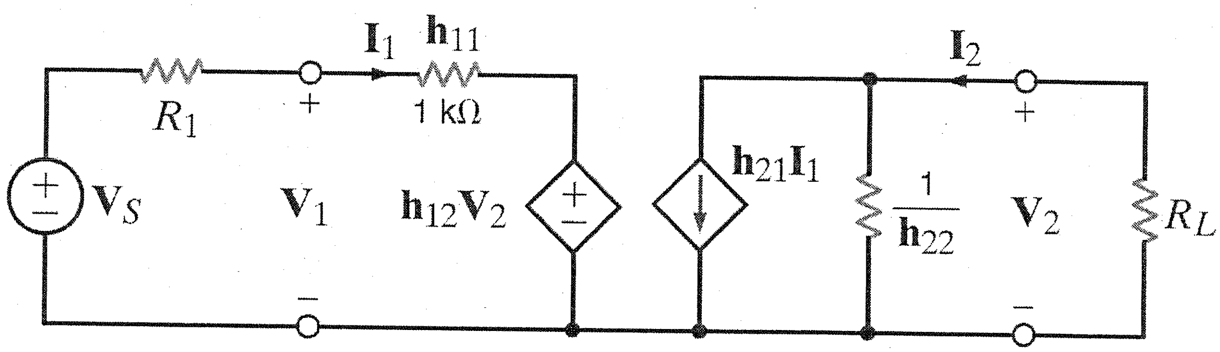


Figure P16.20

SOLUTION:

$$\left.\begin{aligned} V_1 &= h_{11} I_1 + h_{12} V_2 \\ I_2 &= h_{21} I_1 + h_{22} V_2 \end{aligned}\right\} \quad \text{and,} \quad \left\{\begin{aligned} V_2 &= -R_L I_2 \Rightarrow I_2 = -V_2/R_L \\ V_S &= I_1 R_1 + V_1 \Rightarrow V_1 = V_S - I_1 R_1 \end{aligned}\right.$$

Now,

$$V_S = I_1 (h_{11} + R_1) + h_{12} V_2$$

$$0 = h_{21} I_1 + V_2 (h_{22} + 1/R_L)$$

$$V_2 = \frac{\begin{vmatrix} h_{11}+R_1 & V_S \\ h_{21} & 0 \end{vmatrix}}{\begin{vmatrix} h_{11}+R_1 & h_{12} \\ h_{21} & h_{22}+\frac{1}{R_L} \end{vmatrix}} \Rightarrow V_2 = \frac{-V_S h_{21}}{(h_{11}+R_1)\left(h_{22}+\frac{1}{R_L}\right) - h_{12} h_{21}}$$

$$\boxed{\frac{V_2}{V_S} = \frac{h_{21} R_L}{h_{12} h_{21} R_L - (1 + h_{22} R_L)(h_{11} + R_1)}}$$

16.21 Determine the hybrid parameters for the network shown in Fig. P16.21.

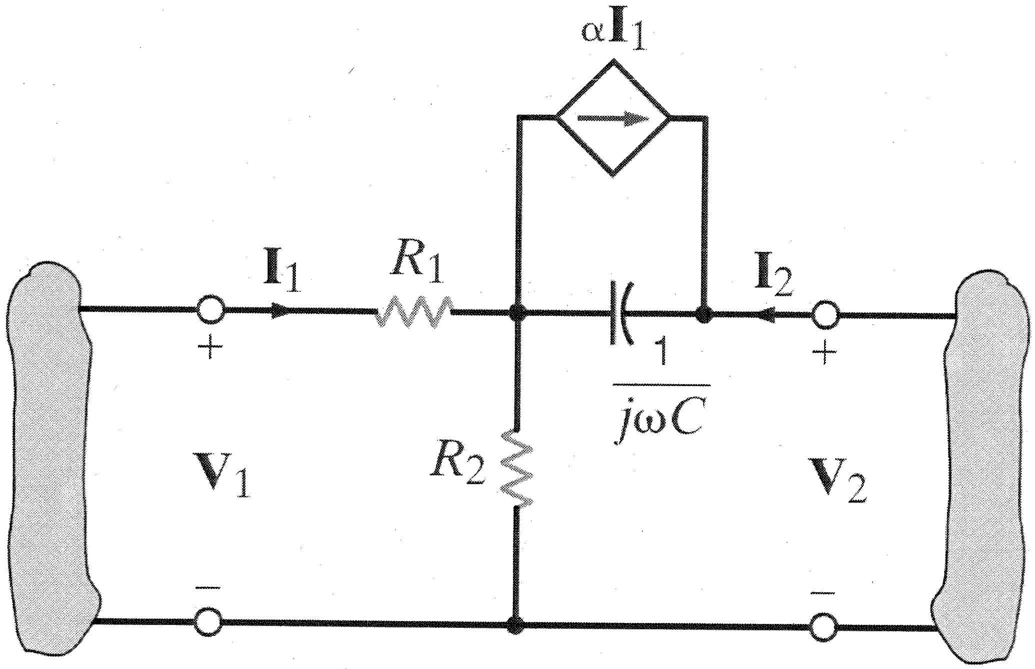
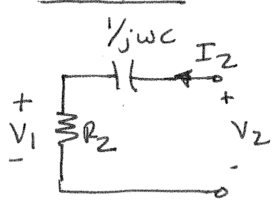


Figure P16.21

SOLUTION:

For $I_1 = 0$



$$* \quad h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} = \frac{j\omega C R_2}{1 + j\omega C R_2}$$

$$* \quad h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} = \frac{j\omega C}{1 + j\omega C R_2}$$

For $V_2 = 0$ Use loop analysis!

$$(I_2 + \alpha I_1)/j\omega C + (I_1 + I_2)R_2 = 0$$

yields $\quad I_1(\alpha + j\omega C R_2) + I_2(1 + j\omega C R_2) = 0$

$$h_{21} = I_2/I_1 = -\frac{\alpha + j\omega C R_2}{1 + j\omega C R_2} \quad *$$

And,

$$V_1 = I_1(R_1 + R_2) + I_2 R_2 = I_1\left\{R_1 + R_2 - R_2\left(\frac{\alpha + j\omega C R_2}{1 + j\omega C R_2}\right)\right\}$$

$$V_1 = I_1\left\{\frac{R_1 + R_2(1-\alpha) + j\omega R_1 R_2 C}{1 + j\omega R_2 C}\right\}$$

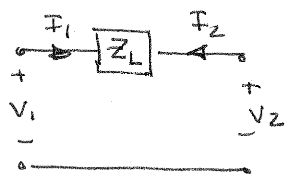
$$h_{11} = \frac{V_1}{I_1} = \frac{R_1 + R_2(1-\alpha) + j\omega R_1 R_2 C}{1 + j\omega R_2 C} \quad *$$

16.22 Find the ABCD parameters for the networks in Fig. P16.1. CS

SOLUTION:

a) $A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = 1$

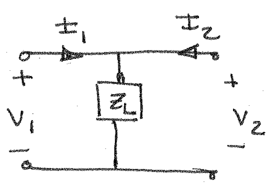
$B = \left.\frac{V_1}{-I_2}\right|_{V_2=0} = Z_L$



$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = 0$

$D = \left.\frac{I_1}{-I_2}\right|_{V_2=0} = 1$

b) $A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = 1$



$B = \left.\frac{V_1}{-I_2}\right|_{V_2=0} = 0$

$C = I_1/V_2\Big|_{I_2=0} = \frac{1}{Z_L}$

$D = \left.\frac{I_1}{-I_2}\right|_{V_2=0} = 1$

16.23 Find the transmission parameters for the network in Fig. P16.23.

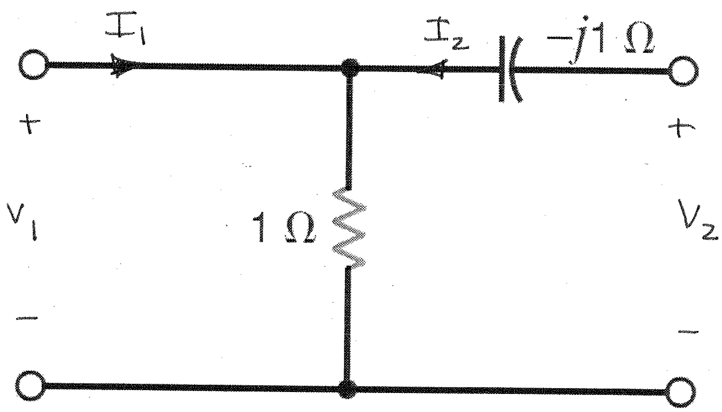


Figure P16.23

SOLUTION:

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = 1$$

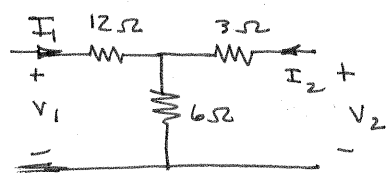
$$B = \left.\frac{V_1}{-I_2}\right|_{V_2=0} = -j1\Omega$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = 1\ S$$

$$D = \left.\frac{I_1}{-I_2}\right|_{V_2=0} \Rightarrow \quad -I_2 = I_1\left(\frac{1}{1-j1}\right) \Rightarrow \quad D = 1-j1$$

16.24 Find the transmission parameters for the network shown in Fig. P16.2. **PSV**

SOLUTION:



$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} \Rightarrow V_2 = \frac{6}{18}V_1 \Rightarrow A = 3$$

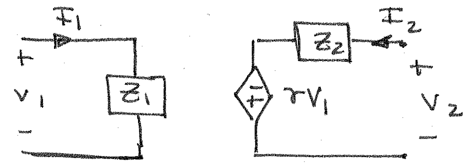
$$B = \left.\frac{V_1}{-I_2}\right|_{V_2=0} \Rightarrow -I_2 = \frac{(3/\!/6)V_1}{12+(3/\!/6)}\left(\frac{1}{3}\right) \Rightarrow B = 21\Omega$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = \frac{1}{6}\,S$$

$$D = \left.\frac{I_1}{-I_2}\right|_{V_2=0} \Rightarrow -I_2 = I_1\left(\frac{6}{6+3}\right) \Rightarrow D = \frac{3}{2}$$

16.25 Find the ABCD parameters for the circuit in Fig. P16.7.

SOLUTION:



$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} \Rightarrow V_2 = -\gamma V_1 \Rightarrow A = -\frac{1}{\gamma}$

$B = \left.\frac{V_1}{-I_2}\right|_{V_2=0} \Rightarrow I_2 = \frac{\gamma V_1}{Z_2} \Rightarrow B = -Z_2/\gamma$

$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} \Rightarrow V_2 = -\gamma V_1 \ \& \ V_1 = I_1 Z_1 \Rightarrow C = -\frac{1}{\gamma Z_1}$

$D = \left.\frac{I_1}{-I_2}\right|_{V_2=0} \Rightarrow I_2 = \frac{\gamma V_1}{Z_2} \ \& \ I_1 = V_1/Z_1 \Rightarrow D = -\frac{Z_2}{\gamma Z_1}$

16.26 Determine the transmission parameters for the network in Fig. P16.26. CS

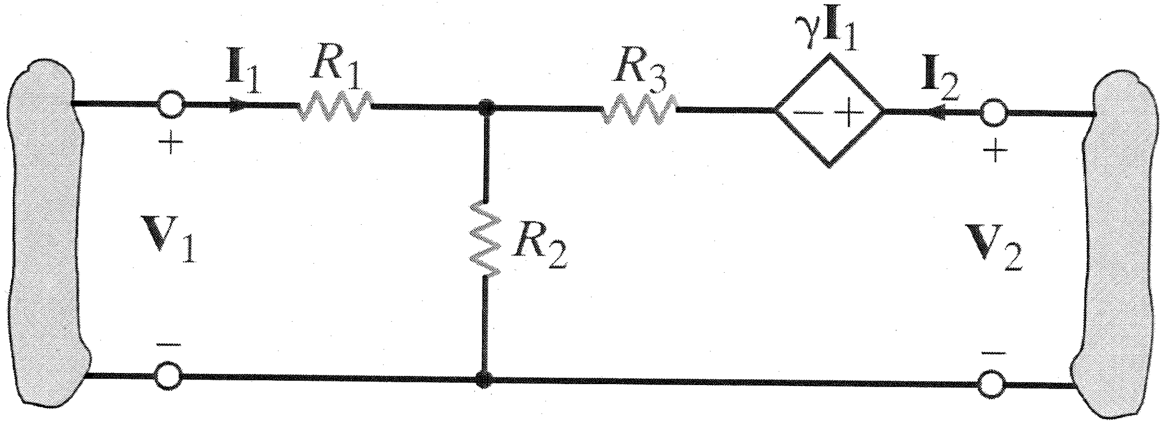


Figure P16.26

SOLUTION: Loop analysis!

$$V_1 = I_1(R_1+R_2) + R_2 I_2$$

$$V_2 = (\gamma + R_2) I_1 + (R_2+R_3) I_2$$

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{R_1+R_2}{\gamma + R_1} \quad *$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = \frac{1}{\gamma + R_2} \quad *$$

$$B = \left.\frac{V_1}{-I_2}\right|_{V_2=0}$$

$$D = \left.\frac{I_1}{-I_2}\right|_{V_2=0} = \frac{R_2+R_3}{\gamma + R_2} \quad *$$

so, $I_1 = -I_2(R_2+R_3)/(\gamma + R_2)$

and, $V_1 = I_2\left[R_2 - (R_1+R_2)\frac{R_2+R_3}{\gamma+R_2}\right] = -I_2\left[\frac{R_1R_2 + R_1R_3 + R_2R_3 - \gamma R_2}{\gamma + R_2}\right]$

$$* \quad B = \frac{R_1R_2 + R_1R_3 + R_2R_3 - \gamma R_2}{\gamma + R_2}$$

16.27 Find the transmission parameters for the circuit in Fig. P16.27.

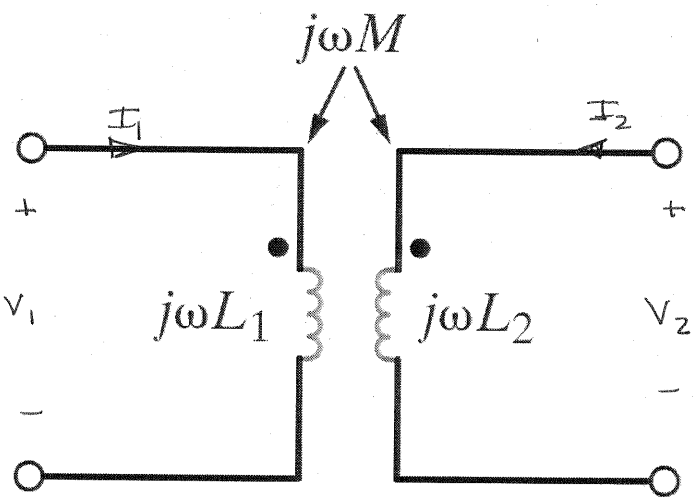


Figure P16.27

SOLUTION:

$V_1 = j\omega L_1 I_1 + j\omega M I_2$

$V_2 = j\omega M I_1 + j\omega L_2 I_2$

$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = L_1/M$ *

$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = \frac{1}{j\omega M}$ *

$B = \left.\frac{V_1}{-I_2}\right|_{V_2=0}$

$D = \left.\frac{I_1}{-I_2}\right|_{V_2=0} = L_2/M$ *

or, $I_1 = -I_2(L_2/M)$

& $V_1 = -I_2\left(\frac{j\omega L_1 L_2}{M} - j\omega M\right) = -I_2\left(\frac{j\omega L_1 L_2 - j\omega M^2}{M}\right)$

* $B = j\omega\left(\frac{L_1 L_2 - M^2}{M}\right)$

16.28 Given the network in Fig. P16.28, find the transmission parameters for the two-port network and then find $\mathbf{I}_o$ using the terminal conditions.

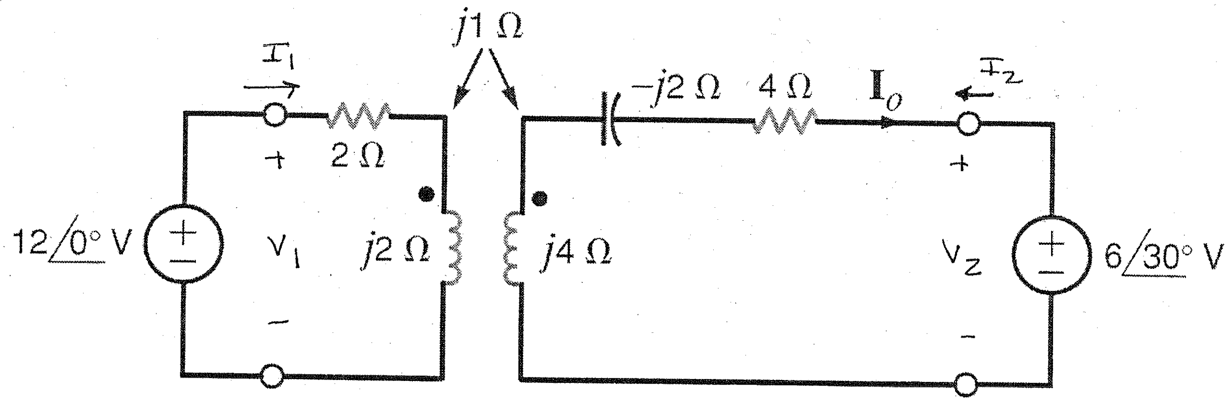


Figure P16.28

SOLUTION:

$V_1 = I_1(2+j2) + j1\,I_2$

$V_2 = j1\,I_1 + I_2(4+j2)$

$A = \left.\dfrac{V_1}{V_2}\right|_{I_2=0} = \dfrac{2+j2}{j1} = 2-j2$ ✱

$D = \left.\dfrac{I_1}{-I_2}\right|_{V_2=0} = \dfrac{4+j2}{j1} = 2-j4$ ✱

$B = \left.\dfrac{V_1}{-I_2}\right|_{V_2=0}$

$C = I_1/V_2\big|_{I_2=0} = -j1\ S$ ✱

now, $I_1 = -I_2(2-j4)$

and $V_1 = -I_2[(2-j4)(2+j2) - j1] = -I_2(12-j5)$

✱ $B = 12 - j5\ \Omega$

Terminal conditions, $V_1 = 12\angle 0° V$ $V_2 = 6\angle 30° V$ $I_o = -I_2$

$$\begin{bmatrix} 2+j2 & -j1 \\ j1 & -4-j2 \end{bmatrix}\begin{bmatrix} I_1 \\ I_o \end{bmatrix} = \begin{bmatrix} 12\angle 0° \\ 6\angle 30° \end{bmatrix} \Rightarrow I_o = 0.48\angle 157.6°\ A$$ ✱

16.29 Find the input admittance of the two-port in Fig. P16.29 in terms of the Y parameters and the load $\mathbf{Y}_L$.

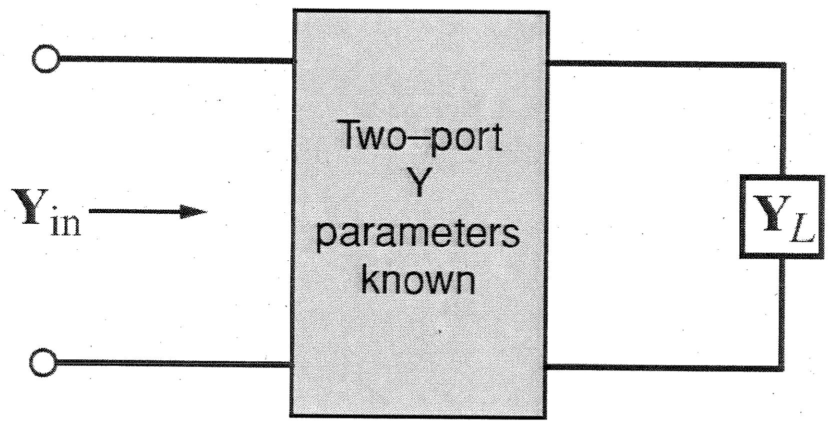


Figure P16.29

SOLUTION:

$$\left.\begin{aligned} I_1 &= V_1 Y_{11} + V_2 Y_{12} \\ I_2 &= Y_{21} V_1 + Y_{22} V_2 \end{aligned}\right\} \quad I_2 = -V_2 Y_L$$

$$Y_{in} = \frac{I_1}{V_1} = Y_{11} + Y_{12}\left(V_2/V_1\right)$$

$$-V_2 Y_L = Y_{21} V_1 + Y_{22} V_2$$

$$0 = Y_{21} V_1 + V_2(Y_{22} + Y_L) \Rightarrow V_2/V_1 = -\frac{Y_{21}}{Y_{22} + Y_L}$$

$$\boxed{Y_{in} = Y_{11} - \frac{Y_{12} Y_{21}}{Y_{22} + Y_L}}$$

16.30 Find the voltage gain $\mathbf{V}_2/\mathbf{V}_1$ for the network in Fig. P16.30 using the Z parameters.

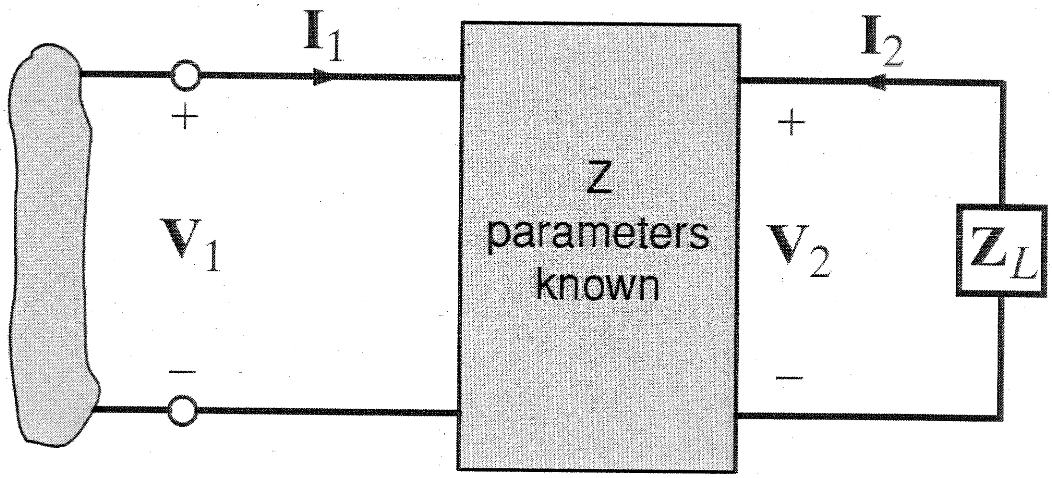


Figure P16.30

SOLUTION:

$V_1 = I_1 Z_{11} + I_2 Z_{12} \qquad V_2 = I_1 Z_{21} + I_2 Z_{22} \qquad V_2 = -I_2 Z_L \Rightarrow I_2 = -\frac{V_2}{Z_L}$

now, $V_1 = I_1 Z_{11} - V_2 \left(Z_{12}/Z_L \right)$

$V_2 = I_1 Z_{21} - V_2 \left(Z_{22}/Z_L \right) \Rightarrow I_1 = V_2 \frac{(Z_L + Z_{22})}{Z_L Z_{21}}$

and, $V_1 = V_2 \left[\frac{Z_{11}(Z_L + Z_{22})}{Z_L \cdot Z_{21}} - \frac{Z_{12}}{Z_L} \right] = V_2 \left[\frac{Z_{11} Z_L + Z_{11} Z_{22} - Z_{12} Z_{21}}{Z_L Z_{21}} \right]$

$$\boxed{\frac{V_2}{V_1} = \frac{Z_{11} Z_L + Z_{11} Z_{22} - Z_{12} Z_{21}}{Z_L Z_{21}}}$$

16.31 Following are the hybrid parameters for a network.

$$\begin{bmatrix} \mathbf{h}_{11} & \mathbf{h}_{12} \\ \mathbf{h}_{21} & \mathbf{h}_{22} \end{bmatrix} = \begin{bmatrix} \frac{11}{5} & \frac{2}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix}$$

Determine the Y parameters for the network.

SOLUTION:

$* \ y_{11} = \frac{1}{h_{11}} = \frac{5}{11} \text{ S}$ $\qquad y_{21} = \frac{h_{21}}{h_{11}} = -\frac{2}{11} \text{ S} \ *$

$* \ y_{12} = \frac{-h_{12}}{h_{11}} = -\frac{2}{11} \text{ S}$

$y_{22} = \frac{\Delta_H}{h_{11}} \qquad \Delta_H = \frac{11}{5}\left(\frac{1}{5}\right) - \left(-\frac{2}{5}\right)\left(\frac{2}{5}\right) = \frac{11}{25} + \frac{4}{25} = \frac{15}{25} = \frac{3}{5}$

$* \ y_{22} = \frac{3}{11} \text{ S}$

16.32 If the Y parameters for a network are known to be

$$\begin{bmatrix} \mathbf{y}_{11} & \mathbf{y}_{12} \\ \mathbf{y}_{21} & \mathbf{y}_{22} \end{bmatrix} = \begin{bmatrix} \frac{5}{11} & -\frac{2}{11} \\ -\frac{2}{11} & \frac{3}{11} \end{bmatrix}$$

find the Z parameters. **CS**

SOLUTION:

$$\Delta_Y = \frac{5}{11}\left(\frac{3}{11}\right) - \left(-\frac{2}{11}\right)^2 = \frac{15}{121} - \frac{4}{121} = \frac{1}{11}$$

$z_{11} = y_{22}/\Delta_Y = 3\,\Omega$ $\qquad$ $z_{21} = -y_{21}/\Delta_Y = 2\,\Omega$

$z_{12} = -y_{12}/\Delta_Y = 2\,\Omega$ $\qquad$ $z_{22} = y_{11}/\Delta_Y = 5\,\Omega$

16.33 Find the Z parameters in terms of the ABCD parameters.

SOLUTION:

ABCD Parameters

$V_1 = AV_2 - BI_2$

$I_1 = CV_2 - DI_2$

Z Parameters

$V_1 = Z_{11} I_1 + Z_{12} I_2$

$V_2 = Z_{21} I_1 + Z_{22} I_2$

$V_2 = I_1/C + (D/C) I_2$

$V_1 = \frac{AI_1}{C} + \frac{AD}{C} I_2 - BI_2 = \frac{A}{C} I_1 + \left(\frac{AD - BC}{C}\right) I_2$

By comparisim,

$Z_{21} = \frac{1}{C}$ $\quad Z_{22} = D/C$ $\quad Z_{11} = A/C$

$Z_{12} = \frac{AD - BC}{C} = \frac{\Delta_T}{C}$

16.34 Find the hybrid parameters in terms of the impedance parameters.

SOLUTION:

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \qquad (1)$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \qquad (2) \qquad \Rightarrow \qquad I_2 = -\frac{Z_{21}}{Z_{22}} I_1 + \frac{1}{Z_{22}} V_2 \qquad (3)$$

Put (3) into (1)

$$V_1 = I_1 \left(Z_{11} - \frac{Z_{12} Z_{21}}{Z_{22}} \right) + \frac{Z_{12}}{Z_{22}} V_2 \qquad (4)$$

from (3) & (4):

$$\boxed{h_{11} = \frac{Z_{11} Z_{22} - Z_{12} Z_{21}}{Z_{22}} \qquad h_{12} = Z_{12}/Z_{22} \qquad h_{21} = -\frac{Z_{21}}{Z_{22}} \qquad h_{22} = \frac{1}{Z_{22}}}$$

16.35 Find the Y parameters for the network in Fig. P16.35.

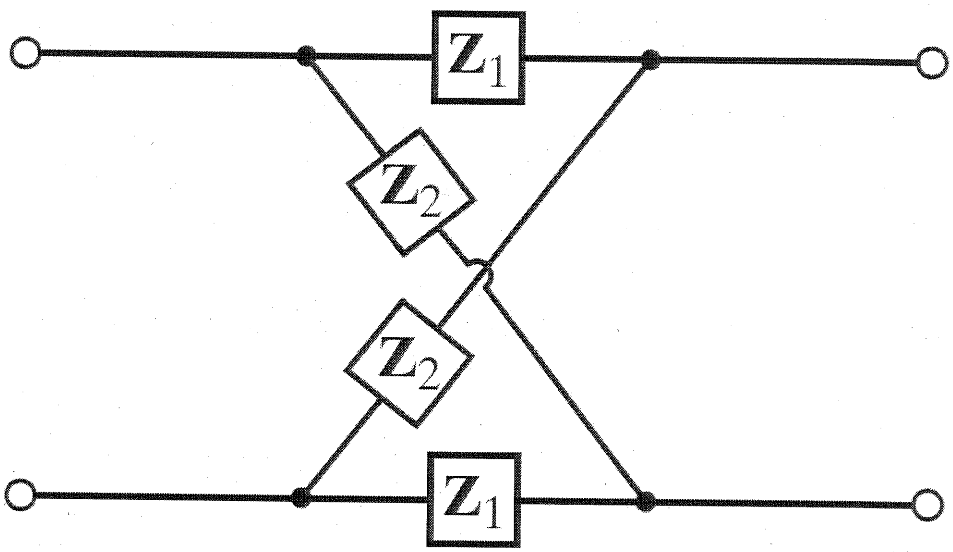
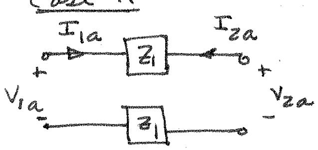


Figure P16.35

SOLUTION: Use parallel connections.

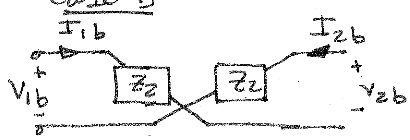
Case A



$$y_{11a} = \frac{1}{2Z_1} \qquad y_{12a} = -\frac{1}{2Z_1}$$

$$y_{21a} = -\frac{1}{2Z_1} \qquad y_{22a} = \frac{1}{2Z_1}$$

Case B



$$y_{11b} = \frac{1}{2Z_2} \qquad y_{12b} = \frac{1}{2Z_2}$$

$$y_{21b} = \frac{1}{2Z_2} \qquad y_{22b} = \frac{1}{2Z_2}$$

Total network: $y_{ij} = y_{ija} + y_{ijb}$

$$y_{11} = \frac{1}{2Z_1} + \frac{1}{2Z_2} \qquad y_{12} = \frac{1}{2Z_2} - \frac{1}{2Z_1}$$

$$y_{21} = \frac{1}{2Z_2} - \frac{1}{2Z_1} \qquad y_{22} = \frac{1}{2Z_1} + \frac{1}{2Z_2}$$

16.36 Determine the Y parameters for the network shown in Fig. P16.36.

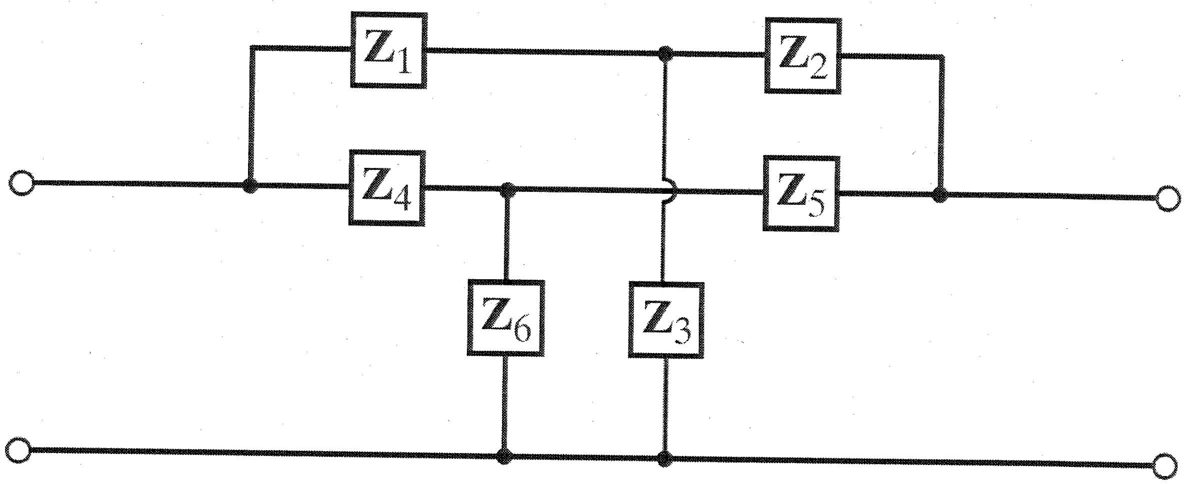
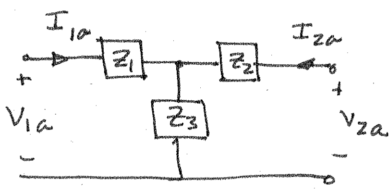


Figure P16.36

SOLUTION: 2 parallel T networks.

Case A



$$y_{11a} = \frac{1}{Z_1 + (Z_2 /\!/ Z_3)} = \frac{Z_2 + Z_3}{Z_1 Z_2 + Z_2 Z_3 + Z_1 Z_3} = \frac{Z_2 + Z_3}{Z_A}$$

$$y_{22a} = \frac{1}{Z_2 + (Z_1 /\!/ Z_3)} = \frac{Z_1 + Z_3}{Z_A}$$

$$y_{12a} = \frac{Z_1 /\!/ Z_3}{Z_2 + (Z_1 /\!/ Z_3)}\left(\frac{-1}{Z_1}\right) = -\frac{Z_3}{Z_A} \qquad y_{21a} = -\frac{Z_3}{Z_A}$$

Similarly for the $Z_4 - Z_5 - Z_6$ T network,

$$y_{11b} = \frac{Z_5 + Z_6}{Z_B} \qquad y_{22b} = \frac{Z_4 + Z_6}{Z_B} \qquad y_{21b} = y_{12b} = -\frac{Z_6}{Z_B} \qquad Z_B = Z_4 Z_5 + Z_5 Z_6 + Z_4 Z_6$$

Total y parameters: $y_{ij} = y_{ija} + y_{ijb}$

$$y_{11} = \frac{Z_2 + Z_3}{Z_A} + \frac{Z_5 + Z_6}{Z_B} \qquad y_{12} = -\frac{Z_3}{Z_A} - \frac{Z_6}{Z_B}$$

$$y_{21} = -\frac{Z_3}{Z_A} - \frac{Z_6}{Z_B} \qquad y_{22} = \frac{Z_1 + Z_3}{Z_A} + \frac{Z_4 + Z_6}{Z_B}$$

16.37 Find the Y parameters of the two-port network in Fig. P16.37. Find the input admittance of the network when the capacitor is connected to the output port. **CS**

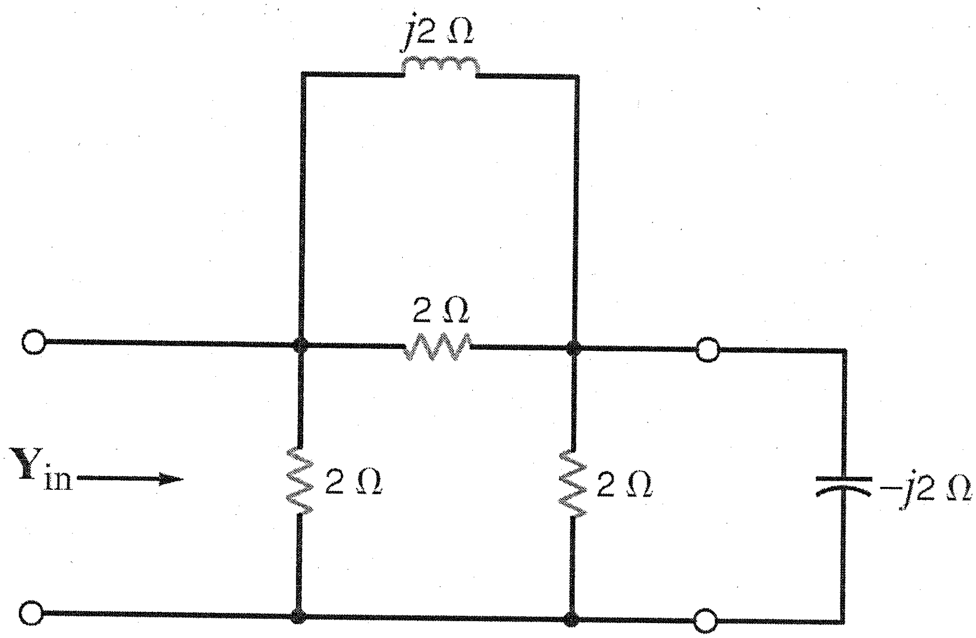
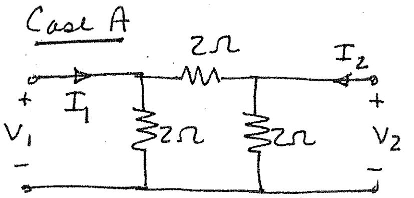


Figure P16.37

SOLUTION: Use 2 parallel networks

Case A

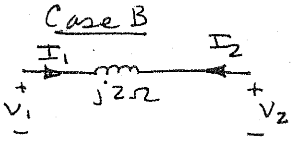


$y_{11a} = \dfrac{1}{2 /\!/ 2} = 1\ S$

$y_{12a} = y_{21a} = -\frac{1}{2}\ S$

$y_{22a} = \dfrac{1}{2/\!/2} = 1\ S$

Case B



$y_{11b} = \dfrac{1}{j2}\ S$

$y_{12b} = y_{21b} = -\dfrac{1}{j2}\ S$

$y_{22} = \dfrac{1}{j2}\ S$

$$Y_{in} = Y_{11} - \frac{Y_{12}Y_{21}}{Y_{22}+Y_L}$$

$$Y_{in} = 1 - j\tfrac{1}{2} - \frac{\frac{1}{4}(1-j1)^2}{(1-j0.5)+j0.5}$$

$$y_{in} = 1 - \frac{j}{2} - \frac{(1-j1)^2}{4}$$

$$\boxed{y_{in} = 1\ S}$$

$y_{11} = 1 + \dfrac{1}{j2} = y_{22}$ $\quad y_{12} = y_{21} = -\dfrac{1}{2} - \dfrac{1}{j2} = -\dfrac{1}{2}(1-j1)$

16.38 Find the Z parameters of the network in Fig. E16.3 by considering the circuit to be a series interconnection of two two-port networks as shown in Fig. P16.38.

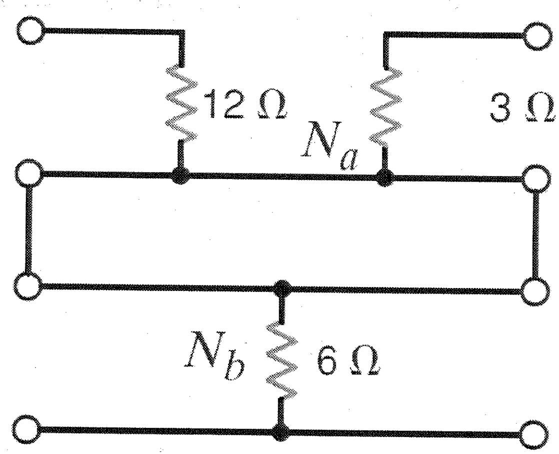
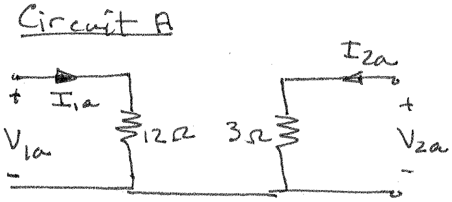


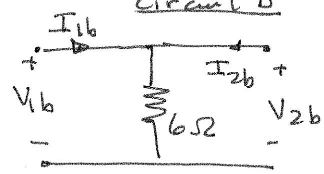
Figure P16.38

SOLUTION: Network consists of 2 series connected subcircuits

Circuit A



Circuit B



$Z_{11a} = 12\Omega$ $\quad Z_{22a} = 3\Omega$

$Z_{12a} = 0\Omega$ $\quad Z_{21a} = 0\Omega$

$Z_{11b} = 6\Omega$ $\quad Z_{22b} = 6\Omega$

$Z_{12b} = 6\Omega$ $\quad Z_{21b} = 6\Omega$

$Z_{ij} = Z_{ija} + Z_{ijb}$

$$\boxed{Z_{11} = 18\Omega \quad Z_{12} = 6\Omega \quad Z_{21} = 6\Omega \quad Z_{22} = 9\Omega}$$

16.39 Find the transmission parameters of the network in Fig. E16.3 by considering the circuit to be a cascade interconnection of three two-port networks as shown in Fig. P16.39. CS

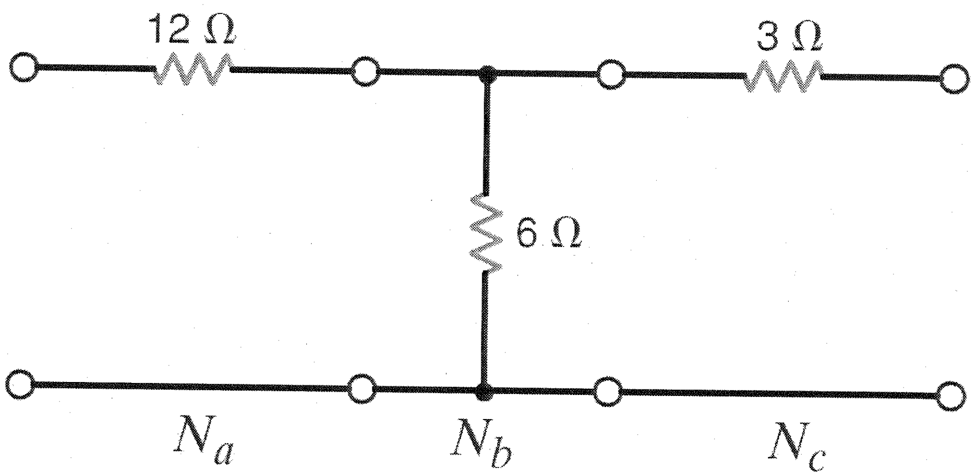
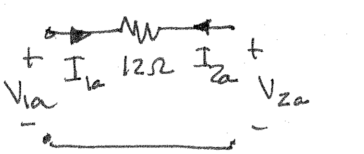


Figure P16.39

SOLUTION: Network consists of 3 two-ports!

<u>Circuit A</u>



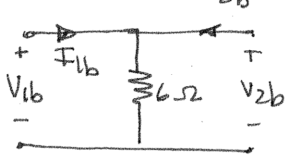
$A_a = \left.\dfrac{V_1}{V_2}\right|_{I_2=0} = 1$

$B_a = V_1/-I_2\Big|_{V_2=0} = 12\Omega$

$C_a = I_1/V_2\Big|_{I_2=0} = 0$

$D_a = \left.\dfrac{I_1}{-I_2}\right|_{V_2=0} = 1$

<u>Circuit B</u> I_{2b}



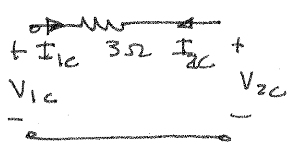
$A_b = 1$

$B_b = 0$ $(I_{2b} = \infty)$

$C_b = 1/6$ S

$D_b = 1$

<u>Circuit C</u>



$A_c = 1$

$B_c = 3\Omega$

$C_c = 0$

$D_c = 1$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 12 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1/6 & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 21 \\ 1/6 & 3/2 \end{bmatrix} \checkmark$$

16.40 Find the ABCD parameters for the circuit in Fig. P16.40.

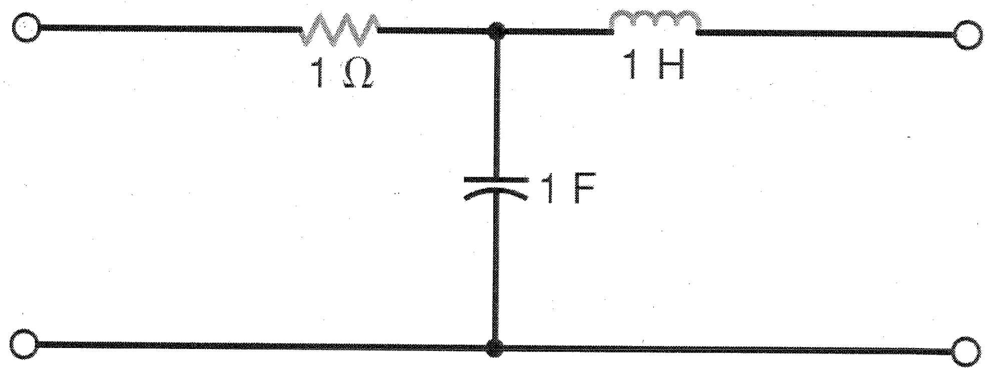
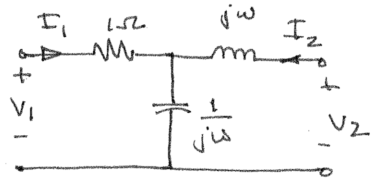


Figure P16.40

SOLUTION:



$A = V_1/V_2\big|_{I_2=0} \Rightarrow \frac{V_2}{V_1} = \frac{1/j\omega}{1+1/j\omega} = \frac{1}{j\omega+1}$

$*\ A = j\omega + 1$

$B = -V_1/I_2\big|_{V_2=0} \Rightarrow -\frac{I_2}{V_1} = \frac{j\omega /\!/ 1/j\omega}{(j\omega /\!/ 1/j\omega)+1}\left(\frac{1}{j\omega}\right) = \frac{1}{1+j\omega-\omega^2}$

$*\ B = 1 + j\omega - \omega^2\ \Omega$

$C = I_1/V_2\big|_{I_2=0} \Rightarrow V_2/I_1 = \frac{1}{j\omega} \Rightarrow C = j\omega\ *$

$D = -I_1/I_2\big|_{V_2=0} \Rightarrow -I_2/I_1 = \frac{1/j\omega}{j\omega+1/j\omega} = \frac{1}{1-\omega^2}$

$*\ D = 1 - \omega^2$

$$\boxed{\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} j\omega+1 & 1+j\omega-\omega^2 \\ j\omega & 1-\omega^2 \end{bmatrix}}$$

16.41 Find the Z parameters for the two-port network in Fig. P16.41 and then determine $\mathbf{I}_o$ for the specified terminal conditions.

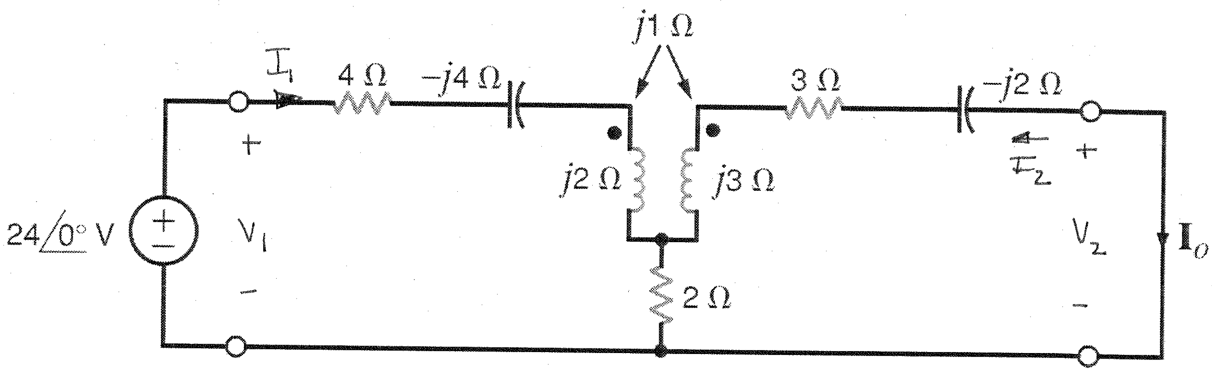


Figure P16.41

SOLUTION:

$V_1 = I_1(6-j2) + I_2(2+j1)$ $\qquad V_2 = I_1(2+j1) + I_2(5+j1)$

$$\boxed{\begin{array}{ll} Z_{11} = 6-j2\,\Omega & Z_{12} = 2+j1\,\Omega \\ Z_{21} = 2+j1\,\Omega & Z_{22} = 5+j1\,\Omega \end{array}}$$

$V_1 = 24\angle 0^\circ$ V $\qquad I_2 = -I_o$

$V_2 = 0$

$$\begin{bmatrix} 6-j2 & 2+j1 \\ 2+j1 & 5+j1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 24\angle 0 \\ 0 \end{bmatrix} \Rightarrow \quad I_2 = 1.78\angle -138^\circ \text{ A}$$

$$\boxed{I_o = 1.78\angle 42^\circ \text{ A}}$$

16.42 Determine the output voltage $\mathbf{V}_o$ in the network in Fig. P16.42 if the Z parameters for the two-port are

$$\mathbf{Z} = \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$$ **PSV**

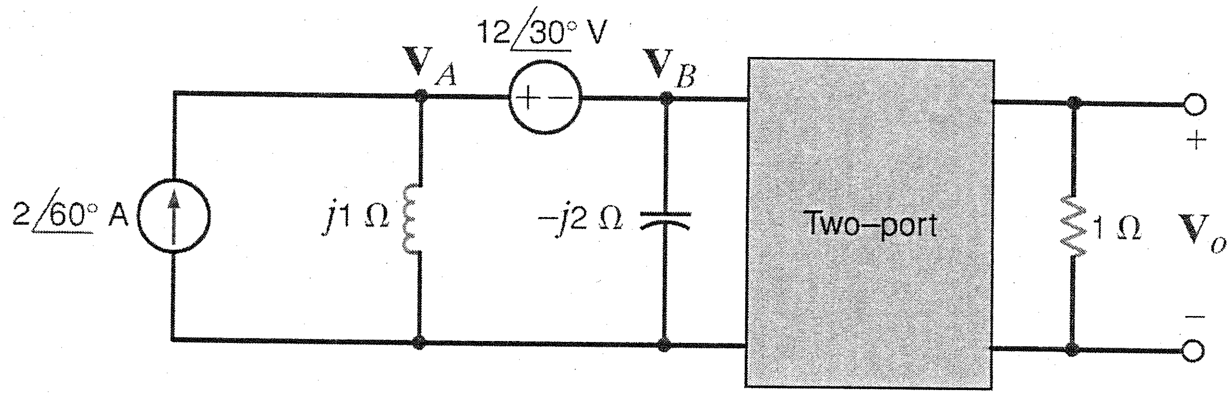
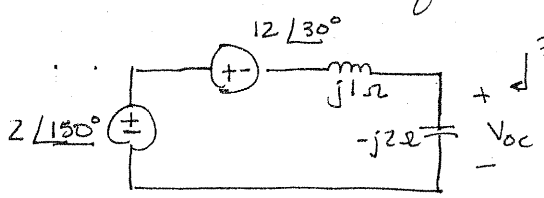


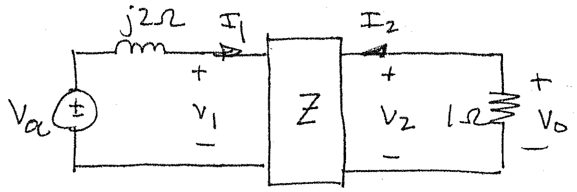
Figure P16.42

SOLUTION: Thevenin eq. at V_B. Use source exchange first.



$Z_{TH} = -j2 \,//\, j1 = j2\,\Omega$

$V_{oc} = (2\angle 150 - 12\angle 30)\left(\dfrac{-j2}{-j2+j1}\right) = 26.2\angle -158°$



$V_1 = V_{oc} - j2 I_1$ $\qquad V_o = V_2$

$V_o = (-1) I_2 = -I_2$

$V_1 = 3I_1 + 2I_2 = V_{oc} - j2I_1 \Rightarrow I_1(3+j2) + 2I_2 = V_{oc}$

$V_2 = 2I_1 + 3I_2 = -I_2 \Rightarrow I_1 + 2I_2 = 0 \Rightarrow I_1 = -2I_2$

now, $[(3-j2)(-2) + 2]I_2 = V_{oc} \Rightarrow I_2 = -V_{oc}/(4+j4)$

$V_o = \dfrac{V_{oc}}{4+j4}$ $\qquad \boxed{V_o = 4.64\angle 157°\ V}$

16FE-1 A two-port network is known to have the following parameters:

$$y_{11} = \frac{1}{14}\text{S} \qquad y_{12} = y_{21} = -\frac{1}{21}\text{S} \qquad y_{22} = \frac{1}{7}\text{S}$$

If a 2-A current source is connected to the input terminals as shown in Fig. 16PFE-1, find the voltage across this current source. CS

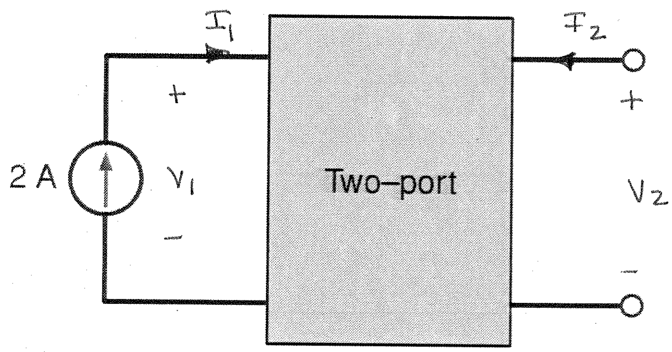


Figure 16PFE-1

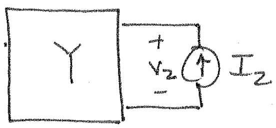
SOLUTION: $I_1 = 2\text{A}$ $I_2 = 0$

$$\left.\begin{aligned} I_1 &= y_{11}V_1 + y_{12}V_2 = 2 \\ I_2 &= y_{21}V_1 + y_{22}V_2 = 0 \end{aligned}\right\} \quad \begin{aligned} &(1/14)V_1 - (1/21)V_2 = 2 \Rightarrow 3V_1 - 2V_2 = 84 \\ &-(1/21)V_1 + (1/7)V_2 = 0 \Rightarrow -V_1 + 3V_2 = 0 \end{aligned}$$

find V_1. $\boxed{V_1 = 36\text{ V}}$

16FE-2 Find the Thévenin equivalent circuit at the output terminals of the network in Fig. 16PFE-1.

SOLUTION:



$$Y = \begin{bmatrix} 1/14 & -1/21 \\ -1/21 & 1/7 \end{bmatrix}$$

$$I_1 = 0 = y_{11}V_1 + y_{12}V_2 \Rightarrow V_1 = -V_2\left(y_{12}/y_{11}\right)$$

$$I_2 = y_{21}V_1 + y_{22}V_2 \Rightarrow I_2 = V_2\left(\frac{y_{11}y_{22} - y_{12}y_{21}}{y_{11}}\right)$$

$$Z_{TH} = \frac{V_2}{I_2} = \frac{y_{11}}{y_{11}y_{22} - y_{12}y_{21}} = \frac{y_{11}}{\Delta_Y}$$

$$\boxed{Z_{TH} = 1.575\,\Omega}$$